**IEOR 240 - Fall 2022**
Homework 1
Due Date: 09/19/2022 (Monday) at 11:59pm

**Homework requirement**

- Homework 1 is due on Monday, September 19th at 11:59pm midnight, electronically on bCourses.
- Submit your homework (typed, scanned or photographed) and the corresponding AMPL files in a single .pdf
- Make sure your homework is legible, and the AMPL files are well organized
- For problems that require a mathematical formulation, please clearly state the definition of decision variables and the indexes of them. Write a sentence to explain each constraint and objective function. Also include an explanation of the optimal solution when you are required to solve the problem in AMPL.
- For problems that require using AMPL please include (1) the model file, (2) the data file (if you used one), (3) the optimal objective and variable values. You can save your values in AMPL by typing the command **"display _objname, _obj, _varname, _var > filename.txt;"** into the console, where "filename" is **unique for each problem**. Combine these files into a single pdf, either by saving them as a pdf or using a screenshot.

1. **(Production planning problem)** National Steel Corporation (NSC), which produces a special-purpose steel that is used in aircraft and aerospace industries. The marketing department of NSC has received orders for 2400, 2000, 2700 and 2500 tons of steel during each of the next four months.: NSC can meet these demands by producing the steel, or by drawing from its inventory, or by any combination thereof.

   The production costs per ton of steel during each of the next four months are projected to be \$7400, \$7500, \$7600, and \$7800. Because of these inflationary costs, it might be advantageous for NSC to produce more steel than it needs in a given month and store the excess, although production capacity can never exceed 4000 tons in any month. All production takes place at the beginning of the month and immediately thereafter the demand is met. The remaining steel is then stored in inventory at a cost of \$120 per ton for each month that it remains there. The inventory at the beginning of the first month is 1000 tons of steel and the inventory level at the end of the fourth month should be at least 1500 tons.

   In addition, if the production level is increased or decreased from one month to the next, then the company incurs a cost for implementing these changes. Specifically, for each ton of increased or decreased production over the previous month, the cost is \$50. The production of the first month is exempt from this cost.

Formulate a production plan for NSC that will minimize the total cost over the next four months. Linearize the absolute value terms in the objective function and solve using AMPL.

2. A bookmaker has quoted odds on a tennis match between players I and II. The match consists of the *best two out of three sets*, i.e., if a player wins the first two sets, the third set is not played and the bet on it is canceled. The bookmaker is giving odds of 5 to 2 that player I will win the match and odds of 3 to 2 that player I will win each set. A bettor has 100 dollars which he can distribute by betting on either player I or II to win the match and any of the sets. All bets are made before the match starts (if there are only two sets, all bets on the third set are returned to the bettor).

   (a) Find a way of placing bets so that no matter what happens the bettor is assured of winning an amount $z$ where $z$ is as large as possible. Formulate this problem as a linear programming and solve it using AMPL.

   (b) What if now we have *best three out of five sets*, i.e., once a player wins three sets, no more sets are played and their corresponding bets are canceled, and everything else keeps the same? Re-solve the problem and compare the answer with part (a).

3. **(Linear regression)** Is there a relationship between IEOR 240 final exam scores and the number of hours spent studying per week? A study was conducted involving 8 students and the data is shown below.

| Hours spent studying per week | IEOR 240 final score |
|---|---|
| 4 | 75 |
| 9 | 82 |
| 10 | 92 |
| 14 | 100 |
| 4 | 68 |
| 7 | 88 |
| 12 | 95 |
| 3 | 77 |

   Find a linear regression model by using the minimum absolute error method. Reformulate it as a linear programming and solve it using AMPL. Give a prediction of the IEOR 240 final score on a student who studied 2 hours a week.

4. **(Solving LP graphically)** Solve the following problems graphically. Comment on whether the problem is infeasible, unbounded, has a unique optimal solution, or has infinitely many optimal solutions. Whenever there exists a unique optimal solution, provide the exact values for the two variables by considering the algebraic expressions of the appropriate constraints.

   (a) $\begin{aligned} \max \quad & 3x + y \\ s.t. \quad & 2x + y \leq 6 \\ & x + y \leq 4 \\ & x \geq 0, y \geq 0 \end{aligned}$

(b) $\begin{aligned} \max \quad & x + y \\ s.t. \quad & 2x + y \leq 6 \\ & x + y \leq 4 \\ & x \geq 0, y \geq 0 \end{aligned}$

(c) $\begin{aligned} \max \quad & x + y \\ s.t. \quad & -x + y \geq 1 \\ & x - y \geq 1 \\ & x \geq 0, y \geq 0 \end{aligned}$

(d) $\begin{aligned} \max \quad & x + y \\ s.t. \quad & -x + y \leq 1 \\ & x - y \leq 1 \\ & x \geq 0, y \geq 0 \end{aligned}$

(e) $\begin{aligned} \min \quad & x + y \\ s.t. \quad & -x + y \leq 1 \\ & x - y \leq 1 \\ & x \geq 0, y \geq 0 \end{aligned}$