The Transition to Turbulence and Couette-Taylor Flow

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Research Question:
How can an investigation of flow regimes in the Couette-Taylor system aid our understanding of turbulence and chaos in fluid flow?

Contents

1 Introduction to fluid flow

In an era of such modern scientific and technological development, it is a wonder that humans cannot fully explain so many fundamental properties of nature. One of these yet-unsolved enigmas is fluid flow, the physics of which has been studied intensively for hundreds of years, yet nevertheless our solutions are so simplistic that they are often unusable in real-world situations. For example, heat transfer from the walls of a pipe to the moving fluid inside is an unsolved theoretical problem that can only be approximated or calculated experimentally.

Fluid flows are generally grouped into two basic states, laminar and turbulent; however, this dichotomy is misleading. Laminar fluid flow is uniform, uni-directional, and mathematically simple. Turbulent flow is irregular and pseudorandom with no consistent direction or visual stability, but the existence of turbulence certainly does not imply the absence of laminar flow. The difference between laminar and non-laminar flow is more correctly described as a transition from one to the other during which both states exist. This will be discussed more rigorously in the following section.

In order to transition to turbulence, a system must be driven away from thermodynamic equilibrium. This can be achieved by imposing a gradient on the velocity, density, or temperature on the system. A velocity gradient (which will be the only instability discussed in this paper) can be represented with a dimensionless value called a Reynolds number (written as Re)

1.1 Determinism and Chaos

Fluids that are laminar and sufficiently close to thermodynamic equilibrium can be represented by unique solutions that are stable to small perturbations, meaning the fluid will return to the same state if disturbed slightly by an outside force (Swinney and Gollub, 1978). As the system is moved further from thermodynamic equilibrium it eventually reaches a state where small perturbations create a variety of different states. Some are ordered patterns and some have no identifiable pattern (turbulence). The Reynolds number at which a fluid transitions from a laminar to a non-laminar state is called the critical Reynolds number (Re_c).

Contrary to intuition, a deterministic flow is not necessarily therefore a predictable one. Lorenz's famous paper (Lorenz, 1963) showed the existence of motion that was deterministic, yet did not follow a predictable pattern. This kind of motion was later termed chaotic. A flow may display other types of deterministic motion, where there is always a predictable structured pattern, such as quasi-periodic motion. Quasi-periodic motion is periodic motion that includes two or more component frequencies. An important difference between non-periodic motion and quasi-periodic motion should be emphasized: the sensitivity of the system to initial conditions (Swinney and Gollub, 1978). Two quasi-periodic systems with very close initial parameters will retain a relative similarity at all times, whereas slightly modified initial conditions in a non-periodic flow can give it a vastly different structure.

1.2 Bifurcation Analysis

A bifurcation in a dynamical system is where smooth changes in certain parameters can result in an immediate qualitative change in the systems behavior (Hilborn, 2000, p. 541). There are two types of bifurcation discussed in this paper. One is an initial instability leading to a patterned, time independent state. The other is where the system adds a periodic component to its motion. This bifurcation is called a Hopf bifurcation.

Landau (1944) argued that turbulence in fluid flow could be solely explained by the repeated bifurcation of a system that started out in a laminar state. In other words, Landau claimed that turbulence, instead of being non-periodic, was actually quasi-periodic with a very large number of component frequencies.

2 The Couette-Taylor system

Because of the complexities of chaotic systems, scientists need simple mechanical systems that can be used to visualize fluid flow using a few easily controlled parameters. One such system that has been used successfully for over one hundred years is the Couette-Taylor system. The apparatus consists of a cylinder placed concentrically inside a translucent outer cylinder and filling the space between the cylinders with a liquid. Then, either or both cylinders are rotated to influence movement of the water.

2.1 Flow Regimes

Several important parameters that describe a particular Couette-Taylor system are usually defined as:

Radius ratio $\eta = \frac{a}{b}$
Angular velocity ratio $\mu = \frac{\Omega_o}{\Omega_i}$
Aspect ratio $\Gamma = \frac{L}{b-a}$
Reynolds number for either cylinder $Re = a(b-a)\Omega/v$

where a is the inner cylinder's radius, b is the outer cylinder's radius, Ω refers to the angular velocity of the cylinder in question, L is the length of the system, and v is the kinematic viscosity. For sufficiently small inner cylinder rotations ($Re_i < Re_c$) and dual rotation where the outer cylinder is rotating significantly faster than the inner cylinder ($|Re_o| \gg Re_i$), the system remains in a state of laminar Couette flow. Under these conditions there is a globally unique solution to the Navier-Stokes equation:

$v_r = v_z = 0$
$v_\theta = Ar + \frac{B}{r}$, where $A = \frac{-\Omega_i(\eta^2-\mu)}{1-\eta^2}$ and $B = \frac{\Omega_i a^2(1-\mu)}{1-\eta^2}$

and where there exists a no-slip boundary condition at the walls of the cylinder. However, these equations also assume the absence of upper and lower-limit boundary conditions, i.e. an annulus of infinite height. This is of course impossible, so in reality high rotation speeds will create certain boundary anomalies that can only be accounted for by building systems with larger aspect ratios.

2.2 Project Goals

The goal of this research is to describe the transition to turbulence through the various states of instability in the Couette-Taylor system, especially in reference to the bifurcations of the fluid flow. This paper will analyze the transition that takes place when only the inner cylinders velocity is increased and the outer cylinder remains still. Then, the implications of the study and several variations of the Couette-Taylor system that could aid in answering modern physics questions will be discussed.

3 Methodology

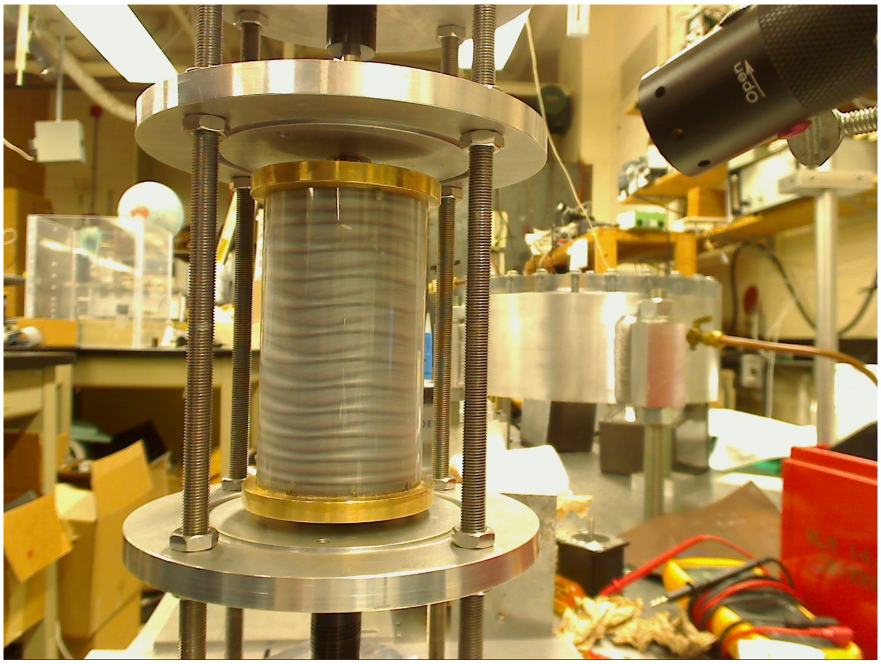

Figure 1: The Couette-Taylor system used for experimentation.

3.1 Experimental Procedures

The system created had the following properties: $\eta = .878$ and $\Gamma \approx 33$. To visualize fluid flow, a 3% concentration of the rheoscopic fluid Kalliroscope AQ1000 was mixed with the water (Andereck et al., 1986). Kalliroscope is composed of very thin crystalline platelets ($25 \times 6 \times .007\mu m$) that act somewhat like tiny mirrors. When the flow of the water is parallel to the observers line of sight, the flakes will allow light to pass around them and the black inner cylinder becomes visible. When the flow moves perpendicular to the observers line of sight, the flakes will reflect light, appearing white. In other words, the Kalliroscope creates a light intensity gradient in the water based on the direction of the flow.

Data was acquired through a video camera hooked up to the apparatus to take videos of size 640×480. The videos were recorded to a hard disk with a Windows XP application called VirtualDub that was used to control frame-rate precisely and split the video into an image sequence.

That image sequence was fed into a MATLAB program that takes a user-specified region of the image sequence and creates a time-dependent log power-spectrum from a discrete Fourier transform of the varying intensity of each pixel (the frequency domains of each pixel are averaged together to create the final spectrum).

3.2 Flow State Arrival

The chaotic nature of phase spaces in the Couette-Taylor system means that the method by which we arrive at certain flow states can influence the stability of those states and the point at which transitions occur. In general, the methodology employed by Andereck et al. (1986) was followed. Our method of flow state arrival was to quickly accelerate the inner cylinder until turbulent Taylor vortices appeared then slowly reduce the speed of the inner cylinder until it reached the desired angular velocity. The purpose of this procedure is to preserve the azimuthal wavenumber, the breakdown of which was the primary reason for the destabilizations of flow states. Generally, due to time constraints the latter method was used, and comparisons of the two systems revealed no discernible difference.

4 Results

4.1 Taylor vortices

The first flow visualization was done to demonstrate the primary bifurcation when only rotating the inner cylinder. As the inner cylinder was slowly rotated up, Taylor vortices began to appear at around $.145 \pm .005$ Hz ($Re_c \approx 127.96$). There was a straight-line distinction between each of the 17 azimuthal modes and no time-dependent waves were visible. The system appears nearly static with the exception of the miniscule specks of Kalliroscope moving inside the fluid. Figure 3 shows the power spectrum created from this state of motion. There are few spikes that would denote prominent frequencies of motion within the fluid; the largest spike increases only .6 above the noise floor. This spectrum has a horizontal component measured proportionally to the frequency of the rotating cylinder. That means the prominent frequency, which has an x-value close to 1, is probably

Figure 2: In Taylor vortex flow, axisymmetric azimuthal helical vortices fully appear at Re_c. $\Omega = .145$ Hz.

due to surface irregularities of the inner cylinder that are picked up by the Fourier transform. This unwanted frequency does, however, show the sensitivity of the webcam and the validity of the process being used. Lastly, because the Fourier transform is discrete rather than continuous, the spectrum necessarily picks up harmonics of the fundamental frequencies picked up. The second spike and the five-or-so visible spikes after it are equally spaced from the first spike, implying that they are harmonics of the fundamental. Thus, no independent periodic fluid flow is visible at this stage.

4.2 Wavy vortex flow

As the inner cylinder is sped up, the system comes to a different flow, named wavy vortex flow. Because the system is brought to rest then spun up differently from the previous configuration, the number of azimuthal modes has changed to 14. All of the toroidal Taylor vortices are still present, but they are separated by sinusoidal boundaries with a wavenumber along the vortex boundaries (as opposed to the axial wavenumber indicating the number of Taylor vortices) of 4. Depending on the way this state is approached this is one of many fluid properties that can change. Figure 5

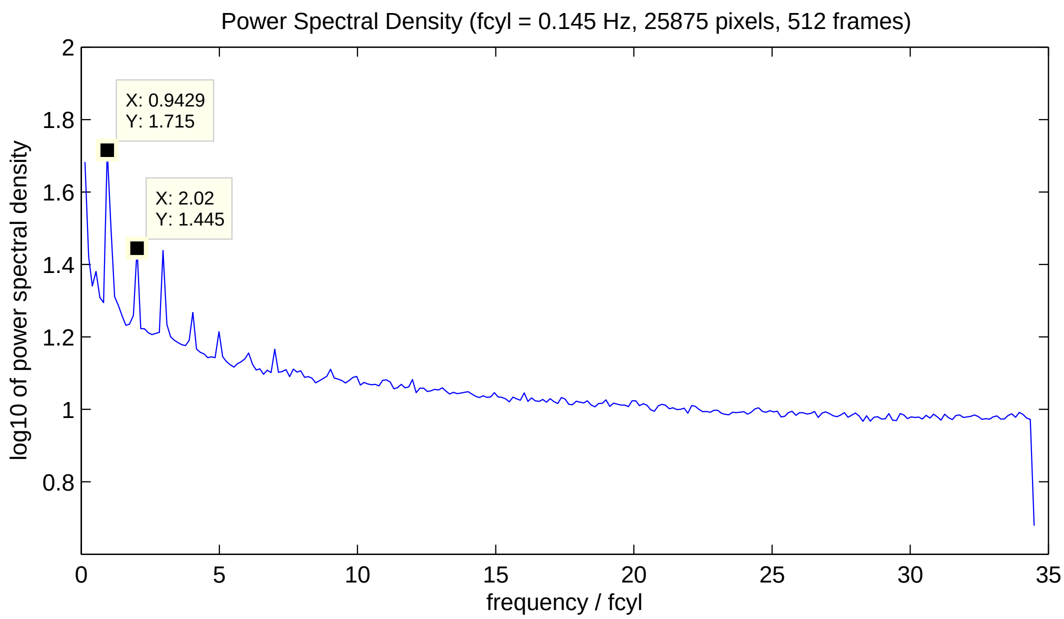

Figure 3: No fundamental frequencies of motion are apparent in Taylor vortex flow. The frequency at .95 is picking up imperfections in the inner cylinder.

shows the power spectrum, which picks up the periodic boundary flow. The frequency of motion is more clearly defined, this time being three orders of magnitude above the noise floor. Up to the tenth harmonic is distinguishable, at which point a phenomenon called folding becomes apparent. This can be explained by the Nyquist theorem, which states that the maximum frequency of a power spectrum is one half the sampling rate (which in this case is 10 fps). Thus frequencies over 5 Hz (22.2 times the inner cylinder frequency) are folded back over and shown within the range 0 - 5 Hz, i.e. the small peak to the left of the tenth harmonic is actually the eleventh harmonic.

4.3 Modulated wavy vortex flow

At a higher rotation rate, the wavy vortices whose individual boundaries were previously sinusoidal develop modulated boundaries. At this stage, the amplitude of the boundaries begins to change periodically, in parts even approaching zero (see right side of figure 6). Whereas Taylor vortices seemed static and wavy vortices moved azimuthally, modulated wavy vortices display a vertical as well as azimuthal periodic motion. Figure 7 is the power spectrum for the modulated wavy vortex state, recorded at an inner cylinder frequency of 2 Hz. This spectrum is drastically more complex than the two previous spectra, because the Fourier analysis detects two fundamental

Figure 4: In wavy vortex flow, a time-dependent wave forms at the boundary of each azimuthal torus. Here the axial wavenumber is 14 and the azimuthal wavenumber is 4. $\Omega = .225$ Hz.

frequencies instead of one. When two fundamental frequencies are present in a discrete system, the power spectrum not only includes harmonics of both frequencies, but also combinations of those frequencies as sums and differences and their harmonics. The major peaks of the spectrum are identified as such.

5 Discussion

5.1 Non-periodicity versus quasi-periodicity

This experiments qualitative analysis of a singly rotating Couette-Taylor system undermines Landaus hypothesis that turbulence is actually a quasi-periodic form of motion. The transition from Taylor vortices to wavy vortex flow is a Hopf bifurcation that creates a stable state of periodic motion, adding a single fundamental frequency to the power spectrum. As Ω_i is raised, this creates a secondary bifurcation where a second fundamental frequency is added to the spectrum. However, this is the last bifurcation that was observed in the system before it approached the turbulent Taylor vortex state. The power spectrum for the turbulent state was drastically different from the

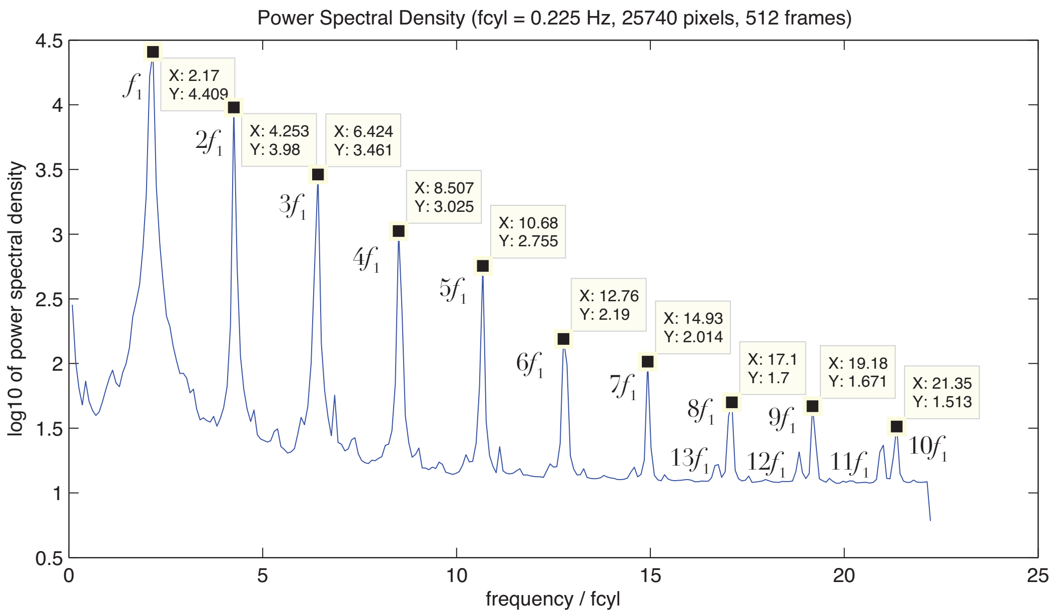

Figure 5: A single fundamental frequency is visible in wavy vortex flow, and its harmonics are labeled. The 11th, 12th, and 13th harmonics have been "folded" over as per the Nyquist theorem.

two previous. Similarly, chaotic systems sometimes undergo such bifurcations before approaching a state of chaotic motion.

5.2 Implications of research and further studies

The major benefit to using Couette-Taylor research is that it provides a closed system with relatively simple parameters to understand general concepts about fluid flow. However, the Couette-Taylor system provides an excellent structure to study the nature of flow past a boundary, which is important to the mechanical construction of hydraulic turbines and watercraft. Several experiments have used the Couette-Taylor system to study torque scaling and velocity structures at very high Reynolds numbers such as $Re = 10^6$ (Lathrop et al., 1992; Lewis and Swinney, 1999).

An interesting variation to normal Couette-Taylor flow involves creating a density gradient from a salinated water solution where water is denser at the bottom of the system and becomes progressively less dense towards the top. The disturbances that form in this environment, called strato-rotational instabilities (SRI), have been studied mathematically (Molemaker et al., 2001; Yavneh et al., 2001; Shalybkov and Rüdiger, 2005) but much less frequently experimentally (Le Bars

Figure 6: It is difficult to portray in a picture, but modulated wavy vortex flow creates a time-dependent amplitude modulation in the waves. Here, two forms of the flow are shown together. $\Omega = 2$ Hz.

and Le Gal, 2007). Using a simple "double bucket" method of mixing, SRIs can be studied in existing Couette-Taylor setups. The implications of stratified Couette-Taylor research are wide reaching. The largest fluid regions on earth, the oceans and the atmosphere, are stratified. Also, it has been postulated that the development of protoplanetary disks are linked to SRIs.

Much is left to be done in the realm of fluid dynamics. With the diversity of research surrounding Couette-Taylor flow, it may be surprising that basic questions have not yet been fully answered. However, the complexity of fluids shows that nature will continue to surprise and inform scientific research for many years. Accordingly, this paper demonstrates several fundamental principles of fluid mechanics that warrant much more in-depth study.

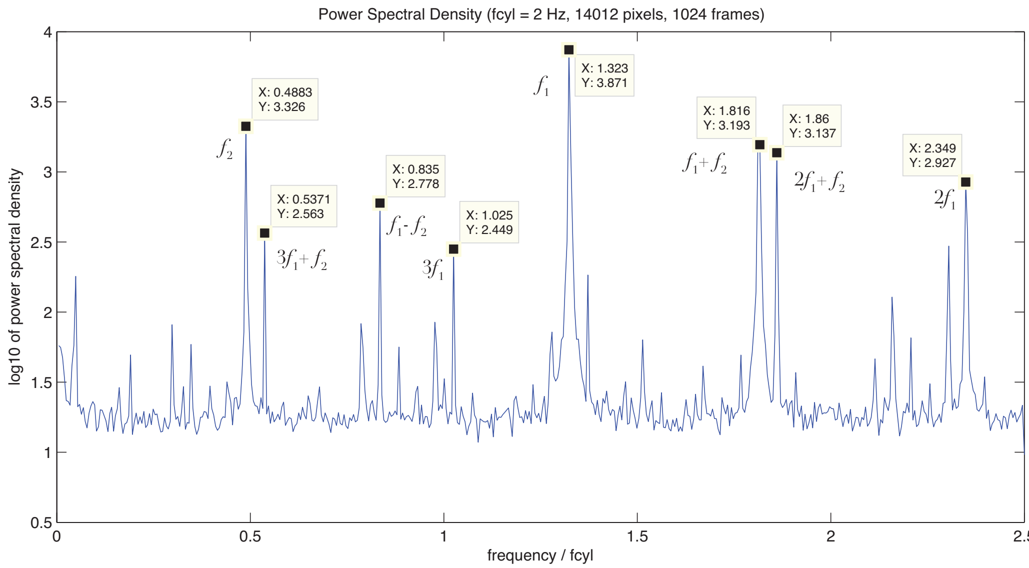

Figure 7: The power spectrum for modulated wavy vortex flow becomes much more confusing than the previous spectra. Two fundamental frequencies as well as their interactions are picked up by the discrete Fourier transform.

References

C.D. Andereck, SS Liu, and H.L. Swinney. Flow regimes in a circular Couette system with independently rotating cylinders. *Journal of Fluid Mechanics*, 164(3):155–183, 1986.

TB Benjamin. Bifurcation Phenomena in Steady Flows of a Viscous Fluid. I. Theory. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences (1934-1990)*, 359 (1696):1–26, 1978.

A. Brandstater and H.L. Swinney. Strange attractors in weakly turbulent Couette-Taylor flow. *Physical Review A (General Physics)*, 35(5):2207–2220, 1987.

H. Faisst and B. Eckhardt. Transition from the Couette-Taylor system to the plane Couette system. *Physical Review E*, 61(6):7227–7230, 2000.

R.C. Hilborn. *Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers.* Oxford University Press, 2000.

L. Landau. On the Problem of Turbulence. In *CR (Dokl.) Acad. Sci. URSS, n. Ser.*, volume 44, pages 311–314, 1944.

D.P. Lathrop, J. Fineberg, and H.L. Swinney. Turbulent flow between concentric rotating cylinders at large Reynolds number. *Physical Review Letters*, 68(10):1515–1518, 1992.

M. Le Bars and P. Le Gal. Experimental Analysis of the Stratorotational Instability in a Cylindrical Couette Flow. *Physical Review Letters*, 99(6):64502, 2007.

G.S. Lewis and H.L. Swinney. Velocity structure functions, scaling, and transitions in high-Reynolds-number Couette-Taylor flow. *Physical Review E*, 59(5):5457–5467, 1999.

E.N. Lorenz. Deterministic Nonperiodic Flow. *Journal of the Atmospheric Sciences*, 20(2):130–141, 1963.

C.K. Mamun and L.S. Tuckerman. Asymmetry and Hopf bifurcation in spherical Couette flow. *Physics of Fluids*, 7(1):80, 1995.

M.J. Molemaker, J.C. McWilliams, and I. Yavneh. Instability and Equilibration of Centrifugally Stable Stratified Taylor-Couette Flow. *Physical Review Letters*, 86(23):5270–5273, 2001.

D. Shalybkov and G. Rüdiger. Non-axisymmetric instability of density-stratified Taylor-Couette flow. In *Journal of Physics: Conference Series*, volume 14, pages 128–137. Institute of Physics Publishing, 2005.

H.L. Swinney and J.P. Gollub. The transition to turbulence. *Physics Today*, 31(8):41–49, 1978.

DJ Tritton. *Physical Fluid Dynamics*. Oxford University Press, USA, 1988.

I. Yavneh, J.C. McWilliams, and M.J. Molemaker. Non-axisymmetric instability of centrifugally stable stratified Taylor–Couette flow. *Journal of Fluid Mechanics*, 448:1–21, 2001.

6 Appendix: construction diagram

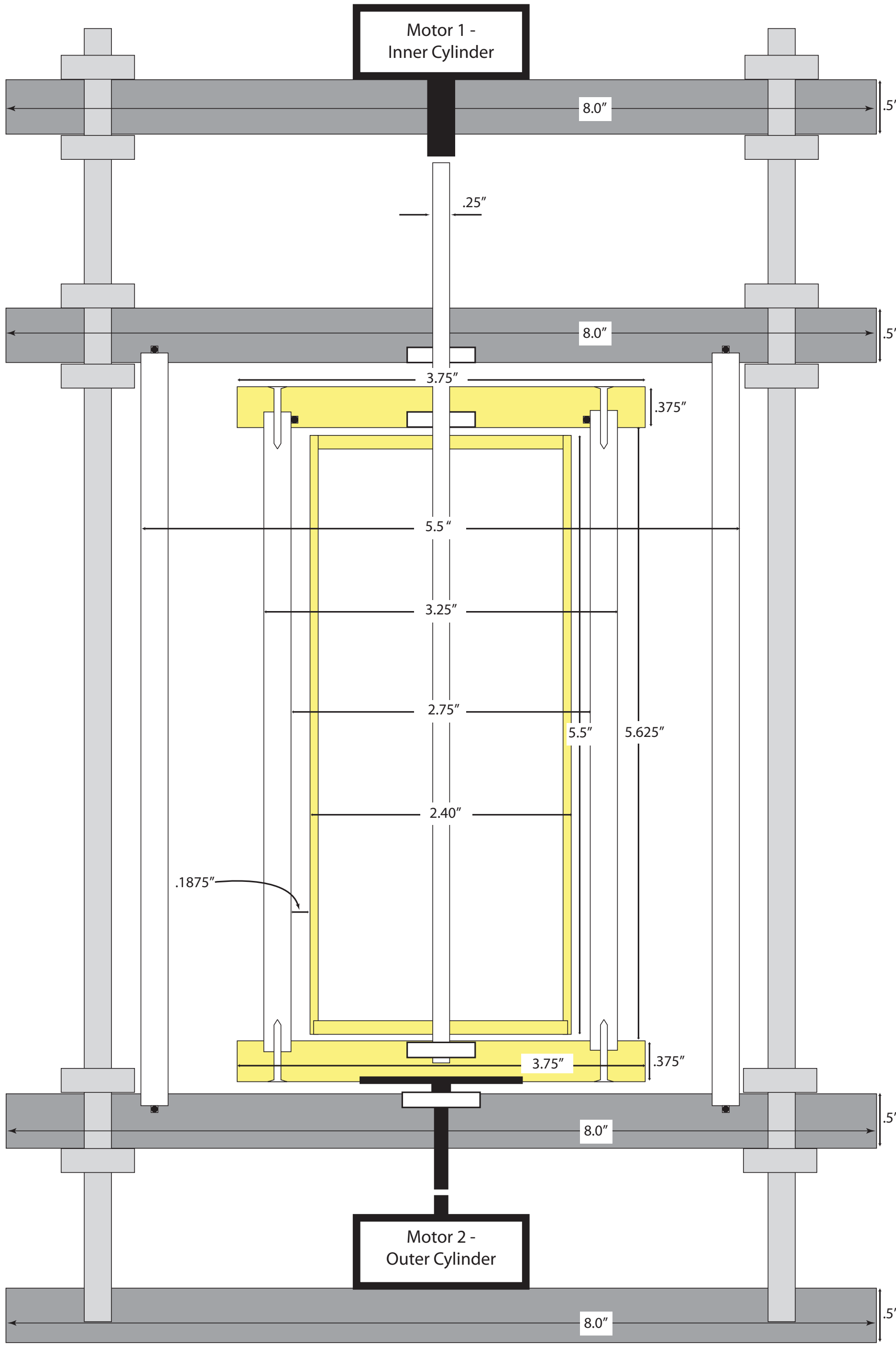

Figure 8: A simple diagram of the system built. The outermost acrylic cylinder is optional (it would be filled with water to fill imperfections in the inner acrylic cylinder, surface).