Customer Churn

Abhay Kulkarni 7/20/2019

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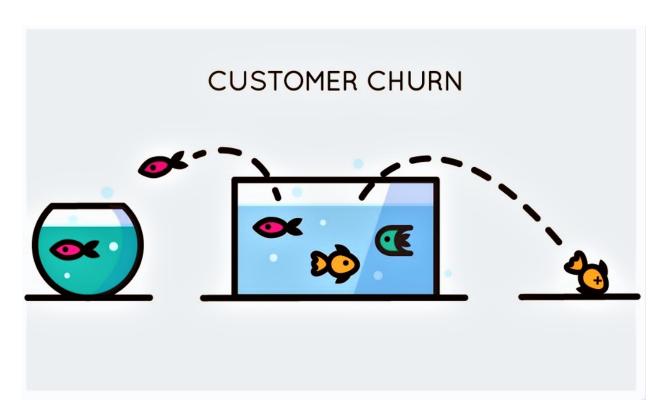


Figure 1: Customer Churn

Customer Churn in Telecom Industry

1 Introduction

1.1 What is Customer Churn

Customer attrition, also known as customer churn, customer turnover, or customer defection, is the loss of clients or customers.

1.2 Why Predict Customer Churn

Current organizations face a huge challenge: to be able to anticipate to customers' abandon in order to retain them on time, reducing this way costs and risks and gaining efficiency and competitivity.

2 Libraries/Packages

```
library(DataExplorer)
## Warning: package 'DataExplorer' was built under R version 3.5.3
library(ggplot2)
## Warning: package 'ggplot2' was built under R version 3.5.3
library(caTools)
## Warning: package 'caTools' was built under R version 3.5.3
library(xlsx)
## Warning: package 'xlsx' was built under R version 3.5.2
library(skimr)
## Warning: package 'skimr' was built under R version 3.5.3
##
## Attaching package: 'skimr'
## The following object is masked from 'package:stats':
##
##
      filter
library(caret)
## Warning: package 'caret' was built under R version 3.5.3
## Loading required package: lattice
## Warning: package 'lattice' was built under R version 3.5.2
library(cowplot)
## Warning: package 'cowplot' was built under R version 3.5.3
##
## *******************************
## Note: As of version 1.0.0, cowplot does not change the
##
    default ggplot2 theme anymore. To recover the previous
    behavior, execute:
##
    theme_set(theme_cowplot())
## ***************
library(caTools)
library(ROSE)
## Warning: package 'ROSE' was built under R version 3.5.3
## Loaded ROSE 0.0-3
library(ROCR)
## Warning: package 'ROCR' was built under R version 3.5.3
## Loading required package: gplots
```

```
## Warning: package 'gplots' was built under R version 3.5.2
##
## Attaching package: 'gplots'
## The following object is masked from 'package:stats':
##
##
       lowess
library(MLmetrics)
## Warning: package 'MLmetrics' was built under R version 3.5.3
##
## Attaching package: 'MLmetrics'
## The following objects are masked from 'package:caret':
##
       MAE, RMSE
##
## The following object is masked from 'package:base':
##
##
       Recall
library(MASS)
## Warning: package 'MASS' was built under R version 3.5.3
library(class)
## Warning: package 'class' was built under R version 3.5.2
library(e1071)
## Warning: package 'e1071' was built under R version 3.5.3
library(car)
## Warning: package 'car' was built under R version 3.5.3
## Loading required package: carData
## Warning: package 'carData' was built under R version 3.5.2
```

3 Project Objective

Customer Churn is a burning problem for Telecom companies. In this project, we simulate one such case of customer churn where we work on a data of postpaid customers with a contract. The data has information about the customer usage behaviour, contract details and the payment details. The data also indicates which were the customers who cancelled their service. Based on this past data, we need to build a model which can predict whether a customer will cancel their service in the future or not.

You are expected to do the following:

3.1 EDA

- How does the data look like, Univariate and bivariate analysis. Plots and charts which illustrate the relationships between variables
- Look out for outliers and missing value
- Check and treat for multicollinearity
- Summarize the insights you get from EDA

3.2 Build Models and compare them to get to the best one

- Logistic Regression
- KNN
- Naive Bayes
- Model Comparison using Model Performance metrics & Interpretation

3.3 Actionable Insights

 $\bullet\,$ Interpretation & Recommendations from the best model

4 Step by Step Approach

4.1 EDA

4.1.1 Set Working Directory

```
setwd("Z:\\Projects\\Predictive\\Customer Churn")
getwd()
```

[1] "Z:/Projects/Predictive/Customer Churn"

4.1.2 Import Data

```
mydata<-read.xlsx2("Cellphone.xlsx", sheetIndex = 2, header = TRUE)</pre>
```

Imported the xlsx datasheet with index 2 as the data is in the 2nd sheet.

4.1.3 Check Dataset

```
skim(mydata)
## Skim summary statistics
   n obs: 3333
##
   n variables: 11
##
##
  -- Variable type:factor -----
##
          variable missing complete
                                       n n_unique
##
      AccountWeeks
                         0
                               3333 3333
                                              212
##
             Churn
                         0
                               3333 3333
                                                2
##
   ContractRenewal
                         0
                               3333 3333
                                                2
     CustServCalls
                         0
                               3333 3333
                                               10
##
##
          DataPlan
                         0
                               3333 3333
                                                2
##
         DataUsage
                         0
                               3333 3333
                                              174
##
          DayCalls
                         0
                               3333 3333
                                              119
           DayMins
                         0
##
                               3333 3333
                                             1667
##
     MonthlyCharge
                         0
                               3333 3333
                                              656
##
        OverageFee
                         0
                               3333 3333
                                             1024
##
          RoamMins
                               3333 3333
                                              162
##
                                    top_counts ordered
##
    105: 43, 87: 42, 101: 40, 93: 40
                                                 FALSE
              0: 2850, 1: 483, NA: 0
##
                                                 FALSE
              1: 3010, 0: 323, NA: 0
##
                                                 FALSE
##
     1: 1181, 2: 759, 0: 697, 3: 429
                                                 FALSE
##
              0: 2411, 1: 922, NA: 0
                                                 FALSE
                          0: 1813, 0.3: 41, 0
##
                                                 FALSE
```

Findings

##

##

##

##

##

102: 78, 105: 75, 107: 69, 95: 69

154: 8, 159: 8, 174: 8, 162: 7

50: 84, 46: 75, 45: 74, 49: 73

8.5: 13, 10

10: 62, 11.: 59, 10

FALSE

FALSE

FALSE

FALSE

FALSE

^{*}There are 3333 observations and 11 variables.

str(mydata)

```
'data.frame':
                    3333 obs. of 11 variables:
##
   $ Churn
                    : Factor w/ 2 levels "0", "1": 1 1 1 1 1 1 1 1 1 1 ...
##
   $ AccountWeeks : Factor w/ 212 levels "1","10","100",...: 33 10 43 196 186 22 26 53 21 47 ...
   $ ContractRenewal: Factor w/ 2 levels "0", "1": 2 2 2 1 1 1 2 1 2 1 ...
##
##
   $ DataPlan
                    : Factor w/ 2 levels "0", "1": 2 2 1 1 1 1 2 1 1 2 ...
                     : Factor w/ 174 levels "0","0.11","0.12",...: 104 141 1 1 1 1 79 1 10 116 ...
##
  $ DataUsage
  $ CustServCalls : Factor w/ 10 levels "0","1","2","3",..: 2 2 1 3 4 1 4 1 2 1 ...
##
                     : Factor w/ 1667 levels "0","100","100.1",...: 1306 477 1176 1442 521 1022 976 435
## $ DayMins
## $ DayCalls
                     : Factor w/ 119 levels "0","100","101",...: 12 25 16 91 15 118 108 99 117 104 ...
  $ MonthlyCharge : Factor w/ 656 levels "100","100.3000000000001",..: 593 530 224 272 125 272 580
   $ OverageFee
                     : Factor w/ 1024 levels "0","1.56","10",...: 1012 1003 652 532 768 105 514 597 517
##
                     : Factor w/ 162 levels "0","1.1","1.3",...: 4 41 26 129 5 126 138 134 150 16 ...
   $ RoamMins
```

Findings

^{*} AccountWeeks, DataUsage, DataUsage, CustServCalls, DayMins, DayCalls, MonthlyCharge, OverageFee, RoamMins are incorrectly imported as factors. Convert factors to numeric data types.

^{*} Churn , ContractRenewal and DataPlan are factor variables.

4.1.4 Creating Backup of the original dataset

```
cleandata <- data.frame(mydata)</pre>
```

4.1.5 AccountWeeks, DataUsage, DataUsage, CustServCalls, DayMins, DayCalls, Monthly-Charge, OverageFee, RoamMins are incorrectly imported as factors. Convert factors to numeric data types.

```
cleandata$AccountWeeks<-as.numeric(as.character(cleandata$AccountWeeks))

cleandata$DataUsage<-as.numeric(as.character(cleandata$DataUsage))

cleandata$CustServCalls<-as.numeric(as.character(cleandata$CustServCalls))

cleandata$DayMins<-as.numeric(as.character(cleandata$DayMins))

cleandata$DayCalls<-as.numeric(as.character(cleandata$DayCalls))

cleandata$DayCalls<-as.numeric(as.character(cleandata$DayCalls))

cleandata$MonthlyCharge<-as.numeric(as.character(cleandata$MonthlyCharge))

cleandata$OverageFee<-as.numeric(as.character(cleandata$OverageFee))

cleandata$RoamMins<-as.numeric(as.character(cleandata$RoamMins))</pre>
```

4.1.6 Checking sturcture of data

str(cleandata)

```
## 'data.frame':
                  3333 obs. of 11 variables:
   $ Churn
                  : Factor w/ 2 levels "0", "1": 1 1 1 1 1 1 1 1 1 ...
## $ AccountWeeks : num 128 107 137 84 75 118 121 147 117 141 ...
## $ ContractRenewal: Factor w/ 2 levels "0","1": 2 2 2 1 1 1 2 1 2 1 ...
## $ DataPlan : Factor w/ 2 levels "0","1": 2 2 1 1 1 1 2 1 1 2 ...
## $ DataUsage
               : num 2.7 3.7 0 0 0 0 2.03 0 0.19 3.02 ...
## $ CustServCalls : num 1 1 0 2 3 0 3 0 1 0 ...
## $ DayMins
                   : num 265 162 243 299 167 ...
## $ DayCalls
                   : num 110 123 114 71 113 98 88 79 97 84 ...
## $ MonthlyCharge : num 89 82 52 57 41 57 87.3 36 63.9 93.2 ...
## $ OverageFee
                   : num 9.87 9.78 6.06 3.1 7.42 ...
  $ RoamMins
                   : num 10 13.7 12.2 6.6 10.1 6.3 7.5 7.1 8.7 11.2 ...
```

4.1.7 Dataset details

introduce(cleandata)

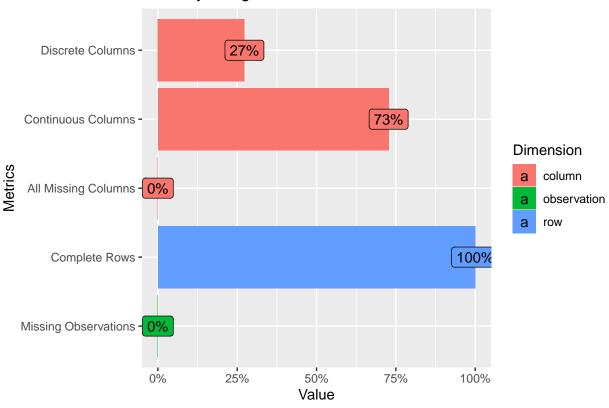
Findings

- There are 3333 Rows and 11 Columns
- 3 discrete colums and 8 continuous columns

4.1.8 Visualize dataset

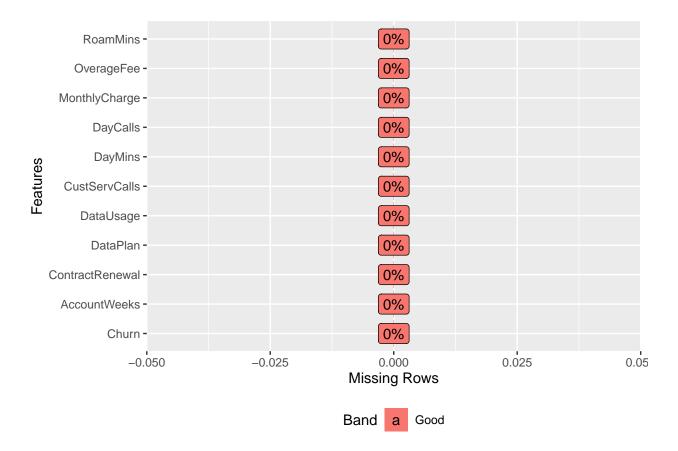
```
plot_intro(cleandata)
```

Memory Usage: 252.1 Kb



4.1.9 Checking for Missing Values

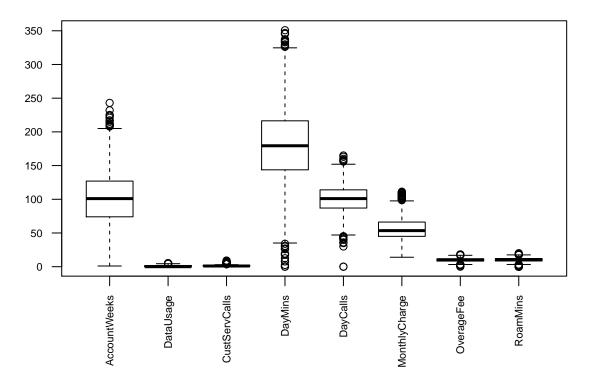
plot_missing(cleandata)



• There are no missing values in the dataset.

4.1.10 Check if there are Outliers

boxplot(scaledata[,c(2,5,6,7,8,9,10,11)],las=2,cex.axis=0.7)



Findings

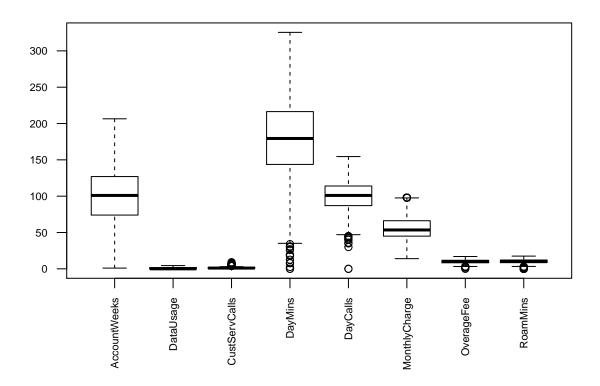
• There are outliers in the dataset.

4.1.11 Outlier Treatment. Winsorizing.

```
summary(scaledata$AccountWeeks)
      Min. 1st Qu. Median
                              Mean 3rd Qu.
                                              Max.
##
                     101.0
                             101.1
                                     127.0
                                             243.0
              74.0
benchAccountWeeks <- 127.0 + 1.5* IQR(scaledata$AccountWeeks)
scaledata$AccountWeeks[scaledata$AccountWeeks>benchAccountWeeks] <- benchAccountWeeks
summary(scaledata$DataUsage)
      Min. 1st Qu. Median
##
                              Mean 3rd Qu.
   0.0000 0.0000 0.0000 0.8165 1.7800 5.4000
benchDataUsage <- 1.78 + 1.5* IQR(scaledata$DataUsage)
scaledata$DataUsage[scaledata$DataUsage>benchDataUsage] <- benchDataUsage
summary(scaledata$DayMins)
##
      Min. 1st Qu. Median
                              Mean 3rd Qu.
                                               Max.
                                             350.8
##
       0.0
            143.7
                    179.4
                             179.8
                                     216.4
benchDayMins <- 216.4 + 1.5* IQR(scaledata$DayMins)
scaledata$DayMins[scaledata$DayMins>benchDayMins] <- benchDayMins</pre>
summary(scaledata$DayCalls)
      Min. 1st Qu. Median
                              Mean 3rd Qu.
##
                    101.0
                                             165.0
              87.0
                             100.4
                                     114.0
benchDayCalls <- 114.0 + 1.5* IQR(scaledata$DayCalls)</pre>
scaledata$DayCalls[scaledata$DayCalls>benchDayCalls] <- benchDayCalls</pre>
summary(scaledata$MonthlyCharge)
##
      Min. 1st Qu. Median
                              Mean 3rd Qu.
##
     14.00
             45.00
                    53.50
                             56.31
                                     66.20 111.30
benchMonthlyCharge <- 66.20 + 1.5* IQR(scaledata$MonthlyCharge)
scaledata$MonthlyCharge[scaledata$MonthlyCharge>benchMonthlyCharge] <- benchMonthlyCharge
summary(scaledata$OverageFee)
      Min. 1st Qu. Median
                              Mean 3rd Qu.
                                              Max.
##
      0.00
              8.33
                    10.07
                             10.05
                                     11.77
                                             18.19
benchOverageFee <- 11.77 + 1.5* IQR(scaledata$OverageFee)
scaledata$0verageFee[scaledata$0verageFee>bench0verageFee] <- bench0verageFee
summary(scaledata$RoamMins)
##
      Min. 1st Qu. Median
                              Mean 3rd Qu.
                                              Max.
##
      0.00
              8.50
                    10.30
                             10.24
                                     12.10
                                             20.00
benchRoamMins <- 12.10 + 1.5* IQR(scaledata$RoamMins)
scaledata$RoamMins[scaledata$RoamMins>benchRoamMins] <- benchRoamMins
```

4.1.12 Outliers Treated.

```
boxplot(scaledata[,c(2,5,6,7,8,9,10,11)],las=2,cex.axis=0.7)
```



4.1.13 Checking Churn data split

```
table(scaledata$Churn)

##

## 0 1

## 2850 483

483/(2850+483)*100
```

[1] 14.49145

Findings

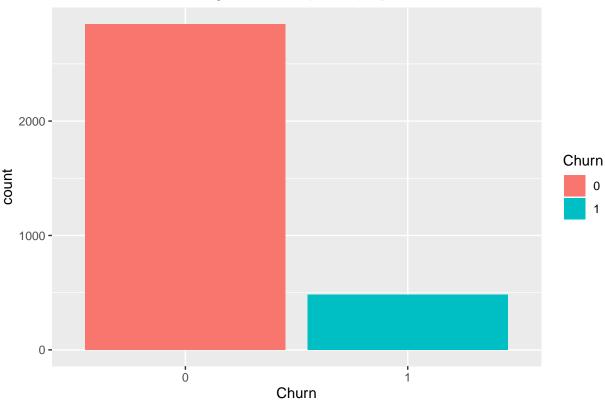
- Minority Class (1) is only 14.49 percent of the total Churn class. It is less than 15%.
- Data will be balanced.

4.1.14 Visualize Dependent Variable(Churn) proportion split

```
ggplot(scaledata) +
aes(x = Churn, fill = Churn) +
geom_bar() +
```

```
scale_fill_hue() +
labs(title = "Target Variable(Churn) Split") +theme(plot.title = element_text(hjust = 0.5))
```

Target Variable(Churn) Split

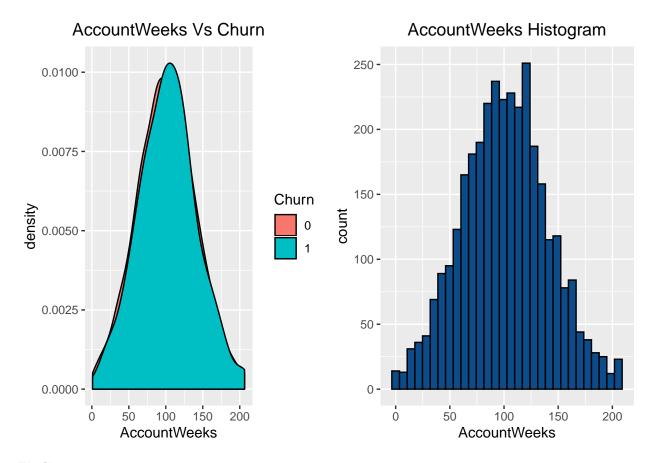


4.1.15 Dependent Variables VS Independent Variables

```
d1 <- ggplot(scaledata) +
   aes(x = AccountWeeks, fill = Churn) +
   geom_density(adjust = 1L) +
   scale_fill_hue() +
   labs(title = "AccountWeeks Vs Churn") +
   theme(plot.title = element_text(hjust = 0.5))

h1 <- ggplot(scaledata) +
   aes(x = AccountWeeks) +
   geom_histogram(bins = 30L, fill = "#0c4c8a",colour="black") +
   labs(title = "AccountWeeks Histogram") +
   theme(plot.title = element_text(hjust = 0.5))

plot_grid(d1,h1)</pre>
```



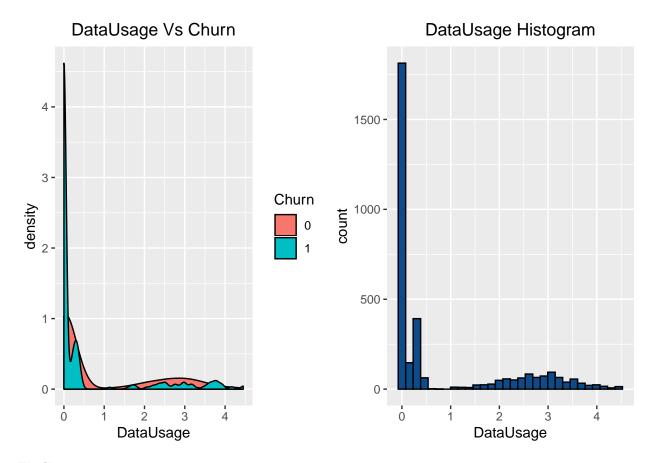
Density plot: AccountWeeks around 100 has more Churn Count.

Histogram: We can say that the frequency distribution for AccountWeeks are normally distributed.

```
d2 <- ggplot(scaledata) +
  aes(x = DataUsage, fill = Churn) +
  geom_density(adjust = 1L) +
  scale_fill_hue() +
  labs(title = "DataUsage Vs Churn") +
  theme(plot.title = element_text(hjust = 0.5))

h2 <- ggplot(scaledata) +
  aes(x = DataUsage) +
  geom_histogram(bins = 30L, fill = "#0c4c8a",colour="black") +
  labs(title = "DataUsage Histogram") +
  theme(plot.title = element_text(hjust = 0.5))

plot_grid(d2,h2)</pre>
```



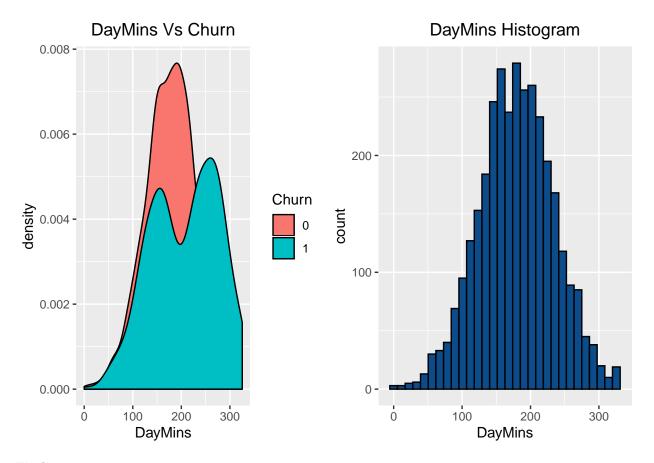
Density plot: There are more ${\bf 0}$ DataUsage in the dataset.

 $\bf Histogram: \ There \ are \ more \ 0 \ Data Usage in the dataset.$

```
d3 <- ggplot(scaledata) +
  aes(x = DayMins, fill = Churn) +
  geom_density(adjust = 1L) +
  scale_fill_hue() +
  labs(title = "DayMins Vs Churn") +
  theme(plot.title = element_text(hjust = 0.5))

h3 <- ggplot(scaledata) +
  aes(x = DayMins) +
  geom_histogram(bins = 30L, fill = "#0c4c8a",colour="black") +
  labs(title = "DayMins Histogram") +
  theme(plot.title = element_text(hjust = 0.5))

plot_grid(d3,h3)</pre>
```



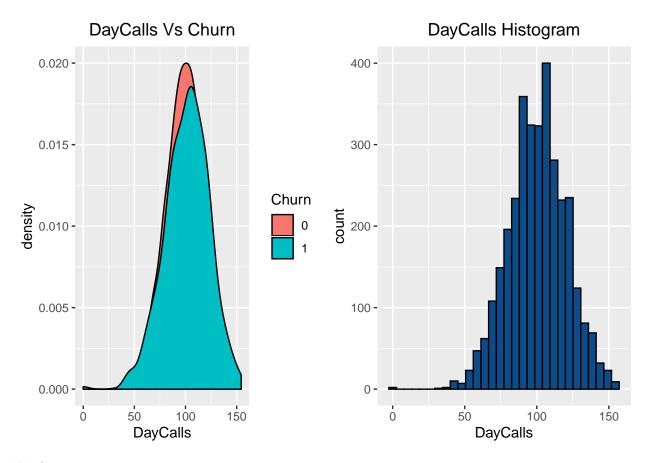
Density plot: Customers have Churned more for DayMins between $150\ and\ 280$.

Histogram: DaysMins is normally Distributed

```
d4 <- ggplot(scaledata) +
  aes(x = DayCalls, fill = Churn) +
  geom_density(adjust = 1L) +
  scale_fill_hue() +
  labs(title = "DayCalls Vs Churn") +
  theme(plot.title = element_text(hjust = 0.5))

h4 <- ggplot(scaledata) +
  aes(x = DayCalls) +
  geom_histogram(bins = 30L, fill = "#0c4c8a",colour="black") +
  labs(title = "DayCalls Histogram") +
  theme(plot.title = element_text(hjust = 0.5))

plot_grid(d4,h4)</pre>
```



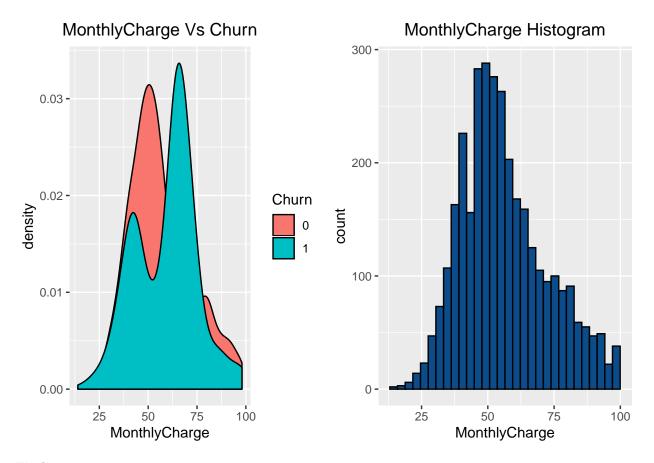
 $\bf Density\ plot:\ Day Calls$ are almost evenly distributed for Churn and Not Churn .

Histogram: DayCalls is normally Distributed

```
d5 <- ggplot(scaledata) +
  aes(x = MonthlyCharge, fill = Churn) +
  geom_density(adjust = 1L) +
  scale_fill_hue() +
  labs(title = "MonthlyCharge Vs Churn") +
  theme(plot.title = element_text(hjust = 0.5))

h5 <- ggplot(scaledata) +
  aes(x = MonthlyCharge) +
  geom_histogram(bins = 30L, fill = "#0c4c8a",colour="black") +
  labs(title = "MonthlyCharge Histogram") +
  theme(plot.title = element_text(hjust = 0.5))

plot_grid(d5,h5)</pre>
```



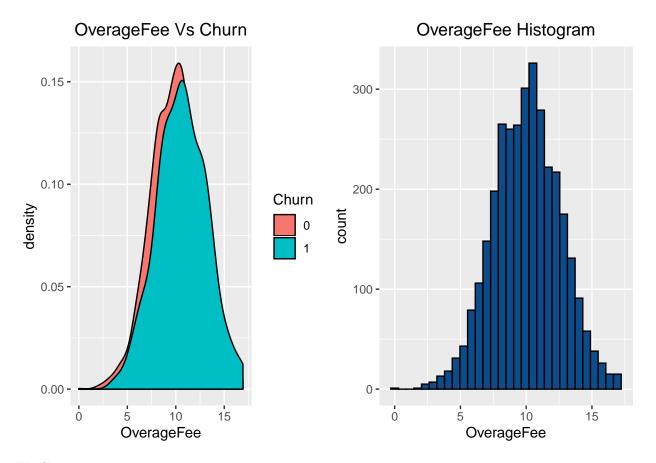
Density plot: Customer with average monthly bill between 50 and 75 is more likely to Churn.

Histogram: MonthlyCharge is normally Distributed

```
d6 <- ggplot(scaledata) +
   aes(x = OverageFee, fill = Churn) +
   geom_density(adjust = 1L) +
   scale_fill_hue() +
   labs(title = "OverageFee Vs Churn") +
   theme(plot.title = element_text(hjust = 0.5))

h6 <- ggplot(scaledata) +
   aes(x = OverageFee) +
   geom_histogram(bins = 30L, fill = "#0c4c8a",colour="black") +
   labs(title = "OverageFee Histogram") +
   theme(plot.title = element_text(hjust = 0.5))

plot_grid(d6,h6)</pre>
```



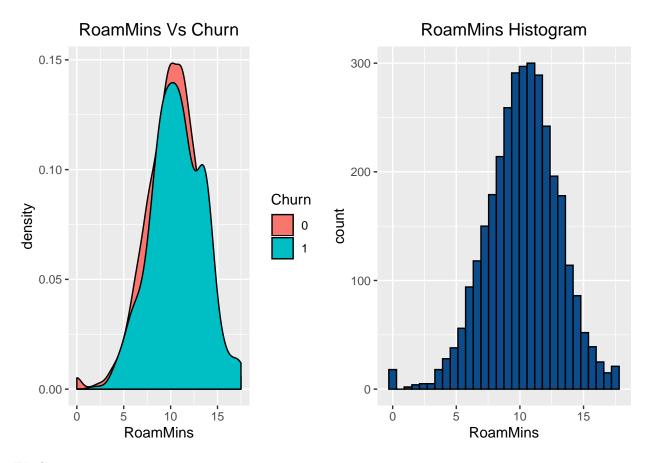
Density plot: Customer with Overcharge fee between 5 and 15 are more likely to cancel the service.

Histogram: Overcharge is normally Distributed

```
d7 <- ggplot(scaledata) +
  aes(x = RoamMins, fill = Churn) +
  geom_density(adjust = 1L) +
  scale_fill_hue() +
  labs(title = "RoamMins Vs Churn") +
  theme(plot.title = element_text(hjust = 0.5))

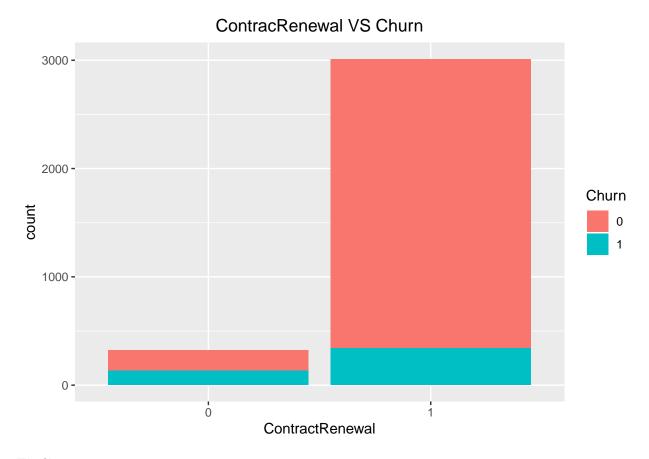
h7 <- ggplot(scaledata) +
  aes(x = RoamMins) +
  geom_histogram(bins = 30L, fill = "#0c4c8a",colour="black") +
  labs(title = "RoamMins Histogram") +
  theme(plot.title = element_text(hjust = 0.5))

plot_grid(d7,h7)</pre>
```

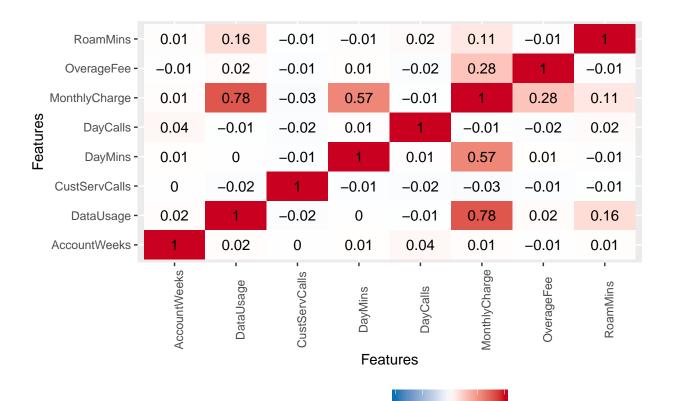


Density plot: Out of Customers who have cancelled the service for RoamMins are most arund 10 minutes **Histogram:** RoamMins is normally Distributed

```
h8<- ggplot(scaledata) +
aes(x = ContractRenewal, fill = Churn) +
geom_bar() +
scale_fill_hue() +
labs(title = "ContracRenewal VS Churn") +
theme(plot.title = element_text(hjust = 0.5))</pre>
```



Bar Chart: Customers who have renewed the plan are more likely to stay with the company plot_correlation(scaledata, type = c("continuous"))



Correlation Meter

Findings

- DataUsage and MonthlyCharge are Highly Correlated.
- MonthlyCharge and DayMins are Highly Correlated.

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5 Build Models

5.1 Pre Process (Train and Test Split)

```
seed <- 101
set.seed(seed)
sample <- sample.split(scaledata,SplitRatio = 0.7)
churn.train <- subset(scaledata,sample == TRUE)
churn.test <- subset(scaledata,sample == FALSE)
nrow(churn.train)

## [1] 2121
nrow(churn.test)

## Balance Dataset with ROSE package
BalancedData=ROSE(Churn~.,data=churn.train,seed=seed)$data</pre>
```

5.1.1 Before Balance VS After Balance

```
table(churn.train$Churn)

##
## 0 1
## 1830 291

table(BalancedData$Churn)

##
## 0 1
## 1062 1059

Findings
```

• Dataset is now BALANCED

5.2 Logistic Regression

Logistic regression is a statistical model that in its basic form uses a logistic function to model a binary dependent variable, although many more complex extensions exist. In regression analysis, logistic regression (or logit regression) is estimating the parameters of a logistic model (a form of binary regression).

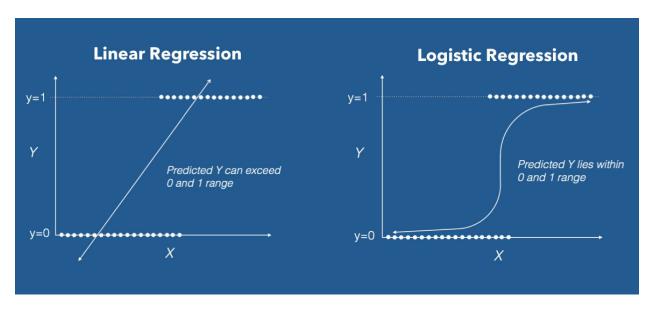


Figure 2: Logistic VS Linear

5.2.1 Full Model for stepwise variable selection and elimination

```
fullmod <- glm(Churn ~.,family=binomial,data = BalancedData)</pre>
summary(fullmod)
##
## Call:
  glm(formula = Churn ~ ., family = binomial, data = BalancedData)
## Deviance Residuals:
##
                    Median
               1Q
                                3Q
                                       Max
  -2.6895 -0.8910 -0.2079
                            0.9166
                                     2.7396
##
## Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
##
                                      -5.389 7.07e-08 ***
## (Intercept)
                  -2.2800535 0.4230727
## AccountWeeks
                  -0.0016946 0.0011316 -1.498 0.134263
## ContractRenewal1 -2.4252907 0.1698024 -14.283 < 2e-16 ***
## DataPlan1
                  -1.2451647 0.2480346
                                       -5.020 5.16e-07 ***
## DataUsage
                  ## CustServCalls
                   ## DayMins
                   0.0060290 0.0009854
                                       6.119 9.45e-10 ***
## DayCalls
                   0.0038169 0.0022627
                                       1.687 0.091635 .
## MonthlyCharge
                   0.0171697
                             0.0046837
                                       3.666 0.000247 ***
## OverageFee
                   0.0899965
                             0.0196366
                                        4.583 4.58e-06 ***
## RoamMins
                   0.0567465
                             0.0167447
                                        3.389 0.000702 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 2940.3 on 2120 degrees of freedom
```

```
## Residual deviance: 2315.4 on 2110 degrees of freedom
## AIC: 2337.4
##
## Number of Fisher Scoring iterations: 4
```

5.2.2 Empty Model for stepwise variable selection and elimination

```
emptyModel<- glm(Churn ~ 1,family=binomial,data = BalancedData)</pre>
summary(emptyModel)
##
## Call:
## glm(formula = Churn ~ 1, family = binomial, data = BalancedData)
## Deviance Residuals:
     Min
              1Q Median
                                      Max
                               3Q
## -1.176 -1.176 -1.179
                                    1.179
## Coefficients:
##
                Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.002829
                          0.043427 -0.065
##
## (Dispersion parameter for binomial family taken to be 1)
##
      Null deviance: 2940.3 on 2120
                                      degrees of freedom
## Residual deviance: 2940.3 on 2120
                                      degrees of freedom
## AIC: 2942.3
##
## Number of Fisher Scoring iterations: 3
```

5.2.3 Baclward Selection of significant variables

```
backwards = step(fullmod)
## Start: AIC=2337.4
## Churn ~ AccountWeeks + ContractRenewal + DataPlan + DataUsage +
      CustServCalls + DayMins + DayCalls + MonthlyCharge + OverageFee +
##
      RoamMins
##
##
                    Df Deviance
                                   ATC
## - DataUsage
                     1 2315.8 2335.8
## <none>
                         2315.4 2337.4
## - AccountWeeks
                     1
                         2317.6 2337.6
## - DayCalls
                     1 2318.2 2338.2
## - RoamMins
                         2327.0 2347.0
                     1
## - MonthlyCharge
                     1
                         2329.0 2349.0
## - OverageFee
                     1
                         2336.7 2356.7
## - DataPlan
                     1
                         2341.2 2361.2
## - DayMins
                         2354.0 2374.0
                     1
## - CustServCalls
                     1
                         2534.1 2554.1
## - ContractRenewal 1
                         2587.1 2607.1
## Step: AIC=2335.84
```

```
## Churn ~ AccountWeeks + ContractRenewal + DataPlan + CustServCalls +
##
      DayMins + DayCalls + MonthlyCharge + OverageFee + RoamMins
##
##
                    Df Deviance
                                   AIC
## <none>
                         2315.8 2335.8
## - AccountWeeks
                         2318.2 2336.2
                     1
## - DayCalls
                         2318.7 2336.7
                     1
## - RoamMins
                         2327.0 2345.0
                     1
## - MonthlyCharge
                     1
                         2329.0 2347.0
                         2337.5 2355.5
## - OverageFee
                     1
## - DayMins
                     1
                         2355.7 2373.7
                     1
## - DataPlan
                         2378.2 2396.2
## - CustServCalls
                     1
                         2535.0 2553.0
## - ContractRenewal 1
                         2587.3 2605.3
```

5.2.4 Forward Slection

```
forwards = step(emptyModel,scope=list(lower=formula(emptyModel),upper=formula(fullmod)), direction="for
```

```
## Start: AIC=2942.33
## Churn ~ 1
##
##
                    Df Deviance
                                  AIC
## + ContractRenewal 1 2736.0 2740.0
## + CustServCalls 1
                        2839.6 2843.6
## + DayMins
                     1
                        2854.5 2858.5
                        2880.7 2884.7
## + DataPlan
                     1
## + DataUsage
                     1
                         2902.3 2906.3
## + OverageFee
                        2907.8 2911.8
                     1
## + RoamMins
                        2920.0 2924.0
                     1
## + MonthlyCharge
                         2929.9 2933.9
                     1
## + DayCalls
                         2937.6 2941.6
                     1
## <none>
                         2940.3 2942.3
                         2940.3 2944.3
## + AccountWeeks
## Step: AIC=2739.97
## Churn ~ ContractRenewal
##
##
                  Df Deviance
                                AIC
## + CustServCalls 1
                      2594.0 2600.0
## + DayMins
                       2630.6 2636.6
## + DataPlan
                      2675.4 2681.4
                   1
## + DataUsage
                   1
                       2692.8 2698.8
## + OverageFee
                   1
                      2697.7 2703.7
## + MonthlyCharge 1
                       2723.6 2729.6
## + RoamMins
                       2729.0 2735.0
                   1
## + DayCalls
                       2731.9 2737.9
                   1
## <none>
                       2736.0 2740.0
## + AccountWeeks 1
                       2735.3 2741.3
## Step: AIC=2599.99
## Churn ~ ContractRenewal + CustServCalls
##
##
                  Df Deviance
                                AIC
```

```
## + DayMins
                        2444.4 2452.4
                    1
## + DataPlan
                        2523.0 2531.0
                    1
                        2543.3 2551.3
## + OverageFee
                    1
## + DataUsage
                        2545.4 2553.4
                    1
## + MonthlyCharge 1
                        2570.2 2578.2
## + RoamMins
                        2585.5 2593.5
                    1
## + DayCalls
                        2589.4 2597.4
                    1
                        2594.0 2600.0
## <none>
## + AccountWeeks
                        2592.9 2600.9
                    1
##
## Step: AIC=2452.39
## Churn ~ ContractRenewal + CustServCalls + DayMins
##
                   Df Deviance
                                  AIC
## + DataPlan
                        2386.2 2396.2
                    1
## + DataUsage
                    1
                        2406.6 2416.6
                        2407.8 2417.8
## + OverageFee
                    1
## + RoamMins
                        2430.8 2440.8
                    1
## + DayCalls
                        2441.4 2451.4
                    1
## <none>
                        2444.4 2452.4
## + AccountWeeks
                    1
                        2442.8 2452.8
## + MonthlyCharge 1
                        2443.4 2453.4
##
## Step: AIC=2396.2
## Churn ~ ContractRenewal + CustServCalls + DayMins + DataPlan
##
##
                   Df Deviance
                                  AIC
## + OverageFee
                        2349.2 2361.2
                    1
## + MonthlyCharge 1
                        2350.8 2362.8
## + RoamMins
                    1
                        2371.8 2383.8
## <none>
                        2386.2 2396.2
## + DayCalls
                        2384.2 2396.2
                    1
## + AccountWeeks
                    1
                        2384.7 2396.7
## + DataUsage
                        2386.0 2398.0
                    1
##
## Step: AIC=2361.25
## Churn ~ ContractRenewal + CustServCalls + DayMins + DataPlan +
##
       OverageFee
##
##
                                  AIC
                   Df Deviance
## + MonthlyCharge 1
                        2331.7 2345.7
## + RoamMins
                        2333.4 2347.4
                    1
## + DayCalls
                        2347.0 2361.0
## <none>
                        2349.2 2361.2
## + AccountWeeks
                        2347.8 2361.8
                    1
## + DataUsage
                        2349.1 2363.1
                    1
##
## Step: AIC=2345.65
## Churn ~ ContractRenewal + CustServCalls + DayMins + DataPlan +
##
       OverageFee + MonthlyCharge
##
                  Df Deviance
##
## + RoamMins
                   1
                       2320.7 2336.7
## + DayCalls
                       2329.3 2345.3
```

```
## + AccountWeeks 1
                       2329.6 2345.6
## <none>
                       2331.7 2345.7
## + DataUsage
                       2331.6 2347.6
##
## Step: AIC=2336.74
## Churn ~ ContractRenewal + CustServCalls + DayMins + DataPlan +
      OverageFee + MonthlyCharge + RoamMins
##
##
##
                  Df Deviance
                                 AIC
## + DayCalls
                   1
                       2318.2 2336.2
## + AccountWeeks 1
                       2318.7 2336.7
                       2320.7 2336.7
## <none>
                       2320.2 2338.2
## + DataUsage
                   1
##
## Step: AIC=2336.16
## Churn ~ ContractRenewal + CustServCalls + DayMins + DataPlan +
       OverageFee + MonthlyCharge + RoamMins + DayCalls
##
##
                  Df Deviance
##
                                 ATC
## + AccountWeeks 1
                       2315.8 2335.8
                       2318.2 2336.2
## <none>
## + DataUsage
                       2317.6 2337.6
##
## Step: AIC=2335.84
## Churn ~ ContractRenewal + CustServCalls + DayMins + DataPlan +
##
       OverageFee + MonthlyCharge + RoamMins + DayCalls + AccountWeeks
##
               Df Deviance
                              AIC
##
                    2315.8 2335.8
## <none>
## + DataUsage 1
                    2315.4 2337.4
```

- From both Forward Selection and Backward Elimination, we see that DataUsage is insignificant feature.
- Build model without DataUsage

5.2.5 Modelling Logistic Regression with Cross Validation

```
fitControl <- trainControl(
  method = "repeatedcv",
  number = 10,
  savePredictions = TRUE
)</pre>
```

5.2.6 Logistic Regression Model

```
## glm(formula = Churn ~ ContractRenewal + CustServCalls + DayMins +
##
      DataPlan + OverageFee + MonthlyCharge + RoamMins + DayCalls +
##
      AccountWeeks, family = "binomial", data = BalancedData)
##
## Deviance Residuals:
##
      Min
                1Q
                     Median
                                  3Q
                                          Max
## -2.6848 -0.8882 -0.2024
                              0.9141
                                       2.7517
##
## Coefficients:
##
                     Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                   -2.2659985 0.4226733 -5.361 8.27e-08 ***
## ContractRenewal1 -2.4223623 0.1695870 -14.284
                                                 < 2e-16 ***
## CustServCalls
                    0.4360201 0.0318788
                                         13.677 < 2e-16 ***
                                           6.214 5.17e-10 ***
## DayMins
                    0.0060928 0.0009805
## DataPlan1
                                         -7.701 1.35e-14 ***
                   -1.3612594 0.1767530
## OverageFee
                    0.0905252
                               0.0196241
                                           4.613 3.97e-06 ***
## MonthlyCharge
                    0.0167706 0.0046417
                                           3.613 0.000303 ***
## RoamMins
                    0.0549779 0.0165200
                                           3.328 0.000875 ***
                    0.0038420 0.0022621
                                           1.698 0.089431 .
## DayCalls
## AccountWeeks
                   -0.0017205 0.0011306 -1.522 0.128091
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
      Null deviance: 2940.3 on 2120 degrees of freedom
## Residual deviance: 2315.8 on 2111
                                      degrees of freedom
## AIC: 2335.8
##
## Number of Fisher Scoring iterations: 4
```

5.2.7 Checking for Multicollinearity

vif(lregmodel)

##	ContractRenewal	CustServCalls	${ t DayMins}$	DataPlan
##	1.105116	1.155222	1.777375	2.125319
##	OverageFee	MonthlyCharge	RoamMins	DayCalls
##	1.146748	2.959555	1.050082	1.008879
##	AccountWeeks			
##	1.012100			

Findings

• Values below 4 indicates that there is no Multicollinearity

5.2.8 Predicting for Train Data

```
logpred<- predict(lregmodel,BalancedData[-1],type = "response")</pre>
```

5.2.9 Probabilities

logpred						
##	1	0	2	1	E	6

```
## 0.34584066 0.19929893 0.05871543 0.24279324 0.42689615 0.08637136
                      8
                                 9
                                           10
           7
                                                      11
## 0.36184384 0.46336282 0.35992386 0.17423454 0.15590466 0.16936702
                     14
                                15
                                           16
                                                      17
## 0.19613675 0.27503938 0.37828311 0.21942511 0.45267783 0.53286568
                                           22
          19
                     20
                                21
                                                      23
## 0.29498312 0.10203806 0.45848804 0.44216439 0.25036871 0.45866522
          25
                     26
                                27
                                           28
                                                      29
## 0.20152063 0.36478359 0.40224855 0.47804593 0.79313361 0.49090121
          31
                     32
                                33
                                          34
                                                      35
## 0.62100035 0.50584914 0.43813658 0.59947232 0.34346128 0.07400533
          37
                     38
                                39
                                           40
                                                      41
## 0.53125875 0.41332439 0.34348894 0.17318168 0.24580521 0.17620273
          43
                     44
                                45
                                           46
                                                      47
## 0.24198814 0.24424332 0.39056003 0.30875867 0.16719006 0.28579054
          49
                     50
                                51
                                           52
                                                      53
                                                                 54
## 0.31124749 0.20923260 0.33965235 0.21683885 0.08490931 0.14589154
                     56
                                57
                                           58
                                                      59
## 0.36440439 0.05009680 0.49542592 0.61039104 0.46311643 0.37831005
                     62
                          63
                                           64
                                                 65
## 0.68890712 0.12844203 0.44795196 0.10221381 0.73580517 0.23183734
                                           70
                     68
                               69
## 0.24827720 0.27487511 0.41421734 0.28658899 0.10968131 0.16900834
          73
                     74
                                75
                                           76
                                                      77
## 0.45467369 0.69312689 0.28030920 0.14508598 0.73434064 0.13838513
          79
                     80
                                81
                                       82
                                                      83
## 0.95373736 0.18942396 0.80110242 0.09165667 0.14669483 0.31974373
          85
                     86
                                87
                                           88
                                                      89
## 0.43142902 0.43068704 0.28602604 0.08876965 0.31474910 0.08338423
          91
                     92
                                93
                                           94
                                                      95
## 0.43466008 0.27082715 0.39563372 0.29699269 0.25099837 0.07679091
          97
                     98
                                99
                                          100
                                                     101
                                                                102
## 0.19452422 0.14048134 0.27058691 0.27042154 0.78590801 0.43965101
         103
                                                     107
                    104
                               105
                                          106
                                                                108
## 0.53761269 0.67635019 0.21628967 0.29400412 0.56659343 0.19677108
                               111
         109
                    110
                                         112
                                                     113
                                                                114
## 0.42501870 0.11144975 0.19336020 0.20223066 0.68140269 0.73593466
                               117
                                          118
         115
                    116
                                                     119
## 0.09239227 0.25132783 0.52597808 0.25795550 0.65821154 0.22459178
         121
                    122
                               123
                                          124
                                                     125
## 0.11465833 0.32199726 0.56818205 0.21646461 0.37960551 0.15445344
         127
                    128
                               129
                                          130
                                                     131
                                                                132
## 0.13349176 0.29185540 0.35614012 0.24209241 0.04826275 0.27135591
                               135
                                          136
         133
                    134
                                                     137
## 0.82087548 0.62110556 0.30355658 0.26128304 0.26756064 0.21972234
                                          142
          139
                    140
                               141
                                                     143
                                                                 144
## 0.25011436 0.18164232 0.75014766 0.91856604 0.77774201 0.22153308
         145
                    146
                               147
                                          148
                                                     149
                                                                 150
## 0.76654872 0.22417625 0.36808600 0.16144712 0.21087695 0.20759384
                    152
                               153
                                          154
                                                     155
                                                                 156
         151
## 0.14002723 0.45804053 0.65869359 0.15221613 0.70083266 0.22300062
                    158
                               159
                                          160
                                                     161
## 0.38289645 0.24794880 0.37257335 0.77978453 0.26708720 0.35264122
##
          163
                    164
                               165
                                          166
                                                     167
```

```
## 0.19501623 0.03147396 0.29860365 0.09145077 0.71434888 0.54102295
##
          169
                     170
                                 171
                                            172
                                                        173
                                                                   174
## 0.12749066 0.29008874 0.18126563 0.60314345 0.34385226 0.19427470
                                            178
                                                                   180
          175
                     176
                                 177
                                                        179
## 0.25763646 0.11367157 0.55589352 0.16876090 0.68932729 0.33185082
          181
                                 183
                     182
                                            184
                                                        185
## 0.57590680 0.05178085 0.20199528 0.54890352 0.55218308 0.19555649
          187
                     188
                                 189
                                            190
                                                        191
                                                                   192
## 0.27912561 0.82178781 0.14010903 0.38008997 0.54439352 0.77130672
          193
                     194
                                 195
                                            196
                                                        197
                                                                   198
## 0.43902509 0.05026143 0.21827082 0.32870849 0.06167798 0.24562087
          199
                     200
                                 201
                                            202
                                                        203
                                                                   204
## 0.39094137 0.97278854 0.34364146 0.31084758 0.45518121 0.11245972
          205
                     206
                                 207
                                            208
                                                        209
## 0.61829524 0.30216885 0.79333448 0.29540366 0.09418310 0.34917558
                     212
                                 213
                                            214
                                                        215
                                                                   216
          211
## 0.17384114 0.25908142 0.21002256 0.14755671 0.32296766 0.27979556
                     218
                                 219
                                            220
                                                        221
                                                                   222
          217
## 0.48500107 0.22393729 0.61590590 0.19395490 0.82193697 0.55199579
          223
                     224
                                 225
                                            226
                                                        227
                                                                   228
## 0.39544801 0.93404579 0.53784371 0.11703283 0.46825815 0.21675191
          229
                     230
                                 231
                                            232
                                                        233
## 0.16175047 0.28408279 0.42703993 0.23387361 0.54205006 0.07143750
          235
                     236
                                 237
                                            238
                                                        239
## 0.48296990 0.24594903 0.17426007 0.56420054 0.26539557 0.27249303
          241
                     242
                                 243
                                            244
                                                        245
                                                                   246
## 0.15433818 0.18666783 0.46292860 0.21858907 0.15337231 0.23834509
          247
                     248
                                 249
                                            250
                                                        251
                                                                   252
## 0.52709350 0.62147197 0.38833533 0.20344177 0.50154886 0.16178468
          253
                     254
                                 255
                                            256
                                                        257
                                                                   258
## 0.43856408 0.39210181 0.24220892 0.08890848 0.25598232 0.46056019
          259
                     260
                                 261
                                            262
                                                        263
                                                                   264
## 0.36653701 0.87720488 0.33609825 0.30049054 0.23878720 0.45920898
          265
                     266
                                 267
                                            268
                                                        269
                                                                   270
## 0.64201649 0.28525184 0.11513109 0.24413295 0.71487122 0.17592230
                                            274
          271
                     272
                                 273
                                                        275
                                                                   276
## 0.37087567 0.33040866 0.23266609 0.77513729 0.22047105 0.55083937
                     278
                                 279
##
          277
                                            280
                                                        281
                                                                   282
## 0.84298048 0.29612890 0.29198178 0.42743991 0.40329909 0.51070958
                                            286
          283
                     284
                                 285
                                                        287
                                                                   288
## 0.35713927 0.42030841 0.46202703 0.43249581 0.30465611 0.24511915
                                            292
          289
                     290
                                 291
                                                        293
                                                                   294
## 0.19336120 0.21728838 0.32000437 0.28003467 0.57141524 0.39766359
          295
                     296
                                 297
                                            298
                                                        299
                                                                   300
## 0.58015373 0.81787996 0.49247617 0.50070975 0.45433231 0.21963799
                     302
                                            304
                                                        305
          301
                                 303
                                                                   306
## 0.27846523 0.30445713 0.22901870 0.29744626 0.62965529 0.29664393
          307
                     308
                                 309
                                            310
                                                        311
                                                                   312
## 0.34356581 0.51919137 0.12733202 0.53734599 0.27148086 0.24856984
          313
                     314
                                 315
                                            316
                                                        317
                                                                   318
## 0.28855603 0.34709918 0.21869549 0.19241859 0.48100463 0.53807754
          319
                     320
                                 321
                                            322
                                                        323
## 0.53844002 0.37662228 0.44958085 0.77096935 0.23787540 0.42561294
##
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## 0.46513891 0.34623008 0.26661116 0.39898448 0.51031721 0.59334057
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## 0.50874982 0.47368609 0.33034824 0.41630234 0.39028293 0.20044680
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## 0.04903479 0.32234887 0.29838277 0.31961006 0.50193921 0.17193159
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## 0.11906910 0.04763083 0.37542391 0.18415340 0.90183737 0.31939341
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## 0.44652706 0.52734867 0.44980190 0.19084769 0.27980858 0.19845090
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## 0.88014145 0.16839264 0.52118378 0.33771813 0.73949250 0.16776376
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## 0.24929271 0.07221404 0.28242691 0.11353183 0.31143616 0.54478081
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## 0.66601684 0.45855883 0.77781981 0.42495911 0.34754233 0.37469605
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## 0.15242819 0.66531872 0.32837905 0.22743591 0.30025211 0.45766673
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## 0.26329224 0.47456593 0.10074962 0.56100155 0.61357211 0.10770252
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## 0.09388195 0.72925358 0.18902388 0.34505316 0.29503798 0.23109144
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## 0.29239949 0.64002643 0.30291066 0.20669054 0.37541306 0.28871878
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## 0.11326691 0.49916891 0.17031941 0.57070504 0.36861106 0.36126549
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## 0.28032371 0.35177895 0.19431272 0.20779514 0.45190662 0.17613644
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## 0.42632983 0.59927880 0.08391001 0.07416958 0.07562769 0.58095842
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## 0.41290591 0.37094630 0.69377142 0.93213240 0.16649906 0.28668888
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## 0.50173349 0.25178102 0.28660802 0.73275841 0.59182592 0.30299504
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## 0.86122808 0.28750907 0.48334806 0.27815951 0.57033667 0.48679372
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## 0.83983236 0.47324645 0.20752720 0.22474201 0.90428302 0.29878166
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## 0.33841118 0.35464431 0.89555574 0.22657489 0.39450973 0.32220958
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## 0.40249692 0.54019412 0.52172723 0.24900996 0.09817625 0.16513447
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## 0.14722543 0.05872109 0.19345062 0.16525182 0.21552530 0.54925918
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## 0.36109115 0.17811739 0.47287236 0.67600070 0.20634962 0.26638782
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## 0.03246218 0.34166304 0.15430194 0.20663788 0.22483713 0.22699018
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## 0.27809411 0.52291721 0.50444561 0.64961752 0.44403886 0.58802731
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## 0.41759616 0.46005742 0.19835607 0.89848734 0.33276889 0.29058748
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## 0.19081635 0.30009312 0.37006322 0.47898758 0.09201452 0.15538646
##
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## 0.76463941 0.38392079 0.44406820 0.72275291 0.02026988 0.08239856
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## 0.30985483 0.28307009 0.30123773 0.40047276 0.24765509 0.15334375
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## 0.51029249 0.46050077 0.53050675 0.05775646 0.03219159 0.19570145
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## 0.19999592 0.63785306 0.47281005 0.13878101 0.30998782 0.50164659
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## 0.46210351 0.27169764 0.59823779 0.35303308 0.09850590 0.70692385
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## 0.28359646 0.73033577 0.17008262 0.67269306 0.40299119 0.12711346
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## 0.30457054 0.47962926 0.66192454 0.24326909 0.36343948 0.55795142
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## 0.20377843 0.21726763 0.53407019 0.18736736 0.64884936 0.29308046
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## 0.59391772 0.18075011 0.58092772 0.64688714 0.33433188 0.85954970
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## 0.09363944 0.34722609 0.11205964 0.28929334 0.15648819 0.23798434
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## 0.45404159 0.47137325 0.04675290 0.23293045 0.18477613 0.44716726
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## 0.14829158 0.66690677 0.24201741 0.43687714 0.29849888 0.64631130
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## 0.33297198 0.19864932 0.61665025 0.56616694 0.28923472 0.09732860
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## 0.43538017 0.03310017 0.26102351 0.37915440 0.35212464 0.38289613
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## 0.23023056 0.51617223 0.36249228 0.19257462 0.18336268 0.36961801
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## 0.49765432 0.33341275 0.38356051 0.16415971 0.52696695 0.13663246
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## 0.31073386 0.13119512 0.18314563 0.24940659 0.20763917 0.21299109
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## 0.18488536 0.35166247 0.71576117 0.14683666 0.19636194 0.10018767
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## 0.57609009 0.36487332 0.43255623 0.21467366 0.41629196 0.43243588
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## 0.36779214 0.38062198 0.42857574 0.65194633 0.30297210 0.20670544
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## 0.63641679 0.74226567 0.29172648 0.37267508 0.36238544 0.74472364
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## 0.34975846 0.73474516 0.42481437 0.17212956 0.24564069 0.44090206
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## 0.19511743 0.05652070 0.46496559 0.09716544 0.25268440 0.30521113
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## 0.41302473 0.40782873 0.48318669 0.41832596 0.08867477 0.31636769
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  0.48357600 0.51040824 0.28583947 0.32308992 0.47970452 0.30484741
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## 0.47283463 0.22372289 0.45411606 0.41799770 0.61816447 0.31890875
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## 0.35600987 0.57271632 0.51283474 0.49187259 0.25541094 0.16686045
##
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## 0.21819859 0.39347422 0.17932623 0.70611044 0.45834087 0.51195018
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## 0.25921349 0.74841169 0.32725088 0.39231546 0.21626637 0.23133315
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## 0.37737166 0.30501503 0.71554825 0.88740905 0.92536428 0.31608937
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## 0.80626259 0.35551452 0.23271174 0.21341428 0.33922134 0.32595899
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## 0.22222714 0.36281871 0.24817699 0.22142976 0.25675467 0.16200025
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## 0.11758738 0.66779841 0.74064743 0.32241571 0.16574790 0.90041778
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## 0.10374611 0.51641342 0.59901133 0.15542955 0.07096092 0.50946621
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## 0.60843479 0.13444025 0.69084234 0.17104351 0.08507809 0.96396333
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## 0.24459756 0.25254638 0.74763922 0.72137828 0.58155095 0.33028515
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## 0.54924923 0.14687552 0.19056997 0.13526343 0.28755461 0.24425528
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## 0.08661234 0.14786192 0.39471318 0.46768524 0.08682624 0.14870271
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## 0.52866743 0.22304274 0.31270565 0.19436564 0.49833026 0.44719156
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## 0.63021733 0.28129826 0.18481710 0.18772378 0.11209265 0.37576525
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## 0.10105833 0.14831683 0.20325049 0.09580351 0.37529356 0.52156964
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## 0.64565424 0.89286458 0.15347268 0.14081545 0.29538871 0.49247319
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## 0.17939799 0.16787541 0.79468335 0.42959274 0.41627859 0.41511599
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## 0.43734411 0.23542468 0.64733400 0.11611937 0.64496403 0.08511272
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## 0.18802329 0.80036739 0.27949334 0.92767460 0.11548658 0.80725988
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## 0.13111764 0.16747376 0.81862468 0.02994937 0.54086961 0.36966345
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## 0.84628633 0.64655148 0.23355683 0.14324576 0.73741535 0.40590763
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## 0.30057084 0.28350602 0.72295326 0.24458696 0.36548180 0.54630220
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## 0.55794923 0.20868348 0.50790268 0.33301151 0.45828201 0.34237745
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## 0.34564874 0.48099018 0.43486372 0.24716317 0.35079866 0.66076193
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## 0.64619822 0.38211143 0.37863392 0.25785833 0.15557398 0.57323716
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## 0.10851721 0.48198048 0.24011059 0.21842380 0.70916102 0.56802617
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## 0.55414684 0.18693136 0.10837618 0.26948894 0.21196136 0.08559393
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## 0.45710847 0.43646910 0.43504994 0.82500443 0.51676872 0.19309128
##
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## 0.70123042 0.22829114 0.40666017 0.43107283 0.03324461 0.42477650
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##
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## 0.64610867 0.30428141 0.35278039 0.64806478 0.15064084 0.46927853
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## 0.38654814 0.65887098 0.53001098 0.36672765 0.43519643 0.20397842
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## 0.24673109 0.23082469 0.24906334 0.15866812 0.14995572 0.39864153
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## 0.24772971 0.28173802 0.51489079 0.15838075 0.76594256 0.61087126
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## 0.51771233 0.64586016 0.23216593 0.26694887 0.25130233 0.23792526
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## 0.15377402 0.55831076 0.31173075 0.49651709 0.20362833 0.14617181
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## 0.47628801 0.55343374 0.25268063 0.26623438 0.31734122 0.22511433
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## 0.19123506 0.51282027 0.23344840 0.44556463 0.81996417 0.21676970
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## 0.64323095 0.24040931 0.28628228 0.24287836 0.12951499 0.21779395
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## 0.47730675 0.22808247 0.44339328 0.22319852 0.19410882 0.13910206
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## 0.40232460 0.62306030 0.14966064 0.19786854 0.12105646 0.29764383
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## 0.83424507 0.45973688 0.11605839 0.38464252 0.85109607 0.35297011
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## 0.22239150 0.30762833 0.62324023 0.30333231 0.30109212 0.71886696
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## 0.44414671 0.64268105 0.13379542 0.20093578 0.13724980 0.17567349
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## 0.15517203 0.21744572 0.07878583 0.48496276 0.56440180 0.50169923
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## 0.60161104 0.37229756 0.46218221 0.61757294 0.58798349 0.14600850
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## 0.07166507 0.35719339 0.33794099 0.66712857 0.30912291 0.68347791
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## 0.52371756 0.20693528 0.54270369 0.37020135 0.17242531 0.25596527
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## 0.24625703 0.21185517 0.32676163 0.33967504 0.46206496 0.56660184
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## 0.06947378 0.68466169 0.87645291 0.14247808 0.78995078 0.34734518
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## 0.45900697 0.22765783 0.55629819 0.18965458 0.15274212 0.10350301
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## 0.20324211 0.19312209 0.08940659 0.57287665 0.21721223 0.44401974
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## 0.18711594 0.32214731 0.16462878 0.32790447 0.17518204 0.46365233
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## 0.85197882 0.15992148 0.13139995 0.41659513 0.29491620 0.37002359
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## 0.11548803 0.95142149 0.01481569 0.34325058 0.39328799 0.11171120
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## 0.82994416 0.28974455 0.43078694 0.42865121 0.15454166 0.64518032
##
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## 0.55694236 0.13382257 0.40668883 0.20316663 0.82831771 0.42083174
##
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## 0.47576716 0.12556231 0.14866109 0.11125076 0.56653879 0.12535385
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## 0.11352374 0.11724897 0.35181953 0.13436548 0.32498148 0.67795321
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## 0.70953910 0.33924216 0.39134132 0.13652903 0.30821240 0.56384299
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## 0.43447362 0.55395433 0.17992096 0.94603000 0.25506712 0.43920888
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## 0.52007317 0.43434769 0.44799938 0.51755514 0.43458523 0.19870775
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## 0.59037209 0.39445498 0.16614249 0.18796127 0.64418936 0.07947131
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## 0.21207680 0.54313061 0.49235524 0.09484359 0.22856256 0.95019288
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## 0.17523237 0.51890287 0.36984288 0.17230941 0.34051191 0.28272115
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## 0.19572939 0.09451496 0.09684715 0.42545217 0.14718406 0.52959191
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## 0.17923403 0.23254593 0.23531173 0.82110792 0.07872139 0.34464525
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## 0.49384185 0.61640245 0.10635898 0.44082935 0.05044867 0.83772805
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## 0.85598867 0.65692415 0.79155826 0.81930649 0.13707331 0.87346514
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## 0.32285172 0.24840973 0.17789422 0.44868590 0.85629681 0.55404493
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## 0.26638500 0.11801231 0.05986661 0.44322460 0.09740258 0.08776564
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## 0.24372879 0.79879020 0.90735761 0.78101838 0.67548273 0.19469561
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## 0.63605039 0.52008232 0.19198692 0.44096944 0.59566613 0.18278445
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## 0.85363838 0.31747618 0.87810250 0.77453296 0.82787623 0.89335049
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## 0.68253276 0.48770570 0.91156668 0.92862392 0.80861149 0.93282363
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## 0.74063207 0.58465648 0.14671688 0.75110168 0.51673207 0.51073354
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## 0.93770233 0.21674230 0.64356455 0.56614689 0.97108034 0.86635407
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## 0.94292510 0.59176061 0.63206749 0.50807293 0.78328588 0.89866981
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## 0.40459465 0.88441469 0.95268297 0.58840360 0.12435205 0.95495338
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## 0.46555046 0.58881523 0.37452312 0.15153556 0.88451949 0.53715038
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## 0.30612616 0.71341989 0.62735620 0.83315124 0.61547116 0.48506901
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         1123
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## 0.64673007 0.29085738 0.29799973 0.50031374 0.76358978 0.95483468
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## 0.08319340 0.58229499 0.53655067 0.36472949 0.12577039 0.45504620
##
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## 0.97195274 0.75042123 0.14751586 0.30676941 0.94506701 0.47136196
##
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## 0.74318848 0.23949375 0.95466258 0.73483359 0.90379685 0.81306477
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## 0.74590490 0.93499789 0.43505138 0.59984600 0.63959152 0.58642762
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## 0.65108950 0.37939685 0.91698002 0.90749731 0.86011258 0.25379104
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## 0.91265740 0.68982809 0.66021266 0.71552734 0.62820312 0.82905940
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## 0.91770576 0.87742287 0.90649914 0.95310319 0.47014682 0.18540461
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## 0.76449041 0.18995383 0.78283825 0.33453956 0.59283584 0.66433189
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         1177
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## 0.38421203 0.40031823 0.20072351 0.89918022 0.66106939 0.56184633
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## 0.79259889 0.14511403 0.57510226 0.63718605 0.79566735 0.54234771
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## 0.29364310 0.84421497 0.83673895 0.69506896 0.80842096 0.25487434
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## 0.23341102 0.89359519 0.68962544 0.57910930 0.77976157 0.52881039
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## 0.66723667 0.55084724 0.93116529 0.96997013 0.58143330 0.88271263
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## 0.65630082 0.70431054 0.41257057 0.70629683 0.50024186 0.57979616
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## 0.62538692 0.58212796 0.37002718 0.17160565 0.70416216 0.41434901
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## 0.24420089 0.70718595 0.50368782 0.71831515 0.29208989 0.55387296
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## 0.60587061 0.71439248 0.50780061 0.77506108 0.80577614 0.73794657
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## 0.13526719 0.64308006 0.51865927 0.32968406 0.96829110 0.59350277
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## 0.69442665 0.69536562 0.46649746 0.45148255 0.64118507 0.37323189
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## 0.91897976 0.41309186 0.49698362 0.98034992 0.92505155 0.72096407
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## 0.37060251 0.70580203 0.74071943 0.72694866 0.65889187 0.56544540
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## 0.97475735 0.26690673 0.08491678 0.96854773 0.35681838 0.88225140
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## 0.80401035 0.48046661 0.83958126 0.96754299 0.98215853 0.33120125
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## 0.93532352 0.71655975 0.38569160 0.90109830 0.60888096 0.86243277
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## 0.17981125 0.71918593 0.87219978 0.93842223 0.86307409 0.87288033
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## 0.64979440 0.37983811 0.63824412 0.47122518 0.38284314 0.69985511
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## 0.37093253 0.51735771 0.80321926 0.63085120 0.67041328 0.30498062
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## 0.94584161 0.95135043 0.52307853 0.91104599 0.11562801 0.83324367
##
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## 0.95426728 0.79502536 0.35302761 0.54716510 0.76195846 0.66398728
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##
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## 0.69867268 0.79913608 0.93843561 0.43019609 0.87199932 0.78723675
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## 0.69380334 0.84837942 0.85502255 0.73857626 0.18159730 0.62265522
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## 0.29135044 0.92612827 0.90988100 0.48180938 0.76807328 0.50048274
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## 0.19903403 0.24520456 0.66779331 0.41610156 0.49132434 0.78930632
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## 0.88380083 0.71510646 0.62294672 0.71436685 0.66017061 0.72030056
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## 0.84242614 0.56616066 0.12118702 0.81148975 0.54912355 0.80475713
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## 0.64434498 0.39280941 0.86774903 0.75724899 0.68041035 0.74495874
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## 0.93727507 0.44641987 0.98296295 0.74590399 0.71285144 0.46888857
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## 0.30917189 0.66780708 0.72996658 0.96074752 0.89492399 0.67564944
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## 0.84689881 0.66762752 0.62217779 0.31938606 0.20618554 0.22874538
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## 0.54609591 0.58932498 0.74025399 0.70484739 0.72723723 0.93119491
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## 0.45277516 0.72529052 0.19139688 0.80723378 0.70340589 0.78518951
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## 0.86505212 0.86384249 0.37548672 0.81804507 0.15231979 0.84120731
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## 0.27521364 0.63965013 0.92641178 0.40003911 0.75696398 0.89376078
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## 0.58287361 0.89514236 0.98352479 0.38855111 0.45931147 0.07200530
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## 0.57609194 0.95064715 0.61512040 0.54799176 0.58232065 0.89667274
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## 0.73364785 0.68182909 0.14182653 0.52183615 0.91718960 0.43468897
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## 0.95775753 0.77229422 0.65599517 0.97117969 0.93687298 0.77671408
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## 0.83501142 0.79465000 0.97504354 0.31710556 0.17362954 0.62389253
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## 0.56778791 0.69115271 0.88119231 0.75543314 0.22271307 0.15632449
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## 0.58621011 0.94577983 0.49259359 0.27648848 0.43927273 0.77059143
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## 0.49435061 0.31073930 0.53183167 0.78989101 0.71897335 0.62384341
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## 0.83680106 0.05575673 0.36285551 0.24095523 0.75290195 0.85474865
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## 0.64206863 0.71331712 0.46385444 0.45561028 0.15020224 0.77485224
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## 0.39105353 0.71638384 0.76895603 0.65894342 0.22780451 0.51897576
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## 0.52068806 0.46985252 0.86606992 0.64591211 0.85461119 0.53192345
##
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## 0.49912787 0.95288133 0.19483351 0.47282409 0.36930556 0.72520014
##
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## 0.32370692 0.46767935 0.38599762 0.41864472 0.60400854 0.90262023
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         1471
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## 0.62956693 0.16987253 0.82372777 0.53374969 0.55858701 0.97834177
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## 0.84528761 0.51071100 0.95864280 0.85587117 0.53284949 0.64012280
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## 0.86181466 0.90909095 0.96352880 0.71405262 0.78409247 0.53116658
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## 0.34472363 0.81042694 0.67653248 0.87876608 0.47849871 0.47164732
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## 0.37074609 0.88428650 0.56008987 0.45381378 0.44797671 0.46750473
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## 0.79840581 0.52530256 0.96612178 0.63560011 0.52431765 0.48690417
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## 0.38144830 0.60363754 0.89516794 0.69600756 0.90815345 0.57618675
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## 0.65585022 0.86551570 0.40525927 0.70549746 0.63009728 0.89920234
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## 0.50017607 0.53696175 0.45922806 0.40146602 0.78823796 0.74391168
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## 0.70540414 0.88124758 0.65169214 0.94684552 0.85825820 0.54127261
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## 0.59368259 0.30646940 0.45181892 0.77119987 0.63516217 0.37611743
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## 0.74265600 0.88170380 0.55728467 0.42119401 0.24569365 0.74102018
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## 0.55787187 0.31411250 0.93852108 0.69333568 0.97009120 0.60249602
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## 0.67072835 0.96729970 0.57386011 0.71688717 0.49024591 0.49929989
##
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## 0.54490874 0.61642608 0.64700272 0.96101978 0.34144232 0.67180790
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## 0.75628111 0.74796697 0.25187668 0.65323862 0.65869126 0.35670623
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## 0.62732793 0.41749793 0.96305880 0.76175352 0.19511882 0.83327964
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## 0.71908408 0.81732686 0.63428671 0.86966801 0.91326473 0.52989831
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## 0.84915022 0.92983725 0.38039968 0.42333959 0.55549579 0.87312220
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## 0.79494367 0.36312098 0.48149148 0.89143405 0.24525395 0.52222168
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## 0.31923026 0.73168362 0.44806304 0.46975862 0.75139586 0.67176286
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## 0.45973570 0.78537667 0.99702231 0.65292379 0.43193361 0.83447784
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## 0.70126536 0.55725462 0.59609737 0.76857328 0.27243802 0.78956604
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## 0.39811949 0.43072404 0.58265605 0.92386269 0.83990436 0.92000510
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## 0.50228284 0.56002258 0.95315074 0.73812852 0.23804117 0.36872849
##
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## 0.84568590 0.90967828 0.26877077 0.85434688 0.95174171 0.55393616
##
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## 0.74744680 0.79185113 0.77919529 0.12002065 0.91140386 0.94065482
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## 0.59147742 0.85160369 0.25974475 0.56336658 0.83806545 0.65858277
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## 0.78013615 0.92598058 0.46921505 0.60028526 0.23406159 0.96522693
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## 0.50087034 0.87012298 0.95333780 0.33190392 0.89785332 0.41821397
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## 0.32170738 0.48814494 0.83320235 0.89299438 0.88080642 0.41695220
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## 0.34380927 0.71210971 0.69903514 0.88853119 0.11448264 0.81112082
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## 0.92314353 0.86836096 0.13989286 0.32197170 0.44822694 0.75174280
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## 0.74137169 0.81709128 0.83715617 0.86340458 0.43012772 0.41512524
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## 0.46560906 0.46974971 0.96722427 0.73642964 0.32995288 0.45757360
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## 0.36888447 0.64512696 0.50026412 0.45789435 0.69572629 0.82316277
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## 0.96350853 0.92439693 0.96396354 0.95584645 0.45452283 0.45912747
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## 0.38299252 0.92173062 0.47580056 0.48959417 0.71742576 0.80675410
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## 0.78730238 0.38775341 0.26056734 0.83515482 0.92829263 0.80148639
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## 0.63844507 0.82453884 0.45019640 0.76426720 0.42944984 0.74521603
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## 0.90898348 0.82149583 0.63894735 0.62104130 0.84986457 0.61902631
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## 0.41991411 0.94704688 0.87839532 0.49981607 0.96816742 0.64362167
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## 0.77332250 0.71838914 0.72265882 0.98678087 0.79196987 0.65234506
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## 0.49689952 0.96161890 0.55548794 0.63885431 0.41010106 0.64402014
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## 0.54149412 0.15024621 0.29791585 0.70620054 0.95697127 0.84381266
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## 0.25383323 0.95622999 0.91278259 0.36640686 0.92835347 0.49132010
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## 0.18688687 0.23808434 0.62112168 0.32746559 0.51727099 0.20307795
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## 0.75952096 0.02269038 0.65067704 0.88934408 0.30274743 0.79789817
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## 0.17779883 0.49978137 0.42032306 0.70016505 0.92030987 0.40972840
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## 0.72048307 0.91574418 0.90146115 0.38730674 0.92949411 0.81087360
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## 0.34011497 0.70881937 0.40973246 0.86292765 0.90777122 0.15797114
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## 0.49521420 0.35604199 0.96778872 0.60106928 0.59904456 0.60010276
##
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## 0.64997923 0.65849773 0.91375795 0.91080665 0.27842510 0.65557266
##
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## 0.98818589 0.45662678 0.33005043 0.70419632 0.86705000 0.66122259
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                                1797
                                           1798
                                                      1799
         1795
## 0.76903000 0.89579269 0.21573015 0.56344912 0.75022968 0.58468860
         1801
                    1802
                                1803
                                           1804
                                                      1805
## 0.81683955 0.63568914 0.61222886 0.63572082 0.69262353 0.36284751
         1807
                    1808
                                1809
                                           1810
                                                      1811
## 0.88919333 0.48703279 0.74485644 0.46038921 0.82625663 0.23835382
         1813
                    1814
                                1815
                                           1816
                                                      1817
## 0.65995267 0.73009807 0.23295473 0.68225797 0.79502116 0.88637929
         1819
                    1820
                               1821
                                           1822
                                                      1823
                                                                 1824
## 0.37592848 0.90645652 0.85516410 0.82580341 0.76547007 0.64243580
         1825
                    1826
                                1827
                                           1828
                                                      1829
## 0.81387047 0.74416539 0.32022166 0.66324973 0.49133186 0.39254702
         1831
                    1832
                                1833
                                           1834
                                                      1835
## 0.81917644 0.84447159 0.26432238 0.33647017 0.20077637 0.92004376
         1837
                    1838
                                1839
                                           1840
                                                      1841
## 0.30719240 0.92986253 0.55248020 0.73760586 0.51267320 0.48537756
         1843
                    1844
                                1845
                                           1846
                                                      1847
                                                                 1848
## 0.26322753 0.55092222 0.73964723 0.44661647 0.39796337 0.47514531
                    1850
                                1851
                                           1852
                                                      1853
## 0.57580388 0.43312879 0.61482642 0.61497297 0.68922498 0.31534168
         1855
                    1856
                                1857
                                           1858
                                                      1859
## 0.54121482 0.62598013 0.87842697 0.63391190 0.78615461 0.43093844
         1861
                    1862
                               1863
                                           1864
                                                      1865
## 0.80103002 0.87922879 0.95344683 0.40174039 0.80472382 0.76554963
         1867
                    1868
                               1869
                                           1870
                                                      1871
## 0.57259880 0.92577807 0.88117509 0.55050163 0.61890595 0.71293413
                    1874
                               1875
                                           1876
                                                      1877
         1873
## 0.58383993 0.71687555 0.58001956 0.97216018 0.31249998 0.31828995
         1879
                    1880
                                1881
                                           1882
                                                      1883
                                                                  1884
## 0.67174542 0.85151487 0.50073433 0.15408353 0.84604691 0.57503356
         1885
                    1886
                                1887
                                           1888
                                                      1889
                                                                  1890
## 0.24872791 0.55433032 0.58129965 0.73563906 0.12507083 0.14990262
                                           1894
         1891
                    1892
                               1893
                                                      1895
                                                                 1896
## 0.64471236 0.95429118 0.91560112 0.93023751 0.55023207 0.96905885
                                1899
                                           1900
         1897
                    1898
                                                      1901
## 0.16720913 0.76510433 0.53827771 0.66206225 0.67196398 0.95394465
         1903
                    1904
                                1905
                                           1906
                                                      1907
## 0.31019183 0.84643488 0.59143510 0.90408222 0.68778430 0.30173429
         1909
                    1910
                                1911
                                           1912
                                                      1913
## 0.79376506 0.89695766 0.88843801 0.63071432 0.69281982 0.46714573
         1915
                    1916
                                1917
                                           1918
                                                      1919
## 0.88451946 0.69503540 0.41562079 0.48106657 0.94082871 0.73067786
                                           1924
         1921
                    1922
                                1923
                                                      1925
                                                                  1926
## 0.49164672 0.49349435 0.95136180 0.38028195 0.63765683 0.66625482
         1927
                    1928
                                1929
                                           1930
                                                      1931
                                                                  1932
## 0.26020089 0.51408286 0.33989021 0.48563324 0.51963881 0.79700060
         1933
                    1934
                                1935
                                           1936
                                                      1937
                                                                  1938
## 0.73214186 0.73712219 0.33931787 0.61735371 0.91834356 0.12228080
         1939
                    1940
                                1941
                                           1942
                                                      1943
## 0.63905528 0.33878116 0.70111076 0.77027679 0.78979717 0.41618678
##
         1945
                    1946
                                1947
                                           1948
                                                      1949
```

```
## 0.70958417 0.67476352 0.26789067 0.13150066 0.93900807 0.67084171
                                           1954
##
         1951
                    1952
                                1953
                                                      1955
                                                                  1956
## 0.13867575 0.94447746 0.70734067 0.72474156 0.79494707 0.53435233
                    1958
                                1959
                                           1960
                                                      1961
         1957
                                                                  1962
## 0.81016558 0.86622307 0.61919754 0.83240303 0.64699876 0.65005600
                                           1966
         1963
                    1964
                               1965
                                                      1967
## 0.69251430 0.63652141 0.73188653 0.95589402 0.71049798 0.85435745
                                           1972
         1969
                    1970
                               1971
                                                      1973
## 0.49007349 0.37337773 0.57008338 0.75101483 0.54211171 0.52744529
         1975
                    1976
                                1977
                                           1978
                                                      1979
## 0.48847258 0.63177177 0.92914921 0.75257020 0.15528047 0.92351130
         1981
                    1982
                               1983
                                           1984
                                                      1985
                                                                  1986
## 0.50611309 0.92434862 0.53698427 0.66577616 0.67856590 0.95959105
         1987
                    1988
                                1989
                                           1990
                                                      1991
## 0.87226501 0.82350504 0.44713441 0.56545646 0.35114337 0.37990932
         1993
                    1994
                                1995
                                           1996
                                                      1997
                                                                  1998
## 0.86676482 0.82309855 0.70294987 0.76460778 0.76743242 0.41478120
         1999
                    2000
                                2001
                                           2002
                                                      2003
                                                                  2004
## 0.82306551 0.86769920 0.87569438 0.95948271 0.77609196 0.96804465
         2005
                    2006
                                2007
                                           2008
                                                      2009
                                                                  2010
## 0.45872076 0.29223317 0.78530329 0.71090012 0.40747672 0.59688588
                    2012
                                2013
                                           2014
                                                      2015
## 0.98511271 0.65308326 0.70341210 0.48133928 0.25926080 0.95474281
         2017
                    2018
                                2019
                                           2020
                                                      2021
## 0.67370066 0.70487006 0.57590747 0.79489137 0.44801441 0.71695130
         2023
                    2024
                                2025
                                           2026
                                                      2027
## 0.61257832 0.78596642 0.40268541 0.78086040 0.80137865 0.65955207
         2029
                    2030
                                2031
                                           2032
                                                      2033
## 0.85292417 0.69810640 0.64650226 0.61899777 0.61446798 0.92967833
         2035
                    2036
                                2037
                                           2038
                                                      2039
## 0.37406193 0.38375491 0.57445351 0.85577267 0.93813827 0.66942534
         2041
                    2042
                                2043
                                           2044
                                                      2045
                                                                  2046
## 0.75288707 0.78369968 0.16995037 0.81135600 0.75604272 0.62001925
                                           2050
         2047
                    2048
                                2049
                                                      2051
                                                                  2052
## 0.96962551 0.34461341 0.44802556 0.85643521 0.56836365 0.27147031
                    2054
                               2055
                                           2056
                                                      2057
         2053
                                                                  2058
## 0.58887776 0.98765678 0.37219134 0.65790322 0.55036261 0.65769635
         2059
                               2061
                                           2062
                    2060
                                                      2063
## 0.86733201 0.53286135 0.60352762 0.36957402 0.91938430 0.45896666
                                           2068
         2065
                    2066
                                2067
                                                      2069
                                                                  2070
## 0.95074899 0.80826542 0.40247190 0.29650393 0.47331786 0.66165167
                    2072
         2071
                                2073
                                           2074
                                                      2075
                                                                  2076
## 0.90585834 0.80010879 0.70089487 0.96227804 0.63658203 0.83106654
                    2078
                                2079
                                           2080
                                                      2081
         2077
## 0.56362285 0.53964653 0.54896697 0.47339930 0.95384206 0.54860025
         2083
                    2084
                                2085
                                           2086
                                                      2087
                                                                  2088
## 0.15844043 0.23418441 0.70741735 0.94918450 0.92585945 0.60191378
         2089
                    2090
                                2091
                                           2092
                                                      2093
                                                                  2094
## 0.69678318 0.76763533 0.97752328 0.92157889 0.10865254 0.96430500
         2095
                    2096
                                2097
                                           2098
                                                      2099
## 0.70172001 0.32111389 0.39665399 0.97814822 0.91221833 0.97077856
                    2102
                                2103
                                           2104
                                                      2105
## 0.13465015 0.31019297 0.74903665 0.72671279 0.57967003 0.73403772
##
         2107
                    2108
                                2109
                                           2110
                                                      2111
```

```
## 0.58582887 0.71015643 0.36319724 0.50856624 0.39177683 0.65871816

## 2113 2114 2115 2116 2117 2118

## 0.78893024 0.54472068 0.76340527 0.89195661 0.53278499 0.66537012

## 2119 2120 2121

## 0.82538750 0.59534387 0.23004189
```

5.2.10 Coverting Probabilities to 0 and 1

##

##

##

```
ypredlog <- ifelse(logpred > 0.5,1,0)
ypredlog
##
             2
                    3
                          4
                                5
                                       6
                                             7
                                                   8
                                                          9
                                                               10
                                                                     11
                                                                           12
                                                                                  13
                                                                                        14
                                                                                              15
       1
##
       0
             0
                    0
                          0
                                0
                                       0
                                             0
                                                   0
                                                          0
                                                                0
                                                                      0
                                                                             0
                                                                                         0
                                                                                                0
                                                                                   0
##
      16
            17
                               20
                                     21
                                            22
                                                  23
                                                               25
                                                                     26
                                                                           27
                                                                                  28
                                                                                        29
                                                                                              30
                   18
                         19
                                                        24
##
       0
             0
                    1
                          0
                                0
                                       0
                                             0
                                                   0
                                                          0
                                                                0
                                                                      0
                                                                            0
                                                                                   0
                                                                                         1
                                                                                                0
##
      31
            32
                   33
                         34
                               35
                                     36
                                            37
                                                  38
                                                        39
                                                               40
                                                                     41
                                                                           42
                                                                                  43
                                                                                        44
                                                                                              45
##
       1
             1
                    0
                          1
                                0
                                      0
                                             1
                                                   0
                                                          0
                                                                0
                                                                      0
                                                                            0
                                                                                   0
                                                                                         0
                                                                                                0
                                            52
                                                  53
##
      46
            47
                   48
                         49
                               50
                                     51
                                                        54
                                                               55
                                                                     56
                                                                           57
                                                                                  58
                                                                                        59
                                                                                              60
```


##	0	0	1	1	0	0	1	0	0	0	0	0	0	1	1
##	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345
##	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
##	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360
##	0	1	0	0	1	0	0	0	0	1	0	1	0	1	0
##	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375
##	0	0	0	0	0	1	1	0	1	0	0	0	0	1	0
##	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390
## ##	0 391	0 392	0 393	0 394	0 395	0 396	1 397	1 398	0 399	0 400	1 401	0 402	0 403	0 404	0 405
##	391	392	393	0	395	396	391	390	399	400	401	402	403	404	405
##	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420
##	0	0	0	0	1	0	0	0	1	0	0	1	1	0	0
##	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435
##	1	0	0	1	1	0	1	0	0	0	1	0	1	0	0
##	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450
##	0	1	0	0	0	1	0	0	0	0	1	1	0	0	0
##	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465
##	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
##	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480
##	0	0	0	0	1	1	1	0	1	0	0	0	1	0	0
##	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495
##	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
##	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510
##	0	0	0	1	0	1	0	0	0	0	1	0	0	0	1
##	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525
##	0	0	1	0	0	1	0	1	0	1	0	0	0	0	1
##	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540
##	0	0	1	0	0	1	0	1	0	1	0	1	1	0	1
##	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555
##	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
##	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570
##	0 571	0 572	1 573	0 574	0 575	1 576	1 577	0 578	0 579	0 580	0 581	0 582	0 583	0 584	0 585
## ##	0	1	0	574 0	0	0	0	0	0	0	1	0	0	0	0
##	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600
##	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
##	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615
##	0	0	0	1	0	0	1	1	0	0	0	1	0	1	0
##	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630
##	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
##	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645
##	0	1	0	0	0	0	0	0	0	0	1	0	0	1	1
##	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660
##	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0
##	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675
##	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0
##	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690
##	0	0	0	0	1	1	0	0	1	0	1	1	0	0	1
##	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705
##	1	0	1	0	0	1	0	0	1	1	1	0	1	0	0
##	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720
##	0	0	0	0	0	0	0	0	0	1	0	0	0	0	725
##	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735

```
0
                                 1
                                      1
                                                 0
           1
                      1
                                          1
                                                      1
                                                            1
                                                                 0
## 1951 1952 1953 1954 1955 1956 1957 1958 1959 1960 1961 1962 1963 1964 1965
                      1
                            1
                                 1
                                      1
                                                            1
  1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980
##
           1
                 1
                      0
                            0
                                 1
                                      1
                                            1
                                                 1
                                                       0
                                                            1
                                                                 1
                                                                       1
## 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995
           1
                            1
                                 1
                                      1
                                            1
                                                 0
                                                       1
                                                            0
                                                                       1
                 1
                      1
## 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010
##
                 0
                                 1
                                      1
                                                 1
                                                       0
                                                            0
      1
           1
                      1
                            1
                                            1
                                                                 1
                                                                       1
                                                                            0
## 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024 2025
                      0
                            0
                                 1
                                                            0
           1
                 1
                                      1
                                            1
                                                 1
                                                       1
                                                                 1
                                                                       1
## 2026 2027 2028 2029 2030 2031 2032 2033 2034 2035 2036 2037 2038 2039 2040
                                                            0
      1
           1
                 1
                      1
                            1
                                 1
                                      1
                                            1
                                                 1
                                                      0
                                                                 1
                                                                       1
                                                                            1
## 2041 2042 2043 2044 2045 2046 2047 2048 2049 2050 2051 2052 2053 2054 2055
##
           1
                 0
                      1
                            1
                                 1
                                      1
                                            0
                                                 0
                                                       1
                                                            1
                                                                 0
                                                                       1
## 2056 2057 2058 2059 2060 2061 2062 2063 2064 2065 2066 2067 2068 2069 2070
##
                                      0
                                                 0
      1
           1
                 1
                            1
                                 1
                                            1
                                                       1
                                                            1
                                                                 0
                                                                       0
                                                                                 1
                      1
## 2071 2072 2073 2074 2075 2076 2077 2078 2079 2080 2081 2082 2083 2084 2085
           1
                 1
                      1
                            1
                                 1
                                      1
                                            1
                                                 1
                                                       0
                                                            1
                                                                 1
## 2086 2087 2088 2089 2090 2091 2092 2093 2094 2095 2096 2097 2098 2099 2100
##
                                 1
                                      1
                                            0
                                                 1
                                                       1
                                                            Λ
                                                                 0
                                                                       1
           1
                 1
                      1
                            1
## 2101 2102 2103 2104 2105 2106 2107 2108 2109 2110 2111 2112 2113 2114 2115
      0
                                      1
                                                 0
                                                       1
                                                            0
                                                                 1
           0
                 1
                      1
                            1
                                 1
                                            1
                                                                       1
## 2116 2117 2118 2119 2120 2121
##
      1
           1
                 1
                      1
```

5.2.11 ConfusionMatrix and Training Model Evaluation

```
ConfusionMatrix(ypredlog, BalancedData$Churn)
```

```
## y_pred
## y_true 0 1
## 0 802 260
## 1 308 751
```

5.2.12 Accuracy of Train, Precision, Recall, Sensitivity and F score

```
accuracy.meas(ypredlog,BalancedData$Churn)

##
## Call:
## accuracy.meas(response = ypredlog, predicted = BalancedData$Churn)
##
## Examples are labelled as positive when predicted is greater than 0.5
##
## precision: 0.477
## recall: 1.000
## F: 0.323
Accuracy(ypredlog,BalancedData$Churn)
```

```
## [1] 0.7322018
Sensitivity(BalancedData$Churn, ypredlog)
```

```
Specificity(BalancedData$Churn,ypredlog)
## [1] 0.7091596
AUC(y_pred = ypredlog,y_true = BalancedData$Churn)
```

5.2.13 Model Evaluation Table

[1] 0.7321692

Accuracy	0.7322018
Sensitivity	0.7551789
Specificity	0.7091596
F Score	0.323
AUC	0.7321692

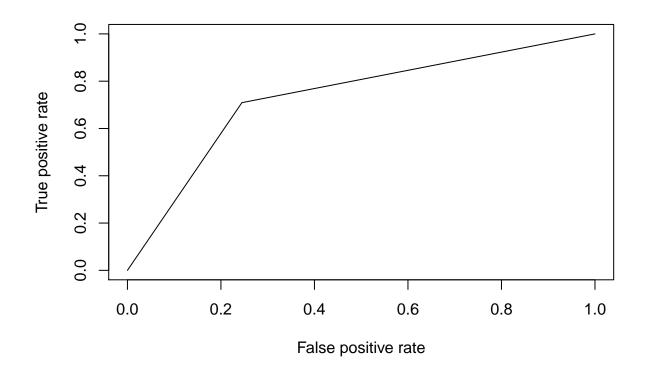
5.2.14 ROC Curve Plot

```
rocrpred2=prediction(ypredlog,BalancedData$Churn)
as.numeric(performance(rocrpred2,"auc")@y.values)

## [1] 0.7321692

pref2=performance(rocrpred2 "tpr" "fpr")
```

```
pref2=performance(rocrpred2, "tpr", "fpr")
plot(pref2)
```



5.2.15 Predicting Test Data

```
logpredtest <- predict(lregmodel,churn.test[-1],type = "response")</pre>
```

5.2.16 Coverting Probabilities to 0 and 1

```
logpredtestprob <- ifelse(logpredtest > 0.5,1,0)
logpredtestprob
```

```
1
                                0
                                     0
                                          0
                                                0
                                                          0
                     1
## 3179 3182 3183 3188 3190 3193 3194 3199 3201 3204 3205 3210 3212 3215 3216
                                0
                                     0
## 3221 3223 3226 3227 3232 3234 3237 3238 3243 3245 3248 3249 3254 3256 3259
           1
                1
                     1
                           0
                                0
                                     0
                                          1
                                                0
                                                     0
                                                          0
                                                               0
                                                                     0
## 3260 3265 3267 3270 3271 3276 3278 3281 3282 3287 3289 3292 3293 3298 3300
                0
                     1
                           0
                                0
                                     0
                                          1
                                                0
                                                     0
                                                          0
                                                               1
## 3303 3304 3309 3311 3314 3315 3320 3322 3325 3326 3331 3333
           0
                0
                     0
                           0
                                0
                                     1
                                          0
                                                0
                                                     0
```

5.2.17 ConfusionMatrix and Test Data Model Evaluation

```
ConfusionMatrix(logpredtestprob, churn.test$Churn)
```

```
## y_pred
## y_true 0 1
## 0 801 219
## 1 51 141
```

${\bf 5.2.18}$ $\,$ Accuracy of Train, Precision , Recall, Sensitivity and F score

```
accuracy.meas(logpredtestprob,churn.test$Churn)
```

```
##
## Call:
## accuracy.meas(response = logpredtestprob, predicted = churn.test$Churn)
##
## Examples are labelled as positive when predicted is greater than 0.5
##
## precision: 0.297
## recall: 1.000
## F: 0.229
Accuracy(logpredtestprob, churn.test$Churn)
```

```
## [1] 0.7772277
Sensitivity(churn.test$Churn, logpredtestprob)
```

```
## [1] 0.7852941
```

```
## [1] 0.7852941

Specificity(churn.test$Churn,logpredtestprob)

## [1] 0.734375
```

```
AUC(y_pred = logpredtestprob,y_true = churn.test$Churn)
```

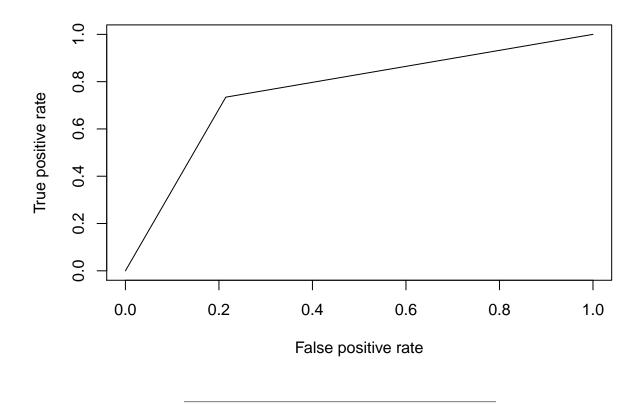
5.2.19 Model Evaluation Table

Accuracy	0.7772277
Sensitivity	0.7852941
Specificity	0.734375
F Score	0.229
AUC	0.7598346

5.2.20 ROC Curve Plot

```
rocrpred=prediction(logpredtestprob,churn.test$Churn)
as.numeric(performance(rocrpred,"auc")@y.values)
```

```
pref=performance(rocrpred,"tpr","fpr")
plot(pref)
```



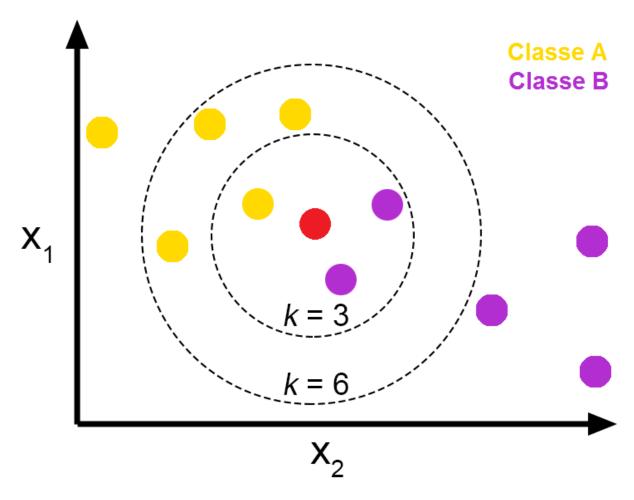


Figure 3: KNN

5.3 KNN

5.3.1 What is KNN

KNN (K — Nearest Neighbors) is one of many (supervised learning) algorithms used in data mining and machine learning, it's a classifier algorithm where the learning is based "how similar" is a data (a vector) from other .

5.3.2 Fitting KNN to the Training set and Predicting the Test set results

```
[171] 1 1 1 1 0 0 0 0 0 1 0 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 1 1 0 0
##
  [205] 0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 0 1 0 0 1 0 1 0 0 1 0 0 1 0 0 0 1 0
  ##
##
  [273] 0 0 0 0 0 0 1 0 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0
  [307] 1 0 1 1 0 0 0 0 0 0 1 0 0 0 0 0 1 1 0 1 1 1 0 0 0 1 1 1 0 0 0 0 1
##
  [341] 0 0 0 0 0 1 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0
  ##
##
  ##
  [477] 1 1 0 0 0 0 0 0 1 1 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 1 1 0 1 1 0 0 0
  [511] 1 0 0 0 0 0 1 0 1 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 1
##
  [545] 0 0 0 1 0 1 0 0 1 0 0 0 1 1 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
  ##
  [613] 0 0 1 0 0 0 0 1 0 1 0 1 0 1 0 1 0 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0
##
##
  [681] 0 1 1 0 1 1 0 0 0 0 0 1 1 0 0 0 0 0 1 1 0 0 0 0 1 1 0 0 0 0 1 0 1 0 0 0
##
##
  [715] 0 1 0 0 1 0 1 0 0 0 0 0 1 1 0 0 0 0 1 0 1 1 0 1 1 0 1 1 0 0 0 0 1 0
  [783] 0 1 1 0 0 0 1 0 1 0 0 0 1 1 1 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 0 1 0 1
## [817] 0 0 1 0 1 1 1 1 1 0 1 1 0 0 0 0 1 0 0 0 0 1 0 0 1 0 0 0 1 1 0 0 0
## [851] 0 1 1 0 0 1 1 0 0 1 1 1 0 1 0 1 0 0 0 0 0 0 1 1 1 1 0 0 1 1 0 1 0 1 0 0
[919] 1 1 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 1 0 1 0 1 0 1 0 1 0 0 0
## [953] 0 1 0 0 0 0 0 0 0 0 0 0 1 0 1 1 0 0 0 1 0 0 0 1 0 0 1 0 0 1 1 1 0
## [1021] 0 0 0 0 0 1 1 0 0 0 1 1 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 1 1 0 0 1
## [1055] 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 1 0 0 0 1 0
## [1089] 1 1 0 1 0 0 0 1 0 1 0 0 0 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 1 0 1
## [1123] 1 1 0 1 0 1 0 1 1 0 0 0 0 1 1 1 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
## [1157] 1 0 1 0 0 1 1 1 0 0 1 1 0 0 0 0 1 1 0 0 0 0 1 1 0 0 0 0 0 0 0 1 1 0 0 0 0
## [1191] 0 1 0 0 0 0 0 0 1 1 1 0 0 0 1 0 1 1 0 0 0 0
## Levels: 0 1
```

5.3.3 Confusion Matrix

```
ConfusionMatrix(ypredKNN, churn.test$Churn)

## y_pred
## y_true 0 1
```

```
## 0 712 308

## 1 100 92

Accuracy(ypredKNN,churn.test$Churn)
```

[1] 0.6633663

5.3.4 Trying Different K value

```
ConfusionMatrix(ypredKNN2, churn.test$Churn)
##
        y_pred
## y_true 0 1
        0 677 343
##
##
        1 100 92
Accuracy(ypredKNN2,churn.test$Churn)
## [1] 0.6344884
5.3.5 Try k 9
ypredKNN3 <- knn(train = BalancedData[,-1],</pre>
                test = churn.test[,-1],
                cl= BalancedData[,1],
                k=9
                )
ConfusionMatrix(ypredKNN3, churn.test$Churn)
##
         y_pred
## y_true 0 1
##
        0 754 266
        1 103 89
Accuracy(ypredKNN3,churn.test$Churn)
## [1] 0.6955446
5.3.6 Try K 15
ypredKNN4 <- knn(train = BalancedData[,-1],</pre>
                test = churn.test[,-1],
                cl= BalancedData[,1],
                k=15
                )
ConfusionMatrix(ypredKNN4, churn.test$Churn)
##
        y_pred
## y_true 0 1
        0 789 231
        1 110 82
Accuracy(ypredKNN4,churn.test$Churn)
## [1] 0.7186469
5.3.7 Try K 25
ypredKNN5 <- knn(train = BalancedData[,-1],</pre>
                test = churn.test[,-1],
                cl= BalancedData[,1],
                k=25
                )
```

```
ConfusionMatrix(ypredKNN5, churn.test$Churn)
##
       y_pred
## y_true 0 1
       0 815 205
##
##
       1 110 82
Accuracy(ypredKNN5,churn.test$Churn)
## [1] 0.740099
5.3.8 Try K 35
ypredKNN6 <- knn(train = BalancedData[,-1],</pre>
               test = churn.test[,-1],
                cl= BalancedData[,1],
                k=35
                )
ConfusionMatrix(ypredKNN6, churn.test$Churn)
        y_pred
## y_true 0 1
##
       0 827 193
       1 109 83
Accuracy(ypredKNN6,churn.test$Churn)
## [1] 0.7508251
Sensitivity(churn.test$Churn, ypredKNN6)
## [1] 0.8107843
Specificity(churn.test$Churn,ypredKNN6)
## [1] 0.4322917
accuracy.meas(ypredKNN6,churn.test$Churn)
##
## Call:
## accuracy.meas(response = ypredKNN6, predicted = churn.test$Churn)
## Examples are labelled as positive when predicted is greater than 0.5
##
## precision: 0.228
## recall: 1.000
## F: 0.185
KNNAUC<- AUC(ypredKNN6,churn.test$Churn)</pre>
KNNAUC
## [1] 0.621538
```

5.3.9 Model Evaluation Table

0.7508251
0.8107843
0.4322917
0.185
0.621538

Naive Bayes



In machine learning, naive Bayes classifiers are a family of simple "probabilistic classifiers" based on applying Bayes' theorem with strong (naive) independence assumptions between the features.

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

using Bayesian probability terminology, the above equation can be written as

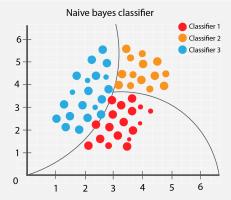


Figure 4: Naive Bayes

5.4 Naive Bayes

5.4.1 What is Naive Bayes

In machine learning, naive Bayes classifiers are a family of simple "probabilistic classifiers" based on applying Bayes' theorem with strong (naive) independence assumptions between the features.

5.4.2 Naive Bayes. Is it applicable here?

YES it is applicable. Naibe Bayes is a **probabilistic classifiers**. Here we'll use it to predict probability if Customer Cancelled(Churn) or Not Cancelled the service.

5.4.3 Fitting Naive Bayes to training set

5.4.4 Prediciting Naive Bayes Test Dataset

```
ypredNB<- predict(classnaive,newdata = churn.test[-1])
ypredNB</pre>
```

```
##
  ##
  ##
  [103] 0 0 0 1 0 0 0 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 1 1 1 0 1 0 1 1 0 0
##
 [137] 0 0 1 0 0 0 0 0 0 0 1 0 0 0 1 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 1 0 0 0 1
 [171] 0 0 0 0 0 0 1 0 1 0 1 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 0 0
##
 ##
 [307] 0 0 0 0 1 0 0 0 0 0 1 0 1 1 0 0 0 0 1 1 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0
```

```
[375] 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 1 1 0 0 0 0 0 1 1
##
## [443] 0 0 0 0 0 1 0 1 0 1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0
  [477] 0 0 0 1 1 0 0 0 0 1 1 0 1 1 1 0 1 0 0 0 0 1 0 0 0 0 0 1 1 1 1 0
## [545] 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## [681] 1 0 1 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
 ##
 [749] 1 0 0 1 0 0 1 0 0 0 1 0 0 0 0 0 0 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 1 0
## [783] 0 1 1 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 1 1 0 0 0 0 0 1 0 1 0 0
## [817] 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 1 0 1 0 1 0 0 0 1
##
  [851] 0 0 1 0 0 1 1 0 1 0 1 0 0 0 0 0 1 1 0 0 1 1 1 1 0 0 0 1 0 1 0 1 0 0 1 0
## [885] 0 0 0 0 1 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0
## [919] 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 1 1 0 0 0 0
## [1055] 0 1 0 0 0 0 0 1 1 0 0 1 0 1 0 0 0 1 1 0 0 0 1 1 0 0 0 0 1 1 0 0 0 1 1
## [1089] 1 0 1 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 1 1 1 0 0 0 0 0 0 1 0 1 0 0
## [1123] 0 0 0 0 0 0 0 0 1 1 0 0 0 0 1 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
## [1191] 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 1 0
## Levels: 0 1
```

5.4.5 ConfusionMatrix

```
ConfusionMatrix(ypredNB,churn.test$Churn)
         y_pred
##
## y_true
          0
        0 886 134
##
        1 44 148
Accuracy(ypredNB,churn.test$Churn)
## [1] 0.8531353
Sensitivity(churn.test$Churn, ypredNB)
## [1] 0.8686275
Specificity(churn.test$Churn,ypredNB)
## [1] 0.7708333
accuracy.meas(ypredNB,churn.test$Churn)
##
## Call:
## accuracy.meas(response = ypredNB, predicted = churn.test$Churn)
## Examples are labelled as positive when predicted is greater than 0.5
##
```

```
## precision: 0.233
## recall: 1.000
## F: 0.189
NBAUC<- AUC(ypredNB,churn.test$Churn)
NBAUC</pre>
```

5.4.6 Model Evaluation Table

Accuracy	0.8531353
Sensitivity	0.8686275
Specificity	0.7708333
F Score	0.189
AUC	0.8197304

5.5 Model Comparison using Model Performance metrics

	Logistic Regression	KNN	Naive Bayes
Accuracy	0.7772277	0.7508251	0.8531353
Sensitivity	0.7852941	0.8107843	0.8686275
Specificity	0.734375	0.4322917	0.7708333
F Score	0.229	0.185	0.189
AUC	0.7598346	0.621538	0.8197304

5.5.1 Interpretation

• From the above model Comparison we can see that Naive Bayes is performing the best.

5.5.2 Conclusion:

- We can see that all the models are performing similarly. However, Naive Bayes is performing the best. Naive Bayes is able to predict Customers who are going to cancel the service.
- ContractRenewal + CustServCalls + DayMins are the main factors which makes customer continue or cancel the service.

5.5.3 Recommendation:

• CustServCalls greatly contribute influence customer's decision to continue or cancel the service. It look like Customer Service has to be improved. There are many customers calling customer service 10 times. Company must invest in training resources and improve their customer service quality and reduce the number of customer service calls.