Number System:- It is method of representing numbers on the numberline with the help of set of Rules and symbol.

Types of Number system

1. Binary
2. Decimal
3. Octal
4. Hexadecimal

Decimal Number System

- Base value 10
- Can go from 0–9 (10 digits)
- eg:- 10285

$(1\times10^4)+(0\times10^3)+(2\times10^2)+(8\times10^1)+(5\times10^0)$

Binary Number System

- Base value 2
- It uses 2 digits (0 and 1)
- eg:- 0, 1, 10, 11, 100, 101, 110, 111, 1000, and 1001.

(0 to 9 in Binary)

Octal Number System

- Base value is 8.
- It uses 8 digits from 0-7.
- eg:- $(135)_{10}$ is $(207)_8$
  $(215)_{10}$ is $(327)_8$

Hexa decimal Number System

- Base value is 16.
- Digits from 0-9 are taken and for 10-15 is represented as A-F.
- Eg:- $(255)_{10}$ can be written as $(FF)_{16}$ and $(1096)_{10}$ as $(448)_{16}$.

Conversion of Number System

Binary to Decimal

Base-2 to Base-10

$(1\ 0\ 1\ 0\ 1\ 0)_2 \longrightarrow (?)_{10}$

$2^5 + 2^4 + 2^3 + 2^1 + 2^0$

$32 + 0 + 8 + 2 + 0 = 42$

(Method - I)

Power of 2's

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	...
1	2	4	8	16	32	64	128	256	512	1024	...

Method - II

$(1\ 0\ 1\ 0\ 1\ 0)_2 \longrightarrow (?)_{10}$

$\boxed{0} \times \boxed{2} + 1$ → Initial value (0)

Base value (2)

$= 1 \times 2 + 0$

$= 2 \times 2 + 1$

$= 5 \times 2 + \boxed{0}$ → Number (Binary)

$= 10 \times 2 + 1$

$= 21 \times 2 + 0$

$= 42$

$(1\ 2\ 3)_{10} \longrightarrow (?)_{10}$

$0 \times 10 + \boxed{1}$ → Number (Decimal)

$= 1 \times 10 + 2$

$= 12 \times 10 + 3$

$= \underline{\underline{123}}$

$(1\ 2\ 3)_{10} \xrightarrow{\boxed{\text{Reverse}}} (3\ 2\ 1)_{10}$

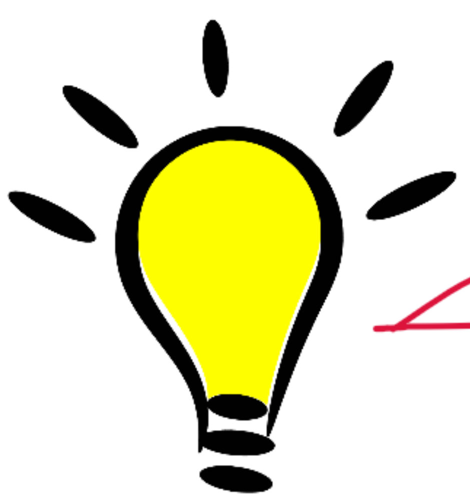
$0 \times 10 + 3$

$= 3 \times 10 + 2$

$= 32 \times 10 + 1$

$= \underline{\underline{321}}$

We can reverse numbers like this



Converting Base-10 to 2

$(42)_{10} \rightarrow (?)_2 \rightarrow (101010)_2$

Method - I

2	42	0
2	21	1
2	10	0
2	5	1
2	2	0
2	1	1
	0	

Method - II

$(42)_{10} \rightarrow$ Can be split into Sum of 2^n

$42 = 32 + 8 + 2$

$32 \rightarrow 2^5$, $8 \rightarrow 2^3$, $2 \rightarrow 2^1$

$$\frac{1}{5}\ \frac{0}{4}\ \frac{1}{3}\ \frac{0}{2}\ \frac{1}{1}\ \frac{0}{0}$$

Decimal Range for n-bit binary number

How many n-bit binary numbers are possible? $\rightarrow 2^n$

$2^2 = 4$, $2^3 = 8$ numbers possible.

1st	0th
0	0
0	1
1	0
1	1

for $2^2 = 4$ number possible

for 3 bit binary, 8 numbers are possible.

2nd	1st	0th
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

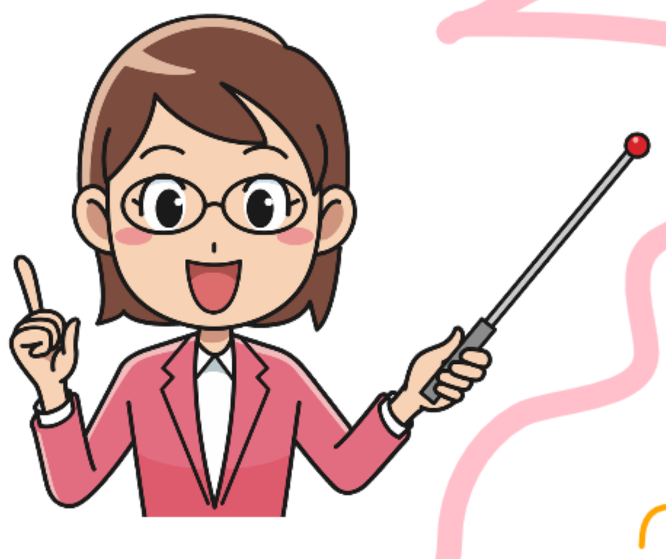
Smallest n-bit binary number

$\hookrightarrow \boxed{(0)_{10}}$

pos.	(n-1)th	(n-2)th	. . .	ith	. . .	2nd	1st	0th
bit	0	0	.	0		0	0	0
wgt.	2^{n-1}	2^{n-2}	. . .	2^i	. .	2^2	2^1	2^0

$\Downarrow$

(largest n-bit binary number)



largest n-bit binary number :-

$$\underbrace{2^0 + 2^1 + 2^2 + \cdots + 2^{n-1}}$$

This forms a gp.

Here, $a = 1$, $r = 2$, $\# = n$

$$\frac{a(r^{\#} - 1)}{r - 1} = \frac{1(2^n - 1)}{2 - 1}$$

$$= \boxed{(2^n - 1)_{10}}$$

largest n bit binary

We know the limits of n-bit binary number which is.

$$2^n \left\{ \begin{array}{l} 0\,0\,0\,0 \cdots 0 \rightarrow (0)_{10} \\ \vdots \\ 1\,1\,1\,1 \cdots 1 \rightarrow (2^n-1) \end{array} \right.$$

$$\xleftarrow{\quad} n \xrightarrow{\quad}$$

least numbers of bits to represent a decimal number.

To deduce the least no of bits used, we know that

$$0 \leq x \leq 2^n - 1$$

$$x \leq 2^n - 1$$

$$2^n \geq x + 1$$

taking $\log_2$ on both sides we get,

$$\log_2 2^n \geq \log_2 (x+1)$$

$$\boxed{n \geq \log_2 (x+1)}$$

formula for least bits to represent decimal number.

for eg:- $x = 12$

$$n \geq \log(12+1)$$

$$n \geq \log_2(13)$$

$$n \geq \underbrace{3.....}_{\downarrow}$$

ceil $= \underline{\underline{4}}$ (least bits)

Complementary Number System