

Informed Search Algorithms

So far we have talked about the uninformed search algorithms which looked through search space for all possible solutions of the problem without having any additional knowledge about search space. But informed search algorithm contains an array of knowledge such as how far we are from the goal, path cost, how to reach to goal node, etc. This knowledge help agents to explore less to the search space and find more efficiently the goal node.

The informed search algorithm is more useful for large search space. Informed search algorithm uses the idea of heuristic, so it is also called Heuristic search.

Heuristics function: Heuristic is a function which is used in Informed Search, and it finds the most promising path. It takes the current state of the agent as its input and produces the estimation of how close agent is from the goal. The heuristic method, however, might not always give the best solution, but it guaranteed to find a good solution in reasonable time. Heuristic function estimates how close a state is to the goal. It is represented by $h(n)$, and it calculates the cost of an optimal path between the pair of states. The value of the heuristic function is always positive.

Admissibility of the heuristic function is given as:

1. $h(n) \leq h^*(n)$

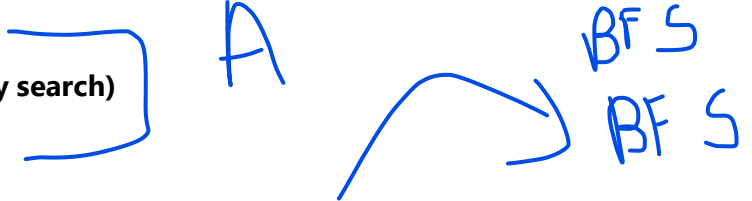
Here $h(n)$ is heuristic cost, and $h^*(n)$ is the estimated cost. Hence heuristic cost should be less than or equal to the estimated cost.

Pure Heuristic Search:

Pure heuristic search is the simplest form of heuristic search algorithms. It expands nodes based on their heuristic value $h(n)$. It maintains two lists, OPEN and CLOSED list. In the CLOSED list, it places those nodes which have already expanded and in the OPEN list, it places nodes which have yet not been expanded.

On each iteration, each node n with the lowest heuristic value is expanded and generates all its successors and n is placed to the closed list. The algorithm continues unit a goal state is found.

-
- **Best First Search Algorithm(Greedy search)**
- **A* Search Algorithm**



1.) Best-first Search Algorithm (Greedy Search):

Greedy best-first search algorithm always selects the path which appears best at that moment. It is the combination of depth-first search and breadth-first search algorithms. It uses the heuristic function and search. Best-first search allows us to take the advantages of both algorithms. With the help of best-first search, at each step, we can choose the most promising node. In the best first search algorithm, we expand the node which is closest to the goal node and the closest cost is estimated by heuristic function, i.e.

1. $f(n) = g(n) + h(n)$

Where, $h(n)$ = estimated cost from node n to the goal.

The greedy best first algorithm is implemented by the priority queue.

Best first search algorithm:

- **Step 1:** Place the starting node into the OPEN list.
- **Step 2:** If the OPEN list is empty, Stop and return failure.
- **Step 3:** Remove the node n , from the OPEN list which has the lowest value of $h(n)$, and places it in the CLOSED list.
- **Step 4:** Expand the node n , and generate the successors of node n .
- **Step 5:** Check each successor of node n , and find whether any node is a goal node or not. If any successor node is goal node, then return success and terminate the search, else proceed to Step 6.
- **Step 6:** For each successor node, algorithm checks for evaluation function $f(n)$, and then check if the node has been in either OPEN or CLOSED list. If the node has not been in both list, then add it to the OPEN list.
- **Step 7:** Return to Step 2.

Advantages:

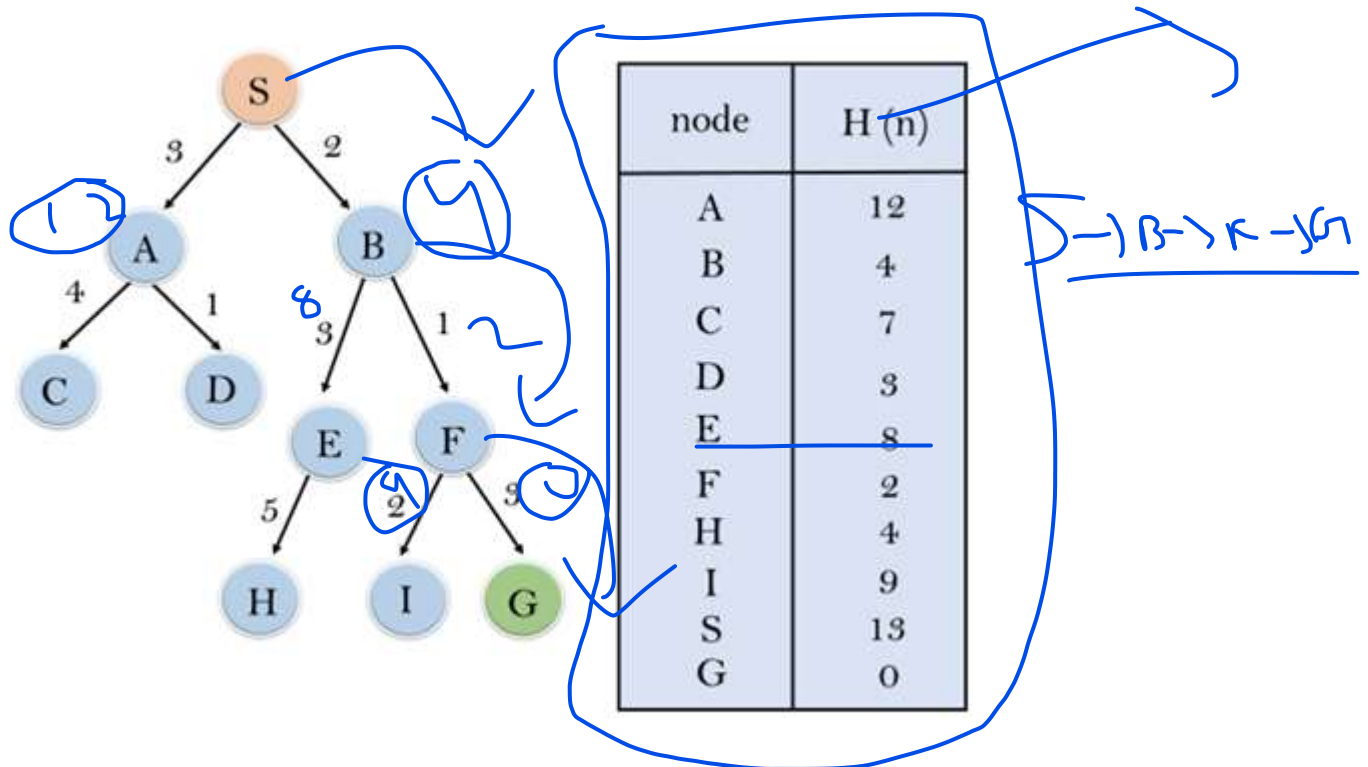
- Best first search can switch between BFS and DFS by gaining the advantages of both the algorithms.
- This algorithm is more efficient than BFS and DFS algorithms.

Disadvantages:

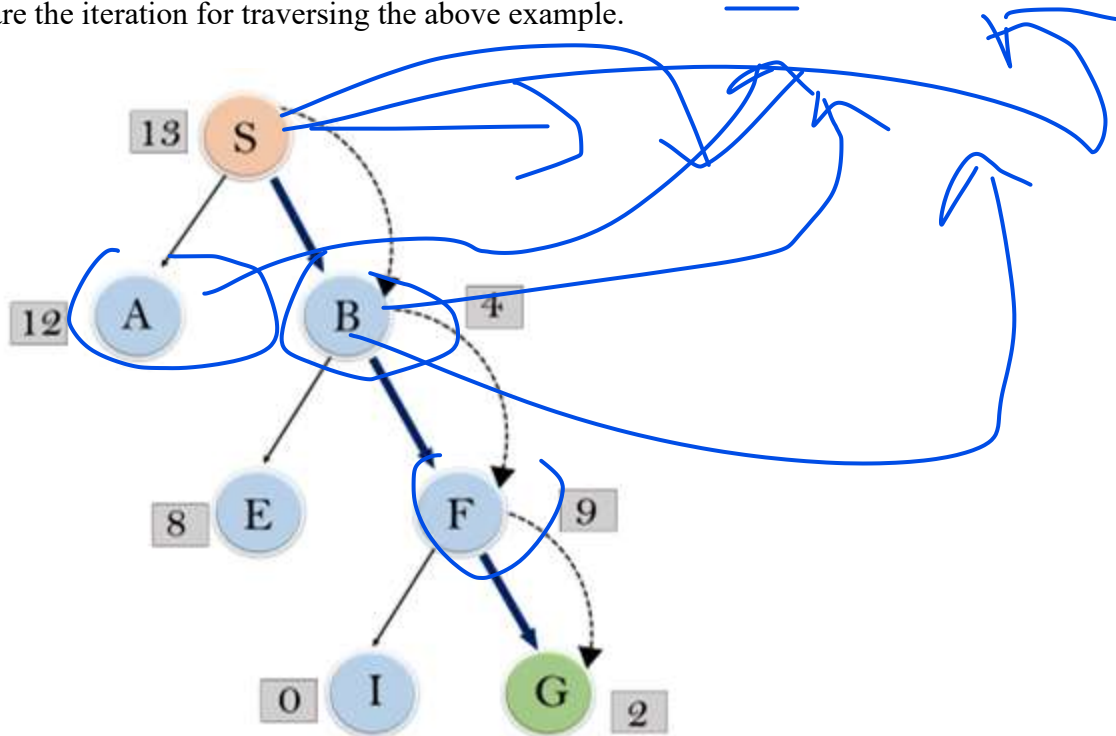
- It can behave as an unguided depth-first search in the worst case scenario.
- It can get stuck in a loop as DFS.
- This algorithm is not optimal.

Example:

Consider the below search problem, and we will traverse it using greedy best-first search. At each iteration, each node is expanded using evaluation function $f(n)=h(n)$, which is given in the below table.



In this search example, we are using two lists which are **OPEN** and **CLOSED** Lists. Following are the iteration for traversing the above example.



Expand the nodes of S and put in the CLOSED list

Initialization: Open [A, B], Closed [S]

Iteration 1: Open [A], Closed [S, B]

Iteration 2: Open [E, F, A], Closed [S, B]
: Open [E, A], Closed [S, B, F]

Iteration 3: Open [I, G, E, A], Closed [S, B, F]
: Open [I, E, A], Closed [S, B, F, G]

Hence the final solution path will be: **S-----> B----->F-----> G**

Time Complexity: The worst case time complexity of Greedy best first search is $O(b^m)$.

Space Complexity: The worst case space complexity of Greedy best first search is $O(b^m)$.
Where, m is the maximum depth of the search space.

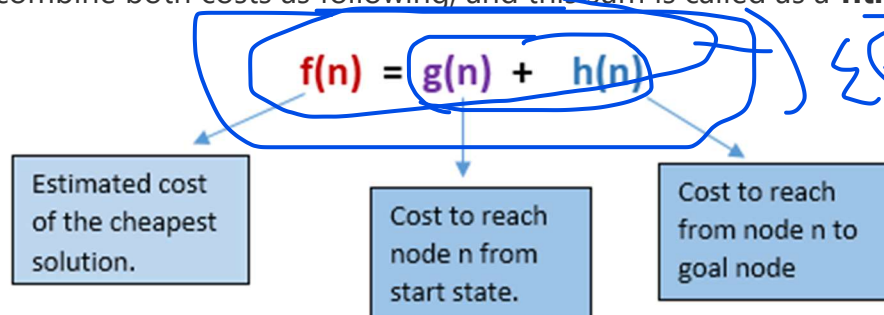
Complete: Greedy best-first search is also incomplete, even if the given state space is finite.

Optimal: Greedy best first search algorithm is not optimal.

2.) A* Search Algorithm:

A* search is the most commonly known form of best-first search. It uses heuristic function $h(n)$, and cost to reach the node n from the start state $g(n)$. It has combined features of UCS and greedy best-first search, by which it solve the problem efficiently. A* search algorithm finds the shortest path through the search space using the heuristic function. This search algorithm expands less search tree and provides optimal result faster. A* algorithm is similar to UCS except that it uses $g(n)+h(n)$ instead of $g(n)$.

In A* search algorithm, we use search heuristic as well as the cost to reach the node. Hence we can combine both costs as following, and this sum is called as a **fitness number**.



At each point in the search space, only those nodes are expanded which have the lowest value of $f(n)$, and the algorithm terminates when the goal node is found.

Algorithm of A* search:

Step1: Place the starting node in the OPEN list.

Step 2: Check if the OPEN list is empty or not, if the list is empty then return failure and stops.

Step 3: Select the node from the OPEN list which has the smallest value of evaluation function ($g+h$), if node n is goal node then return success and stop, otherwise

Step 4: Expand node n and generate all of its successors, and put n into the closed list. For each successor n' , check whether n' is already in the OPEN or CLOSED list, if not then compute evaluation function for n' and place into Open list.

Step 5: Else if node n' is already in OPEN and CLOSED, then it should be attached to the back pointer which reflects the lowest $g(n')$ value.

Step 6: Return to **Step 2**.

Advantages:

- A* search algorithm is the best algorithm than other search algorithms.
- A* search algorithm is optimal and complete.
- This algorithm can solve very complex problems.

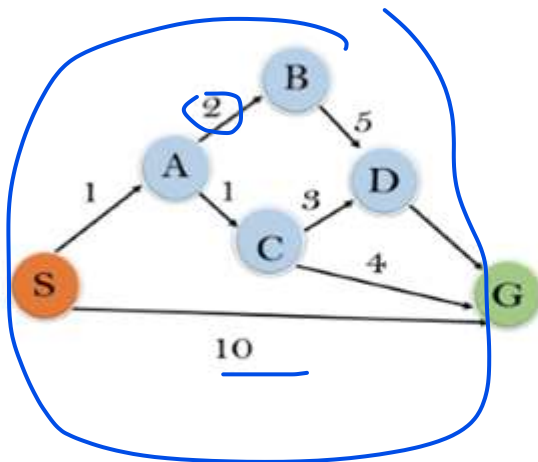
Disadvantages:

- It does not always produce the shortest path as it mostly based on heuristics and approximation.
- A* search algorithm has some complexity issues.
- The main drawback of A* is memory requirement as it keeps all generated nodes in the memory, so it is not practical for various large-scale problems.

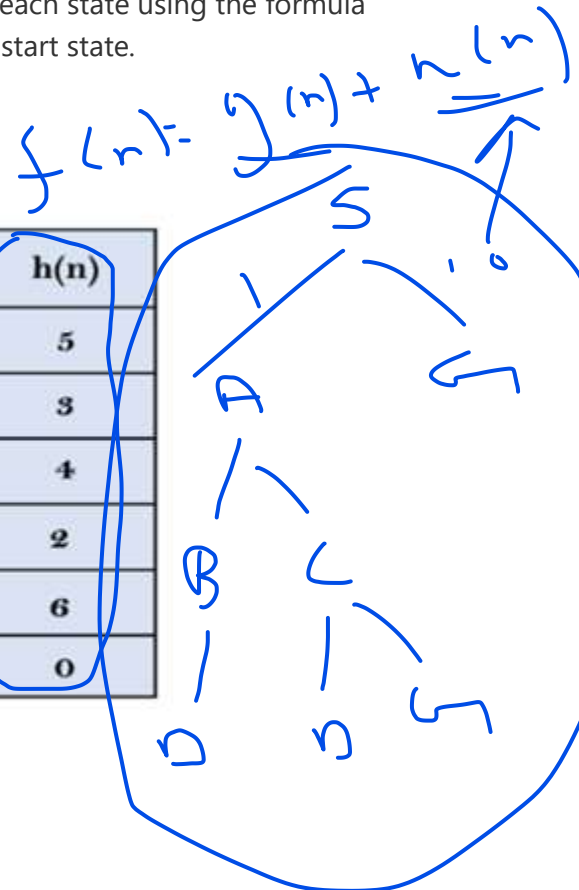
Example:

In this example, we will traverse the given graph using the A* algorithm. The heuristic value of all states is given in the below table so we will calculate the $f(n)$ of each state using the formula $f(n) = g(n) + h(n)$, where $g(n)$ is the cost to reach any node from start state.

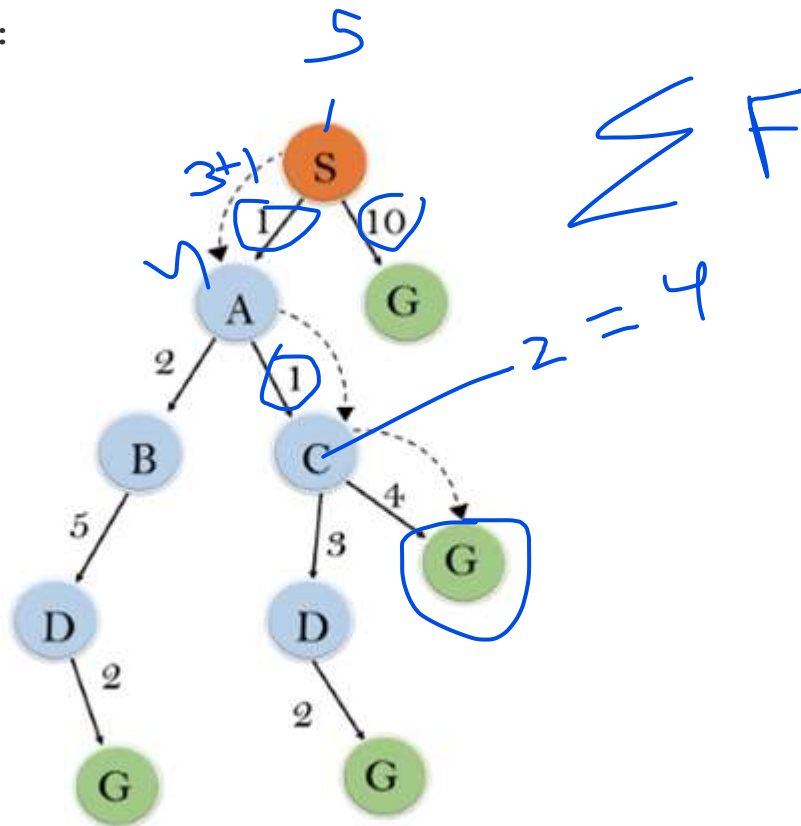
Here we will use OPEN and CLOSED list.



State	$h(n)$
S	5
A	3
B	4
C	2
D	6
G	0



Solution:



Initialization: $\{(S, 5)\}$

Iteration1: $\{(S \rightarrow A, 4), (S \rightarrow G, 10)\}$

Iteration2: $\{(S \rightarrow A \rightarrow C, 4), (S \rightarrow A \rightarrow B, 7), (S \rightarrow G, 10)\}$

Iteration3: $\{(S \rightarrow A \rightarrow C \rightarrow G, 6), (S \rightarrow A \rightarrow C \rightarrow D, 11), (S \rightarrow A \rightarrow B, 7), (S \rightarrow G, 10)\}$

Iteration 4 will give the final result, as $S \rightarrow A \rightarrow C \rightarrow G$ it provides the optimal path with cost 6.

Points to remember:

- A* algorithm returns the path which occurred first, and it does not search for all remaining paths.
- The efficiency of A* algorithm depends on the quality of heuristic.
- A* algorithm expands all nodes which satisfy the condition $f(n) < \infty$

Complete: A* algorithm is complete as long as:

- Branching factor is finite.
- Cost at every action is fixed.

Optimal: A* search algorithm is optimal if it follows below two conditions:

- **Admissible:** the first condition requires for optimality is that $h(n)$ should be an admissible heuristic for A* tree search. An admissible heuristic is optimistic in nature.
- **Consistency:** Second required condition is consistency for only A* graph-search.

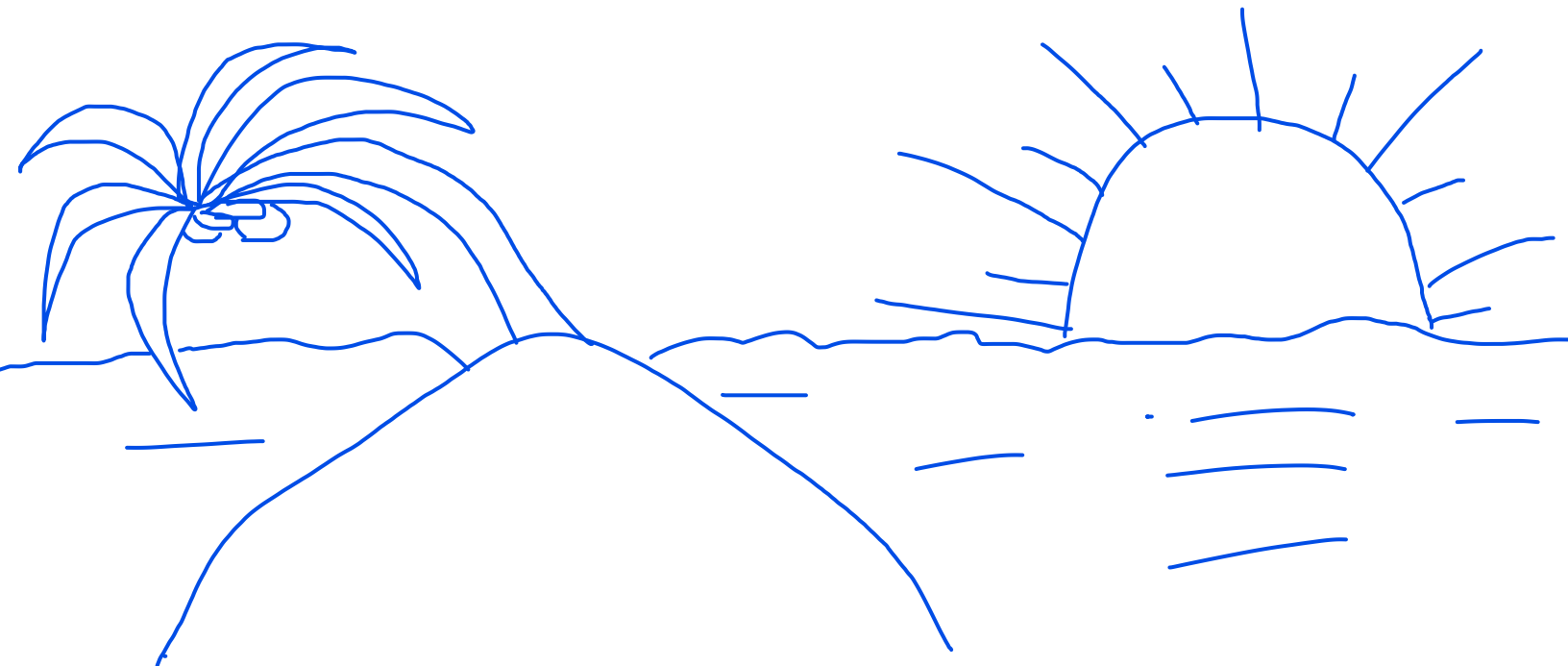
If the heuristic function is admissible, then A* tree search will always find the least cost path.

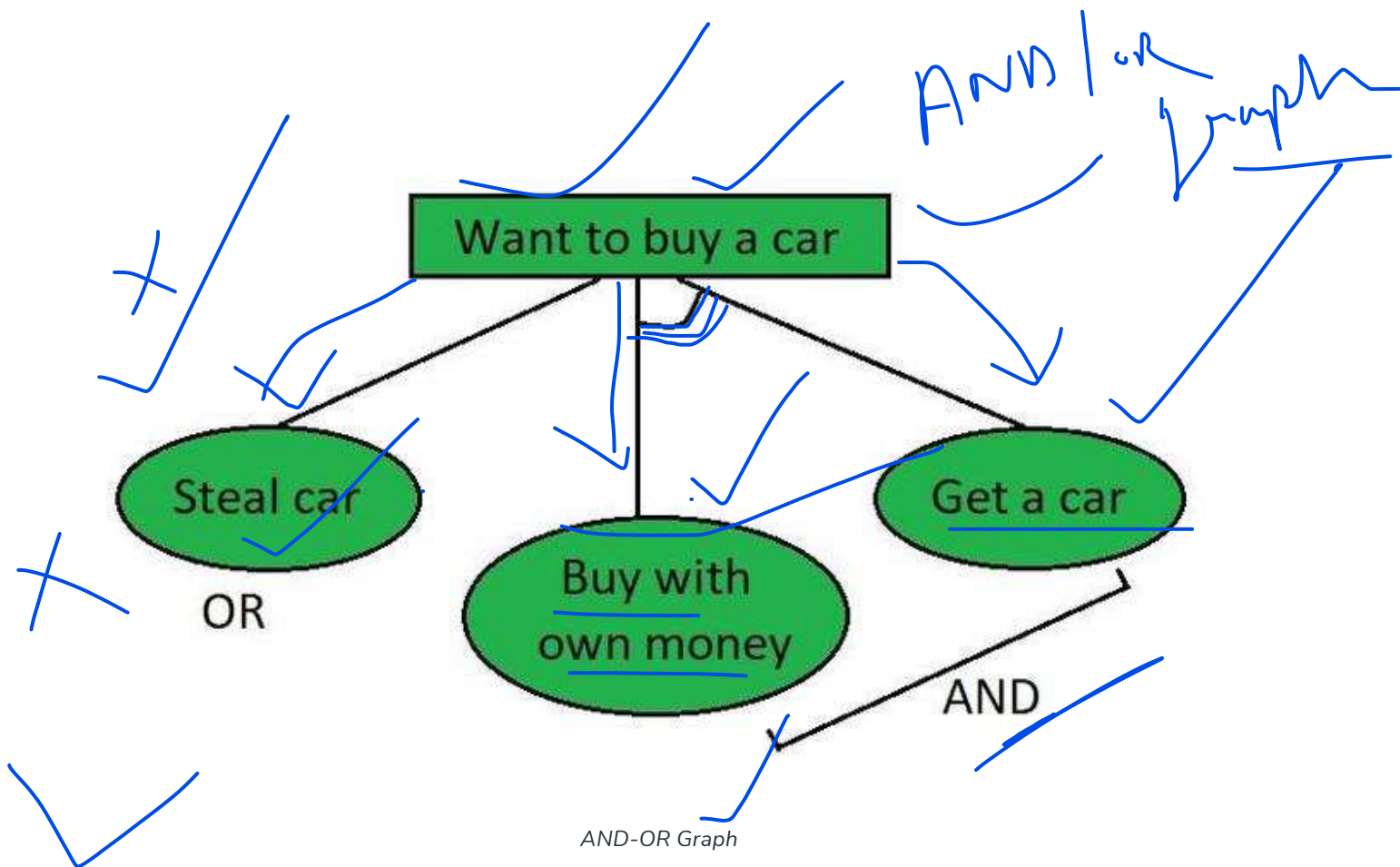
Time Complexity: The time complexity of A* search algorithm depends on heuristic function, and the number of nodes expanded is exponential to the depth of solution d . So the time complexity is $O(b^d)$, where b is the branching factor.

Space Complexity: The space complexity of A* search algorithm is $O(b^d)$

AO* algorithm – Artificial intelligence

Best-first search is what the AO* algorithm does. The AO* method **divides** any given difficult **problem into a smaller group** of problems that are then resolved **using the AND-OR graph concept**. AND OR graphs are specialized graphs that are used in problems that can be divided into smaller problems. The AND side of the graph represents a set of tasks that must be completed to achieve the main goal, while the OR side of the graph represents different methods for accomplishing the same main goal.





In the above figure, the **buying of a car** may be broken down into smaller problems or tasks that can be accomplished **to achieve the main goal** in the above figure, which is an example of a simple AND-OR graph. The other task is to either steal a car that will help us accomplish the main goal or use your own money to purchase a car that will accomplish the main goal. The AND symbol is used to indicate the AND part of the graphs, which refers to the need that all subproblems containing the AND to be resolved before the preceding node or issue may be finished.

The start state and the target state are already known in the knowledge-based search strategy known as the **AO* algorithm**, and the best path is identified by heuristics. The informed search technique considerably reduces the algorithm's **time complexity**. The AO* algorithm is far more effective in searching AND-OR trees **than** the A* algorithm.

Working of AO* algorithm:

The evaluation function in AO* looks like this:

$$f(n) = g(n) + h(n)$$

$$f(n) = \text{Actual cost} + \text{Estimated cost}$$

here,

$f(n)$ = The actual cost of traversal.

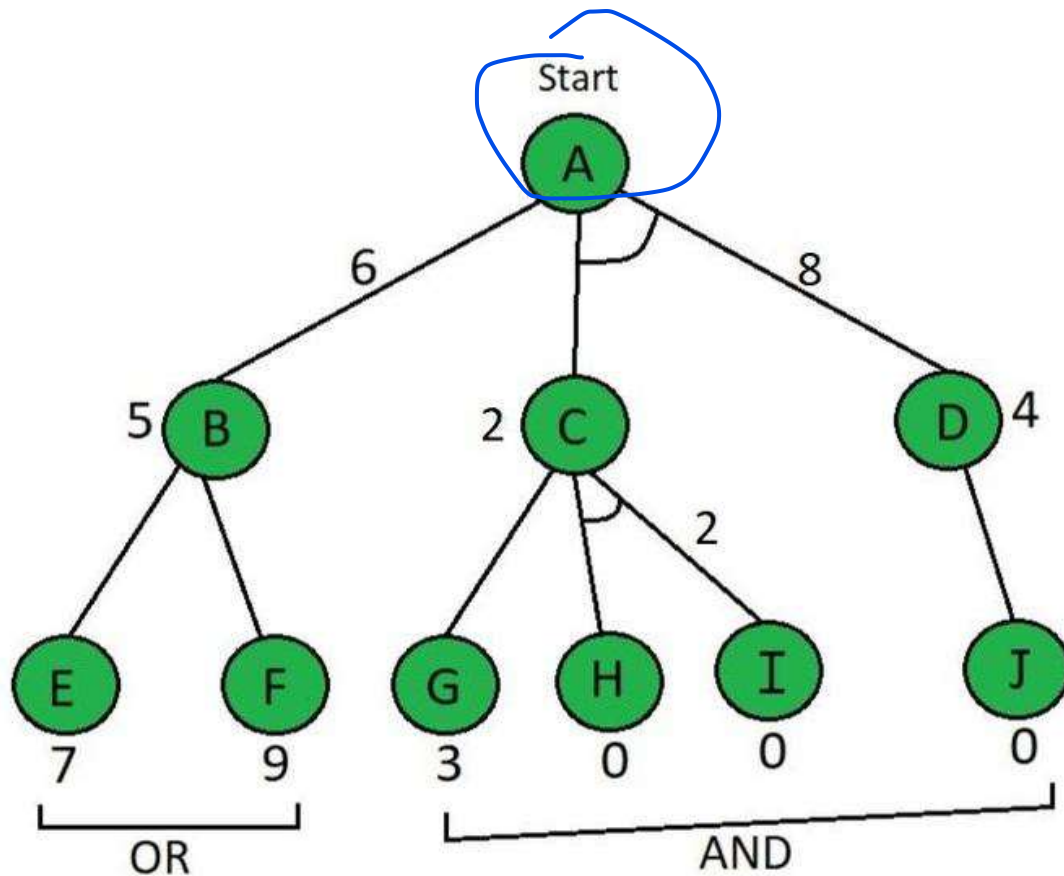
$g(n)$ = the cost from the initial node to the current node.

$h(n)$ = estimated cost from the current node to the goal state.

Difference between the A* Algorithm and AO* algorithm

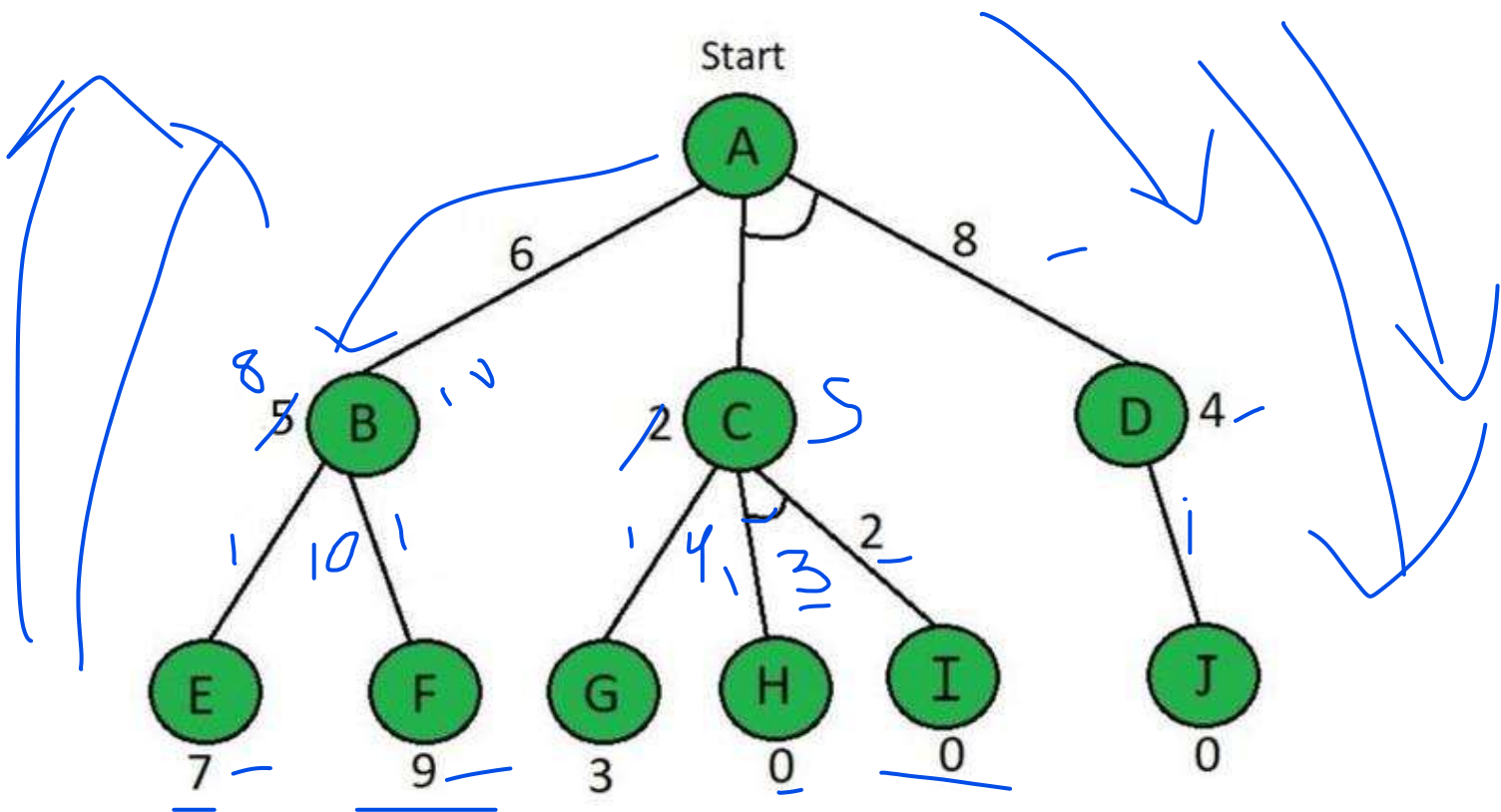
- A* algorithm and AO* algorithm both works on the **best first search**.
- They are both **informed search** and works on given heuristics values.
- **A*** always **gives** the **optimal solution** but AO* doesn't guarantee to give the optimal solution.
- Once AO* got a solution **doesn't explore** all possible paths but A* explores all paths.
- When compared to the A* algorithm, the AO* algorithm uses **less memory**.
- opposite to the A* algorithm, the AO* algorithm cannot go into an endless **loop**.

Example:



Here in the above example below the Node which is given is the heuristic value i.e **$h(n)$** . Edge length is considered as **1**.

Step 1



AO* Algorithm (Step-1)

With help of **$f(n) = g(n) + h(n)$** evaluation function,
Start from node A,

$$\underline{f(A \rightarrow B)} = g(B) + h(B)$$

$$= 1 + 5$$

default for path cost

$$= \underline{6}$$

.....here $g(n)=1$ is taken by

$$f(A \rightarrow C+D) = g(c) + h(c) + g(d) + h(d)$$

$$= 1 + 2 + 1 + 4$$

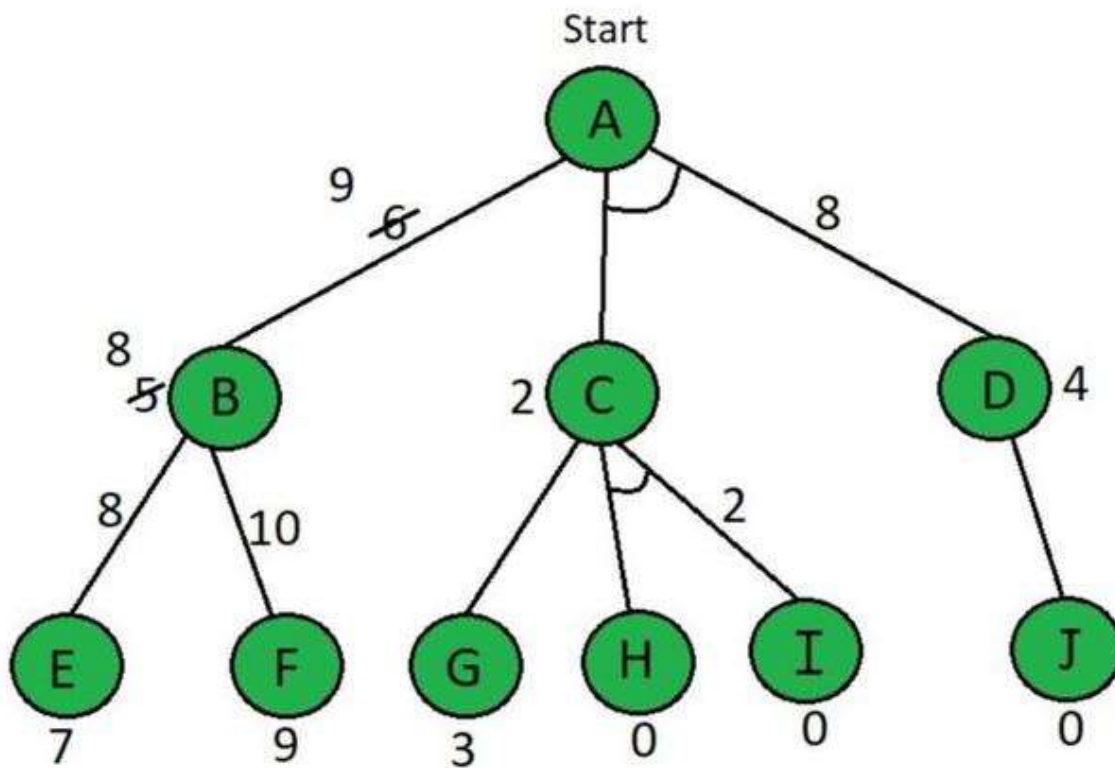
because they are in AND

$$= 8$$

.....here we have added **C & D**

So, by calculation **$A \rightarrow B$** path is chosen which is the minimum path,
i.e **$f(A \rightarrow B)$**

Step 2



AO* Algorithm (Step-2)

According to the answer of step 1, explore node B
Here the value of E & F are calculated as follows,

$$f(B \rightarrow E) = g(e) + h(e)$$

$$f(B \rightarrow E) = 1 + 7$$

$$= 8$$

$$f(B \rightarrow f) = g(f) + h(f)$$

$$f(B \rightarrow f) = 1 + 9$$

$$= 10$$

So, by above calculation B \rightarrow E path is chosen which is minimum path,
i.e $f(B \rightarrow E)$

By comparing $f(A \rightarrow B)$ & $f(A \rightarrow C+D)$
 $f(A \rightarrow C+D)$ is shown to be **smaller**. i.e $8 < 9$
 Now explore $f(A \rightarrow C+D)$

So, the current node is C

$$f(C \rightarrow G) = g(g) + h(g)$$

$$\begin{aligned} f(C \rightarrow G) &= 1 + 3 \\ &= 4 \end{aligned}$$

$$f(C \rightarrow H+I) = g(h) + h(h) + g(i) + h(i)$$

$$f(C \rightarrow H+I) = 1 + 0 + 1 + 0$$

because they are in AND

$$= 2$$

.....here we have added H & I

f(C→H+I) is selected as the path with the lowest cost and the heuristic is also left unchanged because it matches the actual cost. Paths H & I are solved because the heuristic for those paths is **0**, but Path **A→D** needs to be calculated because it has an **AND**.

$$f(D \rightarrow J) = g(j) + h(j)$$

$$\begin{aligned} f(D \rightarrow J) &= 1 + 0 \\ &= 1 \end{aligned}$$

the heuristic of node D needs to be updated to 1.

$$f(A \rightarrow C+D) = g(c) + h(c) + g(d) + h(d)$$

$$= 1 + 2 + 1 + 1$$

$$= 5$$

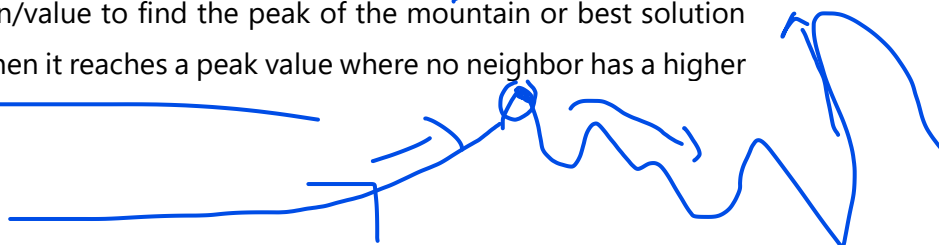
as we can see that path **f(A→C+D)** is get solved and this **tree has become a solved tree** now.

In simple words, the main flow of this algorithm is that we have to find **firstly level 1st** heuristic value and **then level 2nd** and after that **update the values** with going **upward** means towards the root node.

In the above tree diagram, we have updated all the values.

Hill Climbing Algorithm in Artificial Intelligence

- Hill climbing algorithm is a local search algorithm which continuously moves in the direction of increasing elevation/value to find the peak of the mountain or best solution to the problem. It terminates when it reaches a peak value where no neighbor has a higher value.



- Hill climbing algorithm is a technique which is used for optimizing the mathematical problems. One of the widely discussed examples of Hill climbing algorithm is Traveling-salesman Problem in which we need to minimize the distance traveled by the salesman.
- It is also called greedy local search as it only looks to its good immediate neighbor state and not beyond that.
- A node of hill climbing algorithm has two components which are state and value.
- Hill Climbing is mostly used when a good heuristic is available.
- In this algorithm, we don't need to maintain and handle the search tree or graph as it only keeps a single current state.
-

Features of Hill Climbing:

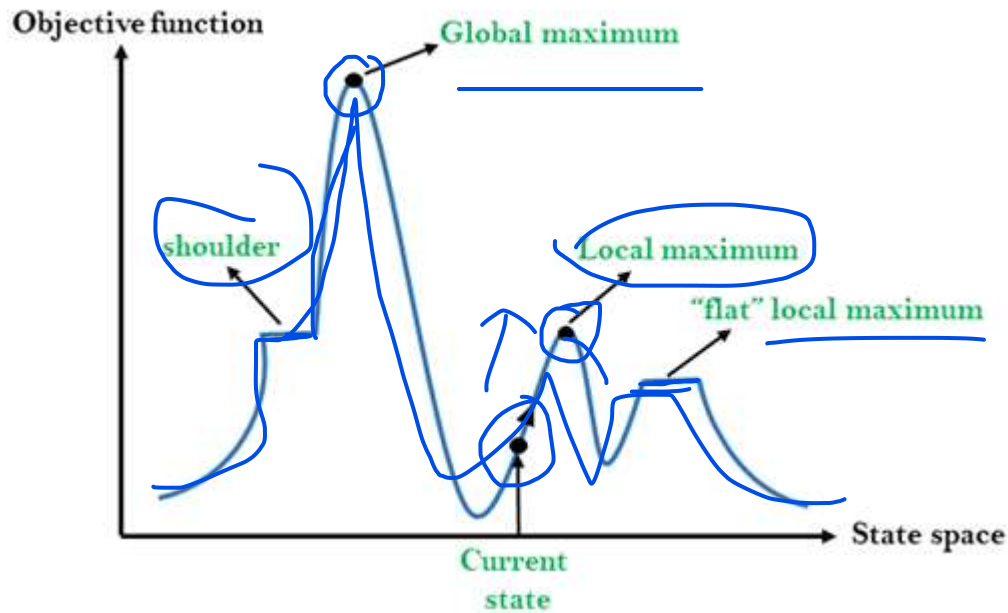
Following are some main features of Hill Climbing Algorithm:

- **Generate and Test variant:** Hill Climbing is the variant of Generate and Test method. The Generate and Test method produce feedback which helps to decide which direction to move in the search space.
- **Greedy approach:** Hill-climbing algorithm search moves in the direction which optimizes the cost.
- **No backtracking:** It does not backtrack the search space, as it does not remember the previous states.

State-space Diagram for Hill Climbing:

The state-space landscape is a graphical representation of the hill-climbing algorithm which is showing a graph between various states of algorithm and Objective function/Cost.

On Y-axis we have taken the function which can be an objective function or cost function, and state-space on the x-axis. If the function on Y-axis is cost then, the goal of search is to find the global minimum and local minimum. If the function of Y-axis is Objective function, then the goal of the search is to find the global maximum and local maximum.



Different regions in the state space landscape:

Local Maximum: Local maximum is a state which is better than its neighbor states, but there is also another state which is higher than it.

Global Maximum: Global maximum is the best possible state of state space landscape. It has the highest value of objective function.

Current state: It is a state in a landscape diagram where an agent is currently present.

Flat local maximum: It is a flat space in the landscape where all the neighbor states of current states have the same value.

Shoulder: It is a plateau region which has an uphill edge.

Types of Hill Climbing Algorithm:

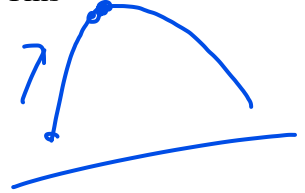
- Simple hill Climbing: ✓
- Steepest-Ascent hill-climbing: ✓
- Stochastic hill Climbing: ✓

1. Simple Hill Climbing:

Simple hill climbing is the simplest way to implement a hill climbing algorithm. **It only evaluates the neighbor node state at a time and selects the first one which**

optimizes current cost and set it as a current state. It only checks its one successor state, and if it finds better than the current state, then move else be in the same state. This algorithm has the following features:

- Less time consuming
- Less optimal solution and the solution is not guaranteed



Algorithm for Simple Hill Climbing:

- **Step 1:** Evaluate the initial state, if it is goal state then return success and Stop.
- **Step 2:** Loop Until a solution is found or there is no new operator left to apply.
- **Step 3:** Select and apply an operator to the current state.
- **Step 4:** Check new state:
 1. If it is goal state, then return success and quit.
 2. Else if it is better than the current state then assign new state as a current state.
 3. Else if not better than the current state, then return to step2.
- **Step 5:** Exit.

2. Steepest-Ascent hill climbing:

The steepest-Ascent algorithm is a variation of simple hill climbing algorithm. This algorithm examines all the neighboring nodes of the current state and selects one neighbor node which is closest to the goal state. This algorithm consumes more time as it searches for multiple neighbors

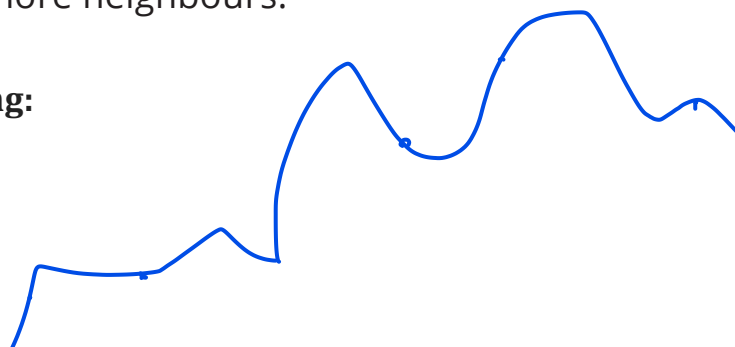
3. Stochastic hill climbing:

Stochastic hill climbing does not examine for all its neighbor before moving. Rather, this search algorithm selects one neighbor node at random and decides whether to choose it as a current state or examine another state.

Steepest-Ascent Hill Climbing

A variant of the straightforward hill-climbing algorithm is the steepest-Ascent algorithm. This method looks at every node that borders the current state and chooses the one that is most near the goal state. This algorithm takes longer since it looks for more neighbours.

Algorithm for Steepest Ascent Hill climbing:

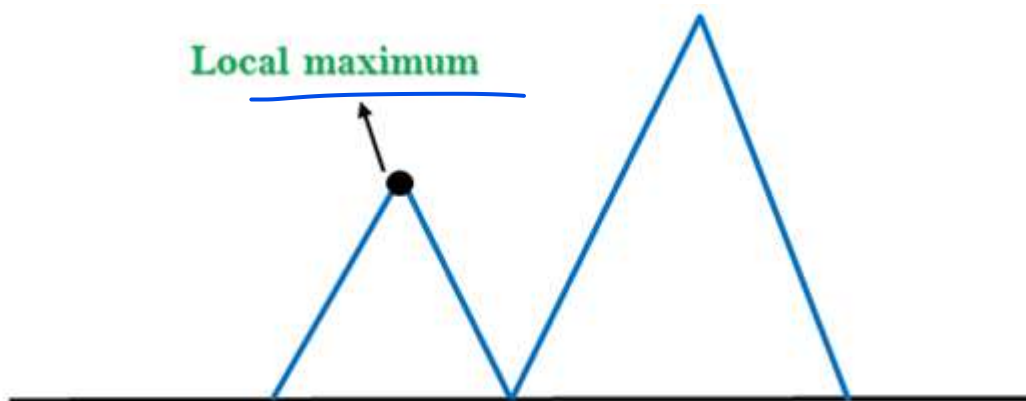


- Analyze the starting situation. Stop and return success if it's a goal state. If not, the initial state should be set as the current state.
- Follow these instructions again and again until a solution is found, or the situation stays the same.
- Choose a state that hasn't yet been used to modify the existing state.
- Create a new "best state" that is initially equivalent to the existing state and then apply it to create the new state.
- Execute these to assess the new state.
- Stop and return success if the current state is a goal state.
- If it is superior to the best state, make it the best state; otherwise, keep going by adding another new state to the loop.
- Set the ideal situation as the current situation.
- Exit

Problems in Hill Climbing Algorithm:

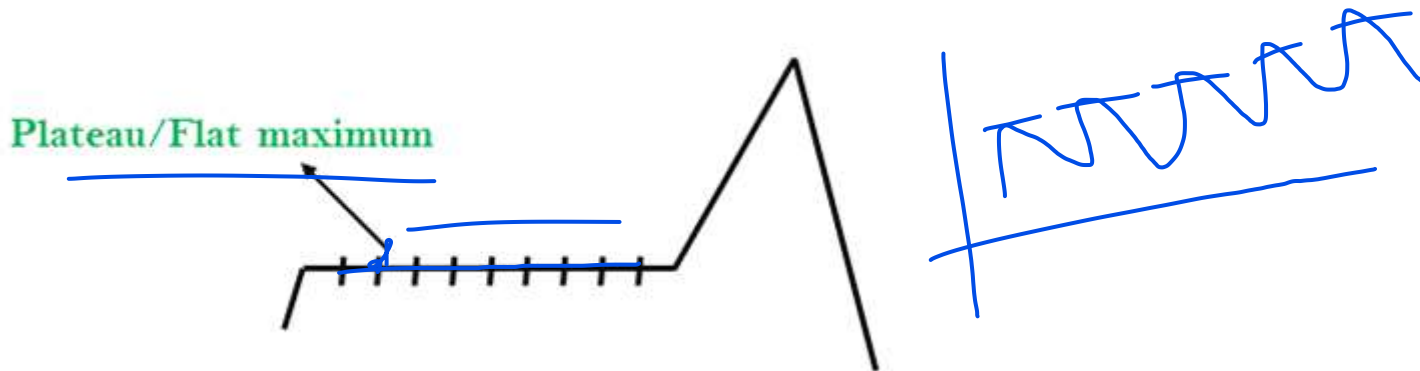
1. Local Maximum: A local maximum is a peak state in the landscape which is better than each of its neighboring states, but there is another state also present which is higher than the local maximum.

Solution: Backtracking technique can be a solution of the local maximum in state space landscape. Create a list of the promising path so that the algorithm can backtrack the search space and explore other paths as well.



2. Plateau: A plateau is the flat area of the search space in which all the neighbor states of the current state contains the same value, because of this algorithm does not find any best direction to move. A hill-climbing search might be lost in the plateau area.

Solution: The solution for the plateau is to take big steps or very little steps while searching, to solve the problem. Randomly select a state which is far away from the current state so it is possible that the algorithm could find non-plateau region.



3. Ridges: Any point on a ridge can appear as a peak since all directions of movement are downhill. As a result, the algorithm terminates in this condition.

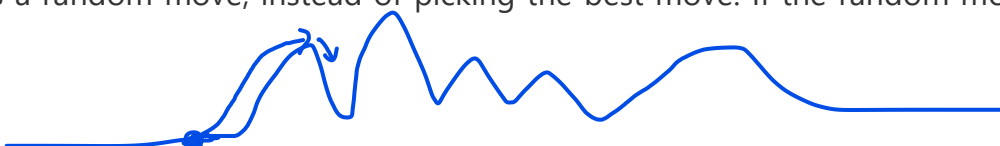
To get over a Ridge: follow two or more rules before being tested. It suggests acting simultaneously in numerous directions.

Solution: With the use of bidirectional search, or by moving in different directions, we can improve this problem.

Simulated Annealing:

A hill-climbing algorithm which never makes a move towards a lower value guaranteed to be incomplete because it can get stuck on a local maximum. And if algorithm applies a random walk, by moving a successor, then it may complete but not efficient. **Simulated Annealing** is an algorithm which yields both efficiency and completeness.

In mechanical term **Annealing** is a process of hardening a metal or glass to a high temperature then cooling gradually, so this allows the metal to reach a low-energy crystalline state. The same process is used in simulated annealing in which the algorithm picks a random move, instead of picking the best move. If the random move improves



the state, then it follows the same path. Otherwise, the algorithm follows the path which has a probability of less than 1 or it moves downhill and chooses another path.