Example 1: Representing Knowledge in FOL

Question:

Convert the following English statements into First-Order Logic:

1. "All humans are mortal."
2. "Socrates is a human."
3. "Therefore, Socrates is mortal."

Solution:

1. **∀x (Human(x) → Mortal(x))** (For all x, if x is a human, then x is mortal.)
2. **Human(Socrates)** (Socrates is a human.)
3. **Mortal(Socrates)** (By applying modus ponens, Socrates is mortal.)

This follows the standard logic of **syllogism**, where we derive a conclusion based on two premises.

Example 2: Family Relationships

Question:

Express the following relationships using FOL:

- "Every parent loves their child."
- "John is the father of Mary."
- "Who does John love?"

Solution:

1. **∀x ∀y (Parent(x, y) → Loves(x, y))**
   (For all x and y, if x is a parent of y, then x loves y.)
2. **Father(John, Mary) → Parent(John, Mary)**
   (If John is Mary's father, then John is her parent.)
3. **Parent(John, Mary)** (From (2))
4. **Loves(John, Mary)** (From (1) and (3), John loves Mary.)

Thus, **John loves Mary** based on the given knowledge.

Example 3: AI-Based Expert System

Question:

Consider an AI expert system for medical diagnosis. Given:

- "If a person has a fever and a cough, they may have the flu."
- "Alice has a fever and a cough."
- "Does Alice have the flu?"

Solution:

1. **∀x (HasFever(x) ∧ HasCough(x) → HasFlu(x))**
   (If x has a fever and a cough, then x has the flu.)
2. **HasFever(Alice) ∧ HasCough(Alice)** (Given fact)
3. **HasFlu(Alice)** (Applying modus ponens)

Thus, **Alice has the flu** based on the given conditions.

Example 4: Animal Classification

Question:

Given the following statements:

- "All birds can fly, except penguins."
- "Tweety is a bird."
- "Penguins cannot fly."
- "Is Tweety able to fly?"

Solution:

1. **∀x (Bird(x) ∧ ¬Penguin(x) → CanFly(x))**
   (For all x, if x is a bird and not a penguin, then x can fly.)
2. **Bird(Tweety)** (Tweety is a bird.)
3. **¬Penguin(Tweety)** (We assume Tweety is not a penguin.)
4. **CanFly(Tweety)** (From (1), (2), and (3), Tweety can fly.)

Thus, **Tweety can fly**.

Example 5: University System

Question:

Convert the following into FOL:

- "Every student takes at least one course."
- "John is a student."
- "Which course does John take?"

Solution:

1. **∀x (Student(x) → ∃y (Course(y) ∧ Takes(x, y)))**
   (For all x, if x is a student, then there exists at least one y such that y is a course and x takes y.)
2. **Student(John)** (Given fact)
3. **∃y (Course(y) ∧ Takes(John, y))** (From (1) and (2), John must take at least one course.)

Thus, **John is taking at least one course**, but we don't have enough information to specify which one.