



# SCHOOL OF ENGINEERING AND TECHNOLOGY



# Unit 3

- **UNIT III:** Reasoning, Introduction to Uncertainty, Bayesian Theory, Bayesian Network, Dempster Shafer Theory. Overview of Planning and its Components. Overview of Learning and basic Techniques. Introduction of Fuzzy Reasoning and Neural Networks.

# Video Link

- <https://www.youtube.com/watch?v=mQ7M-zhiu7U>

- The reasoning is the mental process of deriving logical conclusion and making predictions from available knowledge, facts, and beliefs.
- **"Reasoning is a way to infer facts from existing data."**
- It is a general process of thinking rationally, to find valid conclusions.
- In artificial intelligence, the reasoning is essential so that the machine can also think rationally as a human brain, and can perform like a human.

# Types of Reasoning

In artificial intelligence, reasoning can be divided into the following categories:

- i. Deductive reasoning
- ii. Inductive reasoning
- iii. Abductive reasoning
- iv. Common Sense Reasoning
- v. Monotonic Reasoning
- vi. Non-monotonic Reasoning

**Note: Inductive and deductive reasoning are the forms of propositional logic.**

# Deductive Reasoning

- Deductive reasoning is deducing new information from logically related known information.
- It is the form of valid reasoning, which means the argument's conclusion must be true when the premises are true.
- Deductive reasoning is a type of propositional logic in AI, and it requires various rules and facts.
- It is sometimes referred to as top-down reasoning, and contradictory to inductive reasoning.
- In deductive reasoning, the truth of the premises guarantees the truth of the conclusion.
- Deductive reasoning mostly starts from the general premises to the specific conclusion.

# Deductive Reasoning

## Example

- **Premise-1: All the human eats veggies**
- **Premise-2: Suresh is human.**
- **Conclusion: Suresh eats veggies.**

## *Process for Deductive Reasoning:*





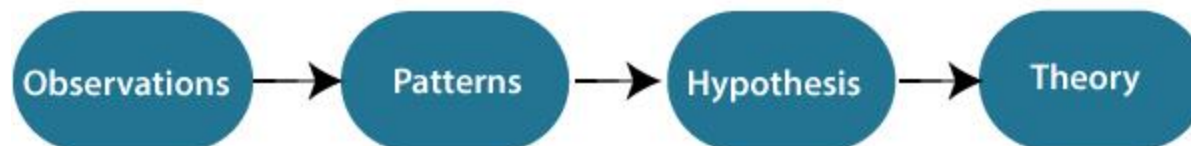
# Inductive Reasoning

- Inductive reasoning is a form of reasoning to arrive at a conclusion using limited sets of facts by the process of generalization. It starts with the series of specific facts or data and reaches to a general statement or conclusion.
- Inductive reasoning is a type of propositional logic, which is also known as cause-effect reasoning or bottom-up reasoning.
- In inductive reasoning, we use historical data or various premises to generate a generic rule, for which premises support the conclusion.
- In inductive reasoning, premises provide probable supports to the conclusion, so the truth of premises does not guarantee the truth of the conclusion.



# Inductive Reasoning

- Example:
- **Premise: All of the pigeons we have seen in the zoo are white.**
- **Conclusion: Therefore, we can expect all the pigeons to be white.**



# Abductive reasoning

Abductive reasoning is a form of logical reasoning which starts with single or multiple observations then seeks to find the most likely explanation or conclusion for the observation.

Abductive reasoning is an extension of deductive reasoning, but in abductive reasoning, the premises do not guarantee the conclusion.

## **Example:**

**Implication:** Cricket ground is wet if it is raining

**Axiom:** Cricket ground is wet.

**Conclusion** It is raining.

# Common Sense Reasoning

- Common sense reasoning is an informal form of reasoning, which can be gained through experiences.
- Common Sense reasoning simulates the human ability to make presumptions about events which occurs on every day.
- It relies on good judgment rather than exact logic and operates on **heuristic knowledge** and **heuristic rules**.
- **Example:**
  - **One person can be at one place at a time.**
  - **If I put my hand in a fire, then it will burn.**
- The above two statements are the examples of common sense reasoning which a human mind can easily understand and assume.

# Monotonic Reasoning

- In monotonic reasoning, once the conclusion is taken, then it will remain the same even if we add some other information to existing information in our knowledge base. In monotonic reasoning, adding knowledge does not decrease the set of prepositions that can be derived.
- To solve monotonic problems, we can derive the valid conclusion from the available facts only, and it will not be affected by new facts.
- Monotonic reasoning is not useful for the real-time systems, as in real time, facts get changed, so we cannot use monotonic reasoning.
- Monotonic reasoning is used in conventional reasoning systems, and a logic-based system is monotonic.
- Any theorem proving is an example of monotonic reasoning.
- **Example:**
- **Earth revolves around the Sun.**
- It is a true fact, and it cannot be changed even if we add another sentence in knowledge base like, "The moon revolves around the earth" Or "Earth is not round," etc.

# Monotonic Reasoning

## Advantages of Monotonic Reasoning:

In monotonic reasoning, each old proof will always remain valid.

If we deduce some facts from available facts, then it will remain valid for always.

## Disadvantages of Monotonic Reasoning:

We cannot represent the real world scenarios using Monotonic reasoning.

Hypothesis knowledge cannot be expressed with monotonic reasoning, which means facts should be true.

Since we can only derive conclusions from the old proofs, so new knowledge from the real world cannot be added.

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# Non-monotonic Reasoning

In Non-monotonic reasoning, some conclusions may be invalidated if we add some more information to our knowledge base.

Logic will be said as non-monotonic if some conclusions can be invalidated by adding more knowledge into our knowledge base.

Non-monotonic reasoning deals with incomplete and uncertain models.

"Human perceptions for various things in daily life, "is a general example of non-monotonic reasoning.

**Example:** Let suppose the knowledge base contains the following knowledge:

**Birds can fly**

**Penguins cannot fly**

**Pitty is a bird**

So from the above sentences, we can conclude that **Pitty can fly**.

However, if we add one another sentence into knowledge base "**Pitty is a penguin**", which concludes "**Pitty cannot fly**", so it invalidates the above conclusion.

# Non-Monotonic Reasoning

- Advantages of Non-monotonic reasoning:
  - For real-world systems such as Robot navigation, we can use non-monotonic reasoning.
  - In Non-monotonic reasoning, we can choose probabilistic facts or can make assumptions.
- Disadvantages of Non-monotonic Reasoning:
  - In non-monotonic reasoning, the old facts may be invalidated by adding new sentences.
  - It cannot be used for theorem **proving**.



# Difference between Inductive and Deductive reasoning

- Deductive reasoning uses available facts, information, or knowledge to deduce a valid conclusion, whereas inductive reasoning involves making a generalization from specific facts, and observations.
- Deductive reasoning uses a top-down approach, whereas inductive reasoning uses a bottom-up approach.
- Deductive reasoning moves from generalized statement to a valid conclusion, whereas Inductive reasoning moves from specific observation to a generalization.
- In deductive reasoning, the conclusions are certain, whereas, in Inductive reasoning, the conclusions are probabilistic.
- Deductive arguments can be valid or invalid, which means if premises are true, the conclusion must be true, whereas inductive argument can be strong or weak, which means conclusion may be false even if premises are true.

# Probabilistic reasoning in Artificial Intelligence

- **Uncertainty:**
  - Knowledge representation using first-order logic and propositional logic with certainty, which means we were sure about the predicates.
  - With this knowledge representation, we might write  $A \rightarrow B$ , which means if A is true then B is true.
  - Consider a situation where we are not sure about whether A is true or not then we cannot express this statement, this situation is called uncertainty.
  - So to represent uncertain knowledge, where we are not sure about the predicates, we need uncertain reasoning or probabilistic reasoning.

# Causes of uncertainty

- Information occurred from unreliable sources.
- Experimental Errors
- Equipment fault
- Temperature variation
- Climate change.

# Probabilistic reasoning

- It is a way of knowledge representation where we apply the concept of probability to indicate the uncertainty in knowledge.
- In probabilistic reasoning, we combine probability theory with logic to handle the uncertainty.
- We use probability in probabilistic reasoning because it provides a way to handle the uncertainty that is the result of someone's laziness and ignorance.
- In the real world, there are lots of scenarios, where the certainty of something is not confirmed:
  - "It will rain today,"
  - "behavior of someone for some situations,"
  - "A match between two teams or two players."
- We can assume that it will happen but not sure about it, so here we use probabilistic reasoning.

# Need of probabilistic reasoning in AI

- When there are unpredictable outcomes.
- When specifications or possibilities of predicates becomes too large to handle.
- When an unknown error occurs during an experiment.
- In probabilistic reasoning, there are two ways to solve problems with uncertain knowledge:
  - **Bayes' rule**
  - **Bayesian Statistics**

# Probability

- Probability can be defined as a chance that an uncertain event will occur.
- It is the numerical measure of the likelihood that an event will occur.
- The value of probability always remains between 0 and 1 that represent ideal uncertainties.
- $0 \leq P(A) \leq 1$ , where  $P(A)$  is the probability of an event  $A$ .
- $P(A) = 0$ , indicates total uncertainty in an event  $A$ .
- $P(A) = 1$ , indicates total certainty in an event  $A$ .
- We can find the probability of an uncertain event by using the below formula.

$$\text{Probability of occurrence} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

# Basic Probability Formula

Let A and B are two events. The probability formulas are listed below:

Probability Range	$0 \leq P(A) \leq 1$
Rule of Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Rule of Complementary Events	$P(A') + P(A) = 1$
Disjoint Events	$P(A \cap B) = 0$
Independent Events	$P(A \cap B) = P(A) \cdot P(B)$
Conditional Probability	$P(A   B) = P(A \cap B) / P(B)$
Bayes Formula	$P(A   B) = P(B   A) \cdot P(A) / P(B)$



# Example :Probability Theory

**Example 1: What is the probability that a card taken from a standard deck, is an Ace?**

**Solution:**

**Total number of cards a standard pack contains = 52**

**Number of Ace cards in a deck of cards = 4**

**So, the number of favourable outcomes = 4**

**Now, by looking at the formula,**

**Probability of selecting an ace from a deck is,**

**$P(\text{Ace}) = (\text{Number of favourable outcomes}) / (\text{Total number of outcomes})$**

# Examples: Probability Theory

**Example 2:** Calculate the probability of getting an odd number if a dice is rolled.

**Solution:**

Sample space (S) = {1, 2, 3, 4, 5, 6}

$$n(S) = 6$$

Let “E” be the event of getting an odd number, E = {1, 3, 5}

$$n(E) = 3$$

So, the Probability of getting an odd number is:

$$P(E) = (\text{Number of favorable outcomes})/(\text{Total number of outcomes})$$

$$= n(E)/n(S)$$

$$= 3/6$$

$$= \frac{1}{2}$$

# Probability Terminologies

- **Event:** Each possible outcome of a variable is called an event.
- **Sample space:** The collection of all possible events is called sample space.
- **Random variables:** Random variables are used to represent the events and objects in the real world.
- **Prior probability:** The prior probability of an event is probability computed before observing new information.
- **Posterior Probability:** The probability that is calculated after all evidence or information has taken into account. It is a combination of prior probability and new information.

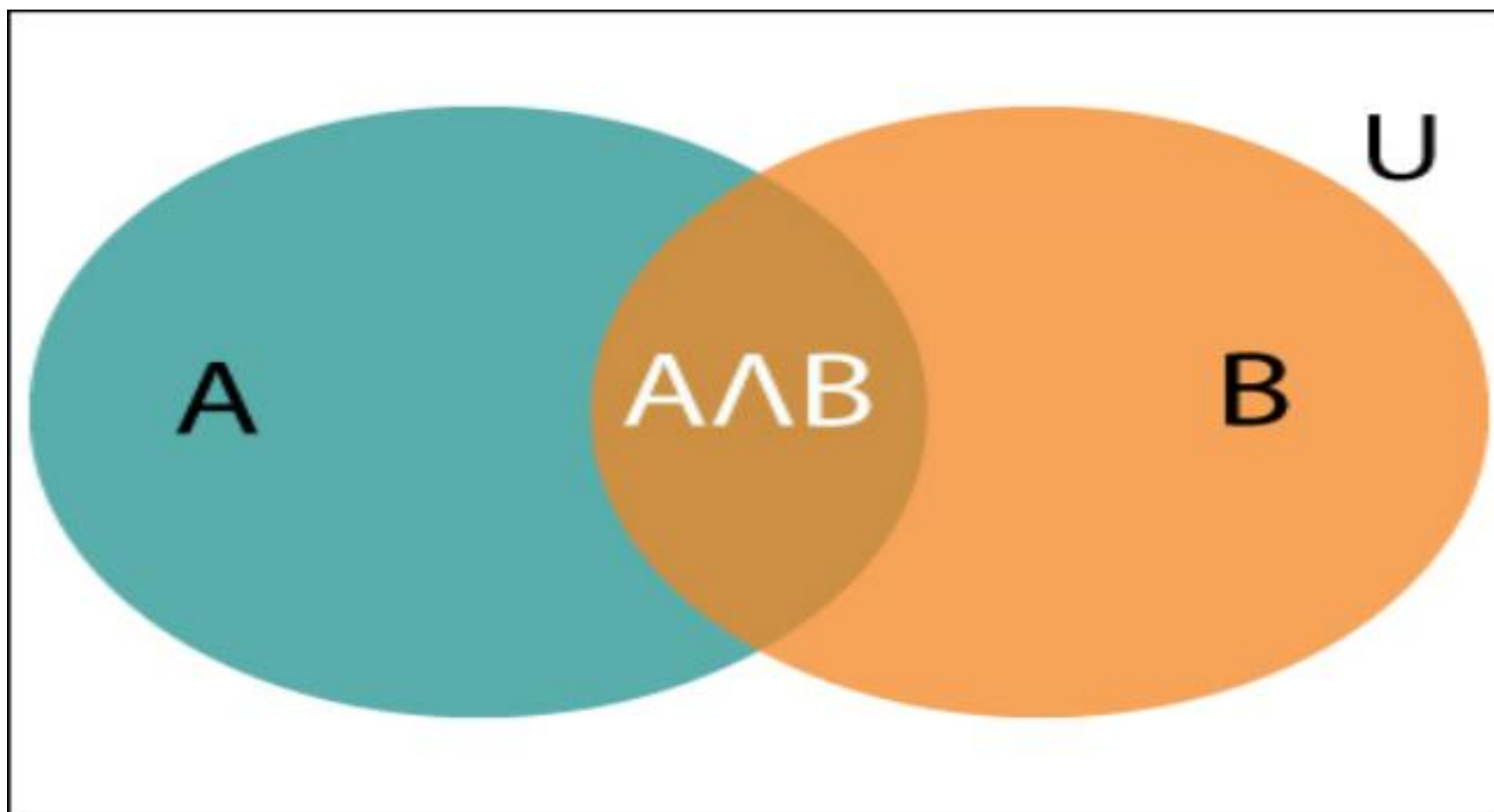
# Conditional Probability

- Conditional probability is a probability of occurring an event when another event has already happened.
- Let's suppose, we want to calculate the event A when event B has already occurred, "the probability of A under the conditions of B", it can be written as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- **Where  $P(A \cap B)$  = Joint probability of a and B**
- **$P(B)$  = Marginal probability of B.**
- If the probability of A is given and we need to find the probability of B, then it will be given as:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$



# Example: Conditional Probability

- **Example:**
- In a class, there are 70% of the students who like English and 40% of the students who like English and mathematics, and then what is the percent of students those who like English also like mathematics?
- **Solution:**
- Let, A is an event that a student likes Mathematics
- B is an event that a student likes English.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.7} = 57\%$$

- **Hence, 57% are the students who like English also like Mathematics.**

# Bayes' Theorem in Artificial Intelligence

- Bayes' theorem is also known as **Bayes' rule**, **Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.
- In probability theory, it relates the conditional probability and marginal probabilities of two random events.
- Bayes' theorem was named after the British mathematician **Thomas Bayes**.
- The **Bayesian inference** is an application of Bayes' theorem, which is fundamental to Bayesian statistics.



# Bayes' Theorem

- It is a way to calculate the value of  $P(B|A)$  with the knowledge of  $P(A|B)$ .
- Bayes' theorem allows updating the probability prediction of an event by observing new information of the real world.
- **Example:** If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age.

# Baye's Rule or Baye's Theorem

- As from product rule we can write:
  - $P(A \cap B) = P(A|B) P(B)$  or
- Similarly, the probability of event B with known event A:
  - $P(A \cap B) = P(B|A) P(A)$

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)} \quad \dots(a)$$

# Baye's Rule

- It shows the simple relationship between joint and conditional probabilities. Here,
- $P(A|B)$  is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.
- $P(B|A)$  is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.
- $P(A)$  is called the **prior probability**, probability of hypothesis before considering the evidence
- $P(B)$  is called **marginal probability**, pure probability of an evidence.
- In the equation (a), in general, we can write  $P(B) = \sum_{i=1}^k P(A_i) \cdot P(B|A_i)$ , hence the Bayes' rule can be written as:

$$P(A_i|B) = \frac{P(A_i) \cdot P(B|A_i)}{\sum_{i=1}^k P(A_i) \cdot P(B|A_i)}$$

- Where  $A_1, A_2, A_3, \dots, A_n$  is a set of mutually exclusive and exhaustive events

# Example: Baye's Theorem

- **Question: what is the probability that a patient has diseases meningitis with a stiff neck?**
- A doctor is aware that disease meningitis causes a patient to have a stiff neck, and it occurs 80% of the time. He is also aware of some more facts, which are given as follows:
- The Known probability that a patient has meningitis disease is 1/30,000.
- The Known probability that a patient has a stiff neck is 2%.
- Let a be the proposition that patient has stiff neck and b be the proposition that patient has meningitis. , so we can calculate the following as:
- $P(a|b) = 0.8$
- $P(b) = 1/30000$
- $P(a) = .02$

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)} = \frac{0.8 * (\frac{1}{30000})}{0.02} = 0.001333333.$$

- Hence, we can assume that 1 patient out of 750 patients has meningitis disease with a stiff neck.

# Example: Baye's Theorem

- Question: From a standard deck of playing cards, a single card is drawn. The probability that the card is king is  $4/52$ , then calculate posterior probability  $P(\text{King}|\text{Face})$ , which means the drawn face card is a king card.
- $P(\text{king})$ : probability that the card is King =  $4/52 = 1/13$
- $P(\text{face})$ : probability that a card is a face card =  $3/13$
- $P(\text{Face}|\text{King})$ : probability of face card when we assume it is a king = 1

$$P(\text{king}|\text{face}) = \frac{1 * (\frac{1}{13})}{(\frac{3}{13})} = 1/3, \text{ it is a probability that a face card is a king card.}$$

# Application of Bayes' theorem in Artificial intelligence:

- It is used to calculate the next step of the robot when the already executed step is given.
- Bayes' theorem is helpful in weather forecasting.
- It can solve the Monty Hall problem.

# Bayesian belief network

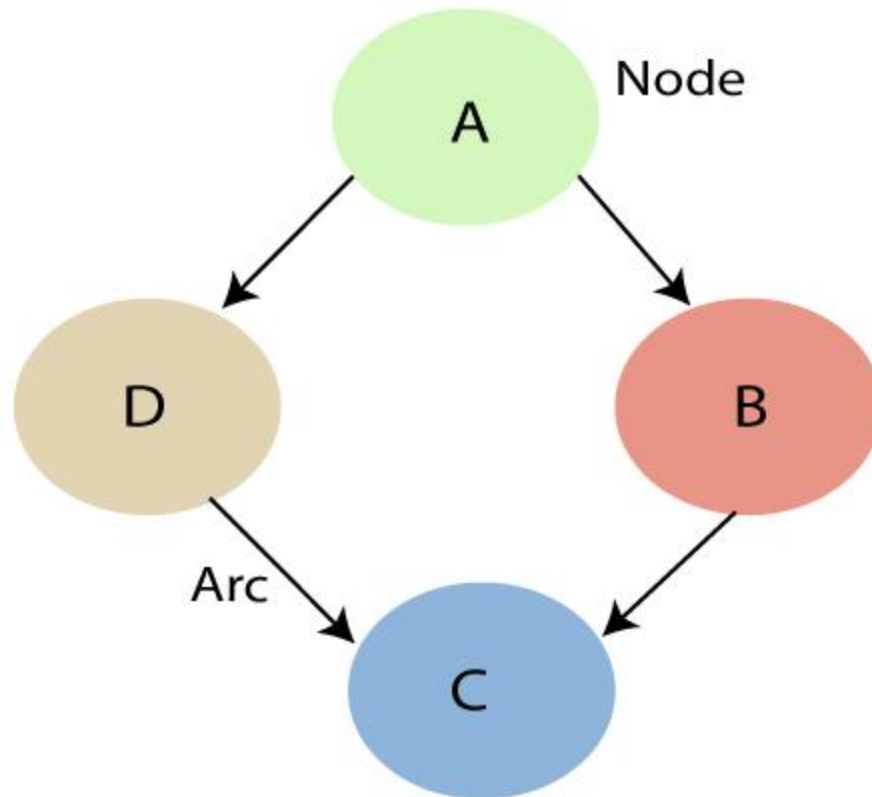
- Bayesian belief network is key computer technology for dealing with probabilistic events and to solve a problem which has uncertainty.
- We can define a Bayesian network as:
- "A Bayesian network is a probabilistic graphical model which represents a set of variables and their conditional dependencies using a directed acyclic graph."
- It is also called a **Bayes network, belief network, decision network, or Bayesian model.**



# Bayesian networks

- Are probabilistic, because these networks are built from a **probability distribution**, and also use probability theory for prediction and anomaly detection.
- Real world applications are probabilistic in nature, and to represent the relationship between multiple events, we need a Bayesian network.
- It can also be used in various tasks including **prediction, anomaly detection, diagnostics, automated insight, reasoning, time series prediction, and decision making under uncertainty**.
- Bayesian Network can be used for building models from data and experts opinions, and it consists of two parts:
  - **Directed Acyclic Graph**
  - **Table of conditional probabilities.**
- The generalized form of Bayesian network that represents and solve decision problems under uncertain knowledge is known as an **Influence diagram**.

# Bayesian Network



# Bayesian Network

- Each **node** corresponds to the random variables, and a variable can be **continuous** or **discrete**.
- **Arc or directed arrows** represent the causal relationship or conditional probabilities between random variables.
- These directed links or arrows connect the pair of nodes in the graph. These links represent that one node directly influence the other node, and if there is no directed link that means that nodes are independent with each other
  - In the above diagram, A, B, C, and D are random variables represented by the nodes of the network graph.
  - If we are considering node B, which is connected with node A by a directed arrow, then node A is called the parent of Node B.
  - Node C is independent of node A.

Note: The Bayesian network graph does not contain any cyclic graph. Hence, it is known as a **directed acyclic graph or DAG**.

# Bayesian Network

- The Bayesian network has mainly two components:
  - **Causal Component**
  - **Actual numbers**
- Each node in the Bayesian network has condition probability distribution  $P(X_i | \text{Parent}(X_i))$ , which determines the effect of the parent on that node.
- Bayesian network is based on Joint probability distribution and conditional probability. So let's first understand the joint probability distribution:

# Joint probability distribution

- If we have variables  $x_1, x_2, x_3, \dots, x_n$ , then the probabilities of a different combination of  $x_1, x_2, x_3 \dots x_n$ , are known as Joint probability distribution.
- $P[x_1, x_2, x_3, \dots, x_n]$ , it can be written as the following way in terms of the joint probability distribution.
- $= P[x_1 | x_2, x_3, \dots, x_n] P[x_2, x_3, \dots, x_n]$
- $= P[x_1 | x_2, x_3, \dots, x_n] P[x_2 | x_3, \dots, x_n] \dots P[x_{n-1} | x_n] P[x_n]$ .
- In general for each variable  $X_i$ , we can write the equation as:
- $P(X_i | X_{i-1}, \dots, X_1) = P(X_i | \text{Parents}(X_i))$

# Bayesian Network Example

- Harry installed a new burglar alarm at his home to detect burglary. The alarm reliably responds at detecting a burglary but also responds for minor earthquakes. Harry has two neighbors David and Sophia, who have taken a responsibility to inform Harry at work when they hear the alarm. David always calls Harry when he hears the alarm, but sometimes he got confused with the phone ringing and calls at that time too. On the other hand, Sophia likes to listen to high music, so sometimes she misses to hear the alarm. Here we would like to compute the probability of Burglary Alarm.

# Example: Bayesian Network

## Problem:

**Calculate the probability that alarm has sounded, but there is neither a burglary, nor an earthquake occurred, and David and Sophia both called the Harry.**

- The Bayesian network for the above problem is given below. The network structure is showing that burglary and earthquake is the parent node of the alarm and directly affecting the probability of alarm's going off, but David and Sophia's calls depend on alarm probability.
- The network is representing that our assumptions do not directly perceive the burglary and also do not notice the minor earthquake, and they also not confer before calling.
- The conditional distributions for each node are given as conditional probabilities table or CPT.
- Each row in the CPT must be sum to 1 because all the entries in the table represent an exhaustive set of cases for the variable.
- In CPT, a boolean variable with k boolean parents contains  $2^k$  probabilities. Hence, if there are two parents, then CPT will contain 4 probability values



# Solution:

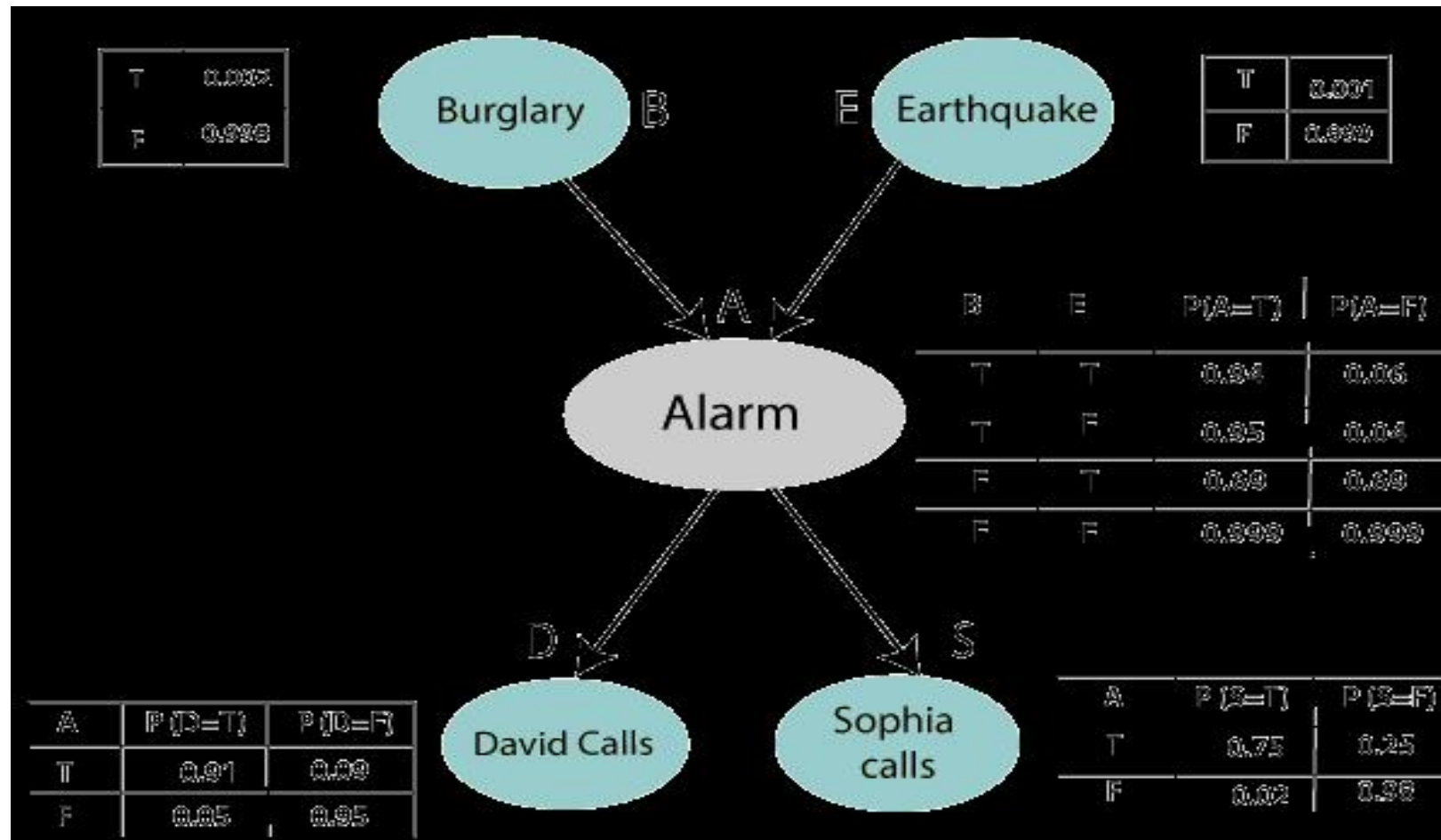
## List of all events occurring in this network:

- Burglary (B)
- Earthquake(E)
- Alarm(A)
- David Calls(D)
- Sophia calls(S)

# Solution:

- We can write the events of problem statement in the form of probability:  $P[D, S, A, B, E]$ , can rewrite the above probability statement using joint probability distribution:
- $P[D, S, A, B, E] = P[D \mid S, A, B, E] \cdot P[S, A, B, E]$
- $= P[D \mid S, A, B, E] \cdot P[S \mid A, B, E] \cdot P[A, B, E]$
- $= P[D \mid A] \cdot P[S \mid A, B, E] \cdot P[A, B, E]$
- $= P[D \mid A] \cdot P[S \mid A] \cdot P[A \mid B, E] \cdot P[B, E]$
- $= P[D \mid A] \cdot P[S \mid A] \cdot P[A \mid B, E] \cdot P[B \mid E] \cdot P[E]$

# Solution



# Solution

- Let's take the observed probability for the Burglary and earthquake component:
  - $P(B = \text{True}) = 0.002$ , which is the probability of burglary.
  - $P(B = \text{False}) = 0.998$ , which is the probability of no burglary.
  - $P(E = \text{True}) = 0.001$ , which is the probability of a minor earthquake
  - $P(E = \text{False}) = 0.999$ , Which is the probability that an earthquake not occurred.

## Conditional probability table for Alarm A:

The Conditional probability of Alarm A depends on Burglar and earthquake:

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B	E	P(A= True)	P(A= False)
True	True	0.94	0.06
True	False	0.95	0.04
False	True	0.31	0.69
False	False	0.001	0.999

**Conditional probability table for David Calls:**

The Conditional probability of David that he will call depends on the probability of Alarm.

A	$P(D = \text{True})$	$P(D = \text{False})$
True	0.91	0.09
False	0.05	0.95

**Conditional probability table for Sophia Calls:**

The Conditional probability of Sophia that she calls is depending on its Parent Node "Alarm."

A	$P(S = \text{True})$	$P(S = \text{False})$
True	0.75	0.25
False	0.02	0.98

# Example

- $P(S, D, A, \neg B, \neg E) = P(S|A) * P(D|A) * P(A|\neg B \wedge \neg E) * P(\neg B) * P(\neg E).$
- $= 0.75 * 0.91 * 0.001 * 0.998 * 0.999$
- $= 0.00068045.$
- Hence, a Bayesian network can answer any query about the domain by using Joint distribution.



# The semantics of Bayesian Network:

There are two ways to understand the semantics of the Bayesian network, which is given below:

**1. To understand the network as the representation of the Joint probability distribution.**

It is helpful to understand how to construct the network.

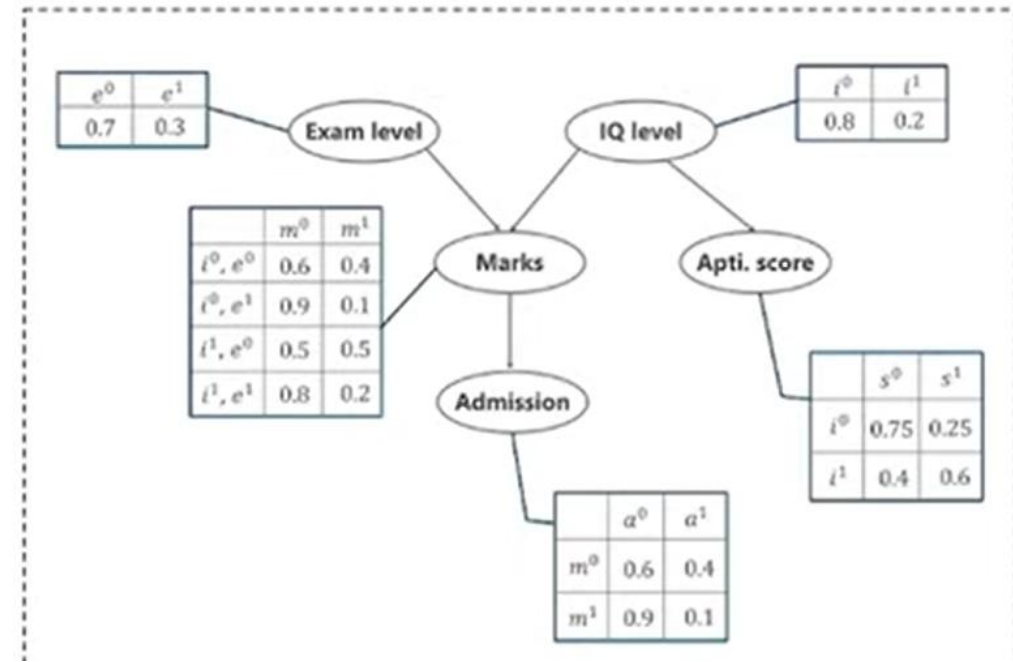
**2. To understand the network as an encoding of a collection of conditional independence statements.**

It is helpful in designing inference procedure.

Create a Bayesian Network that will model the marks ( $m$ ) of a student on his examination.

The marks will depend on:

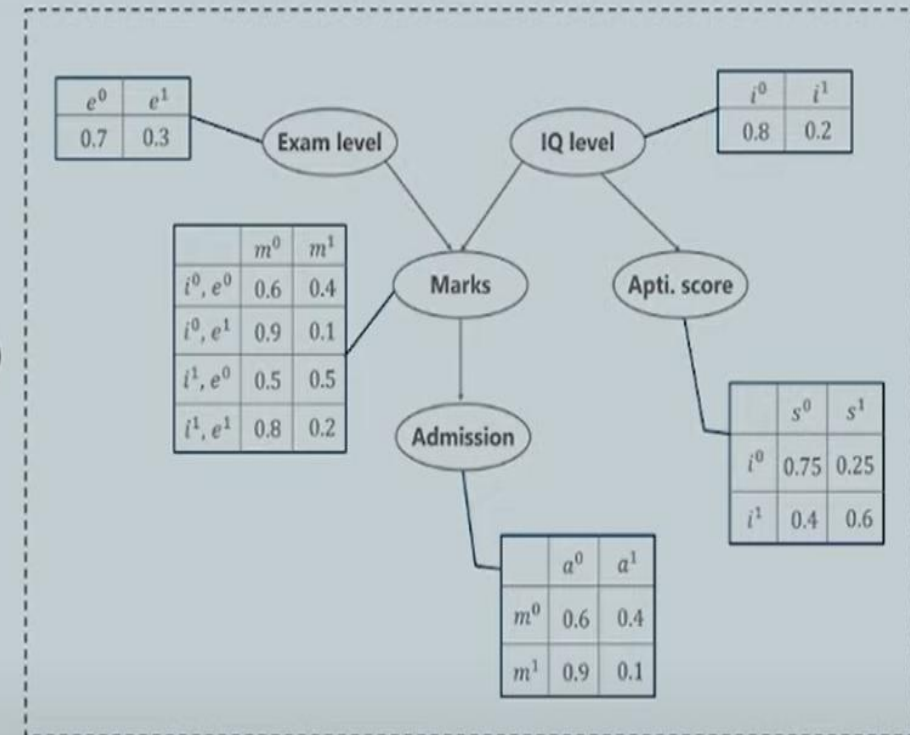
- **Exam level** ( $e$ ): (difficult, easy)
- **IQ** of the student ( $i$ ): (high, low)
- Marks  $\rightarrow$  **admitted** ( $a$ ) to a university
- The IQ  $\rightarrow$  **aptitude score** ( $s$ ) of the student



*Factorizing Joint Probability Distribution:*

$$p(a, m, i, e, s) = p(a | m) p(m | i, e) p(i) p(e) p(s | i)$$

- $p(a | m)$ : CP of student admit -> marks
- $p(m | i, d)$ : CP of the student's marks -> (IQ & exam level)
- $p(i)$ : Probability -> IQ level
- $p(d)$ : Probability -> exam level
- $p(s | i)$  CP of aptitude scores -> IQ level



# Example

- [https://youtu.be/-h\\_h7pnwY8A](https://youtu.be/-h_h7pnwY8A)

# Example 1

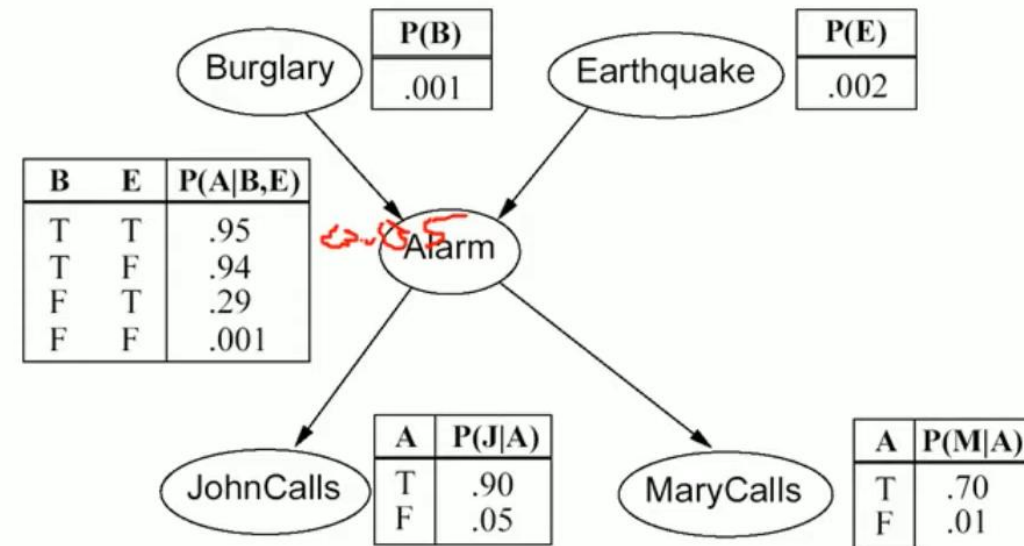
2. What is the probability that John call?

Solution:

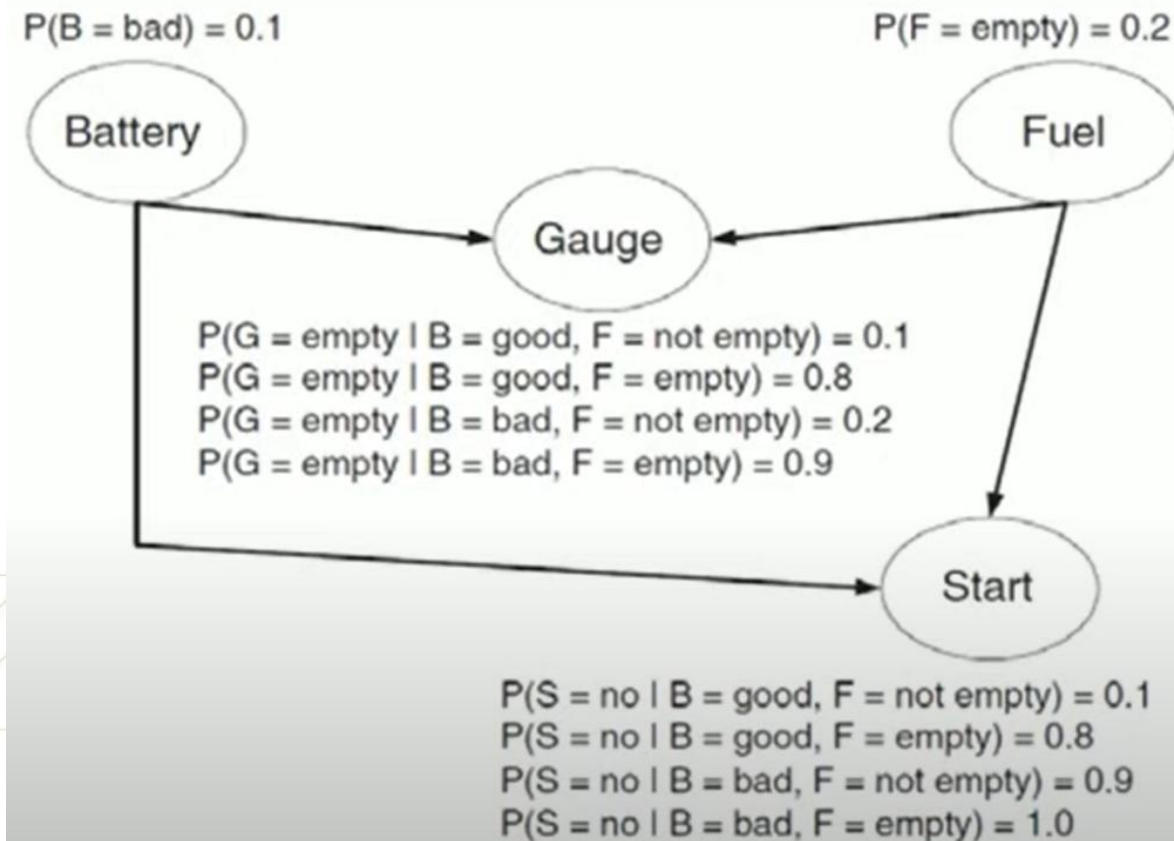
$$P(j) = P(j | a) P(a) + P(j | \neg a) P(\neg a)$$

$$= P(j | a) \{P(a | b, e) * P(b, e) + P(a | \neg b, e) * P(\neg b, e) + P(a | b, \neg e) * P(b, \neg e) + P(a | \neg b, \neg e) * P(\neg b, \neg e)\}$$

$$+ P(j | \neg a) \{P(\neg a | b, e) * P(b, e) + P(\neg a | \neg b, e) * P(\neg b, e) + P(\neg a | b, \neg e) * P(b, \neg e) + P(\neg a | \neg b, \neg e) * P(\neg b, \neg e)\}$$



# Case Study: Starting of Vehicle



- $P(B=\text{Good}, F=\text{Empty}, G=\text{Empty}, S=\text{Yes})$**
- $P(B=\text{Bad}, F=\text{Empty}, G=\text{Not Empty}, S=\text{No})$**
- Given the battery is bad, Compute the probability that the car will start**



# Important

- Cover all the variations of questions covered in class for the following case studies
  - Burglary Alarm
  - Wet Grass due to Sprinkler or Rain
  - Gauge, Vehicle, Fuel, Battery Problem

# THANK YOU

