VIPS
Technical Campus
योग: कर्मसु कौशलम्
IN PURSUIT OF PERFECTION

SCHOOL OF ENGINEERING AND TECHNOLOGY

Unit 3

- **UNIT III:** Introduction of Fuzzy Reasoning and Neural Networks.

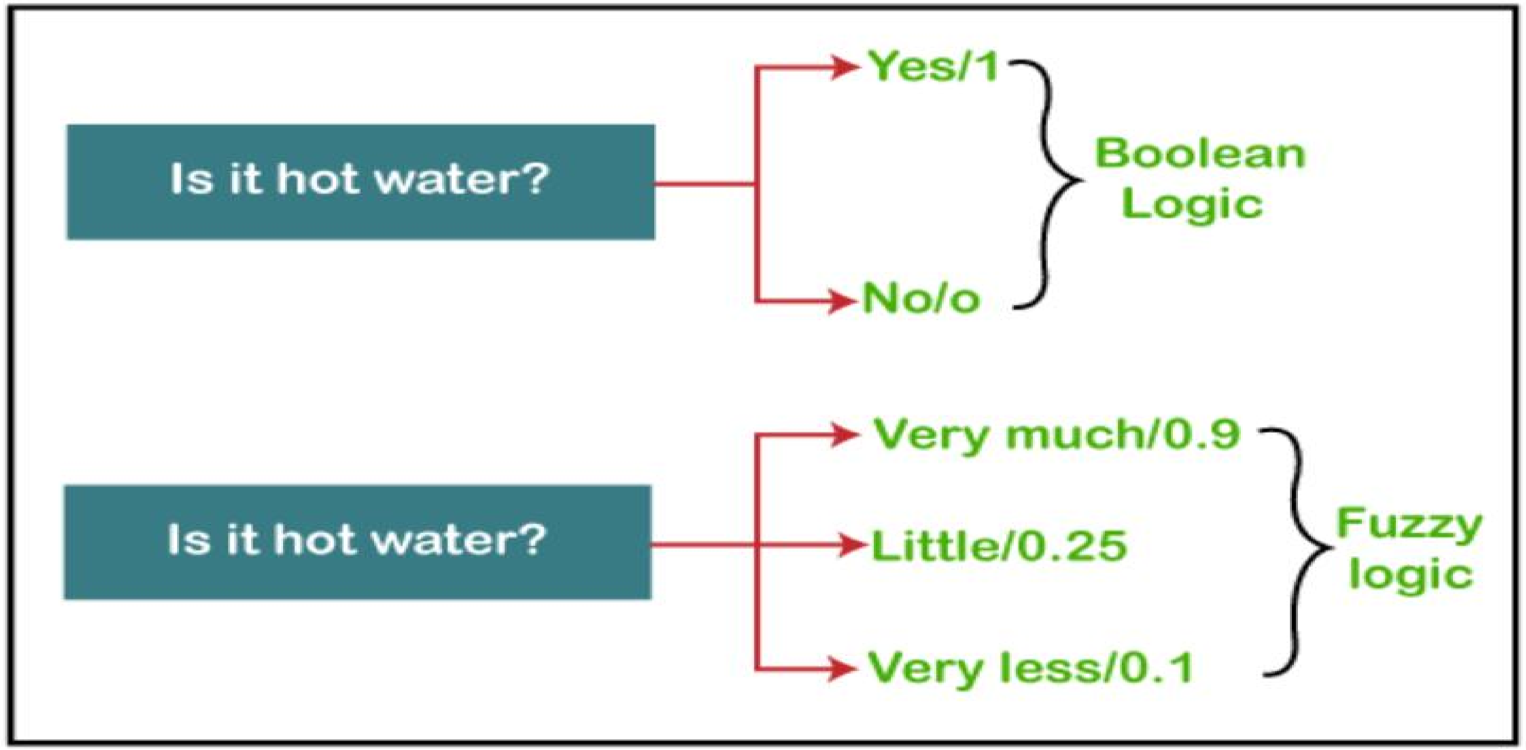
Video Link

- https://youtu.be/f1a6lPPQNgM

Fuzzy Logic

- The 'Fuzzy' word means the things that are not clear or are vague. Sometimes, we cannot decide in real life that the given problem or statement is either true or false. At that time, this concept provides many values between the true and false and gives the flexibility to find the best solution to that problem.

Example of Fuzzy Logic as comparing to Boolean Logic



Fuzzy logic contains the multiple logical values and these values are the truth values of a variable or problem between 0 and 1. This concept was introduced by **Lofti Zadeh** in **1965** based on the **Fuzzy Set Theory**. This concept provides the possibilities which are not given by computers, but similar to the range of possibilities generated by humans.

In the Boolean system, only two possibilities (0 and 1) exist, where 1 denotes the absolute truth value and 0 denotes the absolute false value. But in the fuzzy system, there are multiple possibilities present between the 0 and 1, which are partially false and partially true.

The Fuzzy logic can be implemented in systems such as micro-controllers, workstation-based or large network-based systems for achieving the definite output. It can also be implemented in both hardware or software.

Characteristics of Fuzzy Logic

Characteristics of Fuzzy Logic

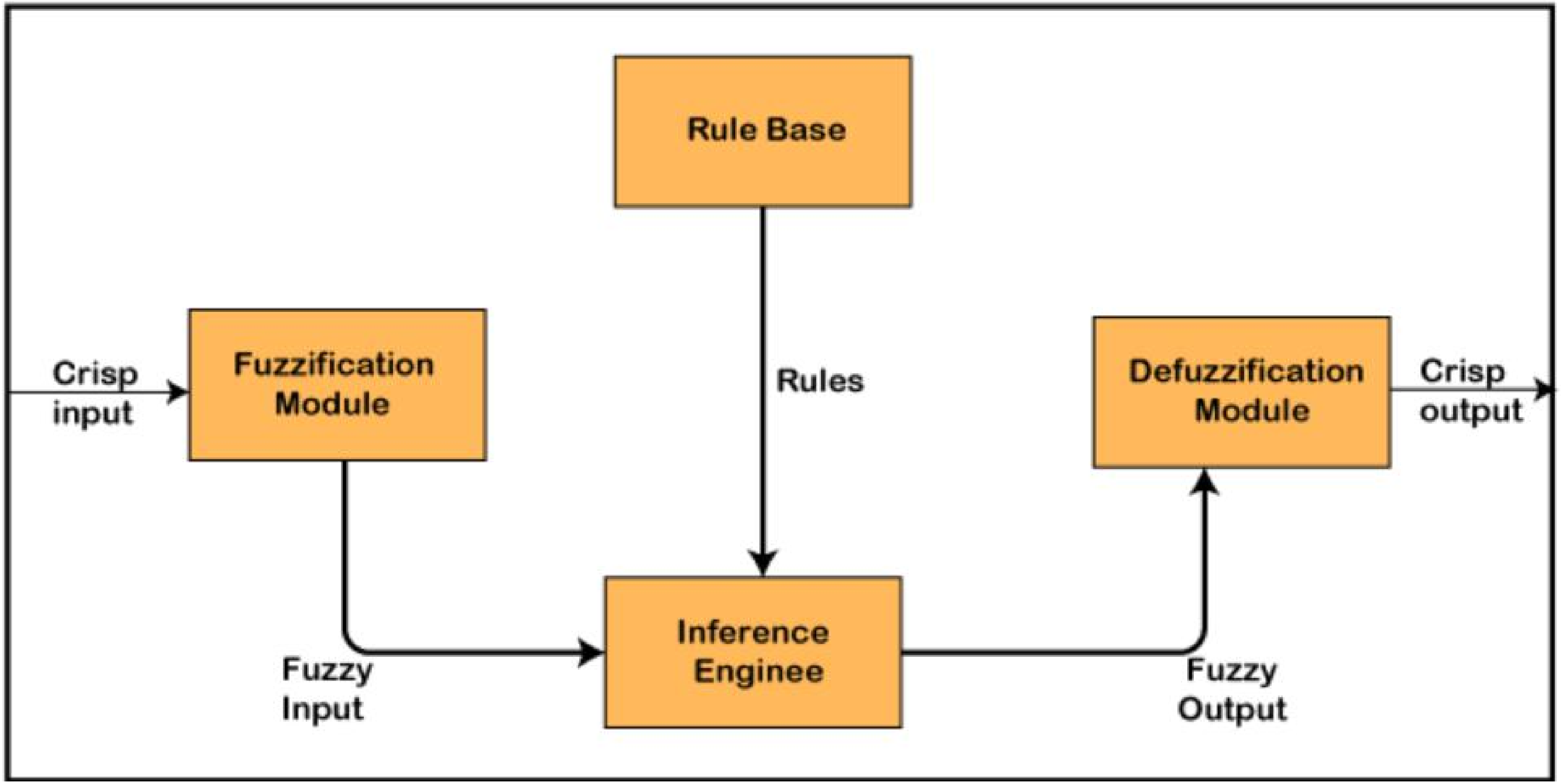
Following are the characteristics of fuzzy logic:

1. This concept is flexible and we can easily understand and implement it.
2. It is used for helping the minimization of the logics created by the human.
3. It is the best method for finding the solution of those problems which are suitable for approximate or uncertain reasoning.
4. It always offers two values, which denote the two possible solutions for a problem and statement.
5. It allows users to build or create the functions which are non-linear of arbitrary complexity.
6. In fuzzy logic, everything is a matter of degree.
7. In the Fuzzy logic, any system which is logical can be easily fuzzified.
8. It is based on natural language processing.
9. It is also used by the quantitative analysts for improving their algorithm's execution.
10. It also allows users to integrate with the programming.

Architecture of a Fuzzy Logic System

In the architecture of the **Fuzzy Logic** system, each component plays an important role. The architecture consists of the different four components which are given below.

1. Rule Base
2. Fuzzification
3. Inference Engine
4. Defuzzification


Rule Base
Rules
Crisp input
Fuzzification Module
Defuzzification Module
Crisp output
Fuzzy Input
Inference Enginee
Fuzzy Output

1. Rule Base

1. Rule Base

Rule Base is a component used for storing the set of rules and the If-Then conditions given by the experts are used for controlling the decision-making systems. There are so many updates that come in the Fuzzy theory recently, which offers effective methods for designing and tuning of fuzzy controllers. These updates or developments decreases the number of fuzzy set of rules.

Fuzzification

Fuzzification is a module or component for transforming the system inputs, i.e., it converts the crisp number into fuzzy steps. The crisp numbers are those inputs which are measured by the sensors and then fuzzification passed them into the control systems for further processing. This component divides the input signals into following five states in any Fuzzy Logic system:

- Large Positive (LP)
- Medium Positive (MP)
- Small (S)
- Medium Negative (MN)
- Large negative (LN)

Inference Engine

This component is a main component in any Fuzzy Logic system (FLS), because all the information is processed in the Inference Engine. It allows users to find the matching degree between the current fuzzy input and the rules. After the matching degree, this system determines which rule is to be added according to the given input field. When all rules are fired, then they are combined for developing the control actions.

Membership Function

The membership function is a function which represents the graph of fuzzy sets, and allows users to quantify the linguistic term. It is a graph which is used for mapping each element of x to the value between 0 and 1.

This function is also known as indicator or characteristics function.

This function of Membership was introduced in the first papers of fuzzy set by **Zadeh**. For the Fuzzy set B, the membership function for X is defined as: $\mu B:X \rightarrow [0,1]$. In this function X, each element of set B is mapped to the value between 0 and 1. This is called a degree of membership or membership value.

Fuzzy Sets (Table from Rich and Knight)

Table 22.1 *Ages and their memberships*

Age	*Infant*	*Child*	*Adolescent*	*Young*	*Adult*	*Old*
2	1	0	0	1	0	0
4	0.1	0.5	0	1	0	0
10	0	1	0.3	1	0	0
15	0	0.8	1	1	0	0
21	0	0	0.1	1	0.8	0.1
30	0	0	0	0.6	1	0.3
35	0	0	0	0.5	1	0.35
40	0	0	0	0.4	1	0.4
45	0	0	0	0.2	1	0.6
60	0	0	0	0	1	0.8
70	0	0	0	0	1	1

The values in the table indicate memberships to the fuzzy sets – *infant, child, adolescent, young, adult* and *old*. Thus a child of age 4 belongs only 50% to the fuzzy set *child* while when he is 10 years he is a 100% member. Note that membership is different from probabilities. Memberships do not necessarily add up to 1. The entries in the table have been made after a manual evaluation of the different ages.

Fuzzy Terminology

Universe of Discourse (U):

This is defined as the range of all possible values that comprise the input to the fuzzy system.

Fuzzy Set

Any set that empowers its members to have different grades of membership (based on a membership function) in an interval [0,1] is a fuzzy set.

Membership function

The membership function μ_A which forms the basis of a fuzzy set is given by

$$\mu_A: U \to [0,1]$$

where the closed interval is one that holds real numbers.

Support of a fuzzy set (S_f)

The support S of a fuzzy set f, in a universal crisp set U is that set which contains all elements of the set U that have a non-zero membership value in f. For instance, the support of the fuzzy set *adult* is

$$S_{adult} = \{21,30,35,40,45,60,70\}$$

Depiction of a fuzzy set

A fuzzy set f in a universal crisp set U, is written as

$$f = \mu_1/s_1 + \mu_2/s_2 + \mu_3/s_3 + \ldots + \mu_n/s_n$$

where μ_i is the membership and s_i is the corresponding term in the *support* set of f i.e. S_f.

This is however only a representation and has *no algebraic implication* (the slash and + signs do not have any meaning).

Accordingly,

$$\text{Old} = 0.1/21 + 0.3/30 + 0.35/35 + 0.4/40 + 0.6/45 + 0.8/60 + 1/70$$

Fuzzy Set Operations

Fuzzy Set Operations

- **Union:** The membership function of the union of two fuzzy sets A and B is defined as the maximum of the two individual membership functions. It is equivalent to the Boolean OR operation.

$$\mu_{A \cup B} = \max(\mu_A, \mu_B)$$

- **Intersection:** The membership function of the intersection of two fuzzy sets A and B is defined as the minimum of the two individual membership functions and is equivalent to the Boolean AND operation.

$$\mu_{A \cap B} = \min(\mu_A, \mu_B)$$

- **Complement:** The membership function of the complement of a fuzzy set A is defined as the negation of the specified membership function: $\mu_{\bar{A}}$. This is equivalent to the Boolean NOT operation

$$\mu_{\bar{A}} = \mu_{A \cup B} = (1 - \mu_A)$$

It may be further noted here that the laws of Associativity, Commutativity , Distributivity and De Morgan's laws hold in fuzzy set theory too.

Fuzzy Set Operations: Union

Example:

Let's suppose A is a set which contains following elements:

$A = \{(X_1, 0.6), (X_2, 0.2), (X_3, 1), (X_4, 0.4)\}$

And, B is a set which contains following elements:

$B = \{(X_1, 0.1), (X_2, 0.8), (X_3, 0), (X_4, 0.9)\}$

then,

$AUB = \{(X_1, 0.6), (X_2, 0.8), (X_3, 1), (X_4, 0.9)\}$

Fuzzy Set Operations: Intersection

Example:

Let's suppose A is a set which contains following elements:

$$A = \{(X_1, 0.3), (X_2, 0.7), (X_3, 0.5), (X_4, 0.1)\}$$

And, B is a set which contains following elements:

$$B = \{(X_1, 0.8), (X_2, 0.2), (X_3, 0.4), (X_4, 0.9)\}$$

then,

$$A \cap B = \{(X_1, 0.3), (X_2, 0.2), (X_3, 0.4), (X_4, 0.1)\}$$

Fuzzy Set Operations: Compliment

Example:

Let's suppose A is a set which contains following elements:

$$A = \{(X_1, 0.3), (X_2, 0.8), (X_3, 0.5), (X_4, 0.1)\}$$

then,

$$\bar{A} = \{(X_1, 0.7), (X_2, 0.2), (X_3, 0.5), (X_4, 0.9)\}$$

Difference Between Fuzzy and Classical Set Theory

Classical Set Theory	Fuzzy Set Theory
1. This theory is a class of those sets having sharp boundaries.	1. This theory is a class of those sets having un-sharp boundaries.
2. This set theory is defined by exact boundaries only 0 and 1.	2. This set theory is defined by ambiguous boundaries.
3. In this theory, there is no uncertainty about the boundary's location of a set.	3. In this theory, there always exists uncertainty about the boundary's location of a set.
4. This theory is widely used in the design of digital systems.	4. It is mainly used for fuzzy controllers.

Applications of Fuzzy Logic

Following are the different application areas where the Fuzzy Logic concept is widely used:

1. It is used in **Businesses** for decision-making support system.
2. It is used in **Automative systems** for controlling the traffic and speed, and for improving the efficiency of automatic transmissions. **Automative systems** also use the shift scheduling method for automatic transmissions.
3. This concept is also used in the **Defence** in various areas. Defence mainly uses the Fuzzy logic systems for underwater target recognition and the automatic target recognition of thermal infrared images.
4. It is also widely used in the **Pattern Recognition and Classification** in the form of Fuzzy logic-based recognition and handwriting recognition. It is also used in the searching of fuzzy images.
5. Fuzzy logic systems also used in **Securities**.

Applications of Fuzzy Logic (Contd.)

6. It is also used in **microwave oven** for setting the lunes power and cooking strategy.
7. This technique is also used in the area of **modern control systems** such as expert systems.
8. **Finance** is also another application where this concept is used for predicting the stock market, and for managing the funds.
9. It is also used for controlling the brakes.
10. It is also used in the **industries of chemicals** for controlling the ph, and chemical distillation process.
11. It is also used in the **industries of manufacturing** for the optimization of milk and cheese production.
12. It is also used in the vacuum cleaners, and the timings of washing machines.
13. It is also used in heaters, air conditioners, and humidifiers.

Advantages of Fuzzy Logic

1. The methodology of this concept works similarly as the human reasoning.
2. Any user can easily understand the structure of Fuzzy Logic.
3. It does not need a large memory, because the algorithms can be easily described with fewer data.
4. It is widely used in all fields of life and easily provides effective solutions to the problems which have high complexity.
5. This concept is based on the set theory of mathematics, so that's why it is simple.
6. It allows users for controlling the control machines and consumer products.
7. The development time of fuzzy logic is short as compared to conventional methods.
8. Due to its flexibility, any user can easily add and delete rules in the FLS system.

Disadvantages of Fuzzy Logic

1. The run time of fuzzy logic systems is slow and takes a long time to produce outputs.
2. Users can understand it easily if they are simple.
3. The possibilities produced by the fuzzy logic system are not always accurate.
4. Many researchers give various ways for solving a given statement using this technique which leads to ambiguity.
5. Fuzzy logics are not suitable for those problems that require high accuracy.
6. The systems of a Fuzzy logic need a lot of testing for verification and validation.

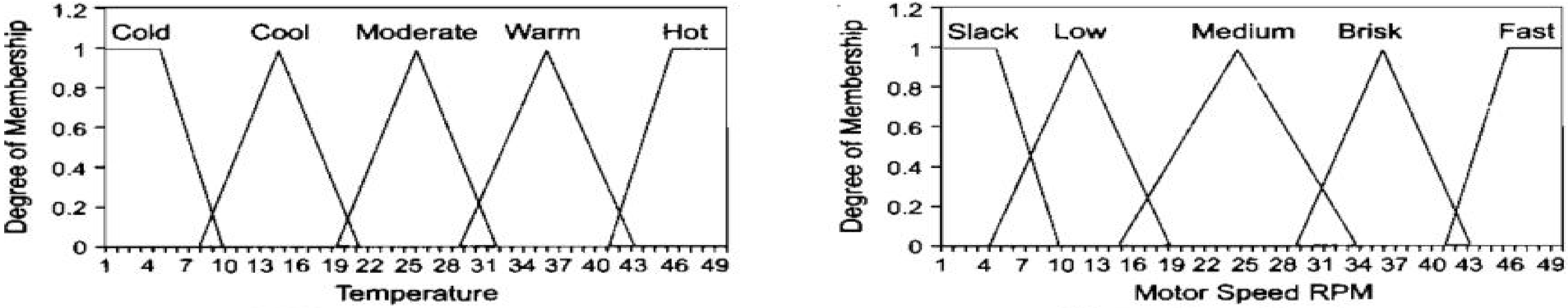
Case Study: Fuzzy Room Cooler

- **Fuzzy Regions**

- **Two parameters decide the water flow rate (Temperature and Pressure)**
  - **Fuzzy Terms for Temperature: Cold, Cool, Moderate, Warm and Hot**
  - **Fuzzy Terms for Fan Motor Speed: Slack, Low, Medium, Brisk, Fast.**

Output of the System, which is the flow-rate of the water controlled by the motorized pump, coould also be defined accordingly by yet another set of fuzzy terms: Strong-Negative, Negative, Low-Negative, Low-Positive and High Positive

Fuzzy Profiles



(a) Temperature *(b) Fan Motor Speed*

Fig. 22.2

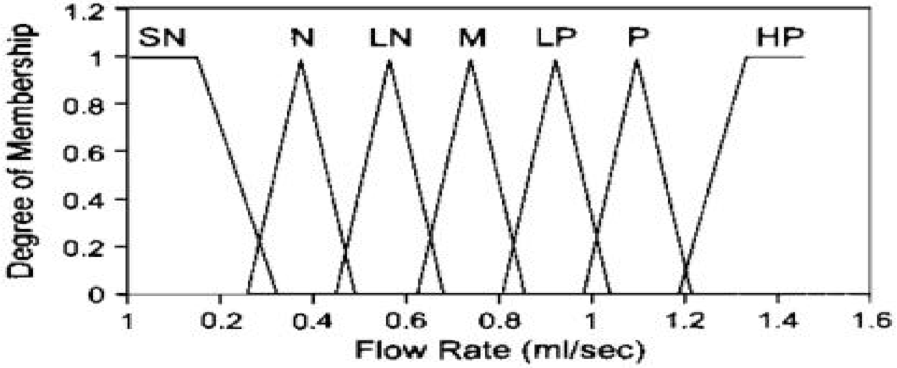


Fig. 22.3 *Water Flow Rate*

Fuzzy Rules

Fuzzy Rules

The fuzzy rules form the triggers of the fuzzy engine. After a study of the system we could write linguistic rules (so akin to natural language) such as –

R1: If temperature is HOT ***and*** fan motor speed is SLACK then flow-rate is HIGH-POSITIVE.
R2: If temperature is HOT ***and*** fan motor speed is LOW then flow-rate is HIGH-POSITIVE
R3: If temperature is HOT ***and*** fan motor speed is MEDIUM then the flow-rate is POSITIVE.
R4: If temperature is HOT ***and*** fan motor speed is BRISK then the flow-rate is HIGH-POSITIVE.
R5: If temperature is WARM ***and*** fan motor speed is MEDIUM then the flow-rate is LOW-POSITIVE,
R6: If temperature is WARM ***and*** fan motor speed is BRISK then the flow-rate is POSITIVE.
R7: If temperature is COOL ***and*** fan motor speed is LOW then flow-rate is NEGATIVE.
R8: If temperature is MODERATE ***and*** fan motor speed is LOW then flow-rate is MEDIUM.

Fuzzification

The fuzzifier forms the heart of the fuzzy engine. Whenever the sensors report the values of temperature and fan speed, they are mapped based on their memberships to the respective fuzzy regions they belong to. For instance if at some instance of time t the temperature is 42 degrees and fan speed is 31 rpm, the corresponding membership values and the associated fuzzy regions are mentioned below

Parameter	*Fuzzy Regions*	*Memberships*
Temperature	warm, hot	0.142, 0.2
Fan speed	medium, brisk	0.25,0.286

From the table, since both temperature and fan speed belong to two regions, it is clear that the rules R3, R4, R5 and R6 are applicable. The rules indicate a conflict. While two of them state that the flow-rate should be POSITIVE, the other two state that it should be LOW-POSITIVE and HIGH-POSITIVE respectively. Though we have resolved the issue of what could be the flow rates, the actual crisp value still eludes us.

Defuzzification

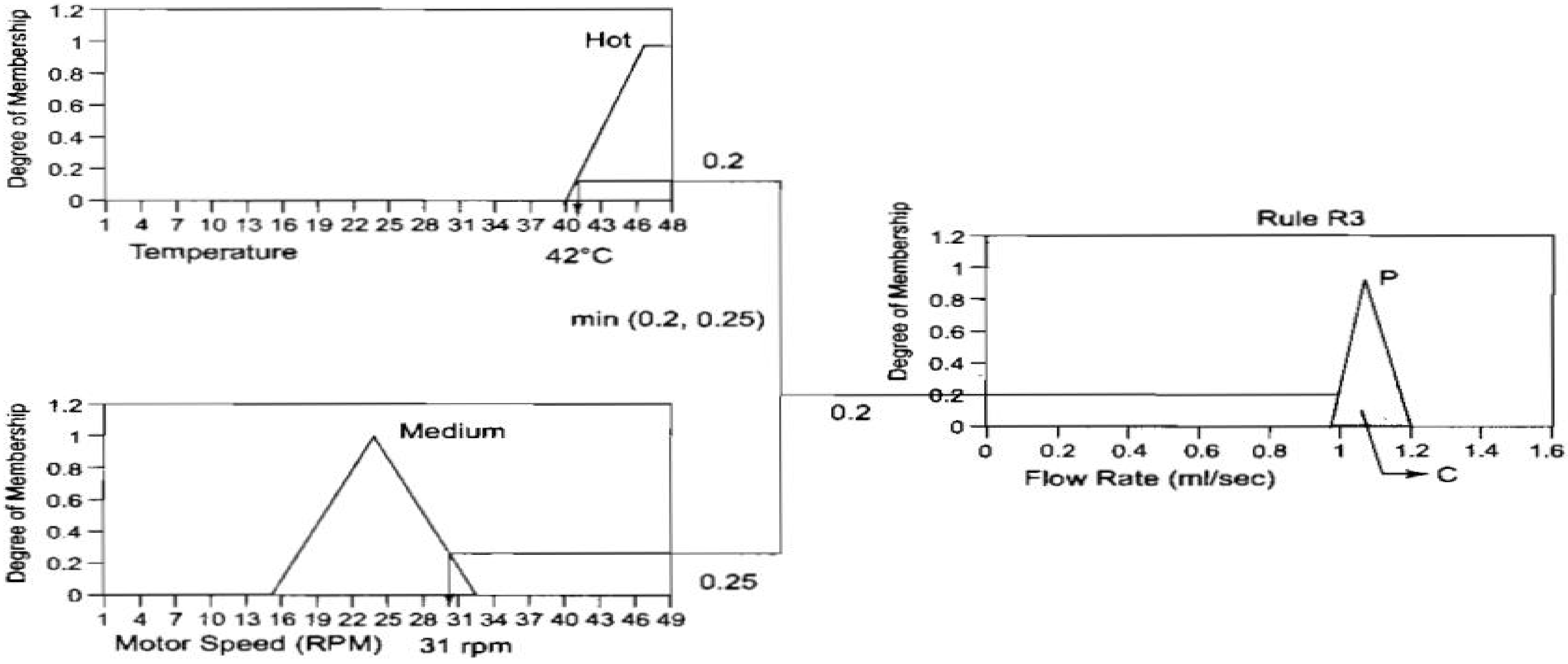
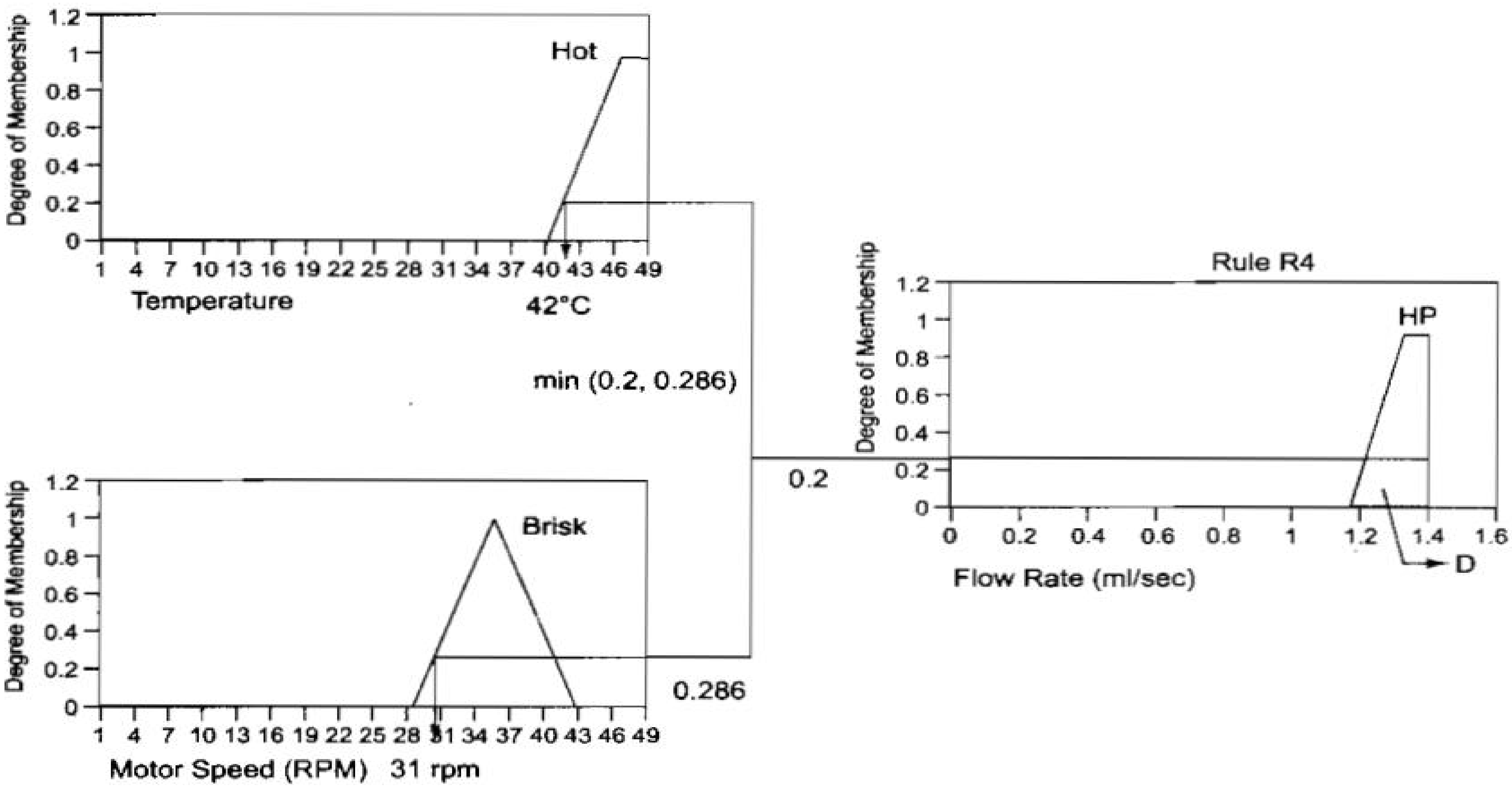


Fig. 22.4 *Defuzzification (contd.)*

Defuzzification (Contd.)



Defuzzification (Contd.)

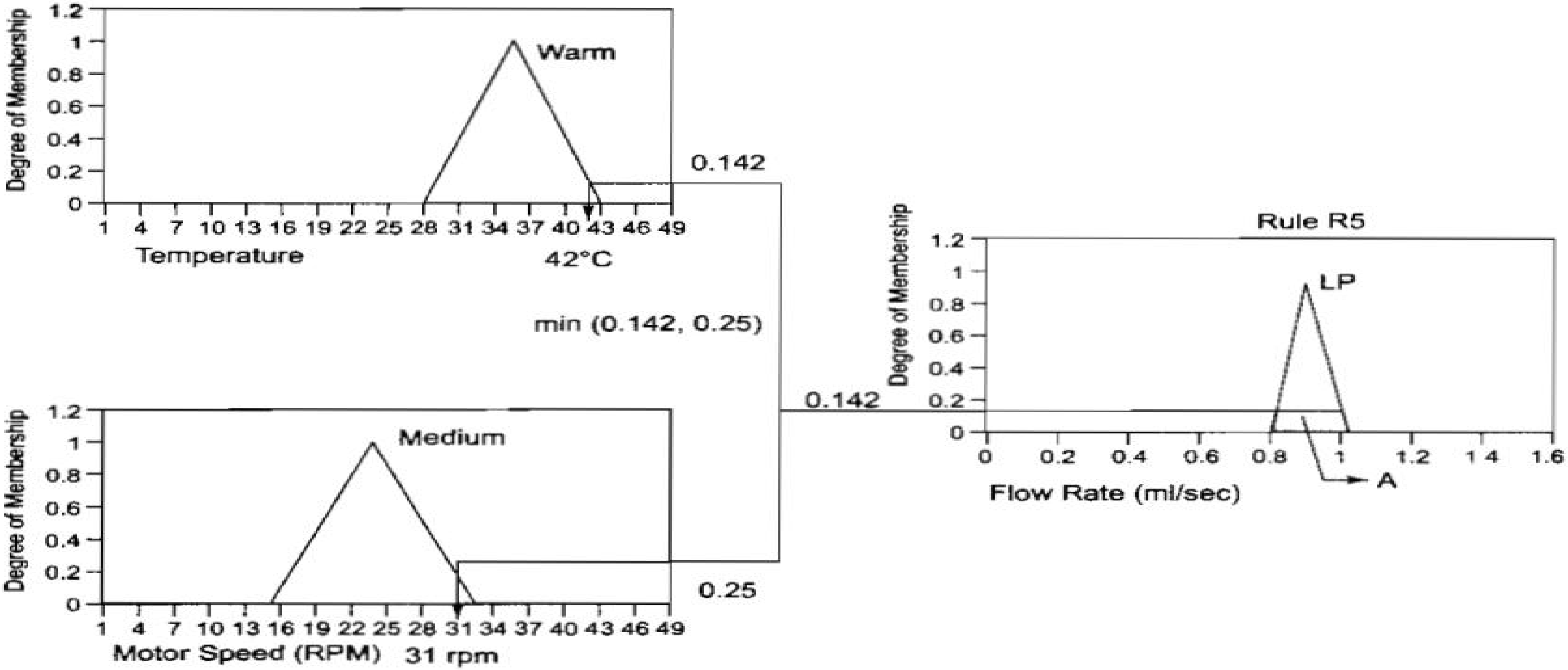
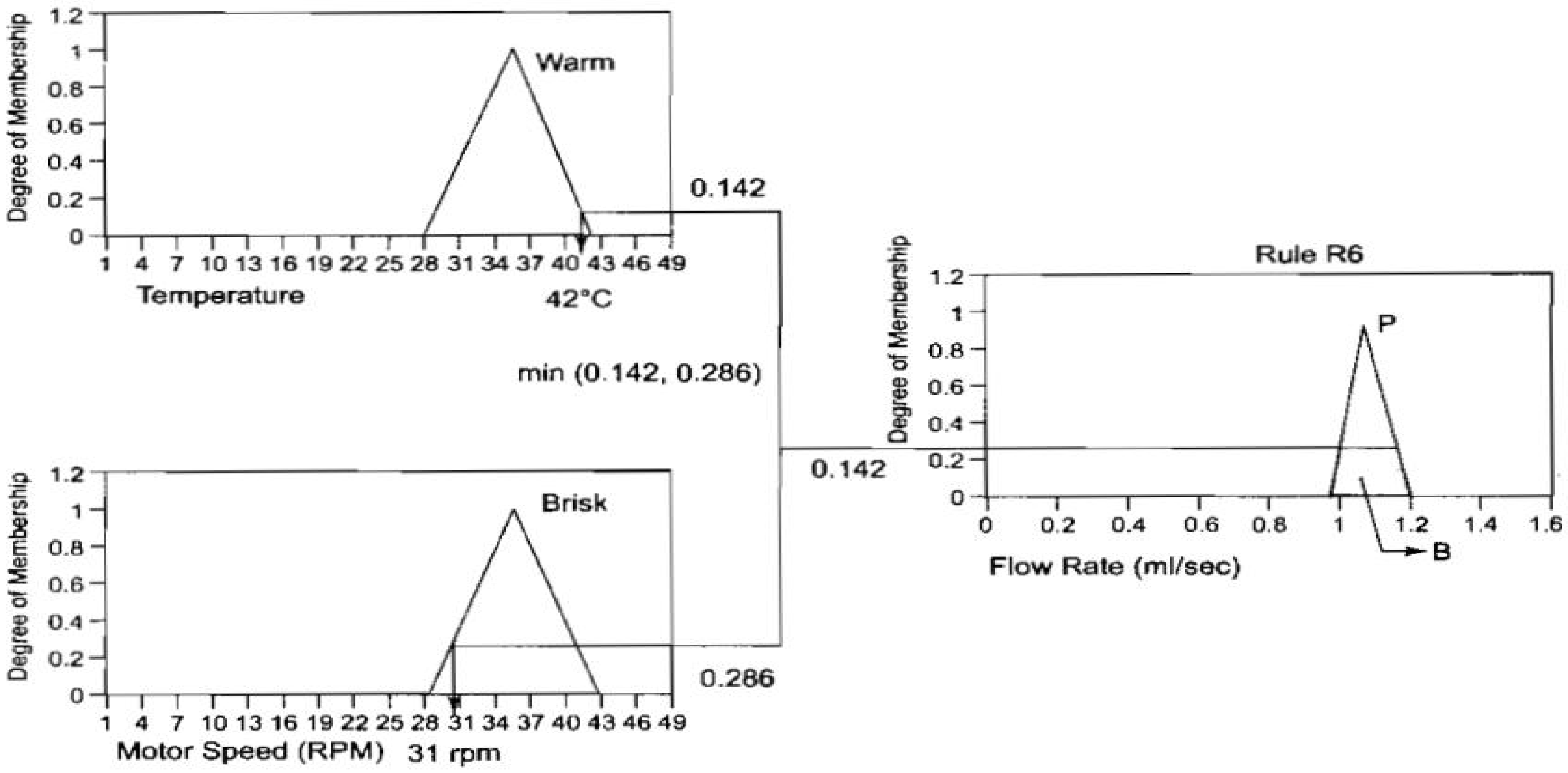


Fig. 22.4 *Defuzzification (contd.)*

Defuzzification (Contd.)



Defuzzification (Contd.)

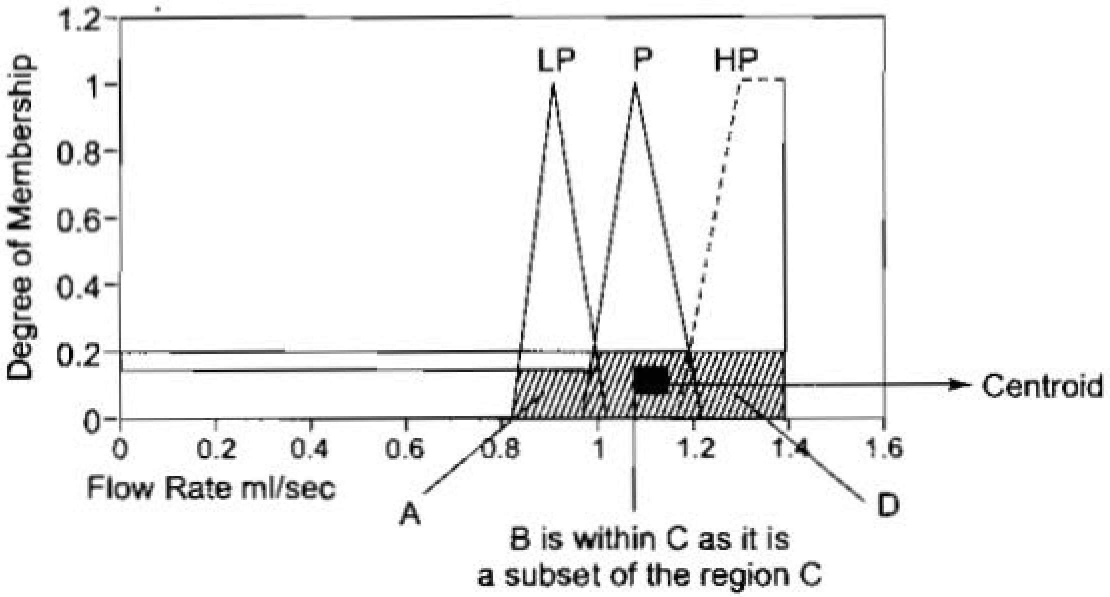


Fig. 22.4 *Defuzzification*

Defuzzifier

This is where we have to demystify these fuzzy terms for the flow rate controller system. In other words, the fuzzy outputs LOW-POSITIVE, POSITIVE and HIGH-POSITIVE are to be converted to a single crisp value which can then be delivered to the final actuator of the pump. This process is called defuzzification. Several methods are used to achieve defuzzification, the most common ones being the Centre of Gravity method and the Composite Maxima method. In both these methods we need to compute the composite region formed by the portions A, B, C and D (See Fig. 22.4) on the output profile. Figure 22.4 shows how this is calculated. In case of parameters whose premises are connected by an *AND*, the minimum of their memberships is first found. This

value is used to cut through the profile of the output fuzzy set (done by drawing a horizontal line). This results in a region (area) on the output surface. For cases where an *OR* relates the premises the maximum membership is taken to work out the output surface. All output surfaces are found to obtain the composite output region.

Depending on the application, either the Centre of Gravity or the Composite Maxima of this region (area) is found and treated as the crisp output. The former method works best for control applications such as the one described herein. The crisp output is the desired flow rate (X-coordinate of the Centroid) and the motorized pump is adjusted accordingly based on this value.

Summary
Steps involved in Fuzzy Logic Based Systems

(i) Formulating Fuzzy regions,

(ii) Fuzzy Rules and

(iii) Embedding a Defuzzification procedure.

Fuzzy logic has also been widely applied to non-control applications as well. Take the case of deciding whether a book on Artificial Intelligence belongs to the domain of Computer Science, Psychology or Civil Engineering. In such situations a crisp numerical output obtained by this defuzzification process used earlier may carry no meaning. The composite maxima is generally used for such problems and the truth depends on whether or not the composite maxima has crossed a predetermined threshold. The defuzzification process however may be tuned to suit the satisfiability of the application at hand.

NeuroFuzzy Room Cooler

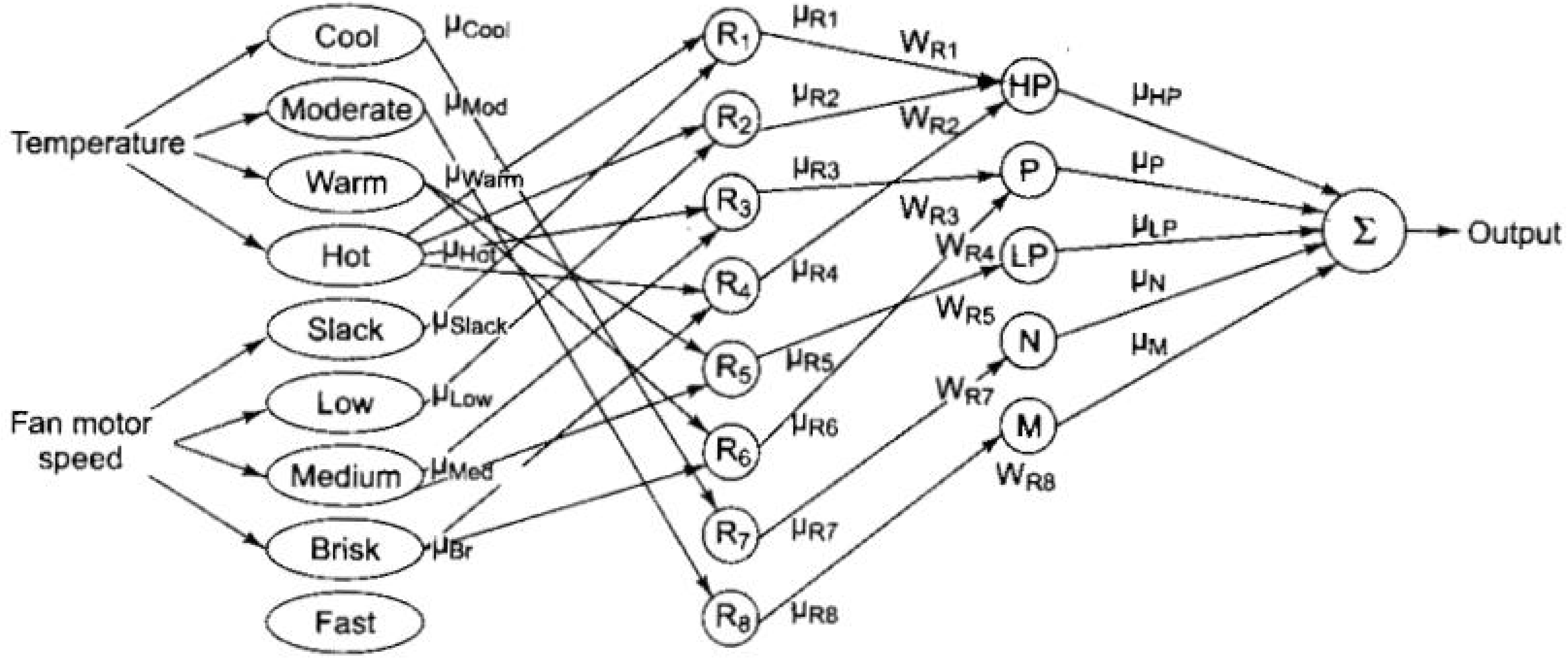


Fig. 22.7 *Neuro Fuzzy Room Cooler*

VIPS
Technical Campus
योग: कर्मसु कौशलम्
IN PURSUIT OF PERFECTION

SCHOOL OF
ENGINEERING AND
TECHNOLOGY

THANK YOU