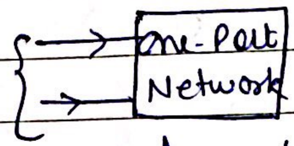
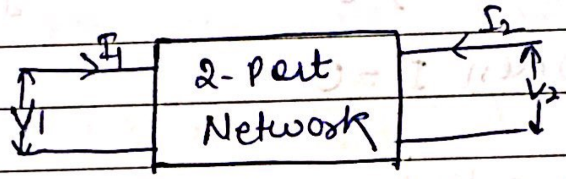
2-Port Networks

Port: A terminal pair is known as 'port' if current entering at one terminal is equal and opposite to the current leaving the other terminal.



2-Terminals → 1 Port.



→ In 2-Port network there are four variables
Two voltages (V_1, V_2)
Two Currents (I_1, I_2)
Out of these four variables, two variables may be considered as 'independent' and two variables as 'dependent'.

→ one of the ports is called the input port, while the other is termed as the o/p port.

① Z-Parameters | open circuit Parameters | Impedance Parameters:-

$$(V_1, V_2) = f(I_1, I_2)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z][I]$$

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — (1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — (2)

→ **Determination of Z-Parameters:-**

Case1: when $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \quad \& \quad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

Case2: when $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} \quad \& \quad Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

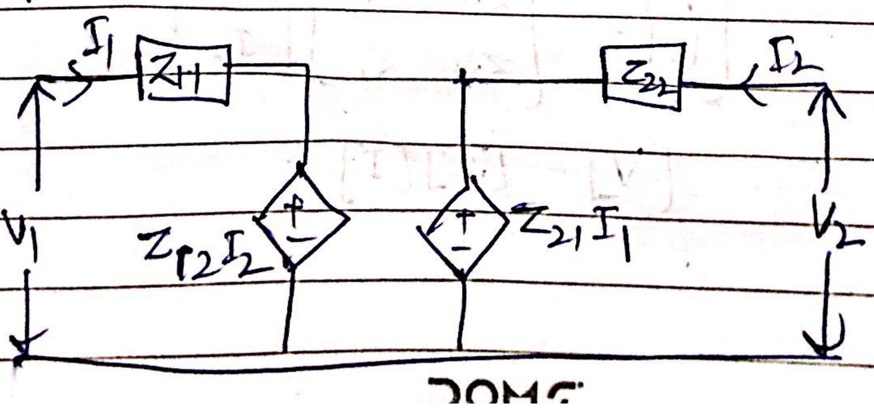
Z_{11} → Input Driving Point Impedance with the output port open circuited.

Z_{21} → forward Transfer Impedance with the output port open circuited.

Z_{12} → Reverse Transfer Impedance with the input port open circuited.

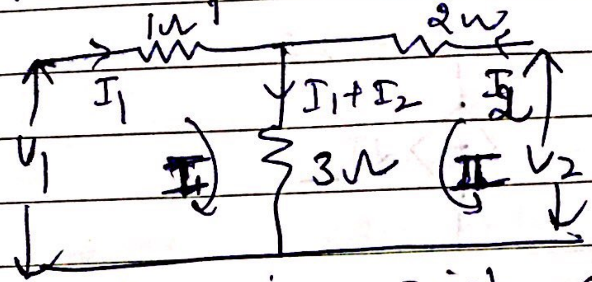
Z_{22} → output Driving Point Impedance with the input port open circuited.

→ Equivalent Circuit of 2-Port n/w in terms of Z-Parameters.



Q.1 for a given network, find z- parameters and also draw the equivalent circuit in terms of z- parameters.

Sol^n



Applying KVL in first and second loop:-

$$V_1 = 1.I_1 + 3(I_1 + I_2)$$

$$\boxed{V_1 = 4I_1 + 3I_2} \quad \text{—— (1)}$$

$$V_2 = 2I_2 + 3(I_1 + I_2)$$

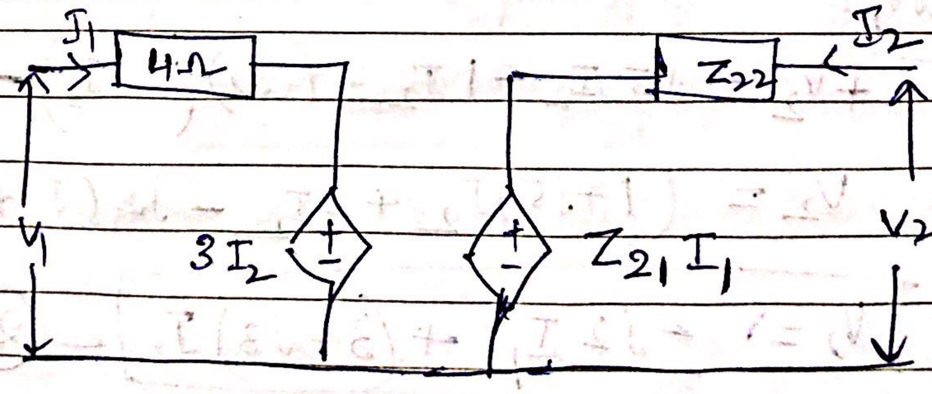
$$\boxed{V_2 = 3I_1 + 5I_2} \quad \text{—— (2)}$$

On comparing eq^n (1) & (2) with the eq^{n}s of z- parameters, we find

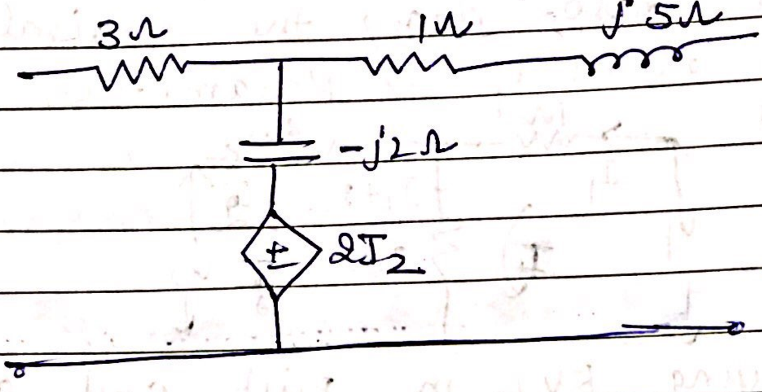
$Z_{11} = 4\Omega$, $Z_{12} = 3\Omega$

$Z_{21} = 3\Omega$, $Z_{22} = 5\Omega$.

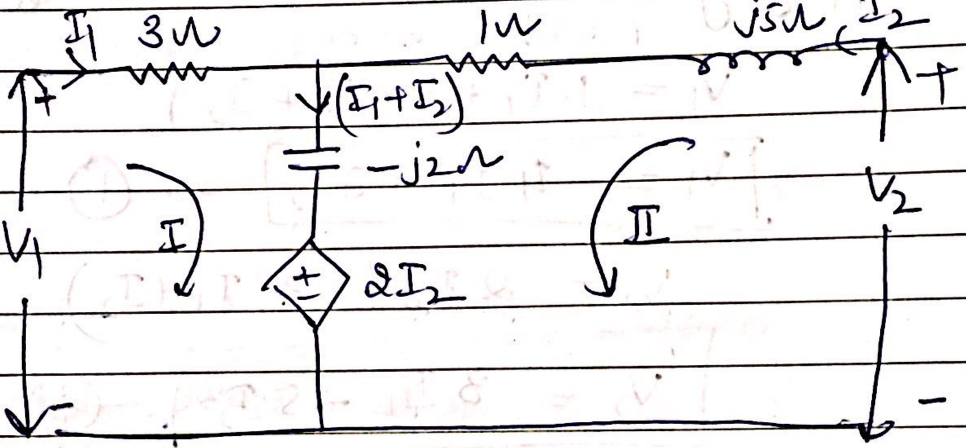
Equivalent circuit



Q.2 Calculate Z-Parameters of the given network.



Solⁿ



from loop 1

$$+V_1 = 3I_1 - (-j2)(I_1+I_2) - 2I_2 = 0$$

$$V_1 - 3I_1 + j2I_1 + j2I_2 - 2I_2 = 0$$

$$\boxed{V_1 = (3-j2)\,I_1 + (2-j2)\,I_2} \quad -(1)$$

from loop 2

$$+V_2 - j5I_2 - 1I_2 - (-j2)(I_1+I_2) - 2I_2 = 0$$

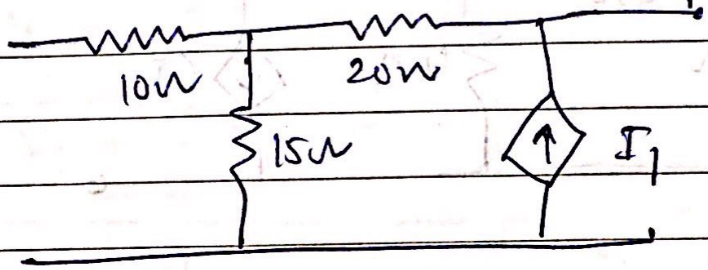
$$V_2 = (1+j5)I_2 + 2I_2 - j2(I_1+I_2)$$

$$\boxed{V_2 = -j2I_1 + (3+j3)\,I_2} \quad -(2)$$

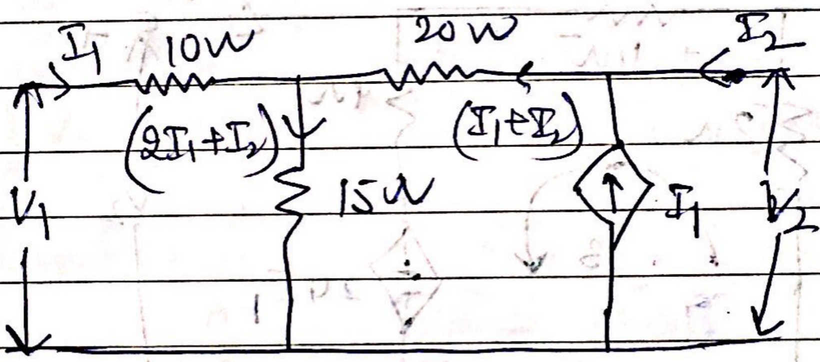
On comparing eqⁿ (1) & (2) with standard eqⁿs of Z-Parameters, we have:-

$$\boxed{\begin{array}{ll} Z_{11} = (3-j2)\,\Omega, & Z_{12} = (2-j2)\,\Omega \\ Z_{21} = (-j2)\,\Omega, & Z_{22} = (3+j3)\,\Omega. \end{array}}$$

Q.3 Determine the Z-Parameters of Given Network.



Solⁿ



$$V_1 = 10I_1 + 15(2I_1 + I_2)$$

$$\boxed{V_1 = 40I_1 + 15I_2} \quad \text{--- (1)}$$

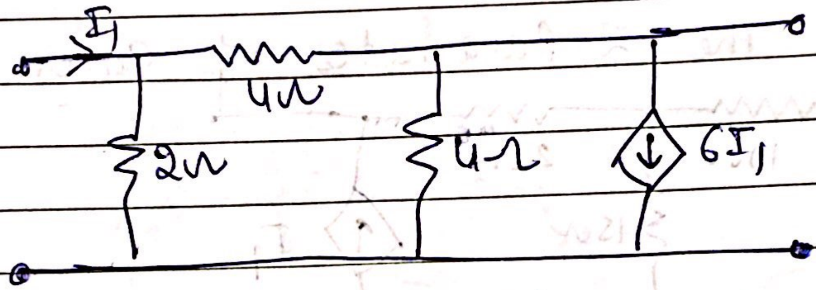
$$V_2 = 20(I_1+I_2) + 15(2I_1+I_2)$$

$$\boxed{V_2 = 50I_1 + 35I_2} \quad \text{--- (2)}$$

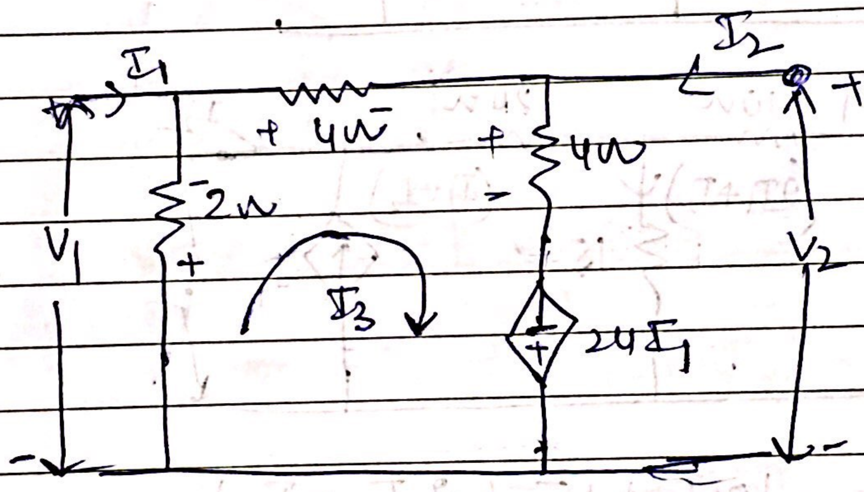
On Comparing these eqⁿs with standard eqⁿs of Z-Parameters.

$$\boxed{\begin{matrix} Z_{11} = 40\Omega, & Z_{12} = 15\Omega \\ Z_{21} = 50\Omega, & Z_{22} = 35\Omega \end{matrix}}$$

Q. 4 Calculate the z- parameters of the given network.



Sol^n



$$V_1 = 2(I_1 - I_3) \quad \text{--- (1)}$$

$$-2(I_3 - I_1) - 4I_3 - 4(I_2 + I_3) + 24I_1 = 0$$

$$-2I_3 + 2I_1 - 4I_3 - 4I_2 - 4I_3 + 24I_1 = 0$$

$$26I_1 - 4I_2 - 10I_3 = 0 \quad \text{--- (2)}$$

$$\boxed{I_3 = 2.6I_1 - 0.4I_2} \quad \text{--- (3)}$$

$+V_2 - 4(I_2 + I_3) + 24 I_1 = 0$

$V_2 = 4(I_2 + I_3) - 24 I_1$

$\boxed{V_2 = -24 I_1 + 4 I_2 + 4 I_3}$ — (4)

Putting value of I_3 from eqn (3) into eqn (B) & (4):

$V_1 = 2 I_1 - 2[2.6 I_1 - 0.4 I_2]$

$\boxed{V_1 = -3.2 I_1 + 0.8 I_2}$ — (5)

$V_2 = -24 I_1 + 4 I_2 + 4[2.6 I_1 - 0.4 I_2]$

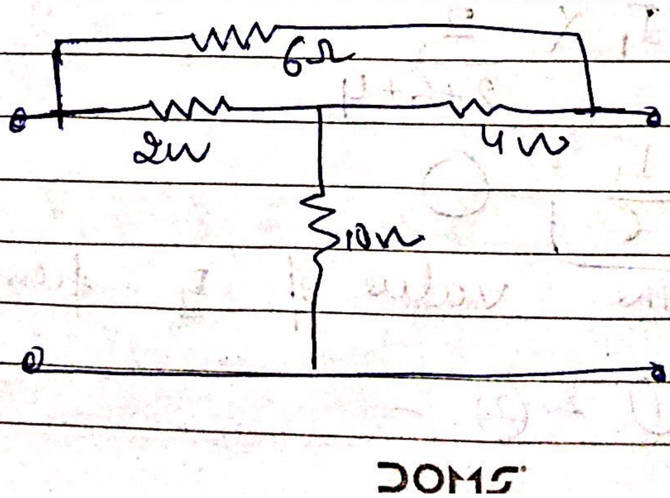
$\boxed{V_2 = -13.6 I_1 + 2.4 I_2}$ — (6)

on comparing eqn (5) & (6) with eqns of 2-P:—

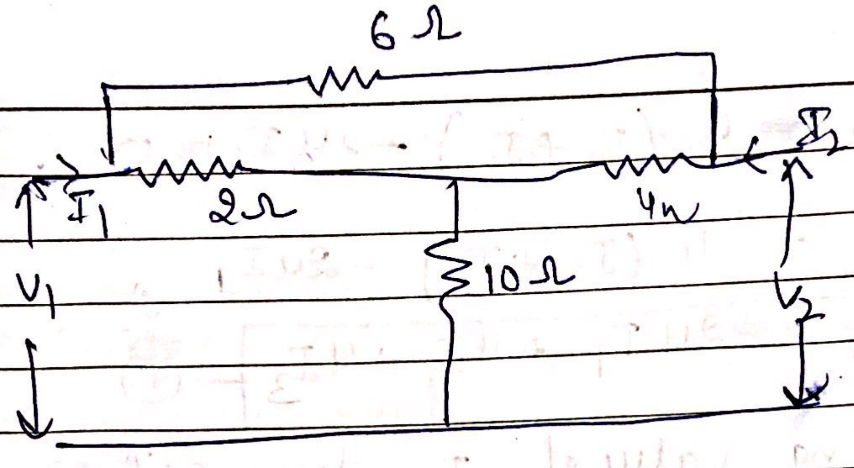
$Z_{11} = -3.2\,\Omega$, $Z_{12} = 0.8\,\Omega$

$Z_{21} = -13.6\,\Omega$, $Z_{22} = 2.4\,\Omega$

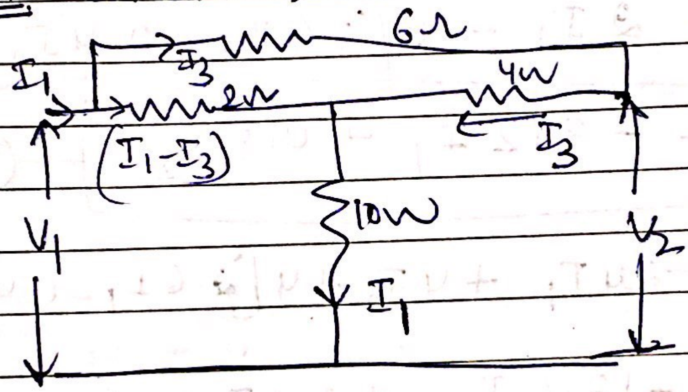
Q.5 obtain the open-circuit parameters of the given network:—



Sol^n



Case 1 : when $I_2 = 0$



$$V_1 = 2(I_1 - I_3) + 10 I_1$$

$$\boxed{V_1 = 12 I_1 - 2 I_3} \quad (1)$$

$$\boxed{V_2 = 4 I_3 + 10 I_1} \quad (2)$$

Using current division law :—

$$I_3 = I_1 \cdot \times \frac{2}{2+6+4}$$

$$\boxed{I_3 = \frac{I_1}{6}} \quad (3)$$

Putting the value of I_3 from eq^n (3) into eq^n (1) & (2) :—

$$V_1 = 12 I_1 - 2\left(\frac{I_1}{6}\right)$$

$$V_1 = \left(\frac{70}{6}\right) I_1$$

$$\boxed{Z_{11} = \frac{V_1}{I_1} = \frac{35}{3}\,\Omega}$$

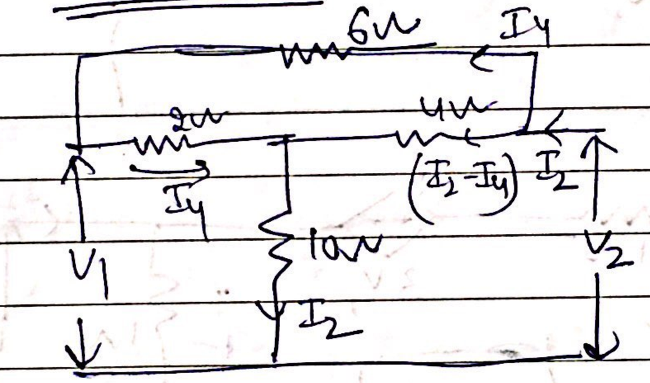
$$V_2 = 4\left(\frac{I_1}{6}\right) + 10 I_1$$

$$V_2 = \frac{2}{3} I_1 + 10 I_1$$

$$V_2 = \frac{32}{3} I_1$$

$$\boxed{Z_{21} = \frac{V_2}{I_1} = \frac{32}{3}\,\Omega}$$

Case 2 : when $I_1 = 0$



$$V_2 = 4(I_2 - I_4) + 10 I_2$$

$$\boxed{V_2 = 14 I_2 - 4 I_4} \quad (4)$$

$$\boxed{V_1 = 2 I_4 + 10 I_2} \quad (5)$$

Using Current Division Law:—

$$I_4 = I_2 \times \frac{4}{6+4+2} = \frac{I_2}{3}$$

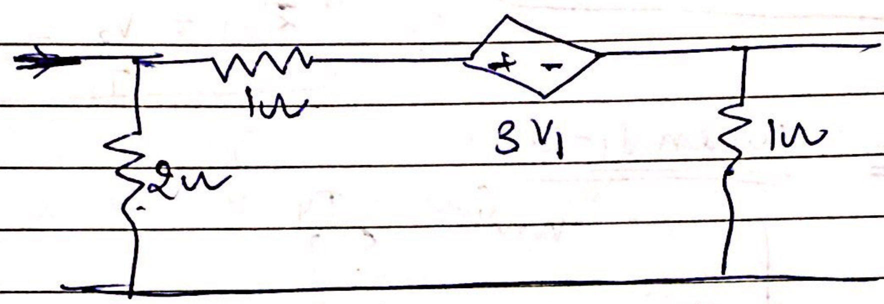
$$V_1 = 2\left(\frac{I_2}{3}\right) + 10 I_2 = \frac{32}{3} I_2$$

$$\boxed{Z_{12} = \frac{V_1}{I_2} = \frac{32}{3}\,\Omega}$$

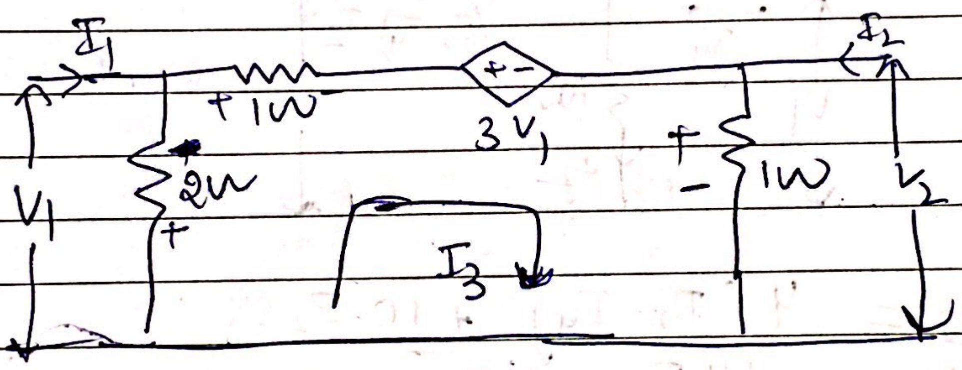
$$V_2 = 14I_2 - 4\left(\frac{I_2}{3}\right) = \frac{38}{3}I_2$$

$$\boxed{Z_{22} = \frac{V_2}{I_2} = \frac{38}{3}\,\Omega}$$

Q.6 find the Z-Parameters of given network:-



Sol^n



$$V_1 = 2(I_1 - I_3) \quad \text{—(1)}$$

$$V_2 = 1(I_2 + I_3) \quad \text{—(2)}$$

$$-2(I_3 - I_1) - 1I_3 - 3V_1 - 1(I_2 + I_3) = 0$$

$$-2I_3 + 2I_1 - I_3 - 3V_1 - I_2 - I_3 = 0$$

$$2I_1 - 4I_3 - 3V_1 - I_2 = 0$$

$$4I_3 = 2I_1 - I_2 - 3V_1$$

$$\text{So } \boxed{I_3 = \left(\frac{I_1}{2} - \frac{I_2}{4} - \frac{3}{4}V_1\right)} \quad \text{--- (3)}$$

Putting value of I_3 from eqn (3) into (1) & (2):—

$$V_1 = 2I_1 - 2\left(\frac{I_1}{2} - \frac{I_2}{4} - \frac{3}{4}V_1\right)$$

$$\boxed{V_1 = -2I_1 - I_2} \quad \text{--- (4)}$$

$$V_2 = I_2 + \left(\frac{I_1}{2} - \frac{I_2}{4} - \frac{3}{4}V_1\right)$$

$$V_2 = \frac{I_1}{2} + \frac{3}{4}I_2 - \frac{3}{4}V_1 \quad \text{--- (5)}$$

Putting value of V_1 from eqn (4) into (5):—

$$V_2 = \frac{I_1}{2} + \frac{3}{4}I_2 + \frac{3}{4}(-2I_1 - I_2)$$

$$= \frac{I_1}{2} + \frac{3}{4}I_2 + \frac{3}{2}I_1 + \frac{3}{4}I_2$$

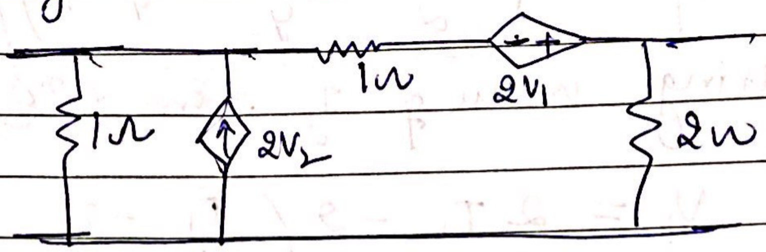
$$\boxed{V_2 = 2I_1 + \frac{3}{2}I_2} \quad \text{--- (6)}$$

On Comparing (5) & (6) with eqn of Z-Parameters

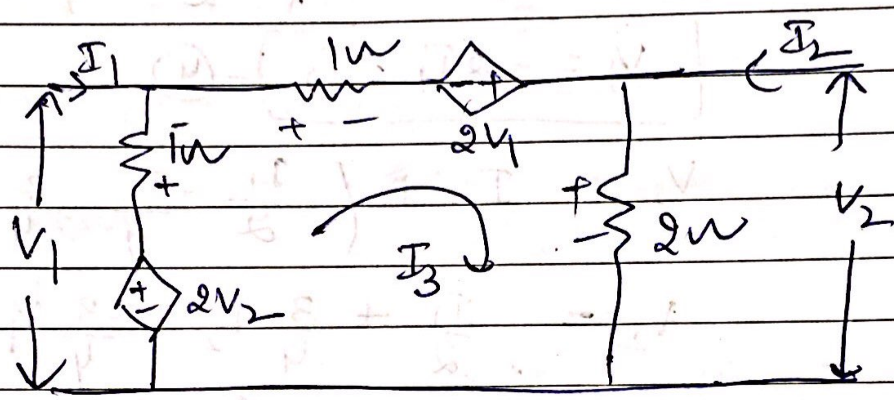
$$\boxed{\begin{array}{ll} Z_{11} = -2\,\Omega, & Z_{12} = -1\,\Omega \\ Z_{21} = 2\,\Omega, & Z_{22} = \frac{3}{2}\,\Omega. \end{array}}$$

Ans.

Q.7 For a given network calculate Z-Parameters.



Sol^n



Applying KVL:—

$$V_1 = 1.(I_1 - I_3) + 2V_2$$

$$V_1 = I_1 - I_3 + 2V_2$$

$$\boxed{I_3 = I_1 + 2V_2 - V_1} \quad —(1)$$

$$\boxed{V_2 = 2(I_2 + I_3)} \quad —(2)$$

$$+2V_2 - 1(I_3 - I_1) - 1\,I_3 + 2V_1 - 2(I_2 + I_3) = 0$$

$$\boxed{2V_1 + 2V_2 = -I_1 + 2I_2 + 4I_3} \quad —(3)$$

Putting the value of I_3 from eq^n (1) into eq^n (2) & (3):—

$$2V_1 + 2V_2 = -I_1 + 2I_2 + 4(I_1 + 2V_2 - V_1)$$

or $\boxed{6V_1 - 6V_2 = 3I_1 + 2I_2}$ — (4).

And $V_2 = 2I_2 + 2(I_1 + 2V_2 - V_1)$

$$\boxed{2V_1 - 3V_2 = 2I_1 + 2I_2} \text{ — (5).}$$

$$-3V_2 = 2I_1 + 2I_2 - 2V_1$$

$$\boxed{-6V_2 = 4I_1 + 4I_2 - 4V_1} \text{ — (6)}$$

Putting value of $-6V_2$ from eq^n (6) into (4):—

$$6V_1 + (4I_1 + 4I_2 - 4V_1) = 3I_1 + 2I_2$$

$$2V_1 = -I_1 - 2I_2$$

$$\boxed{V_1 = -\frac{1}{2}I_1 - I_2} \text{ — (7).}$$

Putting value of V_1 from eq^n (7) into eq^n (4):—

$$6\left[-\frac{1}{2}I_1 - I_2\right] - 6V_2 = 3I_1 + 2I_2$$

$$-3I_1 - 6I_2 - 6V_2 = 3I_1 + 2I_2$$

$$-3I_1 - 6I_2 - 3I_1 - 2I_2 = 6V_2$$

$$-6I_1 - 8I_2 = 6V_2$$

$$\boxed{V_2 = -I_1 - \frac{4}{3}I_2} \text{ — (8)}$$

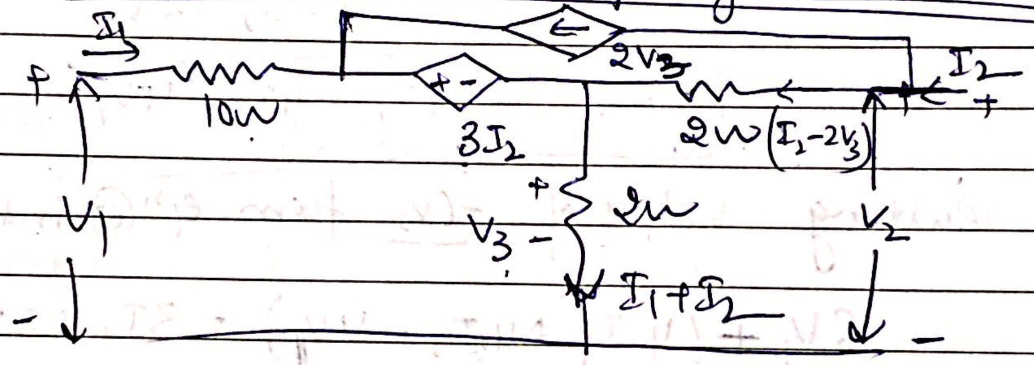
DOMS

So Comparing eqⁿ (7) & (8) with eqⁿ of Z-Parameters;—

$Z_{11} = -\frac{1}{2}\,\Omega$, $Z_{12} = -1\,\Omega$

$Z_{21} = -1\,\Omega$, $Z_{22} = -\frac{4}{3}\,\Omega$

Q.8 find Z-Parameter of given network:—



Solⁿ

$$V_1 = 10I_1 + 3I_2 + 2(I_1+I_2)$$

$$\boxed{V_1 = 12I_1 + 5I_2} \quad \text{— (1)},$$

$$V_2 = 2(I_2 - 2V_3) + 2(I_1 + I_2)$$

$$\boxed{V_2 = 2I_1 + 4I_2 - 4V_3} \quad \text{— (2)},$$

$$\boxed{V_3 = 2(I_1 + I_2)} \quad \text{— (3)}$$

Putting value of V_3 from eqⁿ (3) into eqⁿ (2):—

DOMS

$$V_2 = 2I_1 + 4I_2 - 4[2(I_1+I_2)]$$

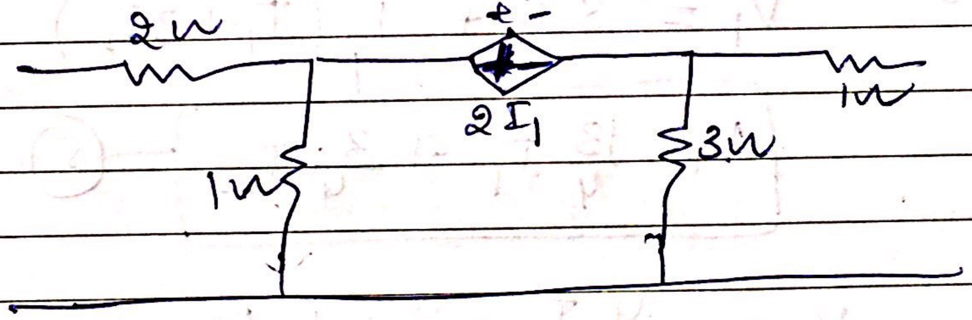
$$V_2 = 2I_1 + 4I_2 - 8I_1 - 8I_2$$

$$\boxed{V_2 = -6I_1 - 4I_2} \quad \text{--- } (4)$$

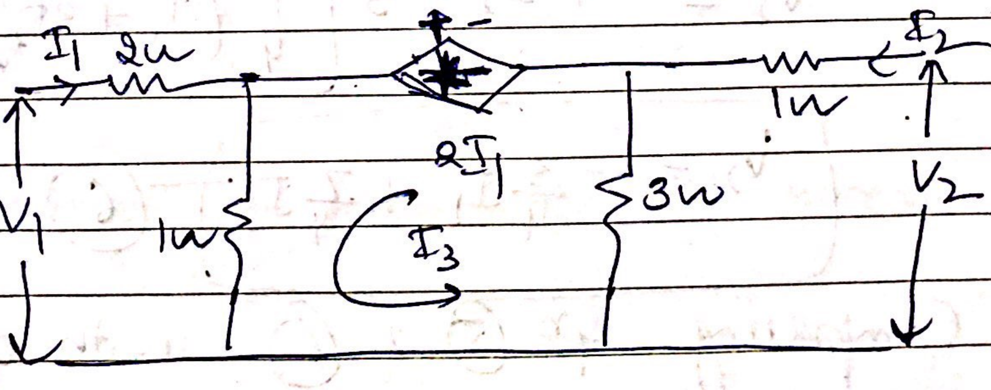
So, comparing eqⁿ (1) & (4) with eqⁿˢ of Z-P:—

$$\boxed{\begin{aligned} Z_{11} &= 12\Omega, \quad Z_{12} = 5\Omega \\ Z_{21} &= -6\Omega, \quad Z_{22} = -4\Omega \end{aligned}}$$ Ans.

Q.9 find Z-Parameters of Given Network:



Solⁿ



$$V_1 = 2I_1 + 1\cdot(I_1 + I_3)$$

$$\boxed{V_1 = 3I_1 + I_3} \quad \text{--- } (1)$$

$$V_2 = 1 \cdot I_2 + 3(I_2 - I_3)$$

$$\boxed{V_2 = 4I_2 - 3I_3} \quad \text{— (2).}$$

$$2I_1 = 1 \cdot (I_1 + I_3) + 3(I_3 - I_2)$$

$$\boxed{0 = -I_1 - 3I_2 + 4I_3} \quad \text{— (3).}$$

from eq^n (3) :—

$$\boxed{I_3 = \frac{1}{4}(I_1 + 3I_2)} \quad \text{— (4).}$$

So
$$V_1 = 3I_1 + \frac{1}{4}(I_1 + 3I_2)$$

$$\boxed{V_1 = \frac{13}{4}I_1 + \frac{3}{4}I_2} \quad \text{— (5).}$$

$$V_2 = 4I_2 - 3\left[\frac{1}{4}(I_1 + 3I_2)\right]$$

$$\boxed{V_2 = -\frac{3}{4}I_1 + \frac{7}{4}I_2} \quad \text{— (6)}$$

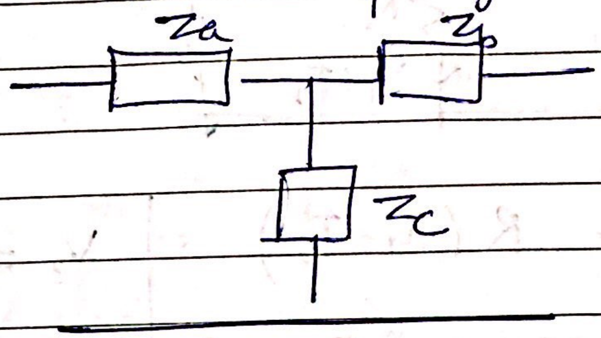
Comparing eq^n (5) & (6) with eq^n of 2-P.

$Z_{11} = \frac{13}{4}\,\Omega$, $Z_{12} = \frac{3}{4}\,\Omega$

$Z_{21} = -\frac{3}{4}\,\Omega$, $Z_{22} = \frac{7}{4}\,\Omega$.

DOMS

Q.9 obtain Z-Parameters of given T-network:—

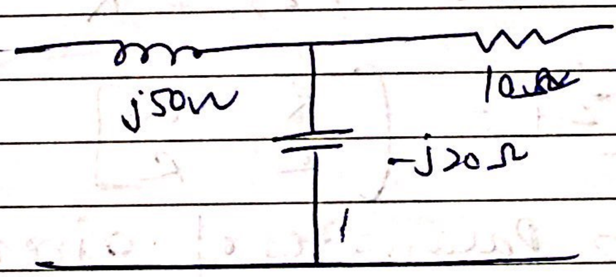


Soln

$$Z_{11} = (Z_a + Z_c), \quad Z_{22} = (Z_b + Z_c)$$

$$Z_{12} = Z_{21} = Z_C$$

Q10



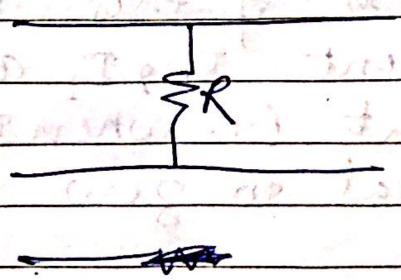
Soln

$$Z_{11} = j50 - j20 = j30\ \Omega$$

$$Z_{12} = Z_{21} = -j20\ \Omega$$

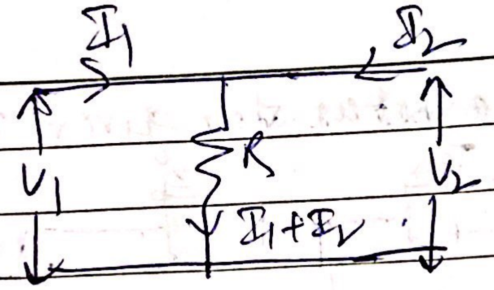
$$Z_{22} = (10 - j20)\ \Omega.$$

Q.11 obtain Z-Parameters of given network



DOMS

Sol^n



$$V_1 = R(I_1+I_2) \qquad V_2 = R(I_1+I_2)$$

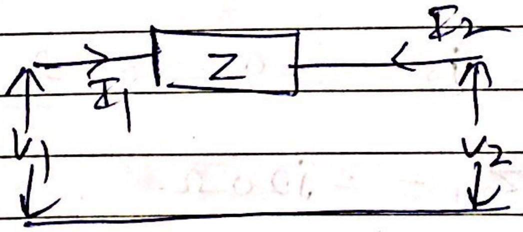
$$V_1 = RI_1 + RI_2 \quad \text{--- (1)} \qquad V_2 = RI_1 + RI_2 \quad \text{--- (2)}$$

So $Z_{11} = R$, $Z_{12} = R$

$Z_{21} = R$, $Z_{22} = R$

$$[Z] = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$

Q. 12 obtain z-Parameters of given network:-



Sol^n

$$V_1 = I_1 Z + V_2$$

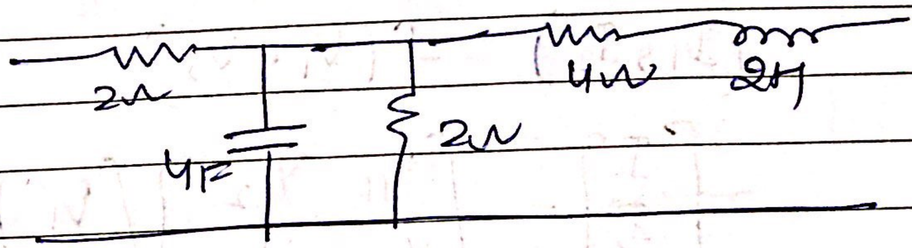
while $I_1 = -I_2$

z-Parameters of this n/w don't exist, since current I_1 & I_2 are not independent i.e. when $I_2 = 0$, I_1 is also equal to zero & vice-versa.

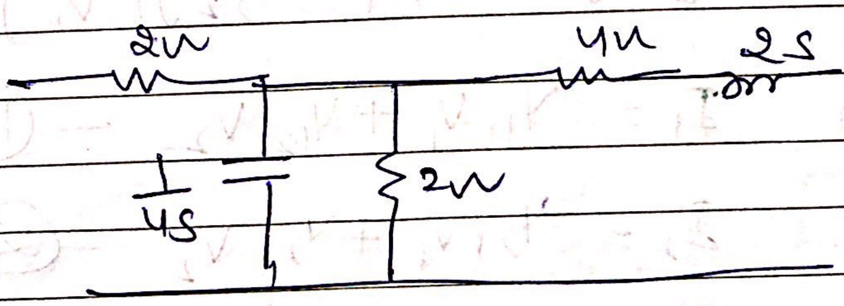
DOMS

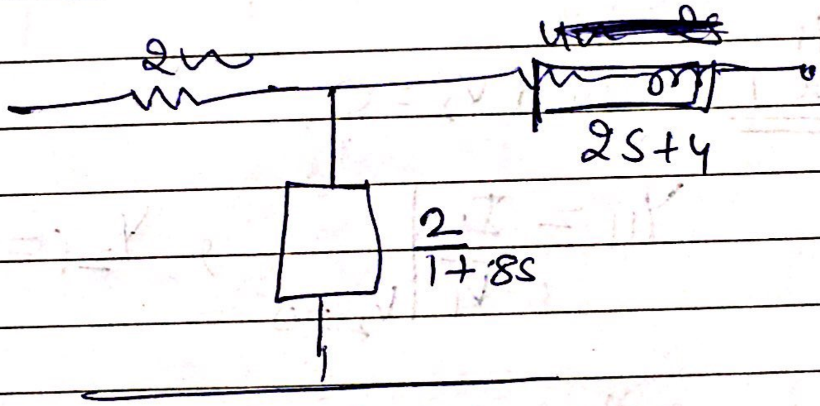
Scanned with CamScanner

Q.13 obtain z-Parameters of given network:-



Soln





$$\text{So } Z_{11} = 2 + \frac{2}{1+8S} = \frac{4+16S}{1+8S}$$

$$Z_{12} = Z_{21} = \frac{2}{1+8S}$$

$$Z_{22} = 2S+4+\frac{2}{1+8S} = \frac{6+34S+16S^2}{1+8S}$$

② Short Circuit | Admittance | Y- Parameters :-

In Y- Parameters :-

$$(I_1, I_2) = f(V_1, V_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{— (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{— (2)}$$

~~Y_{11}~~

Case 1 when $V_2 = 0$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}, \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Case 2 when $V_1 = 0$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}, \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

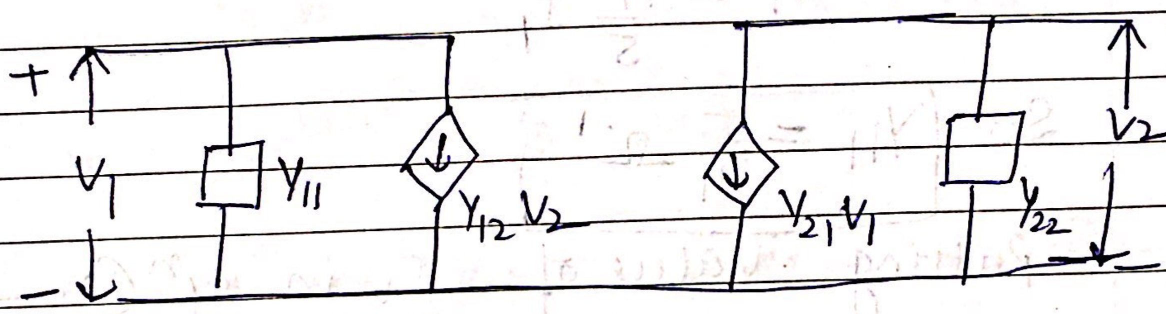
$Y_{11} \rightarrow$ I/P Driving Point Admittance when O/P circuit is short circuited.

$Y_{21} \rightarrow$ forward transfer Admittance when O/P circuit is short circuited.

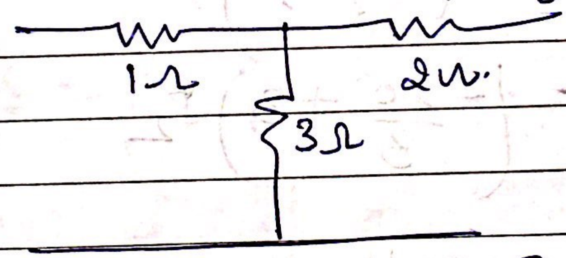
$Y_{22} \rightarrow$ O/P Driving Point Admittance with the I/P circuit short circuited.

Y_{12} → Reverse Transfer Admittance with the I/P short circuited.

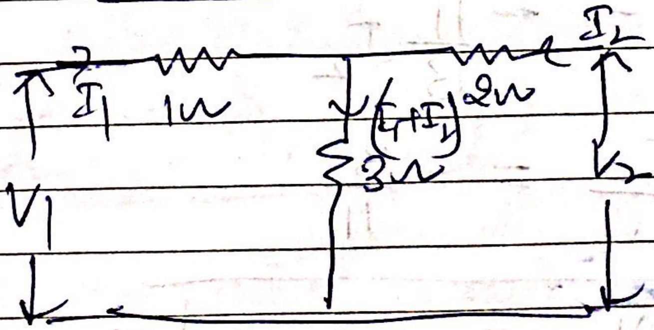
Equivalent circuit of 2-Port n/w in terms of Y- Parameters:-



Q.1 Obtain Y- Parameters of given network:-



Solⁿ



$$V_1 = 4I_1 + 3I_2 \quad \text{--- (1)}$$

$$V_2 = 3I_1 + 5I_2 \quad \text{--- (2)}$$

Case 1:- when $V_2 = 0$

$$3I_1 = -5I_2$$

$\cancel{I_1 = -\frac{5}{3}I_2}$

$I_1 = -\frac{5}{3}I_2$

$I_2 = -\frac{3}{5}I_1$

DOMS

Putting the value of I_2 in eq^n ① :—

$$V_1 = 4I_1 + 3\left(-\frac{3}{5}I_1\right)$$

$$V_1 = 4I_1 - \frac{9}{5}I_1$$

$$V_1 = \frac{11}{5}I_1$$

So $\boxed{Y_{11} = \frac{5}{11}\,\Omega^{-1}}$

Putting value of I_1 in eq^n ① :—

$$V_1 = 4\left(-\frac{5}{3}I_2\right) + 3I_2$$

$$V_1 = \left(-\frac{20}{3} + 3\right)I_2$$

$$V_1 = -\frac{11}{3}I_2$$

So $\frac{I_2}{V_1} = -\frac{3}{11}$

So $\boxed{Y_{21} = -\frac{3}{11}\,\Omega^{-1}}$

Case 2 : when $V_1 = 0$

$$4I_1 = -3I_2$$

$$I_1 = -\frac{3}{4}I_2 \quad \& \quad I_2 = -\frac{4}{3}I_1$$

DOMS

_/_/_

Putting the value of I_2 in eq^n (2):—

$$V_2 = 3I_1 + 5\left(-\frac{4}{3}I_1\right)$$

$$V_2 = 3I_1 - \frac{20}{3}I_1$$

$$V_2 = -\frac{11}{3}I_1$$

$$\boxed{Y_{12} = \frac{I_1}{V_2} = -\frac{3}{11}\ \Omega^{-1}}$$

Putting the value of I_1 in eq^n (2):—

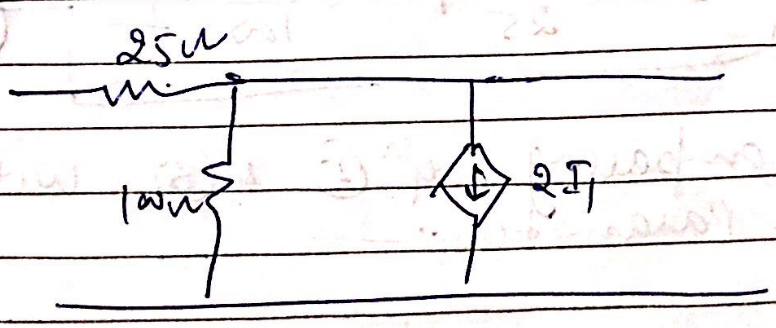
$$V_2 = 3\left(-\frac{3}{4}I_2\right) + 5I_2$$

$$V_2 = \left(-\frac{9}{4} + 5\right)I_2$$

$$V_2 = \frac{11}{4}I_2$$

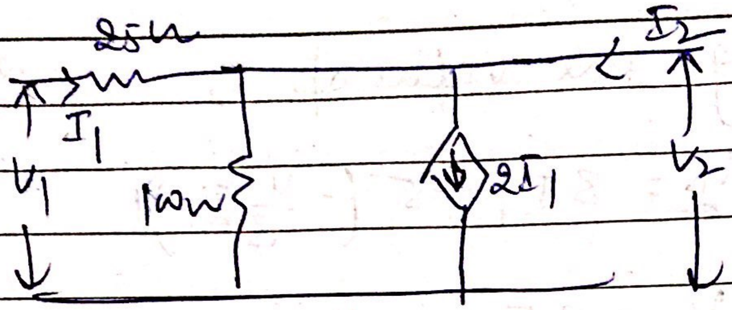
$$\boxed{Y_{22} = \frac{I_2}{V_2} = \frac{4}{11}\ \Omega^{-1}}$$

Q.15 find the short circuit Parameters of given network.



DOMS

Solⁿ



Applying KCL :→

$$I_1 + I_2 = \frac{V_2}{100} + 2I_1$$

$$\boxed{I_1 = -\frac{V_2}{100} + I_2} \quad \text{--- (1)}.$$

And Also

$$I_1 = \frac{V_1 - V_2}{25}$$

$$\boxed{I_1 = \frac{1}{25}V_1 - \frac{1}{25}V_2} \quad \text{--- (2)}.$$

Putting the value of I_1 in eqⁿ (1) :→

$$\frac{V_1 - V_2}{25} = -\frac{V_2}{100} + I_2$$

$$\boxed{I_2 = \frac{1}{25}V_1 - \frac{3}{100}V_2} \quad \text{--- (3)}$$

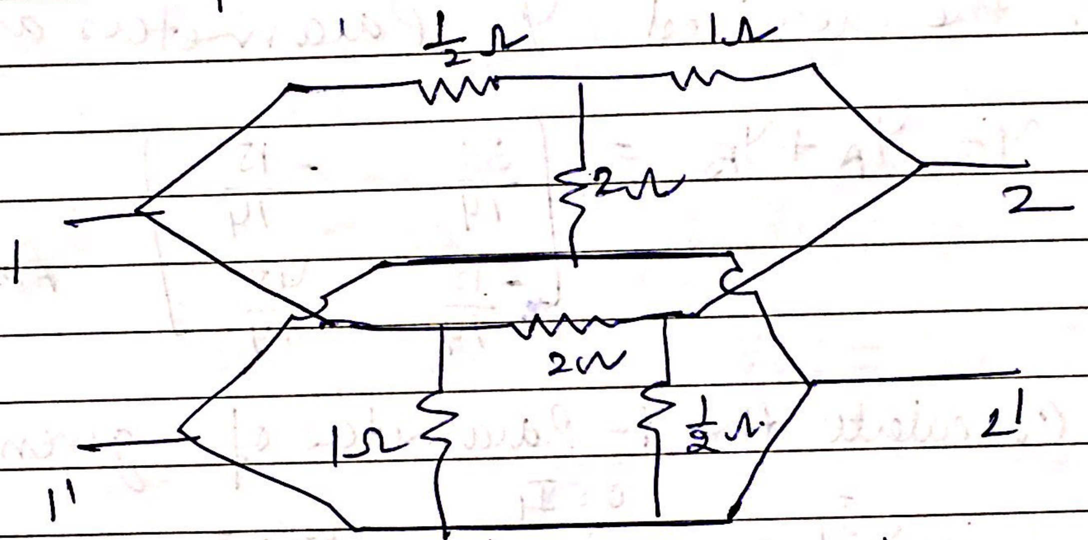
On comparing eqⁿ (2) & (3) with eqⁿ of ~~Y~~-Parameters :→

DOMS

$$Y_{11} = \frac{1}{25}\,\Omega^{-1}, \quad Y_{12} = \frac{-1}{25}\,\Omega^{-1}$$

$$Y_{21} = \frac{1}{25}\,\Omega^{-1}, \quad Y_{22} = \frac{-3}{100}\,\Omega^{-1}$$

Q.16 The given network consists of a resistive-T and a resistive-π network connected in parallel. Calculate Y-Parameters.



Soln Y-Parameter of Ist network :-

$$Y_A = [Z_A]^{-1} = \begin{bmatrix} \frac{1}{2}+2 & 2 \\ 2 & 2+1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{5}{2} & 2 \\ 2 & 3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{6}{7} & \frac{-4}{7} \\ \frac{-4}{7} & \frac{5}{7} \end{bmatrix}$$

DOMS

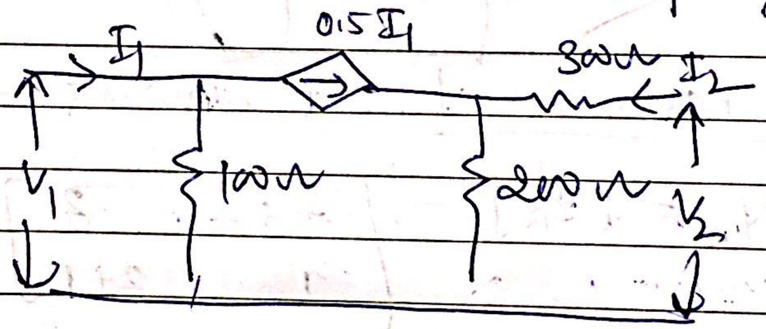
$$Y_B = \begin{bmatrix} \frac{1}{1}+\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}+2 \end{bmatrix}$$

$$Y_B = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}.$$

So the required Y- Parameters are:-

$$Y = Y_A + Y_B = \begin{bmatrix} \frac{33}{14} & \frac{-15}{14} \\ \frac{-15}{14} & \frac{45}{14} \end{bmatrix}$$ Ans.

Q.17 Calculate the Y- Parameters of given network



Sol^n

$$V_1 = 100(I_1 - 0.5 I_1$$

$$\boxed{V_1 = 50 I_1}, \quad \boxed{I_1 = \frac{V_1}{50}} \text{ — (1)}$$

$$V_2 = 300 I_2 + 200(I_2 + 0.5 I_1)$$

$$\boxed{V_2 = 100 I_1 + 500 I_2} \text{ — (2).}$$

Putting the value of I_1 from eq^n (1) into (2):-

$$V_2 = 100\left(\frac{V_1}{50}\right) + 500\, I_2$$

$$V_2 = 2V_1 + 500\, I_2$$

$$500\, I_2 = -2V_1 + V_2$$

$$I_2 = -\frac{2}{500}V_1 + \frac{1}{500}V_2$$

$$\boxed{I_2 = -\frac{1}{250}V_1 + \frac{1}{500}V_2}$$

So

$$\boxed{\begin{array}{l} Y_{11} = \frac{1}{50}\,\Omega^{-1}, \quad Y_{12} = 0 \\ Y_{21} = -\frac{1}{250}\,\Omega^{-1}, \quad Y_{22} = \frac{1}{500}\,\Omega^{-1} \end{array}}$$ Ans

(3) h-Parameters (Hybrid Parameters).

Hybrid parameters are widely used in electronic circuit, especially in constructing models for transistors

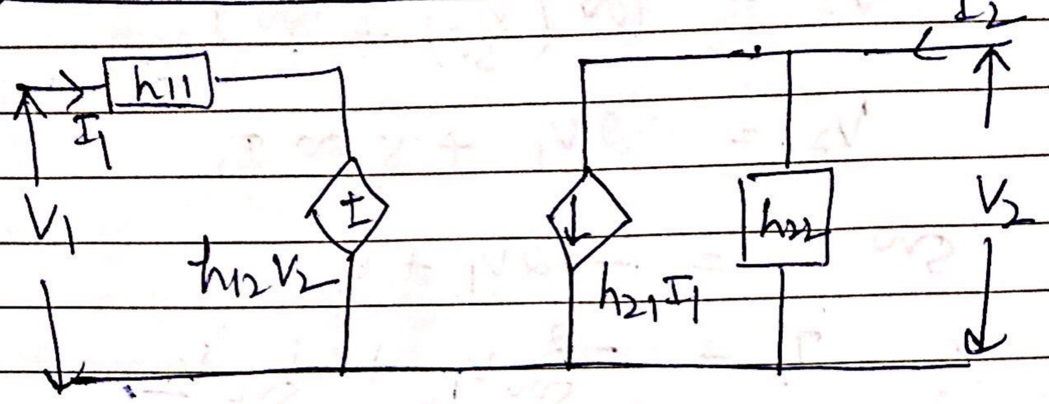
$$(V_1, I_2) = f(I_1, V_2)$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

Equivalent circuit:-



Case1 :- when $V_2 = 0$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad , \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

Case2 when $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad , \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

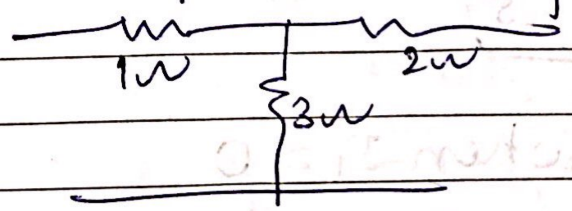
$h_{11} \rightarrow$ I/P impedance with O/P circuit short circuited.

$h_{21} \Rightarrow$ forward Current Gain with O/P Port short circuited.

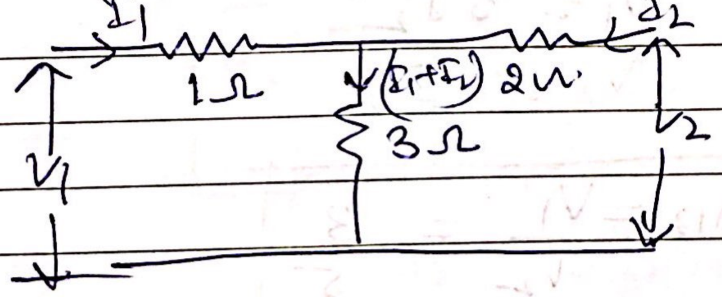
$h_{12} \rightarrow$ Reverse Voltage Gain with the I/P Port open circuited.

$h_{22} \rightarrow$ output Admittance with the input port open circuited.

Q.18 obtain h- parameters of given network.



Sol^n



$V_1 = 4I_1 + 3I_2$ —(1)

$V_2 = 3I_1 + 5I_2$ —(2)

Case I when $V_2 = 0$

$3I_1 = -5I_2$

$I_2 = -\frac{3}{5}I_1$, $I_1 = -\frac{5}{3}I_2$

Putting value of I_2 in eq^n (1):-

$V_1 = 4I_1 + 3\left(-\frac{3}{5}I_1\right)$

$V_1 = 4I_1 - \frac{9}{5}I_1$

$\boxed{V_1 = \frac{11}{5}I_1}$

So $\boxed{h_{11} = \frac{11}{5}\,\Omega}$

$$\boxed{h_{21} = -\frac{3}{5}}$$

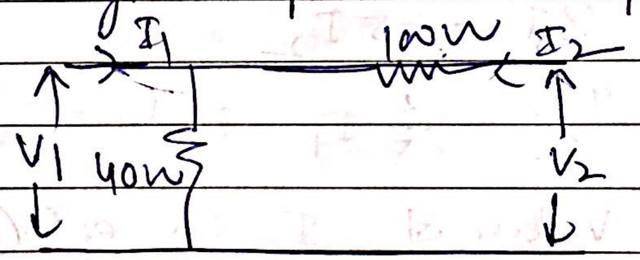
Case 2 :- when $I_1 = 0$

$$V_1 = 3 I_2$$
$$V_2 = 5 I_2$$

so $\boxed{h_{12} = \frac{V_1}{V_2} = \frac{3}{5}}$

$$\boxed{h_{22} = \frac{I_2}{V_2} = \frac{1}{5}\ \Omega^{-1}}$$

Q. 19 find hybrid parameters of given network :-



Sol^n

$$V_1 = 40(I_1 + I_2)$$
$$\boxed{V_1 = 40 I_1 + 40 I_2} \quad (1)$$

$$V_2 = 100 I_2 + 40(I_1 + I_2)$$
$$\boxed{V_2 = 40 I_1 + 140 I_2} \quad (2)$$

DOMS

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Case 1 when $V_2 = 0$

$$40 I_1 = -140 I_2$$

$$\boxed{I_1 = -\frac{7}{2} I_2} \quad (3)$$

$$\boxed{I_2 = -\frac{2}{7} I_1} \quad (3).$$

Putting value of I_2 from (3) in (1):-

$$V_1 = 40 I_1 + 40\left(-\frac{2}{7} I_1\right)$$

$$V_1 = 40 I_1 - \frac{80}{7} I_1$$

$$V_1 = \frac{200}{7} I_1$$

So $\boxed{h_{11} = \frac{200}{7}\ \Omega}$

$$\boxed{h_{21} = \frac{I_2}{I_1} = -\frac{2}{7}}$$

Case 2:- when $I_1 = 0$

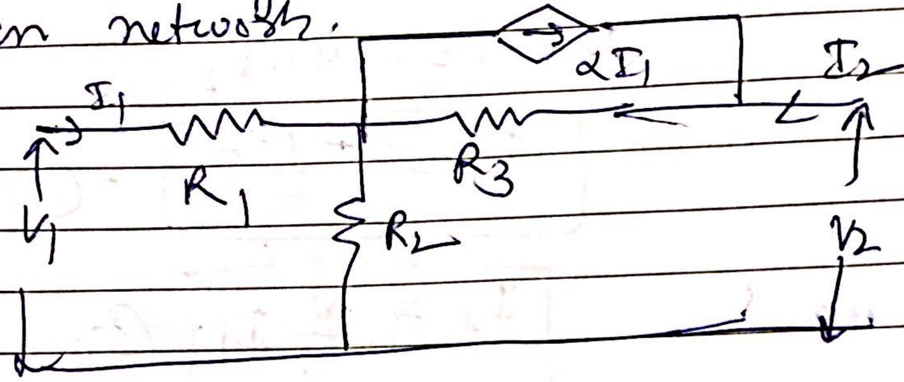
$$V_1 = 40 I_2$$

$$V_2 = 140 I_2$$

$\boxed{h_{12} = \frac{40}{140} = \frac{2}{7}}$, $\boxed{h_{22} = \frac{I_2}{V_2} = \frac{1}{140}\ \Omega^{-1}}$

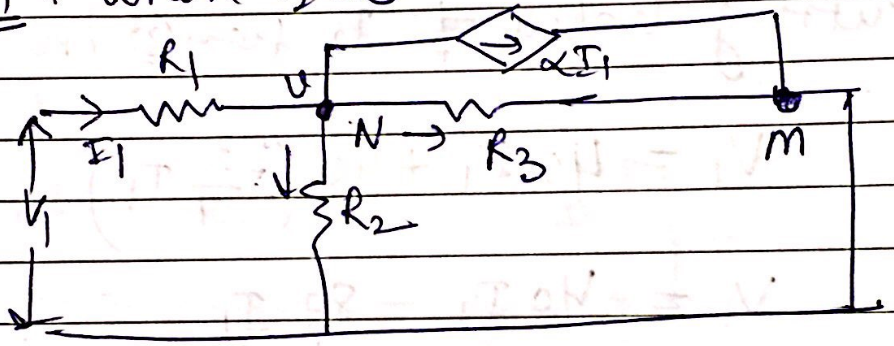
DOMS

Q.20 find the hybrid parameters of the given network.



Sol^n

Case 1 : when $V_2 = 0$



Let v be the voltage at the j^n of R_1, R_2 & R_3. then applying KCL at node N:—

$$I_1 = \frac{v}{R_2} + \frac{v}{R_3} + \alpha I_1$$

this gives:—

$$v = \frac{(1-\alpha) I_1 R_2 R_3}{R_2 + R_3}$$

Applying KVL:—

$$V_1 = I_1 R_1 + v$$

Putting value of v:—

DOMS

$$V_1 = I_1 . R_1 + \frac{(1-\alpha) I_1 R_2 R_3}{R_2+R_3}$$

$$V_1 = I_1 \left[R_1 + \frac{(1-\alpha) R_2 R_3}{R_2+R_2} \right]$$

Applying KCL at node M :-

$$\frac{v}{R_3} + \alpha I_1 + I_2 = 0$$

$$I_2 = -\alpha I_1 - \frac{v}{R_3}$$

$$= -\alpha I_1 - \frac{1}{R_3} \left[\frac{(1-\alpha) I_1 R_2 R_3}{R_2+R_3} \right]$$

$$I_2 = I_1 \left[-\alpha - \frac{(1-\alpha) R_2}{R_2+R_3} \right]$$

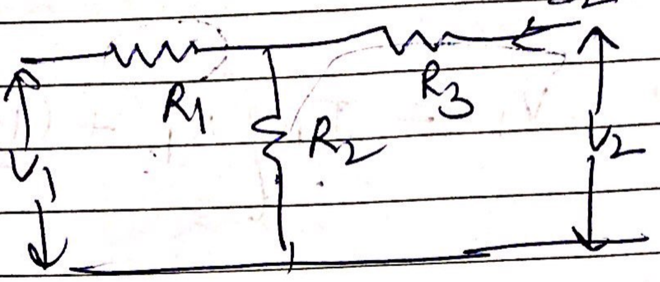
$$= -I_1 \left[\frac{\alpha R_2 + \alpha R_3 - R_2 + \alpha R_2}{R_2+R_3} \right]$$

$$I_2 = -I_1 \left[\frac{\alpha R_3 + R_2}{R_2+R_3} \right]$$

So $h_{11} = \frac{V_1}{I_1} = \left[R_1 + \frac{(1-\alpha) R_2 R_3}{R_2+R_3} \right] \Omega$

$$h_{21} = \frac{I_2}{I_1} = -\left[\frac{\alpha R_3 + R_2}{R_2+R_3} \right]$$

Case 2 :- when $I_1 = 0$



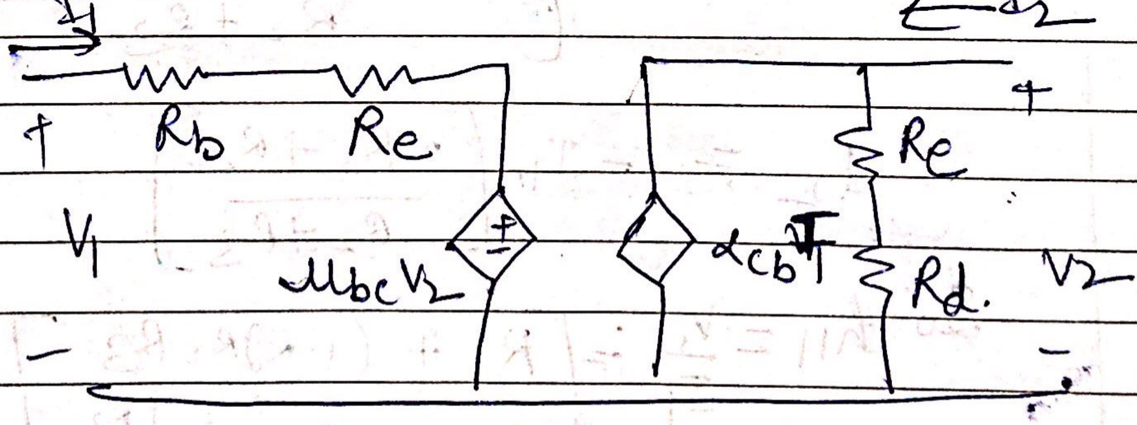
$$V_2 = I_2 R_2 + I_2 R_3$$

$$V_1 = I_2 R_2$$

So $\boxed{h_{12} = \frac{V_1}{V_2} = \frac{R_2}{R_2 + R_3}}$

$\boxed{h_{22} = \frac{I_2}{V_2} = \frac{1}{R_2 + R_3} \ \Omega^{-1}}$

Q.21) Calculate h-parameters of a model of a common-emitter connected transistor circuit.



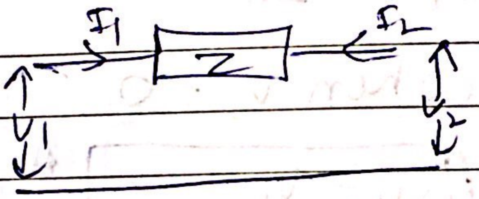
Sol^n) $V_1 = (R_b + R_e) I_1 + \mu_{bc} V_2$ —— (1)

$$I_2 = \alpha_{cb} I_1 + \frac{V_2}{R_e + R_d} \quad — (2)$$

So

$$\boxed{h_{11} = R_b + R_e \ \Omega, \quad h_{12} = \mu_{bc} \qquad h_{21} = \alpha_{cb}, \quad h_{22} = \frac{1}{R_e + \frac{1}{R_d}} \ \Omega^{-1}}$$

Q.22 obtain h-parameters of given network:



$$V_1 = V_2 + I_1 Z \quad — (1)$$

$$I_1 = -I_2$$

~~So h_{11}~~ Case1: when $V_2 = 0$

$$V_1 = I_1 Z$$

$$\boxed{h_{11} = \frac{V_1}{I_1} = Z \ \Omega}, \quad \boxed{h_{21} = \frac{I_2}{I_1} = -1}$$

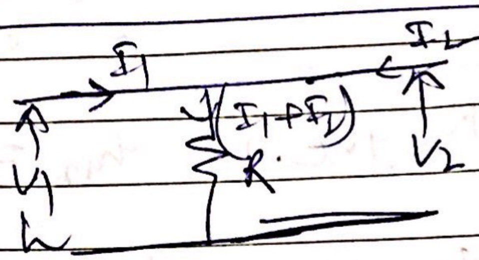
Case2: when $I_1 = 0$

$$V_1 = V_2$$

$$I_1 = -I_2$$

$$\boxed{h_{12} = \frac{V_1}{V_2} = 1}, \quad \boxed{h_{22} = \frac{I_2}{V_2} = 0}$$

Q.23 obtain h-Parameters of given network:-



Sol^n

$V_1 = RI_1 + RI_2$

$V_2 = RI_1 + RI_2$

$V_1 = V_2$

Case1:- when $V_2 = 0$

$$\boxed{h_{11} = \frac{V_1}{I_1} = 0\,\Omega}$$

As $\boxed{V_1 = V_2 = 0}$

$$\boxed{h_{21} = \frac{I_2}{I_1} = -1}$$

Case2:- when $I_1 = 0$

$V_1 = RI_2$

$V_2 = RI_2$

$$\boxed{h_{12} = \frac{V_1}{V_2} = 1}$$

$$\boxed{h_{22} = \frac{I_2}{V_2} = \frac{1}{R}\,\Omega^{-1}}$$

DOMS

(4) Transmission (T) or chain or ABCD Parameters:-

$$(V_1, I_1) = f(V_2, -I_2)$$

T-parameters are used in analysis of power transmission lines. The inputs and output ports are called the sending & receiving ends respectively.

(Since in this case, output port current is considered outward, so the negative sign arises with I_2.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2 \quad \text{---(1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{---(2)}$$

Case 1: when $I_2 = 0$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad \& \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

Case 2: when $V_2 = 0$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \quad \& \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

A → Reverse Voltage Ratio with o/p port open circuited

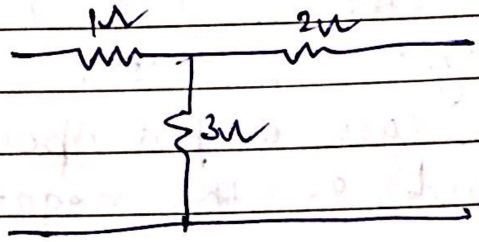
C → Reverse Transfer Admittance with o/p port open circuited.

B → Reverse-transfer Impedance with o/p port short circuited.

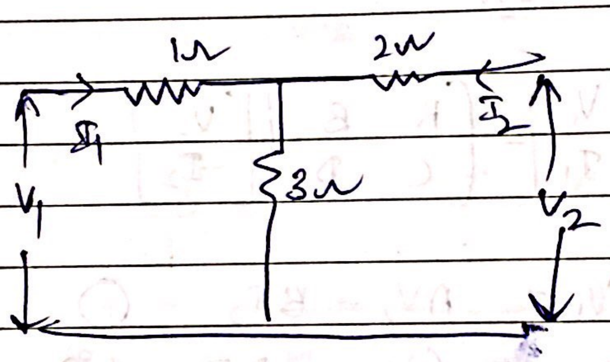
D → Reverse Current Ratio with o/p port short circuited.

★ The equivalent circuit of a two-port network is not possible in terms of T-parameters.

Q.24 obtain T-parameters of a given network



Soln



$$V_1 = 4I_1 + 3I_2 \quad \text{—} (1)$$

$$V_2 = 3I_1 + 5I_2 \quad \text{—} (2)$$

Case1: when $I_2 = 0$

$$V_1 = 4I_1$$

$$V_2 = 3I_1$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{4}{3}, \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{3}\ \text{ohm}^{-1}$$

Case2: when $V_2 = 0$

$$3I_1 = -5I_2 \Rightarrow I_1 = -\frac{5}{3}I_2$$

$$V_1 = 4\left(\frac{-5}{3}I_2\right) + 3I_2$$

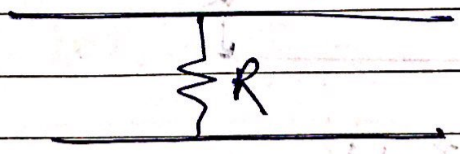
So, $V_1 = -\frac{11}{3}I_2$

$$B = \frac{V_1}{-I_2}\Big|_{V_2=0} = \frac{11}{3}\,\Omega$$

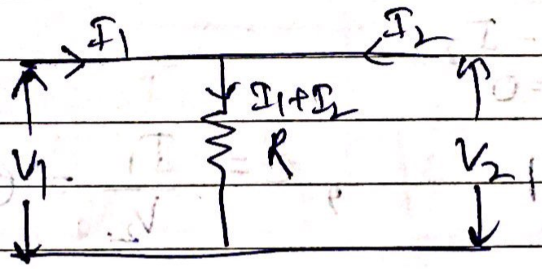
$$D = \frac{I_1}{-I_2}\Big|_{V_2=0} = \frac{5}{3}$$

Q.25 obtain T-parameters of given network.

Q.



Sol^n



$$V_1 = RI_1 + RI_2 \quad -(1)$$

$$V_2 = RI_1 + RI_2 \quad -(2)$$

<u>case 1</u>: when $I_2 = 0$

$$V_1 = RI_1$$

$$V_2 = RI_1$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1, \quad C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{1}{R}$$

<u>case 2</u>: when $V_2 = 0$

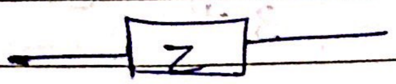
$$RI_1 = -RI_2$$

$$\boxed{I_1 = -I_2}$$

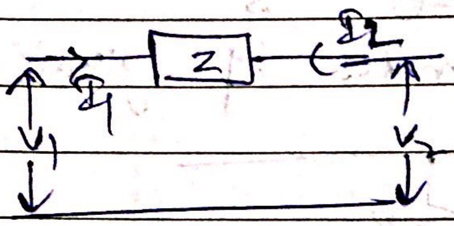
$$V_1 = R(-I_2) + RI_2 = 0, \quad D = \frac{I_1}{-I_2}\Big|_{V_2=0} = 1$$

So $B = \frac{V_1}{-I_2} = 0$ ohms   So $[T] = \begin{bmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{bmatrix}$ Ans.

Q.26 obtain T- parameters of given network



Soln



$$V_1 = I_1 Z + V_2$$

$$I_1 = -I_2$$

Case1: when $I_2 = 0$

So $A = \frac{V_1}{V_2} = 1$, $C = \frac{I_1}{V_2} = 0$

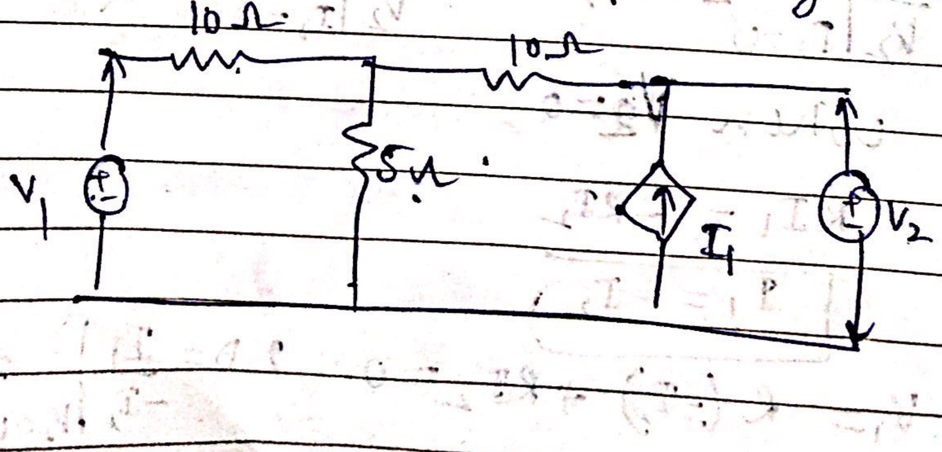
Case2: when $V_2 = 0$

$B = \frac{V_1}{-I_2} = Z$, $D = \frac{I_1}{-I_2} = 1$

So

$$[T] = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \text{Ans.}$$

Q.27 obtain ABCD parameters of the given circuit.



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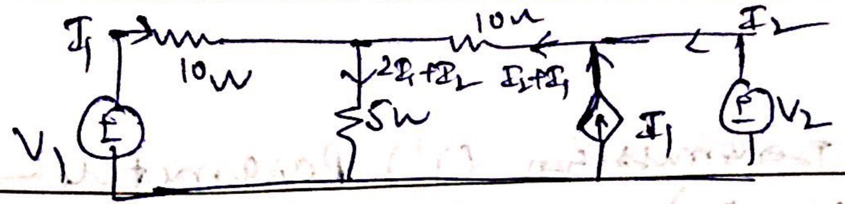


Solⁿ Applying KVL:-

$$V_1 = 10I_1 + 5(2I_1 + I_2)$$

$$\boxed{V_1 = 20I_1 + 5I_2} \quad \text{--- (1)}$$

$$V_2 = 10(I_2 + I_1) + 5(2I_1 + I_2)$$

$$V_2 = 10I_2 + 10I_1 + 10I_1 + 5I_2$$

$$\boxed{V_2 = 20I_1 + 15I_2} \quad \text{--- (2)}$$

Case 1 : when $I_2 = 0$

$$\boxed{A = \frac{V_1}{V_2} = 1}, \quad \boxed{C = \frac{I_1}{V_2} = \frac{1}{20} \text{ ohm}^{-1}}$$

Case 2 : when $V_2 = 0$

$$20I_1 = -15I_2$$

$$I_1 = -\frac{15}{20} I_2$$

$$\boxed{I_1 = -\frac{3}{4} I_2}$$

$$V_1 = 20\left(-\frac{3}{4} I_2\right) + 5I_2$$

$$V_1 = -15I_2 + 5I_2$$

$$V_1 = -10I_2$$

$$B = \frac{V_1}{-I_2}$$

$$B = \frac{-10I_2}{-I_2}$$

$$\boxed{B = 10\Omega}$$

$$D = \frac{I_1}{-I_2} = \frac{15}{20} = \frac{3}{4}$$

$$\boxed{D = \frac{3}{4}}$$

* Inverse Transmission (T') Parameters:-

$$(V_2, I_2) = f(V_1, -I_1)$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

$$V_2 = A'V_1 - B'I_1 \quad \text{--- (1)}$$
$$I_2 = C'V_1 - D'I_1 \quad \text{--- (2)}$$

Case 1: when $I_1 = 0$

$$V_2 = A'V_1$$
$$I_2 = C'V_1$$

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0}$$

$$C' = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$

Case 2:- when $V_1 = 0$

$$V_2 = -B'I_1$$
$$I_2 = -D'I_1$$

$$B' = \left.\frac{V_2}{-I_1}\right|_{V_1=0}$$

$$D' = \left.\frac{I_2}{-I_1}\right|_{V_1=0}$$

$A' \rightarrow$ forward voltage Ratio with I/P Port open circuited.

$C' \rightarrow$ Transfer Admittance with I/P port open circuited.

$B' \rightarrow$ Transfer Impedance with I/P Port Short circuited

$D' \rightarrow$ forward current Ratio with I/P Port Short circuited.

Inverse Hybrid (g) Parameters :-

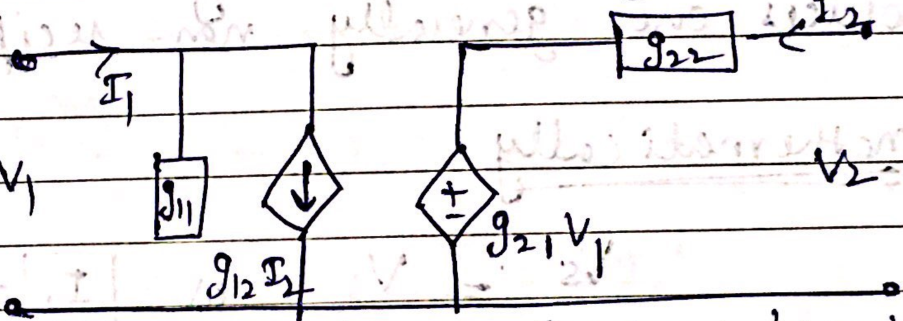
$$[g] = [h]^{-1}$$

$$(I_1, V_2) = f(V_1, I_2)$$

$$I_1 = g_{11} V_1 + g_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \quad \text{--- (2)}$$

Equivalent ckt of 2- Port n/w in terms of g- parameters :-



Case 1 : when $I_2 = 0$

$$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}$$

$$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$$

Case 2 : when $V_1 = 0$

$$g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0}$$

$$g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0}$$

$g_{11} \rightarrow$ I/P Admittance with O/P Port open circuited

$g_{21} \rightarrow$ forward Voltage Gain with O/P Port open circuited

$g_{12} \rightarrow$ Reverse Current Gain with I/P Port Short circuited

$g_{22} \rightarrow$ O/P Impedance with I/P Port short circuited.

* **Condition for Reciprocity :—** A two port network is said to be reciprocal, if the ratio of the excitation to response is invariant to an interchange of the position of excitation and response in the network.

→ Networks containing resistors, inductors & capacitors are generally reciprocal.

→ Networks that additionally have dependent sources are generally non-reciprocal.

Mathematically.

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'} \quad \text{or} \quad \boxed{I_2' = I_1'}$$

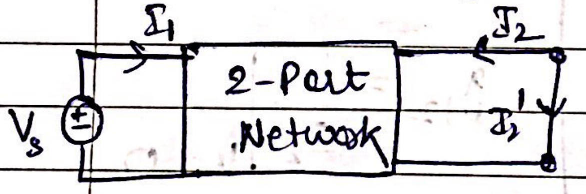


fig (a)

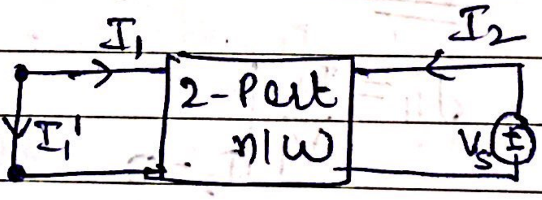


fig (b)

$V_1 = V_s$	$V_2 = V_s$
$I_1 = I_1$	$I_2 = I_2$
$V_2 = 0$	$I_1 = -I_1'$
$I_2 = -I_2'$	$V_1 = 0$

(1) Condition for Reciprocity in terms of Z-Parameters:-

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -(1)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -(2)$$

from fig (a)

$V_1 = V_s, \quad I_1 = I_1, \quad V_2 = 0, \quad I_2 = -I_2'$

Putting these values in eqn (1) & (2):→

$$V_s = Z_{11} I_1 - Z_{12} I_2' \quad -(3)$$
$$0 = Z_{21} I_1 - Z_{22} I_2' \quad -(4)$$

from eqn (4):— $Z_{21} I_1 = Z_{22} I_2'$

$$I_1 = \frac{Z_{22}}{Z_{21}} I_2'$$

Putting value of I_1 in eqn (3):—

$$V_s = Z_{11}\left(\frac{Z_{22}}{Z_{21}} I_2'\right) - Z_{12} I_2'$$

$$= \frac{Z_{11} Z_{22} I_2' - Z_{12} Z_{21} I_2'}{Z_{21}}$$

$$V_s = \frac{(Z_{11} Z_{22} - Z_{12} Z_{21})}{Z_{21}} I_2'$$

$$\boxed{\frac{V_s}{I_2'} = \frac{\Delta Z}{Z_{21}}} \quad -(5)$$

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from fig (b) :-

$V_2 = V_S$, $I_2 = I_2$, $V_1 = 0$, $I_1 = -I_1'$

So from eq^n (1) & (2) :-

$$0 = -Z_{11} I_1' + Z_{12} I_2 \quad \text{--- (6)}$$

$$V_S = -Z_{21} I_1' + Z_{22} I_2 \quad \text{--- (7)}$$

So from eq^n (6) :-

$$Z_{12} I_2 = Z_{11} I_1'$$

$$I_2 = \frac{Z_{11}}{Z_{12}} I_1'$$

Putting value of I_2 in eq^n (7) :-

$$V_S = -Z_{21} I_1' + Z_{22} \left(\frac{Z_{11}}{Z_{12}} I_1' \right)$$

$$V_S = \frac{-Z_{12} Z_{21} I_1' + Z_{11} Z_{22} I_1'}{Z_{12}}$$

$$V_S = \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{12}} \right) I_1'$$

$$\boxed{\frac{V_S}{I_1'} = \frac{\Delta Z'}{Z_{12}}} \quad \text{--- (8)}$$

from eq^n (5) & (8) :-

$$\boxed{Z_{12} = Z_{21}} \checkmark$$

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(2) Condition of Reciprocity in terms of Y-Parameters:-

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—(1)}$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—(2)}.$$

from fig.(a)

$$V_1 = V_S,\ I_1 = I_1,\ V_2 = 0,\ I_2 = -I_2'$$

$$I_1 = Y_{11} V_S$$
$$-I_2' = Y_{21} V_S$$

$$\boxed{\frac{V_S}{I_2'} = -Y_{21}} \quad \text{—(3)}.$$

from fig(b):-

$$V_2 = V_S,\ I_2 = I_2,\ V_1 = 0,\ I_1 = -I_1'$$

$$-I_1' = Y_{12} V_S$$
$$I_2 = Y_{22} V_S$$

$$\boxed{\frac{V_S}{I_1'} = -Y_{12}} \quad \text{—(4)}.$$

on comparing eq^n (3) & (4) :-

$$\boxed{Y_{12} = Y_{21}}$$

this is the condition of Reciprocity in terms of Y- parameters.

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3) In terms of T-Parameters:—

$$V_1 = AV_2 - BI_2 \quad — (1)$$
$$I_1 = CV_2 - DI_2 \quad — (2).$$

from fig(a):— $V_1 = V_s$, $I_1 = I_1$, $V_2 = 0$, $I_2 = -I_2'$

$$V_s = BI_2'$$

$$\boxed{\frac{V_s}{I_2'} = B} \quad — (3).$$

from fig(b):— $V_2 = V_s$, $I_2 = I_2$, $V_1 = 0$, $I_1 = -I_1'$

$$0 = AV_s - BI_2 \quad — (4)$$
$$-I_1' = CV_s - DI_2 \quad — (5).$$

from eq^n (4):—

$$AV_s = BI_2$$

$$I_2 = \left(\frac{A}{B}\right)V_s.$$

Putting value of I_2 in eq^n (5):—

$$-I_1' = CV_s - D\left(\frac{A}{B}V_s\right)$$

$$= \left(\frac{BC - AD}{B}\right)V_s$$

$$I_1' = \left(\frac{AD - BC}{B}\right)V_s.$$

$$\boxed{\frac{V_s}{I_1'} = \frac{B}{AD-BC}} \quad \text{--- (6)}$$

On comparing eqn (3) & (6) :→

$$B = \frac{B}{AD-BC}$$

$$\boxed{\begin{matrix} AD - BC = 1 \\ \text{or} \\ \Delta T = 1 \end{matrix}}$$

← This is the condition of Reciprocity in terms of T- parameters.

(4) <u>In terms of h - Parameters:→</u>

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

<u>from fig (a)</u>:- $V_1 = V_S$, $I_1 = I_1$, $V_2 = 0$, $I_2 = -I_2'$

$$V_S = h_{11} I_1$$

$$-I_2' = h_{21} I_1$$

$$\boxed{\frac{V_S}{I_2'} = -\frac{h_{11}}{h_{21}}} \quad \text{--- (3)}$$

<u>from fig (b)</u>:- $V_2 = V_S$, $I_2 = I_2$, $V_1 = 0$, $I_1 = -I_1'$

$$0 = -h_{11} I_1' + h_{12} V_S \quad \text{--- (4)}$$

$$I_2 = -h_{21} I_1' + h_{22} V_S \quad \text{--- (5)}$$

$$h_{11} I_1' = h_{12} V_S$$

$$\boxed{\frac{V_S}{I_1'} = \frac{h_{11}}{h_{12}}} \quad \text{--- (6)}$$

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on comparing eqn (3) & (6) :—

$$-\frac{h_{11}}{h_{21}} = \frac{h_{11}}{h_{12}}$$

So $\boxed{h_{12} = -h_{21}}$

Similarly

(5) In terms of T' Parameters :—

$$\boxed{\Delta T' = 1}$$

(6) In terms of g Parameters :—

$$\boxed{g_{12} = -g_{21}}$$

★ **Condition of Symmetry :—**

A 2-Port network is said to be symmetrical if the ports can be interchanged without changing the port voltages and currents.

mathematically,

$$\boxed{\left.\frac{V_S}{I_1}\right|_{I_2=0} = \left.\frac{V_S}{I_2}\right|_{I_1=0}}$$

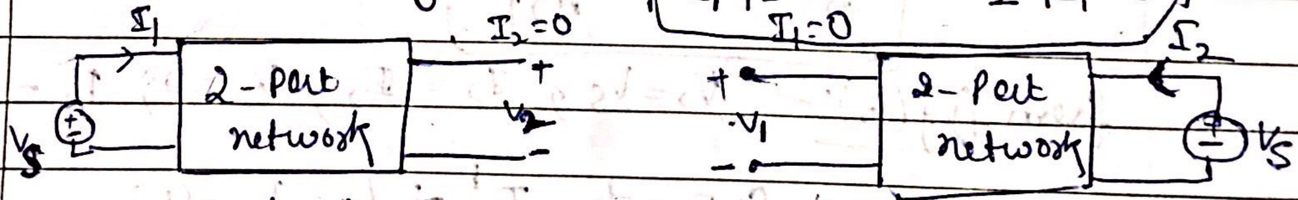

fig.(a)

$$\left[\begin{array}{ll} V_1 = V_S, & I_1 = I_1 \\ I_2 = 0, & V_2 = V_2 \end{array}\right] \qquad \left[\begin{array}{ll} V_2 = V_S, & I_1 = 0 \\ I_2 = I_2, & V_1 = V_1 \end{array}\right]$$

i) In terms of Z-Parameters:-

from fig. (a) $V_1 = V_S$, $I_2 = 0$

$I_1 = I_1$, $V_2 = V_2$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_S = Z_{11} I_1$$

$$V_2 = Z_{21} I_1$$

So $\boxed{\left.\frac{V_S}{I_1}\right|_{I_2=0} = Z_{11}}$

from fig. (b) :-

$V_2 = V_S$, $I_1 = 0$

$I_2 = I_2$, $V_1 = V_1$

$$V_1 = Z_{12} I_2$$

$$V_S = Z_{22} I_2$$

So $\left.\frac{V_S}{I_2}\right|_{I_1=0} = Z_{22}$

from the Definition of Symmetry

$\left.\frac{V_S}{I_1}\right|_{I_2=0} = \left.\frac{V_S}{I_2}\right|_{I_1=0}$ leads to

$$\boxed{Z_{11} = Z_{22}}$$

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ii) In terms of Y-Parameters :-

from fig.(a) :- $V_1 = V_S$, $I_1 = I_1$, $I_2 = 0$, $V_2 = V_2$

e_q^n of Y-P

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad -(1)$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad -(2)$$

$$I_1 = Y_{11} V_S + Y_{12} V_2 \quad -(3)$$
$$0 = Y_{21} V_S + Y_{22} V_2 \quad -(4)$$

$$V_2 = -\frac{Y_{21} V_S}{Y_{22}}$$

Putting value of V_2 in e_q^n (3) :-

$$I_1 = Y_{11} V_S + Y_{12} \left[\frac{-Y_{21} V_S}{Y_{22}} \right]$$

$$I_1 = \left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{22}} \right] V_S$$

$$\frac{V_S}{I_1} = \frac{Y_{22}}{Y_{11} Y_{22} - Y_{12} Y_{21}}$$

$$\boxed{\frac{V_S}{I_1} = \frac{Y_{22}}{\Delta Y}} \quad -(5)$$

from fig. (b) $V_2 = V_S$, $I_2 = I_2$, $I_1 = 0$, $V_1 = V_1$

$$0 = Y_{11} V_1 + Y_{12} V_S$$
$$I_2 = Y_{21} V_1 + Y_{22} V_S.$$

$$\boxed{\frac{V_S}{I_2} = \frac{Y_{11}}{Y_{11} Y_{22} - Y_{12} Y_{21}}} \quad \text{— } (6)$$

Equating eq^n (5) & (6):—

$$\boxed{Y_{11} = Y_{22}}$$

iii) <u>In terms of T- parameters:-</u>

from fig(a).

$V_1 = V_S$ | $I_2 = 0$

$I_1 = I_1$ | $V_2 = V_2$.

<u>eq^n of T-Parameters:-</u>

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$V_S = AV_2$$
$$I_1 = CV_2$$

So

$$\boxed{\frac{V_S}{I_1} = \frac{A}{C}} \quad \text{— } (1)$$

<u>from fig(b)</u> : $V_2 = V_S$, $I_2 = I_2$, $I_1 = 0$, $V_1 = V_1$

$$V_1 = AV_S - BI_2$$
$$0 = CV_S - DI_2$$

So $\boxed{\dfrac{V_s}{I_2} = \dfrac{D}{C}}$ — (2)

from eq^n (1) & (2):—

$$\frac{A}{C} = \frac{D}{C}$$

$$\boxed{A = D}$$

(iv) <u>In terms of h-Parameters:—</u>

from fig. (a) $V_1 = V_s$, $I_1 = I_1$, $I_2 = 0$, $V_2 = V_2$

$$V_s = h_{11} I_1 + h_{12} V_2$$
$$0 = h_{21} I_1 + h_{22} V_2$$

$$\boxed{\frac{V_s}{I_1} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}}$$ — (1)

<u>from fig (b)</u> :— $V_2 = V_s$, $I_2 = I_2$, $I_1 = 0$, $V_1 = V_1$

$$V_1 = h_{12} V_s$$
$$I_2 = h_{22} V_s$$

or $$\boxed{\frac{V_s}{I_2} = \frac{1}{h_{22}}}$$

Therefore the condition for Symmetry :—

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$$ leads to

$$h_{11}h_{22} - h_{12}h_{21} = 1$$

$$\boxed{\Delta h = 1}$$

v) In terms of g- Parameters:-

$$g_{11}g_{22} - g_{12}g_{21} = 1$$

or $\boxed{\Delta g = 1}$

vi) In terms of Inverse T' Parameters:-

$$\boxed{A' = D'}$$

Relationships Between Parameter Sets:-

If we want to express α- parameters in terms of β- Parameters, we have to write β-parameter equations and then by algebric manipulation, rewrite the equations as needed for α- parameters.

① Z-Parameters in terms of Y- Parameters:-

$$[I] = [Y][V]$$

$$[V] = [Z][I]$$

So $$[Z] = [Y]^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta Y}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{12} & Y_{11} \end{bmatrix}$$

So $Z_{11} = \frac{Y_{22}}{\Delta Y}$, $Z_{12} = -\frac{Y_{12}}{\Delta Y}$

$Z_{21} = -\frac{Y_{21}}{\Delta Y}$, $Z_{22} = \frac{Y_{11}}{\Delta Y}$

② Z-Parameters in terms of T-Parameters:—

$$V_1 = AV_2 - BI_2 \quad \text{—①}$$
$$I_1 = CV_2 - DI_2 \quad \text{—②}$$

$$\left[\begin{array}{l} V_1 = Z_{11}I_1 + Z_{12}I_2 \\ V_2 = Z_{21}I_1 + Z_{22}I_2 \end{array}\right]$$

Rewriting eqn ②:—

$$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2 \quad \text{—③}$$

Putting the value of V_2 from eqn ③ into eqn ①:—

$$V_1 = A\left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right] - BI_2$$

$$V_1 = \frac{A}{C}I_1 + \frac{AD-BC}{C}I_2$$

$$\boxed{V_1 = \frac{A}{C}I_1 + \frac{\Delta T}{C}I_2} \quad \text{—④}$$

Comparing eqn ③ & ④ with eqns of Z-Parameters—

$$\boxed{\begin{array}{ll} Z_{11} = \frac{A}{C}, & Z_{21} = \frac{1}{C} \\ Z_{12} = \frac{\Delta T}{C}, & Z_{22} = \frac{D}{C} \end{array}}$$

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(3) Z-Parameters in terms of h-parameters :-

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}.$$

$$\begin{bmatrix} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{bmatrix}$$

Rewriting eq^n (2) :-

$$\boxed{V_2 = -\frac{h_{21}}{h_{11}} I_1 + \frac{1}{h_{22}} I_2} \quad \text{--- (3)}$$

Putting value of V_2 from eq^n (3) into eq^n (1) :-

$$V_1 = h_{11} I_1 + h_{12} \left[-\frac{h_{21}}{h_{11}} I_1 + \frac{1}{h_{22}} I_2 \right]$$

$$V_1 = \left[h_{11} - \frac{h_{12} h_{21}}{h_{22}} \right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$\boxed{V_1 = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2} \quad \text{--- (4)}$$

comparing eq^n (3) & (4) with eq^ns of Z-Parameters :

$$\boxed{\begin{aligned} Z_{11} &= \frac{\Delta h}{h_{22}}, & Z_{12} &= \frac{h_{12}}{h_{22}} \\ Z_{21} &= \frac{-h_{21}}{h_{22}}, & Z_{22} &= \frac{1}{h_{22}} \end{aligned}}$$

(4) Z-Parameters in terms of g-Parameters :-

$$Z_{11} = \frac{1}{g_{11}}, \quad Z_{12} = -\frac{g_{12}}{g_{11}}, \quad Z_{21} = \frac{g_{21}}{g_{11}} \quad \& \quad Z_{22} = \frac{\Delta g}{g_{11}}$$

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★ Y-Parameters in terms of other Parameters:—

(5) Y-Parameters in terms of Z-Parameters:—

$$\therefore [Y] = [Z]^{-1}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = \frac{-Z_{12}}{\Delta Z}, \quad Y_{21} = \frac{-Z_{21}}{\Delta Z} \ \& \ Y_{22} = \frac{Z_{11}}{\Delta Z}.$$

(6) Y-Parameters in terms of T-Parameters:—

$$\begin{bmatrix} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2 \quad \text{—(1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{—(2)}$$

Rewriting eqn (1):—

$$\boxed{I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2} \quad \text{—(3)}.$$

Putting the value of I_2 in eqn (2):—

$$I_1 = CV_2 - D\left[-\frac{1}{B}V_1 + \frac{A}{B}V_2\right]$$

$$I_1 = CV_2 + \frac{D}{B}V_1 - \frac{AD}{B}V_2$$

$$I_1 = \frac{D}{B}V_1 + \left(C - \frac{AD}{B}\right)V_2$$

$$\boxed{I_1 = \frac{D}{B}V_1 - \frac{\Delta T}{B}V_2} \quad \text{—(4)}.$$

on comparing eqn. (3) & (4) with eqns of Y-P:—

$$\boxed{\begin{matrix} Y_{11} = \frac{D}{B}, & Y_{12} = \frac{-\Delta T}{B} \\ Y_{21} = -\frac{1}{B}, & Y_{22} = \frac{A}{B} \end{matrix}}$$

⑦ Y- Parameters in terms of h- parameters:→

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -②.$$

$$\begin{bmatrix} I_1 = Y_{11} V_1 + Y_{12} V_2 \\ I_2 = Y_{21} V_1 + Y_{22} V_2 \end{bmatrix}$$

Rewriting the eqn ①:—

$$\boxed{I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2} \quad -③$$

$$I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right] + h_{22} V_2$$

$$\boxed{I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{\Delta h}{h_{11}} V_2} \quad -④$$

on comparing eqn ③ & ④ with eqns of Y-P:—

$$\boxed{\begin{array}{ll} Y_{11} = \frac{1}{h_{11}}, & Y_{12} = \frac{-h_{12}}{h_{11}} \\ Y_{21} = \frac{h_{21}}{h_{11}}, & Y_{22} = \frac{\Delta h}{h_{11}} \end{array}}$$

⑧ Y-Parameters in terms of g-Parameters:—

$$\boxed{\begin{array}{ll} Y_{11} = \frac{\Delta g}{g_{22}}, & Y_{12} = \frac{g_{12}}{g_{22}} \\ Y_{21} = -\frac{g_{21}}{g_{22}}, & Y_{22} = \frac{1}{g_{22}} \end{array}}$$

$\_/\_/\_$

★ T-Parameters in terms of other Parameters:—

(1). T-Parameters in terms of Z-Parameters:—

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

$$\left[\begin{array}{l} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{array}\right]$$

Rewriting eqn (2):—

$$I_1 = \frac{1}{Z_{21}} V_2 + \frac{Z_{22}}{Z_{21}} (-I_2)$$

$$\boxed{I_1 = \frac{1}{Z_{21}} V_2 + \left(\frac{Z_{22}}{Z_{21}}\right)(-I_2)} \quad \text{--- (3)}$$

Putting value of I_1 in eqn (1):—

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{11} Z_{22}}{Z_{21}} I_2 + Z_{12} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 + \left(\frac{Z_{12} Z_{21} - Z_{11} Z_{22}}{Z_{21}}\right) I_2$$

$$\boxed{V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{\Delta Z}{Z_{21}} I_2} \quad \text{--- (4)}$$

Comparing eqn (3) & (4) with eqns of T-P:—

$$\boxed{\begin{array}{ll} C = \dfrac{1}{Z_{21}}, & D = \dfrac{Z_{22}}{Z_{21}} \\ A = \dfrac{Z_{11}}{Z_{21}}, & B = \dfrac{\Delta Z}{Z_{21}} \end{array}}$$

(10) T- Parameters in terms of Y-Parameters:—

$$A = \frac{-Y_{22}}{Y_{21}} , \quad B = \frac{-1}{Y_{21}}$$

$$C = \frac{-\Delta Y}{Y_{21}} , \quad D = \frac{-Y_{11}}{Y_{21}}$$

(11) T-Parameters interms of h-Parameters:—

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—(1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—(2)}$$

$$\left[\begin{array}{l} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{array} \right]$$

Rewriting eqn (2):—

$$\boxed{I_1 = \frac{-h_{22}}{h_{21}} V_2 + \left(\frac{-1}{h_{21}}\right)(-I_2)} \quad \text{—(3)}$$

Putting value of I_1 from eqn (3) into eqn (1):—

$$V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2\right] + h_{12} V_2$$

$$\boxed{V_1 = \frac{-\Delta h}{h_{21}} V_2 - \left(\frac{h_{11}}{h_{21}}\right)(-I_2)} \quad \text{—(4)}$$

On comparing eqn (3) & (4) with eqns of T-Parameters.

$$\boxed{\begin{array}{ll} A = \frac{-\Delta h}{h_{21}} , & B = \frac{-h_{11}}{h_{21}} \\ C = \frac{-h_{22}}{h_{21}} , & D = \frac{-1}{h_{21}} \end{array}}$$

(12) T-Parameters in terms of g-Parameters:-

$$A = \frac{1}{g_{21}}, \quad B = \frac{g_{22}}{g_{21}}, \quad C = \frac{g_{11}}{g_{21}} \quad \& \quad D = \frac{\Delta g}{g_{21}}$$

★ h-Parameters in terms of other Parameters:-

(13) h-Parameters in terms of Z-Parameters:-

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—(1)}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—(2)}$$

$$\left[\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}\right]$$

Rewriting the eqn (2):-

$$\boxed{I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2} \quad \text{—(3)}$$

Putting value of I_2 from eqn (3) into (1):-

$$V_1 = Z_{11} I_1 + Z_{12}\left(-\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2\right)$$

$$\boxed{V_1 = \left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}}\right) I_1 + \frac{Z_{12}}{Z_{22}} V_2} \quad \text{—(4)}$$

Comparing eqn (3) & (4) with eqns of h-Parameters

$$\boxed{\begin{aligned} h_{11} &= \frac{\Delta Z}{Z_{22}}, \quad h_{12} = \frac{Z_{12}}{Z_{22}} \\ h_{21} &= -\frac{Z_{21}}{Z_{22}}, \quad h_{22} = \frac{1}{Z_{22}} \end{aligned}}$$

(14) h-Parameters in terms of Y-Parameters:-

$$h_{11} = \frac{1}{Y_{11}}, \quad h_{12} = \frac{-Y_{12}}{Y_{11}}, \quad h_{21} = \frac{Y_{21}}{Y_{11}} \quad \& \quad h_{22} = \frac{\Delta Y}{Y_{11}}$$

(15) h-Parameters in terms of T-Parameters.

$$V_1 = AV_2 - BI_2 \quad \text{---(1)}$$
$$I_1 = CV_2 - DI_2 \quad \text{---(2)}$$

$$\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases}$$

Rewriting eq^n (2):-

$$\boxed{I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2} \quad \text{---(3)}$$

Putting value of I_2 in eq^n (1):-

$$V_1 = AV_2 + B\left[\frac{1}{D} I_1 - \frac{C}{D} V_2\right]$$

$$\boxed{V_1 = \frac{B}{D} I_1 + \left(A - \frac{BC}{D}\right) V_2} \quad \text{---(4)}$$

on comparing eq^n (3) & (4) with eq^ns of h-Parameters:-

$$\boxed{\begin{array}{ll} h_{11} = \frac{B}{D}, & h_{12} = \frac{\Delta T}{D} \\ h_{21} = -\frac{1}{D}, & h_{22} = \frac{C}{D} \end{array}}$$

(16) h-Parameters in terms of g-Parameters:-

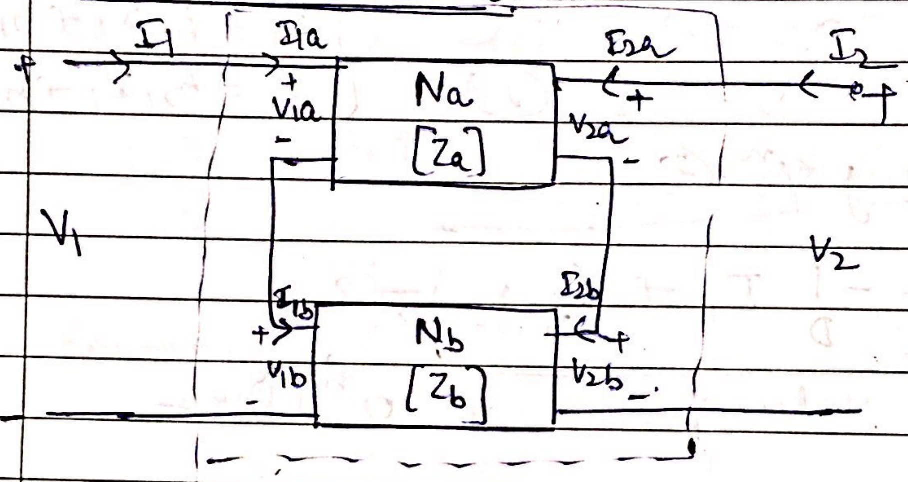
$$[h] = [g]^{-1}$$

$$h_{11} = \frac{g_{22}}{\Delta g}, \quad h_{12} = -\frac{g_{12}}{\Delta g}, \quad h_{21} = \frac{-g_{21}}{\Delta g}, \quad h_{22} = \frac{g_{11}}{\Delta g}$$

* Interconnections of Two Port Networks:—

Two port networks may be interconnected in various configurations, such as series, Parallel, Cascade, Series-Parallel, Parallel-Series Connection, resulting in new 2-Port networks.

① Series Connection:— (Series-Series Connection). (Z-Parameters)



for Network, Na:—

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

for Network, Nb:—

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

then, Series Connection requires that:—

$$I_1 = I_{1a} = I_{1b}, \quad I_2 = I_{2a} = I_{2b}$$

$$V_1 = V_{1a} + V_{1b}, \quad V_2 = V_{2a} + V_{2b}.$$

$$V_1 = \left[Z_{11a} I_{1a} + Z_{12a} I_{2a}\right] + \left[Z_{11b} I_{1b} + Z_{12b} I_{2b}\right]$$

$$V_2 = \left[Z_{21a} I_{1a} + Z_{22a} I_{2a}\right] + \left[Z_{21b} I_{1b} + Z_{22b} I_{2b}\right]$$

Since $I_1 = I_{1a} = I_{1b}$ &

$I_2 = I_{2a} = I_{2b}$.

$$V_1 = (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b}) I_2$$

$$V_2 = (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$$

So $Z_{11} = Z_{11a} + Z_{11b}$

$Z_{12} = Z_{12a} + Z_{12b}$

$Z_{21} = Z_{21a} + Z_{21b}$

$Z_{22} = Z_{22a} + Z_{22b}$

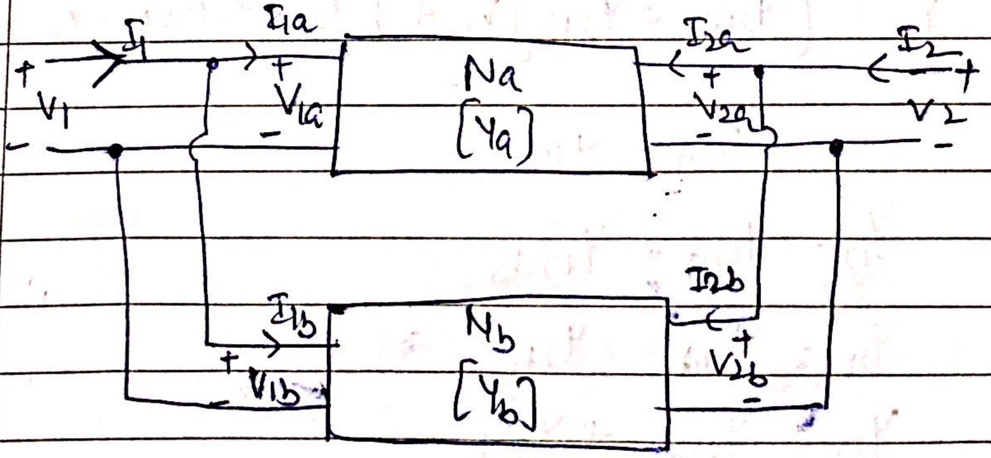
or in matrix form:—

$$[Z] = [Z_a] + [Z_b]$$

So,

The overall Z-parameter matrix for series connected 2-port networks is simply the sum of Z-parameter matrices of each individual 2-port network connected in series.

② Parallel Connection :— (Y-Parameters)



for Network N_a:—

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

For Network N_b:-

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

For Parallel Connection:-

$V_1 = V_{1a} = V_{1b}$ | $I_1 = I_{1a} + I_{1b}$

$V_2 = V_{2a} = V_{2b}$ | $I_2 = I_{2a} + I_{2b}$.

Now

$$I_1 = I_{1a} + I_{1b}$$

$$I_1 = [Y_{11a} V_{1a} + Y_{12a} V_{2a}] + [Y_{11b} V_{1b} + Y_{12b} V_{2b}]$$

$$I_2 = I_{2a} + I_{2b}$$

$$I_2 = [Y_{21a} V_{1a} + Y_{22a} V_{2a}] + [Y_{21b} V_{1b} + Y_{22b} V_{2b}]$$

Since

$$V_1 = V_{1a} = V_{1b} \quad \& \quad V_2 = V_{2a} = V_{2b}.$$

$$I_1 = [Y_{11a} + Y_{11b}] V_1 + [Y_{12a} + Y_{12b}] V_2$$

$$I_2 = [Y_{21a} + Y_{21b}] V_1 + [Y_{22a} + Y_{22b}] V_2$$

So

$$Y_{11} = Y_{11a} + Y_{11b}$$

$$Y_{12} = Y_{12a} + Y_{12b}$$

$$Y_{21} = Y_{21a} + Y_{21b}$$

$$Y_{22} = Y_{22a} + Y_{22b}.$$

So

$$[Y] = [Y_a] + [Y_b]$$

Hence,

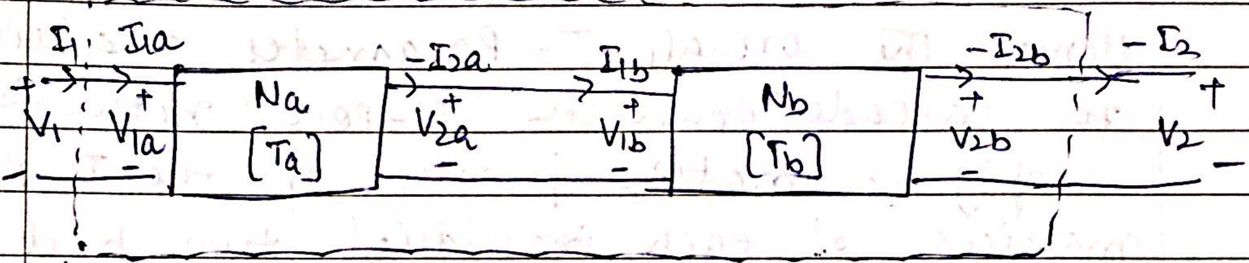
The overall Y-Parameters matrix for

DOMS

Parallel connected 2-Port networks is simply the sum of Y-parameter matricies of each individual 2-Port network connected in parallel.

③ Cascade Connection :- (Tandem Connection)

Two two port networks are said to be connected in cascade if the o/p port of first becomes the I/P port of the second.



for N_a

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

for N_b

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

Cascade Connection Requires that:—

$I_1 = I_{1a}$	$-I_{2a} = I_{1b}$	$I_{2b} = I_2$
$V_1 = V_{1a}$	$V_{2a} = V_{1b}$	$V_{2b} = V_2$

Now,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

So $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

or in eq^n form:—

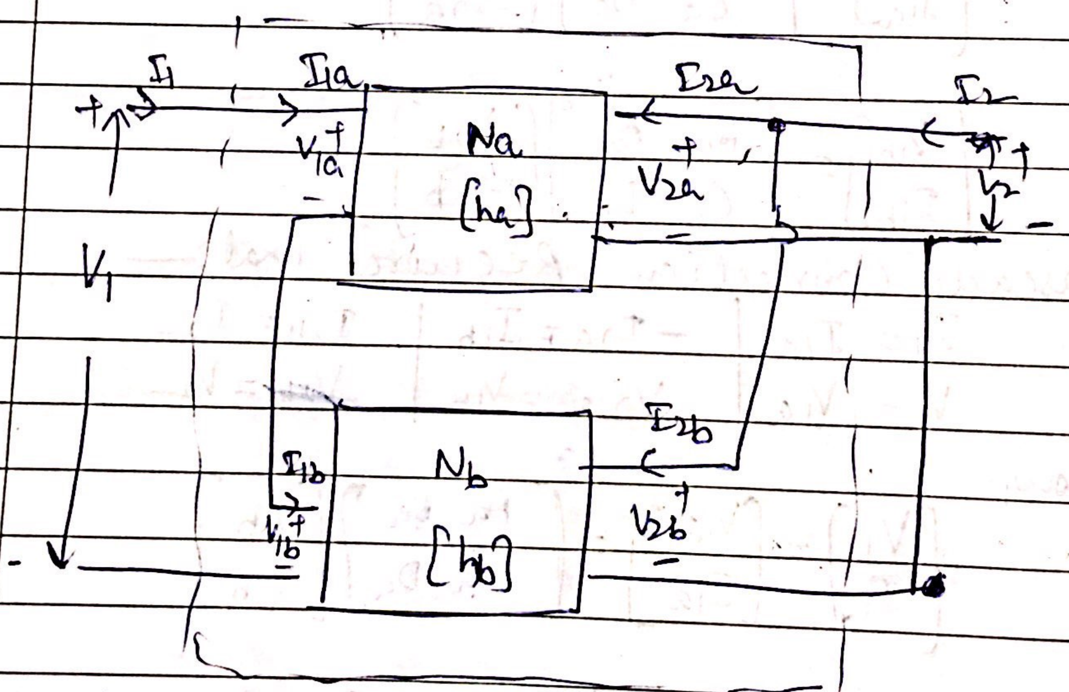
$$A = A_a A_b + B_a C_b$$
$$B = A_a B_b + B_a D_b$$
$$C = C_a A_b + D_a C_b$$
$$D = C_a B_b + D_a D_b.$$

or in the matrix form:—

$$[T] = [T_a] \cdot [T_b]$$

Hence, the overall T-Parameter matrix for cascade connected 2-port networks is simply the matrix product of the T-Parameter matrices of each individual two port network in cascade.

4. Series - Parallel Connection:-



Series - Parallel Connection Requires:—

$$V_1 = V_{1a} + V_{1b} \qquad V_2 = V_{2a} \pm V_{2b}$$
$$I_1 = I_{1a} = I_{1b} \qquad I_2 = I_{2a} + I_{2b}.$$

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for Network N_a:—

$$\begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} h_{11a} & h_{12a} \\ h_{21a} & h_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix}$$

for Network N_b:—

$$\begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} h_{11b} & h_{12b} \\ h_{21b} & h_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

$$V_1 = V_{1a} + V_{1b}$$

$$V_1 = [h_{11a} I_{1a} + h_{12a} V_{2a}] + (h_{11b} I_{1b} + h_{12b} V_{2b})$$

$$I_2 = I_{2a} + I_{2b}$$

$$I_2 = [h_{21a} I_{1a} + h_{22a} V_{2a}] + [h_{21b} I_{1b} + h_{22b} V_{2b}]$$

As $I_1 = I_{1a} = I_{1b}$ &

$V_2 = V_{2a} = V_{2b}$.

$$V_1 = (h_{11a} + h_{11b}) I_1 + (h_{12a} + h_{12b}) V_2$$

$$I_2 = (h_{21a} + h_{21b}) I_1 + (h_{22a} + h_{22b}) V_2$$

So $h_{11} = h_{11a} + h_{11b}$

$h_{12} = h_{12a} + h_{12b}$

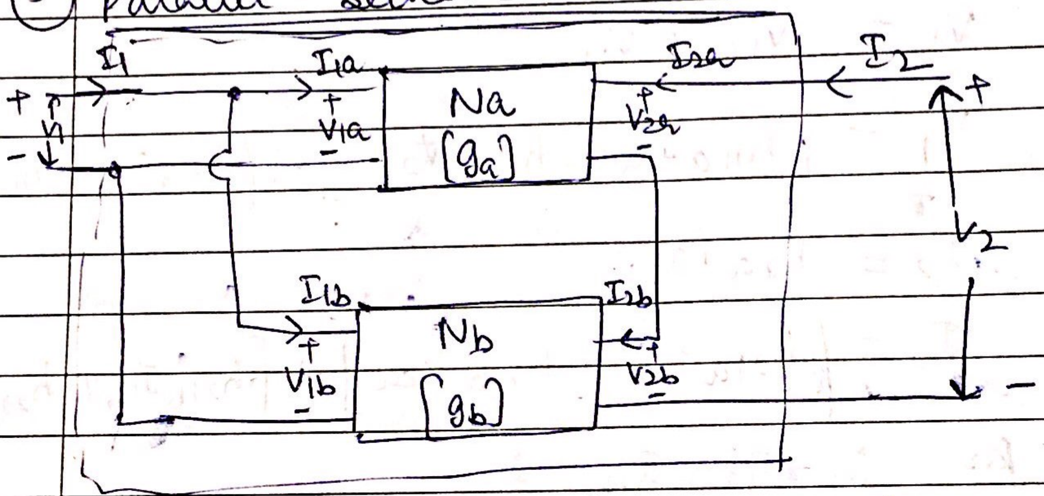
$h_{21} = h_{21a} + h_{21b}$

$h_{22} = h_{22a} + h_{22b}$

or $[h] = [h_a] + [h_b]$

So, the overall h-Parameter matrix for series-parallel connected 2-Port networks is simply the sum of h-Parameter matrices for each individual 2-Port network connected in series-parallel connection.

(5) Parallel - Series Connection:-



The connection requires that:-

$$V_1 = V_{1a} \pm V_{1b}$$

$$I_1 = I_{1a} + I_{1b}$$

$$V_2 = V_{2a} + V_{2b}$$

$$I_2 = I_{2a} = I_{2b}.$$

Similar to previous case:-

$$g_{11} = g_{11a} + g_{11b}$$

$$g_{12} = g_{12a} + g_{12b}$$

$$g_{21} = g_{21a} + g_{21b}$$

$$g_{22} = g_{22a} + g_{22b}.$$

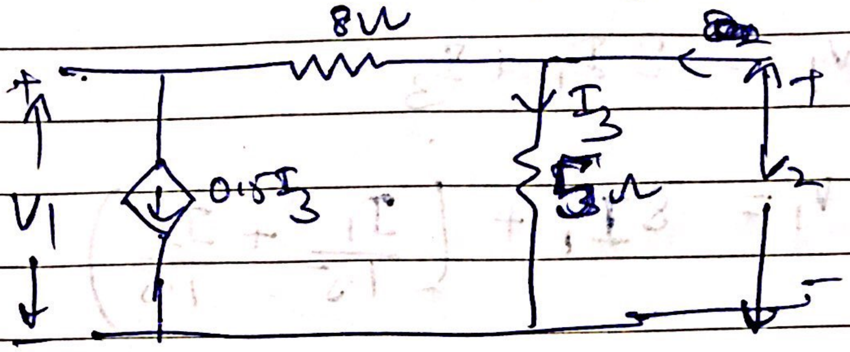
or in the matrix form:—

$$[g] = [g_a] + [g_b]$$

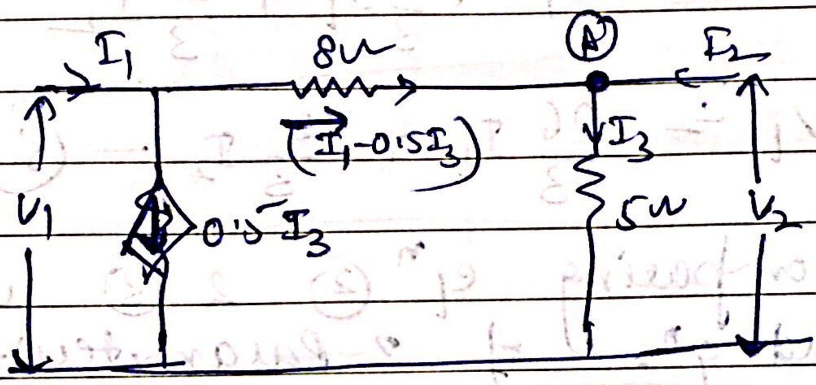
So

The overall g-parameter matrix for parallel-series connected 2-Port networks is simply the sum of g-parameter matrices of each individual 2-Port network connected in parallel Series Connection.

Q.28 obtain z-Parameter of given network.



Sol^n



Applying KCL at Node (A) :-

$$I_1 - 0.5I_3 + I_2 = I_3$$

$$1.5I_3 = I_1 + I_2$$

$$I_3 = \frac{1}{1.5}[I_1 + I_2]$$

$$\boxed{I_3 = \frac{1}{1.5}I_1 + \frac{1}{1.5}I_2} \quad \text{--- (1)}$$

DOMS

$V_2 = 5 I_3$

$= 5 \left[\frac{I_1}{1.5} + \frac{I_2}{1.5} \right]$

$$\boxed{V_2 = \frac{10}{3} I_1 + \frac{10}{3} I_2} \text{ — (2).}$$

$V_1 = 8(I_1 - 0.5 I_3) + 5 I_3$

$= 8 I_1 - 4 I_3 + 5 I_3$

$V_1 = 8 I_1 + I_3$

$V_1 = 8 I_1 + \left(\frac{I_1}{1.5} + \frac{I_2}{1.5} \right)$

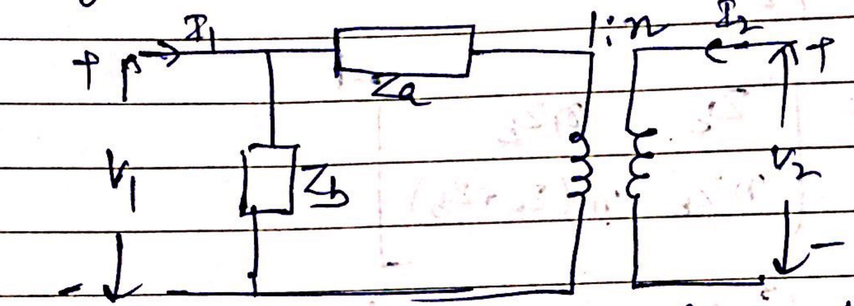
$= 8 I_1 + \frac{2}{3} I_1 + \frac{2}{3} I_2$

$$\boxed{V_1 = \frac{26}{3} I_1 + \frac{2}{3} I_2} \text{ — (3)}$$

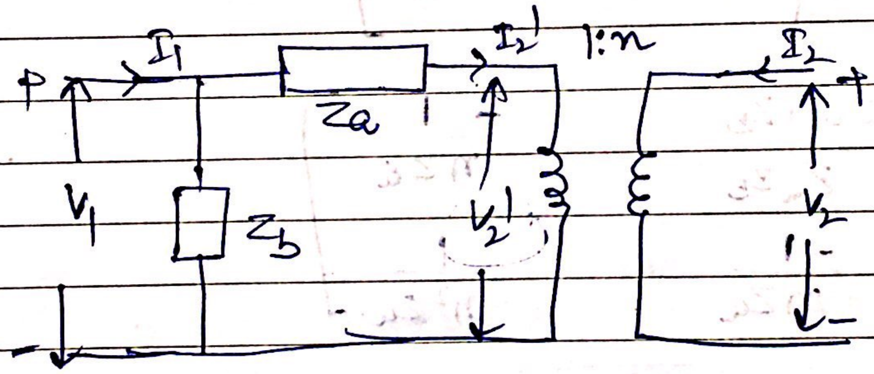
on comparing eqn (2) & (3) with standard eqns of 2-Parameters –

$$\boxed{\begin{aligned} Z_{11} &= \frac{26}{3} = 8.67\,\Omega \\ Z_{12} &= \frac{2}{3} = 0.67\,\Omega \\ Z_{21} &= \frac{10}{3} = 3.33\,\Omega \\ Z_{22} &= \frac{10}{3} = 3.33\,\Omega. \end{aligned}}$$

Q.29 For the given network find.
1.) Impedance and Admittance Parameters.
2.) Transmission and Inverse Transmission Parameters.
3.) Hybrid & Inverse hybrid Parameters



Sol^n Let V_2' & I_2' be the I/P voltage and I/P current of the Transformer respectively.



$$V_1 = Z_b(I_1 - I_2') \quad \text{— (1)}$$

$$V_2 = nV_2' \quad \text{and} \quad I_2 = -\frac{1}{n} I_2'$$

$$V_2 = n[V_1 - Z_a I_2']$$

So $$V_1 = Z_b(I_1 + nI_2)$$

$$\boxed{V_1 = Z_b I_1 + nZ_b I_2} \quad \text{— (2)}$$

$$V_2 = n[Z_b I_1 + nZ_b I_2 + nZ_a I_2]$$

$$\boxed{V_2 = nZ_b I_1 + n^2(Z_a + Z_b) I_2} \quad \text{— (3)}$$

(1). So $Z_{11} = Z_b$, $Z_{12} = n Z_b$

$Z_{21} = n Z_b$, $Z_{22} = n^2 (Z_a + Z_b)$

$$[Y] = [Z]^{-1}$$

$$= \begin{bmatrix} Z_b & n Z_b \\ n Z_b & n^2(Z_a + Z_b) \end{bmatrix}^{-1}$$

$$= \frac{1}{n^2 Z_a Z_b} \begin{bmatrix} n^2(Z_a+Z_b) & -n Z_b \\ -n Z_b & Z_b \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{Z_a + Z_b}{Z_a Z_b} & -\frac{1}{n Z_a} \\ \frac{-1}{n Z_a} & \frac{1}{n^2 Z_a} \end{bmatrix}$$

So

$Y_{11} = \frac{Z_a + Z_b}{Z_a Z_b}$, $Y_{12} = \frac{-1}{n Z_a}$

$Y_{21} = \frac{-1}{n Z_a}$, $Y_{22} = \frac{1}{n^2 Z_a}$

(2) $A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{n}$, $C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{n Z_b}$

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = n Z_a$, $D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = n\left(\frac{Z_a + Z_b}{Z_b}\right)$

$$A' = \frac{D}{\Delta T} = \frac{n(Z_a+Z_b)}{Z_b}$$

$$B' = \frac{B}{\Delta T} = n Z_a$$

$$C' = \frac{C}{\Delta T} = \frac{1}{n Z_b}$$

$$D' = \frac{A}{\Delta T} = \frac{1}{n}$$

(3). $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{Z_a Z_b}{Z_a+Z_b}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-1}{n}\left(\frac{Z_b}{Z_a+Z_b}\right)$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{Z_b}{n(Z_a+Z_b)}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{n^2(Z_a+Z_b)}$$

and

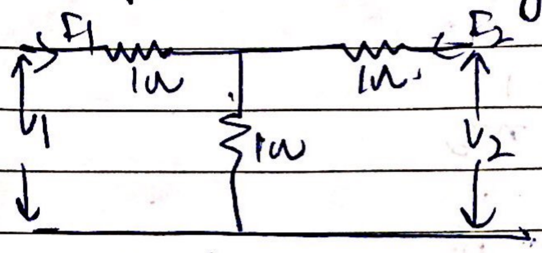
$$g_{11} = \frac{h_{22}}{\Delta h} = \frac{1}{Z_b}$$

$$g_{12} = \frac{-h_{21}}{\Delta h} = -n$$

$$g_{21} = \frac{-h_{12}}{\Delta h} = n$$

$$g_{22} = \frac{h_{11}}{\Delta h} = n^2 Z_a$$

Q.30 Two Identical sections of the n/w shown in figure are cascaded. Calculate the ABCD parameters of the resulting network.



Solⁿ For one network:—

$$V_1 = 1 \cdot I_1 + 1 \cdot (I_1 + I_2)$$

$$\boxed{V_1 = 2I_1 + I_2} \text{ — (1)}$$

$$V_2 = 1 I_2 + 1 (I_1 + I_2)$$

$$\boxed{V_2 = 2I_2 + I_1} \text{ — (2)}$$

Case 1: when $I_2 = 0$

$$V_1 = 2I_1$$

$$V_2 = I_1$$

So $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0} = 2$

$C = \dfrac{I_1}{V_2} = 1 \text{ ohm}^{-1}$

Case 2: when $V_2 = 0$

$$2I_2 = -I_1$$

$$I_2 = \frac{-I_1}{2} \text{ and } I_1 = -2I_2$$

$B = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0} = 3$

$D = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0} = 2.$

So $[T] = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

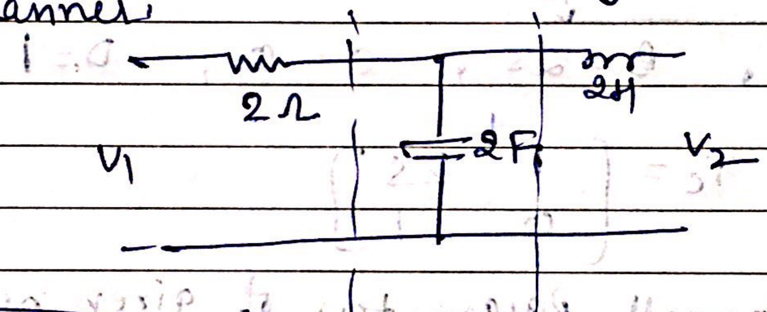
Similarly, for second network, parameters will be $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

So, The overall T-parameters of the cascaded network is given by:-

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

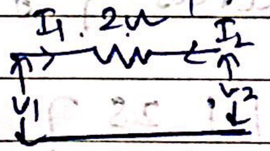
$$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \text{ Ans.}$$

Q.31 Determine the transmission of a T-network as shown in figure. Considering three sections as shown in fig, assuming connected in cascade manner.



Sol^n

for Na



$V_1 = 2I_1 + V_2$

$I_1 = -I_2$

Case 1: when $I_2 = 0$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0$$

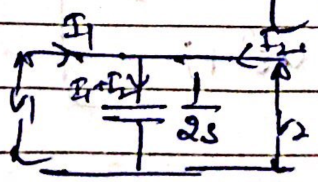
Case 2: when $V_2 = 0$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 2$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$$

So $T_a = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

for Nb



$$V_1 = \frac{1}{2s} I_1 + \frac{1}{2s} I_2$$

$$V_2 = \frac{1}{2s} I_1 + \frac{1}{2s} I_2$$

DOMS

Case 1: when $I_2 = 0$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1 \quad , \quad C = \frac{I_1}{V_2}\Big|_{I_2=0} = 2S$$

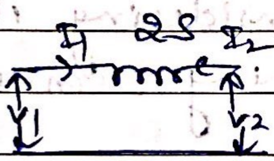
Case 2: when $V_2 = 0$

$$B = 0 \quad , \quad D = 1$$

So

$$T_B = \begin{bmatrix} 1 & 0 \\ 2S & 1 \end{bmatrix}$$

$\boxed{\text{for } N_C}$



$$A = 1 \quad , \quad B = 2S \quad , \quad C = 0 \quad , \quad D = 1$$

So

$$T_C = \begin{bmatrix} 1 & 2S \\ 0 & 1 \end{bmatrix}$$

So, the overall parameters of given network

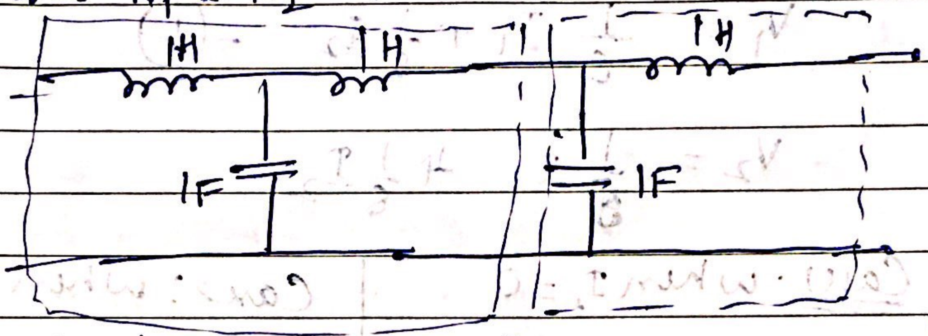
$$T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2S & 1 \end{bmatrix} \begin{bmatrix} 1 & 2S \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4S & 2 \\ 2S & 1 \end{bmatrix} \begin{bmatrix} 1 & 2S \\ 0 & 1 \end{bmatrix}$$

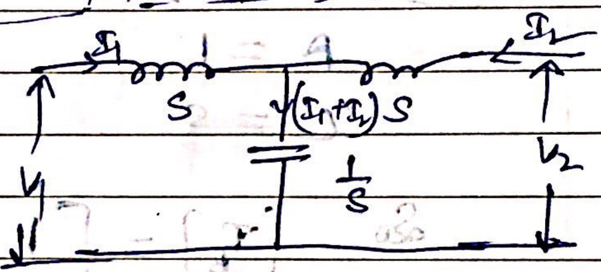
$$= \begin{bmatrix} 1+4S & 2S(1+4S)+2 \\ 2S & 2S(2S)+1 \end{bmatrix}$$

$$T = \begin{bmatrix} 4S+1 & 2(4S^2+S+1) \\ 2S & 4S^2+1 \end{bmatrix}$$ Ans.

Q.32 Determine the Transmission parameters of the given network as shown in figure, using the concept of interconnection of two two-port networks N_1 & N_2 in cascade.



Sol^n for Network N_1 :-



$$V_1 = S I_1 + \frac{1}{S}(I_1 + I_2)$$

$$\boxed{V_1 = \left(\frac{S^2+1}{S}\right) I_1 + \frac{1}{S} I_2} \quad \text{—— (1)}$$

$$V_2 = S I_2 + \frac{1}{S}(I_1 + I_2)$$

$$\boxed{V_2 = \frac{1}{S} I_1 + \left(\frac{S^2+1}{S}\right) I_2} \quad \text{—— (2).}$$

So, Case 1 : when $I_2 = 0$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = S^2+1$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = S$$

Case 2 : when $V_2 = 0$

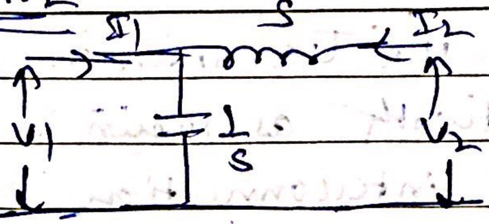
$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 2S + S^3$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1+S^2$$

$$[T_A] = \begin{bmatrix} 1+S^2 & 2S+S^3 \\ S & 1+S^2 \end{bmatrix}$$

DOMS

for Network N_2



$$V_1 = \frac{1}{s} I_1 + \frac{1}{s} I_2 \quad \text{—(1)}$$

$$V_2 = \frac{1}{s} I_1 + \frac{1}{s} I_2$$

Case 1: when $I_2 = 0$

$$A = 1$$
$$C = s$$

Case 2: when $V_2 = 0$

$$B = s$$
$$D = 1+s^2$$

So
$$[T_B] = \begin{bmatrix} 1 & s \\ s & 1+s^2 \end{bmatrix}$$

So overall Parameters:—

$$T = [T_A] \cdot [T_B]$$

$$= \begin{bmatrix} 1+s^2 & 2s+s^3 \\ s & 1+s^2 \end{bmatrix} \begin{bmatrix} 1 & s \\ s & 1+s^2 \end{bmatrix}$$

$$T = \begin{bmatrix} 1+3s^2+s^4 & 3s+4s^3+s^5 \\ 2s+s^3 & 1+3s^2+s^4 \end{bmatrix}$$ Ans.