Chapter 9

Network Functions and Responses

Inside this Chapter

9.1. INTRODUCTION

The concept of transform impedance and transform admittance is discussed in this chapter. Further more, a function relating currents or voltages at different parts of the network, called a *transfer function*, is found to be mathematically similar to the transform impedance function. These functions are called network functions. The definitions and locations (in the s-plane) of poles and zeros are also introduced in this chapter.

9.2. CONCEPT OF COMPLEX FREQUENCY

As we have seen earlier, the solution of differential equations for the networks is of the form

$$x(t) = K e^{s_n t} \qquad \text{...(1)}$$

This $x(t)$ may be a voltage $v(t)$ or a current $i(t)$. Generally, $x(t)$ is a function of time. And s_n is a complex number, which may be expressed as

$$s_n = \sigma_n + j\omega_n \qquad \text{...(2)}$$

where ω_n, the imaginary part of s_n, is called as angular frequency (radian frequency) and it uses in time-domain equations in the forms of cos $\omega_n t$ or sin $\omega_n t$. The dimension of the radian frequency is radian per second. The radian frequency may be expressed as

$$\omega_n = 2\pi f_n = \frac{2\pi}{T} \qquad \text{...(3)}$$

where f_n is the frequency, in hertz (Hz) and T is the time period, in seconds.

From equation (2), we can easily see that σ_n and ω_n must have identical dimensions. The dimension of ω_n is (time)$^{-1}$, since the radian is a dimensionless quantity (being length of arc per length of radius). The dimension of σ_n can be determined as σ_n appears as an exponential factor, *i.e.*,

$$V = V_o e^{\sigma_n t} \qquad \text{...(4)}$$

or

$$\sigma_n = \frac{1}{t}\ln\frac{V}{V_o}$$

Since the usual unit for the natural logarithm is the neper, therefore, the dimension of σ_n is neper per second.

Now, the complex sum

$$s_n = \sigma_n + j\omega_n$$

is defined as the complex frequency. The real part of the complex frequency is neper frequency, corresponds to exponential decay or exponential increase (depending on sign) and the imaginary part of the complex frequency is the radian frequency (or real frequency), corresponds to oscillations.

9.2.1. Terminal Pairs or Ports

In figure 9.1(*a*) is shown a representation of a one-port network. The pair of terminals is connected to an energy source which is the driving force for the network, so that the pair of terminals is known as the driving point of the network.

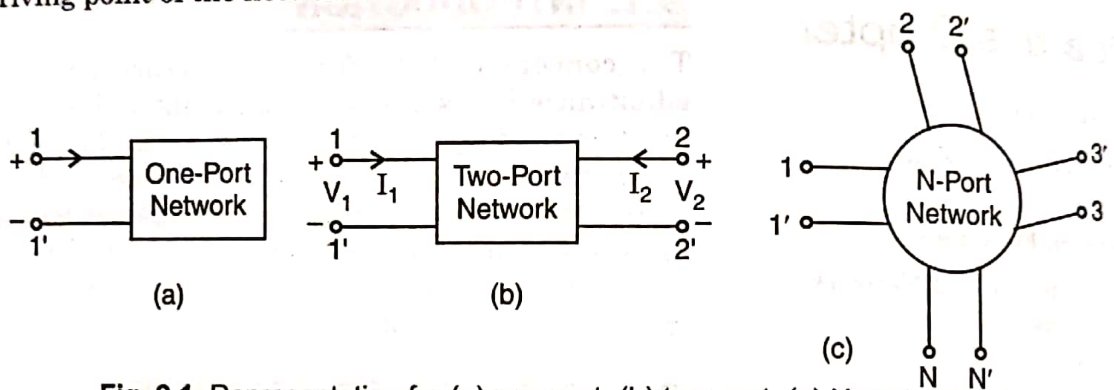


Fig. 9.1. Representation for (a) one-port, (b) two-port, (c) N-port networks

Figure 9.1(*b*) shows a two-port network. The port 1–1' is assumed to be connected to the driving force (as an input), and port 2--2' is connected to a load (as an output). In (*c*) of figure 9.1 is shown a representation of an *N*-port network for the general case. Our emphasis now will be on one-port and two-port networks.

9.3. NETWORK FUNCTIONS

(A) The *transform impedance* at a port has been defined as the ratio of voltage transform to current transform. Thus we write

$$Z(s) = \frac{V(s)}{I(s)}$$

Similarly, the *transform admittance* is defined as the ratio of current transform to voltage transform, *i.e.*,

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)}$$

The transform impedance and transform admittance must relate to the source port 1–1' or 2–2' in figure 9.1(*a*) and (*b*). The impedance (or admittance) found at a given port is called a *driving-point impedance* (*or admittance*) *i.e.*, transform impedances (or admittances) of ports 1–1' and 2–2' are also called as input and output driving point impedances (or admittances) respectively.

Because of the similarity of impedance and admittance, these two quantities are assigned one name "*immittance*" (a combination of *im*pedance and ad*mittance*). An immittance is thus an impedance or an admittance. Table 9.1 shows the immittance of circuit elements, and the immittance functions for some networks are given in Table 9.2.

Table ...

Elements	Impedance function Z(s)	Admittance function Y(s)
Resistance (R) ; in Ω	R	$1/R = G$
Inductance (L) ; in H	sL	$1/sL$
Capacitance (C) ; in F	$1/sC$	sC

Therefore,

$$Z(s) \equiv R \equiv sL \equiv \frac{1}{sC} \quad \text{and} \quad Y(s) \equiv \frac{1}{R} \equiv \frac{1}{sL} \equiv sC$$

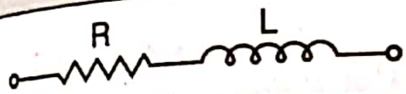
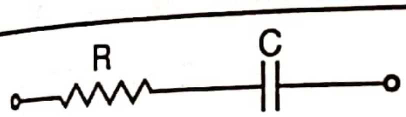
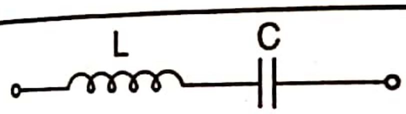
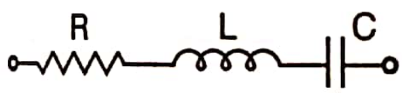
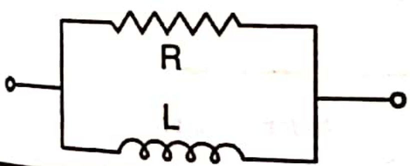
Note :

The impedance function $Z(s)$ and admittance function $Y(s)$ are easily determined for series a... parallel circuit respectively. As,

(i) For series circuit ; $Z(s) = Z_1(s) + Z_2(s) + Z_3(s)$

(ii) For parallel circuit ; $Y(s) = Y_1(s) + Y_2(s) + Y_3(s)$

Table 9.2. Immittance functions for some simple networks

Networks	Impedance function Z(s)	Admittance function Y(s)
	$R + sL$	$\frac{1}{R + sL}$
	$\frac{sRC + 1}{sC}$	$\frac{sC}{sRC + 1}$
	$\frac{s^2LC + 1}{sC}$	$\frac{sC}{s^2LC + 1}$
	$\frac{sRC + s^2LC + 1}{sC}$	$\frac{sC}{sRC + s^2LC + 1}$
	$\frac{sRL}{sL + R}$	$\frac{sL + R}{sRL}$

(B) The transfer function is used to describe networks which have at least two ports. In general, the transfer function relates the transform of a quantity at one port to the transform of another quantity at another port. Thus transfer functions have the following possible forms :

(*i*) The ratio of one voltage to another current or one current to another voltage ; Z(s) or Y(s).

(*ii*) The ratio of one voltage to another voltage, or the voltage transfer function ; G(s).

(*iii*) The ratio of one current to another current, or the current transfer function ; α(s).

It is conventional, to define transfer functions as the ratio of an output quantity to an input quantity. In terms of the two-port network of figure 9.1 (*b*), the output quantities are $V_2(s)$ and $I_2(s)$ and the input quantities are $V_1(s)$ and $I_1(s)$. Using this scheme, these are only four basic transfer functions for the two port network and these are given as :

(1) Transfer impedance function ; $Z_{21}(s) = \dfrac{V_2(s)}{I_1(s)}$

(2) Transfer admittance function ; $Y_{21}(s) = \dfrac{I_2(s)}{V_1(s)}$

(3) Voltage transfer function ; $G_{21}(s) = \dfrac{V_2(s)}{V_1(s)}$

(4) Current transfer function ; $\alpha_{21}(s) = \dfrac{I_2(s)}{I_1(s)}$

Note :

The ratio of an input quantity to an output quantity is termed as the *'inverse transfer function'* i.e.,

$\dfrac{V_1(s)}{I_2(s)} = Z_{12}(s)$; the inverse transfer impedance function.

$\dfrac{I_1(s)}{V_2(s)} = Y_{12}(s)$; the inverse transfer admittance function.

$\dfrac{V_1(s)}{V_2(s)} = G_{12}(s)$; the inverse voltage transfer function.

$\dfrac{I_1(s)}{I_2(s)} = \alpha_{12}(s)$; the inverse current transfer function.

EXAMPLE 9.1 Obtain the driving point impedance functions or transform impedances Z(s) for the networks shown in figure 9.2 (a) to (c), in which transformed impedance marked for each element.

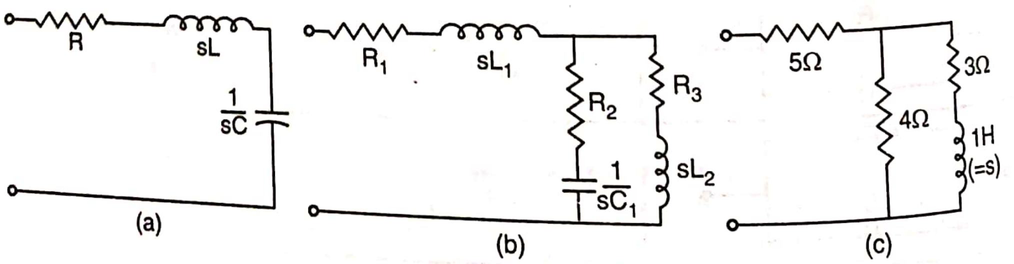


Fig. 9.2.

Solution : (*a*) As an illustration, the driving point impedance function $Z(s)$ for the series *R-L-C* network of figure 9.2(*a*) is

$$Z(s) = R + sL + \frac{1}{sC} = \frac{RCs + LCs^2 + 1}{sC}$$

(b) Network shown in figure 9.2(b) contains series parallel branches only.

Hence,
$$Z(s) = (R_1 + sL_1) + \left[\left(R_2 + \frac{1}{sC_1}\right) \Big\| (R_3 + sL_2) \right]$$

$$= (R_1 + sL_1) + \left[\frac{\left(R_2 + \frac{1}{sC_1}\right) \cdot (R_3 + sL_2)}{R_2 + \frac{1}{sC_1} + R_3 + sL_2} \right] = \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_2 s^2 + b_1 s + b_0}$$

Where, $a_3 = L_1 L_2 C_1$; $b_2 = L_2\, C_1$,

$a_2 = [(R_2 + R_3)\, L_1 + (R_1 + R_2)\, L_2]\, C_1$; $b_1 = (R_2 + R_3)\, C_1$

$a_1 = (L_1 + L_2) + (R_1 R_2 + R_2 R_3 + R_3 R_1) C_1$; $b_0 = 1$

$a_0 = R_1 + R_3$

(c) For figure 9.2(c) ; The impedance function for 3 Ω, 1*H* series circuit is $Z_1(s) = 3 + s$. The branch $Z_1(s)$ is in parallel with branch $Z_2(s) = 4\ \Omega$.

Hence,
$$Z(s) = 5 + [(3 + s) \,|\, 4]$$

$$= 5 + \frac{(3+s).\,4}{3 + s + 4} = \frac{9s + 47}{s + 7} = 9 \cdot \left(\frac{s + \frac{47}{9}}{s + 7} \right)$$

EXAMPLE 9.2 **Determine the driving point admittance functions or Transform admittances $Y(s)$ for the network shown in figure 9.3 (*a*) to (*c*).**

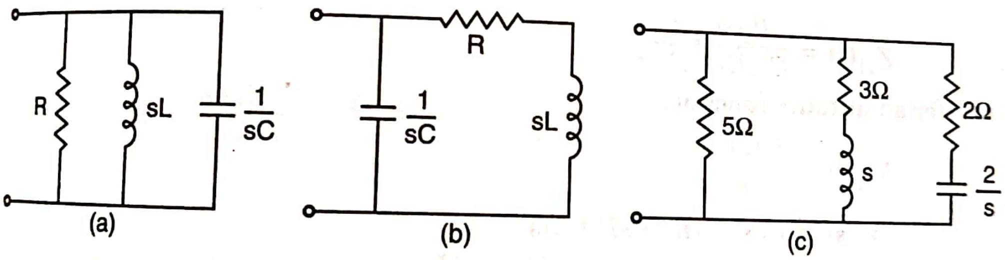


Fig. 9.3.

Solution : (a) $Y(s) = \frac{1}{R} + \frac{1}{sL} + sC = \frac{sL + R + s^2 RLC}{sRL}$

(b) The series combination of sL and R is (sL + R), which is in parallel with $\frac{1}{sC}$.

Therefore
$$Y(s) = \frac{1}{(sL + R)} + sC = \frac{1 + s^2 LC + sRC}{sL + R}$$

(c)
$$Y_1(s) = \frac{1}{5} = 0.2,\ Y_2(s) = \frac{1}{3 + s},$$

$$Y_3(s) = \frac{1}{2 + \frac{2}{s}} = \left(\frac{s}{2s + 2} \right) = \frac{0.5s}{s + 1}$$

Hence,
$$Y(s) = 0.2 + \frac{1}{3 + s} + \frac{0.5s}{s + 1} = \frac{1.6 + 3.3s + 0.7s^2}{3 + 4s + s^2}$$

EXAMPLE 9.3 **For the circuit shown in figure 9.4. Determine the network transfer function.**

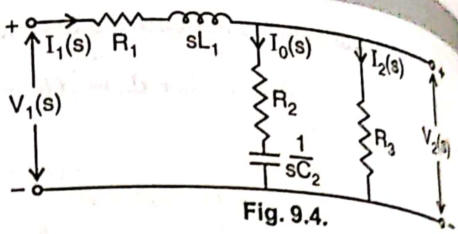


Fig. 9.4.

Solution : The network equations are

$$V_2(s) = R_3 I_2(s) \qquad \text{...(1)}$$

$$= \left(R_2 + \frac{1}{sC_2}\right) I_0(s) \qquad \text{...(2)}$$

or
$$I_0(s) = \frac{V_2(s)}{R_2 + \dfrac{1}{sC_2}}$$

$$I_1(s) = I_2(s) + I_0(s) \qquad \text{...(3)}$$

$$V_1(s) = V_2(s) + (R_1 + sL_1) I_1(s) \qquad \text{...(4)}$$

Transfer impedance function,

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

$$V_2(s) = \left(R_2 + \frac{1}{sC_2}\right) I_0(s)$$

$$I_1(s) = I_2(s) + I_0(s) = I_0(s)\left[\frac{I_2(s)}{I_0(s)} + 1\right]$$

$$= I_0(s)\left[\frac{R_2}{R_3} + \frac{1}{sC_2R_3} + 1\right] \qquad \text{[From Equation (1)]}$$

$$\therefore \quad Z_{21}(s) = \frac{R_3(sC_2R_2 + 1)}{C_2(R_2 + R_3)s + 1}$$

(ii) Transfer admittance function,

$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

$$V_1(s) = V_2(s) + (R_1 + sL_1)\, I_1(s)$$

$$= R_3 I_2(s) + (R_1 + sL_1)\,[I_2(s) + I_0(s)] \qquad \text{[From Equations (1) and (3)]}$$

$$= I_2(s)\left[R_3 + (R_1 + sL_1) + (R_1 + sL_1)\left(\frac{R_3}{R_2 + \dfrac{1}{sC_2}}\right)\right]$$

$$= I_2(s)\left[\frac{(R_3 + R_1 + sL_1)(sC_2R_2 + 1) + (R_1 + sL_1).\,R_3.sC_2}{(sC_2R_2 + 1)}\right]$$

$$\therefore \; Y_{21}(s) = \frac{sC_2R_2 + 1}{L_1C_2\,(R_2 + R_3)\,s^2 + \left[C_2(R_1R_2 + R_2R_3 + R_3R_1) + L_1\right]s + (R_1 + R_3)}$$

(iii) Voltage transfer function,

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{V_2(s)}{I_1(s)} \cdot \frac{I_2(s)}{V_1(s)} \cdot \frac{I_1(s)}{I_2(s)} = Z_{21}(s) \cdot Y_{21}(s) \cdot \frac{I_1(s)}{I_2(s)}$$

$$I_1(s) = I_2(s) + I_0(s) = I_2(s)\left[1 + \frac{I_0(s)}{I_2(s)}\right]$$

$$= I_2(s)\left[1+\frac{R_3}{R_2+\frac{1}{sC_2}}\right] = I_2(s)\left[\frac{[(sR_2C_2+1)]+sC_2R_3}{(sR_2C_2+1)}\right]$$

$$\frac{I_1(s)}{I_2(s)} = \frac{C_2(R_2+R_3)\,s+1}{sR_2C_2+1}$$

$$\therefore G_{21}(s) = \frac{R_3(sC_2R_2+1)}{L_1C_2(R_2+R_3)s^2+[C_2(R_1R_2+R_2R_3+R_3R_1)+L_1]s+(R_1+R_3)}$$

(*iv*) Current transfer function,

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)} = \frac{sR_2C_2+1}{C_2(R_2+R_3)s+1}$$

EXAMPLE 9.4 Find the voltage transfer functions of the networks shown in figure 9.5 (*a*) to (*c*).

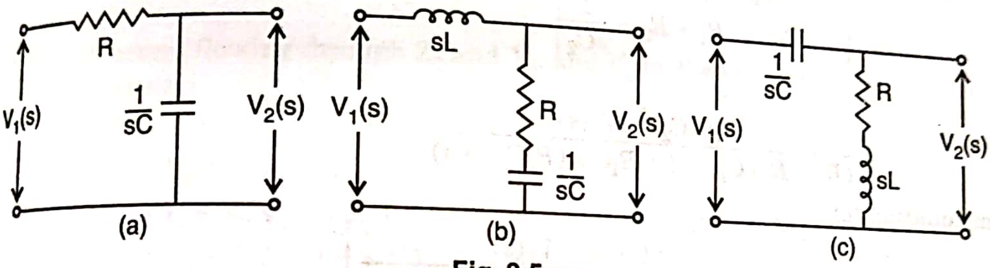


Fig. 9.5.

Solution :

(*a*) $$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{sRC+1}$$

(*b*) $$G_{21}(s) = \frac{R+\frac{1}{sC}}{R+\frac{1}{sC}+sL} = \frac{sRC+1}{s^2LC+sRC+1}$$

(*c*) $$G_{21}(s) = \frac{R+sL}{R+sL+\frac{1}{sC}} = \frac{sC\,(R+sL)}{s^2LC+sRC+1}$$

EXAMPLE 9.5 Find the expression of voltage transfer function for the network shown in figure 9.6(*a*).

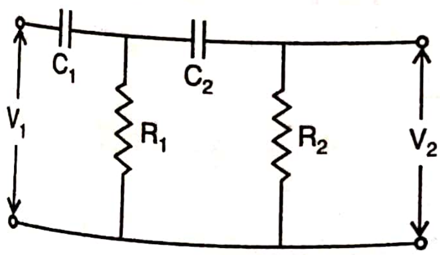


Fig. 9.6(a).

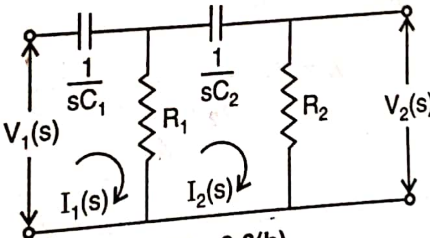


Fig. 9.6(b).

Solution : Redrawing the circuit in Laplace (*s*-domain) as shown in figure 9.6(*b*).

From first (leftmost) loop,

$$V_1(s) = I_1(s)\left[R_1 + \frac{1}{sC_1}\right] - R_1 I_2(s) \quad \text{...(a)}$$

From second loop,

$$-R_1 I_1(s) + \left(R_1 + R_2 + \frac{1}{sC_2}\right) I_2(s) = 0 \quad \text{...(b)}$$

And from third (rightmost) loop,

$$V_2(s) = R_2 I_2(s) \quad \text{...(c)}$$

Using Cramer's rule from equations (*a*) and (*b*),

$$I_2(s) = \frac{\begin{vmatrix} R_1 + \dfrac{1}{sC_1} & V_1(s) \\ -R_1 & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + \dfrac{1}{sC_1} & -R_1 \\ -R_1 & R_1 + R_2 + \dfrac{1}{sC_2} \end{vmatrix}} = \frac{R_1 . V_1(s)}{\left(R_1 + \dfrac{1}{sC_1}\right)\left(R_1 + R_2 + \dfrac{1}{sC_2}\right) - R_1^2}$$

$$= \frac{R_1 C_1 C_2 s^2 \cdot V_1(s)}{(R_1 + R_2)\, C_2 s + 1 + R_1 C_1 s\,(R_2 C_2 s + 1)}$$

From equation (*c*),

$$V_2(s) = R_2 \cdot \left[\frac{R_1 C_1 C_2 s^2 \cdot V_1(s)}{(R_1 + R_2)\, C_2 s + 1 + R_1 C_1 s\,(R_2 C_2 s + 1)}\right]$$

Therefore,

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \left[\frac{R_1 R_2 C_1 C_2 s^2}{(R_1 + R_2)\, C_2 s + 1 + R_1 C_1 s\,(R_2 C_2 s + 1)}\right]$$

(C) The Calculation of Network Functions for Ladder Networks

If each immittance of the network of figure 9.7 represents one element, the network is known as a simple ladder network. We follow the practice of characterising series arms by their impedances and shunt (parallel) arms by their admittances.

We begin our computations at the port other than the one for which the driving point immittance is being found. Thus in figure 9.7 (with only six arms being considered), we begin with Y_6. It is first inverted and combined with Z_5. Next, this sum is inverted and combined with Y_4. This pattern is continued until the process terminates. The driving point impedance or transform impedance at port 1–1' will then be

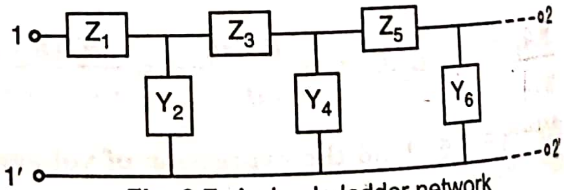


Fig. 9.7. A simple ladder network

$$Z = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6 + \dots}}}}}$$

This equation is known as continued fraction. It may be simplified, to determine Z for a given ladder network.

EXAMPLE 9.6 Determine the open-circuit driving-point impedance (Z_{11}), transfer impedance (Z_{21}) and voltage transfer function (G_{21}) for the network shown in figure 9.8.

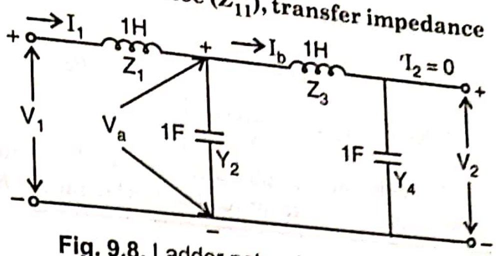


Fig. 9.8. Ladder network of example 9.6

Solution : First, immittances are converted in s-domain. As,

$$Z_1 = s \;;\; Y_2 = s \;;\; Z_3 = s \;;\; Y_4 = s$$

Therefore,
$$Z_{11}(s) = s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{s}}}$$

This equation is reduced by starting at the last term and combining terms step by step, giving

$$Z_{11}(s) = s + \cfrac{1}{s + \cfrac{s}{s^2+1}} = s + \frac{s^2+1}{s^3+2s} = \frac{s^4+3s^2+1}{s^3+2s}$$

Let I_b be the current flowing through Z_3 and Y_4, and V_a be the voltage across Y_2. Then, we write the following equations.

$$I_b = Y_4 V_2 = sV_2$$
$$V_a = V_2 + I_b Z_3 = V_2 + sV_2 \cdot s = (s^2+1)V_2$$
$$I_1 = I_b + Y_2 V_a = sV_2 + s \cdot (s^2+1)V_2 = (2s + s^3)V_2$$
$$V_1 = V_a + Z_1 I_1 = (s^2+1)V_2 + s \cdot (2s+s^3)V_2 = (s^4 + 3s^2 + 1)V_2$$

From above equations,

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = \frac{1}{s^3+2s}$$

and
$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{s^4+3s^2+1}$$

Alternative way : (Check) :

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = \frac{V_2(s)}{I_1(s)} \cdot \frac{V_1(s)}{V_2(s)} = Z_{21}(s) \cdot \frac{1}{G_{21}(s)} = \frac{s^4+3s^2+1}{s^3+2s}$$

9.4. POLES AND ZEROS OF NETWORK FUNCTIONS

We have shown that in linear *RLC* networks, all network functions $T(s)$ are the rational functions of s and may be expressed as the ratio of two polynomials namely, $N(s)$, the numerator polynomial and $D(s)$, the denominator polynomial as

$$T(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1}s^{n-1} + \dots\dots\dots + a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + \dots\dots\dots + b_1 s + b_0} \quad \text{...(1)}$$

or
$$T(s) = \frac{N(s)}{D(s)} = K \frac{s^n + c_{n-1}s^{n-1} + \dots\dots\dots + c_1 s + c_0}{s^m + d_{m-1}s^{m-1} + \dots\dots\dots + d_1 s + d_0} \quad \text{...(2)}$$

Where $K = \frac{a_n}{b_m}$; is a positive constant known as scalar factor, the cofficients a, b, c and d are real and positive for passive network and no dependent sources.

The polynomial $N(s) = 0$ has n roots, they are called as *zeros* of the network function $T(s)$, and the polynomial $D(s) = 0$ has m roots, they are called as *poles* of the $T(s)$.

Writing equation (2) in terms of the roots in a factored form, we obtain

$$T(s) = K\frac{(s-z_1)(s-z_2)\,........\,(s-z_n)}{(s-p_1)(s-p_2)\,........\,(s-p_m)} \quad ...(3)$$

Where z_i, the zeros of $T(s)$, and
p_j, the poles of $T(s)$.

The value of the poles and zeros of $T(s)$ and hence their location in s-plane completely specify the network function except for the scalar factor K.

Note :

In the complex s-plane, a pole is denoted by a small cross (×) and a zero by a small circle (o).

Case I : When r poles ($r < m$) or r zeros ($r < n$) in equation (3) have the same value, the pole or zero is said to be of *multiplicity* r.

Case II : When $n > m$, we have *poles at infinity* of multiplicity or degree of $(n-m)$.

Case III : When $n < m$, we then have *zeros at infinity* of multiplicity or degree of $(m-n)$.

For any rational network function,
the number of finite poles + the number of poles at infinity
the number of finite poles + the number of zeros at infinity *i.e.*, the total number of poles
= the number of finite zeros + the number of zeros at infinity
= the total number of zeros.

EXAMPLE 9.7 Obtain the pole-zero location for the function

$$T(s) = \frac{(2s+4)(s+4)}{s(s+1)(s+3)}$$

Solution : The poles are at $s = 0$, $s = -1$, $s = -3$
and the zeros are at $s = -2$, $s = -4$, $s = \infty$
The poles and zeros of $T(s)$ in the s-plane are shown in figure 9.9.

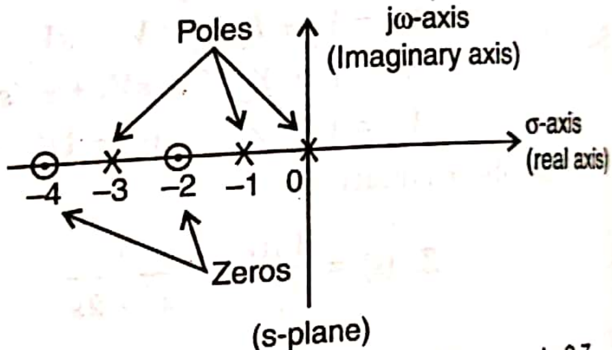


Fig. 9.9. Pole-zero diagram of example 9.7

EXAMPLE 9.8 Obtain the pole-zero locations of the following functions.

(i) $f_1(t) = e^{-\sigma t}$ **; a decaying exponential**

(ii) $f_2(t) = \cos \omega t$ **; a sinusoid**

(iii) $f_3(t) = e^{-\sigma t} \cos \omega t$ **; a damped sinusoid**

Solution :

(i) $f_1(t) = e^{-\sigma t}$

$$F_1(s) = £[f(t)] = \frac{1}{s+\sigma}$$; has a pole at $s = -\sigma$

(ii) $f_2(t) = \cos \omega t$

$$F_2(s) = \frac{s}{s^2+\omega^2}$$; has a pair of simple conjugate poles at $s = \pm j\omega$ and a zero at origin

(iii) $f_3(t) = e^{-\sigma t} \cos \omega t$

$$F_3(s) = \frac{(s+\sigma)}{(s+\sigma)^2+\omega^2}$$;

has a zero at $s = -\sigma$ and a pair of complex conjugate poles at $s = -\sigma \pm j\omega$

The pole-zero locations of $f_1(t)$, $f_2(t)$ and $f_3(t)$ are shown in figure 9.10(*a*), (*b*) and (*c*) respectively.

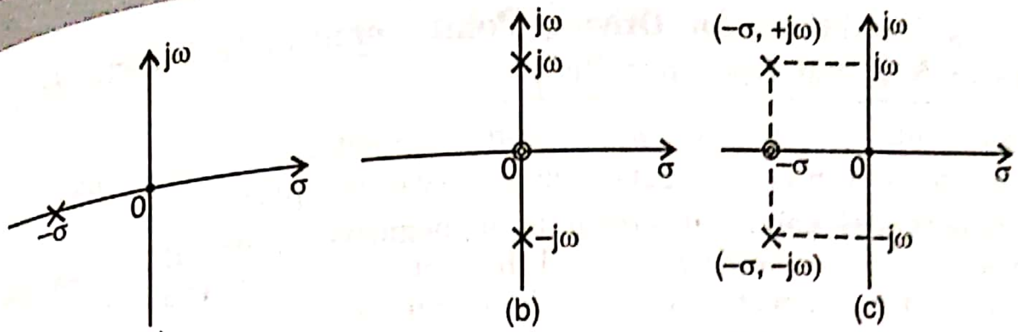


Fig. 9.10. Poles and zeros of example 9.8

EXAMPLE 9.9 **Find the open circuit driving point impedance at terminals 1-1′ of the ladder network shown in figure 9.11.**

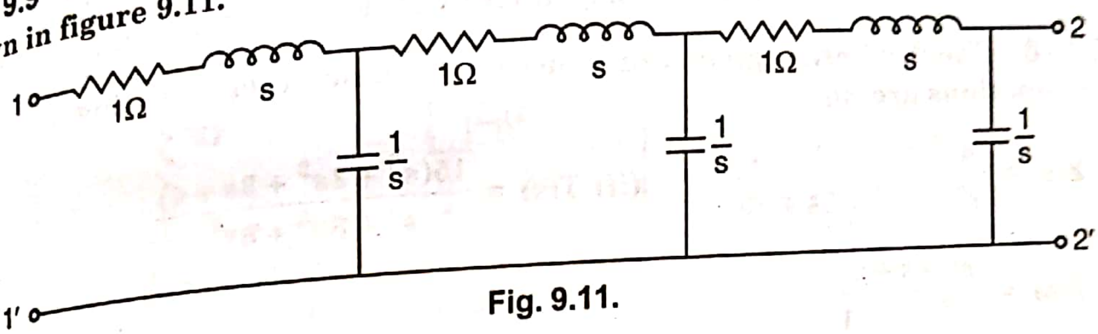


Fig. 9.11.

Solution : From figure 9.11,

$$Z_1(s) = s+1,\ Y_2(s) = s,\ Z_3(s) = s+1,\ Y_4 = s,\ Z_5 = s+1,\ Y_6 = s.$$

Therefore,

$$Z_{11}(s) = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6}}}}}$$

$$= (s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \cfrac{1}{s}}}}}$$

$$= (s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \cfrac{1}{s + \cfrac{s}{s^2+s+1}}}}$$

$$= (s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \cfrac{s^2+s+1}{s^3+s^2+2s}}}$$

$$= \frac{s^6 + 6s^5 + 11s^4 + 13s^3 + 10s^2 + 5s + 1}{s^5 + 5s^4 + 5s^3 + 3s^2 + 3s}$$

9.4.1. Necessary Conditions for Driving Point Immittance Functions (with common factors in $N(s)$ and $D(s)$ cancelled) :

1. The coefficients in the polynomials $N(s)$ and $D(s)$ must be real and positive.
2. Poles and zeros must be conjugate if imaginary or complex.
3. The real part of all poles and zeros must be negative or zero, if the real part is zero, then that pole or zero must be simple. *i.e.* all the roots of $N(s) = 0$ and $D(s) = 0$ lie on the left half of s-plane and simple roots may lie on the imaginary or $j\omega$-axis.
4. The polynomials $N(s)$ and $D(s)$ may not have missing terms between those of highest and lowest degrees, unless all even or all odd terms are missing.
5. The highest degree of $N(s)$ and $D(s)$ may differ by either zero or one only.
6. The lowest degree of $N(s)$ and $D(s)$ may differ by either zero or one only.

EXAMPLE 9.10 Check whether given functions are suitable in representing the driving point immittance functions are not

(i) $Z(s) = \dfrac{4s^4 + s^2 - 3s + 1}{s^3 + 2s^2 + 2s + 40}$ (ii) $Y(s) = \dfrac{15(s^3 + 2s^2 + 3s + 4)}{s^4 + 8s^3 + 6s^2}$

(iii) $Z(s) = \dfrac{s^2 + s + 2}{2s^2 + s + 1}$

Solution : (i) No ; one coefficient is missing and one is negative.
(ii) No ; The lowest degrees of N(s) and D(s) differ by two.
(iii) Yes ; all conditions are satisfied.

9.4.2. Necessary Conditions for Transfer Functions (with common factors in $N(s)$ and $D(s)$ cancelled)

1. The coefficients in the polynominals $N(s)$ and $D(s)$ of $T = N/D$ must be real and those for $D(s)$ must be positive.
2. Poles and zeroes must be conjugate if imaginary or complex.
3. The real part of poles must be negative or zero, if the real part is zero, then that pole must be simple. This includes the origin.
4. The polynominal $D(s)$ may not have any missing term between that of highest and lowest degrees, unless all even or all odd terms are missing.
5. The polynominal $N(s)$ may have terms missing, and some of the coefficients may be negative.
6. The degree of $N(s)$ may be as small as zero independent of the degree of $D(s)$.
7. (a) for G and α : The maximum degree of $N(s)$ is equal to the degree of $D(s)$.
   (b) For Z and Y : The maximum degree of $N(s)$ is equal to the degree of $D(s)$ plus one.

EXAMPLE 9.11 Check whether given functions are suitable in representing the transfer functions are not.

(i) $G_{21}(s) = \dfrac{3s + 2}{5s^3 + 4s^2 + 1}$ (ii) $\alpha_{21}(s) = \dfrac{2s^2 + 5s + 1}{s + 7}$

(iii) $Z_{21}(s) = \dfrac{1}{s^3 + 2s}$ (iv) $G_{21}(s) = \dfrac{2s^2 + 5}{3s^2 + 9s + 1}$

Solution : (i) No ; coefficient is missing in polynominal $D(s)$.
(ii) No ; the degree of $N(s)$ is greater then $D(s)$.
(iii) Yes ; all conditions are satisfied.
(iv) Yes ; all conditions are satisfied.

EXAMPLE 9.12 **For the circuit shown in figure 9.12(a), obtain driving point admittance $Y_{11} = I_1/V_1$.**

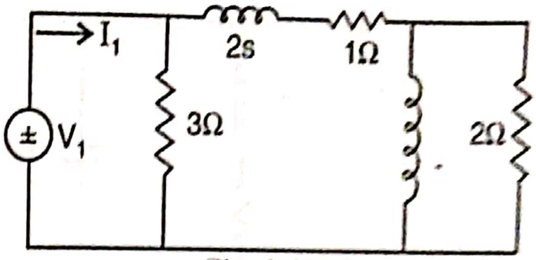


Fig. 9.12(a).

Solution : By circuit reduction technique, s and 2Ω are in parallel their equivalent as shown in figure 9.12(b).

$$Z_1(s) = s \,||\, 2 = \frac{2s}{s+2}$$

$Z_1(s)$ and a series combination of $2s$ and 1Ω are in series, their equivalent is shown in figure 9.12(c).

$$Z_2(s) = Z_1(s) + (2s+1) = \frac{2s}{s+2} + (2s+1) = \frac{2s^2+7s+2}{s+2}$$

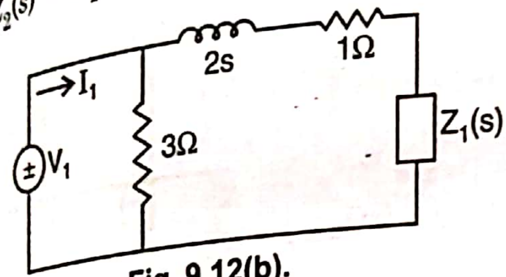


Fig. 9.12(b).

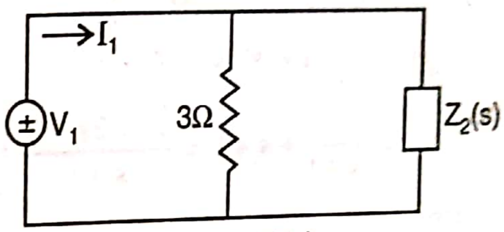


Fig. 9.12(c).

Now, $Z_2(s)$ and 3Ω are in parallel, their admittance equivalent is

$$Y_{eq}(s) = \frac{1}{3} + \frac{1}{Z_2(s)}$$

or

$$Y_{11}(s) = Y_{eq}(s) = \frac{1}{3} + \frac{s+2}{2s^2+7s+2} = \frac{2s^2+10s+8}{3(2s^2+7s+2)}$$

$$= \frac{2}{3} \cdot \frac{(s^2+5s+4)}{(2s^2+7s+2)}$$

EXAMPLE 9.13 **Find the transfer impedance function $Z_{21}(s) = V_2(s)/I_1(s)$ of the network shown in figure 9.13.**

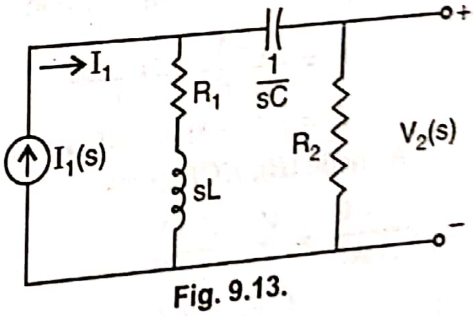


Fig. 9.13.

Solution : Applying current division rule,

Current in impedance $\left(R_2 + \frac{1}{sC}\right)$ is given by

$$I'(s) = I_1(s) \cdot \frac{R_1 + sL}{R_1 + sL + R_2 + \frac{1}{sC}}$$

$$= I_1(s) \cdot \frac{(R_1 + sL) \cdot sC}{sC \cdot (R_1 + R_2 + sL) + 1}$$

Then, $V_2(s) = I'(s) \cdot R_2 = I_1(s) \cdot \dfrac{(R_1 + sL) sC \cdot R_2}{sC(R_1 + R_2 + sL) + 1}$

Therefore,

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = \frac{R_2(s^2LC + sR_1C)}{s^2LC + sC(R_1+R_2)+1}$$

EXAMPLE 9.14 **Find the driving point impedance of the network shown in figure 9.14(a). Find the poles and zeros of the network and locate them in the s-plane.**

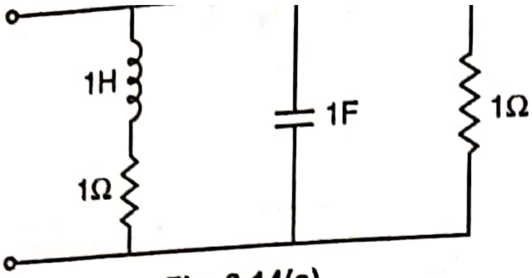


Fig. 9.14(a).

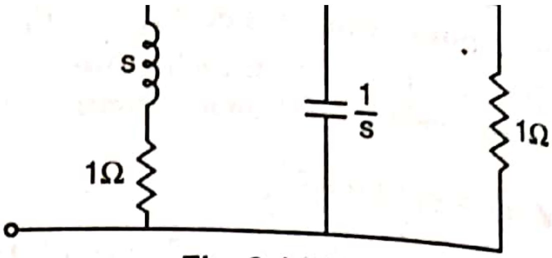


Fig. 9.14(b).

Solution : Let us first transform the impedances into s-domain [figure 9.14(b)]

There are 3-parallel branches having impedances,

$$Z_1(s) = s + 1 \quad ; \quad Z_2(s) = \frac{1}{s} \quad ; \qquad Z_3 = 1$$

So, $\quad Y(s) = Y_1(s) + Y_2(s) + Y_3(s)$

$$= \frac{1}{s+1} + s + 1 = \frac{s^2 + 2s + 2}{s+1}$$

Therefore,

$$Z(s) = \frac{1}{Y(s)} = \frac{s+1}{s^2 + 2s + 2}$$

$Z(s)$ being the driving point impedance

or $\quad Z(s) = \dfrac{s+1}{(s+1)^2 - (j1)^2}$

$$= \frac{s+1}{(s+1+j)(s+1-j)}$$

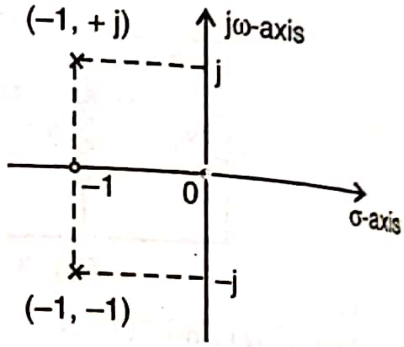


Fig. 9.14. (c) Pole-zero location of Z(s) of example 9.14

Thus, the poles are at $(-1-j)$ and $(-1+j)$ and there is one zero at -1, as located in figure 9.14(c).

EXAMPLE 9.15 **Find driving point impedance $Z_{11}(s)$, transfer impedance $Z_{21}(s)$ and voltage transfer function $G_{21}(s)$ for the circuit of figure 9.15.**

Solution : Let the voltage at node B be V. Applying KCL at node (A) gives,

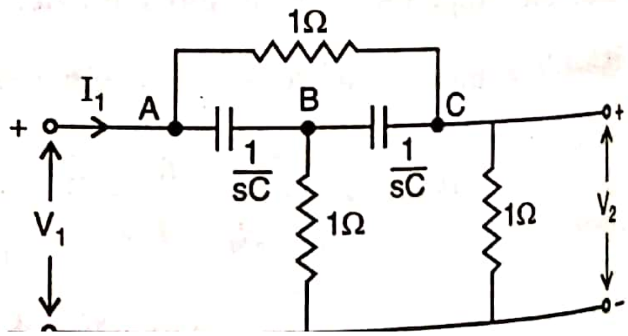
$$I_1 = \frac{V_1 - V}{1/sC} + \frac{V_1 - V_2}{1}$$

$$I_1 = (sC + 1)V_1 - sC\,V - V_2 \qquad \text{...(1)}$$

At node (B), KCL gives,

$$\frac{V_1 - V}{1/sC} = \frac{V}{1} + \frac{V - V_2}{1/sC}$$

$$sC\,V_1 - (2sC + 1)V + sC\,V_2 = 0 \qquad \text{...(2)}$$



At node (C), KCL gives

Putting the value of V from equation (5) in equation (2),

$$sCV_1 - (2sC+1)\left\{\left(1+\frac{2}{sC}\right)V_2 - \frac{1}{sC}V_1\right\} + sCV_2 = 0$$

$$\left(sC+2+\frac{1}{sC}\right)V_1 = \left(2sC+4+1+\frac{2}{sC}-sC\right)V_2$$

$$(s^2C^2+2sC+1)V_1 = (s^2C^2+5sC+2)V_2 \qquad ...(6)$$

or

$$V_1 = \left(\frac{s^2C^2+5sC+2}{s^2C^2+2sC+1}\right)V_2 \qquad ...(6a)$$

From equations (4) and (6a),

$$I_1 = \left[(sC+2)\left[\frac{s^2C^2+5sC+2}{s^2C^2+2sC+1}\right]-(sC+3)\right]V_2 = \left[\frac{2s^2C^2+5sC+1}{s^2C^2+2sC+1}\right]V_2$$

or

$$Z_{21}(s) = \frac{V_2}{I_1} = \left(\frac{\boldsymbol{s^2C^2+2sC+1}}{\boldsymbol{2s^2C^2+5sC+1}}\right)$$

And from equation (6),

$$V_2 = \left(\frac{s^2C^2+2sC+1}{s^2C^2+5sC+2}\right)V_1 \qquad ...(6b)$$

Now from equations (4) and (6b),

$$I_1 = \left[(sC+2)-(sC+3)\left\{\frac{s^2C^2+2sC+1}{s^2C^2+5sC+2}\right\}\right]V_1 = \left[\frac{2s^2C^2+5sC+1}{s^2C^2+5sC+2}\right]V_1$$

or
$$Z_{11}(s) = \frac{V_1}{I_1} = \frac{\boldsymbol{s^2C^2+5sC+2}}{\boldsymbol{2s^2C^2+5sC+1}}.$$

And
$$G_{21}(s) = \frac{V_2}{V_1} = \frac{\boldsymbol{s^2C^2+2sC+1}}{\boldsymbol{s^2C^2+5sC+2}}$$

EXAMPLE 9.16 For the network of figure 9.16(a) and the element values specified, find the current ratio transfer function given by α = (I_2/I_1). *(I.P. Univ., 2001)*

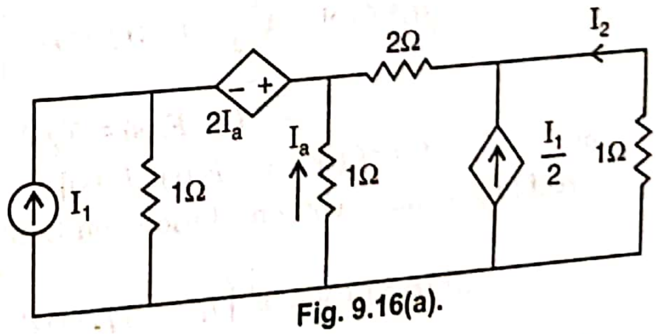


Fig. 9.16(a).

Solution : First, we convert current source into equivalent voltage source as shown in figure 9.16(b). Let I be the current in first (leftmost) loop. Then applying KVL,

$$I_1 = I - 2I_a - I_a \qquad ...(i)$$

where $I = -\left(I_a + \frac{I_1}{2} + I_2\right)$ ...(ii)

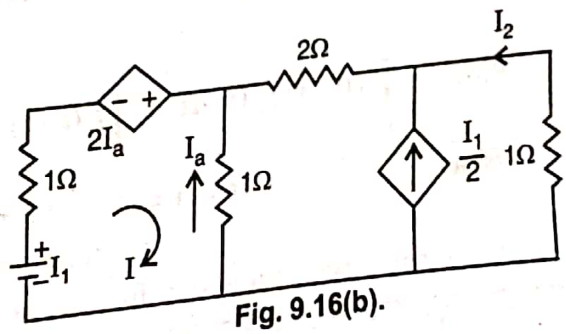


Fig. 9.16(b).

From equations (i) and (ii), eliminating I, we get

$$I_1 = -\left(I_a + \frac{I_1}{2} + I_2\right) - 2I_a - I_a$$

or

$$I_a = -\frac{1}{4}\left(\frac{3I_1}{2} + I_2\right) \qquad ...(iii)$$

Now, Applying KVL, in loop consisting 1Ω, 2Ω and 1Ω. As,

$$1\cdot I_2 + 2\left(I_2 + \frac{I_1}{2}\right) - I_a = 0$$

or

$$3I_2 + I_1 - I_a = 0 \qquad ...(iv)$$

From equations (*iii*) and (*iv*), eliminating I_a, we get

$$3I_2 + I_1 + \frac{1}{4}\left(\frac{3I_1}{2} + I_2\right) = 0$$

$$3I_2 + \frac{1}{4}I_2 + I_1 + \frac{3}{8}I_1 = 0$$

or

$$\frac{13}{4}I_2 + \frac{11}{8}I_1 = 0 \quad \text{or} \quad \alpha = \frac{I_2}{I_1} = -\frac{11}{26}$$

9.5. CONVOLUTION INTEGRAL

The convolution integral finds applications in many fields. One important application is that it enables us to evaluate the response of an electrical network to any arbitrary input in terms of the impulse response of the network. Our following discussion is based upon the important *convolution theorem* of Laplace transform.

If $f_1(t)$ and $f_2(t)$ are two functions of time which are zero for $t < 0$, and if their Laplace transforms are $F_1(s)$ and $F_2(s)$, respectively, then the convolution theorem states that the Laplace transform of the convolution of $f_1(t)$ and $f_2(t)$ is given by $F_1(s).\ F_2(s)$. The convolution of $f_1(t)$ and $f_2(t)$ is denoted by a special notation [by putting a star between $f_1(t)$ and $f_2(t)$] as

$$f_1(t) * f_2(t) = \int_0^t f_1(\tau)\, f_2(t-\tau)\, d\tau = \int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau = f_2(t) * f_1(t)$$

Where τ is a dummy veriable for t.

Hence, the convolution theorem may be given as

$$£[f_1(t) * f_2(t)] = £\left[\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau\right] = £\left[\int_0^t f_1(\tau)\, f_2(t-\tau)\, d\tau\right]$$

$$= F_1(s)\cdot F_2(s) = F_2(s)\cdot F_1(s)$$

or

$$f_1(t) * f_2(t) = £^{-1}[F_1(s)\cdot F_2(s)]$$

Proof : By definition of convolution and Laplace transform,

$$£[f_1(t) * f_2(t)] = £\left[\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau\right] = \int_0^\infty\left[\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau\right] e^{-st}\, dt$$

Now, $$\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau = \int_0^\infty f_1(t-\tau)\, U(t-\tau)\, f_2(\tau)\, d\tau$$

Since $$U(t-\tau) = 1 \text{ for } t > \tau \text{ or } \tau < t$$
$$= 0 \text{ for } t < \tau \text{ or } \tau > t$$

then, the integrand is zero for values of $\tau > t$.

Note : The maximum overshoot is often used to measure the relative stability of a control system. A system with large overshoot is usually undesirable.

...(iv)

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Therefore,

$$\pounds[f_1(t) * f_2(t)] = \int_0^\infty \left[\int_0^\infty f_1(t-\tau)\, U(t-\tau)\, f_2(\tau)\, d\tau \right] e^{-st} dt$$

Putting $\quad t - \tau = x,\ dt = dx$

and $\quad t = 0 \Rightarrow x = -\tau$

and $\quad t = \infty \Rightarrow x = \infty$

Therefore,

$$\pounds[f_1(t)*f_2(t)] = \int_0^\infty \left[\int_{-\tau}^\infty f_1(x)\, U(x)\, f_2(\tau)\, d\tau \right] e^{-s(\tau+x)}\, dx$$

$$= \int_{-\tau}^\infty f_1(x)\, U(x)\, e^{-sx}\, dx \int_0^\infty f_2(\tau)\, e^{-s\tau} d\tau$$

$$= \int_0^\infty f_1(x)\, e^{-sx}\, dx \int_0^\infty f_2(\tau)\, e^{-s\tau} d\tau$$

[as $U(x) = 0$ for $x < 0$]

$$= F_1(s) \cdot F_2(s)$$

EXAMPLE 9.17 **Find the convolution $f(t)$ of two functions $f_1(t)$ and $f_2(t)$ which are given as**

$$f_1(t) = \begin{cases} 1; |t| < 1 \\ 0; |t| > 1 \end{cases} \quad \text{and} \quad f_2(t) = \begin{cases} 1; |t| < \frac{1}{2} \\ 0; |t| > \frac{1}{2} \end{cases}$$

Solution : First, we plot the functions $f_1(t)$ and $f_2(t)$ used in finding the function $f(t) = (f_1(t) * f_2(t))$ as shown in figure 9.17(*a*).

Figure 9.17(*b*) shows $f_1(\tau)$ and $f_2(-\tau)$, whereas figure 9.27(c) shows $f_1(\tau)$ and $f_2(t-\tau)$, (which is $f_2(-\tau)$ shifted by t). Because the edges of $f_2(-\tau)$ are at $\tau = -\frac{1}{2}$ and $\frac{1}{2}$, the edges of $f_2(t-\tau)$ are at $-\frac{1}{2}+t$ and $\frac{1}{2}+t$.

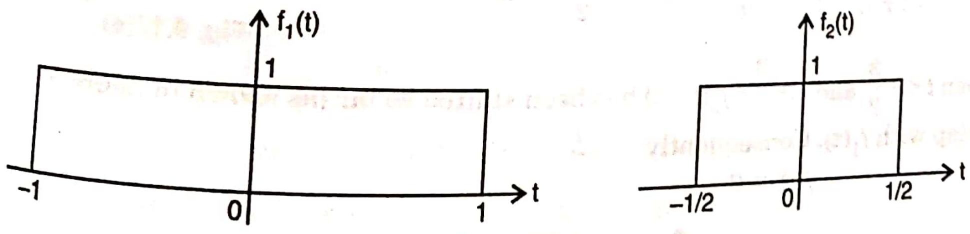


Fig. 9.17(a).

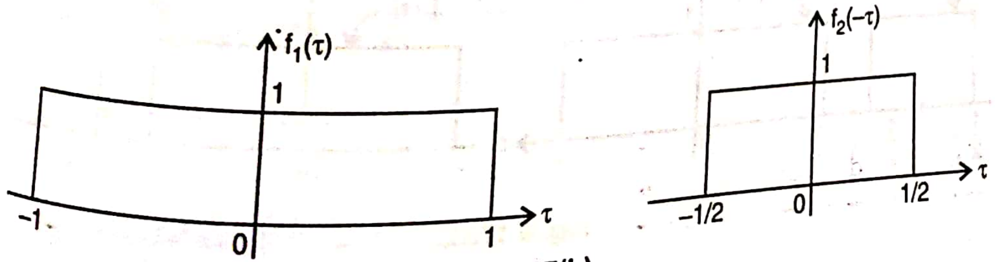


Fig. 9.17(b).

(*i*) When $-\frac{3}{2} \le t \le -\frac{1}{2}$; the two functions overlap over the interval $\left(-1, \frac{1}{2}+t\right)$ (shaded interval), so that as shown in figure 9.17(*c*).

$$f(t) = \int_{-1}^{\frac{1}{2}+t} f_1(\tau) f_2(t-\tau)\, d\tau = \int_{-1}^{\frac{1}{2}+t} 1.d\tau$$

$$= \tau\Big|_{-1}^{\frac{1}{2}+t} = \frac{1}{2}+t-(-1) = \frac{3}{2}+t \qquad ...(1)$$

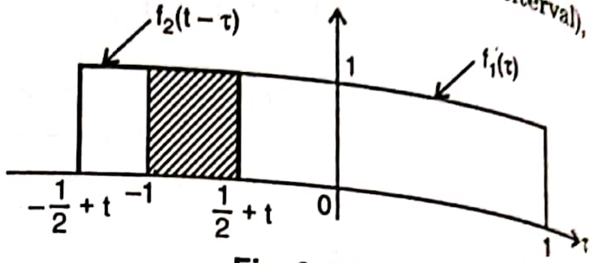


Fig. 9.17(c).

(*ii*) When $-\frac{1}{2} < t < \frac{1}{2}$; the situation is as illustrated in figure 9.17(*d*). The two functions overlap over the range $-\frac{1}{2}+t$ to $\frac{1}{2}+t$ (shaded interval).

Note that the expressions for $f_1(\tau)$ and $f_2(t-\tau)$ do not change; only the range of integration changes. Therefore,

$$f(t) = \int_{-\frac{1}{2}+t}^{\frac{1}{2}+t} 1.\,dt = \tau\Big|_{-1/2+t}^{1/2+t}$$

$$= \frac{1}{2}+t-\left(-\frac{1}{2}+t\right) = 1 \qquad ...(2)$$

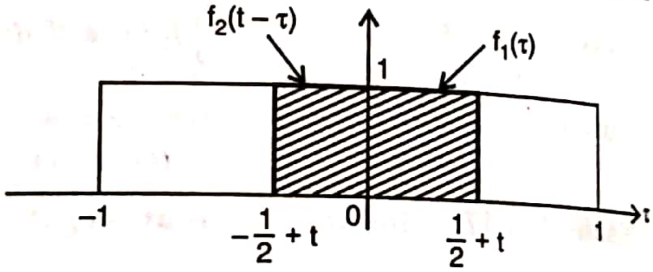


Fig. 9.17(d).

(*iii*) When $\frac{1}{2} \le t \le \frac{3}{2}$; the situation is as shown in figure 9.17(*e*). The functions $f_1(\tau)$ and $f_2(t-\tau)$ overlap the interval from $-\frac{1}{2}+t$ to 1 (shaded interval), so that

$$f(t) = \int_{-1/2+t}^{1} 1.d\tau$$

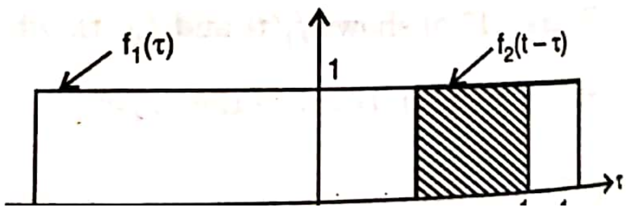


Figure 9.17(g) shows $f(t)$ plotted according to equations (1) through (4).

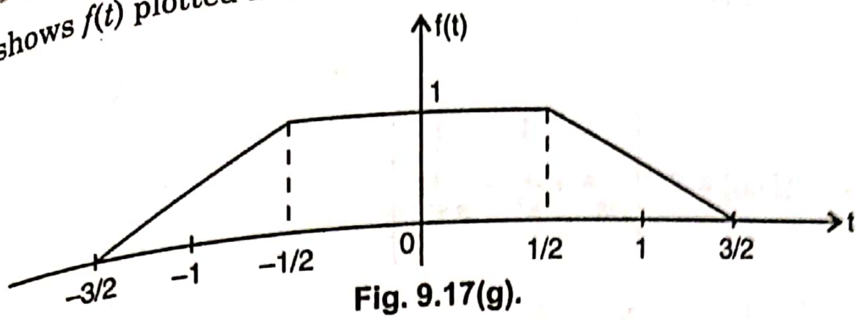


Fig. 9.17(g).

EXAMPLE 9.18 Find the inverse Laplace transform of the given function (U.P.T.U., 2002)

$F(s) = \dfrac{1}{s^3(s+2)}$**, using convolution integral.**

Solution:

$$F(s) = \frac{1}{s^2(s+2)}$$

Let

$$F(s) = F_1(s) \cdot F_2(s) = \frac{1}{s^2(s+2)}$$

Where

$$F_1(s) = \frac{1}{s^2} \Rightarrow f_1(t) = t$$

and

$$F_2(s) = \frac{1}{s+2} \Rightarrow f_2(t) = e^{-2t}$$

We know that

$$f(t) = \pounds^{-1}[F(s)]$$

$$f(t) = \pounds^{-1}[F_1(s)F_2(s)] = f_1(t) * f_2(t)$$

$$f(t) = \int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau = \int_0^t (t-\tau)\, e^{-2\tau} d\tau$$

(as $f_1(t) = t$ and $f_2(t) = e^{-2t}$)

$$= t\int_0^t e^{-2\tau}\, d\tau - \int_0^t \tau\, e^{-2\tau}\, d\tau = t \cdot \frac{e^{-2\tau}}{-2}\Bigg|_0^t - \left[\tau \cdot \frac{e^{-2\tau}}{-2}\Bigg|_0^t - \int_0^t 1 \cdot \frac{e^{-2\tau}}{-2}\, d\tau\right]$$

$$= \frac{t}{2}(1-e^{-2t}) + \frac{t}{2}(e^{-2t}) - \frac{1}{2} \cdot \frac{e^{-2\tau}}{-2}\Bigg|_0^t$$

$$= \frac{t}{2}(1-e^{-2t}) + \frac{t}{2}(e^{-2t}) + \frac{1}{4}(e^{-2t}-1) = -\frac{1}{4}(1-2t-e^{-2t})$$

Let us verify the result using partial fraction expansion as

$$F(s) = \frac{1}{s^2(s+2)} = \frac{K_1}{s} + \frac{K_2}{s^2} + \frac{K_3}{s+2}$$

$$K_2 = s^2F(s)\Big|_{s=0} = \frac{1}{2}$$

$$K_1 = \frac{d}{ds}[s^2F(s)]\Big| \quad -\frac{1}{\;\;\;^2}\Big| \quad = -\frac{1}{4}$$

$$K_3 = (s+2)F(s)\big|_{s=-2} = \frac{1}{4}$$

Therefore,

$$f(t) = £^{-1}[F(s)] = £^{-1}\left[\frac{-\frac{1}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{\frac{1}{4}}{s+2}\right]$$

$$f(t) = -\frac{1}{4} + \frac{1}{2}t + \frac{1}{4}e^{-2t} = -\frac{1}{4}(1 - 2t - e^{-2t})$$

EXAMPLE 9.19 **Find the response with e^{-2t} as input to the transfer function $H(s) = \frac{V_2(s)}{V_1(s)}$ $= \frac{1}{s+3}$.**

Solution : $H(s) = \dfrac{1}{s+3} \Rightarrow h(t) = £^{-1}[H(s)] = e^{-3t}$

$$v_2(t) = v_1(t) * h(t) = \int_0^t v_1(t-\tau)\, h(\tau)\, d\tau$$

$$= \int_0^t e^{-2(t-\tau)}\, e^{-3\tau}\, d\tau = e^{-2t}\int_0^t e^{-\tau}\, d\tau$$

$$= e^{-2t}(1 - e^{-t}) = e^{-2t} - e^{-3t}$$

Again, the result may be verified by the partial-fraction expansion technique as

$$V_2(s) = H(s)\, V_1(s) = \frac{1}{s+3}\cdot\frac{1}{s+2} = \frac{1}{s+2} - \frac{1}{s+3}$$

therefore,

$$v_2(t) = £^{-1}[V_2(s)] = £^{-1}\left[\frac{1}{s+2}\right] - £^{-1}\left[\frac{1}{s+3}\right] = e^{-2t} - e^{-3t}$$

EXAMPLE 9.20 **Verify that the convolution between two functions $f_1(t) = 2\,U(t)$ and $f_2(t) = e^{-3t}\, U(t)$ is $\frac{2}{3}(1 - e^{-3t})\, U(t)$; where $U(t)$ is a unit step function.**

Solution : $f_1(t) * f_2(t) = \displaystyle\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau$

$$= \int_0^t 2\,U(t-\tau)\cdot e^{-3\tau}\, U(\tau)\, d\tau = 2\int_0^t e^{-3\tau}\, d\tau = 2\cdot\frac{e^{-3\tau}}{-3}\Bigg|_0^t$$

$$= \frac{2}{3}(1 - e^{-3t})\, U(t)$$

EXAMPLE 9.21 **Find the convolution between $f_1(t) = U(t)$ and $f_2(t) = e^{-t}U(t)$.** *(I.P. Univ., 2001)*

Solution : Convolution of $f_1(t)$ and $f_2(t)$ is given by

$$f_1(t) * f_2(t) = \int_0^t f_1(\tau)\cdot f_2(t-\tau)\, d\tau$$

$$= \int_0^t U(\tau)\cdot e^{-(t-\tau)}\, U(t-\tau)\, d\tau = e^{-t}\int_0^t e^{\tau}\, U(t-\tau) d\tau$$

$$= e^{-t}\, e^{\tau}\Big|_0^t = e^{-t}\,(e^t - e^0) = (1-e^{-t})\, U(t)$$

Alternative ways :

$$F_1(s) = £[f_1(t)] = \frac{1}{s}$$

and

$$F_2(s) = £\,[f_2(t)] = \frac{1}{s+1}$$

Then

$$f_1(t) * f_2(t) = £^{-1}[F_1(s).\, F_2(s)] = £^{-1}\left[\frac{1}{s}\cdot\frac{1}{s+1}\right]$$

$$= £^{-1}\left[\frac{1}{s}-\frac{1}{s+1}\right]$$

$$= (1-e^{-t})\, U(t)$$ (Using partial fraction expansion)

EXAMPLE 9.22 **Find the current transfer ratio (I_2/I_1) and voltage transfer ratio (V_2/V_1) of the network shown in figure 9.18.**

Solution : Let the voltage at node B be V.

Applying KCL at node A gives,

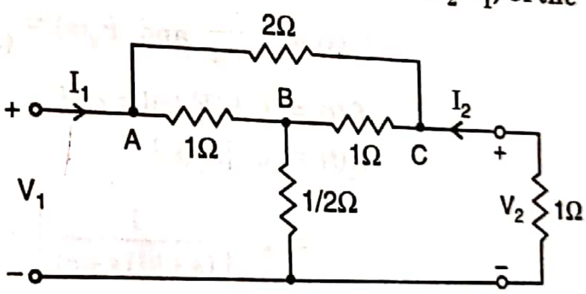


Fig. 9.18.

$$I_1 = \frac{V_1 - V}{1} + \frac{V_1 - V_2}{2}$$

or

$$I_1 = \frac{3}{2}V_1 - V - \frac{1}{2}V_2 \quad ...(1)$$

At node B, applying KCL gives,

$$\frac{V_1 - V}{1} = \frac{V}{(1/2)} + \frac{V-V_2}{1}$$

$$V_1 - V = 2V + V - V_2$$

or

$$4V = V_1 + V_2 \quad ...(2)$$

At node C, applying KCL gives,

$$\frac{V_1 - V_2}{2} + \frac{V - V_2}{1} = \frac{V_2}{1}$$

$$\frac{1}{2}V_1 - \frac{5}{2}V_2 = -V \quad ...(3)$$

or

$$V = -\frac{1}{2}V_1 + \frac{5}{2}V_2 \quad ...(4)$$

And from the network,

$$V_2 = 1\cdot(-I_2)$$

or

$$V_2 = -I_2$$

From equations (2) and (3),

$$4\left(-\frac{1}{2}V_1+\frac{5}{2}V_2\right) = V_1 + V_2$$

or $$9V_2 = 3\,V_1 \quad \text{or} \quad 3V_2 = V_1$$

or $$\frac{V_2}{V_1} = G_{21} = \frac{1}{3}$$

Now, from equations (1) and (3),

$$I_1 = \frac{3}{2}V_1 - \left(-\frac{1}{2}V_1+\frac{5}{2}V_2\right)-\frac{1}{2}V_2$$

$$I_1 = 2V_1 - 3V_2 = 6V_2 - 3V_2 \qquad \text{(Since } V_1 = 3V_2\text{)}$$

or $$I_1 = 3V_2 = 3(-I_2) \qquad \text{[From equation (4)]}$$

or $$\frac{I_2}{I_1} = \alpha_{21} = -\frac{1}{3}$$

EXAMPLE 9.23 **Find inverse Laplace of $F_1(s)\cdot F_2(s)$ by convolution integral for the following:**

(i) $F_1(s) = \dfrac{1}{s+a}$ $\qquad$ $F_2(s) = \dfrac{1}{(s+b)\,(s+c)}$

(ii) $F_1(s) = \dfrac{s}{s+1}$, $\qquad$ $F_2(s) = \dfrac{1}{s^2+1}$ (U.P.T.U., 2001)

Solution :

(i) $$F_1(s) = \frac{1}{s+a} \text{ and } F_2(s) = \frac{1}{(s+b)\,(s+c)}$$

$$f_1(t) = £^{-1}[F_1(s)] = e^{-at}$$

$$f_2(t) = £^{-1}[F_2(s)]$$

$$= £^{-1}\left[\frac{1}{(s+b)\,(s+c)}\right] = £^{-1}\left[\frac{1}{c-b}\cdot\frac{1}{s+b}+\frac{1}{b-c}\cdot\frac{1}{s+c}\right]$$

or $$f_2(t) = \frac{1}{c-b}(e^{-bt}-e^{-ct})$$

$$£^{-1}[F(s)] = f_1(t) * f_2(t) = \int_0^t f_1(t-\tau)\cdot f_2(\tau)\,d\tau$$

$$= \int_0^t e^{-a(t-\tau)}.\frac{(e^{-b\tau}-e^{-c\tau})\,d\tau}{c-b} = \frac{e^{-at}}{c-b}\int_0^t e^{a\tau}(e^{-b\tau}-e^{-c\tau})\,d\tau$$

$$= \frac{e^{-at}}{c-b}\left[\int_0^t e^{(a-b)\tau}d\tau - \int_0^t e^{(a-c)\tau}\,d\tau\right]$$

$$= \frac{e^{-at}}{c-b}\left[\frac{1}{a-b}e^{(a-b)\tau}\Big|_0^t - \frac{1}{a-c}e^{(a-c)\tau}\Big|_0^t\right]$$

$$= \frac{e^{-at}}{c-b}\left[\frac{1}{a-b}\cdot(e^{(a-b)t}-1) - \frac{1}{a-c}\cdot(e^{(a-c)t}-1)\right]$$

$$= \frac{e^{-at}}{c-b}\left[\frac{e^{(a-b)t}}{a-b} - \frac{e^{(a-c)t}}{a-c} - \left\{\frac{1}{a-b} - \frac{1}{a-c}\right\}\right]$$

$$= \frac{-e^{at}}{c-b}\left[\frac{e^{(a-b)t}}{a-b} - \frac{e^{(a-c)t}}{a-c} - \left\{\frac{a-c-a+b}{(a-b)(a-c)}\right\}\right]$$

$$= \frac{e^{-bt}}{(c-b)(a-b)} - \frac{e^{-ct}}{(c-b)(a-c)} + \frac{1}{(a-b)(a-c)}$$

$$= \frac{e^{-bt}}{(a-b)(c-b)} + \frac{e^{-ct}}{(a-c)(b-c)} + \frac{1}{(b-a)(c-a)}$$

(ii)

$$F_1(s) = \frac{s}{s+1} \quad \text{and} \quad F_2(s) = \frac{1}{s^2+1}$$

$$f_1(t) = \pounds^{-1}\left[\frac{s}{s+1}\right] = \pounds^{-1}\left[1 - \frac{1}{s+1}\right] = \delta(t) - e^{-t}$$

$$f_2(t) = \pounds^{-1}\left[\frac{1}{s^2+1}\right] = \sin t$$

$$\pounds^{-1}[F(s)] = \int_0^t [\delta(\tau) - e^{-\tau}]\cdot \sin(t-\tau)\,d\tau$$

$$= \sin(t-\tau)\Big|_{\tau=0} - \int_0^t e^{-\tau}\sin(t-\tau)\,d\tau$$

$$= \sin t - \int_0^t e^{-\tau}\cdot\left\{\frac{e^{j(t-\tau)} - e^{-j(t-\tau)}}{2j}\right\}d\tau$$

$$= \sin t - \frac{1}{2j}\int_0^t e^{jt}e^{-(1+j)\tau}\,d\tau + \frac{1}{2j}\int_0^t e^{-jt}\cdot e^{-(1-j)\tau}\,d\tau$$

$$= \sin t - \frac{e^{jt}}{2j}\left\{\frac{e^{-(1+j)\tau}}{-(1+j)}\right\}\Bigg|_0^t + \frac{e^{-jt}}{2j}\left\{\frac{e^{-(1-j)\tau}}{-(1-j)}\right\}\Bigg|_0^t$$

$$= \sin t - \frac{e^{jt}}{2j}\left\{\frac{e^{-(1+j)t}-1}{-(1+j)}\right\} + \frac{e^{-jt}}{2j}\left\{\frac{e^{-(1-j)t}-1}{-(1-j)}\right\}$$

$$= \sin t - \frac{1}{2j}\left[\frac{e^{-t}-e^{jt}}{-(1+j)} - \frac{e^{-t}-e^{-jt}}{-(1-j)}\right]$$

$$= \sin t - \frac{1}{2j}\left[\frac{(1-j)(e^{-t}-e^{jt}) - (1+j)(e^{-t}-e^{-jt})}{-(1+j)(1-j)}\right]$$

$$= \sin t - \frac{1}{2j}\left[\frac{e^{-t}-e^{jt}-j\,e^{-t}+j\,e^{jt}-e^{-t}+e^{-jt}-j\,e^{-t}+j\,e^{-jt}}{-2}\right]$$

$$= \sin t + \frac{1}{2j}\left[\frac{-(e^{jt}-e^{-jt}) - j(e^{-t}+e^{-t}) + j(e^{jt}+e^{-jt})}{2}\right]$$

$$= \sin t + \frac{1}{2}\left[\frac{-(e^{jt}-e^{-jt})}{2j} - \frac{2je^{-t}}{2j} + \frac{e^{jt}+e^{-jt}}{2}\right]$$

$$= \sin t + \frac{1}{2}\left[-\sin t - e^{-t} + \cos t\right] = \frac{1}{2}\left[\sin t + \cos t - e^{-t}\right]$$

For checking, make the partial fraction expansion of

$$F(s) = F_1(s)\cdot F_2(s) = \frac{s}{(s+1)(s^2+1)}$$

$$= -\frac{1}{2}\cdot\frac{1}{s+1} + \frac{1}{2}\cdot\frac{(s+1)}{(s^2+1)} = -\frac{1}{2}\cdot\frac{1}{s+1} + \frac{1}{2}\cdot\frac{s}{s^2+1} + \frac{1}{2}\cdot\frac{1}{s^2+1}$$

Therefore,

$$£^{-1}[F(s)] = -\frac{1}{2}e^{-t} + \frac{1}{2}\cos t + \frac{1}{2}\sin t$$

EXAMPLE 9.24 **The impulse response of a network (system) is given as ($e^{-t} + e^{-2t}$). Find the transfer function. Determine the input/excitation required to produce an output/response as te^{-2t}.**

Solution : Transfer function ;

$$H(s) = £[h(t)] = £[e^{-t} + e^{-2t}]$$

$$= \frac{1}{s+1} + \frac{1}{s+2} = \frac{2s+3}{(s+1)(s+2)}$$

The output $y(t)$ (required) $= te^{-2t}$

or $$Y(s) = \frac{1}{(s+2)^2}$$

As we know that,

$$H(s) = \frac{Y(s)}{X(s)}$$

or $$X(s) = \frac{Y(s)}{H(s)} = \frac{s+1}{(s+2)(2s+3)}$$

Using partial fraction expansion, we have

$$X(s) = \frac{1}{s+2} - \frac{1}{2(s+1.5)}$$

Therefore, input $x(t) = £^{-1}[X(s)] = e^{-2t} - \frac{1}{2}e^{-1.5t}$

EXAMPLE 9.25 **The unit-step response of a system is given by ($1 - e^{-bt}$). Determine the impulse response $h(t)$ of the system.**

Solution : Here, $y(t) = 1 - e^{-bt}$ for the input $x(t) = U(t)$

or $$Y(s) = \frac{1}{s} - \frac{1}{s+b} = \frac{b}{s(s+b)} \qquad \text{For } X(s) = \frac{1}{s}$$

Since, $$H(s) = Y(s)/X(s)$$

Therefore, $$H(s) = \frac{b}{s+b}$$

Hence, impulse response ;

$$h(t) = b\,e^{-bt}\,U(t)$$

EXAMPLE 9.26 For the network shown in figure 9.19(*a*). Compute (*a*) $G_{21} = V_2/V_1$, (*b*) $\alpha_{21} = I_2/I_1$.

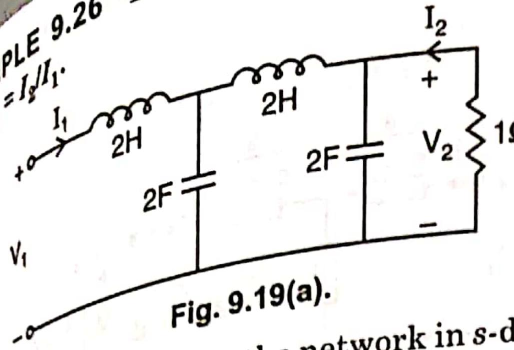


Fig. 9.19(a).

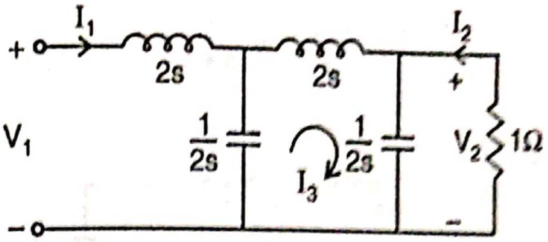


Fig. 9.19(b).

Solution : First, redrawing the network in *s*-domain in figure 9.19(*b*). The loop equations become.

$$V_1 = 2sI_1 + \frac{1}{2s}(I_1 - I_3)$$

$$V_1 = \left(2s + \frac{1}{2s}\right) I_1 - \frac{1}{2s} I_3 \quad ...(i)$$

or

$$\frac{1}{2s}(I_3 - I_1) + 2s\,(I_3) + \frac{1}{2s}(I_3 + I_2) = 0 \quad ...(ii)$$

$$I_1 = (2 + 4s^2)\, I_3 + I_2 \quad ...(iii)$$

or

$$V_2 = 1 \cdot (-I_2) = -I_2 \quad ...(iv)$$

$$V_2 = \frac{1}{2s}(I_2 + I_3)$$

And

From equations (*iii*) and (*iv*),

$$I_3 = -(2s + 1)\, I_2 \quad ...(v)$$

From equations (*ii*) and (*v*),

$$I_1 = -(2 + 4s^2)(2s + 1)\, I_2 + I_2 \quad ...(vi)$$

$$I_1 = -(8s^3 + 4s^2 + 4s + 1)\, I_2$$

or

Now, from equations (*i*), (*v*) and (*vi*), we have

$$V_1 = -\left[(8s^3 + 4s^2 + 4s + 1)\left(2s + \frac{1}{2s}\right) - \frac{1}{2s}(2s + 1)\right] I_2 \quad ...(vii)$$

or $\quad V_1 = -[16s^4 + 8s^3 + 12s^2 + 4s + 1]\, I_2$

Therefore, from equations (*iii*) and (*vii*), we have

$$G_{21} = \frac{V_2}{V_1} = \frac{1}{16s^4 + 8s^3 + 12s^2 + 4s + 1}$$

And from equation (*vi*), we have

$$\alpha_{21} = \frac{I_2}{I_1} = -\frac{1}{8s^3 + 4s^2 + 4s + 1}$$

EXAMPLE 9.27 For the network shown in figure 9.20(*a*), find $G_{21} = V_2/V_1$.

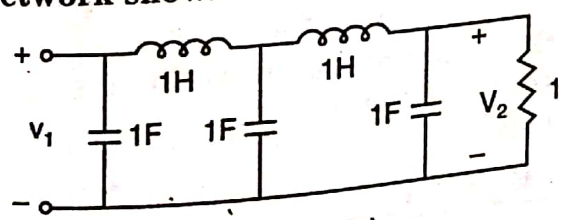


Fig. 9.20(a).

Solution : Redrawing the network in s-domain as sho... ... and 1Ω are in parallel, *i.e.*,

$$\frac{1}{s} \| 1 = \frac{1}{s+1}$$

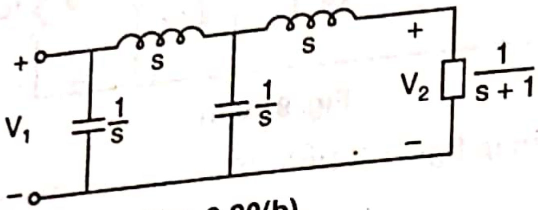


Fig. 9.20(b).

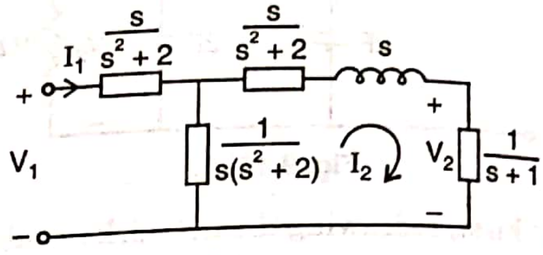


Fig. 9.20(c).

Applying the $\Delta - Y$ transformation, network becomes as shown in figure 9.20 (*c*).

Now, applying KVL,

$$V_1 = \left(\frac{s}{(s^2+2}\right) I_1 + \frac{1}{s(s^2+2)}(I_1 - I_2) \qquad \text{...(i)}$$

And $\left(\frac{s}{(s^2+2} + s + \frac{1}{s+1}\right) I_2 + \frac{1}{s(s^2+2)}(I_2 - I_1) = 0$

or
$$I_1 = \left\{ s(s^2+2)\left[\frac{s}{s^2+2} + s + \frac{1}{s+1}\right] + 1 \right\} I_2$$

or
$$I_1 = \left[s^2 + s^2(s^2+2) + \frac{s(s^2+2)}{s+1} + 1 \right] I_2 \qquad \text{...(ii)}$$

From equations (*i*) and (*ii*), we have

$$V_1 = \left\{ \frac{s^2+1}{s(s^2+2)}\left[s^2 + s^2(s^2+2) + \frac{s(s^2+2)}{s+1} + 1\right] - \frac{1}{s(s^2+2)} \right\} I_2$$

After solving, we get

$$V_1 = \frac{(s^4 + s^3 + 3s^2 + 2s + 1)}{s+1} I_2$$

Now,
$$V_2 = \frac{1}{s+1} I_2$$

Therefore,
$$G_{21} = \frac{V_2}{V_1} = \frac{1}{s^4 + s^3 + 3s^2 + 2s + 1}$$

EXAMPLE 9.28 **Determine the driving point admittance and transfer admittance for the bridged T-network shown in figure 9.21(*a*), with a 2Ω load resistor connected across port 2.** *(U.P.T.U., 2003 C.O.)*

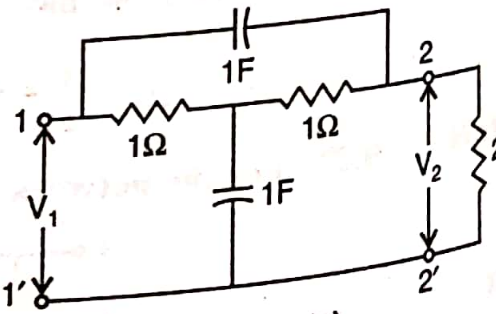


Fig. 9.21 (a).

Solution : Redrawn the network in s-domain as shown in figure 9.21 (*b*). And let the V be the voltage at node B.

Applying KCL at node A gives, ...(i)

$$I_1 = \frac{V_1 - V}{1} + \frac{V_1 - V_2}{1/s} = (s+1)V_1 - sV_2 - V$$

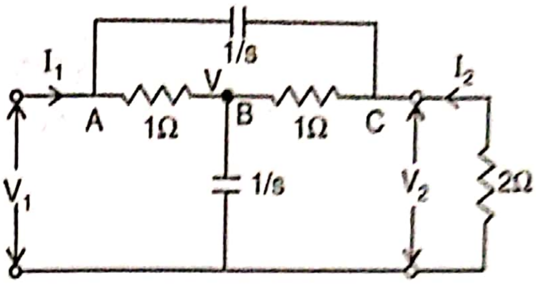


Fig. 9.21 (b).

At node B, applying KCL,

$$\frac{V_1 - V}{1} = \frac{V}{1/s} + \frac{V - V_2}{1}$$

$$(s+2)V = V_1 + V_2 \qquad ...(ii)$$

Similarly at node C, KCL gives

$$\frac{V - V_2}{1} + \frac{V_1 - V_2}{1/s} = \frac{V_2}{2}$$

$$V = -sV_1 + \left(s + \frac{3}{2}\right) V_2 \qquad ...(iii)$$

From equations (ii) and (iii),

$$(s+2)\left\{-sV_1 + \left(s+\frac{3}{2}\right)V_2\right\} = V_1 + V_2$$

$$\left[(s+2)\left(s+\frac{3}{2}\right) - 1\right] V_2 = V_1\,[1 + s(s+2)]$$

$$\left(s^2 + \frac{7}{2}s + 2\right) V_2 = (s^2 + 2s + 1)\, V_1 \qquad ...(iv)$$

From equations (i), (iii) and (iv), we have

$$I_1 = (s+1)\,V_1 - s\,V_2 + sV_1 - \left(s + \frac{3}{2}\right) V_2$$

$$= (2s+1)\,V_1 - \left(2s + \frac{3}{2}\right)\left[\frac{s^2 + 2s + 1}{s^2 + \frac{7}{2}s + 2}\right] V_1$$

$$I_1 = \frac{(2s+1)\left(s^2 + \frac{7}{2}s + 2\right) - \left(2s + \frac{3}{2}\right)(s^2 + 2s + 1)}{\left(s^2 + \frac{7}{2}s + 2\right)} V_1$$

Therefore, driving point impedance,

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{2} \cdot \frac{5s^2 + 5s + 1}{s^2 + \frac{7}{2}s + 2} \qquad ...(v)$$

Also $\qquad V_2 = -2I_2$

So, transfer admittance,

$$Y_{21} = \frac{I_2}{V_1} = -\frac{V_2}{2} \cdot \left(\frac{s^2 + 2s + 1}{s^2 + \frac{7}{2}s + 2}\right) \cdot \frac{1}{V_2}$$

[From equation (iv)]

$$= -\frac{1}{2} \cdot \frac{s^2 + 2s + 1}{s^2 + \frac{7}{2}s + 2}$$

EXAMPLE 9.29 **Find open circuit transfer impedance V_2/I_1 and open circuit voltage ratio V_2/V_1 for the ladder network shown in figure 9.22 (a).** *(U.P.T.U., 2003)*

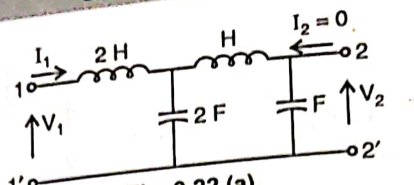


Fig. 9.22 (a).

Solution : (By taking H is 1H and F is 1F). The network in transform-domain is as shown in figure 9.22 (*b*).

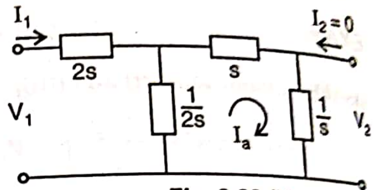


Fig. 9.22 (b).

Applying KVL,

$$V_1 = 2s.I_1 + \frac{1}{2s}(I_1 - I_a) \qquad \ldots(i)$$

$$V_2 = \frac{1}{s} I_a \qquad \ldots(ii)$$

And

$$I_a = \frac{I_1 \cdot \frac{1}{2s}}{\frac{1}{2s} + s + \frac{1}{s}} = \frac{I_1}{2s^2 + 3} \qquad \ldots(iii)$$

From equations (*ii*) and (*iii*), we have

$$\frac{V_2}{I_1} = \frac{1}{s(2s^2 + 3)}$$

From equations (*i*) and (*iii*),

$$V_1 = \left[\left(2s + \frac{1}{2s}\right)(2s^2 + 3) - \frac{1}{2s}\right] I_a = \left[\frac{(4s^2 + 1)(2s^2 + 3) - 1}{2s}\right] I_a$$

$$= \left(\frac{4s^4 + 7s^2 + 1}{s}\right) I_a \qquad \ldots(iv)$$

From equations (*ii*) and (*iv*),

$$\frac{V_2}{V_1} = \frac{1}{4s^4 + 7s^2 + 1}$$

EXAMPLE 9.30 **Determine Inverse Laplace Transform of the following function using Convolution integral :**

$$F(s) = F_1(s).F_2(s) = \frac{s + 1}{s(s^2 + 4)} \qquad \text{(U.P.T.U., 2003 C.O.)}$$

Solution :

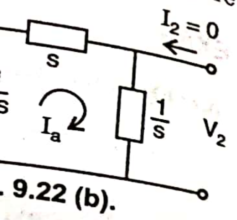
$$F(s) = F_1(s).F_2(s) = \frac{s+1}{s(s^2+4)}$$

Let $\quad F_1(s) = \frac{1}{s}$ and $F_2(s) = \frac{s+1}{s^2+4}$

Then $\quad f_1(t) = U(t)$ and $f_2(t) = \cos 2t + \frac{1}{2}\sin 2t$

$$f(t) = \int_0^t f_1(t-\tau) \cdot f_2(\tau)\, d\tau = \int_0^t U(t-\tau)\left(\cos 2\tau + \frac{1}{2}\sin 2\tau\right) d\tau$$

as shown in figure



. 9.22 (b).

...(iii)

...(iv)

nction using

U., 2003 C.O.)

$$= \int_0^t \cos 2\tau \, d\tau + \frac{1}{2}\int_0^t \sin 2\tau \, d\tau = \frac{\sin 2\tau}{2}\Big|_0^t + \frac{1}{2}\,\frac{-\cos 2\tau}{2}\Big|_0^t$$

or $$f(t) = \frac{1}{2}\sin 2t - \frac{1}{4}(\cos 2t - 1) = \frac{1}{4} + \frac{1}{2}\sin 2t - \frac{1}{4}\cos 2t$$

$$= \frac{1}{4}(1 + 2\sin 2t - \cos 2t)$$

EXAMPLE 9.31 **Find the transfer function $\left(\frac{V_2}{V_1}\right)$ of the network given in figure 9.23(a). Also sketch pole-zero configuration.**

(U.P.T.U., 2006)

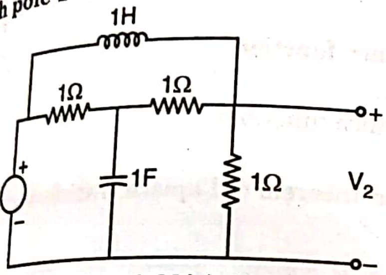


Fig. 9.23(a).

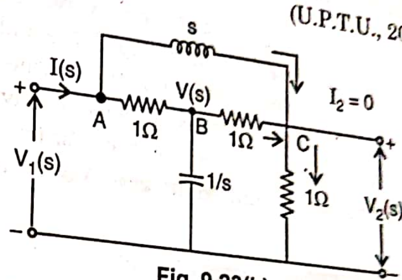


Fig. 9.23(b).

Solution : Suppose the voltage of point B is $V(s)$ as shown in figure 9.23(b).

Apply KCL at 'A';

$$\frac{V_1(s) - V(s)}{1} + \frac{V_1(s) - V_2(s)}{s} = I_1(s)$$

or $$\left(1 + \frac{1}{s}\right)V_1(s) - \frac{1}{s}V_2(s) - V(s) = I_1(s) \quad \text{...(1)}$$

Apply KCL at 'B';

$$\frac{V_1(s) - V(s)}{1} + \frac{V_2(s) - V(s)}{1} = s\,V(s)$$

or $$V(s) = \frac{1}{(s+2)}V_1(s) + \frac{1}{(s+2)}V_2(s) \quad \text{...(2)}$$

and apply KCL at 'C';

$$\frac{V_2(s) - V_1(s)}{s} + \frac{V_2(s) - V(s)}{1} + \frac{V_2(s)}{1} = 0$$

$$(1 + 2\,s)V_2(s) - V_1(s) - sV(s) = 0 \quad \text{...(3)}$$

From equations (2) and (3), we get

$$(1+2s)\,V_2(s) - V_1(s) - \frac{s}{s+2}V_1(s) - \frac{s}{s+1}V_2(s) = 0$$

or, $$\left[(1+2s) - \frac{s}{s+2}\right]V_2(s) = \left[1 + \frac{s}{s+2}\right]V_1(s)$$

or, $$\frac{2s^2 + 5s + 2 - s}{s+2}V_2(s) = \frac{2s+2}{s+2}V_1(s)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{2s+2}{2s^2+4s+2}$$

$$= \frac{(s+1)}{(s^2+2s+1)} = \frac{(s+1)}{(s+1)^2} = \frac{1}{s+1}$$

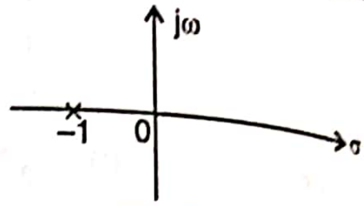


Fig. 9.23(c).

Zero at $s = \infty$
Pole $s = -1$
The pole-zero plot is as shown in figure 9.23(c).

EXERCISES

9.1. Explain the concept of complex frequency.
9.2. Differentiate transform impedance and transfer impedance function.
9.3. Define all the transfer functions of the two port network.
9.4. Write the necessary conditions for driving point immittance functions.
9.5. Write the necessary conditions for transfer functions.
9.6. What is convolution integral. Also derive the convolution theorem of Laplace transform.

PROBLEMS

9.1. Find driving point impedance $Z_{11}(s)$ and voltage transfer function $G_{21}(s) = V_2(s)/V_1(s)$ in Laplace domain in the circuit of figure P.9.1.

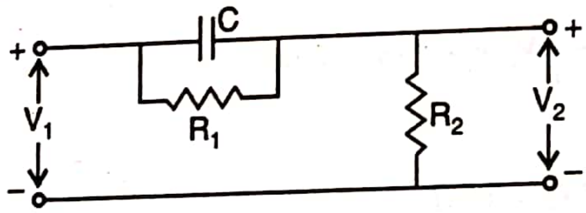
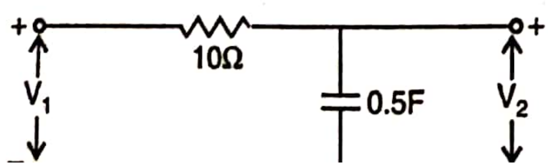
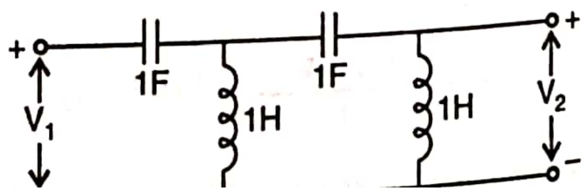


Fig. P.9.1.

9.2. Obtain the voltage transfer function $G_{21}(s) = V_2(s)/V_1(s)$ of the network shown in figure P.9.2. Find $v_2(t)$ when $v_1(t) = 10\, e^{-2t}$.





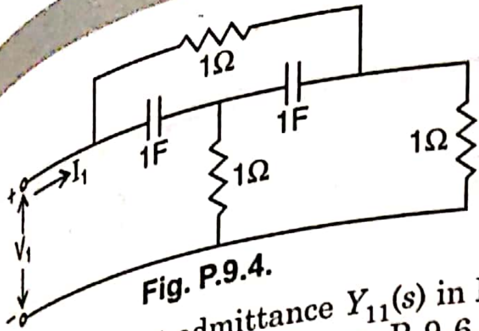


Fig. P.9.4.

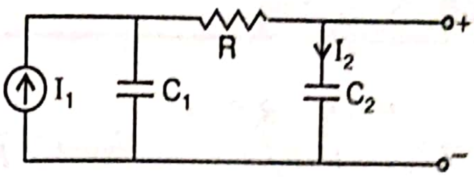


Fig. P.9.5.

5. Find driving point admittance $Y_{11}(s)$ in Laplace domain for the circuit of figure P.9.5.
6. For network shown in figure P.9.6, Compute $G_{21}(s) = V_2(s)/V_1(s)$.

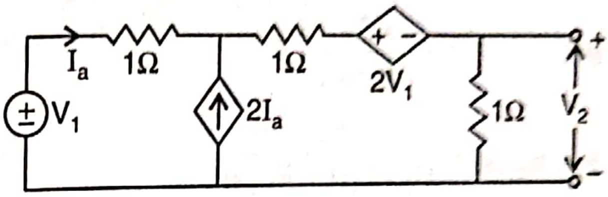


Fig. P.9.6.

7. Show that the inverse Laplace transform of $F(s) = \frac{s+5}{(s+2)^2}$ by convolution integral becomes $e^{-2t} + 3t\,e^{-2t}$.
8. Show that the convolution integral of $f_1(t) = e^{-at}$ and $f_2(t) = t$ is $\frac{1}{a^2}(at-1+e^{-at})$.
9. Show that the inverse Laplace transform of $F(s) = \frac{1}{(s^2+a^2)^2}$ by convolution integral becomes $\frac{1}{2a^3}(\sin at - at\cos at)$.
10. A unit step input is applied to the *R-L* series circuit as shown in figure P.9.10. Verify the current in the circuit by the application of convolution integral as $(1-e^{-t})$.

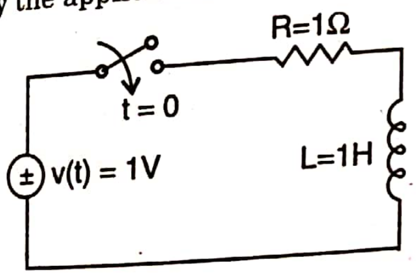


Fig. P.9.10.

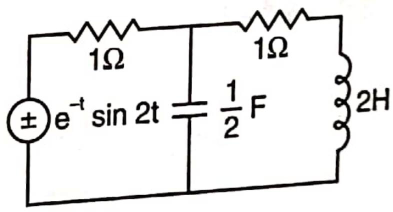


Fig. P.9.12.

11. Find the transform current $I(s)$ in the generator of figure P.9.12.
12. For the network as shown in figure P.9.13 with port 2 open. Find (*a*) the input impedance Z_{11}, (*b*) the voltage-ratio transfer function, $G_{21} = V_2/V_1$ for the two port network.
13. For the network shown in figure P.9.14. Calculate the voltage-ratio transfer function, $G_{21} = \frac{V_2}{V_1}$.

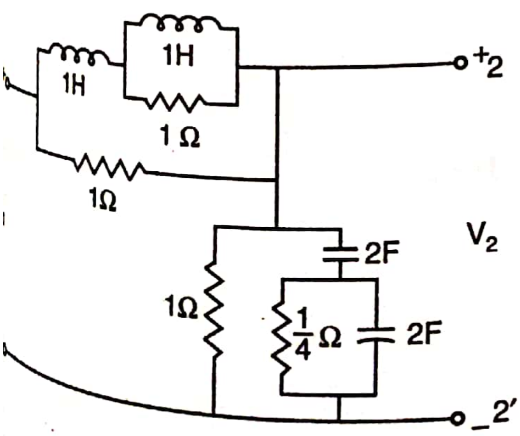


Fig. P.9.13.

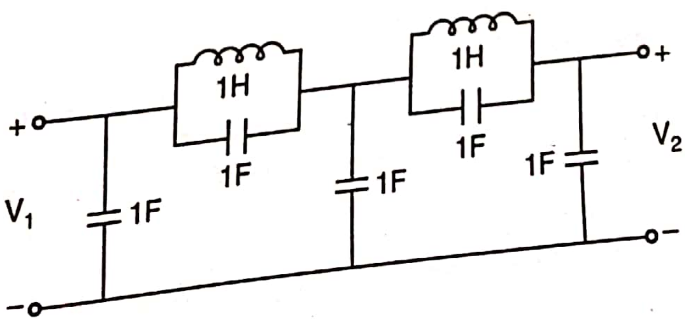


Fig. P.9.14.

9.1. $Z_{11}(s) = \dfrac{R_1 + R_2 + R_1 R_2 Cs}{1 + R_1 Cs}$; $G_{21}(s) = \dfrac{R_2(1 + R_1 Cs)}{R_1 + R_2 + R_1 R_2 Cs}$

9.2. $G_{21}(s) = \dfrac{1}{5s+1}$; $v_2(t) = -1.11\,(e^{-2t} - e^{-0.2t})$

9.3. $G_{21} = \dfrac{s^4}{s^4 + 3s^2 + 1}$

9.4. $\alpha_{21}(s) = \dfrac{C_2}{C_2 + C_1(1 + RC_2 s)}$

9.5. $Y_{11}(s) = \dfrac{2s^2 + 5s + 1}{s^2 + 5s + 2}$

9.6. $G_{21}(s) = -\dfrac{3}{7}$.

9.12. $I(s) = \dfrac{2(2s^2 + s + 2)}{(s^2 + 2s + 5)(2s^2 + 5s + 4)}$

9.13. $Z_{11} = 1$; $G_{21} = \dfrac{s+1}{s^2 + 3s + 1}$

9.14. $G_{21} = \dfrac{(s^2+1)^2}{5s^4 + 5s^2 + 1}$

□□□