UNIT-1

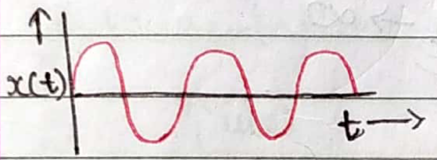
Circuits and system

- Introduction to signals : Their Classifications

Refrence Input or Excitation $\longrightarrow$ $\boxed{\text{System}}$ $\longrightarrow$ Responce (Output)

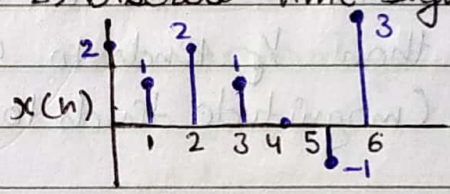
- Signals are classified into two categories :-

1) Continuous Time Signals



expressed mathematically as $A \sin \omega t$

2) Discrete Time Signals



$$x[n] = \{2, 1, \underset{\substack{\uparrow \\ n=0}}{2}, 1, 0, -1, 3\}$$

- The basic Continuous Time System are:-

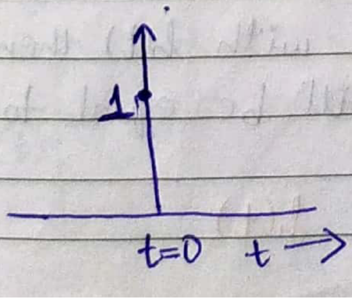
1) Unit Impulse
2) Unit Step
3) Unit Ramp
4) Unit Parabolic

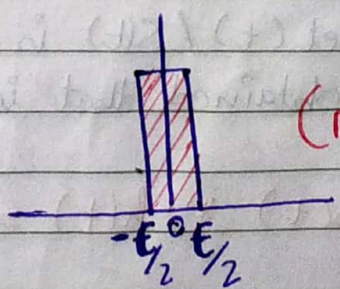
All are inter related

1) Unit Impulse signal

It is denoted by $\delta(t)$ and represented as

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$ (Representation)





(Meaning)

$\epsilon \Rightarrow$ Small interval of time (Defination)

- Unit Impulse signal is an Even signal having very large magnitude at intifintisimally small interval of time.
- The interval ϵ and the area under the curve being unity indicates that magnitude of this impulse is $1/\epsilon$
- When ϵ tends to 0 $\quad \epsilon \rightarrow 0$
  than $1/\epsilon$ tends to ∞ $\quad 1/\epsilon \rightarrow \infty$
  (magnitude tends to ∞)

The output / response do not depends on the type of input wether it is +ve or -ve

$\delta(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$

$\delta(-t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$

- **Response**

When an impulse signal as an excitation is incident on a system, the output obtained is known as Unit Impulse Response.

If the system is represented as $h(t)$

$\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

When del (t) / $\delta(t)$ is convolved with $h(t)$ then the output obtained that is $y(t)$ will be equal to $h(t)$ itself

$$\delta(t) * h(t) = y(t) = h(t)$$

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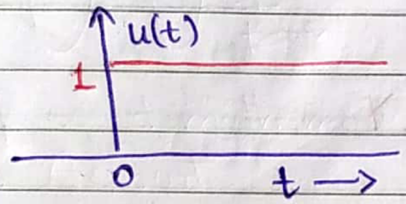
∴ Unit ~~Respose~~ Impulse Response is the characteristic of the system.

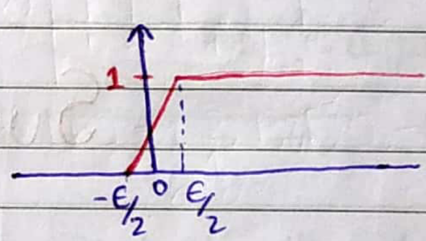
2) Unit Step signal
Basically it is a DC signal having constant magnitude for all time, greater than and equal to 0.
And becomes zero for all time less than 0.

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Representation :-



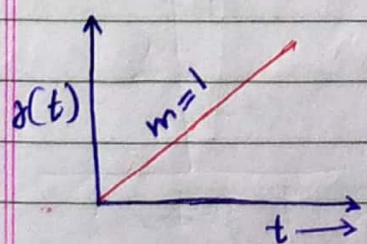
More Accuratly



Second representation is more accurate because energy cannot be transfered from one system to another in zero time, it will take some time no matter how small it will be.

Note:- On differentiation of a Unit Step signal, we obtain Unit Implse signal.

3) Unit Ramp Signal
It is denoted by $r(t)$
Its slope is unity



Mathematically

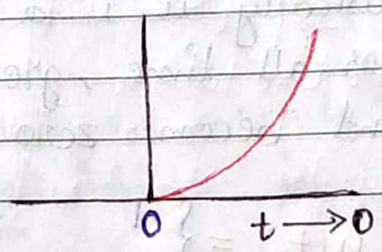
$$r(t) \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

On differentiation of unit Ramp Signal we obtain unit step signal.

④ Unit Parabolic Signal

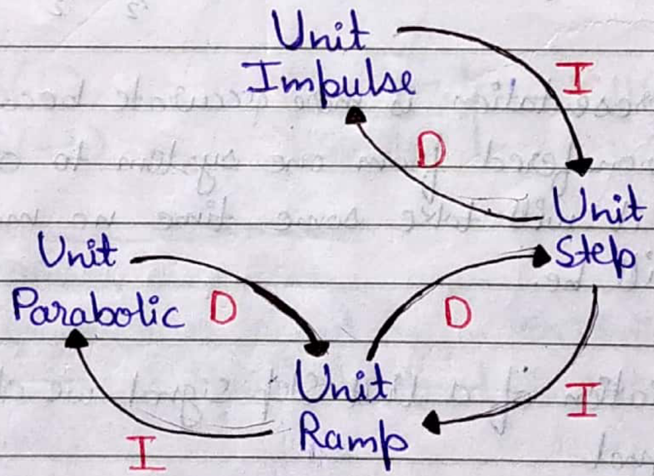
Represented by $p(t)$

$$p(t) = \begin{cases} t^2/2 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



On differentiation of unit Parabolic Signal we obtain unit Ramp signal.

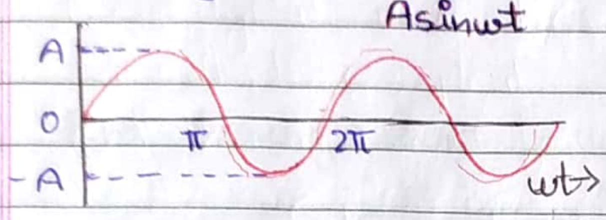
SUMMARY



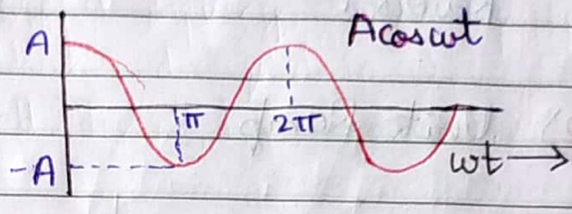
Some more Signals :-

1) Sinosoidal Signal

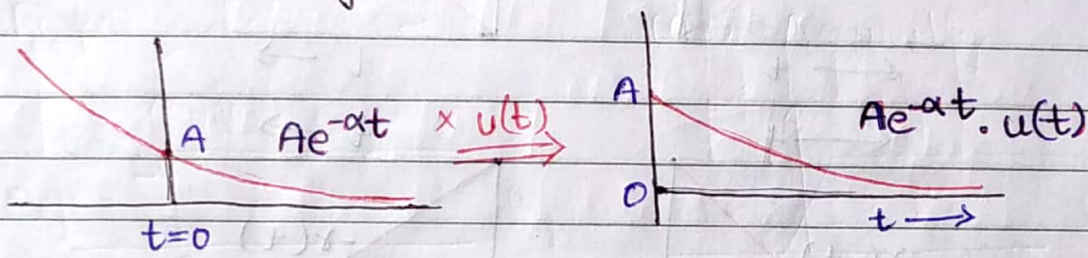
Sine Signal



Cosine Signal

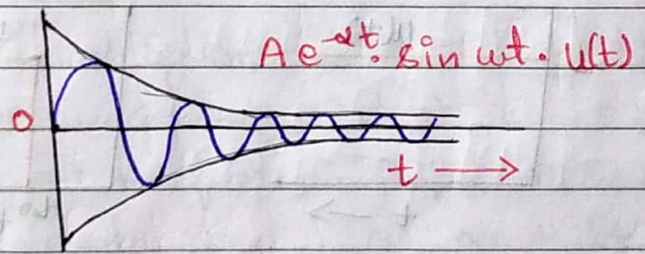


2) Exponential Signal



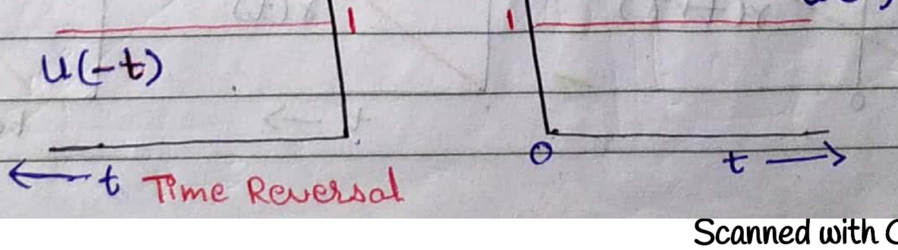
We multiplied it with u(t) so that it will only exist for $t > 0$.

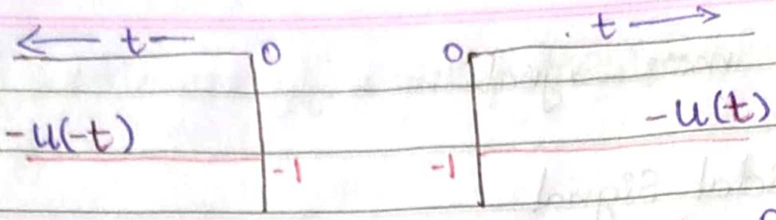
3) Exponentially decaying Sinosoidal
This is a product of above two signals.



Time Shifting and Time Reversal of Signal

I) Unit Step

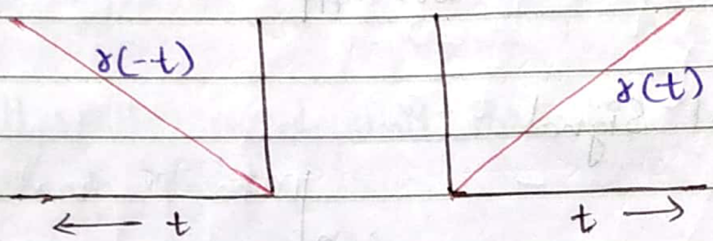


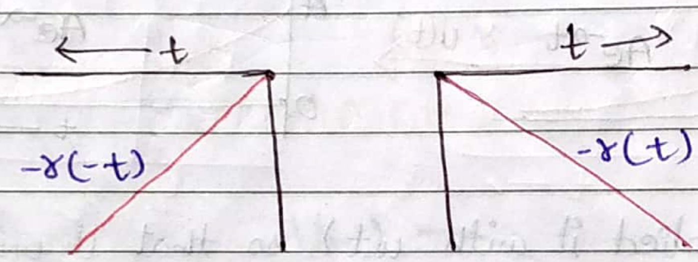


Both time and magnitude Reversal

Magnitude Reversal

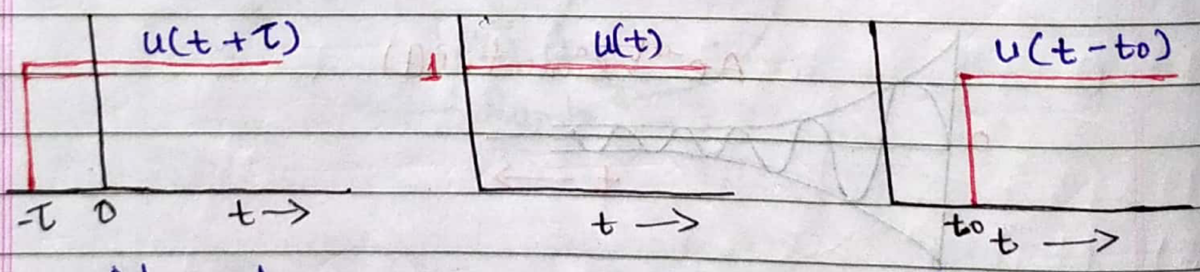
2) Unit Ramp





Time delayed and advanced

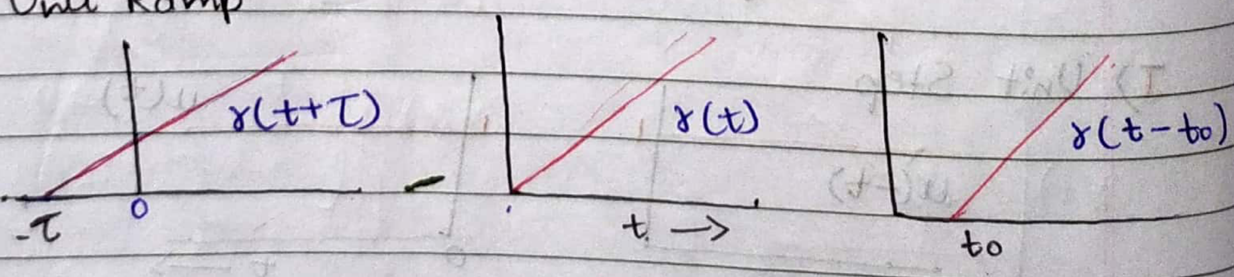
1) Unit Step Signal



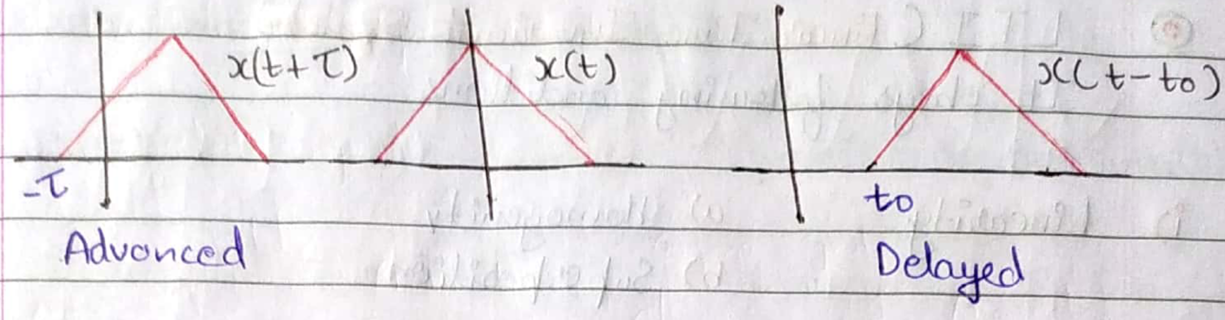
Advonced

Delayed

2) Unit Ramp



ex)



- Signal transformation is used in signal Synthesis.

- ## Systems

Types

I (1) CTS
(2) DTS

II (1) Causal System
Response is dependent on present and past signal of input

I/p → $\boxed{C}$ → O/p

(2) Anti Causal System
Response is dependent on present and future input signal

I/P → $\boxed{AC}$ → O/P

Properties

Linear
i) Homogenety
ii) Superposition

Non-linear
i) No Homogeneity
ii) No Superposition

Time Invariant
$x(t) \to \boxed{ht} \to y(t)$
$x(t-t_0) \to \boxed{ht} \to y(t-t_0)$
Output will be delayed acc. to delay in input

Time Varying
$x(t) \to \boxed{ht} \to y(t)$
$x(t-t_0) \to \boxed{ht} \to y(t-t_0')$
$t_0' > t_0$ Sluggish
$t_0' < t_0$
fast acting Time varying system

LTI (Linear Time Invariant System)

It obeys following conditions

i) Linearity — a) Homogenity
b) Superposition

a) Homogenity

If input is multiplied by an specific magnitude then the output should be multiplied by the same factor.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$x(t) \longrightarrow \boxed{\alpha} \xrightarrow{\alpha \cdot x(t)} \boxed{h(t)} \longrightarrow y'(t) = \alpha \cdot y(t)$$

α = scalar

b) Superposition

When there are 2 inputs $x_1(t)$ and $x_2(t)$ being applying onto a system the output which is obtained is the sum of output of both the signals acting independently.

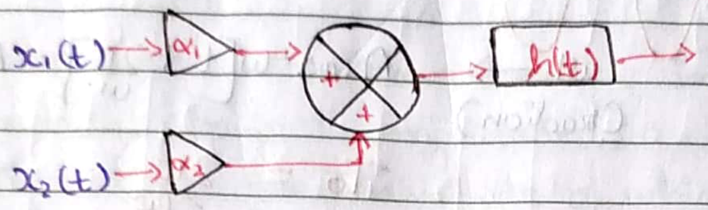
$$x_1(t) \longrightarrow \boxed{h(t)} \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow \boxed{h(t)} \longrightarrow y_2(t)$$

$$x_1(t) + x_2(t) \longrightarrow \boxed{h(t)} \longrightarrow y'(t) = y_1(t) + y_2(t)$$

superposition principle

- Both Homogeneity and Superposition



$\rightarrow y_{new}(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$

LTI System

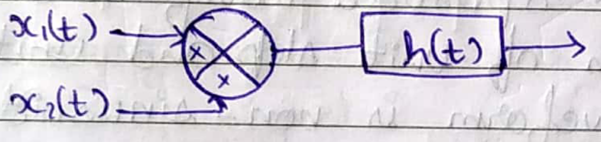
Q) A system having $y(t) = [x(t)]^2$. LTI = ?

$x_1(t) \longrightarrow \boxed{h(t)} \longrightarrow [x_1(t)]^2$

$x_2(t) \longrightarrow \boxed{h(t)} \longrightarrow [x_2 t]^2$

Sum of Output

$[x_1(t)]^2 + [x_2(t)^2]$ —— ①



$\rightarrow [x_1(t)]^2 + 2\,x_1(t)\,x_2(t) + [x_2(t)]^2$ —— ②

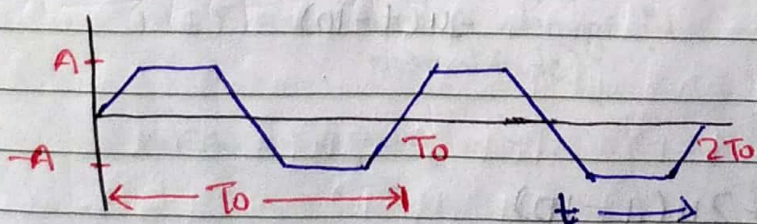
Eq^n ① & ② are not equal so this is not a LTI System

Periodic Waveforms

A periodic waveform is one which repeats itself after a fixed interval of time

$$f(t) = f(t+T_0)$$ T_0 = fundamental Period

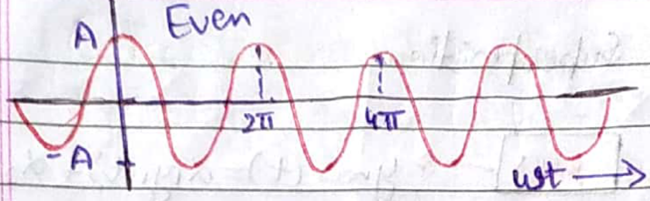
e.g.)



Trapezoidal Waveform

To

2) Even

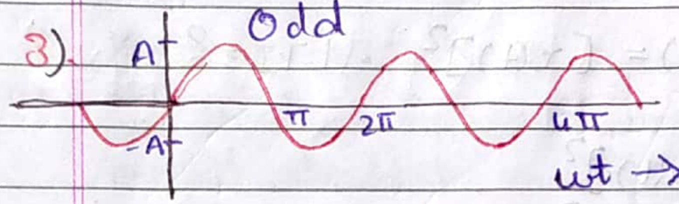


$A\cos\omega t = A\cos(\omega t + 2\pi)$

$A\cos\omega\left[t + \frac{2\pi}{\omega}\right] = A\cos\omega(t+T_0)$

$T_0 = 2\pi/\omega$

3) Odd



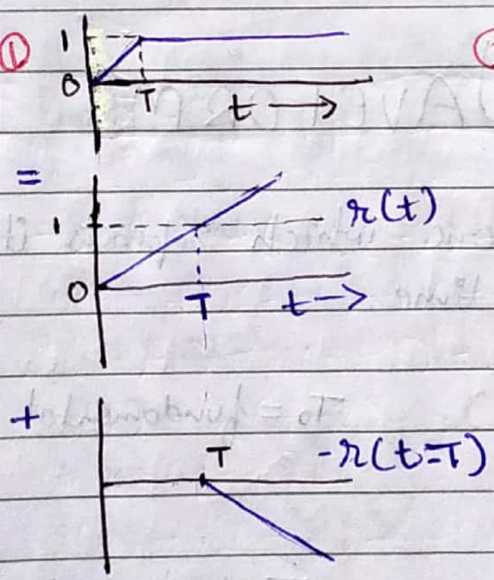
$A\sin\omega t = (\omega t + 2\pi)$

$T_0 = 2\pi/\omega$

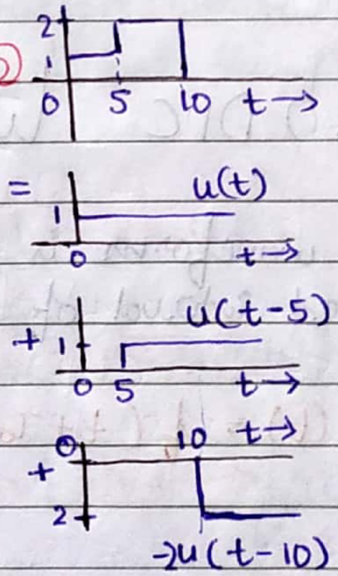
WAVEFORM SYNTHESIS

Any periodic or aperiodic waveform can be expressed as a combination of unit step and unit ramp, provided the waveform is non-sinusoidal and constructed with straight lines. This is known as Waveform Synthesis.

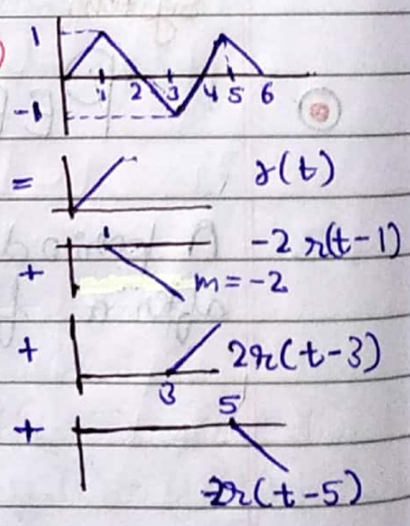
eg) ①



②



③



① $r(t) - r(t-T)$

② $u(t) + u(t-5) - 2u(t-10)$

③ $r(t) - 2r(t-1) + 2r(t-3) - 2r(t-5) - - - -$

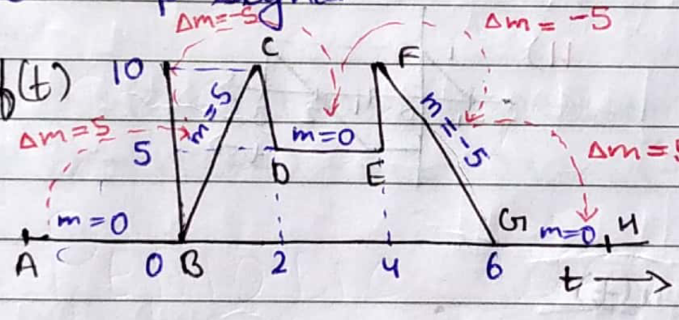
3-Rules of Waveform Synthesis

1) Whenever there is a change in magnitude (rather "sudden" change in magnitude), a step signal exists at that time, the magnitude of which is equal to the difference in final magnitude and intial magnitude.

2) Whenever there is a "sudden" change in slope at time $t=\tau$, there exists a ramp signal, having slope equals to the difference of slopes after $t=\tau$ and before $t=\tau$

3) Whenever both magnitude and slope are changing simontaneously at $t=\tau$, their exists both Step Signal and Ramp signal

Q



$m_{AB} = 0$

$m_{BC} = \frac{10-0}{2-0} = 5$

$m_{CD} = \infty$ (sudden change in magnitude)

$m_{DE} = 0$

$m_{EF} = \infty$ (sudden change in magnitude)

$m_{FG} = \frac{0-10}{6-4} = -5$

$m_{GH} = 0$

4 Ramp

$5rt - 5r(t-2) - 5(t-4) + 5r(t-6)$

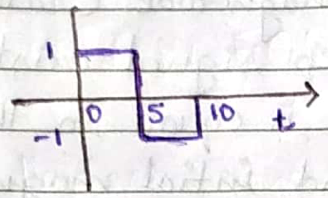
2 Step

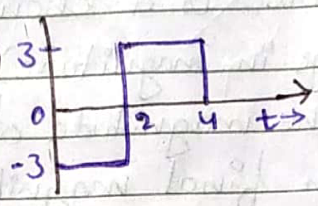
$-5u(t-2) + 5u(t-4)$

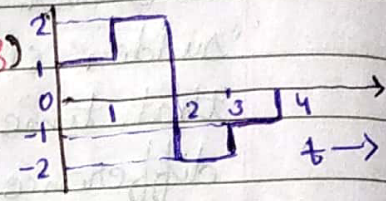
$f(t) = 5rt - 5r(t-2) - 5u(t-2) + 5u(t-4) + 5r(t-4) + 5r(t-6)$

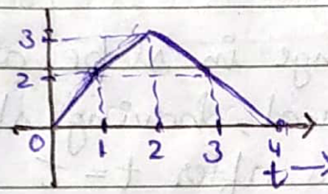
Question on Waveform Synthesis

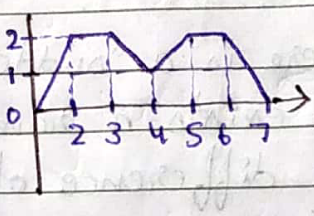
Synthesize using Step & Ramp Signal only

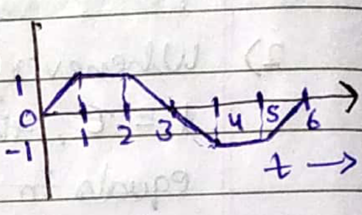
1) 

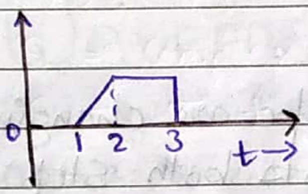
2) 

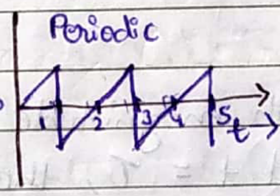
3) 

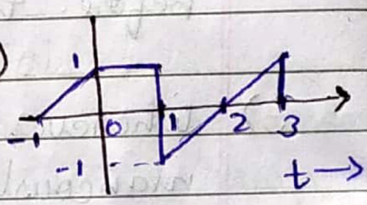
4) 

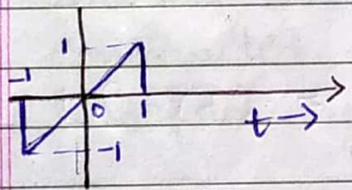
5) 

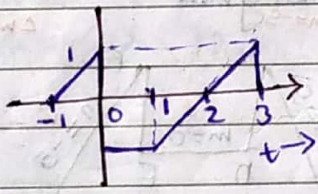
6) 

7) 

8) 

9) 

10) 

11) 

① $u(t) - 2u(t-5) + u(t-10)$

② $-3u(t) + 6u(t-2) - 3u(t-4)$

③ $u(t) + u(t-1) - 4u(t-2) + u(t-3) + u(t-4)$

④ $2r(t) - r(t-1) - 2r(t-2) - r(t-3) + r(t-4)$

⑤ $r(t) - r(t-2) - r(t-3) + 2r(t-4) - r(t-5) - 2r(t-6)$
$+ 2r(t-7)$

⑥ $r(t) - r(t-1) - r(t-2) + r(t-4) + r(t-5) - r(t-6)$

⑦ $r(t-1) - r(t-2) - u(t-3)$

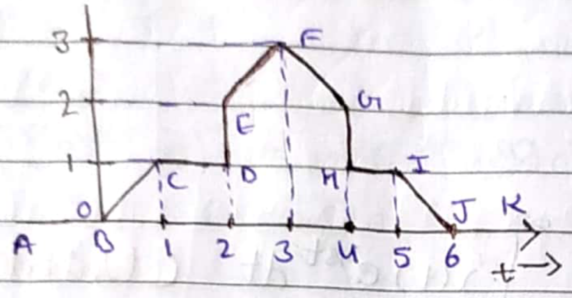
⑧ $r(t) - 2u(t-1) - 2u(t-3) - 2u(t-5) - - - -$

⑨ $r(t+1) - r(t) - 2u(t-1) + r(t-1) - r(t-3) - u(t-3)$

⑩ $-u(t+1) + r(t+1) - r(t-1) - u(t-1)$

(11) $r(t+1) - \cancel{2u(t)} - r(t) - 2u(t) + r(t-1) - r(t-3) + u(t-3)$

Eg.)



$m_{AB} = 0$ $\Delta m = 1$

$m_{BC} = 1$ $\Delta m = -1$

$m_{CD} = 0$ $\Delta m = 1$

$m_{EF} = 1$ $\Delta m = -2$

$m_{FG} = -1$ $\Delta m = 1$

$m_{HI} = 0$ $\Delta m = -1$

$m_{IJ} = -1$ $\Delta m = 1$

$m_{JK} = 0$

2 step signals
$u(t-2) - u(t-4)$

Ramp signals $= r(t) - r(t-1) + r(t-2) - 2r(t-3) + r(t-4) - r(t-5) + r(t-6)$

$$f(t) = r(t) - r(t-1) + r(t-2) + u(t-2) - 2r(t-3) + r(t-4) - u(t-4) - r(t-5) + r(t-6)$$

PROPERTIES & APPLICATIONS OF LAPLACE TRANSFORM

- Laplace Transform :-

$$L[x(t)] = X(s) = \int_0^{\infty} x(t)\, e^{-st}\, dt \quad \text{(Unilateral)}$$

Under the condition $\int_0^{\infty} |x(t) e^{-\sigma t}|\, dt < \infty$

where σ = Real part of s
S = complex frequency (L.T. variable)
$S = \sigma + j\omega$

- Laplace Transform of Basic Signals

1) $L\{\delta(t)\} = \int_0^{\infty} \delta(t) e^{-st}\, dt \qquad \delta(t) = \begin{cases} 1, & t=0 \\ 0, & t \neq 0 \end{cases}$

$= \boxed{1 \cdot 1 = 1}$

2) $L\{u(t)\} = \int_0^{\infty} u(t)\, e^{-st}\, dt \qquad u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t<0 \end{cases}$

$= \int_0^{\infty} 1\, e^{-st}\, dt$

$= \left[\frac{e^{-st}}{-s}\right]_0^{\infty} = \frac{-1}{s}[e^{-\infty} - e^{0}] = \frac{-1}{s}[0-1]$

$= \boxed{\frac{1}{s}}$

3) $L\{r(t)\} = \int_0^{\infty} t \cdot e^{-st}\, dt \qquad r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t<0 \end{cases}$

$= t \cdot \frac{e^{-st}}{-s} - \int_0^{\infty} \frac{1 \cdot e^{-st}}{-s}\, dt$

$$= 0 + \frac{1}{s}\left[\int_0^\infty e^{-st}\,dt\right] = \frac{1}{s}\cdot\frac{1}{s}$$

$$= \boxed{\frac{1}{s^2}}$$

4) $L\{e^{-\alpha t}u(t)\} = \int_0^\infty e^{-\alpha t}\, \cancel{u(t)}\, e^{-st}\,dt$

$$= \int_0^\infty e^{-(s+\alpha)t}\,dt = \left[\frac{e^{-(s+\alpha)t}}{-(s+\alpha)}\right]_0^\infty$$

$$= \frac{-1}{s+\alpha}\left[e^{-\infty} - e^{0}\right] = \boxed{\frac{1}{s+\alpha}}$$

5) $L\{\sin \omega t \cdot u(t)\}$ & $L\{\cos \omega t\ u(t)\}$

$L[e^{-j\omega t}] = \int_0^\infty e^{-j\omega t}\cdot e^{-st} = \int_0^\infty e^{-(s+j\omega)t}\,dt$

$L[e^{-j\omega t}] = \frac{1}{s+j\omega}$

We know $e^{-j\omega t} = \cos\omega t - j\sin\omega t$

So, $L[\cos\omega t - j\sin\omega t] = \frac{1}{s+j\omega} = \frac{s-j\omega}{(s-j\omega)(s+j\omega)}$

$$= \frac{s-j\omega}{s^2+\omega^2}$$

$\boxed{L[\cos\omega t] = \frac{s}{s^2+\omega^2}}$ $\boxed{L[\sin\omega t] = \frac{\omega}{s^2+\omega^2}}$

eg.) ① $L[\sin 3t] = 3/s^2+9$

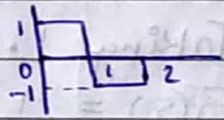
② $L[\cos 6t] = s/s^2+36$

③ $L[5\sin 2t] = 5\cdot\frac{2}{s^2+4} = \frac{10}{s^2+4}$

④ $L[-7\cos 4t] = \frac{-7s}{s^2+16}$

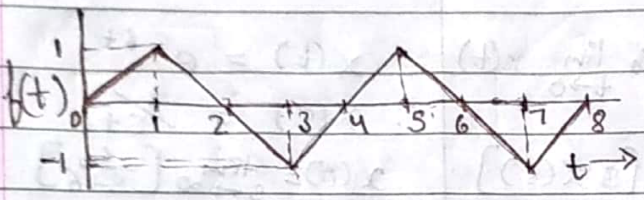
PROPERTIES OF LAPLACE TRANSFORM

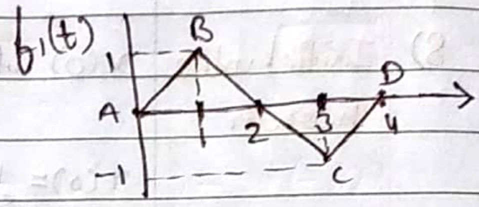
S. No.	Property	Formulae	Illustration
1)	Linearity	$L[\alpha x(t) + \beta y(t)] = \alpha X(S) + \beta Y(S)$	$L[3u(t) + 2e^{-\alpha t}]$ $= \frac{3}{S} + \frac{2}{S+\alpha}$ $= \frac{5S+3\alpha}{S(S+\alpha)}$
2)	Time-shifting	$L[x(t-t_0)] = e^{-St_0} \cdot X(S)$ $L[x(t+t_0)] = e^{St_0} \cdot X(S)$	$L[u(t-3)] = e^{-3S} \cdot \frac{1}{S}$ $L[-\gamma(t-2)] = -\frac{e^{2S}}{S^2}$
3)	Frequency Shifting	$x(t) \longleftrightarrow X(S)$ $e^{-\alpha t} x(t) \longleftrightarrow X(S+\alpha)$ $e^{\alpha t} x(t) \longleftrightarrow X(S-\alpha)$	$L[e^{-4t}] = \frac{1}{S+4}$ $L[e^{-5t} \cdot \gamma(t)] = \frac{1}{(S+5)^2}$
4)	Differentiation Property	$x(t) \longleftrightarrow X(S)$ $L\left[\frac{dx(t)}{dt}\right] = S X(S) - x(0)$ $L\left[\frac{d^2x(t)}{dt^2}\right] = S^2 X(S) - S x(0) - \dot{x}(0)$ $\dot{x}(0) = \lim_{t \to 0}\left[\frac{dx(t)}{dt}\right]$	$L[e^{-2t}] = \frac{1}{S+2}$ $L\left[\frac{d}{dt}(e^{-2t})\right] = \frac{S}{S+2} - [e^{-2 0}]$
5)	Integration Property	$x(t) \longleftrightarrow X(S)$ $L\left[\int_{\tau=0}^{t} x(\tau)\, d\tau\right] = \frac{X(S)}{S}$	$L\left[\int_{0}^{t} e^{-2\tau} d\tau\right] = \frac{1}{S(S+2)}$
6)	Multiplication by t	$x(t) \longleftrightarrow X(S)$ $t\,x(t) \longleftrightarrow -\frac{d}{dS}[X(S)]$	$L[\sin t] = \frac{1}{S^2+1}$ $L[t \sin t] = -\frac{d}{dS}\left[\frac{1}{S^2+1}\right]$ $= -\frac{1}{(S^2+1)^2}$

7) Convolution Property	$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$ $\int x(\tau)\, h(t-\tau)\, d\tau = y(t)$ Then, $X(s) \cdot H(s) = Y(s)$	$x(t) * h(t) = y(t)$ $X(s) \cdot H(s) = Y(s)$ $u(t) \longrightarrow \boxed{e^{-\alpha t}} \rightarrow y(t)$ $1/s \rightarrow \boxed{\frac{1}{s+\alpha}} \rightarrow Y(s) = \frac{1}{s} \cdot \frac{1}{s+\alpha} = \frac{1}{s(s+\alpha)}$
8) Initial value Theorem	$x(0)$ which is $\lim_{t \to 0} x(t)$ is $x(0) = \lim_{s \to \infty} [s\, X(s)]$	$x(t) = e^{-6t}$ $X(s) = 1/s+6$ $x(0) = \lim_{s \to \infty} \left[\frac{s}{s+6}\right]$ $= \lim_{s \to \infty} \left(\frac{1}{1+\frac{6}{s}}\right) = \frac{1}{1+0} = 1$
9) Final value Theorem	$x(\infty)$ which is $\lim_{t \to \infty} x(t)$ is $x(\infty) = \lim_{s \to 0} [s\, X(s)]$	$x(t) = e^{-4t}$ $X(s) = \frac{1}{s+4}$ $x(\infty) = \lim_{s \to 0} \left(\frac{s}{s+4}\right) = 0$
10) Laplace Transform of Periodic Waveforms	If $f_1(t+T_0) = f_1(t)$ and $L[f_1(t)] = F_1(s)$ then laplace Transform of $f(t)$ which is periodic function of $f_1(t)$ is $F(s) = \left(\frac{1}{1-e^{-sT}}\right) F_1(s)$	$f_1(t)$  $f_1(t) = u(t) - 2u(t-1) + u(t-2)$ $F_1(s) = \frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}$ $= \frac{1}{s}(1 - 2e^{-s} + e^{-2s})$ $= \frac{(1-e^{-s})^2}{s}$ $F(s) = \frac{1}{1-e^{-2s}} \cdot \frac{(1-e^{-s})^2}{s}$ $= \frac{(1-e^{-s})^2}{s(1-e^{-2s})} = \frac{1-e^{-s}}{s(1+e^{-s})}$

APPLICATION OF LAPLACE TRANSFORM COMPLEX WAVEFORM

Part A : Periodic Waveform

① 



$T = 4\ s$

$m_{AB} = 1 \quad m_{BC} = -1$

$m_{CD} = +1 \quad m_{DE} = 0$

$f(t) = f_1(t) + f_1(t-4) + f_1(t-8) + \cdots$

$F(s) = \dfrac{F_1(s)}{1-e^{-sT}}$; $T = 4$ $\therefore$ $F(s) = \dfrac{F_1(s)}{1-e^{-4s}}$ — ①

$f_1(t) = r(t) - 2r(t-1) + 2r(t-3) - r(t-4)$ — ②

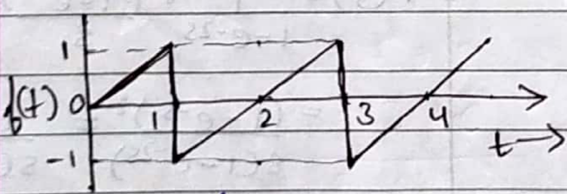
Taking L.T.

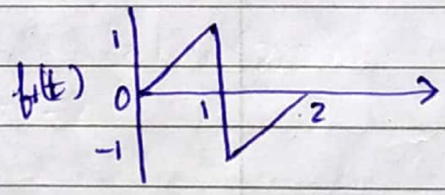
$F_1(s) = \dfrac{1}{s^2} - \dfrac{2}{s^2}e^{-s} + \dfrac{2}{s^2}e^{-3s} - \dfrac{1}{s^2}e^{-4s}$

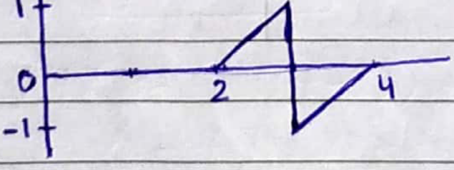
$= \dfrac{1}{s^2}\left(1 - 2e^{-s} + 2e^{-3s} - e^{-4s}\right)$ — ③

Putting ③ in ①

$F(s) = \dfrac{1 - 2e^{-s} + 2e^{-3s} - e^{-4s}}{s^2(1-e^{-4s})}$

② 



$f_1(t-2)$ 

$+ f_1(t-4) + f_1(t-6) + \cdots$

$f_1(t) = r(t) - 2u(t-1) - r(t-2)$

So, $F_1(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s} - \frac{e^{-2s}}{s^2}$

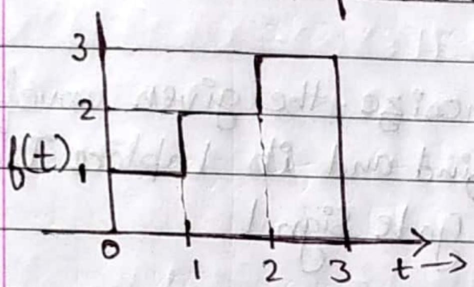
$F(s) = \frac{F_1(s)}{1-e^{-sT}}$; here $T=2$

So $F(s) = \frac{F_1(s)}{1-e^{-sT}}$

Therefore, $F(s) = \left[\frac{1}{s^2} - \frac{2e^{-s}}{s} - \frac{e^{-2s}}{s^2}\right] \cdot \frac{1}{1-e^{-2s}}$

$= \frac{1}{s^2(1-e^{-2s})}(1-2se^{-s}-e^{-2s})$ Ans

Part B: Aperiodic Wave form

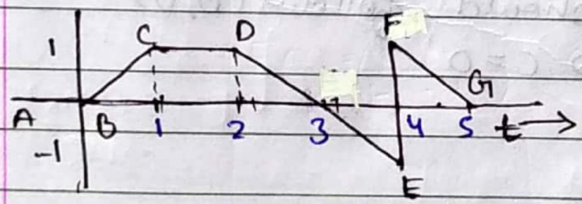
① 

$f(t) = u(t) + u(t-1) + u(t-2) - 3u(t-3)$

on taking L.T.

$F(s) = \frac{1}{s} + \frac{1}{s}e^{-s} + \frac{1}{s}e^{-2s} - \frac{3e^{-3s}}{s}$

$= \frac{1}{s}\left[1 + e^{-s} + e^{-2s} - 3e^{-3s}\right]$

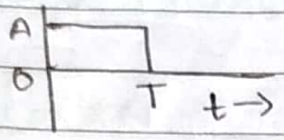
② 

$f(t) = r(t) - r(t-1) - r(t-2) + 2u(t-4) + r(t-5)$

Taking L.T.

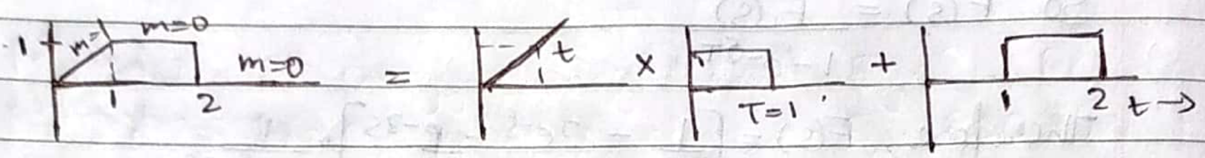
$F(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s^2} + \frac{2e^{-4s}}{s} + \frac{e^{-5s}}{s^2}$

Gate Signal (Do if Asked)



$A[u(t) - u(t-T)]$

$A = 1$



$$f(x) = t[u(t) - u(t-1)] + u(t-1) - u(t-2)$$
$$= tu(t) - (t-1)u(t-1) - u(t-2)$$
$$= r(t) - r(t-1) - u(t-2)$$

$$F(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}$$

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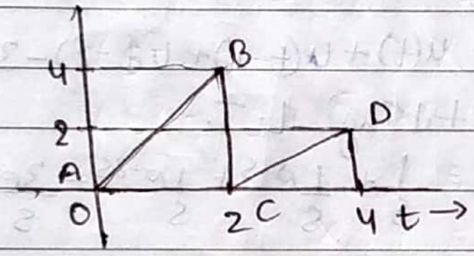

Synthesize the given waveform and find and its Laplace Transform using Gate Signal

Eqn of line AB : $m = \frac{4-0}{2-0} = 2$

$y = 2t + c$ should follow (0,0)

$0 = 0 + c \Rightarrow c = 0$

$\boxed{y = 2t}$

Eqn of line CD : $m = \frac{4-0}{4-0} = 1$

$y = t + c'$ should follow (2,0)

$0 = 2 + c'$ $\quad c' = -2$

$\boxed{y = t - 2}$

$$f(t) = 2t(u(t) - u(t-2)) + (t-2)(u(t-2) - u(t-4))$$
$$= 2tu(t) - 2tu(t-2) + tu(t-2) - 2u(t-2) - tu(t-4) + 2u(t-4)$$
$$= 2r(t) - tu(t-2) - 2u(t-2) - (t-4+4)u(t-4) + 2u(t-4)$$

#

$= 2r(t) - (t-2+2)u(t-2) - 2u(t-2) - (t-4)u(t-4) - 4u(t-4)$
$+ 2u(t-4)$

$= 2r(t) - r(t-2) - 2u(t-2) - 2u(t-2) - r(t-4) - 2u(t-4)$

$f(t) = 2r(t) - r(t-2) - 4u(t-2) - r(t-4) - 2u(t-4)$

$$F(s) = \frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{4}{s}e^{-2s} - \frac{1}{s^2}e^{-4s} - \frac{2}{s}e^{-4s}$$ Ans