Chapter 10

Network Synthesis

Inside this Chapter

10.1. INTRODUCTION

In the preceding chapter we have been primarily concerned with the problem of determining the response, given the excitation in the network ; this problem lies in the domain of network analysis. In this chapter we will be dealing with the problem of synthesizing a network given the excitation and response.

10.2. ELEMENTS OF REALIZABILITY THEORY

The starting point for any network synthesis problem is the network function which is the ratio of response $R(s)$ to the excitation $E(s)$, *i.e.*

$$T(s) = \frac{R(s)}{E(s)}$$

Our task is to synthesize a network from a given network function.

10.2.1. Causality and Stability

The first step in a synthesis procedure is to determine whether $T(s)$ can be realized as a physical passive network. There are two important considerations : causality and stability. By causality we mean that a voltage cannot appear between any pair of terminals in the network before a current is impressed, or vice-verse. In other words, the response of the network must be zero for $t < 0$.

In order for network to be stable, the following three conditions on its network function $T(s)$ must be satisfied.

(*i*) $T(s)$ cannot have poles in the right half of s-plane.

(*ii*) $T(s)$ cannot have multiple poles in the imaginary ($j\omega$) -axis.

(*iii*) The degree of the numerator of $T(s)$ cannot exceed the degree of denominator by more than unity.

10.2.2. Hurwitz Polynomial

Another element of realizability is a class of polynomial known as Hurwitz polynomial which is, in fact, the denominator polynomial of the network function satisfying certain conditions.

In brief, A polynomial $P(s)$ is said to be Hurwitz if the following conditions are satisfied :

(1) $P(s)$ is real when s is real.

(2) The roots of $P(s)$ have real parts which are zero or negative.

Properties of Hurwitz Polynomial

As a result of above conditions (1) and (2), Hurwitz polynomial $P(s)$ have the following properties:

(i) If the polynomial $P(s)$ can be written as

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots\ldots\ldots + a_1 s + a_0$$

then, All the coefficients a_i must be positive. A corollary is that between the highest order term in s and the lowest order term, none of the coefficients may be zero unless the polynomial is even or odd. In other words, $a_{n-1}, a_{n-2}, \ldots, a_2, a_1$ must not be zero if the polynomial is neither even nor odd.

(ii) Both the odd and even parts of a Hurwitz polynomial $P(s)$ have roots on the $j\omega$ – axis only. If we denote the even part of $P(s)$ as $M(s)$ and the odd part as $N(s)$, so that

$$P(s) = M(s) + N(s)$$

then $M(s)$ and $N(s)$ both have roots on the $j\omega$-axis only.

(iii) As a result of property (ii), if $P(s)$ is either even or odd, all its roots are on the $j\omega$-axis (including origin).

(iv) The continued fraction expansion of the ratio ($\psi(s)$ of the odd to even parts ($N(s)/M(s)$) or the even to odd parts ($M(s)/N(s)$) of a Hurwitz polynomial yields all positive quotient terms. As,

$$\psi(s) = \frac{N(s)}{M(s)} \text{ or } \frac{M(s)}{N(s)}$$

$$= q_1 s + \cfrac{1}{q_2 s + \cfrac{1}{q_3 s + \cfrac{1}{\cdots}}} + \frac{1}{q_n s}$$

where the quotients $q_1, q_2 \ldots, q_n$ must be positive if the polynomial $P(s) = M(s) + N(s)$ is Hurwitz.

(v) If $P(s)$ is Hurwitz polynomial and $W(s)$ is a multiplicative factor. Then $P_1(s) = P(s).W(s)$ is also Hurwitz polynomial, if $W(s)$ is Hurwitz polynomial.

(vi) In case the polynomial is either only even or only odd, it is not possible to obtain the continued fraction expansion in such cases, the polynomial $P(s)$ is Hurwitz if the ratio of $P(s)$ and its derivative $P'(s)$ gives a continued fraction expansion.

Note :

Generally, we can check the given polynomial is Hurwitz or not, by using properties (i) and (iv)/(v)/(vi) for conditions (1) and (2) respectively.

Procedure for Obtaining the Continued Fraction Expansion

To obtain the continued fraction expansion, we must perform a series of long division. Suppose $\psi(s)$ is

$$\psi(s) = \frac{M(s)}{N(s)}$$

Where $M(s)$ is of one higher degree than $N(s)$. Then, we obtain a single quotient and a remainder.

$$\psi(s) = q_1 s + \frac{R_1(s)}{N(s)}$$

The degree of the term $R_1(s)$ is one lower than the degree of $N(s)$. Therefore if we invert the second term and divide, we have

$$\frac{N(s)}{R_1(s)} = q_2 s + \frac{R_2(s)}{R_1(s)}$$

Inverting and dividing again, we obtain

$$\frac{R_1(s)}{R_2(s)} = q_3 s + \frac{R_3(s)}{R_2(s)}$$

and so on.

There is a theorem which states that the continued fraction expansion of the even to odd or odd to even parts of a polynomial must be finite in length.

EXAMPLE 10.1 Check whether the given polynomial

$$P(s) = s^4 + s^3 + 5s^2 + 3s + 4 \text{ is Hurwitz or not.}$$

Solution:

Condition (1): Since all coefficients of $P(s)$ are positive, so $P(s)$ is real for s real.

Condition (2): The even and odd parts of $P(s)$ are

$$M(s) = s^4 + 5s^2 + 4$$
$$N(s) = s^3 + 3s$$

Continued fraction expansion of $\psi(s) = \dfrac{M(s)}{N(s)}$ is given by

$$s^3 + 3s \,\big)\, s^4 + 5s^2 + 4 \,\big(\, s$$
$$\underline{s^4 + 3s^2}$$
$$2s^2 + 4 \,\big)\, s^3 + 3s \,\big(\, \tfrac{1}{2}s$$
$$\underline{s^3 + 2s}$$
$$s \,\big)\, 2s^2 + 4 \,\big(\, 2s$$
$$\underline{2s^2}$$
$$4 \,\big)\, s \,\big(\, \tfrac{1}{4}s$$
$$\underline{s}$$
$$\times$$

So that the continued fraction expansion of $\psi(s)$ is

$$\psi(s) = \frac{M(s)}{N(s)}$$
$$= s + \cfrac{1}{\cfrac{s}{2} + \cfrac{1}{2s + \cfrac{1}{s/4}}}$$

Since all the quotient terms of the continued fraction expansion are positive, $P(s)$ is Hurwitz.

EXAMPLE 10.2 Check whether the given polynomial

$$P(s) = s^4 + s^3 + 2s^2 + 4s + 1 \text{ is Hurwitz or not.}$$

Solution:

Condition (1): is satisfied (since all a_i are positive).

Condition (2): Even and odd parts of $P(s)$ are

$$M(s) = s^4 + 2s^2 + 1$$
$$N(s) = s^3 + 4s$$

So, continued fractison expansion of $\psi(s) = {}^{M(s)}/_{N(s)}$ is given as,

$$s^3 + 4s \,\big)\, s^4 + 2s^2 + 1 \,\big(\, s$$
$$\underline{s^4 + 4s^2}$$
$$-2s^2 + 1 \,\big)\, s^3 + 4s \,\big(\, -\frac{1}{2}s$$
$$\underline{s^3 - \frac{s}{2}}$$
$$\frac{9}{2}s \,\big)\, -2s^2 + 1 \,\big(\, -\frac{2}{9} \cdot 2s = -\frac{4}{9}s$$
$$\underline{-2s^2}$$
$$1 \,\big)\, \frac{9}{2}s \,\big(\, \frac{9}{2}s$$
$$\underline{\frac{9}{2}s}$$
$$\times$$

The given polynomial is not Hurwitz because of presence of the negative quotient terms in the continued fraction expansion.

EXAMPLE 10.3 **Check whether the given polynomial $P(s) = s^3 + 2s^2 + 3s + 6$ is Hurwitz or not.**

Solution :

Condition (1) : is satisfied (since all a_i are positive).

Condition (2) : Even and odd parts of $P(s)$ are

$$M(s) = 2s^2 + 6$$
$$N(s) = s^3 + 3s$$

So, continued fraction expansion of $\psi(s) = \dfrac{N(s)}{M(s)}$ is given as

$$2s^2 + 6 \,\big)\, s^3 + 3s \,\big(\, \frac{1}{2}s$$
$$\underline{s^3 + 3s}$$
$$\times$$

we see that the division has been terminated abruptly (suddenly) by a common factor $s^3 + 3s$. Thus

$$P(s) = (s^3 + 3s)\left(1 + \frac{2}{s}\right) \equiv W(s).\, P_1(s)$$

We know that the term $P_1(s) = \left(1 + \dfrac{2}{s}\right)$ is Hurwitz. Now check for

$$W(s) = s^3 + 3s = s(s^2 + 3) = s\left(s + j\sqrt{3}\right)\left(s - j\sqrt{3}\right) \text{ is Hurwitz}$$

Alternatively : since $W(s)$ is an odd function, therefore the continued fraction expansion of $\psi'(s) = {}^{W(s)}/_{W'(s)}$ [using property (*vi*)] is given as

$$3s^2 + 3 \,\big)\, s^3 + 3s \,\big(\, \frac{1}{3}s$$
$$\underline{s^3 + s}$$
$$2s \,\big)\, 3s^2 + 3 \,\big(\, \frac{3}{2}s$$
$$\underline{3s^2}$$
$$3 \,\big)\, 2s \,\big(\, \frac{2}{3}s$$
$$\underline{2s}$$
$$\times$$

Since $W(s)$ is Hurwitz, from property (v),

$P(s)$ is Hurwitz.

EXAMPLE 10.4 **Check whether given polynomial.**

$P(s) = s^7 + 2s^6 + 2s^5 + s^4 + 4s^3 + 8s^2 + 8s + 4$ **is Hurwitz or not.**

Solution : Condition (1) : is satisfied (since all a_i are positive).

Condition (2) : The continued fraction expansion of $\psi(s)$ is given as

$$2s^6 + s^4 + 8s^2 + 4 \,\big)\, s^7 + 2s^5 + 4s^3 + 8s \,\big(\, \frac{1}{2}s$$
$$\underline{s^7 + \frac{1}{2}s^5 + 4s^3 + 2s}$$
$$\frac{3}{2}s^5 + 6s \,\big)\, 2s^6 + s^4 + 8s^2 + 4 \,\big(\, \frac{2}{3}\cdot 2s = \frac{4}{3}s$$
$$\underline{2s^6 + 8s^2}$$
$$s^4 + 4 \,\big)\, \frac{3}{2}s^5 + 6s \,\big(\, \frac{3}{2}s$$
$$\underline{\frac{3}{2}s^5 + 6s}$$
$$\times$$

Thus we see that the term $(s^4 + 4)$, which can be factored into

$$(s^4 + 4) = (s^2 + 2)^2 - 4s^2 = (s^2 + 2 + 2s)(s^2 + 2 - 2s)$$

First factor $(s^2 + 2s + 2)$ is Hurwitz* and second factor is not Hurwitz. As,

$$s^2 + 2 - 2s = (s - 1)^2 + (1)^2 = (s - 1 - j1)(s - 1 + j1)$$

or

$$s = 1 + j1,\ 1 - j1 \text{ (roots lie on right half of } s\text{-plane).}$$

Now, It is clear that $P(s)$ is not Hurwitz.

EXAMPLE 10.5 **Find the range of values of a so that**

$P(s) = s^4 + s^3 + as^2 + 2s + 3$ **is Hurwitz.**

Solution : Condition (1) : All a_i must be positive, therefore, $a > 0$.

Condition (2) : Forming $M(s)/N(s)$ and obtaining the continued fraction expansion and requiring the quotients to be positive.

$$s^3 + 2s \,\big)\, s^4 + as^2 + 3 \,\big(\, s$$
$$\underline{s^4 + 2s^2}$$
$$(a - 2)s^2 + 3 \,\big)\, s^3 + 2s \,\big(\, \frac{1}{a - 2}s$$
$$\underline{s^3 + \frac{3}{a - 2}s}$$
$$\left(2 - \frac{3}{a - 2}\right)s \,\big)\, (a - 2)s^2 + 3 \,\big($$
$$\ldots$$
$$\ldots$$

*Quadratic term with positive a_i are always Hurwitz, since roots of quadratic equation lie on left half of s-plane. If any coefficient a_i is negative then quadratic term is not Hurwitz, since atleast one root on right half of σ-plane.

therefore, $a-2>0$ and $2-\dfrac{3}{a-2}>0$

or $a>2$ and $a>3.5$

Thus the range of a is $a > 3.5$

EXAMPLE 10.6 **Check whether the given polynomical is Hurwitz or not.**

$$P(s) = s^5 + 7s^4 + 6s^3 + 9s^2 + 8s.$$

Solution : The polynomial $P(s)$ is not Hurwitz because a root of $P(s)$ lies at the origin*. (which is not permitted according to condition of Hurwitz polynomial).

10.2.3. Positive Real Functions

These functions are important because they represent physically realizable passive driving point immittances. A function $T(s) = \dfrac{N(s)}{D(s)}$ is positive real (*p.r.*) if the following conditions are satisfied :

(1) $T(s)$ is real for s real, *i.e.* $T(\sigma)$ is purely real.

(2) $D(s)$ is Hurwitz ploynomial.

(3) $T(s)$ may have poles on the $j\omega$-axis. Thses poles are simple and the residues there of are real and positive.

(4) The real part of $T(s)$ is greater than or equal to zero for the real part of s is greater then or equal to zero, *i.e.*,

$$Re[T(s)] \geq 0 \quad \text{for } Re\ s \geq 0.$$

or $$Re[T(s)] \geq 0 \quad \text{for } Re\ s = 0$$

and $$Re[T(s)] \geq 0 \quad \text{for } Re\ s > 0 \text{ (same as condition 1)}$$

Therefore,

$$Re[T(j\omega)] \geq 0 \quad \text{for all } \omega$$

A simplification of condition (4) is possible. Let

$$T(s) = \frac{N(s)}{D(s)} = \frac{M_1(s)+N_1(s)}{M_2(s)+N_2(s)}$$

where $M_i(s)$ is an even function and $N_i(s)$ is an odd function.

Rationalising,

$$T(s) = \frac{M_1+N_1}{M_2+N_2}\cdot\frac{M_2-N_2}{M_2-N_2} = \frac{M_1M_2-N_1N_2}{M_2^{\,2}-N_2^{\,2}} + \frac{N_1M_2-M_1N_2}{M_2^{\,2}-N_2^{\,2}}$$

We see that the products M_1M_2 and N_1N_2 are even functions, while N_1M_2 and M_1N_2 are odd functions. Therefore, the even part of $T(s)$ is

$$Ev\,[T(s)] = \frac{M_1M_2-N_1N_2}{M_2^{\,2}-N_2^{\,2}}$$

and odd part of $T(s)$ is

$$\text{odd}\,[T(s)] = \frac{N_1M_2-M_1N_2}{M_2^{\,2}-N_2^{\,2}}$$

If we let $s = j\omega$ (since $\sigma = 0$), we see that the even part of any polynomial is real, while the odd part of the polynomial is imaginary *i.e.*,

$$Re[T(j\omega)] = Ev\,[T(s)]\big|_{s=j\omega}$$

and $$j\,Im\,[T(j\omega)] = odd\,[T(s)]\big|_{s=j\omega}$$

It is clear that

$$T(j\omega) = Re[T(j\omega)] + j\,Im[T(j\omega)]$$

*Roots of $P(s)$ are not permitted at the origin except in case of odd function (polynomial) since one root of odd part $N(s)$ of polynomial is always present at the origin.

Therefore, to test for the condition (4) for positive realness, we determine the real part of $T(j\omega)$ by finding the even part of $T(s)$ and then putting $s = j\omega$. We then check to see whether $Re[T(j\omega)] \geq 0$ for all ω.

The denominator of $Re\,[T(j\omega)]$ is always a positive quantity because

$$[M_2(j\omega)]^2 - [N_2\,(j\omega)]^2 = M_2\,(\omega^2) + N_2(\omega^2) \geq 0$$

That is, there is an extra j or imaginary term in $N_2(j\omega)$, which when squared, gives -1, so that denominator of $Re[T(j\omega)]$ is the sum of two squared numbers and is always positive. Therefore, our task resolves into the problem of determining whether

$$A(\omega^2) \equiv M_1\,(j\omega).\,M_2(j\omega) - N_1(j\omega).\,N_2\,(j\omega) \geq 0$$

Properties of Positive Real Function (p.r.f.)

(i) If $T(s)$ is *p.r.*, then $^1/_{T(s)}$ is also *p.r.* This property implies that if a driving point impedance $Z(s)$ is *p.r.*, then its reciprocal $(^1/_{Z(s)})$, the driving point admittance $Y(s)$, is also *p.r.*

(ii) The sum of *p.r.* functions is *p.r.* from an immittance stand point, we see that if two impedances are connected in series or two admittances are connected in parallel, the resultant impedance or admittance is *p.r.* (Note that the difference of two *p.r.* functions is not necessarily *p.r.*).

(iii) The poles and zeros of a *p.r.f* cannot have positive real parts, *i.e.*, they cannot be in the right half of the *s*-plane. In addition to this, only simple poles with real positive residues can exist on the $j\omega$-axis.

(iv) The poles and zeros of a *p.r.f.* are real or occur in conjugate pairs.

(v) The highest powers of the numerator $N(s)$ and denominator $D(s)$ polynomial may differ atmost by unity. This condition prohibits multiple poles or zeros at $s = \infty$.

(vi) The lowest power of $D(s)$ and $N(s)$ polynomials may differ by atmost unity. This condition prevents the possibility of multiple poles or zeros at $s = 0$.

Note :

The necessary and sufficient conditions for a rational function $T(s)$ with real coefficients to be *p.r.* are :

Condition (1) : $T(s)$ must have no poles in the right half of *s*-plane, *i.e.*, Denominator $D(s)$ of $T(s)$ must be Hurwitz polynomial. This condition can check through a continued fraction expansion of the odd to even parts or even to odd parts of the $D(s)$ in which quotients must be positive.

Condition (2) : This condition is checked only when the poles of $T(s)$ are on the $j\omega$-axis, otherwise not, (If denominator $D(s)$ of $T(s)$ has a factor of the type $s^2 + a$; where a is positive real constant, then we can say, the poles of $T(s)$ are on the $j\omega$-axis). $T(s)$ may have only simple poles on the imaginary axis ($j\omega$-axis) with real and positive residues. This condition is tested by making a partial fraction expansion of $T(s)$ and checking whether the residues of the poles on the $j\omega$-axis are positive and real.

Condition (3) : $Re[T(j\omega)] \geq 0$, for all ω

or $A(\omega^2) \equiv M_1\,(j\omega).\,M_2\,(j\omega) - N_1\,(j\omega).\,N_2\,(j\omega) \geq 0$; for all ω

EXAMPLE 10.7 **Determine whether the function**

$$Z(s) = \frac{2s^2+5}{s\,(s^2+1)} \text{ is } p.r. \text{ or not.}$$

Solution :

Condition (1) : $D(s) = s^3 + s$, then $D'(s) = 3s^2 + 1$

$$3s^2+1 \overline{)\, s^3 + s \,}\left\lfloor \tfrac{1}{3}s \right.$$

$$\underline{s^3 + \tfrac{1}{3}s}$$

$$\tfrac{2}{3}s \overline{)\, 3s^2 + 1 \,}\left\lfloor \tfrac{3}{2}\cdot 3s = \tfrac{9}{2}s \right.$$

$$\underline{3s^2}$$

$$1 \overline{)\, \tfrac{2}{3}s \,}\left\lfloor \tfrac{2}{3}s \right.$$

$$\underline{\tfrac{2}{3}s}$$

$$\times$$

Therefore, $D(s)$ is Hurwitz polynomical.

Condition (2) : We find that $Z(s)$ has a pair of poles $s = \pm j1$

The partial fraction expansion of $Z(s)$ is

$$Z(s) = \frac{-3s}{s^2+1} + \frac{5}{s}$$

which shows that the residue of the poles at $s = \pm j1$ is negative.

Therefore, $Z(s)$ is not *p.r.f.*

EXAMPLE 10.8 Show that the function $F(s) = \dfrac{(s+2)(s+4)}{(s+1)(s+3)}$ is p.r.

Solution : $F(s) = \dfrac{s^2+6s+8}{s^2+4s+3}$

Condition (1) : We see that all roots of $D(s)$ lie on the negative real axis or left half of s-plane.

Therefore, $D(s)$ is Hurwitz polynomial

Condition (2) : doesnot exist.

Condition (3) : $M_1 = s^2 + 8$, $N_1 = 6s$, $M_2 = s^2 + 3$, $N_2 = 4s$.

$$M_1M_2 - N_1N_2 \geq 0$$

$$(s^2+8)\cdot(s^2+3) - (6\,s)\cdot(4\,s) \geq 0$$

$$s^4 - 13\,s^2 + 24 \geq 0$$

$$A(\omega^2) \equiv \omega^4 + 13\,\omega^2 + 24 \geq 0 \text{ for all } \omega.$$

Therefore, $F(s)$ is *p.r.f.*

EXAMPLE 10.9 Test for positive real

$$F(s) = \frac{2s^2+2s+1}{s^3+2s^2+s+2}$$

Solution : Condition (1) : $M(s) = 2s^2 + 2$, $N(s) = s^3 + s$

$$2s^2+2 \overline{)\, s^3 + s \,}\left\lfloor \tfrac{s}{2} \right.$$

$$\underline{s^3 + s}$$

$$\times$$

The process terminates prematurely. The divisor $(2s^2 + 2)$ is a factor of $M(s)$ and $N(s)$ of the denominator polynomial.

Condition (2) : $D(s) = (2s^2 + 2)\left(\frac{s}{2}+1\right) = (s^2 + 1)(s + 2)$; $D(s)$ has a root on $j\omega$ axis at $s = \pm j$.

We have to determine the residue.

$$F(s) = \frac{As+B}{s^2+1} + \frac{C}{s+2}$$

By the partial fraction expansion, we have

$$F(s) = \frac{s}{s^2+1} + \frac{1}{s+2}$$

So, we can say residue is positive.

Condition (3) :

$$M_1(s) = 2s^2 + 1 \; ; N_1(s) = 2s$$

$$M_2(s) = 2s^2 + 2, \; N_2(s) = s^3 + s$$

$$\begin{aligned} A(\omega^2) &\equiv M_1M_2 - N_1N_2 \geq 0 \\ &= (2s^2 + 1)(2s^2 + 2) - (2s)(s^3 + s) \geq 0 \\ &= (2s^2 + 1)\, 2(s^2 + 1) - 2s^2(s^2 + 1) \geq 0 \\ &= 2(s^2 + 1)(2s^2 + 1 - s^2) \geq 0 = 2(s^2 + 1)^2 \geq 0 \\ &= 2(-\omega^2 + 1)^2 \geq 0 \text{ (By putting } j\omega) \\ &= 2(1 - \omega^2)^2 \geq 0 \text{ for all } \omega \end{aligned}$$

Therefore, $F(s)$ is positive real function.

Note :

When all coefficients of $A(\omega^2)$ are not positive, Sturm's test has to be used, which states that : The number of zeros of $A(x)$ where $x = \omega^2$ in the interval $0 < x < \infty$ is equal to $s_\infty \sim s_0$ where s_0 and s_∞ are the number of sign changes in the set $(A_0, A_1, A_2, ..., A_n)$ evaluated at $x = 0$ and $x = \infty$, respectively.

where

$$A_0(x) = A(x) \; ;$$

$$A_1(x) = \frac{dA_0(x)}{dx}$$

and the subsequent functions are

$$\frac{A_{i-2}x}{A_{i-1}x} = (K_1x - K_0) - \frac{A_ix}{A_{i-1}x}$$

where K_1 and K_0 are constants. The procedure of finding $A_i(x)$ will continue till $A_n(x)$ is a constant. Every time $A(x)$ goes through a zero, the sign of $A(x)$ changes. If there are no zeros of $A(x)$ in the range $0 < x < \infty$, the condition of sturm is satisfied.

EXAMPLE 10.10 **Test whether the following function is a p.r.f. or not.**

$$F(s) = \frac{s^3+s^2+3s+5}{s^2+6s+8}$$

Solution :

Condition (1) : $M(s) = s^2 + 8$, $N(s) = 6s$

Therefore, $D(s)$ is Hurwitz polynomial.

Note : If all the coefficients of $A(\omega^2)$, are positive, then $A(\omega^2)$ is positive for all values of ω.

Condition (2) : There are no poles of given function $F(s)$ lie on $j\omega$-axis. So this condition is ignored.

Condition (3) : $M_1 = s^2 + 5,\ M_2 = s^2 + 8,\ N_1 = s^3 + 3s,\ N_2 = 6s$

$$A(\omega^2) = M_1 M_2 - N_1 N_2 \geq 0$$

$$(s^2 + 5)\,(s^2 + 8) - (s^3 + 3s)\,(6s) \geq 0$$

$$s^4 + 13\,s^2 + 40 - 6\,s^4 - 18\,s^2 \geq 0$$

$$-5\,s^4 - 5\,s^2 + 40 \geq 0$$

$$-5\omega^4 + 5\omega^2 + 40 \geq 0 \text{ (By putting } s = j\omega)$$

Since all coefficients of $A(\omega^2)$ are not positive. Using sturm's test as

$$A_0(x) = -5x^2 + 5x + 40$$

$$A_1(x) = \frac{dA_0(x)}{dx} = -10x + 5$$

$$\frac{A_0(x)}{A_1(x)} = (K_1 x - K_0) - \frac{A_2(x)}{A_1(x)}$$

$$-10x + 5\ \big)\ -5x^2 + 5x + 40\ \big(\ x/2 - 1/4$$

$$\underline{-5x^2 + \frac{5}{2}x}$$

$$\frac{5}{2}x + 40$$

$$\underline{\frac{5}{2}x - \frac{5}{4}}$$

$$\frac{165}{4}$$

Therefore, $$\frac{A_0(x)}{A_1(x)} = \left(\frac{x}{2} - \frac{1}{4}\right) + \frac{\frac{165}{4}}{-10x+5} = \left(\frac{x}{2} - \frac{1}{4}\right) - \frac{\frac{-165}{4}}{-10x+5}$$

$$A_2(x) = -\frac{165}{4}$$

	A_0	A_1	A_2	No. of sign changes
$x = 0$	+	+	–	$s_0 = 1$
$x = \infty$	–	–	–	$s_\infty = 0$

Now, $s_\infty \sim s_0 = 1$

Therefore, $A(\omega^2) \ngeq 0$ for all ω.

Hence, given function is not a *p.r.f.*

(i) Foster – I or Foster series form, as shown in figure 10.1(*a*).
(ii) Foster – II or Foster parallel form, as shown in figure 10.1(*b*).
(iii) Cauer – I form, and
(iv) Cauer – II form, as shown in figure 10.1(*c*).

The Foster I and II forms are obtained by partial fraction expansion of $Z(s)$ and $Y(s)$ respectively, and the Cauer I and II forms by Continued–fraction expansion of immittance function by arranging both numerator and denominator in **descending** and **ascending** order respectively.

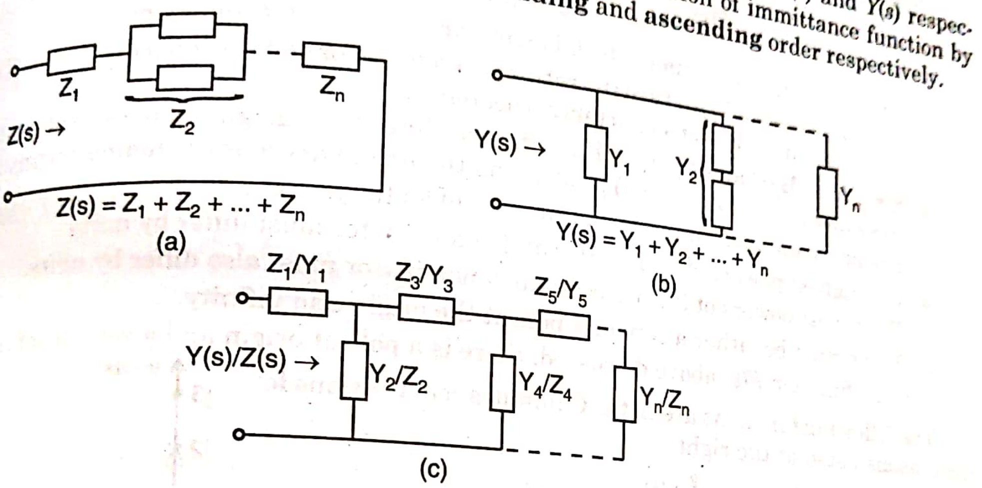

Fig. 10.1. General representation of (a) Foster – I, (b) Foster – II, and (c) Cauer I and II forms

10.4. L-C IMMITTANCE FUNCTION

Consider the impedance $Z(s)$ of a passive one port network. Let us represent $Z(s)$ as

$$Z(s) = \frac{M_1(s)+N_1(s)}{M_2(s)+N_2(s)}$$

Where M_1, M_2 are even parts of the numerator and denominator, and N_1, N_2 are odd parts.

The average power dissipated by the one-port passive network is

$$\text{Average power} = \frac{1}{2}\, Re[Z(j\omega)].\,|I|^2$$

Where I is the input current. For a pure *L-C* (reactive) network, it is known that the power dissipated is zero. Therefore, real part of $Z(j\omega)$ is zero, *i.e.,*

$$Re[Z(j\omega)] = 0$$

Since we known that (from condition of positive real functions)

$$Re[Z(j\omega)] = Ev\,[Z(j\omega)]$$

$$= \frac{M_1(j\omega).M_2(j\omega)-N_1(j\omega)\cdot N_2(j\omega)}{M_2^{\,2}(j\omega)-N_2^{\,2}(j\omega)}$$

In order to $Re[Z(j\omega)] = 0$, *i.e.,*

$M_1(j\omega).\,M_2(j\omega) - N_1(j\omega).\,N_2(j\omega) = 0$

For existence of the function $Z(s)$, either of the following cases must hold:

(a) $M_1 = 0$ and $N_2 = 0$

(b) $M_2 = 0$ and $N_1 = 0$

In case (a), $Z(s)$ is $Z(s) = \dfrac{N_1(s)}{M_2(s)}$

and in case (b), $Z(s) = \dfrac{M_1(s)}{N_2(s)}$

Consider the example of an L-C immittance function given by

$$F(s) = \frac{a_4s^4 + a_2s^2 + a_0}{b_5s^5 + b_3s^3 + b_1s} = \frac{K(s^2+\omega_0^2)(s^2+\omega_2^2)}{s(s^2+\omega_1^2)(s^2+\omega_3^2)}$$

We see from this development the following properties of L-C functions :

1. $Z_{LC}(s)$ and hence $Y_{LC}(s)$ are the ratio of even to odd or odd to even polynomials. (This property is also called as "Foster's Reactance Theorem").
2. Since both $M_i(s)$ and $N_i(s)$ are Hurwitz, they have only imaginary roots, and it follow that the poles and zeros of $Z_{LC}(s)$ or $Y_{LC}(s)$ are on the imaginary axis (including origin).
3. The poles and zeros interlace (or alternate) on the $j\omega$-axis.
4. The highest powers of Numerater and denominator must differ by unity.
5. The lowest powers of Numerator and denominator must also differ by unity.
6. There must be either a zero or a pole at the origin and infinity.

(For the function $F(s)$ above discussed, there is a pole at origin and a zero at infinity).

The following functions are not L-C immittance functions for the reasons listed at the right.

(i) $F(s) = \dfrac{K(s^2+1)(s^2+9)}{(s^2+2)(s^2+10)}$ (1)

(ii) $F(s) = \dfrac{s^5+3s^3+9s}{7s^4+6s^2}$ (2)

(iii) $F(s) = \dfrac{Ks(s^2+4)}{(s^2+1)(s^2+3)}$ (3)

On the other hand, the function $F(s) = \dfrac{2(s^2+1)(s^2+9)}{s(s^2+4)}$, whose pole zero diagram is shown in figure 10.2, is an L-C immittance.

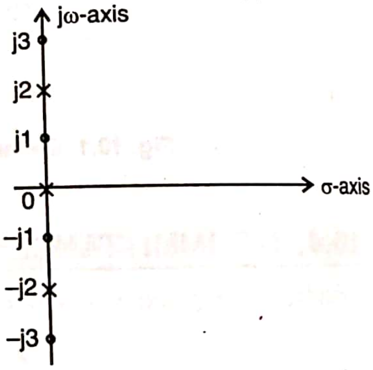

Fig. 10.2. Pole-zero diagram for example 10.11

Note :

Case I : When L and C are in series as shown in figure 10.3(a).

$$Z(s) = sL + \frac{1}{sC} = \frac{s^2LC+1}{sC} = \frac{s^2+\frac{1}{LC}}{\frac{1}{L}s}$$

and $$Y(s) = \frac{1}{Z(s)} = \frac{\frac{1}{L}s}{s^2+\frac{1}{LC}}$$

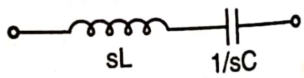

Fig. 10.3(a).

Case II : When L and C are in parallel as shown in figure 10.3(b).

$$Y(s) = sC + \frac{1}{sL} = \frac{s^2LC+1}{sL} = \frac{s^2+\frac{1}{LC}}{\frac{1}{C}s}$$

and $$Z(s) = \frac{1}{Y(s)} = \frac{\frac{1}{C}s}{s^2+\frac{1}{LC}}$$

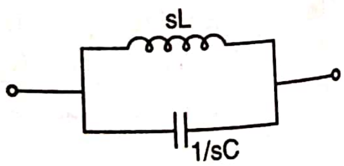

Fig. 10.3(b).

EXAMPLE 10.11 **An impedance function has the pole zero pattern shown in the figure 10.2. If $Z(-2) = -\frac{130}{16}$, synthesize the impedance in**

(a) Foster – I and II forms,

(b) Cauer – I and II forms.

Solution : From the pole zero diagram

$$Z(s) = \frac{K(s^2+1)(s^2+9)}{s(s^2+4)}$$

By putting

$$Z(-2) = -\frac{130}{16} = \frac{K\cdot 5\cdot 13}{-2\cdot 8} \text{ gives } K = 2$$

Therefore,

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

(a) Foster – I Form :

$$\begin{array}{r|l|l} s^3+4s & 2s^4+20s^2+18 & 2s \\ & 2s^4+8s^2 & \\ \hline & 12s^2+18 & \end{array}$$

$$Z(s) = 2s + \frac{12s^2+18}{s(s^2+4)}$$

Using partial fraction expansion,

$$\frac{12s^2+18}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4}$$

$$A = 9/2,\ C = 0 \text{ and } B = \frac{15}{2}$$

Therefore,

$$Z(s) = 2s + \frac{\frac{9}{2}}{s} + \frac{\frac{15}{2}s}{s^2+4}$$

We then obtain the synthesized network in figure 10.4 (*a*).

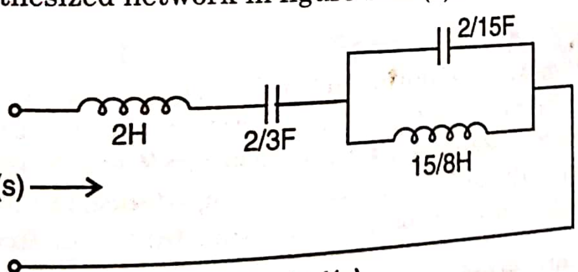

Fig. 10.4(a).

Foster – II form :

$$Y(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$$

Using partial fraction expansion,

$$\frac{s(s^2+4)}{2(s^2+1)(s^2+9)} = \frac{1}{2}\left[\frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+9}\right]$$

$$A = \frac{3}{8},\ B = D = 0,\ C = \frac{5}{8}$$

Therefore,

$$Y(s) = \frac{1}{2}\left[\frac{\frac{3}{8}s}{s^2+1} + \frac{\frac{5}{8}s}{s^2+9}\right]$$

$$= \frac{\frac{3}{16}s}{s^2+1} + \frac{\frac{5}{16}s}{s^2+9}$$

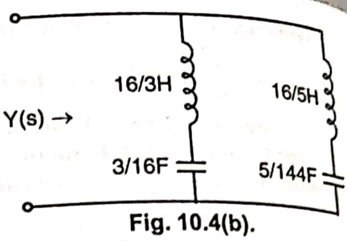

Fig. 10.4(b).

Hence, synthesized network is shown in figure 10.4 (*b*).

(*b*) Cauer – I form :

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)} = \frac{2s^4+20s^2+18}{s^3+4s}$$

The continued fraction expansion is

$$s^3+4s \,\big)\, 2s^4+20s^2+18 \,\big(\, 2s \leftrightarrow Z_1$$
$$\underline{2s^4+8s^2}$$
$$12s^2+18 \,\big)\, s^3+4s \,\big(\, \frac{1}{12}s \leftrightarrow Y_2$$
$$\underline{s^3+\frac{3}{2}s}$$
$$\frac{5}{2}s \,\big)\, 12s^2+18 \,\big(\, \frac{2}{5}\times 12s = \frac{24}{5}s \leftrightarrow Z_3$$
$$\underline{12\,s^2}$$
$$18 \,\big)\, \frac{5}{2}s \,\big(\, \frac{1}{18}\cdot\frac{5}{2}s = \frac{5}{36}s \leftrightarrow Y_4$$
$$\underline{\frac{5}{2}s}$$
$$\times$$

Note :

We know that the quotients of the continued fraction expansion give the elements of the network in Cauer I and II forms. Because the continued fraction expansion always inverts each remainder and divides, the successive quotients alternate between Z and Y and then Z again, as shown in the preceding expansion. If the initial function is an impedance, the first quotient must necessarily be an impedance. When the initial function is an admittance, the first quotient is an admittance.

Therefore, the final synthesized network is shown in figure 10.4(*c*).

Cauer – II form :

$$Z(s) = \frac{2s^4+20s^2+18}{s^3+4s}$$

$$= \frac{18+20s^2+2s^4}{4s+s^3}$$

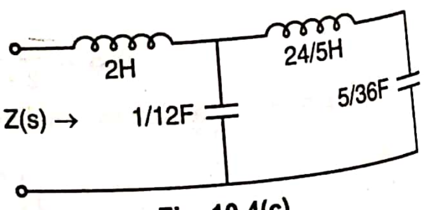

Fig. 10.4(c).

The Continued fraction expansion is

$$4s+s^3 \,\big)\, 18+20s^2+2s^4 \,\big(\, \frac{18}{4s}=\frac{9}{2s} \leftrightarrow Z_1$$
$$18+\frac{9}{2}s^2$$
$$\frac{31}{2}s^2+2s^4 \,\big)\, 4s+s^3 \,\big(\, \frac{2}{31}\cdot\frac{4}{s}=\frac{8}{31s} \leftrightarrow Y_2$$
$$4s+\frac{16}{31}s^3$$
$$\frac{15}{31}s^3 \,\big)\, \frac{31}{2}s^2+2s^4 \,\big(\, \frac{31}{15}\cdot\frac{31}{2s}=\frac{961}{30s} \leftrightarrow Z_3$$
$$\frac{31}{2}s^2$$
$$2s^4 \,\big)\, \frac{15}{31}s^3 \,\big(\, \frac{1}{2}\cdot\frac{15}{31\,s}=\frac{15}{62s} \leftrightarrow Y_4$$
$$\frac{15}{31}s^3$$
$$\times$$

Therefore, the final synthesized network is shown in figure 10.4(*d*).

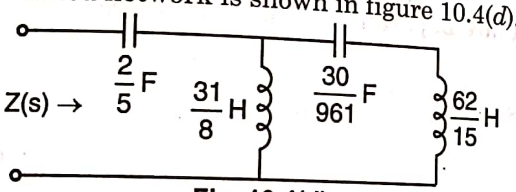

Fig. 10.4(d).

Note :

The number of elements of the synthesized LC network is always same as the higher degree of the two polynomials.

10.5. R-C IMPEDANCE OR R-L ADMITTANCE FUNCTION

The R-C impedance or R-L admittance function has following properties :

1. The poles and zeros lie on the nagative real axis (included origin) of the complex *s*-plane.
2. The poles and zero interlace (or alternate) along the negative real axis.
3. (*a*) The residues of the poles of $Z_{R\text{-}C}(s)$ or $Y_{R\text{-}L}(s)$ must be real and positive.
   (*b*) The residues of the poles of the $Y_{R\text{-}C}(s)$ or $Z_{R\text{-}L}(s)$ are real and negative, however, the residues of the poles of $\frac{Y_{R-C}(s)}{s}$ or $\frac{Z_{R-L}(s)}{s}$ must be real and positive.
4. The singularity nearest to (or at) the origin must be a pole, *i.e.*, function $Z_{R\text{-}C}(s)$ or $Y_{R\text{-}L}(s) \rightarrow \infty$ with $s \rightarrow 0$.
5. The singularity nearest to (or at) the minus infinity ($-\infty$) must be a zero, *i.e.*, function $Z_{RC}(s)$ or $Y_{R\text{-}L}(s) \rightarrow 0$ with $s \rightarrow \infty$.

The following functions are not $Z_{RC}(s)$ or $Y_{R\text{-}L}(s)$ functions for the reasons listed at the right.

(i)
$$F(s) = \frac{(s+1)(s-3)}{(s+4)(s+8)} \qquad (1)$$

$$F(s) = \frac{(s+2)(s+5)}{s(s+1)} \quad (2)$$

(ii)

$$F(s) = \frac{s(s+8)}{(s+1)(s+9)} \quad (4, 5)$$

(iii)

On the otherhand, the function $F(s) = \dfrac{(s+1)(s+4)}{s(s+2)(s+6)}$ is a $Z_{R\text{-}C}(s)$ or $Y_{R\text{-}L}(s)$ function.

Note :

An R-C impedance, $Z_{R\text{-}C}(s)$, also can be realized as an R-L admittance, $Y_{R\text{-}L}(s)$. All the properties of R-L admittances are the same as the properties of R-C impedances. It is therefore important to specify whether a function is to be realized as an R-C impedance or an R-L admittance.

Case I : when R and C are in series as shown in figure 10.5(a).

$$Z(s) = R + \frac{1}{sC} = \frac{sRC+1}{sC} = \frac{s+\frac{1}{RC}}{\frac{1}{R}s}$$

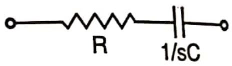

Fig. 10.5(a).

and

$$Y(s) = \frac{\frac{1}{R}s}{s+\frac{1}{RC}}$$

Case II : When R and C are in parallel as shown in figure 10.5(b).

$$Y(s) = \frac{1}{R} + sC = \frac{1+sRC}{R} = \frac{s+\frac{1}{RC}}{\frac{1}{C}}$$

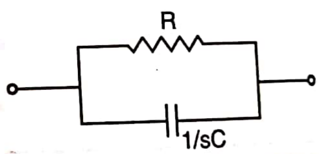

Fig. 10.5(b).

and

$$Z(s) = \frac{\frac{1}{C}}{s+\frac{1}{RC}}$$

EXAMPLE 10.12 **An impedance function is given by**

$$\mathbf{Z(s)} = \frac{(s+1)(s+4)}{s(s+2)(s+5)}$$

Find the R-C representation of (a) Foster – I and II forms, (b) Cauer - I and II forms.

Solution : (a) Foster - I form :

Using partial fraction expansion,

$$Z(s) = \frac{(s+1)(s+4)}{s(s+2)(s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$A = s\cdot Z(s)\Big|_{s=0} = \frac{(1)\cdot(4)}{(2)\cdot(5)} = \frac{2}{5}$$

$$B = (s+2).Z(s)\Big|_{s=-2} = \frac{(-1)\cdot(2)}{(-2)\cdot(3)} = \frac{1}{3}$$

$$C = (s+5).Z(s)\Big|_{s=-5} = \frac{(-4)\cdot(-1)}{(-5)\cdot(-3)} = \frac{4}{15}$$

Therefore,

$$Z(s) = \frac{\frac{2}{5}}{s} + \frac{\frac{1}{3}}{s+2} + \frac{\frac{4}{15}}{s+5}$$

And, synthesized network is shown in figure 10.6(a).

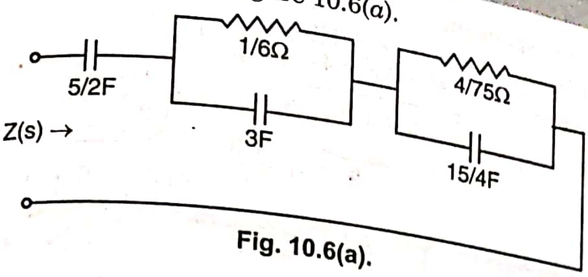

Fig. 10.6(a).

Foster – II form :

$$Y(s) = \frac{s(s+2)(s+5)}{(s+1)(s+4)} = \frac{s^3+7s^2+10s}{s^2+5s+4}$$

$$\frac{Y(s)}{s} = \frac{s^2+7s+10}{s^2+5s+4}$$

$$s^2+5s+4\,)\,s^2+7s+10\,(\,1$$
$$\underline{s^2+5s+4}$$
$$2s+6$$

$$= 1+\frac{2s+6}{(s+1)(s+4)}$$

Using partial fraction expansion,

$$\frac{2s+6}{(s+1)(s+4)} = \frac{A}{s+1}+\frac{B}{s+4}$$

$$A = \left.\frac{2s+6}{s+4}\right|_{s=-1} = \frac{-2+6}{-1+4} = \frac{4}{3}$$

$$B = \left.\frac{2s+6}{s+1}\right|_{s=-4} = \frac{-8+6}{-4+1} = \frac{2}{3}$$

Then,

$$\frac{Y(s)}{s} = 1+\frac{\frac{4}{3}}{s+1}+\frac{\frac{2}{3}}{s+4}$$

or

$$Y(s) = s+\frac{\frac{4}{3}s}{s+1}+\frac{\frac{2}{3}s}{s+4}$$

And, synthesized network is shown in figure 10.6(b).

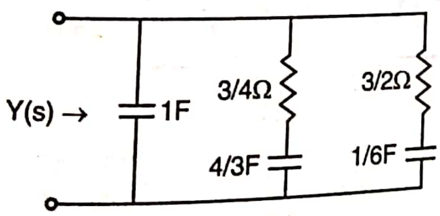

Fig. 10.6(b).

(b) Cauer – I form :

$$Z(s) = \frac{(s+1)(s+4)}{s(s+2)(s+5)} = \frac{s^2+5s+4}{s^3+7s^2+10s}$$

$$Y(s) = \frac{s^3+7s^2+10s}{s^2+5s+4}$$

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The continued fraction expansion is

$$
\begin{array}{l}
s^2+5s+4 \,\big)\, s^3+7s^2+10s \,\big(\, s \leftrightarrow Y_1 \\
\qquad\qquad\quad s^3+5s^2+4s \\
\hline
\qquad\qquad 2s^2+6s \,\big)\, s^2+5s+4 \,\big(\, \dfrac{1}{2} \leftrightarrow Z_2 \\
\qquad\qquad\qquad\qquad s^2+3s \\
\hline
\qquad\qquad\qquad\qquad 2s+4 \,\big)\, 2s^2+6s \,\big(\, s \leftrightarrow Y_3 \\
\qquad\qquad\qquad\qquad\qquad\quad 2s^2+4s \\
\hline
\qquad\qquad\qquad\qquad\qquad\quad 2s \,\big)\, 2s+4 \,\big(\, 1 \leftrightarrow Z_4 \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad 2s \\
\hline
\qquad\qquad\qquad\qquad\qquad\qquad\quad 4 \,\big)\, 2s \,\big(\, \dfrac{1}{2}s \leftrightarrow Y_5 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2s \\
\hline
\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times
\end{array}
$$

Therefore, the final synthesized network is shown in figure 10.6(*c*).

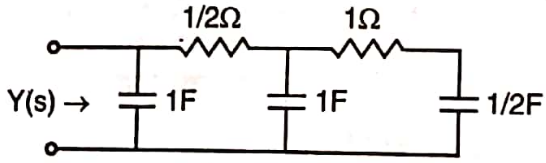


Fig. 10.6(c).

Cauer – II form :

$$Z(s) = \frac{s^2+5s+4}{s^3+7s^2+10s} = \frac{4+5s+s^2}{10s+7s^2+s^3}$$

The continued fraction expansion is

$$
\begin{array}{l}
10s+7s^2+s^3 \,\big)\, 4+5s+s^2 \,\big(\, \dfrac{4}{10s} = \dfrac{2}{5s} \leftrightarrow Z_1 \\
\qquad\qquad\quad 4+\dfrac{14}{5}s+\dfrac{2}{5}s^2 \\
\hline
\qquad \dfrac{11}{5}s+\dfrac{3}{5}s^2 \,\big)\, 10s+7s^2+s^3 \,\big(\, \dfrac{5}{11}\cdot 10 = \dfrac{50}{11} \leftrightarrow Y_2 \\
\qquad\qquad\qquad 10s+\dfrac{30}{11}s^2 \\
\hline
\qquad\qquad \dfrac{47}{11}s^2+s^3 \,\big)\, \dfrac{11}{5}s+\dfrac{3}{5}s^2 \,\big(\, \dfrac{11}{47}\cdot\dfrac{11}{5s} = \dfrac{121}{235s} \leftrightarrow Z_3 \\
\qquad\qquad\qquad\qquad \dfrac{11}{5}s+\dfrac{121}{235}s^2 \\
\hline
\qquad\qquad\qquad \dfrac{20}{235}s^2 \,\big)\, \dfrac{47}{11}s^2+s^3 \,\big(\, \dfrac{235}{20}\cdot\dfrac{47}{11} = \dfrac{2209}{44} \leftrightarrow Y_4 \\
\qquad\qquad\qquad\qquad\quad \dfrac{47}{11}s^2 \\
\hline
\qquad\qquad\qquad\qquad\quad s^3 \,\big)\, \dfrac{20}{235}s^2 \,\big(\, \dfrac{20}{235s} = \dfrac{4}{47s} \leftrightarrow Z_5 \\
\qquad\qquad\qquad\qquad\qquad \dfrac{20}{235}s^2 \\
\hline
\qquad\qquad\qquad\qquad\qquad \times
\end{array}
$$

Therefore, the synthesized network is shown in figure 10.6(d).

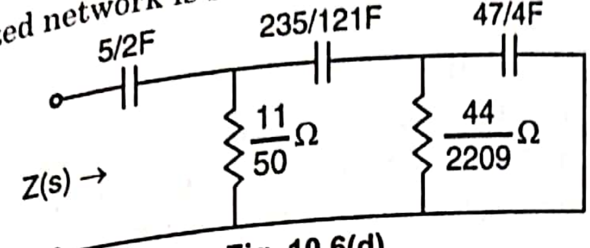


Fig. 10.6(d).

10.6. R-L IMPEDANCE OR R-C ADMITTANCE FUNCTION

R-L impedance or R-C admittance function has following properties :

1. The poles and zeros lie on the negative real axis (included origin) of the complex s-plane.
2. The poles and zeros interlace along the negative real axis.
3. (a) The residues of the poles of $Z_{R\text{-}L}(s)$ or $Y_{R\text{-}C}(s)$ are real and negative. However, the residues of the poles of $\frac{Z_{R-L}(s)}{s}$ or $\frac{Y_{R-C}(s)}{s}$ must be real and positive.

   (b) The residues of the poles of $Y_{R\text{-}L}(s)$ or $Z_{R\text{-}C}(s)$ must be real and positive.
4. The singularity nearest to (or at) the origin must be a zero, *i.e.*, function $Z_{R\text{-}L}(s)$ or $Y_{R\text{-}C}(s) \to 0$ with $s \to 0$.
5. The singularity nearest to (or at) the minus infinity $(-\infty)$ must be a pole *i.e.*, the function $Z_{R\text{-}L}(s)$ or $Y_{R\text{-}C}(s) \to \infty$ with $s \to \infty$.

The following functions are not $Z_{R\text{-}L}(s)$ or $Y_{R\text{-}C}(s)$ functions for the reasons listed at the right.

(i) $$F(s) = \frac{(s+4)(s+8)}{(s+2)(s-5)} \qquad (1)$$

(ii) $$F(s) = \frac{s(s+1)}{(s+2)(s+5)} \qquad (2)$$

(iii) $$F(s) = \frac{(s+1)(s+8)(s+12)}{s(s+2)(s+10)} \qquad (4,5)$$

On the other hand, the function $F(s) = \frac{s(s+2)(s+6)}{(s+1)(s+4)}$ is a $Z_{R\text{-}L}(s)$ or $Y_{R\text{-}C}(s)$ function.

Note :

An R-L impedance, $Z_{R\text{-}L}(s)$, also can be realized as an R-C admittance, $Y_{R\text{-}C}(s)$. All the properties of R-C admittances are the same as the properties of R-L impedances. It is therefore important to specify whether a function is to be realized as an R-L impedance or an R-C admittance.

Case I : When R and L are in series as shown in figure 10.7(a).

$$Z(s) = R + sL = L\left(s + \frac{R}{L}\right)$$

Fig. 10.7(a).

and

$$Y(s) = \frac{\frac{1}{L}}{s + \frac{R}{L}}$$

Case II : When R and L are in parallel as shown in figure 10.7(b).

$$Y(s) = \frac{1}{R} + \frac{1}{sL} = \frac{sL+R}{sRL} = \frac{1}{R} \cdot \frac{s + \frac{R}{L}}{s}$$

Fig. 10.7(b).

and

$$Z(s) = \frac{Rs}{\ldots}$$

EXAMPLE 10.13 An impedance function is given by

$$Z(s) = \frac{s(s+2)(s+5)}{(s+1)(s+4)}$$

Find the R-L representation of (*a*) Foster - I and II forms (*b*) Cauer-I and II forms.

Solution : (*a*) Foster – I form : Since we know that the residues of poles of $Z_{R\text{-}L}(s)$ are real and negative. So we determine the residues of $\frac{Z(s)}{s}$ as :

$$\frac{Z(s)}{s} = \frac{(s+2)(s+5)}{(s+1)(s+4)}$$

$$s^2+5s+4 \,\overline{)\, s^2+7s+10 \,}\, 1$$
$$\underline{s^2+5s+4}$$
$$2s+6$$

$$= 1+\frac{2s+6}{(s+1)(s+4)}$$

Using partial fraction expansion,

$$\frac{Z(s)}{s} = 1+\frac{\frac{4}{3}}{s+1}+\frac{\frac{2}{3}}{s+4}$$

or,
$$Z(s) = s+\frac{\frac{4}{3}s}{s+1}+\frac{\frac{2}{3}s}{s+4}$$

Therefore, synthesized network is shown in figure 10.8(*a*).

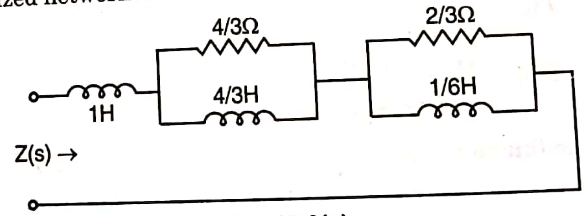


Fig. 10.8(a).

Foster – II form : $Y(s) = \frac{(s+1)(s+4)}{s(s+2)(s+5)}$

Using partial fraction expansion, we have

$$Y(s) = \frac{\frac{2}{5}}{s}+\frac{\frac{1}{3}}{s+2}+\frac{\frac{4}{15}}{s+5}$$

Therefore, synthesized network is shown in figure 10.8(*b*).

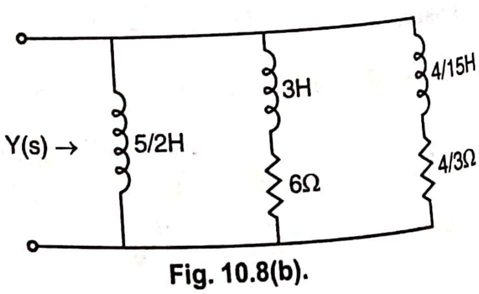


Fig. 10.8(b).

(*b*) Cauer – I form :

$$Z(s) = \frac{s^3+7s^2+10s}{s^2+5s+4}$$

As found in previous example,

$$Z_1 = s,\ Y_2 = \frac{1}{2},$$

$$Z_3 = s,\ Y_4 = 1,\ Z_5 = \frac{1}{2}s$$

Therefore, the synthesized network is shown in figure 10.8(c).

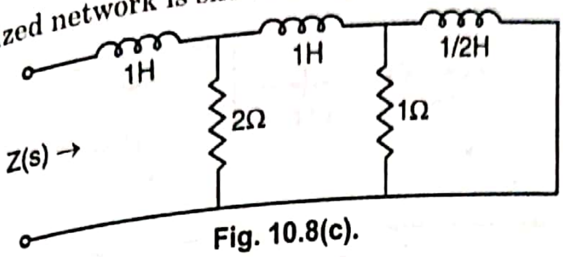


Fig. 10.8(c).

r - II form :

$$Z(s) = \frac{s^3 + 7s^2 + 10s}{s^2 + 5s + 4} = \frac{10s + 7s^2 + s^3}{4 + 5s + s^2}$$

$$Y(s) = \frac{4 + 5s + s^2}{10s + 7s^2 + s^3}$$

As found in previous example,

$$Y_1 = \frac{2}{5s},\ Z_2 = \frac{50}{11},\ Y_3 = \frac{121}{235s},\ Z_4 = \frac{2209}{44},\ Y_5 = \frac{4}{47s}$$

Therefore, the synthesized network is shown in figure 10.8(*d*).

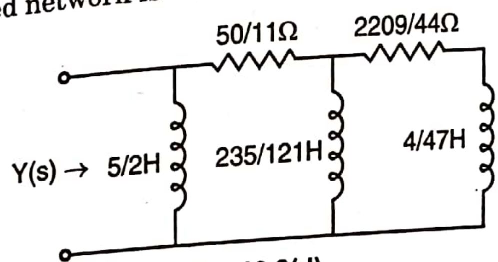


Fig. 10.8(d).

OTHER EXAMPLES

AMPLE 10.14 Which of the following functions are L-C driving point impedances ? Why?

$$Z_1(s) = \frac{s(s^2+4)(s^2+16)}{(s^2+9)(s^2+25)}$$

$$Z_2(s) = \frac{(s^2+1)(s^2+8)}{s(s^2+4)}$$

And, synthesize the realizable impedances in a Foster and a Cauer forms.

tion : $Z_1(s)$ is not *L-C* driving point impedance, since poles and zeros are not interlacing on the axis. On the other hand, $Z_2(s)$ is *L-C* driving point impedance.

er - I form :

$$Z_2(s) = \frac{(s^2+1)(s^2+8)}{s(s^2+4)}$$

Using partial fraction expansion,

$$Z_2(s) = s + \frac{2}{s} + \frac{3s}{s^2+4}$$

Therefore, the synthesized network in Foster – I form is shown in figure 10.9.

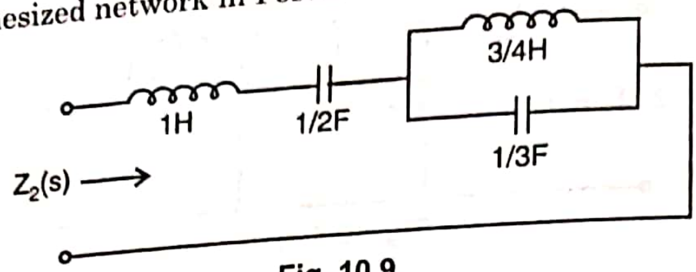


Fig. 10.9.

Cauer-II Form :

$$Z_2(s) = \frac{8+9s^2+s^4}{4s+s^3}$$

The continued fraction expansion is

$$4s + s^3 \,)\, 8 + 9s^2 + s^4 \,(\, \frac{2}{s} \leftrightarrow Z_1$$

$$8 + 2s^2$$

$$7s^2 + s^4 \,)\, 4s + s^3 \,(\, \frac{4}{7s} \leftrightarrow Y_2$$

$$4s + \frac{4}{7}s^3$$

$$\frac{3}{7}s^3 \,)\, 7s^2 + s^4 \,(\, \frac{7}{3}\cdot\frac{7}{s} = \frac{49}{3s} \leftrightarrow Z_3$$

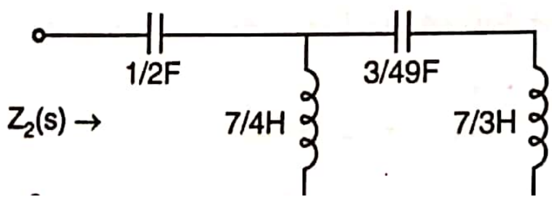
$$7s^2$$

$$s^4 \,)\, \frac{3}{7}s^3 \,(\, \frac{3}{7s} \leftrightarrow Y4$$

$$\frac{3}{7}s^3$$

$$\times$$

Therefore, the synthesized network in Cauer – II form is shown in figure 10.10.



$$Z_{in} = \frac{2s^2+2}{s^3+2s^2+2s+2}$$

$$= \frac{1}{1+\dfrac{1\cdot(s^3+2s)}{(2s^2+2)}}$$

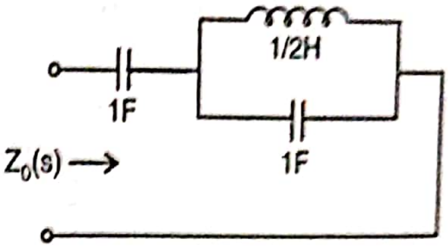


Fig. 10.11(b).

Therefore,

$$Z_0 = \frac{2s^2+2}{s^3+2s} = \frac{2s^2+2}{s(s^2+2)}$$

Using partial fraction expansion,

$$Z_0(s) = \frac{1}{s}+\frac{s}{s^2+2}$$

Therefore, the synthesized network in Foster – *I* form is shown in figure 10.11(*b*).

EXAMPLE 10.16 For the network shown in figure 10.12(*a*). Find Y when

$$\frac{V_2}{V_0} = \frac{1}{2+Y}$$

$$= \frac{s(s^2+3)}{2s^3+s^2+6s+1}$$

Synthesize *Y* as the L-C admittance.

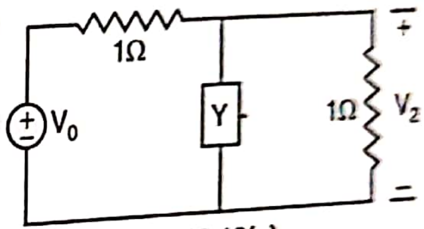


Fig. 10.12(a).

Solution :

$$\frac{V_2}{V_0} = \frac{1}{2+Y} = \frac{1}{2+\dfrac{s^2+1}{s(s^2+3)}}$$

Therefore,

$$Y(s) = \frac{s^2+1}{s(s^2+3)}$$

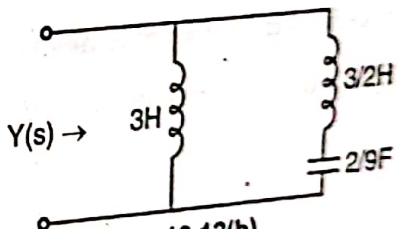


Fig. 10.12(b).

Using partial fraction expansion,

$$Y(s) = \frac{\frac{1}{3}}{s}+\frac{\frac{2}{3}s}{s^2+3}$$

Hence, Synthesized network is shown in figure 10.12(*b*).

EXAMPLE 10.17 Synthesize *Z*(*s*) in Cauer – II form.

$$Z(s) = \frac{8s^3+10s}{s^4+6s^2+5}$$

Solution :

$$Z(s) = \frac{8s^3+10s}{s^4+6s^2+5}$$

$$Z(s) = \frac{10s+8s^3}{5+6s^2+s^4}$$

So,

$$Y(s) = \frac{5+6s^2+s^4}{10s+8s^3}$$

The continued fraction expansion is

$$10s + 8s^3 \overline{)\,5 + 6s^2 + s^4\,}\left(\frac{1}{2s} \leftrightarrow Y_1\right.$$
$$5 + 4s^2$$
$$2s^2 + s^4 \overline{)\,10s + 8s^3\,}\left(\frac{5}{s} \leftrightarrow Z_2\right.$$
$$10s + 5s^3$$
$$3s^3 \overline{)\,2s^2 + s^4\,}\left(\frac{2}{3s} \leftrightarrow Y_3\right.$$
$$2s^2$$
$$s^4 \overline{)\,3s^3\,}\left(\frac{3}{s} \leftrightarrow Z_4\right.$$
$$3s^3$$
$$\times$$

Therefore, the required synthesized network is shown in figure 10.13.

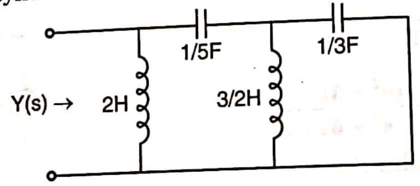


Fig. 10.13.

EXAMPLE 10.18 **Synthesize the network, if** $Z(s) = \dfrac{s^5 + 5s^3 + 4s}{s^4 + 3s^2 + 1}$ **as Cauer – I form.**

Solution : The continued fraction exapnsion is

$$s^4 + 3s^2 + 1 \overline{)\,s^5 + 5s^3 + 4s\,}\left(s \leftrightarrow Z_1\right.$$
$$s^5 + 3s^3 + s$$
$$2s^3 + 3s \overline{)\,s^4 + 3s^2 + 1\,}\left(\frac{s}{2} \leftrightarrow Y_2\right.$$
$$s^4 + \frac{3}{2}s^2$$
$$\frac{3}{2}s^2 + 1 \overline{)\,2s^3 + 3s\,}\left(\frac{4}{3}s \leftrightarrow Z_3\right.$$
$$2s^3 + \frac{4}{3}s$$
$$\frac{5}{3}s \overline{)\,\frac{3}{2}s^2 + 1\,}\left(\frac{3}{5}\cdot\frac{3}{2}s = \frac{9}{10}s \leftrightarrow Y_4\right.$$
$$\frac{3}{2}s^2$$
$$1 \overline{)\,\frac{5}{3}s\,}\left(\frac{5}{3}s \leftrightarrow Z_5\right.$$
$$\frac{5}{3}s$$
$$\times$$

The synthesized network is shown in figure 10.14.

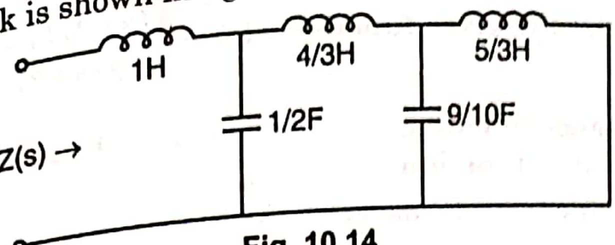


Fig. 10.14.

EXAMPLE 10.19 **Synthesize the given impedance function in Foster – I and II forms.**

$$Z(s) = \frac{8(s^2+4)(s^2+25)}{s(s^2+16)}$$

Solution : Foster – I form : Using partial fraction expansion,

$$Z(s) = 8s + \frac{50}{s} + \frac{54s}{s^2+16}$$

Therefore, the synthesized network is shown in figure 10.15(*a*).

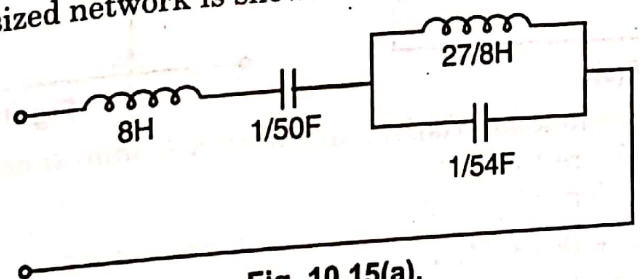


Fig. 10.15(a).

Foster – II form :

$$Y(s) = \frac{s(s^2+16)}{8(s^2+4)(s^2+25)}$$

Using partial fraction expansion,

$$Y(s) = \frac{s(s^2+16)}{8(s^2+4)(s^2+25)} = \frac{As}{s^2+4} + \frac{Bs}{s^2+25} \qquad ...(i)$$

Comparing coefficients of s^3 : $A + B = \frac{1}{8}$ ...(*ii*)

Comparing coefficients of s : $25A + 4B = 2$

From equations (*i*) and (*ii*), we have

$$A = \frac{1}{14},\ B = \frac{3}{56}$$

Therefore,

$$Y(s) = \frac{\frac{1}{14}s}{s^2+4} + \frac{\frac{3}{56}s}{s^2+25}$$

Synthesized network is shown in figure 10.15(*b*).

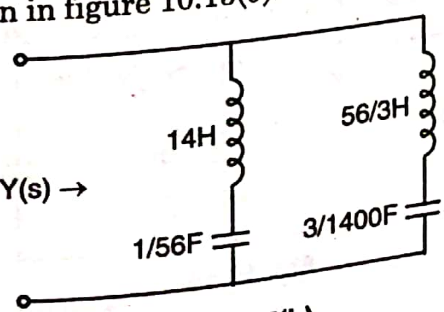


Fig. 10.15(b).

EXAMPLE 10.20 Synthesize the given function $F(s) = \dfrac{3(s+2)(s+4)}{s(s+3)}$ in a Foster and a Cauer forms, if

(i) If $F(s)$ is an impedance function.

(ii) If $F(s)$ is an admittance function.

Solution : (A) Using partial fraction expansion,

$$F(s) = 3 + \frac{8}{s} + \frac{1}{s+3}$$

(i) If $F(s)$ is an impedance function $Z(s)$, it must be an R-C impedance function and it is realized in the Foster – I form as in figure 10.16(*a*).

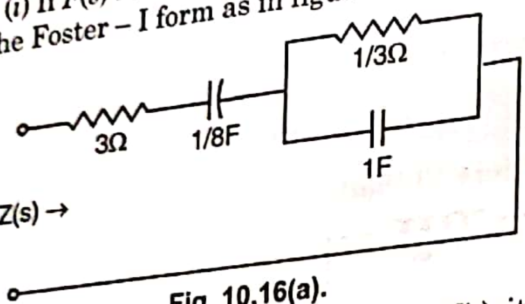


Fig. 10.16(a).

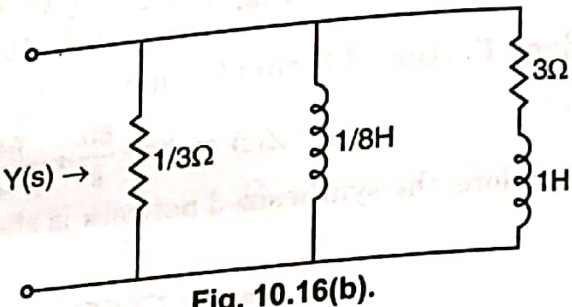


Fig. 10.16(b).

(ii) If $F(s)$ is an admittance function $Y(s)$, it must be an R-L admittance function and it is realized in the Foster–II form as in figure 10.16(*b*).

(*B*) Using continued fraction expansion,

$$s^2 + 3s \,\big)\, 3s^2 + 18s + 24 \,\big(\, 3$$
$$\underline{3s^2 + 9s}$$
$$9s + 24 \,\big)\, s^2 + 3s \,\big(\, \frac{s}{9}$$
$$\underline{s^2 + \frac{24}{9}s}$$
$$\frac{1}{3}s \,\big)\, 9s + 24 \,\big(\, 27$$
$$\underline{9s}$$
$$24 \,\big)\, \frac{1}{3}s \,\big(\, \frac{s}{72}$$
$$\underline{\frac{1}{3}s}$$
$$\times$$

(*i*) If $F(s)$ is an impedance function, then

$$Z_1 = 3,\ Y_2 = \frac{s}{9},\ Z_3 = 27,\ Y_4 = \frac{s}{72}$$

i.e., $F(s)$ is an R-C impedance function and it is realized as in figure 10.16(*c*).

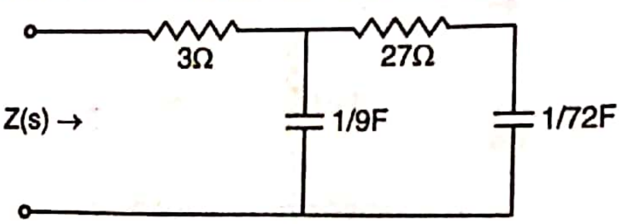


Fig. 10.16(c).

(ii) If $F(s)$ is an admittance function, then

$$Y_1 = 3,\quad Z_2 = \frac{s}{9},\ Y_3 = 27,\ Z_4 = \frac{s}{72}$$

i.e., $F(s)$ is an R-L admittance function and it is realized as in figure 10.16(d).

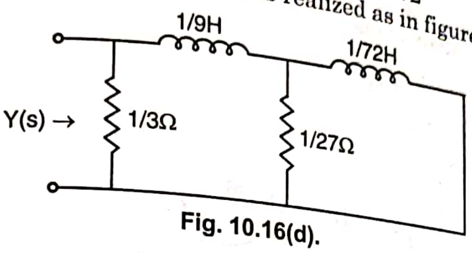


Fig. 10.16(d).

EXAMPLE 10.21 If $Z(s) = \dfrac{(s+4)(s+6)}{(s+3)(s+5)}$, find Cauer–I and II forms of $Z(s)$ (if possible).

Solution : (i) Cauer–I form :

$$Z(s) = \frac{s^2+10s+24}{s^2+8s+15}$$

Continued fraction expansion is given as

$$s^2+8s+15\ \overline{)\ s^2+10s+24\ }\ 1 \leftrightarrow Z_1$$
$$\underline{s^2+8s+15}$$
$$2s+9\ \overline{)\ s^2+8s+15\ }\ \frac{s}{2} \leftrightarrow Y_2$$
$$\underline{s^2+\frac{9}{2}s}$$
$$\frac{7}{2}s+15\ \overline{)\ 2s+9\ }\ \frac{4}{7} \leftrightarrow Z_3$$
$$\underline{2s+\frac{60}{7}}$$
$$\frac{3}{7}\ \overline{)\ \frac{7}{2}s+15\ }\ \frac{49}{6}s \leftrightarrow Y_4$$
$$\underline{\frac{7}{2}s}$$
$$15\ \overline{)\ \frac{3}{7}\ }\ \frac{1}{35} \leftrightarrow Z_5$$
$$\underline{\frac{3}{7}}$$
$$\times$$

The synthesized network in Cauer – I form is shown in figure 10.17(a).

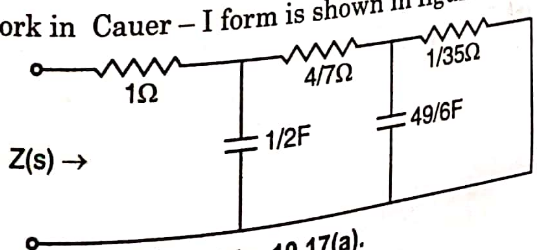


Fig. 10.17(a).

(ii) Cauer–II form :

Now, $$Y(s) = \frac{15+8s+s^2}{24+10s+s^2}$$

The continued fraction expansion is

$$24+10s+s^2 \,\big)\, 15+8s+s^2 \,\big(\, \frac{5}{8} \leftrightarrow Y_1$$
$$15+\frac{25}{4}s+\frac{5}{8}s^2$$
$$\frac{7}{4}s+\frac{3}{8}s^2 \,\big)\, 24+10s+s^2 \,\big(\, \frac{96}{7s} \leftrightarrow Z_2$$
$$24+\frac{36}{7}s$$
$$\frac{34}{7}s+s^2 \,\big)\, \frac{7}{4}s+\frac{3}{8}s^2 \,\big(\, \frac{49}{136} \leftrightarrow Y_3$$
$$\frac{7}{4}s+\frac{49}{136}s^2$$
$$\frac{1}{68}s^2 \,\big)\, \frac{34}{7}s+s^2 \,\big(\, \frac{2312}{7s} \leftrightarrow Z_4$$
$$\frac{34}{7}s$$
$$s^2 \,\big)\, \frac{1}{68}s^2 \,\big(\, \frac{1}{68} \leftrightarrow Y_5$$
$$\frac{1}{68}s^2$$
$$\times$$

The synthesized network in Cauer-II form is shown in figure 10.17(*b*).

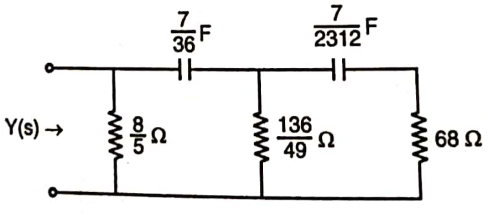


Fig. 10.17(b).

EXAMPLE 10.22 **Realize the R-C admittance in Cauer – I and Foster – II forms.**

$$Y(s) = \frac{s^2+7s+6}{s+2}$$

Solution : (a) Cauer – I form : The continued fraction expansion is

$$\begin{array}{l} s+2\,\big)\, s^2+7s+6 \,\big(\, s \leftrightarrow Y_1 \\ \qquad\quad s^2+2s \\ \hline \qquad 5s+6\,\big)\, s+2 \,\big(\, \frac{1}{5} \leftrightarrow Z_2 \\ \qquad\qquad s+\frac{6}{5} \\ \hline \qquad\qquad \frac{4}{5}\,\big)\, 5s+6 \,\big(\, \frac{25}{4}s \leftrightarrow Y_3 \\ \qquad\qquad\qquad 5s \\ \hline \qquad\qquad\qquad 6\,\big)\, \frac{4}{5} \,\big(\, \frac{2}{15} \leftrightarrow Z_4 \\ \qquad\qquad\qquad\quad \frac{4}{5} \\ \hline \qquad\qquad\qquad\quad \times \end{array}$$

Therefore, the synthesized network is shown in figure 10.18(*a*).

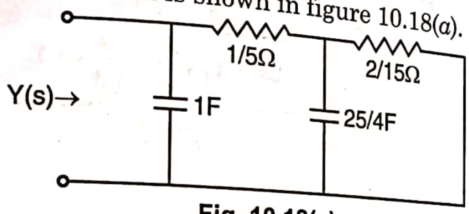


Fig. 10.18(a).

(b) Foster – II form :

$$Y(s) = \frac{s^2+7s+6}{s+2} = s+\frac{5s+6}{s+2} = s+3+\frac{2s}{s+2}$$

And, the synthesized network is shown in figure 10.18(*b*).

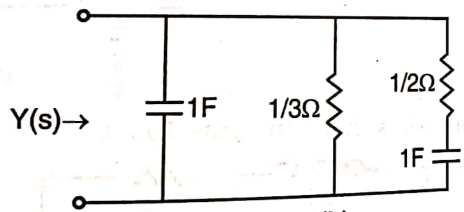


Fig. 10.18(b).

EXAMPLE 10.23 Show that function $F(s) = \dfrac{s\,(3s+8)}{(s+1)\,(s+3)}$ represents an R-L impedance. Realize the impedance in Foster – I form.

Solution :

$$F(s) = \frac{3s\left(s+\frac{8}{3}\right)}{(s+1)\,(s+3)}$$

Zeros at $s = 0$, $s = -\frac{8}{3}$ interlaced by poles at $s = -1$, $s = -3$. The first singularity at the origin. Hence $F(s)$ is a *R-C* admittance or *R-L* impedance. Hence,

$$Z_{RL}(s) = Y_{RC}(s) = \frac{3s\left(s+\frac{8}{3}\right)}{(s+1)\,(s+3)} = \frac{\frac{5}{2}s}{s+1} + \frac{\frac{s}{2}}{s+3}$$

Therefore, given function is an R-L impedance function, and synthesized network in Foster – I form is shown in figure 10.19.

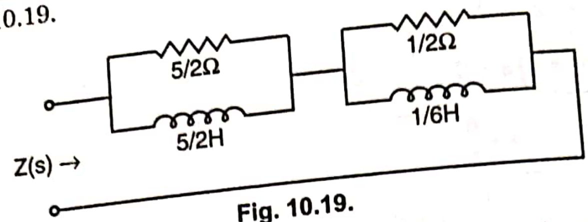


Fig. 10.19.

EXAMPLE 10.24 **(Synthesis of R-L-C function). Synthesize by continued fractions the function**

$$Y(s) = \frac{s^3+2s^2+3s+1}{s^3+s^2+2s+1}$$

Solution :

$$s^3+s^2+2s+1 \,\big)\, s^3+2s^2+3s+1 \,\big(\, 1 \leftrightarrow Y_1$$
$$\underline{s^3+s^2+2s+1}$$
$$s^2+s \,\big)\, s^3+s^2+2s+1 \,\big(\, s \leftrightarrow Z_2$$
$$\underline{s^3+s^2}$$
$$2s+1 \,\big)\, s^2+s \,\big(\, \frac{s}{2} \leftrightarrow Y_3$$
$$\underline{s^2+\frac{s}{2}}$$
$$\frac{s}{2} \,\big)\, 2s+1 \,\big(\, 4 \leftrightarrow Z_4$$
$$\underline{2s}$$
$$1 \,\big)\, \frac{s}{2} \,\big(\, \frac{s}{2} \leftrightarrow Y_5$$
$$\underline{\frac{s}{2}}$$
$$\times$$

Therefore the synthesized network is shown in figure 10.20.

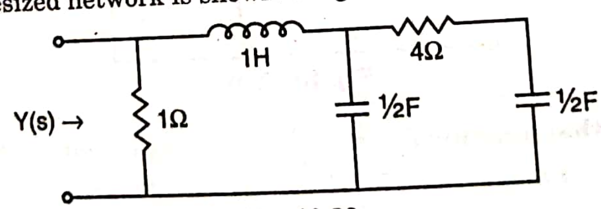


Fig. 10.20.

EXAMPLE 10.25 **Test whether the following polynomial is Hurwitz.**

(I.P. Univ., 2001)

$$P(s) = s^3 + 2s^2 + s + 2$$

Solution : Condition (1) : is satisfied (since all coefficients of $P(s)$, *i.e.*, a_i are positive).

Condition (2) : Even and odd parts of $P(s)$ are

$$M(s) = 2s^2 + 2$$
$$N(s) = s^3 + s$$

So continued fraction expansion of $\Psi(s) = \dfrac{N(s)}{M(s)}$ is given as

$$2s^2+2\overline{)s^3+s}\left(\frac{1}{2}s\right.$$
$$\underline{s^3+s}$$
$$\times$$

We see that the division has been terminated prematurely. Thus $(2s^2+2)$ factor is common in $M(s)$ and $N(s)$. As

$$P(s) = (2s^2+2)\left(1+\frac{1}{2}s\right) \equiv W(s).\, P_1(s)$$

We know that the term $P_1(s)$ is Hurwitz and $W(s)$ is also Hurwitz. Therefore, $P(s)$ is Hurwitz polynomial.

Alternatively :
$$P(s) = (2\,s^2+2)\left(1+\frac{1}{2}s\right) = 2\,(s^2+1)\left(1+\frac{1}{2}s\right)$$
$$= (s+j\omega)\,(s-j\omega)\,(2+s)$$

We see that No roots of $P(s)$ lie on the right half of s-plane. Therefore, $P(s)$ is Hurwitz polynomial.

EXAMPLE 10.26 **Test whether the given function $F(s)$ represents a p.r.f.**

$$F(s) = \frac{s+3}{s^2+5s+1}$$

(I.P. Univ., 2001)

Solution : Condition (1) : $M(s) = s^2+1$, $N(s) = 5s$

$$5s\overline{)s^2+1}\left(\frac{1}{5}s\right.$$
$$\underline{s^2}$$
$$1\overline{)5s}\left(5s\right.$$
$$\underline{5s}$$
$$\times$$

All quotients are positive. Therefore, Denominator of $F(s)$ is Hurwitz polynomial.

Condition (2) : There are no poles of given function $F(s)$ lie on $j\omega$-axis, so this condition does not exist.

Condition (3) : $M_1 = 3$, $M_2 = s^2+1$, $N_1 = s$, $N_2 = 5s$

$$A(\omega^2) \equiv M_1M_2 - N_1\,N_2 \geq 0$$
$$3\,(s^2+1) - s\cdot(5s) \geq 0$$
$$3\,s^2+3-5s^2 \geq 0$$
$$-2s^2+3 \geq 0$$
$$2\omega^2+3 \geq 0 \qquad \text{(By putting } s=j\omega\text{)}$$

Hence, $A(\omega) \geq 0$ for all ω.

Therefore, given function $F(s)$ is a **positive real function.**

EXAMPLE 10.27 **Realize the following RC driving point impedance function in (i) Foster-I form, (ii) Cauer-II form.** (U.P.T.U., 2002, 2003(C.O.))

$$Z(s) = \frac{s^2+4s+3}{s^2+2s}$$

Solution : (i) Foster-I form :

Using partial fraction expansion, we have

$$Z(s) = \frac{(s+1)(s+3)}{s(s+2)} = 1+\frac{A}{s}+\frac{B}{s+2} = 1+\frac{3}{2s}+\frac{\frac{1}{2}}{s+1}$$

Therefore, the synthesized network is shown in figure 10.21(a).

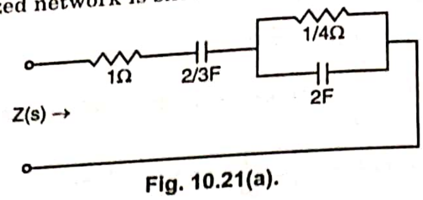


Fig. 10.21(a).

(ii) Cauer-II form :

$$Z(s) = \frac{3+4s+s^2}{2s+s^2}$$

The continued fraction expansion is

$$2s+s^2 \,\big)\, 3+4s+s^2 \,\big(\, \frac{3}{2s} \leftrightarrow Z_1$$

$$3+\frac{3}{2}s$$

$$\frac{5}{2}s+s^2 \,\big)\, 2s+s^2 \,\big(\, \frac{2}{5/2} = \frac{4}{5} \leftrightarrow Y_2$$

$$2s+\frac{4}{5}s^2$$

$$\frac{1}{5}s^2 \,\big)\, \frac{5}{2}s+s^2 \,\big(\, \frac{5}{s}\cdot\frac{5}{2} = \frac{25}{2s} \leftrightarrow Z_3$$

$$\frac{5}{2}s$$

$$s^2 \,\big)\, \frac{1}{5}s^2 \,\big(\, \frac{1}{5} \leftrightarrow Y_4$$

$$\frac{1}{5}s^2$$

$$\times$$

Therefore, the synthesized network is shown in figure 10.21(b).

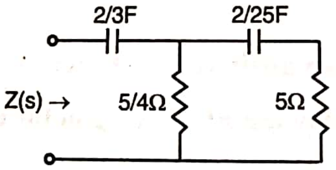


Fig. 10.21(b).

EXAMPLE 10.28 **Determine the range of β such that the polynomical**

$$P(s) = s^4 + s^3 + 4s^2 + \beta s + 3 \text{ is Hurwitz.}$$

(I.P. Univ., 2000)

Solution : (1) All coefficients of $P(s)$ must be positive, therefore $\beta > 0$.

(2) $M(s) = s^4 + 4s^2 + 3$, $N(s) = s^3 + \beta s$

Continued fraction expansion of $\frac{M(s)}{N(s)}$ is given as

$$s^3+\beta s \overline{)\, s^4+4s^2+3\,}\, (s$$
$$\underline{s^4+\beta s^2}$$
$$(4-\beta)\,s^2+3 \overline{)\, s^3+\beta s\,}\, \left(\frac{s}{4-\beta}\right.$$
$$\underline{s^3+\frac{3}{4-\beta}s}$$
$$\left(\beta-\frac{3}{4-\beta}\right)s$$

All quotients must be positive, *i.e.*

(i) $4-\beta>0$ or $\beta<4$

(ii) $\beta-\dfrac{3}{4-\beta}>0$ or $-\beta^2+4\beta-3>0$

or $\beta^2-4\beta+3<0$

or $(\beta-1)(\beta-3)<0$

i.e. $1<\beta<3$

Therefore the required range of β is

$$1<\beta<3$$

EXAMPLE 10.29 **Which of the following function is R-L driving point impedance? Why? Synthesize the realizable impedance in Foster's first form [F$_1$].**

$$F_1(s)=\frac{(s+1)(s+8)}{(s+2)(s+4)}, \qquad F_2(s)=\frac{(s+2)(s+4)}{(s+3)(s+5)}$$

(I.P.Univ., 2000)

Solution : $F_1(s)$ is not driving point impedance, since poles and zeros are not interlacing at negative real axis. But $F_2(s)$ is an R-L driving point impedance, since poles and zeros are interlacing at negative real axis and first singularity nearest to origin is a zero. Therefore,

$$F_2(s)=Z_{R\text{-}L}(s)=\frac{(s+2)(s+4)}{(s+3)(s+5)}$$

And
$$\frac{Z_{R-L}(s)}{s}=\frac{(s+2)(s+4)}{s(s+3)(s+5)}$$

Using partial fraction expansion,

$$\frac{Z_{R-L}(s)}{s}=\frac{A}{s}+\frac{B}{s+3}+\frac{C}{s+5}$$

$$A=\left.\frac{(s+2)(s+4)}{(s+3)(s+5)}\right|_{s=0}=\frac{8}{15}$$

$$B=\left.\frac{(s+2)(s+4)}{s(s+5)}\right|_{s=-3}=\frac{-1}{-6}=\frac{1}{6}$$

$$C=\left.\frac{(s+2)(s+4)}{s(s+3)}\right|_{s=-5}=\frac{3}{10}$$

Therefore,
$$Z_{R\text{-}L}(s)=\frac{8}{15}+\frac{\frac{1}{6}s}{s+3}+\frac{\frac{3}{10}s}{s+5}$$

And synthesized network is shown in figure 10.22.

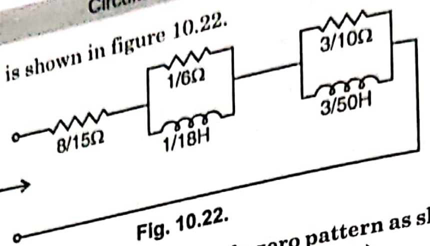


Fig. 10.22.

EXAMPLE 10.30 **An impedance function has the pole-zero pattern as shown in figure 10.23(*a*). If $Z(-2) = 3$, synthesize the impedance in Foster second form (F_{II})** *(I.P. Univ., 2000)*

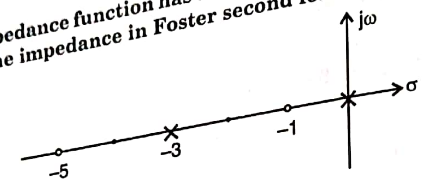


Fig. 10.23(a).

Solution : From the pole-zero diagram,

$$Z(s) = \frac{K(s+1)(s+5)}{s(s+3)}$$

$$Z(-2) = 3 = \frac{K(-1)(3)}{(-2)(1)} = \frac{3}{2}K$$

or $$K = 2$$

Therefore, $$Z(s) = \frac{2(s+1)(s+5)}{s(s+3)}$$

Since singularity nearest to origin is a pole, therefore, given function is *RC* impedance function.

$$Y(s) = \frac{s(s+3)}{2(s+1)(s+5)}$$

and

or $$\frac{Y(s)}{s} = \frac{(s+3)/2}{(s+1)(s+5)}$$

Using partial fraction expansion,

$$\frac{Y(s)}{s} = \frac{A}{s+1} + \frac{B}{s+5}$$

$$A = \left.\frac{(s+3)/2}{s+5}\right|_{s=-1} = \frac{1}{4}$$

$$B = \left.\frac{(s+3)/2}{s+1}\right|_{s=-5} = \frac{-1}{-4} = \frac{1}{4}$$

Therefore, $$Y(s) = \frac{\frac{1}{4}s}{s+1} + \frac{\frac{1}{4}s}{s+5}$$

and synthesized network is shown in figure 10.23(*b*).

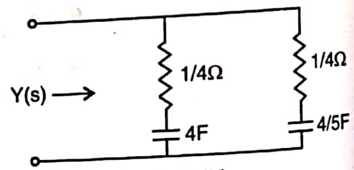
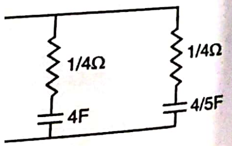


Fig. 10.23(b).

n as shown in figure 10.23(a).
(I.P. Univ., 2000)

is RC impedance function.



ig. 10.23(b).

...LE 10.31 An LC impedance function for one port network is given by

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

...thesize the network in
... Foster's type – I form
... Cauer's type – II form.
...: Same as example 10.11.
(I.P. Univ., 2001)

...LE 10.32 Check the positive realness of the following functions :

... $\frac{s^2+s+6}{s^2+s+1}$

... $\frac{s^2+6s+5}{s^2+9s+14}$

(U.P. T.U., 2001)

...:

$$F(s) = \frac{s^2+s+6}{s^2+s+1}$$

...dition (1): $M(s) = s^2 + 1,\ N(s) = s$

$$s\,)\,s^2+1\,(\,s$$
$$\underline{s^2}$$
$$1\,)\,s\,(\,s$$
$$\underline{s}$$
$$\times$$

...efore, $D(S)$ is Hurwitz polynomial
...dition (2) : doesnot exist.
...dition (3) : $M_1(s) = s^2 + 6,\ M_2(s) = s^2 + 1$

$$N_1(s) = s,\ N_2(s) = s$$

$$M_1M_2 - N_1N_2 \geq 0$$

$$(s^2+6)(s^2+1) - s \cdot s \geq 0$$

$$s^4 + 7s^2 + 6 - s^2 \geq 0$$

$$s^4 + 6s^2 + 6 \geq 0$$

$$A(\omega^2) = \omega^4 - 6\omega^2 + 6 \geq 0$$

...e all coefficients of $A(\omega^2)$ are not positive. Using sturm's test as

$$A(x) = A_0(x) = x^2 - 6x + 6 \qquad (\text{putting } \omega^2 = x)$$

$$A_1(x) = \frac{d\,A_0(x)}{dx} = 2x - 6$$

$$2x-6\,)\,x^2-6x+6\,(\,\frac{x}{2}-\frac{3}{2}$$
$$\underline{x^2-3x}$$
$$-3x+6$$
$$\underline{-3x+9}$$
$$-3$$

$$\frac{A_0(x)}{A_1(x)} = \left(\frac{x}{2}-\frac{3}{2}\right) - \frac{3}{2x-6}$$

$$A_2(x) = 3$$

	A_0	A_1	A_2	No. of Sign Changes
$x = 0$	+	–	+	$s_0 = 2$
$x = \infty$	+	+	+	$s_\infty = 0$

Now $\qquad s_\infty \sim s_0 = 2$

Therefore, $A(\omega^2) \ngeq 0$ for all ω

Hence, given function is not a **p.r.f.**

(ii) $$F(s) = \frac{s^2+6s+5}{s^2+9s+14} = \frac{(s+1)(s+5)}{(s+2)(s+7)}$$

Condition (1) : $M(s) = s^2 + 14$; $N(s) = 9s$

$$9s\,)\,s^2+14\,\left(\frac{s}{9}\right.$$
$$s^2$$
$$14\,)\,9s\,\left(\frac{9s}{14}\right.$$
$$9s$$
$$\times$$

Therefore, $D(s)$ is Hurmitz polynomial.

Condtion (2): doesnot exist.

Condition (3): $M_1(s) = s^2 + 5$, $M_2(s) = s^2 + 14$

$$N_1(s) = 6s,\ N_2(s) = 9s$$

$$M_1M_2 - N_1N_2 \geq 0$$

$$(s^2 + 5)(s^2 + 14) - (6s)(9s) \geq 0$$

$$s^4 + 19s^2 + 70 - 54s^2 \geq 0$$

$$s^4 - 35s^2 + 70 \geq 0$$

$$A(\omega^2) = \omega^4 + 35\omega^2 + 70 \geq 0$$

Since all coefficients of $A(\omega^2)$ are positive therefore, $A(\omega^2) \geq 0$ for all ω.

Hence given function is a **p.r.f.**

EXAMPLE 10.33. **Synthesize**

$$Z(s) = \frac{(s+1)(s+3)}{s(s+2)}$$ **in Cauer's I form.** *(U.P.T.U., 2001, 2003(C.O.))*

Solution : $$Z(s) = \frac{s^2+4s+3}{s^2+2s}$$

$$s^2+2s \,\big)\, s^2+4s+3 \,\big(\, 1 \leftrightarrow Z_1$$
$$\underline{s^2+2s}$$
$$2s+3 \,\big)\, s^2+2s \,\big(\, \frac{s}{2} \leftrightarrow Y_2$$
$$\underline{s^2+\frac{3}{2}s}$$
$$\frac{1}{2}s \,\big)\, 2s+3 \,\big(\, 4 \leftrightarrow Z_3$$
$$\underline{2s}$$
$$3 \,\big)\, \frac{1}{2}s \,\big(\, \frac{1}{6}s \leftrightarrow Y_4$$
$$\underline{\frac{1}{2}s}$$
$$\times$$

Therefore, the synethesized network is shown in figure 10.24.

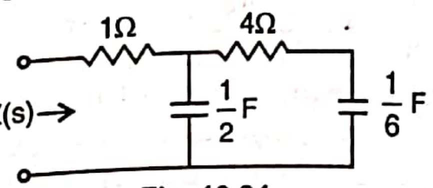


Fig. 10.24.

EXAMPLE 10.34 **Synthesize**

$$Z(s) = \frac{(s+5)}{(s+1)(s+6)}$$ **in Foster's II form.** *(U.P.T.U., 2001)*

Solution :

$$Z(s) = \frac{s+5}{(s+1)(s+6)}$$

$$Y(s) = \frac{(s+1)(s+6)}{(s+5)}$$

$$s+5 \,\big)\, s^2+7s+6 \,\big(\, s+2$$
$$\underline{s^2+5s}$$
$$2s+6$$
$$\underline{2s+10}$$
$$-4$$

$$Y(s) = s+2+\frac{-4}{s+5}$$

$$\frac{Y(s)}{s} = \frac{s^2+7s+6}{s^2+5s}$$

$$s^2+5s \,\big)\, s^2+7s+6 \,\big(\, 1$$
$$\underline{s^2+5s}$$
$$2s+6$$

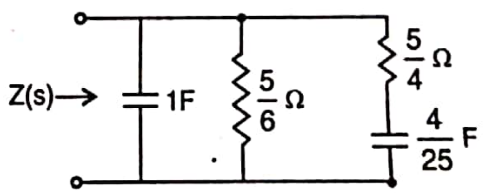


Fig. 10.25.

$$\frac{Y(s)}{s} = 1+\frac{2s+6}{s(s+5)}$$

Using partial fraction expansions,

$$\frac{Y(s)}{s} = 1+\frac{\frac{6}{5}}{s}+\frac{\frac{4}{5}}{s+5}$$

So $$Y(s) = s+\frac{6}{5}+\frac{\frac{4}{5}s}{s+5}$$

Therefore, the synthesized network is shown in figure 10.25.

EXAMPLE 10.35 **Realize the following RC driving point impedance function in (*i*) First Foster form (*ii*) First Cauer form :**

$$Z(s) = \frac{s^2+6s+8}{s^2+4s+3}$$ *(U.P.T.U., 2003)*

Solution : (*i*) First Foster form :

$$Z(s) = \frac{s^2+6s+8}{s^2+4s+3} = 1+\frac{2s+5}{(s+1)(s+3)} = 1+\frac{3/2}{s+1}+\frac{1/2}{s+3}$$

Therefore, first Foster form is shown in figure 10.26(*a*).

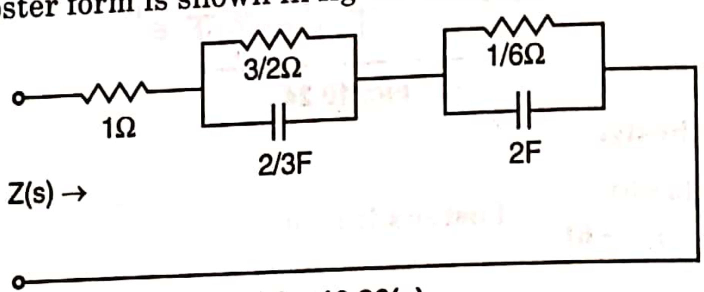


Fig. 10.26(a).

(*ii*) First Cauer form :

$$s^2+4s+3 \,\big)\, s^2+6s+8 \,\big(\, 1 \rightarrow Z_1$$
$$\underline{s^2+4s+3}$$
$$2s+5 \,\big)\, s^2+4s+3 \,\big(\, \frac{1}{2}s \leftrightarrow Y_2$$
$$\underline{s^2+\frac{5}{2}s}$$
$$\frac{3}{2}s+3 \,\big)\, 2s+5 \,\big(\, \frac{4}{3} \leftrightarrow Z_3$$
$$\underline{2s+4}$$
$$1 \,\big)\, \frac{3}{2}s+3 \,\big(\, \frac{3}{2}s \leftrightarrow Y_4$$
$$\underline{\frac{3}{2}s}$$
$$3 \,\big)\, 1 \,\big(\, \frac{1}{3} \leftrightarrow Z_5$$
$$\underline{1}$$
$$\times$$

Therefore, first Cauer form is shown in figure 10.26(b).

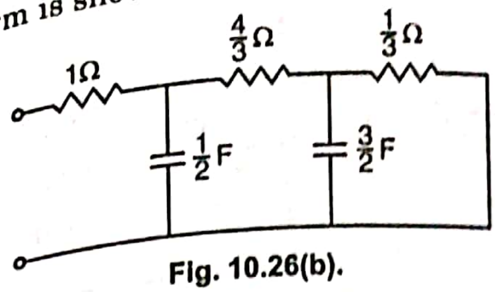


Fig. 10.26(b).

EXAMPLE 10.36 **Realize the following functions in First and Second Foster form.** *(U.P.T.U., 2004)*

(i) $Z(s) = \dfrac{(s^2+1)(s^2+16)}{s(s^2+9)}$

(ii) $Z(s) = \dfrac{(s+2)(s+5)}{(s+1)(s+4)}$

Solution : (i) **First Foster form :**

$$Z(s) = \frac{(s^2+1)(s^2+16)}{s(s^2+9)} = s + \frac{8s^2+16}{s(s^2+9)}$$

$$= s + \frac{A}{s} + \frac{Bs}{s^2+9} = s + \frac{16/9}{s} + \frac{(56/9)s}{s^2+9}$$

Therefore, the first Foster form of the given $Z(s)$ is shown in figure 10.27(a).

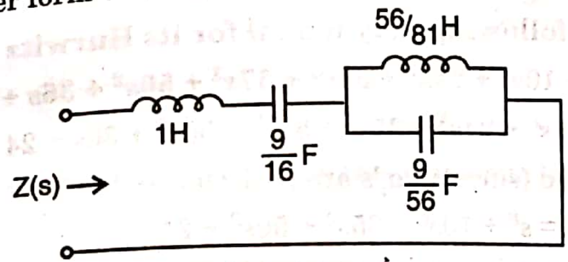


Fig. 10.27(a).

Second Foster form :

$$Y(s) = \frac{s(s^2+9)}{(s^2+1)(s^2+16)} = \frac{As}{s^2+1} + \frac{Bs}{s^2+16} = \frac{(8/15)s}{s^2+1} = \frac{(7/15)s}{s^2+16}$$

Therefore, the second Foster form is shown in figure 10.27(b).

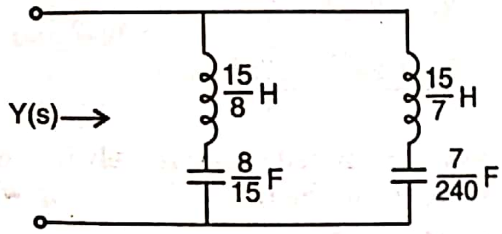


Fig. 10.27(b).

(ii) **First Foster form:**

$$Z(s) = \frac{(s+2)(s+5)}{(s+1)(s+4)} = 1 + \frac{2s+6}{(s+1)(s+4)} = 1 + \frac{A}{s+1} + \frac{B}{s+4} = 1 + \frac{4/3}{s+1} + \frac{2/3}{s+4}$$

Therefore, the first Foster form of the given $Z(s)$ is shown in Fig. 10.27(c).

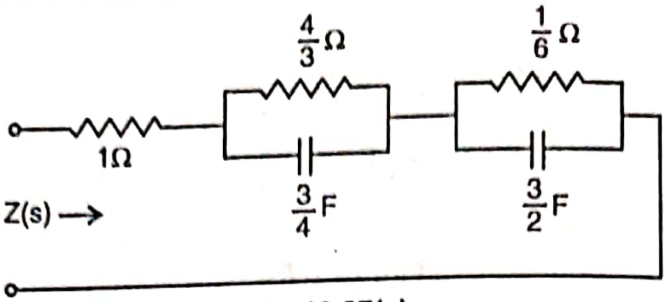


Fig. 10.27(c).

Second Foster form :

$$\frac{Y(s)}{s} = \frac{(s+1)(s+4)}{s(s+2)(s+s)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$Y(s) = \frac{2}{s} + \frac{\left(\frac{1}{3}\right)s}{s+2} + \frac{\left(\frac{4}{15}\right)s}{s+5}$$

Therefore, the second Foster form is shown in figure 10.27(*d*).

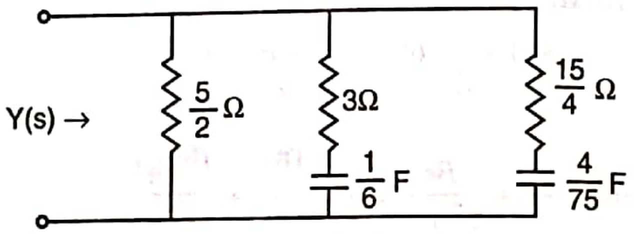


Fig. 10.27(d).

EXAMPLE 10.37 **Test the following polynomial for its Hurwitz character.**

$$P(s) = s^8 + 3s^7 + 10s^6 + 24s^5 + 35s^4 + 57s^3 + 50s^2 + 36s + 24. \qquad \text{(U.P.T.U., 2004)}$$

Solution : $P(s) = s^8 + 3s^7 + 10s^6 + 24s^5 + 35s^4 + 57s^3 + 50s^2 + 36s + 24$

Condition (1) : is satisfied (since all a_i's are positive).

Condition (2) : $M(s) = s^8 + 10s^6 + 35s^4 + 50s^2 + 24$

$N(s) = 3s^7 + 24s^5 + 57s^3 + 36s$

So continued fraction expansion is given as

$$3s^7 + 24s^5 + 57s^3 + 36s \,\Big)\, s^8 + 10s^6 + 35s^4 + 50s^2 + 24 \,\Big(\, \frac{s}{3}$$
$$\underline{s^8 + 8s^6 + 19s^4 + 12s^2}$$
$$2s^6 + 16s^4 + 38s^2 + 24 \,\Big)\, 3s^7 + 24s^5 + 57s^3 + 36s \,\Big(\, \frac{3}{2}s$$
$$\underline{3s^7 + 24s^5 + 57s^3 + 36s}$$
$$\times$$

We see that the division has been terminated pre-maturely by a common factor $3s^7 + 24s^5 + 57s^3 + 36s$. Thus our task resolves into the problem of determining whether.

$P_1(s) = 3s^7 + 24s^5 + 57s^3 + 36s$ is Hurwitz.

or $\quad P_1(s) = 3s\,(s^6 + 8s^4 + 19s^2 + 12) = W(s)\,.\,P(s)$

or $\quad P(s) = s^6 + 8s^4 + 19s^2 + 12$

Since $P(s)$ is an even polynomial. Therefore, the ratio of $P(s)$ and $P'(s)$ gives a continued fraction expansion as

$$6s^5+32s^3+38s \,\big)\, s^6+8s^4+19s^2+12 \,\big(\, \frac{1}{6}s$$

$$s^6+\frac{16}{3}s^4+\frac{19}{3}s^2$$

$$\frac{8}{3}s^4+\frac{38}{3}s^2+12 \,\big)\, 6s^5+32s^3+38s \,\big(\, \frac{9}{4}s$$

$$6s^5+\frac{57}{2}s^3+27s$$

$$\frac{7}{2}s^3+11s \,\big)\, \frac{8}{3}s^4+\frac{38}{3}s^2+12 \,\big(\, \frac{16}{21}s$$

$$\frac{8}{3}s^4+\frac{176}{21}s^2$$

$$\frac{30}{7}s^2+12 \,\big)\, \frac{7}{2}s^3+11s \,\big(\, \frac{49}{60}s$$

$$\frac{7}{2}s^3+\frac{49}{5}s$$

$$\frac{6}{5}s \,\big)\, \frac{30}{7}s^2+12 \,\big(\, \frac{25}{7}s$$

$$\frac{30}{7}s^2$$

$$12 \,\big)\, \frac{6}{5}s \,\big(\, \frac{6}{60}s$$

$$\frac{6}{5}s$$

$$\times$$

[S]ince, all quotient terms are positive, therefore the polynomial $P(s)$ is Hurwitz and hence given [poly]nomial is Hurwitz.

[EXA]MPLE 10.38 **An impedance is given by**

$$Z(s)=\frac{8(s^2+1)(s^2+3)}{s(s^2+2)(s^2+4)}$$

Realize the network in (*i*) Foster-I form and (*ii*) Cauer-II form. *(U.P.T.U., 2004)*

[Solu]tion : $Z(s)=\dfrac{8(s^2+1)(s^2+3)}{s(s^2+2)(s^2+4)}=\dfrac{8(s^4+4s^2+3)}{s(s^2+2)(s^2+4)}$

[(i)] Foster-I form : Using partial fraction expansion,

$$Z(s)=\frac{A}{s}+\frac{Bs}{s^2+2}+\frac{Cs}{s^2+4}$$

$$8s^4+32s^2+24 \equiv A(s^2+2)(s^2+4)+Bs^2(s^2+4)+Cs^2(s^2+2)$$

$$s^4 \rightarrow A+B+C=8$$

$$s^2 \rightarrow 6A+4B+2C=32$$

$$\text{Constt.} \rightarrow 8A=24$$

Gives,

$$A=3$$
$$B=2$$
$$C=3$$

Therefore, $Z(s) = \dfrac{3}{s} + \dfrac{2s}{s^2+2} + \dfrac{3s}{s^2+4}$

We then obtained the synthesized network in Foster-I form as shown in figure 10.28(*a*).

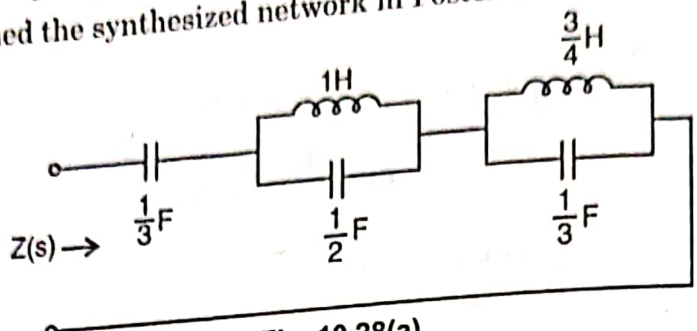


Fig. 10.28(a).

(*ii*) **Cauer-II form :** $Z(s) = \dfrac{24+32s^2+8s^4}{8s+6s^3+s^5}$

$$
\begin{array}{l}
8s+6s^3+s^5 \,\big)\, 24+32s^2+8s^4 \,\big(\, \dfrac{3}{s} \leftrightarrow Z_1 \\
\qquad\qquad\qquad\; 24+18s^2+3s^4 \\
\hline
14s^2+5s^4 \,\big)\, 8s+6s^3+s^5 \,\big(\, \dfrac{8}{14s} = \dfrac{4}{7s} \leftrightarrow Y_2 \\
\qquad\qquad\quad\; 8s+\dfrac{20}{7}s^3 \\
\hline
\dfrac{22}{7}s^3+s^5 \,\big)\, 14s^2+5s^4 \,\big(\, \dfrac{7}{22}\cdot\dfrac{14}{s} = \dfrac{49}{11s} \leftrightarrow Z_3 \\
\qquad\qquad\quad\; 14s^2+\dfrac{49}{11}s^4 \\
\hline
\dfrac{6}{11}s^4 \,\big)\, \dfrac{22}{7}s^3+s^5 \,\big(\, \dfrac{11}{6}\cdot\dfrac{22}{7s} = \dfrac{121}{21s} \leftrightarrow Y_4 \\
\qquad\quad\; \dfrac{22}{7}s^3 \\
\hline
s^5 \,\big)\, \dfrac{6}{11}s^4 \,\big(\, \dfrac{6}{11s} \leftrightarrow Z_5 \\
\qquad \dfrac{6}{11}s^4 \\
\hline
\qquad \times
\end{array}
$$

Therefore, the synthesized network in Cauer-II form as shown in figure 10.28(*b*).

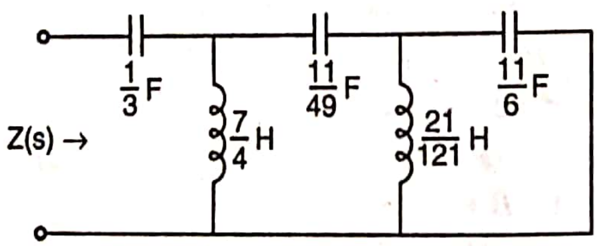


Fig. 10.28(b).

EXAMPLE 10.39 **Of the two pole-zero diagrams shown in figures 10.29(*a*) and (*b*), pick the diagram that represents an *RL* impedance function and synthesize by first Foster form.**

(U.P.T.U., 2005)

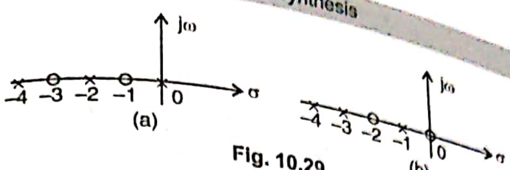


Fig. 10.29.

Solution: The pole-zero diagram of figure 10.29(*a*) represents an *R-C* impedance function, since there is a pole at origin. The pole-zero diagram of figure 10.29(*b*) shows that two consecutive poles at ($s = -3$ and $s = -4$). This is not permitted in any immittance function. The correct pole-zero diagram must be as shown in figure 10.29(*c*). This represents an *R-L* impedance function, since there is a zero at origin and poles and zeros are interlaced on the negative real axis.

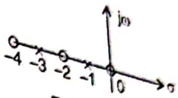


Fig. 10.29. (c)

$$Z_{R\text{-}L}(s) = \frac{s(s+2)(s+4)}{(s+1)(s+3)}$$

Foster's I form:

$$\frac{Z(s)}{s} = \frac{(s+2)(s+4)}{(s+1)(s+3)}$$

$$\frac{Z(s)}{s} = \frac{s^2+6s+8}{s^2+4s+3} = 1 + \frac{2s+5}{(s+1)(s+3)}.$$

or

$$\frac{Z(s)}{s} = 1 + \frac{\frac{3}{2}}{s+1} + \frac{\frac{1}{2}}{s+3} \qquad \text{(Using partial fraction expansion)}$$

or

$$Z(s) = s + \frac{\frac{3}{2}s}{s+1} + \frac{\frac{1}{2}s}{s+3}$$

Therefore, synthesized network is shown in figure 10.29(*d*).

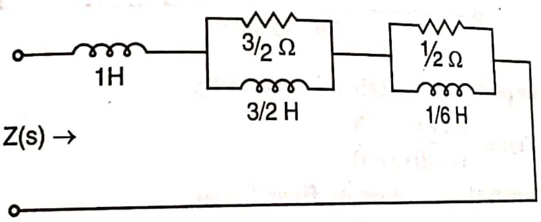


Fig. 10.29(d).

EXAMPLE 10.40 Realize $Z(s) = \dfrac{s(s^2+2)(s^2+4)}{(s^4+1)(s^2+3)(s^2+5)}$ **in first Cauer form.**

(U.P.T.U., 2005)

Solution: There is a misprint in this question. The power of *s* in the first factor of the denominator of $Z(s)$ must be 2.

$$Y(s) = \frac{(s^2+1)(s^2+3)(s^2+5)}{s(s^2+2)(s^2+4)}$$

$$= \frac{s^6+9s^4+23s^2+15}{s^5+6s^3+8s}$$

The continued fraction expansion is

$$s^5+6s^3+8s \;\big)\; s^6+9s^4+23s^2+15 \;\big(\; s \leftrightarrow Y_1$$
$$s^6+6s^4+8s^2$$
$$3s^4+15s^2+15 \;\big)\; s^5+6s^3+8s \;\big(\; \frac{s}{3} \leftrightarrow Z_2$$
$$s^5+5s^3+5s$$
$$s^3+3s \;\big)\; 3s^4+15s^2+15 \;\big(\; 3s \leftrightarrow Y_3$$
$$3s^4+9s^2$$
$$6s^2+15 \;\big)\; s^3+3s \;\big(\; \frac{s}{6} \leftrightarrow Z_4$$
$$s^3+\frac{5}{2}s$$
$$\frac{1}{2}s \;\big)\; 6s^2+15 \;\big(\; 12s \leftrightarrow Y_5$$
$$6s^2$$
$$15 \;\big)\; \frac{1}{2}s \;\big(\; \frac{s}{30} \leftrightarrow Z_6$$
$$\frac{1}{2}s$$
$$\times$$

Therefore, synthesized network is shown in figure 10.30.

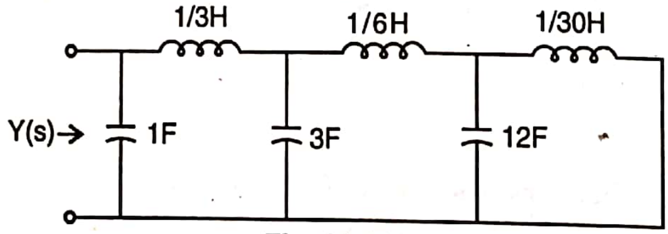


Fig. 10.30.

EXAMPLE 10.41 **An impedance function is given by**

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+4)}$$

Find the *R-L* representation of Foster first form. *(U.P.T.U., May, 2006)*

Solution : $Z(s) = \dfrac{2(s+1)(s+3)}{(s+2)(s+4)}$

Hence, $\dfrac{Z(s)}{s} = \dfrac{2(s+1)(s+3)}{s(s+2)(s+4)} = \dfrac{3/4}{s} + \dfrac{1/2}{s+2} + \dfrac{3/4}{s+4}$ (Using partial fraction expansion)

or $Z(s) = \dfrac{3}{4} + \dfrac{(1/2)s}{s+2} + \dfrac{(3/4)s}{s+4}$

Therefore, the synthesized network is shown in figure 10.31.

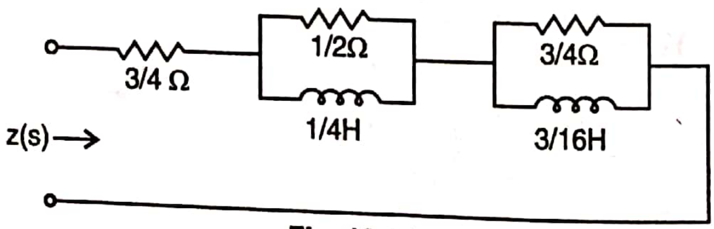


Fig. 10.31.

[EXA]MPLE 10.42 **Diagnose whether the following impedance function represents a RL [or R]C network and find its first Cauer form.**

$$Z(s) = \frac{(s+4)(s+6)}{(s+3)(s+5)}$$

(U.P.T.U, 2006)

[Sol]ution : $Z(s) = \left(\frac{(s+4)(s+6)}{(s+3)(s+5)}\right)$

Zeros: $s = -4, -6$

Poles : $s = -3, -5$

Since singularities nearest to origin and farthest from origin (or [neare]st to infinity) are pole and zero respectively as shown in figure [10.32](a). Therefore the given network function (Impedance) represents RC network.

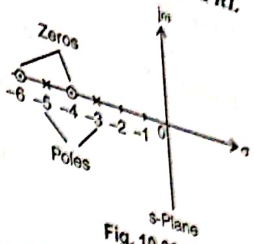


Fig. 10.32(a).

$$Z(s) = \frac{s^2+10s+24}{s^2+8s+15}$$

First Cauer form:

$$s^2+8s+15 \,\big)\, s^2+10s+24 \,\big(\, 1 \leftrightarrow Z_1$$
$$\underline{s^2+8s+15}$$
$$2s+9 \,\big)\, s^2+8s+15 \,\big(\, \frac{s}{2} \leftrightarrow Y_2$$
$$\underline{s^2+\frac{9}{2}s}$$
$$\frac{7}{2}s+15 \,\big)\, 2s+9 \,\big(\, \frac{4}{7} \leftrightarrow Z_3$$
$$\underline{2s+\frac{60}{7}}$$
$$\frac{3}{7} \,\big)\, \frac{7}{2}s+15 \,\big(\, \frac{49}{6}s \leftrightarrow Y_4$$
$$\underline{\frac{7}{2}s}$$
$$15 \,\big)\, \frac{3}{7} \,\big(\, \frac{1}{35} \leftrightarrow Z_5$$
$$\underline{\frac{3}{7}}$$
$$\times$$

$Z_1 = 1\Omega \Rightarrow R_1 = 1\,\Omega$

$Y_2 = \frac{s}{2}\mho \Rightarrow C_2 = \frac{1}{2}F$

$Z_3 = \frac{4}{7}\Omega \Rightarrow R_3 = \frac{4}{7}\,\Omega$

$Y_4 = \frac{49}{6}s\,\mho \Rightarrow C_4 = \frac{49}{6}F$

$Z_5 = \frac{1}{35}\Omega \Rightarrow R_5 = \frac{1}{35}\,\Omega$

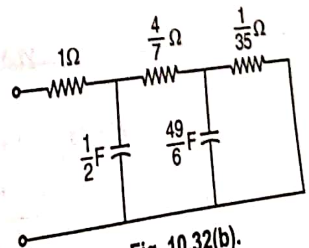


Fig. 10.32(b).

[Th]erefore, the first Cauer form is as shown in figure 10.32(b).

EXAMPLE 10.43 Realize the function $Z(s) = \dfrac{s(s^2+4)}{2(s^2+1)(s^2+9)}$ in both Foster forms LC network.

(U.P.T.U, 2006)

Solution : Foster–I Form: Using partial fraction expansion,

$$Z(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)} = \frac{K_1 s}{(s^2+1)} + \frac{K_2 s}{(s^2+9)}$$

$$K_1 = \left.\frac{(s^2+4)}{2(s^2+9)}\right|_{s^2=-1} = \frac{3}{16}$$

$$K_2 = \left.\frac{(s^2+4)}{2(s^2+1)}\right|_{s^2=-9} = \frac{5}{16}$$

$$\therefore \quad Z(s) = \frac{\frac{3}{16}s}{(s^2+1)} + \frac{\frac{5}{16}s}{(s^2+9)}$$

$$Z(s) = \frac{\frac{1}{C_1}s}{s^2+\frac{1}{L_1C_1}} + \frac{\frac{1}{C_2}s}{s^2+\frac{1}{L_2C_2}}$$

$$\therefore \quad C_1 = \frac{16}{3}F, \frac{1}{L_1C_1} = 1 \Rightarrow L_1 = \frac{3}{16}H$$

$$C_2 = \frac{16}{5}F,\ L_2C_2 = \frac{1}{9} \Rightarrow L_2 = \frac{5}{144}H$$

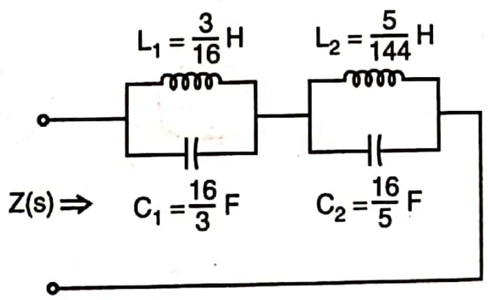


Fig. 10.33(a).

Therefore, Foster's I form as shown in figure 10.33(a).

Foster–II Form:

$$Y(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)} = \frac{2s^4+20s^2+18}{s^3+4s}$$

$$\begin{array}{r|l}
s^3+4s & 2s^4+20s^2+18 \;(\;2s \\
 & 2s^4+8s^2 \\ \hline
 & \quad 12s^2+18
\end{array}$$

$$\therefore \quad Y(s) = 2s + \frac{12s^2+18}{s(s^2+4)}$$

n both Foster forms LC

(U.P.T.U. 2006)

Using partial fraction expansion,

$$\frac{12s^2+18}{s(s^2+4)} = \frac{K_1}{s} + \frac{K_2 s}{s^2+4}$$

$$K_1 = \left.\frac{12s^2+18}{(s^2+4)}\right|_{s=0} = \frac{9}{2}$$

$$K_2 = \left.\frac{12s^2+18}{s^2}\right|_{s^2=-4} = \frac{15}{2}$$

$$Y(s) = 2s + \frac{\frac{9}{2}}{s} + \frac{\frac{15}{2}s}{s^2+4}$$

$$Y(s) = C_1 s + \frac{1}{L_2 s} + \frac{\frac{1}{L_3}s}{s^2 + \frac{1}{L_3 C_3}}$$

$$C_1 = 2F$$

$$L_2 = \frac{2}{9}H$$

$$L_3 = \frac{2}{15}H$$

$$L_3 C_3 = \frac{1}{4} \quad \Rightarrow C_3 = \frac{15}{8}F$$

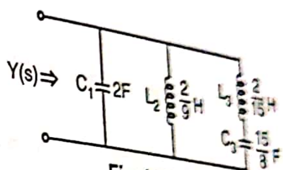


Fig. 10.33(b).

Therefore, Foster's II from as shown in figure 10.33(*b*).

EXERCISES

1. Define Hurwitz polynomial and write its properties.
2. Define positive real function and write its properties.
3. Define Foster's reactance theorem.
4. Write the properties of
   (*a*) *L-C* immittance functions
   (*b*) *R-C* impedance or *R-L* admittance functions
   (*c*) *R-L* impedance or *R-C* admittance functions
5. What are the procedures to obtain (*a*) Foster's I form (*b*) Foster's II form (*c*) Cauer's I form, (*d*) Cauer's II form.

PROBLEMS

1. Check whether the given polynomials are Hurwitz or not.

(*a*) $P(s) = s^4 + s^3 - 21s^2 + 8s + 2$

(*b*) $P(s) = s^5 + s^3 + 2s^2 + 3s + 1$

(*c*) $P(s) = s^4 + s^3 + 2s^2 + 3s + 2$

(*d*) $P(s) = s^7 + 3s^5 + 2s^3 + s$

(*e*) $P(s) = s^4 + 3s^2 + 2$

(*f*) $P(s) = s^4 + 11s^3 + 39s^2 + 51s + 20$

(*g*) $P(s) = s^5 + s^3 + s$

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(*h*) $P(s) = s^4 + 3s^2 + 4$

(*i*) $P(s) = s^5 + 12s^4 + 45s^3 + 44s + 48$

(*j*) $P(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 4s + 8$

(*k*) $P(s) = s^6 + 2s^5 + 14s^4 + 26s^3 + 49s^2 + 72s + 36$

(*l*) $P(s) = s^5 + 7s^4 + 5s^3 + s^2 + 2s$

10.2. Find the range of values a in $P(s)$, so that $P(s) = 2s^4 + s^3 + as^2 + s + 2$ is Hurwitz.

10.3. Check whether the given functions are *p.r.* or not.

(*a*) $F(s) = \dfrac{s^2+10s+4}{s+2}$ (*b*) $F(s) = \dfrac{s^2+s+1}{s^2+s+4}$

(*c*) $F(s) = \dfrac{s^3+5s^2+9s+3}{s^3+4s^2+7s+9}$ (*d*) $F(s) = \dfrac{4s+1}{s+2}$

(*e*) $F(s) = \dfrac{2s^4+7s^3+11s^2+12s+4}{s^4+5s^3+9s^2+11s+6}$ *Hint:* Using Sterm's test.

10.4. Synthesize the network function $Z(s) = \dfrac{s(s^2+4)}{2(s^2+1)(s^2+9)}$ as a Foster–I form and also as a Foster–II form.

10.5. Synthesize the function $Z(s) = \dfrac{2s^5+12s^3+16s}{s^4+4s^2+3}$ as Cauer–I form.

10.6. Realize the network function $Z(s) = \dfrac{(s^2+1)(s^2+3)}{s(s^2+2)}$ as Cauer–II form.

10.7. Realize $Z_{RC}(s) = \dfrac{s^2+4s+1}{4s^2+5s+1}$ in Cauer–II form.

10.8. Realize $Z_{RL}(s) = \dfrac{4s^2+5s+1}{s^2+4s+1}$ in Cauer–II form.

10.9. Realize $Z(s) = \dfrac{3(s+2)}{(s+1)(s+4)}$ in Cauer–II form.

10.10. Realize $Z(s) = \dfrac{(s+2)}{(s+1)(s+3)}$ in Foster–II form.

10.11. Given $F(s) = \dfrac{6(s+2)(s+4)}{s(s+3)}$, find the continued fraction expansion and hence synthesize the network for the case when (*a*) $F(s)$ is an impedance $Z(s)$, (*b*) $F(s)$ is an admittance $Y(s)$.

ANSWERS

10.1.	(*a*) (No)	(*b*) (No)	(*c*) (No)
	(*d*) (Yes)	(*e*) (Yes)	(*f*) (Yes)
	(*g*) (No)	(*h*) (No)	(*i*) (Yes)
	(*j*) (No)	(*k*) (Yes)	(*l*) (No)
10.2.	($a > 4$)		
10.3.	(*a*) (Yes)	(*b*) (Yes)	(*c*) (Yes)
	(*d*) (Yes)	(*e*) (Yes)*Hint:* Using Sterm's test.	

rwitz.

d also as a Foster–

synthesize the

$Y(s)$.

Refer figure A-10.4 (a) and (b).

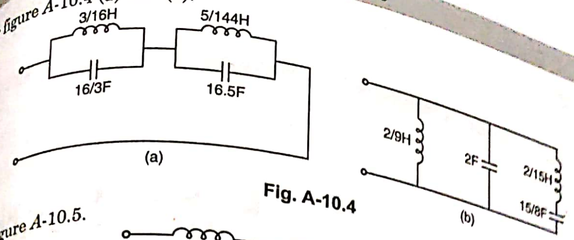

Fig. A-10.4

Refer figure A-10.5.

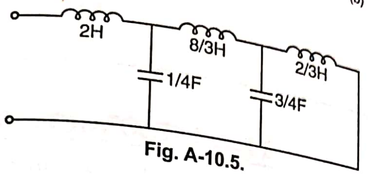

Fig. A-10.5.

Refer figure A-10.6.

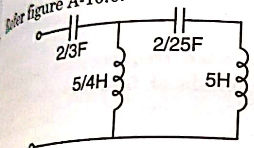

Fig. A-10.6.

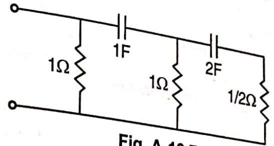

Fig. A-10.7.

Refer figure A-10.7.

Refer figure A-10.8.

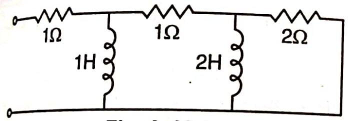

Fig. A-10.8.

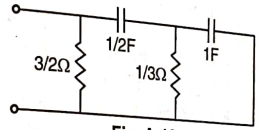

Fig. A-10.9.

Refer figure A-10.9.

Refer figure A-10.10.

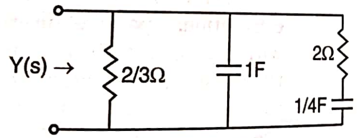

Fig. A-10.10.

(a) refer figure A-10.11(a), (b) refer figure A-10.11(b).

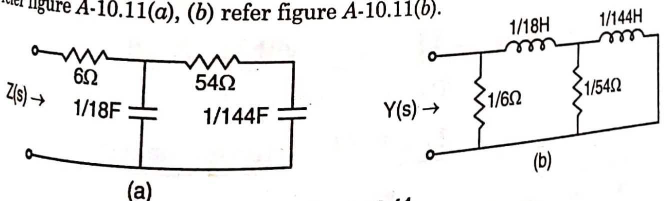

Fig. A-10.11.

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