

FIRST TERM EXAMINATION [FEB. 2015]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME-110]

Time. 1 Hour

MM : 30

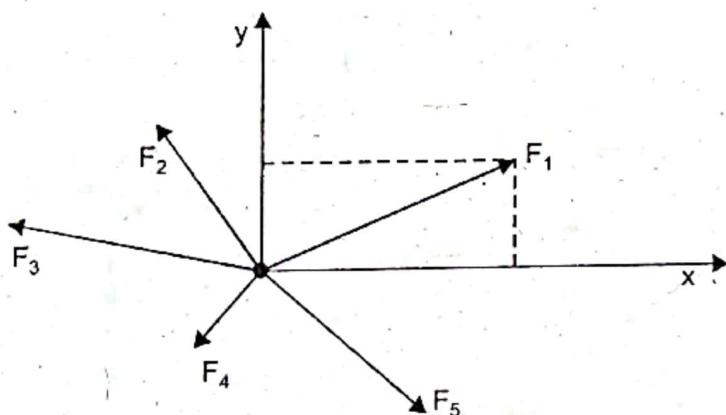
Note: Q.No.1 is compulsory. Attempt any two more questions from the rest.

Q.1. (a) Explain the term "principle of equilibrium" with conditions of equilibrium. (2×5=10)

Ans. If no. of forces acting on a pt. 'O' (i.e. F_1, F_2, F_3, F_4 & F_5)

If their resultant is zero. The particle is said to be in equilibrium

$$R = \sqrt{R_x^2 + R_y^2} = 0$$



i.e.,

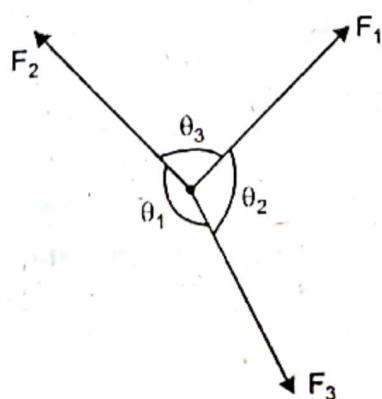
$$\Sigma F_x = 0 \text{ or } \Sigma F_y = 0$$

These are equations or condition of eqn. If no. of concurrent forces laying in a plane are in equilibrium.

Q.1. (b) State and explain the lami's theorem.

(2)

Ans. Lami's Theorem: If three concurrent of co-planer forces are acting on a point 'O' Then the ratio of force is sine of angle of opposite two forces.

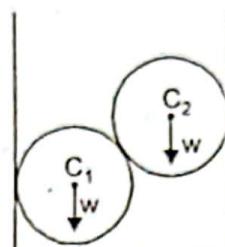
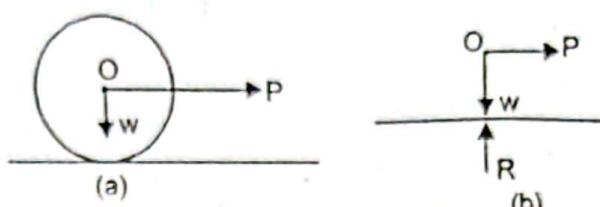


$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$$

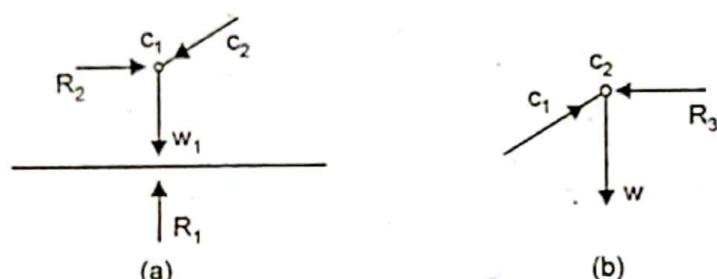
Q.1. (c) Explain the term "Free body diagram" with examples.

Ans. F.B.D.: → The Free body diagram may be defined as the all forces acting on a body (direct or indirect) are shown individually or freely.

For e.g.

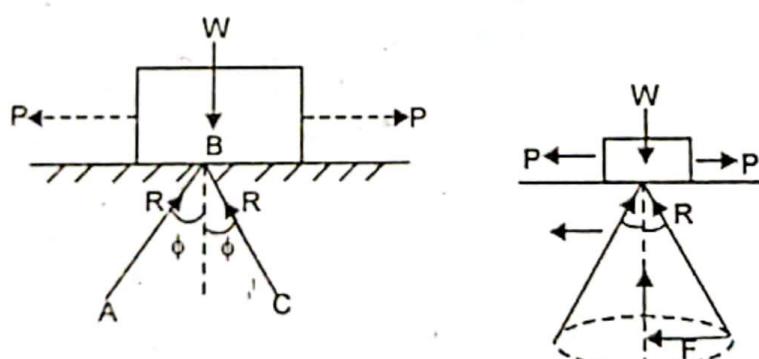


(c)



Q.1. (d) Define and explain "cone of friction" with neat sketch.

Ans. Cone of friction: Consider a block of weight W resting on horizontal surface and acting upon by force P. When we consider coplanar forces, in order for motion not to occur in any direction, resultant R must lie within angle ABC, where ϕ is angle of friction

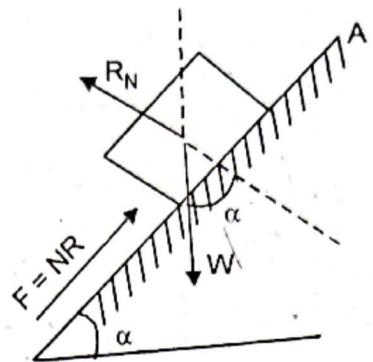


Consider force P is gradually changed through 360° . For motion not to occur, resultant reaction R must be contained with in the cone generated by revolving line AB about normal BN.

The inverted so formed with semi-central angle equal to angle of friction ϕ is called cone of friction. Now, for motion to occur resultant R will be on the surface of cone.

Q.1. (e) Explain "Angle of repose" and show that angle of repose is equal to the angle of friction.

Ans. Angle of Repose: Consider a block of weight W resting on an inclined plane OA making an angle α with the horizontal. Let angle α be increased gradually till the block is just at the point of sliding.



The block is in equilibrium under various forces.

Resolving forces along and perpendicular to plane

$$\mu R = W \sin \alpha$$

$$R = W \cos \alpha$$

$$\mu = \tan \alpha$$

In terms of angle of friction, coefficient of friction is given as $\mu = \tan \phi$

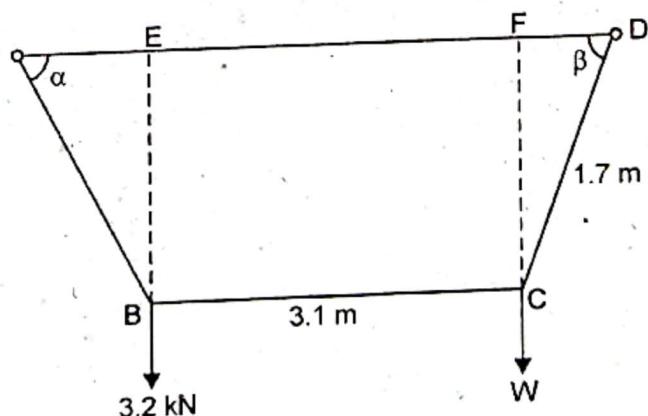
$$\text{eqn. (1)} = \text{eqn. (2)}$$

$$\tan \alpha = \tan \phi$$

$$\phi = \alpha$$

The angle of inclined plane at which block resting on it is about to slide down the plane is called angle of repose.

Q.2. (a) Two weights are suspended from points B and C of a rope as shown in fig. (a). If distance AD is 6m, how much will be the magnitude of W to maintain the equilibrium.



Ans. From ABE

$$2.3^2 = L^2 + (2.9 - a)^2 \quad \dots(1)$$

From DCF

$$1.7^2 = L^2 + a^2 \quad \dots(2)$$

from eqn (1) - (2)

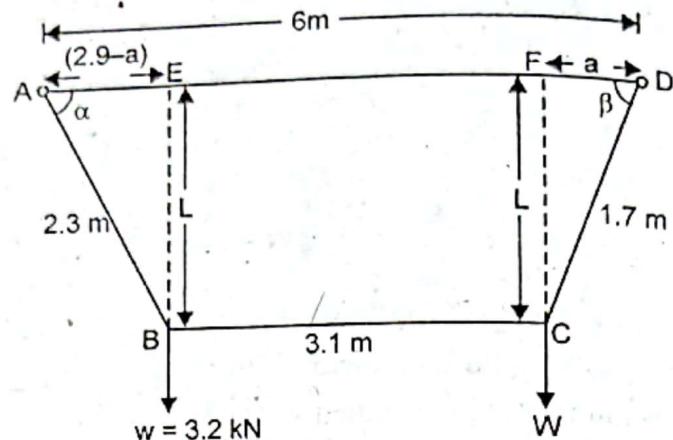
$$2.3^2 - 1.7^2 = L^2 + (2.9 - a)^2 - L^2 - a^2$$

$$2.4 = 2.9^2 + a^2 - 2 \times 2.9 \times a - a^2$$

$$2.4 - 8.41 = -5.8a$$

$$-6.01 = 5.8a$$

$$a = 1.04m = DF$$



$$AE = 2.9 - 1.04 = 1.86 \text{ m}$$

$$\cos \alpha = \frac{AE}{AB} = \frac{1.86}{2.3} = 0.8086$$

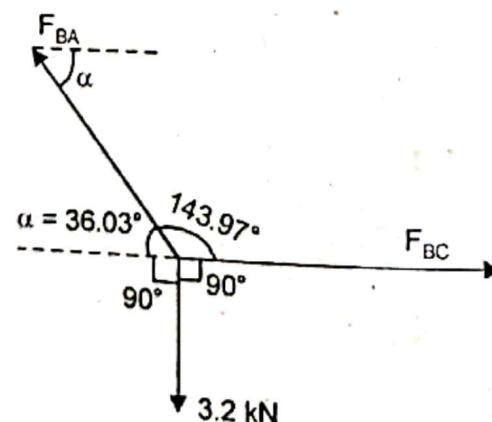
$$\alpha = \cos^{-1}(0.8086)$$

$$\alpha = 36.03^\circ$$

$$\cos \beta = \frac{DF}{CD} = \frac{1.09}{1.7} = 0.612$$

$$\beta = \cos^{-1}(0.612)$$

$$\beta = 52.26^\circ$$



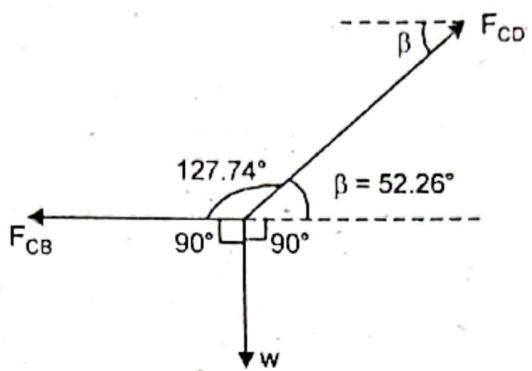
$$\frac{F_{BA}}{\sin 90^\circ} = \frac{F_{BC}}{\sin 126.03^\circ} = \frac{3.2}{\sin 143.97^\circ}$$

$$F_{BA} = \frac{3.2}{\sin 143.97^\circ} \times \sin 90^\circ$$

$$F_{BA} = 5.44 \text{ kN}$$

$$F_{BC} = \frac{3.2}{\sin 143.97^\circ} \times \sin 126.03^\circ$$

$$F_{BC} = 4.39 \text{ kN}$$



$$\frac{F_{CB}}{\sin(90^\circ + 52.26^\circ)} = \frac{W}{\sin 127.74^\circ} = \frac{F_{CD}}{90^\circ}$$

$$W = \frac{F_{CB}}{\sin 142.26^\circ} \times \sin 127.74^\circ$$

$$W = \frac{4.39}{\sin 142.26^\circ} \times \sin 127.74^\circ$$

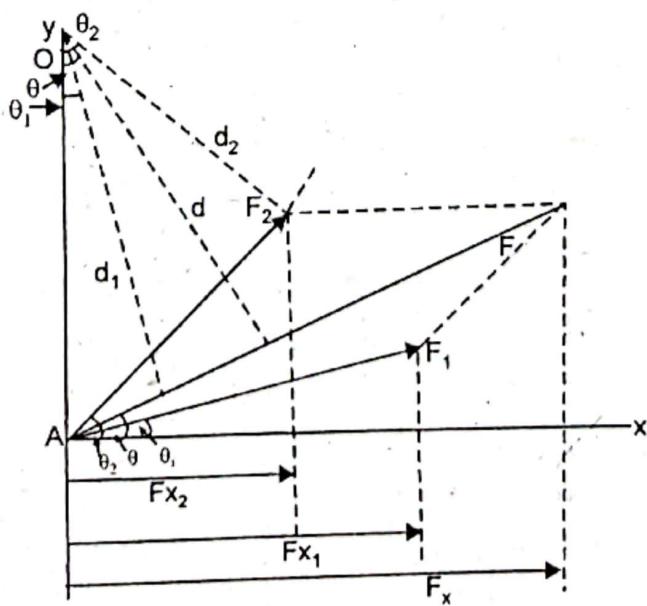
$\Rightarrow W = 5.67 \text{ kN}$ Ans.

Q.2. (b) State and prove "Varignon's Theorem".

Ans. Varignon's Theorem: or (Principle of Moment's): Varignon's theorem states that the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.

OR

Principle of moment states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces.



Theorem of Varignon: "The moment of force about an axis is equal to the sum of the moment of its components about the same axis".

Moment of force F about O:-

$$Fd = F(OA \cos \theta) = F \cos \theta \times OA$$

$$Fd = OAF_x$$

Moment of force F, about O

$$\begin{aligned} F_1 d_1 &= F_1 (OA \cos \theta) = F_1 \cos \theta \times OA \\ &= Fx_1 \times OA \end{aligned} \quad \dots(i)$$

Moment of force F_2 about O:-

$$F_2 d_2 = Fx_2 \times OA \quad \dots(ii)$$

adding eqⁿ (i) and (ii)

$$F_1 d_1 + F_2 d_2 = OA(Fx_1 + Fx_2)$$

$$F_x = Fx_1 + Fx_2$$

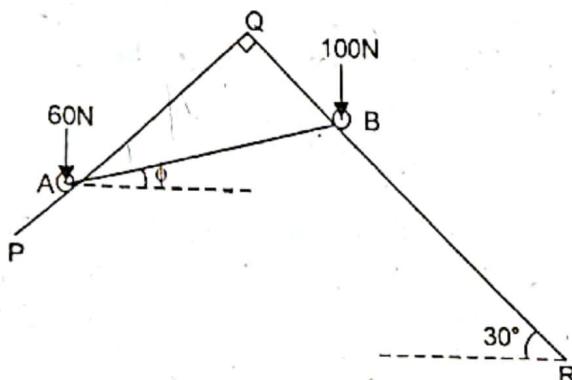
x component of resultant F = The sum of x components of forces F_1 and F_2

$$F_1 d_1 \times F_2 d_2 = OA \times F_x$$

$$F_1 d_1 \times F_2 d_2 = F_x d$$

So the moment of force about an axis is equal to sum of moment of its components about the same axis.

Q.3. (a) Two spheres weighing 60 N and 100 N are connected by flexible string AB and rest on two mutually perpendicular planes PQ and QR as shown in fig. (b). Find the tension in the string which passes freely through slots in smooth inclined planes.

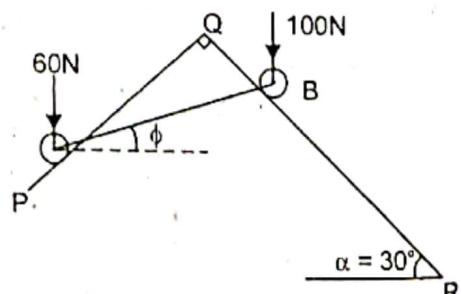
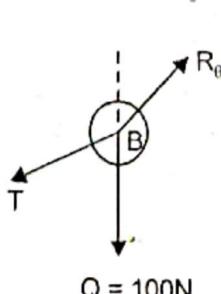
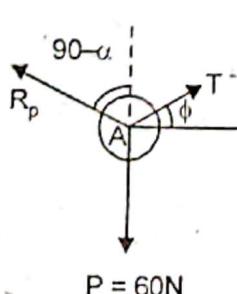


Ans. P = 60N, Q = 100N, $\alpha = 30^\circ$

Consider the eq^m. on Roller 'P'

$$\Sigma F_x = T \cos \phi - R_p \sin(90 - \alpha) = 0$$

$$T \cos \phi - R_p \cos \alpha \quad \dots(1)$$



$$\Sigma F_y = 0 \quad R_p \cos(90 - \alpha) - P + T \sin \phi = 0$$

...(2)

$$R_p \sin \alpha - P + T \sin \phi = 0$$

Consider the eq^m. on Roller Q.

$$\Sigma F_x = RQ \sin \alpha - T \cos \phi = 0 \quad \dots(3)$$

$$F_y = RQ \cos \alpha - \theta - T \sin \theta = 0 \quad \dots(4)$$

From eqⁿ. 1.

$$R_p = \frac{T \cos \phi}{\cos \alpha}$$

Put in equation 2

$$\frac{T \cos \phi}{\cos \alpha} \sin \alpha - P + T \sin \phi = 0$$

$$P = T(\sin \phi + \cos \phi \tan \alpha) \quad \dots(5)$$

From equation 3.

$$R_\theta = \frac{T \cos \alpha}{\sin \alpha}$$

$$\text{Put in equation 4. } \frac{T \cos \phi}{\sin \alpha} - \theta - T \sin \phi = 0$$

$$-Q = T(\cos \phi \cos \alpha - \sin \phi) \quad \dots(6)$$

divide (5) by (6)

$$\frac{\theta}{P} = \frac{(\cos \phi \cot \alpha - \sin \phi)}{(\cos \phi \tan \alpha + \sin \phi)}$$

$$\tan \phi = \frac{P \cot \alpha - Q \tan \alpha}{P + Q}$$

$$\tan \phi = \frac{\left(60\sqrt{3} - 100 \frac{1}{\sqrt{3}}\right)}{100 + 60} = \frac{1}{3\sqrt{3}}$$

$$\boxed{\phi = 10.9^\circ}$$

$$\text{From equation (6)} \quad T = \frac{\theta}{\theta \cos \phi \cot \alpha - \sin \phi} = \frac{100}{\cos 10.9^\circ \cos 30^\circ - \sin 109^\circ}$$

$$\boxed{T = 66.2 \text{ N}}$$

Q. 3.(b) A ladder 8 m long weighing 200 N is resting against a rough vertical wall. A man of 720 N climbs the ladder. At what position will he induce slipping? Take μ for both the contact surfaces of ladder 0.25.

Ans.

$$l = 8 \text{ m}$$

$$w_1 = 200 \text{ N}$$

3

C

T

V

V

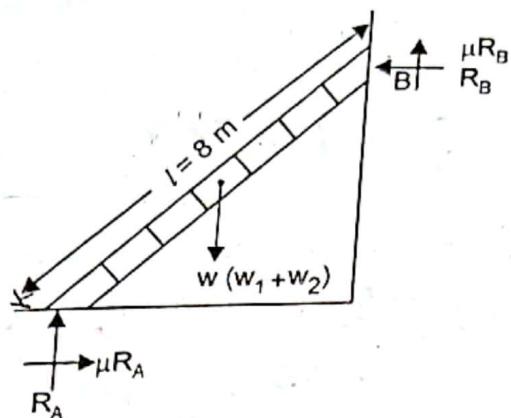
$$\text{men weight } (w_2) = 720 \text{ N}$$

$$\mu = 0.25$$

F.B.D

$$\sum F_x = 0$$

$$\mu R_A - R_B = 0 \Rightarrow 0.25 R_A = R_B$$



$$\sum F_y = 0$$

$$R_A + \mu R_B - W = 0$$

$$R_A + 0.25 R_A = 200 + 720 \text{ N}$$

$$R_A = \frac{920}{1.25} = 3680 \text{ N}$$

$$R_B = 920 \text{ N}$$

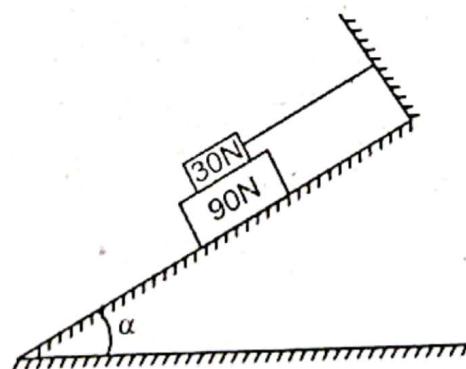
Taking moment about A

$$R_B \times 8 \sin 45^\circ + 0.25 R_B (8 \cos 45) - 920 (4 \cos 45) = 0 \\ = 5204 + 1301 - 2602$$

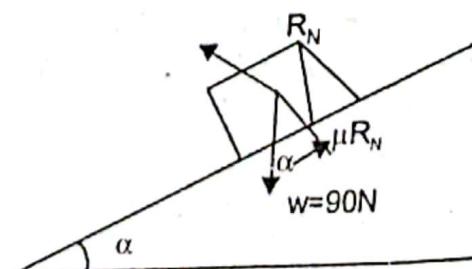
$$l = 3.903 \text{ m}$$

At distance 3.903 m the ladder will slip.

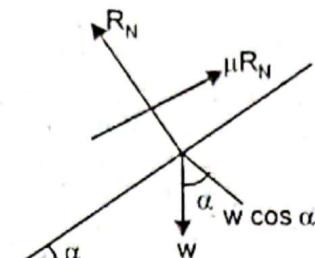
Q.4. (a) What should be the value of angle α in fig. (c), so that the motion of 90 N block impends down the plane. Take μ for all the surfaces to be 0.33.



Ans.



F.B.D



$$\Sigma F_x = 0$$

and

$$\Sigma F_y = 0$$

$$\mu R_N = w \sin \alpha \quad \dots(1)$$

$$R_N = w \cos \alpha \quad \dots(2)$$

$$R_N \times 0.33 = 90 \sin \alpha$$

$$\sin \alpha = \frac{0.33}{90}$$

$$R_N = 272.7 \sin \alpha$$

$$R_N = 90 \cos \alpha$$

$$90 = 272.7 \tan \alpha$$

$$\frac{90}{272.7} = \tan \alpha$$

$$\alpha = \tan^{-1} 0.33$$

$$\boxed{\alpha = 18.2^\circ}$$

Q.4. (b) For a flat belt drive, prove that: $T_1/T_2 = e^{\mu\theta}$

where T_1 = tension on the tight side of the belt, T_2 = tension on the slack side of the belt, μ = coefficient of friction and θ = Angle of contact.

Ans. Derive the relationship:

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

T_1 = Tension Tight side

T_2 = Tension and slack side

β = angle of contact

Let

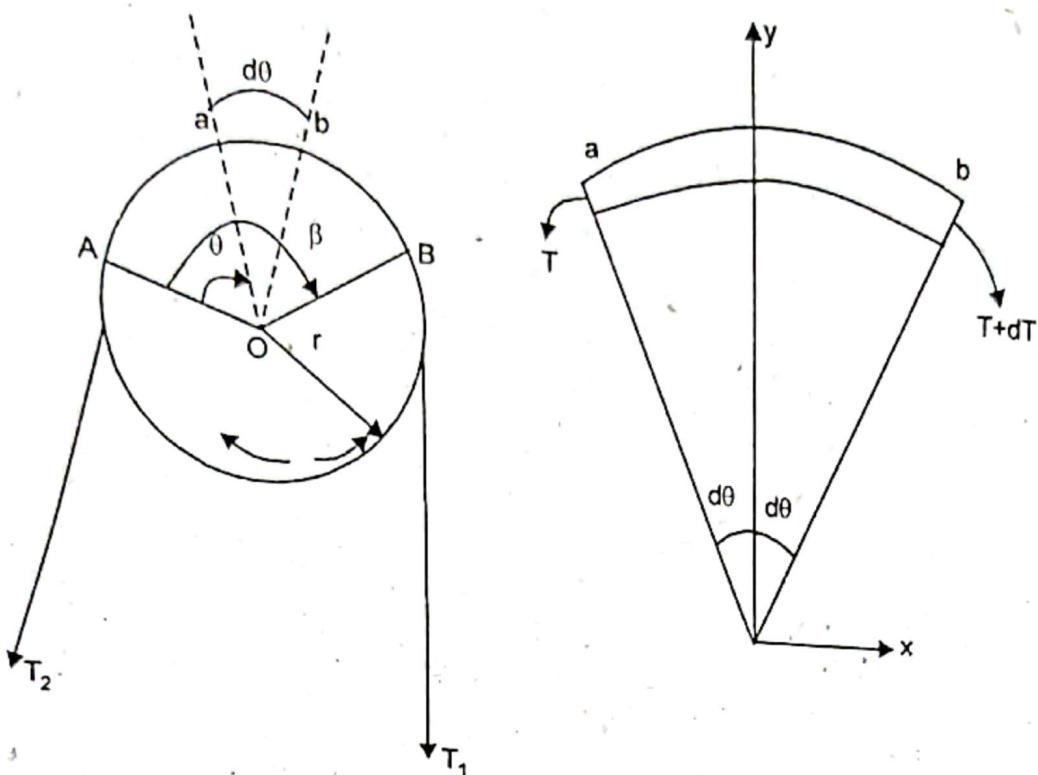
Tension at a = T

Tension at b = $T + dT$

Normal reaction = dN

Frictional force = μdN

μ = coefficient of friction



Equation of equilibrium

$$\Sigma F_x = 0$$

$$T \cos \frac{d\theta}{2} + dF - (T + dT) \cos \frac{d\theta}{2} = 0$$

$$\Sigma F_y = 0$$

$$AN = T \sin \frac{d\theta}{2} - (T + dT) \sin \frac{d\theta}{2} = 0$$

angle $d\theta$ is very small put

$$\cos \frac{d\theta}{2} \approx 1 \text{ and } \sin \frac{d\theta}{2} \approx \frac{d\theta}{2}$$

$$\frac{dQ}{2} \approx \frac{dQ}{2}$$

$$dF = \mu dN \text{ is above equation}$$

$$T + \mu dN - (T - dT) = 0$$

$$dT - \mu dN = 0$$

and $dN - T \frac{d\theta}{2} (T + dT) \frac{d\theta}{2} = 0$

$$dN - T d\theta = 0 \text{ after neglecting } \left(dT \frac{d\theta}{2} \right)$$

Eliminate dN in equation (1) and (2)

$$dT = \mu T d\theta \text{ Integrating both side}$$

$$\frac{dT}{T} = \mu d\theta \text{ Integrating both side}$$

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^\beta \mu d\theta = [\log_e T]_{T_2}^{T_1} = \mu \beta$$

$$\log \frac{T_1}{T_2} = \ln \frac{T_1}{T_2} = \mu \beta$$

or

$$\boxed{\frac{T_1}{T_2} = e^{\mu \beta}} \text{ also } \boxed{2.3 \log_{10} \frac{T_1}{T_2} = \mu \beta}$$

SECOND TERM EXAMINATION [APRIL 2015]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME-110]

Time. 1 Hour

MM : 30

Note: Q.No.1 is compulsory. Attempt any two more questions from the rest.

Q.1. (a) Explain the term "statically determinate truss" and also give mathematical condition for rigid or perfect truss. (2x5=10)

Ans. A truss is statically determinate if the equation of static equilibrium alone are sufficient to determine the axial force's in the member's without the need of considering their deformation.

condition for perfect truss i.e. \rightarrow

$$m = 2J - 3$$

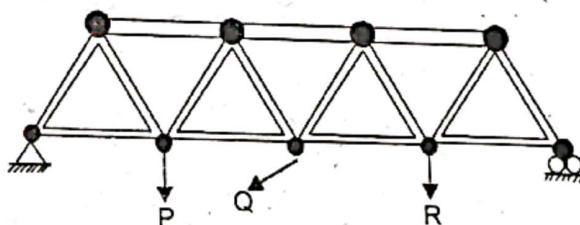
Member's Joints

Q.1. (b) Explain the difference between frame, truss and machine.

Ans. Frame and Truss:

Frame: it is structure consisting of several bars or members pinned together and in which one or more than one of its member is subjected to more than two forces. They are designed to support loads and stationary structures.

Truss: It is a system of uniform bar's or members joined together at their end's by riveting or welding and constructed to support loads. Every member of a truss is a two force member.



Machine: The machines are structures designed to transmit and modify forces and contains same moving members.

Q.1.(c) State the basic assumptions for the perfect truss ?

Ans. Refer Q.No.1(e) First Term Exam 2018 (Page No.: 2-2018)

Q.1. (d) Explain the difference between centroid and centre of gravity.

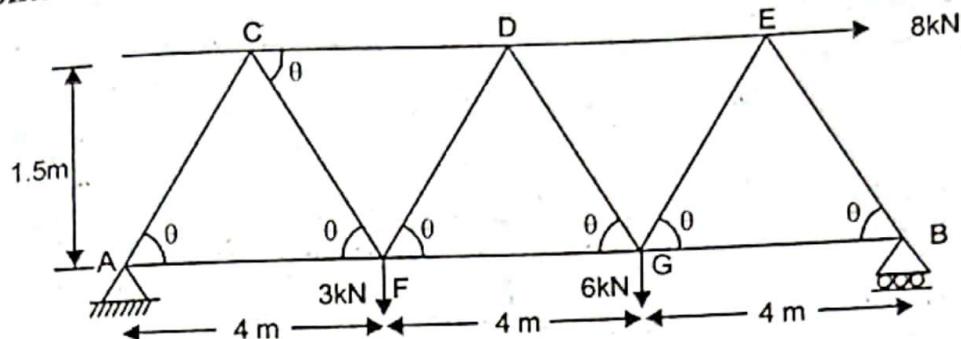
Ans. Center of gravity: C.G. is that point through which the resultant of the system of parallel forces formed by the weight of all the particle's of the body passes.

Centroid: Basically centroid and C.G. are same but C.G. is for 3D body and centroid is for one dimensional body or line segments

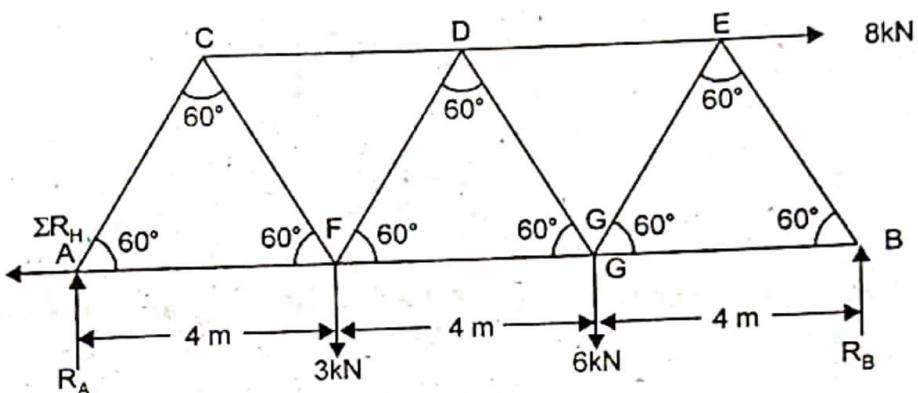
Q.1. (e) Explain that moment of area of any plane figure about a line passing through its centroid is zero.

Ans. The moment of area of a plane figure about a line passing through its centroid is zero because the center of gravity of any plane figure lies on its reference line which is zero.

Q.2. Determine the forces in the truss shown in fig. below. which is subjected to horizontal and vertical loads. Mention the nature of forces also. (10)



Ans.



$$\sum F_y = 0$$

$$R_A + R_B = 9kN$$

$$\sum M_x = 0$$

$$R_B \times 12 = 6 \times 8 + 3 \times 4$$

$$R_B = \frac{60}{12} = 5kN$$

$$R_A = 4kN$$

$$\sum R_H = 8kN$$

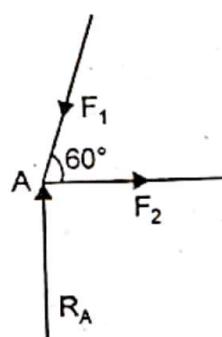
$$R_B = \frac{60}{12} = 5kN$$

$$\sum F_x = 0$$

$$F_1 = \frac{R_A}{\sin 60} = \frac{4k}{0.866} = 4.6kN \quad \dots(T)$$

$$F_2 = F_1 \cos 60 = 2.3 kN \quad \dots(C)$$

At Point A



At Point B

$$\begin{aligned} \sum F_x &= 0 \\ F_7 \cos 60 - F_8 &= 0 \\ R_B - F_7 \sin 60^\circ &= 0 \end{aligned}$$

$$F_7 = \frac{R_B}{\sin 60^\circ} = 5.77kN(C)$$

$$F_8 = \frac{F_7}{\cos 60^\circ} = 11.5 \text{ kN(T)}$$

At Point F:

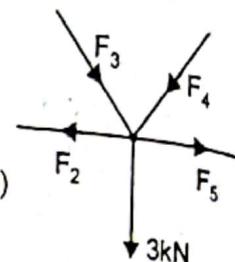
$$F_5 \cos 60^\circ + F_3 \cos 60^\circ - F_3 = 0$$

$$F_3 \sin 60^\circ - F_4 \sin 60^\circ - 3 = 0$$

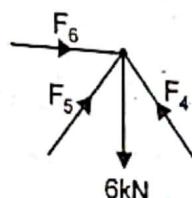
$$F_7 = 5.7 \text{ kN (T)}$$

$$F_3 = \frac{3 \times 5.7 \sin 60^\circ}{\sin 60^\circ} = 17.1 \text{ (C)}$$

$$F_4 = 4.3 \text{ kN(C)}$$



At Point E:



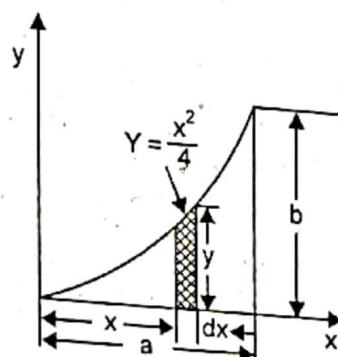
$$F_6 = F_4 \cos 60^\circ + F_5 \cos 60^\circ$$

$$F_6 = 4.3 \cos 60^\circ + 6.2 \cos 60^\circ$$

$$F_6 = 5.25 \text{ kN (T)}$$

Q.3. Determine the coordinates of the C.G. of the shaded area between the parabola $Y = x^2/4$ and the straight line $Y = X$.
Ans.

$$Y = \frac{x^2}{4} \quad \dots(1)$$



Equation of Parabola

$$b = Ra^2 \text{ or } R = \frac{b}{a^2}$$

$$y = \frac{b}{a^2} \times \frac{x^2}{4} \text{ or } x = \frac{a \sqrt{y}}{\sqrt{b}}$$

By Vertical strip:

$$dA = y dx$$

$$n_c = \frac{\int x dA}{\int dA} = \frac{\int x \cdot y dx}{\int y dx} = \frac{\int_0^a n \left(\frac{b}{a^2} x^2 \right) dx}{\int_0^a \frac{b}{a^2} x^2 dx}$$

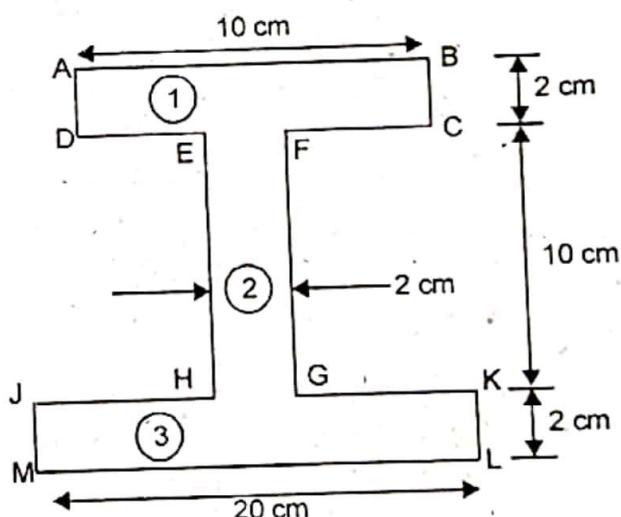
$$X_c = \frac{\left[\frac{b}{a^2} \frac{x^4}{x} \right]_0^a - \left[\frac{b}{a^2} \frac{x^3}{3} \right]_0^a}{\int dA} \Rightarrow X_c = \frac{3}{4}a$$

$$Y_c = \frac{\int y dA}{\int dA}$$

$$Y_c = \frac{\int \frac{y}{2} y dx}{\int y dx} = \frac{\int_0^a \frac{1}{2} y^2 dx}{\int_0^a y dx} = \frac{\left[\frac{b^2}{2a^4} \frac{x^5}{5} \right]_0^a}{\left[\frac{b}{a^2} \frac{x^3}{3} \right]_0^a} = \frac{3}{10}b$$

$$Y_c = \frac{3}{10}b$$

Q.4. Find the moment of inertia of the section shown in fig. below, about the centroidal axis X-X, perpendicular to the web.



Ans.

$$Y = \frac{A_1 Y_1 + A_2 Y_2 + A_3 Y_3}{A_1 + A_2 + A_3}$$

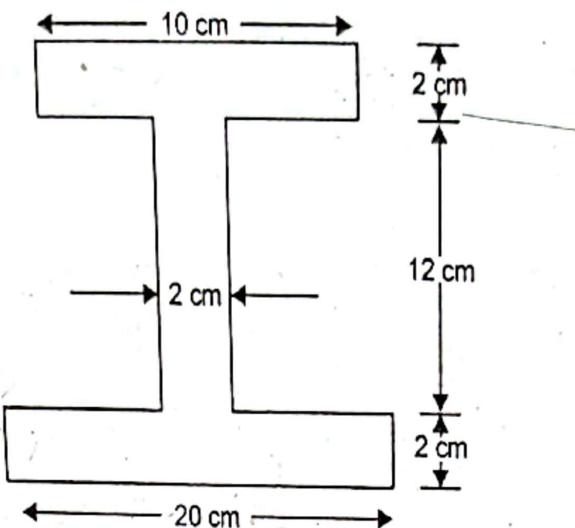
$$Y = \frac{20 + 2 \times 1 + 2 \times 12 \times 8 + 10 \times 2 \times 15}{20 \times 2 + 12 \times 2 + 8 \times 2}$$

$$= \frac{40 + 192 + 300}{40 + 24 + 16}$$

$$Y = 6.65 \text{ cm.}$$

$$I_{xx} = \sum I_{nn} + \sum A_n Y_n^2$$

$$I_{nn} = I_{11} + I_{22} + I_{33} + A_1 Y_1^2 + A_2 Y_2^2 + A_3 Y_3^2$$



Y_1, Y_2 and Y_3 are measured from C.G.

$$Y_1 = 15 - 6.65 = 8.35 \text{ cm}$$

$$Y_2 = 8 - 6.65 = 1.35 \text{ cm}$$

$$Y_3 = 6.65 - 1 = 5.65 \text{ cm}$$

$$I_{gg} = \frac{1}{12} bd^3$$

$$I_{xx} = \left[\frac{1}{12} \times 8 \times (2)^3 + \frac{1}{12} \times 2 \times 12^3 + \frac{1}{12} \times 12 \times 2^3 \right]$$

$$I_{xx} = 2212.33 \text{ cm}^4$$

$$I_{yy} = \left[\frac{1}{12} \times 2 \times 8^3 + \frac{1}{12} \times 12 \times 2^3 + \frac{1}{12} \times 12 \times 2^3 \right]$$

$$I_{yy} = 381.33 \text{ cm}^4$$

Q.5. What is polar moment of inertia

Ans. Polar M.O.I. is defined as the M.O.I. of an area of a plane fig. with respect to an axes perpendicular to the x-y plane and passing through a pole $J = I_{xx} + I_{yy}$

FIRST TERM EXAMINATION [FEB. 2016]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME-110]

Time : 1½ Hrs.

M.M. : 30

Note: Attempt any three questions including Q no. 1 which is compulsory. Select one question from each unit.

Q.1. What is difference between Resultant and Equilibrium?

Ans.

Resultant	Equilibrium
When there are number of forces acting at a point, we can replace all forces by a single force which can produce same effect of all the forces acting together. This single force is resultant of forces	Equilibrium force is the force needed to bring system in equilibrium. Therefore it is equal but opposite to resultant of all forces.

Q.1. (b) What is essential conditions for three, non parallel, Co- planer forces to be in equilibrium? Explain with example.

Ans. The condition for three non-parallel coplanar force's which is in equilibrium condition is Lamie's Theorem which is

$$\frac{P}{\sin \alpha} = \frac{\theta}{\sin \beta} = \frac{R}{\sin \gamma}$$

For e.g.:

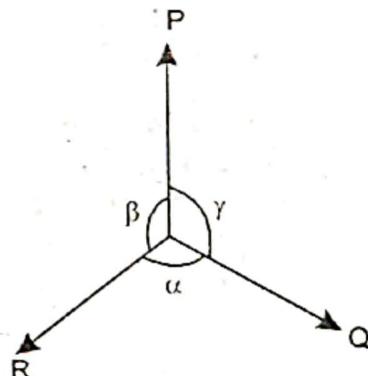
$$P = 100 \text{ N}, \theta = 50^\circ$$

$$R = ? (\text{unknown})$$

$$\text{where } \alpha = 90^\circ, \beta = 120^\circ, \gamma = 150^\circ$$

$$\frac{100}{\sin 90} = \frac{50}{\sin 120} = \frac{R}{\sin 150^\circ}$$

$$R = 50 \text{ N}$$

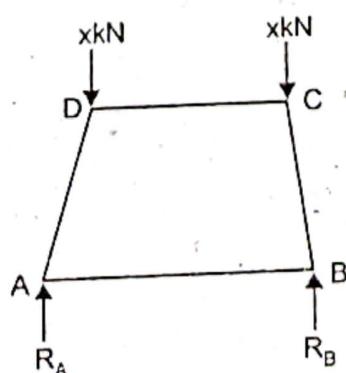


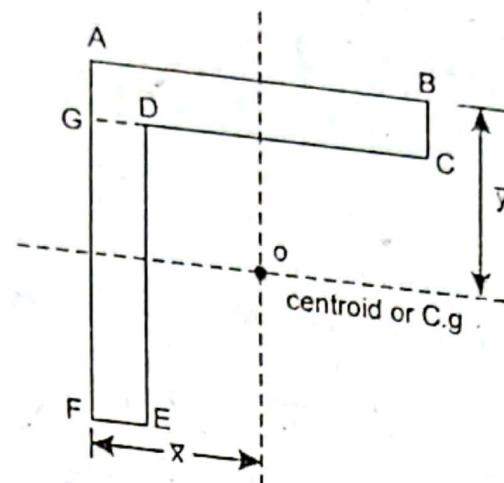
Q.1. (c) If a given area has two or more symmetry where will its centroid be located? Explain with example.

Ans.

$$\bar{X} = \frac{a_1 X_1 + a_2 X_2}{a_1 + a_2}$$

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2}{Y_1 + Y_2}$$





Q.1. (d) Why is coefficient of static friction greater than coefficient of kinetic friction.

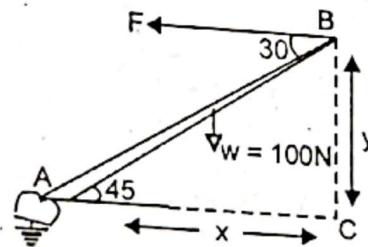
Ans. The value of static friction (μ_s) is always greater than the value of kinetic friction (μ_k) because at this time the maximum frictional force exerted at the time of impending motion i.e. the motion is just about to begin hence the maximum effect observed under the static condition that why its value is higher.

Q.1. (e) Give 2 advantages of method of section.

- Ans.** (a) It is used to find forces in any membrane directly.
 (b) It is short and quick method.

UNIT-I

Q.2. (a) A bar AB of weight 100N is hinged at A and is pulled by a cable attached at B by Force F. Find force F and magnitude and direction of reaction A if bar is in equilibrium as shown in Fig.



Ans. Taking moment at C

$$R_A \sin 45 \times x - f \cos 15 \times y = w \times \frac{x}{2}$$

$$\tan \theta = 45$$

$$\tan 45 = \frac{y}{x}$$

$$y = x$$

$$R_A \sin 45 - F \cos 15 = \frac{100}{2}$$

$$R_A \sin 45 - F \cos 15 = 50$$

$$\sum f_x = 0$$

$$F \cos 15^\circ = R \cos 45^\circ$$

$$F = R \frac{\cos 45^\circ}{\cos 15^\circ}$$

$$F = .732 R$$

$$\Sigma M_c = 0$$

$$R_A \sin 45^\circ - F \cos 15^\circ = 50$$

$$R_A \sin 45^\circ - .732 R_A \cos 15^\circ = 50$$

$$R_A \times .707 - .732 \times .965 R_A = 50$$

$$.707 R_A = .706 R_A$$

$$.001 R_A = 50$$

$$R_A = 50 \text{ KN}$$

$$F = .732 \times 50000$$

$$F = 36.6 \text{ KN.}$$

Q.2. (b) Two identical iron spheres, each of radius 5 cm and weight 150 N are connected with string of length 16 cm and rest on horizontal smooth floor. Another sphere of radius 6 cm and weight 200 N rest over them. Determine tension in string and reaction at all contact surface.

Ans.

$$r_c = r_B = 5 \text{ cm}; l_{Bc} = 6 \text{ cm}$$

$$r_A = 6 \text{ cm}$$

Solving for Cylinder 1

$$\Sigma f_y = 0$$

$$R_D \sin \theta + R_\epsilon \sin \theta = 200$$

$$2R_D \sin \theta = 200 (R_D = R_\epsilon)$$

$$R_D \sin \theta = 100$$

$$\sin \theta = \frac{AH}{AB} = \frac{\sqrt{11^2 - 8^2}}{11} = .686$$

$$\sin \theta = .686$$

$$R_D = \frac{100}{\sin \theta} = 145.77$$

$$R_D = R_\epsilon = 146.$$

Solving for Cylinder 2

$$\Sigma f_y = 0$$

$$T - R_D \cos \theta = 0$$

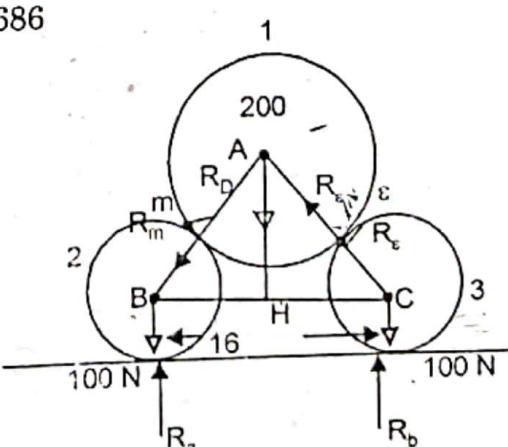
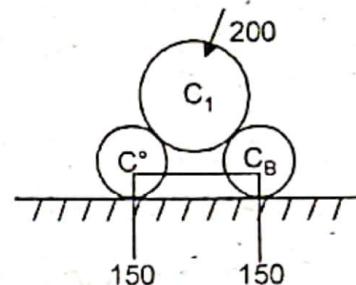
$$T = R_D \cos \theta = 146 \times \frac{8}{11} = 106N$$

$$\Sigma f_y = 0$$

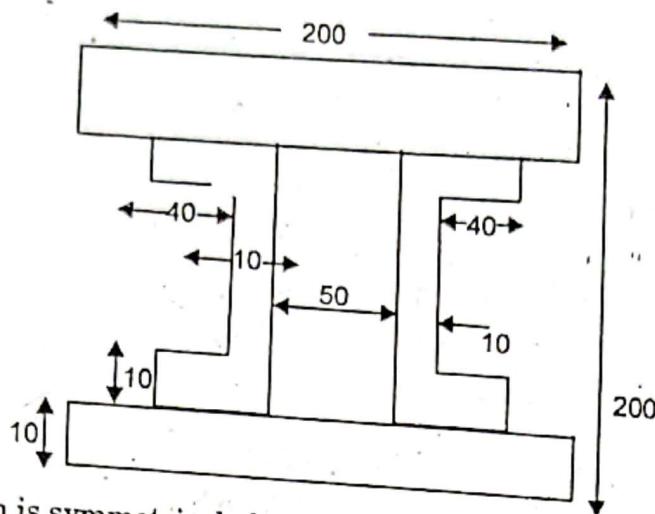
$$R_G - 100 - R_D \sin \theta = 0$$

$$R_G - 100 - 146 \times .686 = 0$$

$$R_F = R_G = 200 \text{ N.}$$



Q.3. (a) Find MOI about horizontal axis passing through CG of given fig.



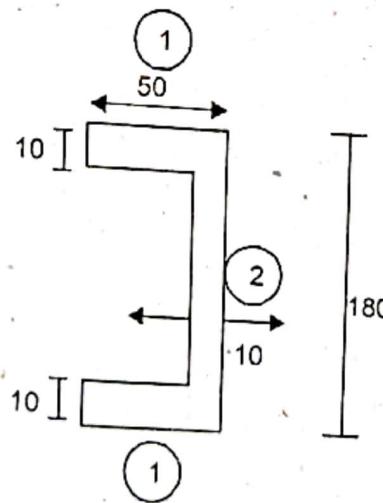
Ans. As section is symmetrical about xx axis
MOI of one top or bottom plate about axis through its C_G and \perp to $x-x$ axis,

$$I_{G1} = \frac{200 \times (10)^3}{12} = 16666$$

Distance between C_G of plates from $x-x$ axis

M.O.I of top and bottom plate about $x-x$ axis

$$2 [7500 + 200 \times 10 \times 95^2] = 36115000$$



M. O. I of Part (1) of one channel section about an axis through C_G and \perp to $x-x$ axis

$$I_{G2} = \frac{50 \times (10)^3}{12} = 4166$$

Distance of C_G of this part from $x-x$ axis

$$h_2 = 90 - 5 = 85$$

M.O.I of part (1) about $x-x$

$$\begin{aligned} I_{x2} &= I_{G2} + ah_2^2 \\ &= 4 [4166 + 50 \times 10 \times 85^2] \\ &= 14466.664 \end{aligned}$$

Similarly M.O.I of Part 2 of channel about axis through C_G and || to xx

$$I_{G3} = 2 \times \left[\frac{10 \times 160^3}{12} \right] = I_{G3} = 6826666$$

M.O.I of whole built up section about axis C_G & || to $x-x$ axis

$$I_{xx} = I_{xx1} + I_{xx2} + I_{G3}$$

$$36115000 + 14466664 + 6826666 = 57408330$$

Q.3. (b) Determine Centroid of bent wire ABCDE

Ans.

$$\text{Length of } AB = l_1 = 80$$

C_1 is at centre of line

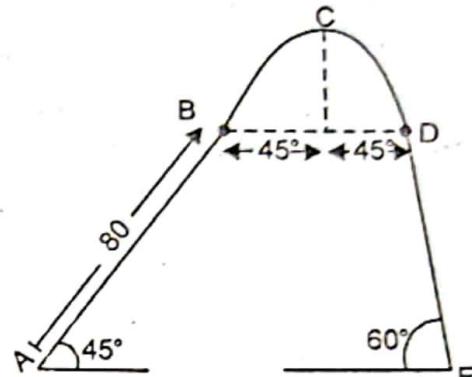
$$x_1 = 80 \cos 45 = 56.56$$

$$y_1 = 80 \sin 45 = 56.56$$

Length of semicircular BC

$$l_2 = \pi \times r = \pi \times 45 = 141.37$$

$$\text{Centroid } C_2 = \frac{2r}{\pi} = \frac{2 \times 45}{\pi} = 28.247$$



Coordinate of Centroid C_2 of semi-Circular portion

$$x_2 = 80 \cos 45 + 45 = 101.56$$

$$y_2 = 80 \sin 45 + 45 = 101.65$$

$$x_3 = 80 \cos 45 + 90 + 80 \cos 60 = 186.56$$

$$y_3 = 80 \sin 45 + 90 + 80 \sin 60 = 215.8$$

$$\bar{x} = \frac{x_1 l_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3}$$

$$\bar{x} = \frac{56.56 \times 80 + 141.37 \times 101.56 + 80 \times 186.56}{80 + 80 + 141.37}$$

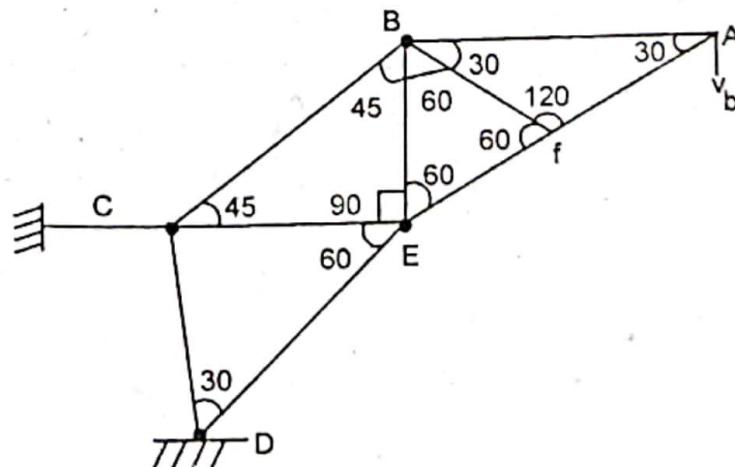
$$\bar{x} = 112.178$$

$$\bar{y} = y_1 l_1 + y_2 l_2 + y_3 l_3 / (l_1 + l_2 + l_3)$$

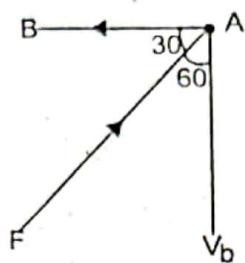
$$= \frac{56.56 \times 80 + 101.56 \times 141.37 + 215.8 \times 80}{80 + 80 + 141.37}$$

$$\bar{y} = 119.94.$$

Q.4. (a) A truss is loaded and supported as shown in Fig. Compute axial force in all the members of truss.



Ans. Joint A



$$\Sigma H = 0$$

$$F_{AB} = F_{AF} \cos 30$$

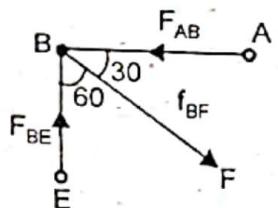
$$F_{AB} = 10.39$$

$$\Sigma V = 0$$

$$6 = F_{AF} \cos 60$$

$$F_{AF} = \frac{6}{\cos 60} = 12$$

Joint B



$$\Sigma x = 0$$

$$F_{AB} = F_{BF} \cos 30$$

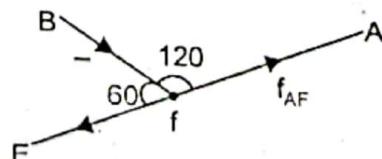
$$F_{BF} = 12$$

$$\Sigma V = 0$$

$$F_{BE} = F_{BF} \cos 60$$

$$F_{BE} = 6$$

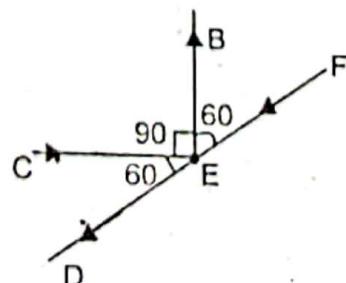
Joint F



$$F_{BF} = F_{AF}$$

$$12 = 12$$

Joint E



$$\Sigma V = 0$$

$$F_{BE} = F_{FE} \cos 60 + F_{ED} \sin 60$$

$$6 = 12 \times .5 + F_{ED} \sin 60^\circ$$

$$\beta' = \beta + F_{ED} \sin 60^\circ$$

$$F_{ED} = 0$$

$$F_{CD} = 0$$

$$\Sigma x = 0$$

$$F_{CE} = F_{FE} \sin 60^\circ + F_{ED} \cos 60^\circ$$

$$F_{CE} = 12 \sin 60^\circ + F_{ED} \times .5$$

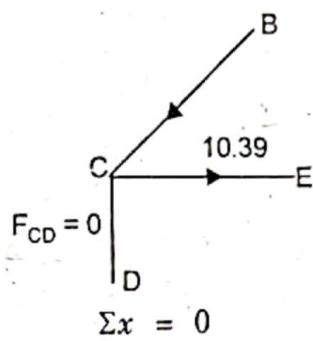
$$F_{CE} = 10.39 + F_{ED} \times .5$$

$$F_{CE} = 10.39$$

[As $F_{ED} = 0$]

Ans

Point C



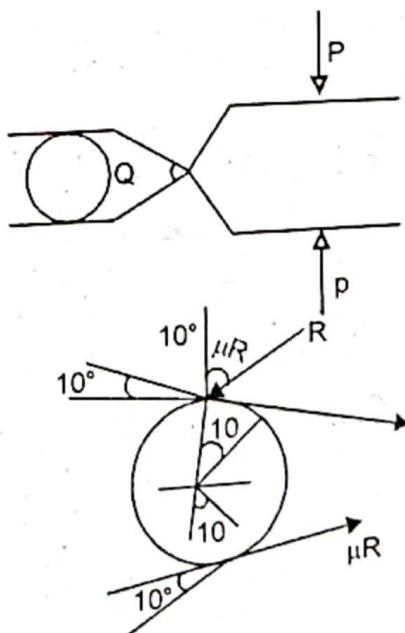
$$\Sigma x = 0$$

$$F_{BC} \cos 45^\circ = 10.39$$

$$F_{BC} = 14.69$$

Q.4. (b) The tongs are used to handle hot steel tubes that are being heat treated in an oil bath. For a 20° jaw opening, what is the coefficient of static friction between the jaws & the tube that will enable the tongs to grip the tube without slipping? (3)

Ans.



$$\Sigma f_x = 0$$

$$\mu R \cos 10^\circ + \mu R \cos 10^\circ - R \sin 10^\circ - R \sin 10^\circ = 0$$

$$2\mu R \cos 10^\circ - 2R \sin 10^\circ = 0$$

$$\mu = \tan(10)$$

$$\mu = .176$$

**FIRST TERM EXAMINATION [FEB. 2017]
SECOND SEMESTER [B.TECH.]
ENGINEERING MECHANICS
[ETME-110]**

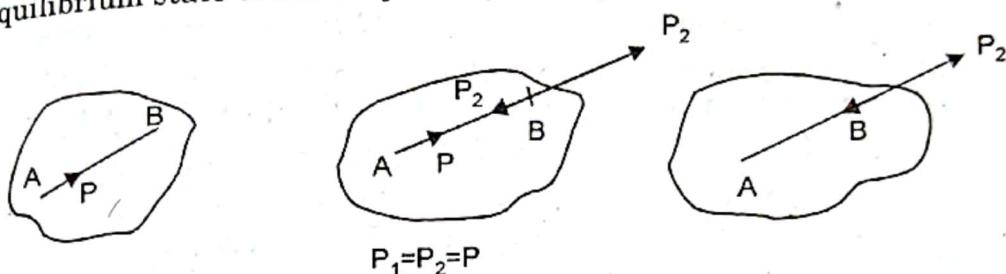
M.M. : 30

Time : 1½ hrs.

Note: Q.No. 1 is compulsory. Attempt any two more Questions from the rest.

Q.1. (a) Describe the 'principle of transmissibility of force' with an example. (2)

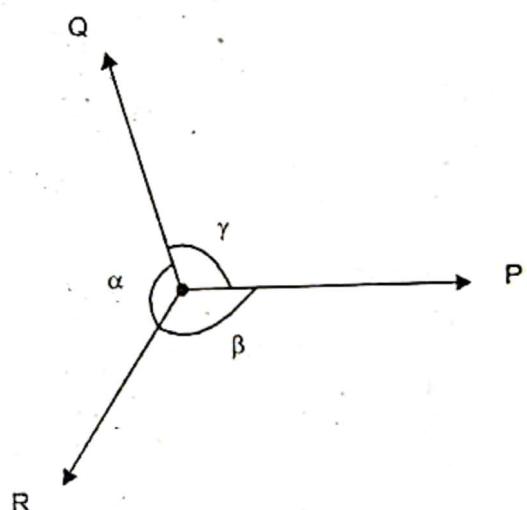
Ans. When the point of application of force acting on a body is shifted to any other point on the line of action of the force without changing its direction, there is no change in the equilibrium state of the body



Q.1. (b) State the Lame's theorem for the equilibrium of a body. (2)

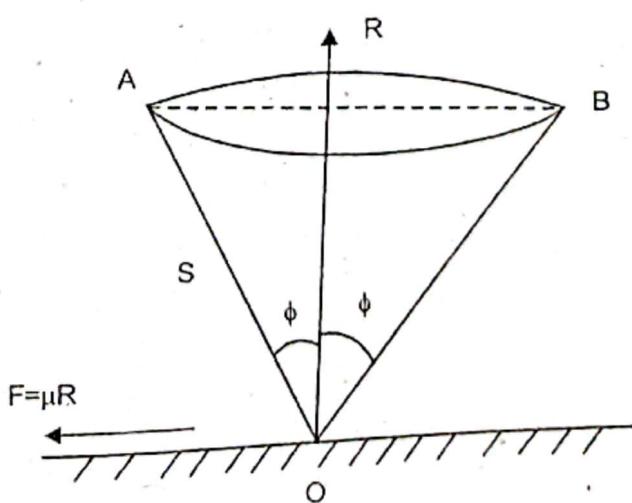
Ans. If three forces acting a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces.

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$



Q.1. (c) Describe "cone of friction".

Ans.



Normal reaction (R) and frictional force (F) meet at O . $R = \sqrt{F^2 + R^2}$ and it makes an angle ϕ with the normal reaction. O is the vertex of cone whose axis is R and semi vertex angle is ϕ . Such a cone is called the cone of friction.

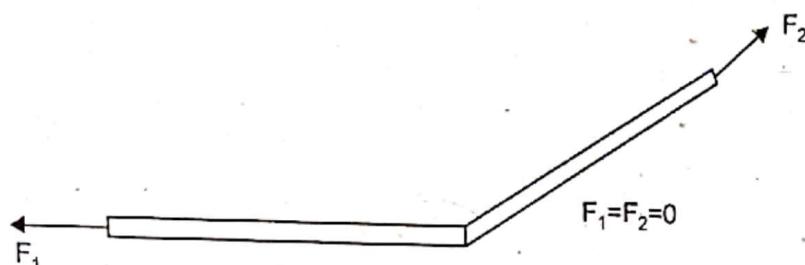
Q.1. (d) State the assumptions in the analysis of perfect truss.

Ans. Assumptions of a truss

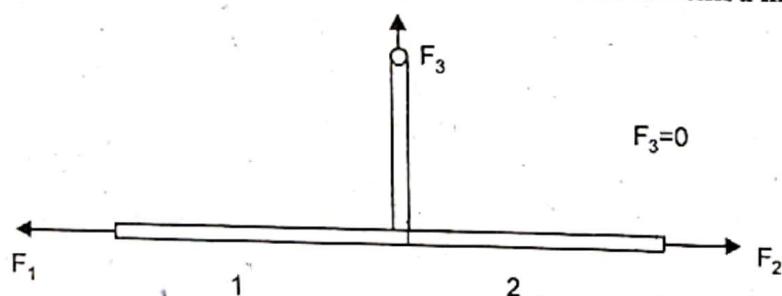
- (1) Load is applied only at the joints
- (2) All the members are straight two force members. Forces are collinear with centerline of the members
- (3) Connection b/w members are frictionless and pin connections
- (4) Truss is statically determinate
- (5) Weight of the members is negligibly small : unless mentioned.

Q.1. (e) What is a zero force member in a truss and how it is identified?

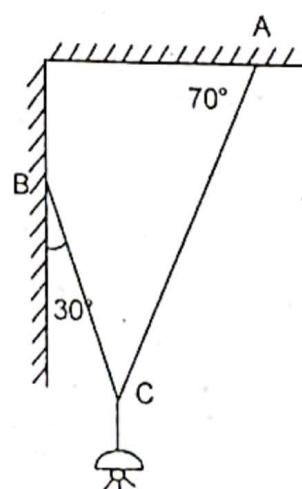
Ans. (1) When two members meeting at a joint are not collinear and there is no external force acting on the joint, then the forces in both the members are zero.



(2) When there are 3 members meeting at a joint of which two are collinear and the third is at an angle and if there is no load at the joint the force in the third member



Q.2. (a) An electric light fixture weighing 20N hangs from point C, by two strings AC and BC as shown in Fig. Determine the tension in the strings AC and BC.



Ans.

$$\Sigma F_x = 0$$

$$T_1 \cos 70^\circ - T_2 \cos 60^\circ = 0$$

$$T_1 \cos 70^\circ = T_2 \cos 60^\circ$$

$$T_1 = T_2 \frac{\cos 60^\circ}{\cos 20^\circ}$$

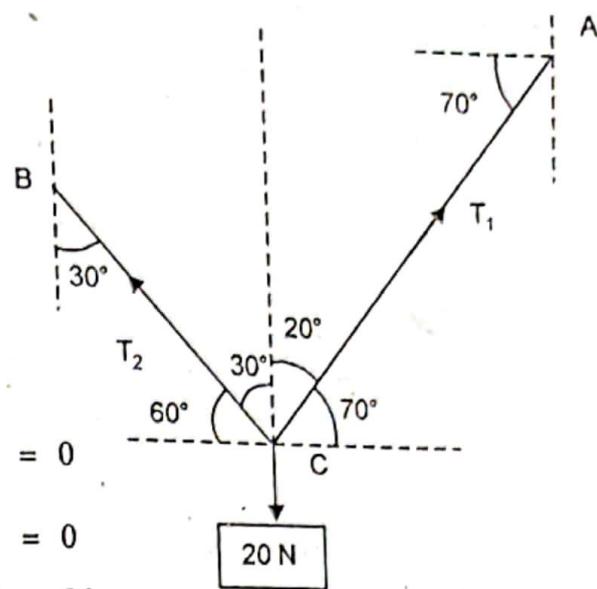
$$T_1 = 8.9297 \times \frac{0.5}{0.3420} \\ = 13.055 \text{ N}$$

$$\Sigma F_y = 0$$

$$T_1 \sin 70^\circ + T_2 \sin 60^\circ - 20 = 0$$

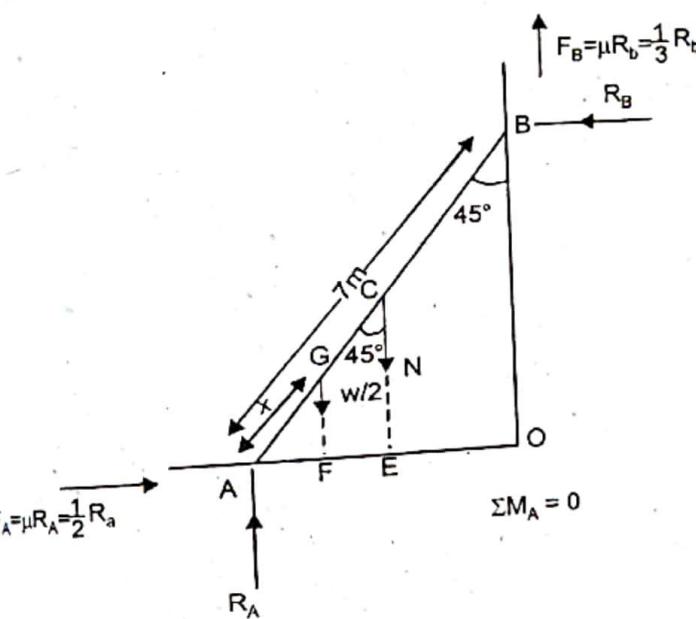
$$\frac{T_2 \cos 60^\circ}{\cos 70^\circ} \times \sin 70^\circ + T_2 \sin 60^\circ - 20 = 0$$

$$T_2 \times 0.5 \times 2.7474 + T_2 \times 8660 = 20 \\ T_2 = 20/2.2397 \\ = 8.9297 \text{ N}$$



Q.2. (b) A 7.0 m long ladder rests against a vertical wall with it makes an angle of 45° and on floor, if a man whose weight is one half of that of the ladder climbs it, at what distance along the ladder will he be, when the ladder is about to slip. $\mu = 1/3$ for wall and ladder and $\mu = 1/2$ for ladder and floor. (7)

Ans.



$$\cos 45^\circ = \frac{OB}{AB} \\ OB = 7 \times \cos 45^\circ \\ = 4.9497 \text{ N}$$

$$\sin 45^\circ = \frac{OA}{AB} \\ OA = 4.9497 \text{ N}$$

$$\sin 45^\circ = \frac{AE}{AC} \\ AE = 2.4748$$

$$\sin 45^\circ = \frac{AF}{AC} \\ AF = x \sin 45^\circ \\ = .707x$$

$$-R_b \times OB - \frac{1}{3} R_b \times OA = W \times 2.4748 + \frac{W}{2} \times .707x$$

$$-R_b \times 4.9497 - \frac{1}{3} R_b \times 4.9497 = 2.4748 \times W + \frac{W}{2} \times .707x \quad \dots(1)$$

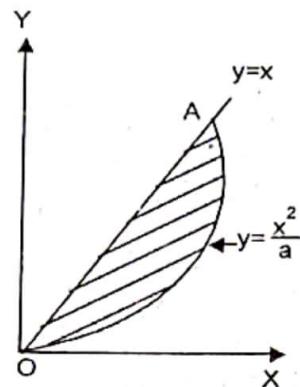
$$-6.5996 R_b = 2.4748 \times W + \frac{W}{2} \times .707x$$

$$R_A + \frac{1}{3} R_b = W + \frac{W}{2} = \frac{3W}{2} \quad \dots(2)$$

$$\frac{3R_A + R_B}{3} = \frac{3W}{2}$$

Q.3. (a) Determine the coordinates of the centroid of the shaded area formed by the intersection of a straight line $y = x$ and a parabola $y = \frac{x^2}{a}$, as shown in Fig.

(10)

**Ans.**

$$y = x \quad \dots(1)$$

$$y = \frac{x^2}{a} \quad \dots(2)$$

$$\text{Area of strip} = (pq)dx$$

$y_1 \rightarrow$ lies on straight line OA

$y_1 \rightarrow$ lies on parabola oA

$$dA = y'dx$$

$$dA = (y_1 - y_2)dx$$

$$y_1 = x; y_2 = \frac{x^2}{a}$$

$$\text{Area of strip } dA = \left(x - \frac{x^2}{a}\right)dx$$

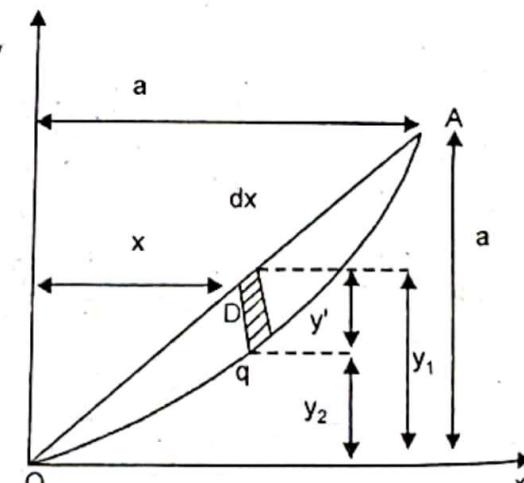
Distance of the centroid of the strip from the y-axis = x

$$x_c = \frac{\int x dA}{\int dA} = \frac{\int_0^a x \left(x - \frac{x^2}{a}\right) dx}{\int_0^a \left(x - \frac{x^2}{a}\right) dx} = \frac{a}{2}$$

$$y_c = \frac{\int y dA}{\int dA} < \frac{\int \left(\frac{y_1 + y_2}{2}\right) (y_1 - y_2) dx}{\int (y_1 - y_2) dx}$$

$$y_1 = x, y_2 = \frac{x^2}{a}$$

$$y_c = \frac{\int \frac{1}{2} \left(x - \frac{x^2}{a}\right) \left(x + \frac{x^2}{a}\right) dx}{\int_0^a \left(x - \frac{x^2}{a}\right) dx} = \frac{2a}{5}$$



Q.4. Determine the forces in each members of the truss as shown in fig.3.
(10)

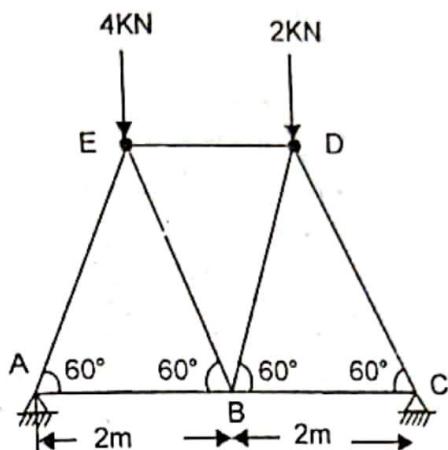
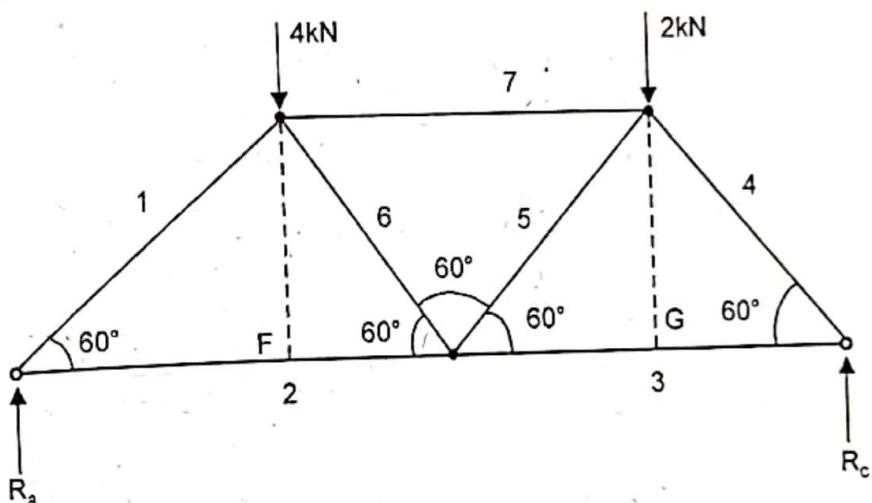


fig.3

Ans.

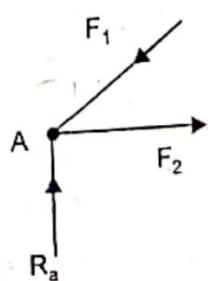
$$AF = AE \cos 60^\circ = 2 \times 0.5 = 1 \text{ m}$$

$$\begin{aligned} AG &= AB + BG = AB \perp BD \cos 60^\circ \\ &= 2 \perp 2 \times 0.5 = 3 \end{aligned}$$

$$\sum m_n = 0$$

$$R_c \times 4 = 2 \times 3 + 4 \times 1 = 10 \quad R_c = 2.5 \text{ kN}$$

$$R_c = (4 + 2) - 2.5 = 3.5 \text{ kN}$$

Joint A

$$\sum F_x = 0 \quad F_2 - F_1 \cos 60^\circ = 0$$

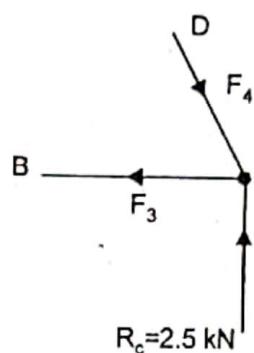
$$\sum F_y = 0 \quad R_a - F_1 \sin 60^\circ = 0$$

$$F_1 = R_a / \sin 60^\circ = 3.5 / 0.866$$

$= 4.04 \text{ kN}$ (Compressive)

$$F_1 = F_1 \cos 60^\circ = 4.04 \times 0.5 = 2.02 \text{ kN}$$
 (Tensile)

Joint C

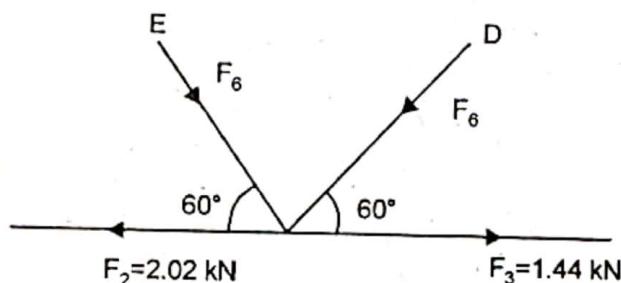


$$\begin{aligned}\Sigma F_x &= 0 \\ F_4 \cos 60^\circ - F_3 &= 0 \\ R_c - F_4 \sin 60^\circ &= 0\end{aligned}$$

$$F_4 = \frac{R_c}{\sin 60^\circ} = \frac{2.5}{0.866} = 2.88 \text{ kN} \text{ (compressive)}$$

$$F_3 = F_4 \cos 60^\circ = 2.88 \times 0.5 = 1.44 \text{ kN} \text{ (Tensile)}$$

Joint B



$$\begin{aligned}\Sigma F_x &= 0 \\ 1.44 - 2.02 + F_5 \cos 60^\circ + F_6 \cos 60^\circ &= 0\end{aligned}$$

$$F_5 + F_6 = 1.16$$

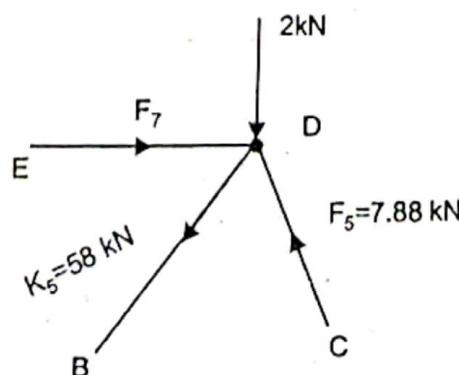
$$\Sigma F_y = 0$$

$$F_5 \sin 60^\circ - F_6 \sin 60^\circ = 0$$

$$F_5 = F_6$$

$$F_5 = F_6 = 0.58 \text{ KN}$$

Joint D



$$\begin{aligned}\Sigma F_x &= 0 \\ F_7 - 0.58 \cos 60^\circ - 2.88 \cos 60^\circ &= 0 \\ F_7 &= 1.73 \text{ kN} \text{ (compressive)}\end{aligned}$$

FIRST TERM EXAMINATION [FEB. 2018]

SECOND SEMESTER [B.TECH]

ENGINEERING MECHANICS [ETME-110]

Time : 1.5 hrs.

M.M. : 30

Note: Q. No. 1 which is compulsory. Attempt any two more questions from the rest.

Q. 1. (a) State and explain Lami's theorem. (2)

Ans. Refer Q. 1. (b) First Term Examination 2017.

Q. 1. (b) State "principle of moments" (or Varignon's principle). (2)

Ans. Refer Q. 1. (b) End Term Examination 2017.

Q. 1. (c) Explain the terms: perfect frame, imperfect frame, deficient frame and redundant frame. (2)

Ans. Perfect, deficient and redundant frames: The structure is said to be perfect if the number of members is just sufficient to prevent its distortion of shape when subjected to external loads. For a perfect frame, the following correlation exists between the number of joints j and the number of members m ,

$$m = 2j - 3$$

The number of joints and the number of members in the frames shown in Fig. 1 satisfy the identity $m = 2j - 3$ and as such these are perfect structures (frames). A perfect structure is statically determinate. Equations of static equilibrium are sufficient to determine the forces in its members without the need of considering their deformation.

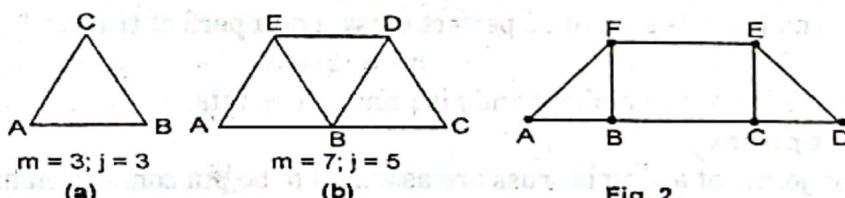


Fig. 2.

A structure is termed imperfect or deficient frame if the number of members in it is less than that required for a perfect frame. Refer Fig. 2. which shows a frame having 6 joints and 8 members. The number of members required for the frame to be perfect is

$$m = 2j - 3 = 2 \times 6 - 3 = 9$$

Since there are only 8 members (one less than that required for a perfect frame), the given frame is deficient or imperfect. An imperfect frame is under rigid or collapsible. Such a frame cannot prevent geometrical distortion when loaded.

A structure is termed redundant frame if the number of members in it is more than that required for a perfect frame. Refer Fig. (2) which shows a frame which has 7 joints and 12 members. The number of members required for the frame to be perfect is

$$m = 2j - 3 = 2 \times 7 - 3 = 11$$

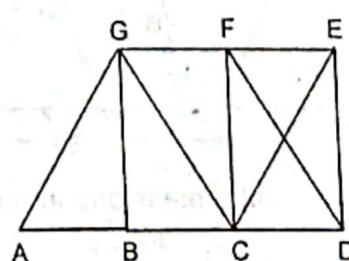


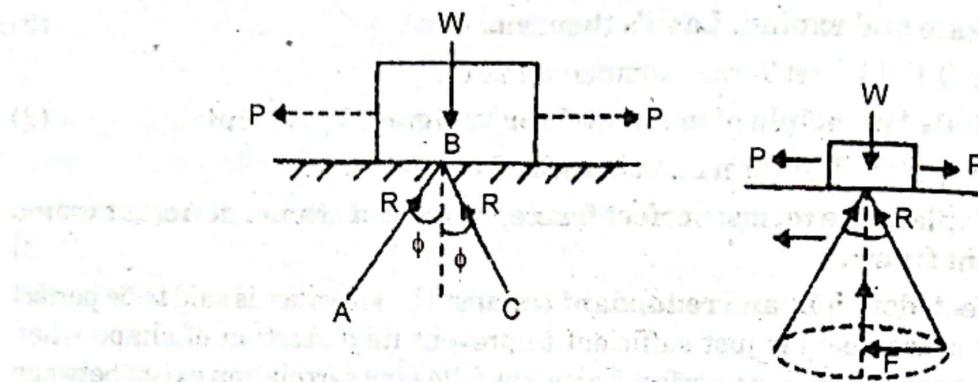
Fig. 3.

indeterminate. Further, each extra member adds one degree of indeterminacy. Thus the frame depicted in Fig. 3. is redundant to a single degree because one member is extra.

The frame depicted in Fig. 3. would remain stable even if one of the members CE or DF is removed.

Q. 1. (d) Explain "Cone of friction".

Ans. Cone of friction: Consider a block of weight W resting on horizontal surface and acting upon it a force P . When we consider coplanar forces, in order for motion not to occur in any direction, resultant R must lie within angle ABC, where ϕ is angle of friction



Consider force P is gradually changed through 360° . For motion not to occur, resultant reaction R must be contained within the cone generated by revolving line AB about normal BN .

The inverted so formed with semi-central angle equal to angle of friction ϕ is called cone of friction. Now, for motion to occur resultant R will be on the surface of cone.

Q. 1. (e) State the assumptions made for the analysis of truss.

Ans. Perfect Truss: When the truss is non collapsible on the removal of external supports, the truss is said to be perfect truss. For a perfect truss.

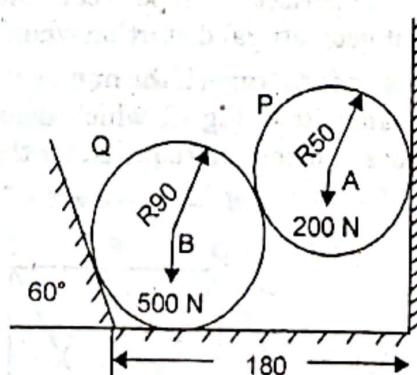
$$m = 2j - 3$$

where m is no. of members and j is number of joints.

Assumptions:

1. The joints of a simple truss are assumed to be pin connection and frictionless.
2. The loads on the truss are applied at joints only.
3. The members of a truss are straight two force members with the forces acting collinear with the centre line of the members.
4. The weights of the members are negligible unless otherwise mentioned.
5. The truss is statically determinate.

Q. 2. (a) Two cylinders P and Q rest in a channel as shown in Fig. 1. Determine the forces at all the four points of contact.



All dimensions in mm

Fig. 1

Ans. First of all, consider the equilibrium of the cylinder P. It is in equilibrium under the action of the following three forces which must pass through A i.e. the centre of the cylinder P as shown in Fig. 1. (a).

1. Weight of the cylinder (200 N) acting downwards.

2. Reaction (R_1) of the cylinder P at the vertical side.

3. Reaction (R_2) of the cylinder P at the point of contact with the cylinder Q.

From the geometry of the figure, we find that

$$ED = \text{Radius of cylinder P} = \frac{100}{2} = 50 \text{ mm}$$

Similarly

$$BF = \text{Radius of cylinder Q} = \frac{180}{2} = 90 \text{ mm}$$

and

$$\angle BCF = 60^\circ$$

$$CF = BF \cot 60^\circ = 90 \times 0.577 = 52 \text{ mm}$$

$$FE = BG = 180 - (52 + 50) = 78 \text{ mm}$$

and

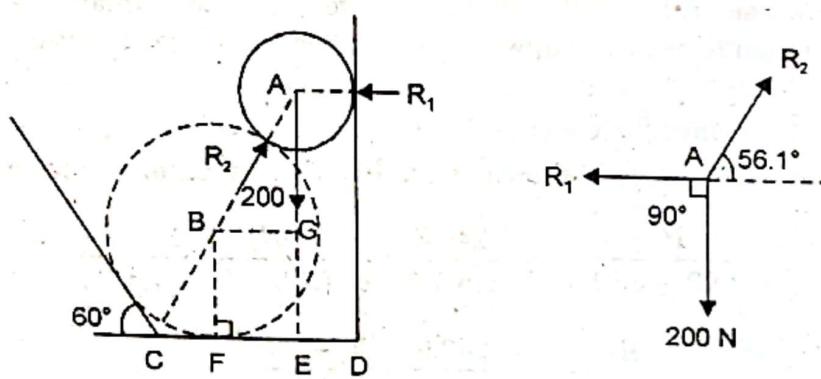
$$AB = 50 + 90 = 140 \text{ mm}$$

$$\therefore \cos \angle ABG = \frac{BG}{AB} = \frac{78}{140} = 0.5571$$

or

$$\angle ABG = 56.1^\circ$$

The system of forces at A is shown in Fig. 1. (b).



(a) Free body diagram

(b) Force diagram

Fig. 1.

Applying Lami's equation at A,

$$\frac{R_1}{\sin(90^\circ + 56.1^\circ)} = \frac{R_2}{\sin 90^\circ} = \frac{200}{\sin(180^\circ - 56.1^\circ)}$$

$$\frac{R_1}{\cos 56.1^\circ} = \frac{R_2}{1} = \frac{200}{\sin 56.1^\circ}$$

$$R_1 = \frac{200 \cos 56.1^\circ}{\sin 56.1^\circ} = \frac{200 \times 0.5571}{0.830} = 134.2 \text{ N}$$

and

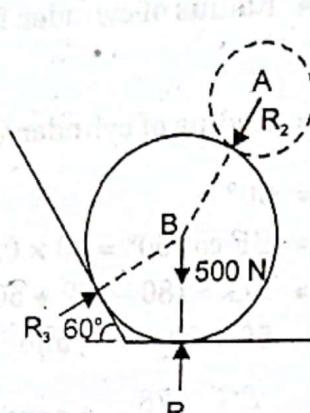
$$R_2 = \frac{200}{\sin 56.1^\circ} = \frac{200}{0.830} = 240.8 \text{ N}$$

Now consider the equilibrium of the cylinder Q. It is in equilibrium under the following four forces, which must pass through the centre of the cylinder as shown in Fig. 2. (a).

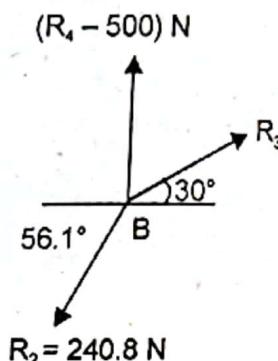
1. Weight of the cylinder Q (500 N) acting downwards.
2. Reaction R_2 equal to 240.8 N of the cylinder P on cylinder Q.
3. Reaction R_3 of the cylinder Q on the inclined surface.
4. Reaction R_4 of the cylinder Q on the base of the channel.

Q. 3. (a) A block of mass 10 kg rests on a vertical wall. A horizontal force of 100 N is applied to the wedge. Determine the reaction at the base.

Ans.



(a) Free body diagram



(b) Force diagram

F.B.D. of

Fig. 2.

A little consideration will show, that the weight of the cylinder Q is acting downwards and the reaction R_4 is acting upwards. Moreover, their lines of action also coincide with each other.

$$\therefore \text{Net downward force} = (R_4 - 500) \text{ N}$$

The system of forces is shown fig. 2. (b) Applying Lami's equation at B.

Applying

$$\frac{R_3}{\sin(90^\circ + 56.1^\circ)} = \frac{240.8}{\sin 60^\circ} = \frac{R_4 - 500}{\sin(180^\circ + 30^\circ - 56.1^\circ)}$$

$$\frac{R_3}{\cos 56.1^\circ} = \frac{240.8}{\sin 60^\circ} = \frac{R_4 - 500}{\sin 26.1^\circ}$$

$$\therefore R_3 = \frac{240.8 \times \cos 56.1^\circ}{\sin 60^\circ} = \frac{240.8 \times 0.5577}{0.866} = 155 \text{ N}$$

$$\text{and } R_4 - 500 = \frac{240.8 \times \sin 26.1^\circ}{\sin 60^\circ} = \frac{240.8 \times 0.399}{0.866} = 122.3 \text{ N}$$

$$R_4 = 122.3 + 500 = 622.3 \text{ N}$$

From

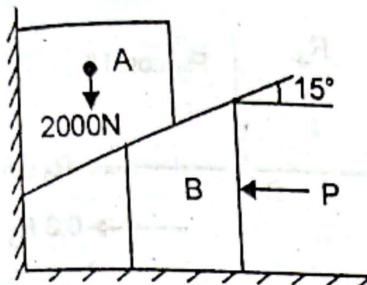
Q. 2. (b) For a flat belt drive, prove that $\frac{T_1}{T_2} = e^{\mu \theta}$.

where T_1 = Tension on the tight side of the belt, T_2 = Tension on the slack side of the belt, μ = coefficient of friction between the belt and the pulley surface and θ = angle of contact between belt and pulley.

Ans. Refer Q. 2. of End Term Exam. 2017.

Q.3. (a) A block overlying a 15° wedge on a horizontal floor and leaning against a vertical wall having weight 2000 N is to be raised by applying a horizontal force to the wedge. Assume coefficient of friction between all the surfaces to be 0.3, determine the minimum horizontal force to be applied to raise the block. (5)

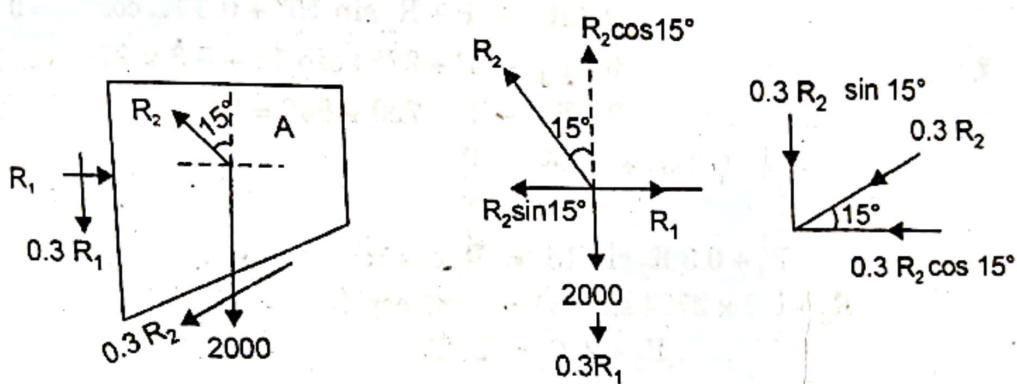
Ans.



Given,

$$\mu = 0.3, \theta = 15^\circ, W_A = 2000, W_B = 0$$

F.B.D. of block A.



Applying the condition of equilibrium.

$$\sum f_x = 0$$

$$R_1 - 0.3 R_2 \cos 15^\circ - R_2 \sin 15^\circ = 0$$

$$R_1 - 0.289 R_2 - R_2 (0.25) = 0$$

$$R_1 = 0.539 R_2$$

...(i)

$$\sum f_y = 0$$

$$R_2 \cos 15^\circ - 0.3 R_2 \sin 15^\circ - 2000 - 0.3 R_1 = 0$$

$$R_2 \cos 15^\circ - 0.078 R_2 - 2000 - 0.3 R_1 = 0$$

$$0.88 R_2 - 2000 = 0.3 R_1$$

From eq. (i)

$$0.88 R_2 - 2000 = 0.3 \times 0.539 R_2$$

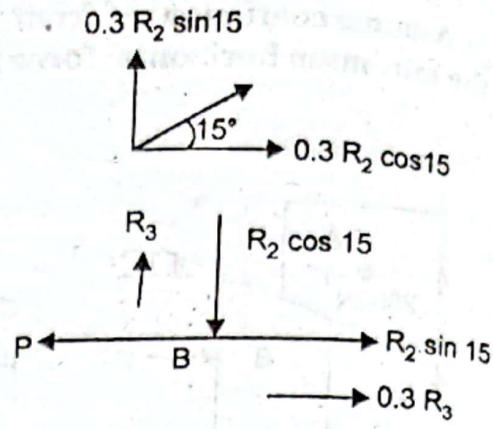
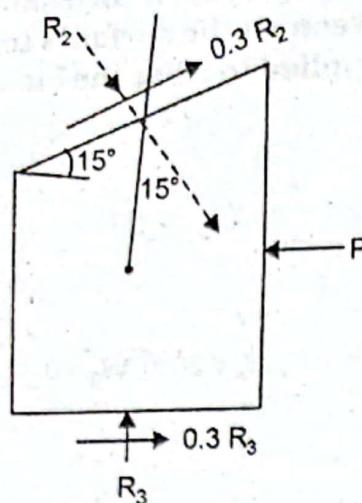
$$0.88 R_2 - 2000 = 0.1617 R_2$$

$$0.718 R_2 = 2000$$

$$R_2 = 2784 \text{ N}$$

$$R_1 = 750 \text{ N}$$

Block B



$$\sum F_x = 0.$$

$$0.3 R_3 - P + R_2 \sin 15^\circ + 0.3 R_2 \cos 15 = 0$$

$$0.3 R_3 - P + 2784 \sin 15 + 0.3 \times 2784 \cos 15 = 0$$

$$0.3 R_3 - P + 720 + 806 = 0$$

$$0.3 R_3 + 1526 = P$$

$$\sum F_y = 0.$$

$$R_3 + 0.3 R_2 \sin 15 = R_2 \cos 15$$

$$R_3 + 0.3 \times 2784 \sin 15 = 2784 \cos 15$$

$$R_3 + 216 = 2689$$

$$R_3 = 2473 \text{ N}$$

From eq. (ii)

$$P = 2268 \text{ N}$$

Q. 3. (b) A beam is loaded as shown in fig. 2. Determine the reaction at both ends. (5)

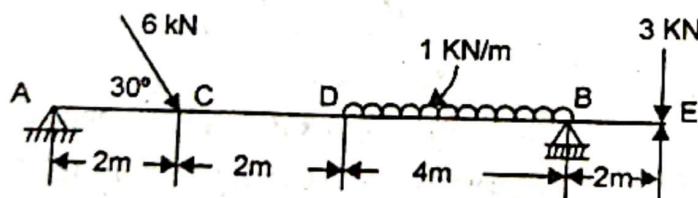


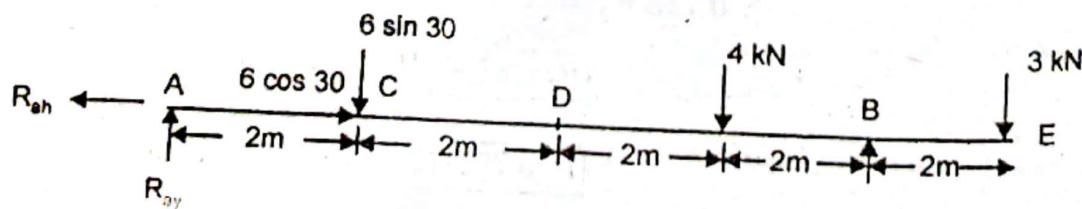
Fig. 3.

$$R_{ah} = 5.19 \text{ kN}$$

$$R_{ay} = 2.5 \text{ kN}$$

$$R_B = 7.5 \text{ kN}$$

Ans. Free body diagram of the beam



Applying the condition of equilibrium. [\because converting U.D.L. to point Load from D to B.]

$$\Sigma M = 0; \Sigma F_x = 0; \Sigma F_y = 0$$

Taking moment about point A.

$$6 \sin 30 \times 2 + 4 \times 6 - R_B \times 8 + 3 \times 10 = 0$$

$$6 + 24 - 8R_B + 30 = 0$$

$$R_B = \frac{60}{8} = 7.5$$

$$R_B = 7.5 \text{ kN}$$

...(i)

$$\Sigma F_x = 0;$$

$$R_{ah} = 6 \cos 30^\circ = 5.19 \text{ kN}$$

$$\Sigma F_y = 0;$$

$$R_{ay} - 6 \sin 30 - 4 + R_B - 3 = 0$$

or

$$R_{ay} + R_B = 6 \sin 30 + 7$$

$$= 10 \text{ kN}$$

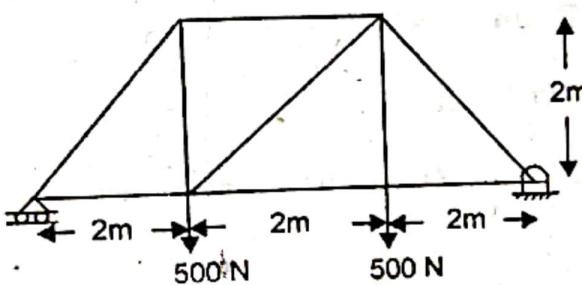
From equation (i)

$$R_{ay} = 10 - 7.5 = 2.5$$

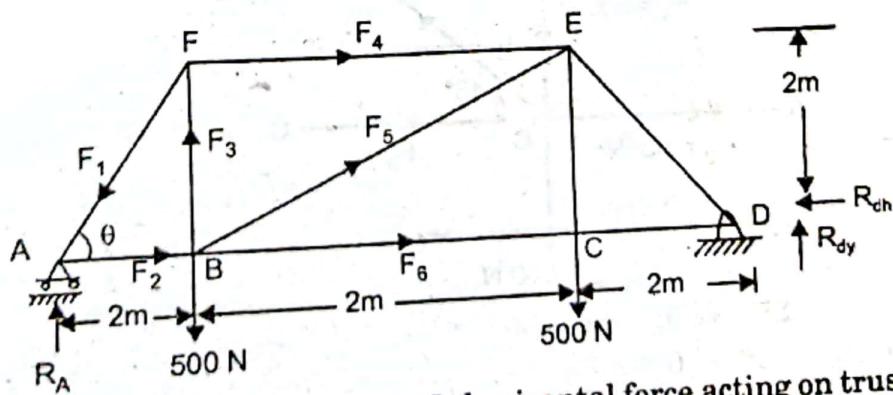
i)

$$R_{ay} = 2.5 \text{ kN}, R_{ah} = 5.19 \text{ kN}, R_B = 7.5 \text{ kN}$$

Q. 4. (a) Determine the reactions and the forces in each member of a truss, supporting two load as shown in fig. 3. (5)



Ans.



In hinged support, $R_{Dh} = 0$ because No horizontal force acting on truss. So both the reaction point A and D acting vertically. Now, condition of equilibrium.

Taking moments about point A.

$$0 + 500 \times 2 + 500 \times 4 = R_{Dy} \times 6$$

$$R_{Dy} = \frac{3000}{6} = 500 \text{ kN}$$

$$R_A = (1000 - 500) = 500 \text{ kN} [\because R_A + R_{Dy} = 500 + 500]$$

$R_{Dy} \rightarrow$ Reaction at D point in vertical direction]

In ΔAFB ,

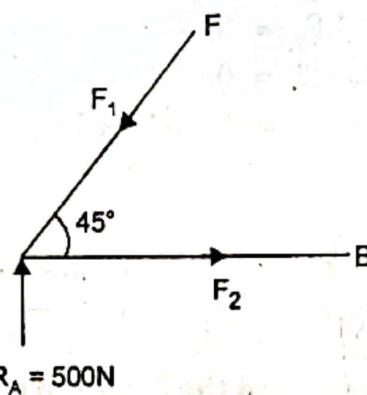
$$\tan \theta = \frac{FB}{AB} = \frac{2}{2} = 1$$

$$\theta = 45^\circ$$

Now, Finding the forces in each members. Consider the joint method of the truss.

Joint A. (F.B.D.)

from equation of equilibrium



$$R_A = 500 \text{ N}$$

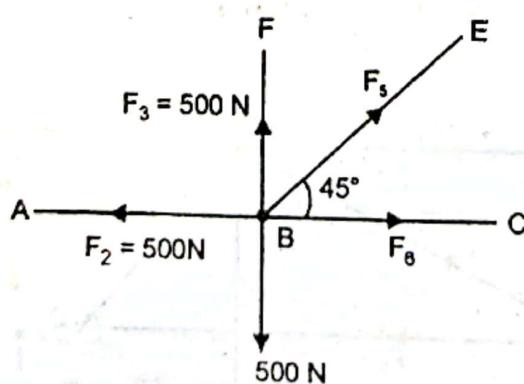
$$\sum F_x = 0; F_2 - F_1 \cos 45^\circ = 0$$

$$\sum F_y = 0; R_A - F_1 \sin 45^\circ = 0$$

$$\therefore F_1 = \frac{R_A}{\sin 45^\circ} = 707.21 \text{ N} \quad (\text{Compression})$$

$$F_2 = F_1 \cos 45^\circ = 707.21 \times 0.707 = 500 \text{ N} \quad (\text{Tension})$$

Joint B.



$$\sum F_x = 0;$$

$$F_6 + F_5 \cos 45^\circ - 500 = 0$$

$$\sum F_y = 0; 500 - 500 + F_5 \sin 45^\circ = 0$$

$$\therefore F_5 = 0$$

Now,

$$F_6 = 500 \text{ N} \quad (\text{Tension})$$

The zero value for force F_5 signifies that no force is induced in the member BE.

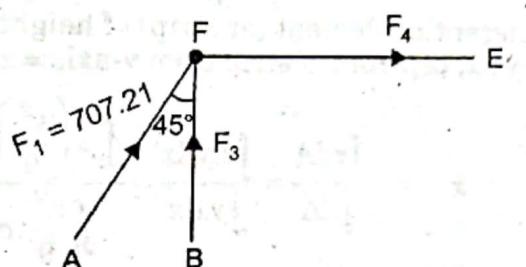
Member	AF, DE	AB, CD	BF, CE	FE	BE	BC
Force	707.21 N	500 N	500 N	500 N	0	500 N
Nature	C	T	T	C	-	T

Since the Truss is symmetrical, we have
Force in member CE = force in member BF = 500 N (T)

$$F_{CD} = F_{AB} = 500 \text{ (T)}$$

$$F_{DE} = 707.21 \text{ (C).}$$

Joint F



$$\Sigma F_x = 0;$$

$$F_4 + 707.21 \sin 45^\circ = 0$$

$$F_4 = -707.21 \times 0.707 = -500 \text{ N}$$

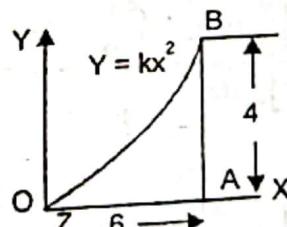
$$\Sigma F_y = 0; F_3 + 707.21 \cos 45^\circ = 0$$

$$F_3 = -707.21 \times 0.707 = -500 \text{ N}$$

Note: The -ve sign with the magnitude of forces F_3 and F_4 shows that a wrong choice has been made while assuming their directions. Obviously the assumed direction of the forces in members FB and FE need to be reversed

Therefore the member FB is a tension member and the member FE is a compression member.

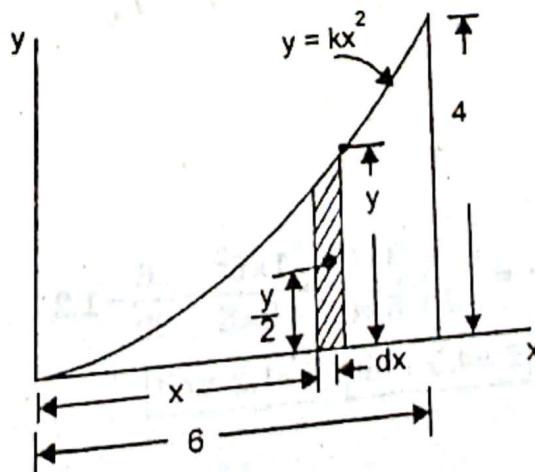
Q. 4. (b) Determine the coordinates of the C.G of the area OAB shown in fig. 4, if the curve OB represents the equation of parabola, given by $y = kx^2$ in which $OA = 6$ units and $AB = 4$ units. (5)



Ans. The equation of parabola is

$$y = kx^2$$

...(i)



Now, The value of constant k is determined by substituting the coordinates $(6, 4)$ a point on the curve in equation of parabola.

$$4 = k6^2$$

or,

$$k = \frac{4}{36} = \frac{1}{9}$$

Now from eq. (i),

$$y = k \cdot x^2$$

$$y = \frac{1}{9} \cdot x^2$$

Or,

$$x^2 = 9y \Rightarrow x = 3\sqrt{y}$$

Consider a vertical differential element (or strip) of height y and width dx .
Area of the strip $dA = y \cdot dx$, centroid of strip from y -axis = x

$$\bar{x} = \frac{\int x dA}{\int dA} = \frac{\int x \cdot y dx}{\int y \cdot dx} = \frac{\int_0^6 x \left(\frac{x^2}{9}\right) dx}{\int_0^6 \frac{x^2}{9} \cdot dx}$$

$$\begin{aligned} &= \frac{\left[\frac{1}{9} \cdot \frac{x^4}{4}\right]_0^6}{\int_0^6 y \cdot dx} = \frac{\frac{1}{9} \left(\frac{6^4}{4}\right)}{\int_0^6 y \cdot dx} = \frac{3}{4} \left[\frac{6^4}{6^3}\right] \\ &= \frac{3}{4} \cdot 6^1 \\ &= \frac{18}{4} = 4.5 \end{aligned}$$

Distance of the centroid of the strip from the x -axis = $\frac{y}{2}$

$$\bar{y} = \frac{\int \frac{y}{2} \cdot y dx}{\int y \cdot dx} = \frac{\int_0^6 \frac{1}{2} y^2 dx}{\int_0^6 y dx} = \frac{\int_0^6 \frac{1}{2} \left(\frac{x^2}{9}\right)^2 dx}{\int_0^6 \left(\frac{x^2}{9}\right) \cdot dx}$$

$$\begin{aligned} &= \frac{\int_0^6 \frac{1}{2} \cdot \frac{x^4}{81} dx}{\int_0^6 \frac{x^2}{9} dx} = \frac{\frac{1}{2} \left[\frac{1}{81} \left(\frac{x^5}{5}\right)\right]_0^6}{\left[\frac{1}{9} \left(\frac{x^3}{3}\right)\right]_0^6} \\ &= \frac{\frac{1}{2} \times \frac{6^5}{81} \cdot \frac{5}{6^3}}{\frac{1}{9} \cdot 6^2} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \times \frac{3 \times 6^5}{5 \times 6^3} \cdot \frac{5}{6^3} = \frac{1 \times 6^2}{6 \times 5} = \frac{36}{30} = 1.2 \end{aligned}$$

Now,

$$\boxed{\bar{x} = 4.5 \text{ unit}} \quad \boxed{\bar{y} = 1.2 \text{ unit}}$$

MID TERM EXAMINATION [FEB-2019]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME-110]

M.M. : 30

Time : 1.30 hrs.

Note :- Attempt Q.No. 1. which is compulsory and any two more question from remaining.

Q.1. Define the following :

Q.1. (a) Derive an expression for center of gravity of a triangular lamina of height (H) and base (B), about its base using method of integration. (2.5)

Ans. Let ABC be the triangle of base width b and height h . Consider an elementary strip of width l , thickness dy and located at distance y from base BC of the triangle. For this elemental strip,

$$\begin{aligned} \text{area} &= l dy \\ \text{moment about } x\text{-axis} &= l dy \times y \end{aligned}$$

Since the integration is to be done with respect to y within the limits 0 to h , it is necessary to express l in terms of y . For that we obtain the following correlation from the similarity of triangles ADE and ABC,

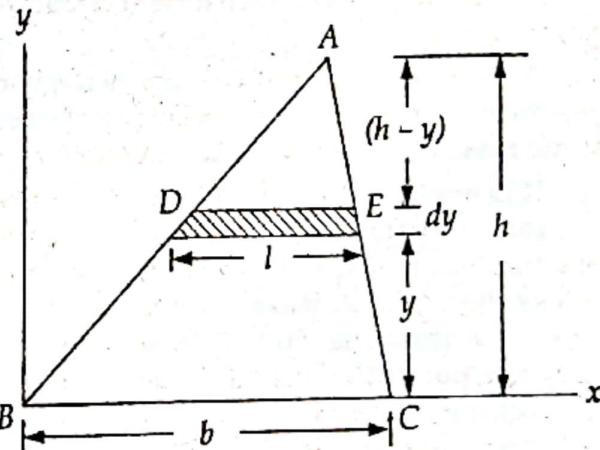
$$\frac{l}{b} = \frac{h-y}{h}; l = b\left(1 - \frac{y}{h}\right)$$

\therefore Moment of elemental strip about x -axis

$$= b\left(1 - \frac{y}{h}\right)y dy$$

$$\text{area of triangle ABC} = \frac{1}{2}bh$$

If \bar{y} is the distance of the centroid from the base, then from the moment principle



$$\frac{1}{2}bh \times \bar{y} = \int_0^h b\left(1 - \frac{y}{h}\right)y dy = b \int_0^h \left(y - \frac{y^2}{h}\right) dy$$

$$= b \left[\frac{y^2}{2} - \frac{y^3}{3h} \right]_0^h = b \left(\frac{h^2}{2} - \frac{h^2}{3} \right) = \frac{bh^2}{6}$$

$$\therefore \bar{y} = \frac{bh^2}{6} \times \frac{2}{bh} = \frac{h}{3}$$

Thus the centroid of a triangle of height h is at a distance $h/3$ from the base or $2h/3$ from the apex.

Q.1. (b) Angle of friction and Cone of friction.

(2.5)

Ans. Angle of friction : The angle of friction ϕ is a measure of the limiting position of total reaction between the two contacting surfaces. It is defined as the angle which the resultant of normal reaction and limiting force of friction makes with the normal reaction.

With reference to Fig.

R = normal reaction

F = limiting force of friction

$$S = \sqrt{R^2 + F^2}$$

= total or resultant reaction

$$\tan \phi = \frac{F}{R} \text{ where } f \text{ is the angle of friction}$$

The ratio F/R is also called the coefficient of friction, μ .

$$\therefore \mu = \tan \phi \quad \dots(1)$$

The above correlation does suggest that angle of friction in radians is equal to the coefficient of friction provided the angle has a small value.

$$\mu = \tan \phi \approx \phi \text{ in radians}$$

Cone of friction: With reference to Fig., the lines of action of normal reaction R and the frictional force F meet at point O . The magnitude of total reaction is $\sqrt{F^2 + R^2}$ and it makes an angle ϕ with the normal reaction.

Obviously O is the vertex of a cone whose axis is R and semi-vertex angle is ϕ . Such a right circular cone is called the cone of friction.

Apparently the cone of friction is the imaginary cone AOB generated in the case of non-coplanar forces by revolving the static resultant reaction S about the normal OR .

It is worthwhile to point out that in case of impending motion, the friction force is maximum and the total reaction lies on the surface of cone. When the friction force is less than the limiting friction, the total reaction would lie within the cone. In that case, the forces acting on the body are not sufficient to cause motion. This aspect forms the working principle for self-locking mechanisms.

Q.1. (c) Define Pappus and Guldinus theorem.

(2.5)

Ans. Pappus-Guldinus Theorems: When a plane curve rotates about a fixed axis, it generates a surface called the surface of revolution.

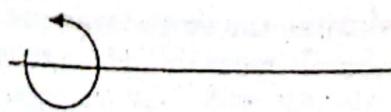
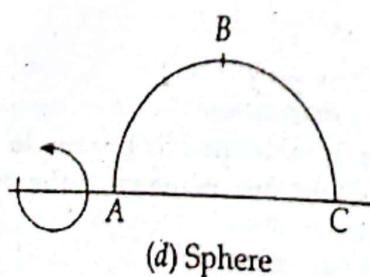
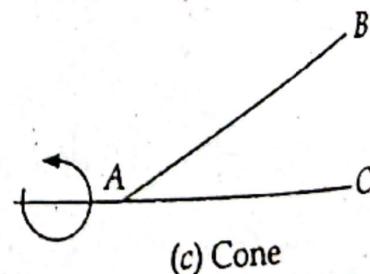
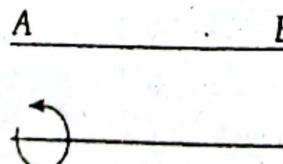
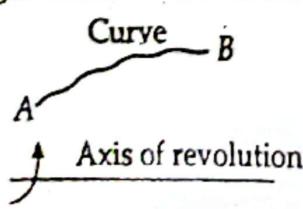
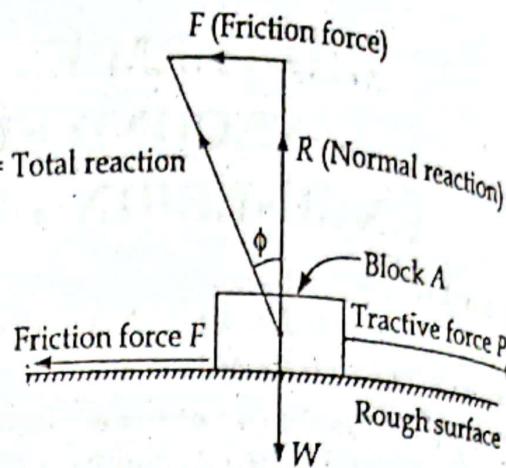


Fig: Generation of surface of revolution



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Q.1. (supports

Theorem I. The area of the surface generated by rotating any plane curve about a non-intersecting axis in its plane is equal to the product of the length of the curve and the distance travelled by its centroid.

Let AB be any plane curve of length l that lies in the x - y plane and does not intersect the x -axis. Consider an elemental length dl_i so short that it may be thought to be a minute portion of a straight line. Let y -coordinate of its mid point be y_i and that of the centroid of the entire curve be \bar{y} .

When this curve is rotated about the x -axis, then area generated by the elementary length $= 2\pi y_i dl_i$

$$\text{area generated by the entire line} = \int 2\pi y_i dl_i$$

The integral $\int y_i dl_i$ is equal to $\bar{y} l$ and therefore area generated $= l \times 2\pi \bar{y}$,

where $2\pi \bar{y}$ is the distance travelled by the centroid of the curve

$$\therefore \text{area generated} = (\text{length of the generating curve}) \times (\text{distance travelled by centroid of the curve})$$

It is to be noted that the generating curve does not cross the axis about which it is rotated. If it does so, then the two sections on either side of the axis would generate areas of opposite signs and the theorem would not apply.

Theorem II. The volume of the solid generated by rotating any plane figure about a non-intersecting axis in its plane is equal to the product of area of the figure and the distance travelled by its centroid.

Consider area A of any plane figure divided into a large number of very thin strips parallel to the x -axis. Let the y -coordinate of the middle of any strip of area dA_i be y_i and the centroid of the entire curve be \bar{y} .

When the plane of the figure is rotated about the x -axis, then

$$\text{Volume generated by the elementary area} = 2\pi y_i dA_i$$

$$\text{Volume generated by the entire area} = \int 2\pi y_i dA_i$$

The integral $\int y_i dA_i$ is equal to $\bar{y} A$ and therefore

$$\text{Volume generated } V = A \times 2\pi \bar{y}$$

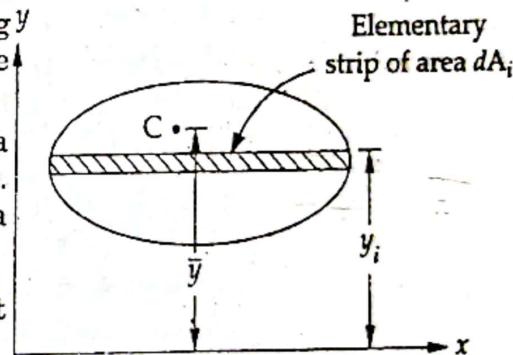
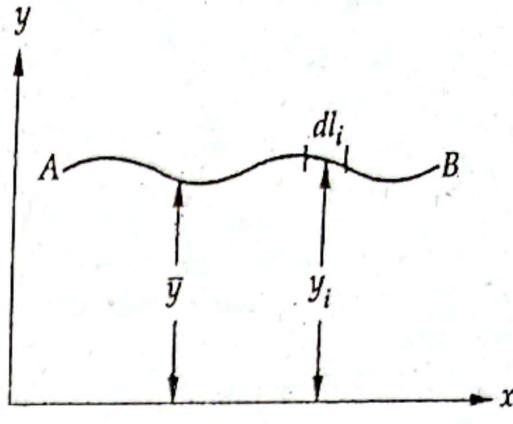
The parameter $(2\pi \bar{y})$ is the distance travelled by the centroid of the area being rotated.

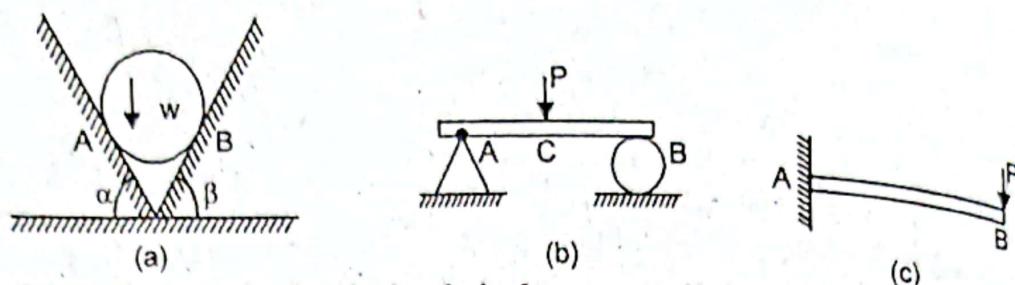
$$\therefore \text{Volume generated} = (\text{area of the figure}) \times (\text{distance travelled by the centroid})$$

Utility: The theorems of Pappus-Guldinus offer a simple way to compute the areas of surfaces of revolution and the volumes of bodies of revolution.

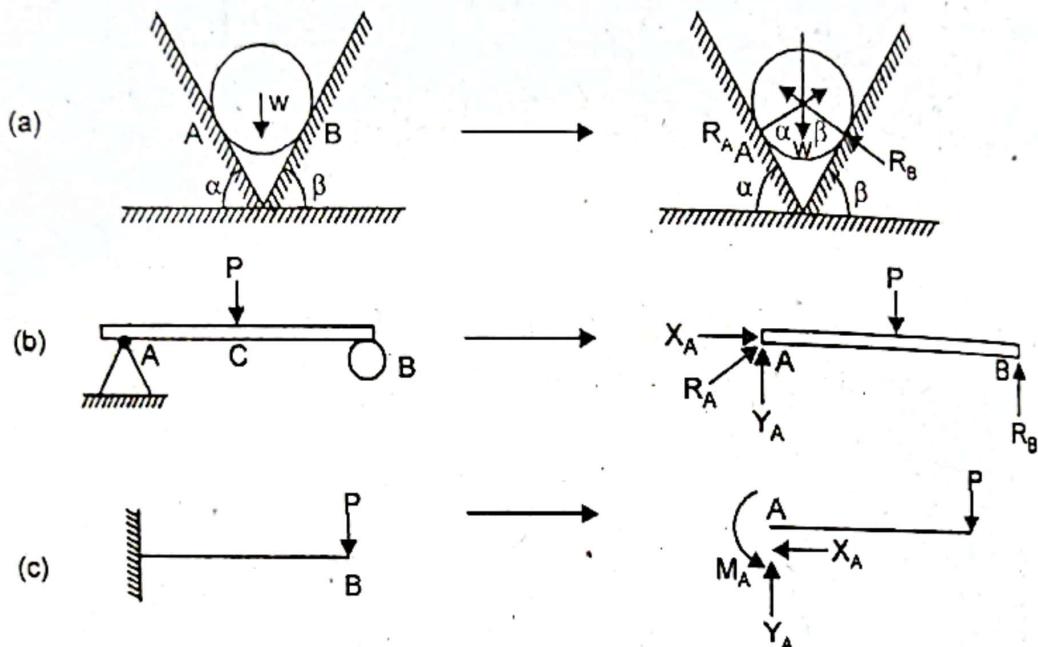
Conversely their application lies in determining the centroid of a plane curve when the area of the surface generated by the curve is known or to determine the centroid of a plane area when the volume of the body generated by the area is known.

Q.1. (d) Define free body diagram and draw the free body diagram for the supports as shown in Fig. (2.5)





Ans. FBD - To draw the free body of a body remove all the supports (like wall, floor, hinge or any other body) and replace them by the reactions which these supports exert on the body.



Q.2. The spanner shown in Fig. is used to rotate a shaft. A pin fits in a hole at A, while a flat frictionless surface support the shaft at B. If a 300 N force P is exerted on the spanner at D, find the reactions at A and B. (10)

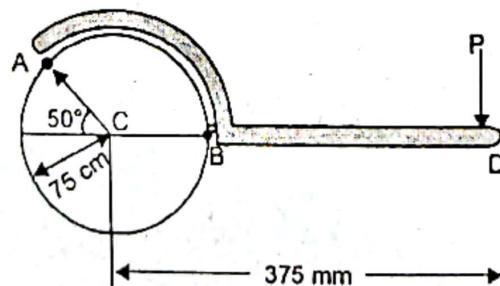
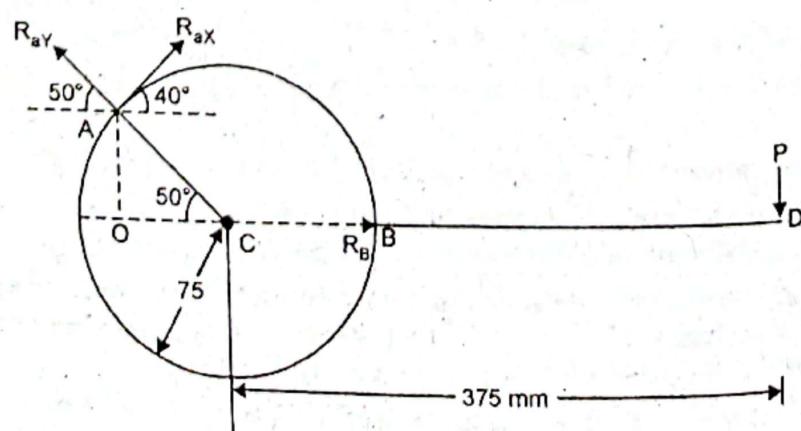


Fig.

Ans.From $\triangle AOC$,

$$AO = 57.4 \text{ mm}; OC = 48.2 \text{ mm}$$

$$\Sigma f_x = 0$$

$$R_{ax} \cos 40 + R_B = R_{ay} \cos 50 \quad \dots(1)$$

$$\Sigma f_y = 0,$$

$$R_{ax} \sin 40 + R_{ay} \sin 50 = 300 = p \quad \dots(2)$$

$$\Sigma M_A = 0$$

$$R_B \times AO = P(48.2 + 375)$$

$$R_B = \frac{300(423.2)}{57.4} = 2211$$

$$\boxed{R_B = 2211 \text{ N}}$$

$$\Sigma M_C = 0$$

$$R_{ax} \times 75 = P \times 375$$

$$\boxed{R_{ax} = 1500 \text{ N}}$$

From eq (2)

$$1500 \sin 40 + R_{ay} \sin 50 = 300$$

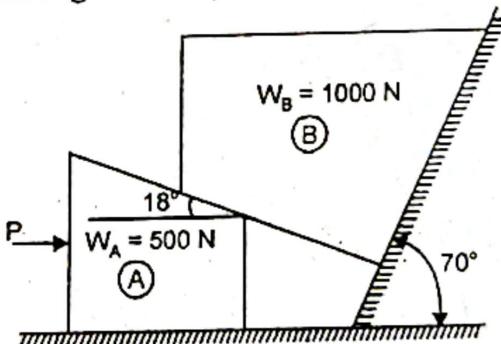
$$964.1 + R_{ay} \sin 50 = 300$$

$$R_{ay} = -\frac{664.1}{\sin 50} = -867 \text{ N}$$

$$R_A = \sqrt{R_{ax}^2 + R_{ay}^2} = \sqrt{1500^2 + 867^2}$$

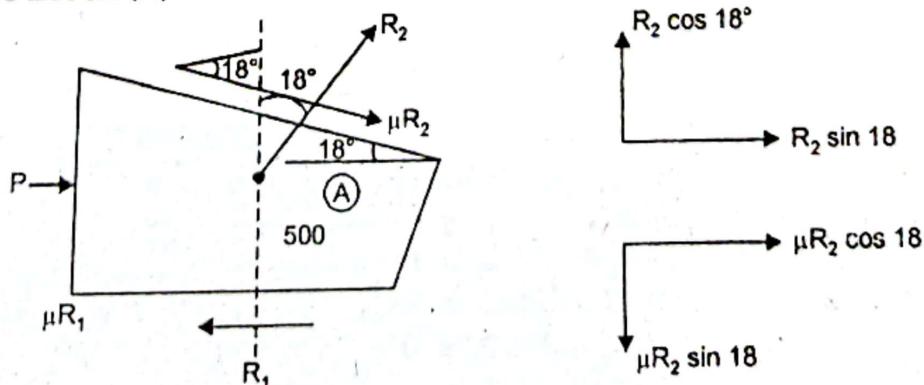
$$= \boxed{R_A = 1732 \text{ N}}$$

Q.3. Determine the force P required to be applied on wedge A ($W_A = 500 \text{ N}$) to lift the load B as shown in Fig.3. Take $\mu = 0.24$ at all contact surface. (10)



Ans. Given, $\mu = 0.24$ at contact surface; $W_A = 500 \text{ N}$; $W_B = 1000 \text{ N}$

FBD for Block (A)



$$\Sigma f_x = 0,$$

$$P - \mu R_1 + \mu R_2 \cos 18 + R_2 \sin 18 = 0$$

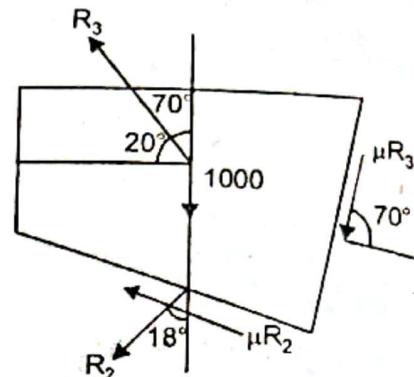
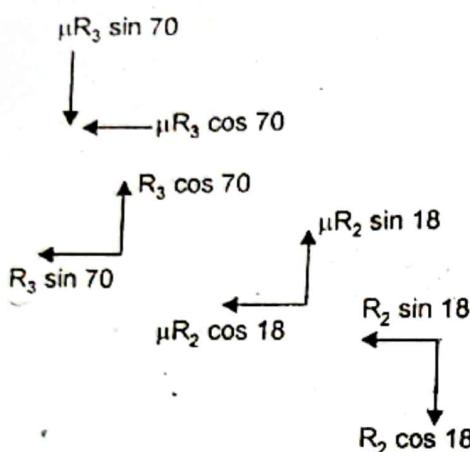
$$P - 0.24 R_1 + 0.24 R_2 \cos 18 + R_2 \sin 18 = 0$$

$$P - 0.24 R_1 + 0.53 R_2 = 0 \quad \dots(1)$$

$$\Sigma f_y = 0$$

$$\begin{aligned} R_1 + R_2 \cos 18 - \mu R_2 \sin 18 &= 500 \\ R_1 + 0.95 R_2 - 0.074 R_2 &= 500 \\ R_1 + 0.878 R_2 &= 500 \end{aligned} \quad \dots(2)$$

FBD of Block B



$$\Sigma f_x = 0$$

$$\mu R_3 \cos 70 + R_3 \sin 70 + \mu R_2 \cos 18 + R_2 \cos 18 = 0$$

$$0.24 R_3 \cos 70 + R_3 \sin 70 + 0.24 R_2 \cos 18 + R_2 \sin 18 = 0$$

$$0.08 R_3 + 0.528 R_2 = 0$$

$$R_3 = -\frac{0.528 R_2}{0.08} = -6.6 R_2 \quad \dots(3)$$

$$\Sigma f_y = 0$$

$$R_3 \cos 70 - \mu R_3 \sin 70 - R_2 \cos 18 + \mu R_2 \sin 18 - 1000 = 0$$

$$0.34 R_3 - 0.24 R_3 \sin 70 - 0.95 R_2 + 0.24 R_2 \sin 18 = 1000$$

$$0.34 R_3 - 0.225 R_3 - 0.95 R_2 + 0.06 R_2 = 1000$$

$$0.12 R_3 - 0.89 R_2 = 1000$$

$$R_3 - 7.41 R_2 = 8333.3 \quad \dots(4)$$

From eq (3)

$$-6.6 R_2 - 7.41 R_2 = 8333.3$$

$$-14.01 R_2 = 8333.3$$

$$R_2 = -594.8 \text{ N}$$

$$R_3 = 3925 \text{ N}$$

From eq (2)

$$R_1 + 0.878 R_2 = 500$$

$$R_1 = 500 - 0.878 (-594.8)$$

$$R_1 = 1022 \text{ N}$$

From eq. (1)

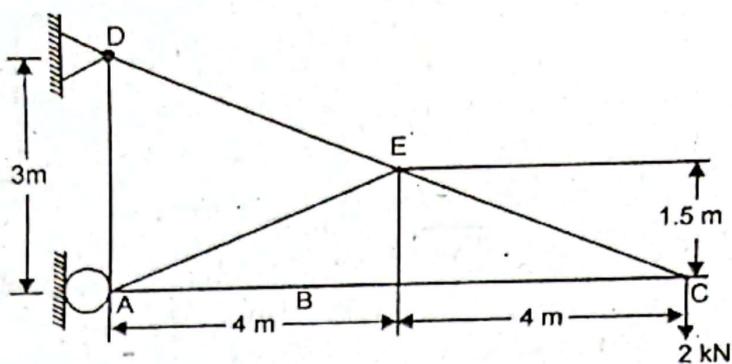
$$P - 0.24 R_1 + 0.53 R_2 = 0$$

$$P - 0.24 (1022) + 0.53 (-594.8) = 0$$

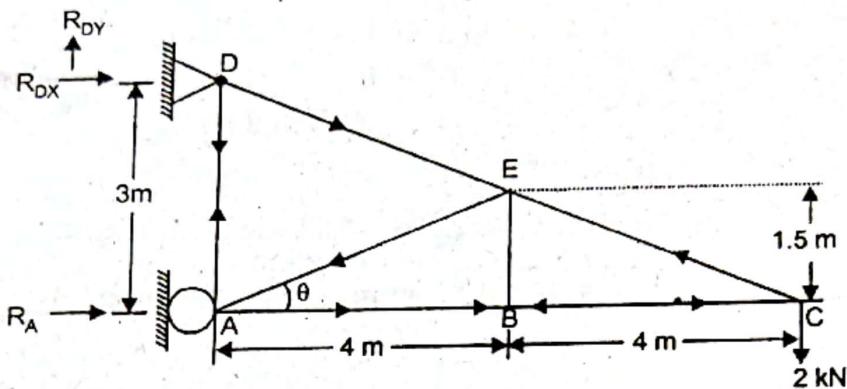
$$P - 245.28 - 315.2 = 0$$

$$P = 560 \text{ N}$$

Q.4. Find the forces induced in all the members due to a load of 2 KN acting at joint C of the truss as shown in the Fig. (10)



Ans.

 ΔAEB ,

$$\tan \theta = \frac{1.5}{4}$$

$$\Rightarrow \theta = 20.5^\circ$$

$$\begin{aligned} \sum f_x &= 0 \\ R_A + R_{DX} &= 0 \end{aligned} \quad \dots(1)$$

$$\begin{aligned} \sum f_y &= 0 \\ R_{DY} &= 2 \text{ KN} \end{aligned} \quad \dots(2)$$

$M_D = 0$ (Taking moment about D point)

$$R_A \times 3 = 2 \times 8$$

$$R_A = 16/3 = 5.33 \text{ KN}$$

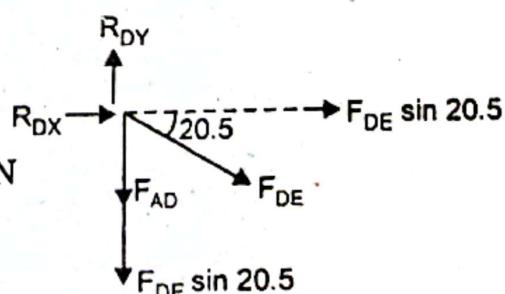
$$\Rightarrow R_A = 5.33 \text{ KN}$$

$$\Rightarrow R_{DX} = -5.33 \text{ KN}$$

Joint D

$$\begin{aligned} \sum f_x &= 0 \\ R_{DX} + F_{DE} \cos 20.5^\circ &= 0 \\ -5.33 + F_{DE} \cos 20.5^\circ &= 5.69 \text{ KN} \end{aligned}$$

$$F_{DE} = 5.69 \text{ KN}$$



$$\sum f_y = 0$$

$$R_{DY} = F_{AD} + F_{DE} \sin 20.5^\circ$$

$$2 = F_{AD} + 5.69 \sin 20.5$$

$$2 = F_{AD} + 2$$

$$F_{AD} = 0$$

Joint A

$$\Sigma f_x = 0$$

$$F_{AE} \cos 20.5^\circ = R_A + F_{AB}$$

$$\Sigma f_y = 0$$

$$F_{AD} = F_{AE} \sin 20.5^\circ$$

$$0 = F_{AE} \sin 20.5^\circ$$

$$F_{AE} = 0$$

$$R_A = -F_{AB}$$

$$F_{AB} = -5.33 \text{ KN}$$

Joint C

$$\Sigma f_x = 0,$$

$$-F_{CE} \cos 20.5^\circ = F_{BC}$$

$$\Sigma f_y = 0,$$

$$2 = F_{CE} \sin 20.5^\circ$$

$$F_{CE} = 5.71 \text{ KN}$$

Now,

$$F_{BC} = -5.71 \cos 20.5^\circ$$

$$F_{BC} = -5.34 \text{ KN} \quad (\text{wrong chosen assumption})$$

Joint B

$$\Sigma f_y = 0,$$

$$F_{BE} = 0$$

$$\Sigma f_x = 0$$

$$F_{BC} = F_{AB}$$

$$= -5.34 \text{ KN.}$$

