

## SYNTAX ANALYSIS

1.

### \* Context free Grammars

$$CFG = \{ S, P, T, N \}$$

S (start symbol)  $\rightarrow$  symbol from which the production starts.

P (Productions)  $\rightarrow$  specify the manner in which terminals

& non terminals can be combined to form strings.

T (Terminals)  $\rightarrow$  basic symbols which forms string

N (Non Terminals)  $\rightarrow$  Syntactic Variables

### Example

$$S \rightarrow ab | bA$$

$$A \rightarrow a | as | bAA$$

S  $\rightarrow$  Start symbol

T  $\rightarrow$  a, b

N  $\rightarrow$  S, A,

P  $\rightarrow$  S  $\rightarrow$  ab, S  $\rightarrow$  bA, A  $\rightarrow$  a, A  $\rightarrow$  as, A  $\rightarrow$  bAA

## \* Derivations

↓  
Leftmost

Rightmost

### Example

$$E \rightarrow E+E | E*E | -E | (E) | id.$$

derive -(id+id)

$$\begin{aligned} E &\rightarrow -E \\ &\rightarrow -(E) \\ &\rightarrow -(E+E) \\ &\rightarrow -(\underline{id}+E) \\ &\rightarrow -(id+id) \end{aligned}$$

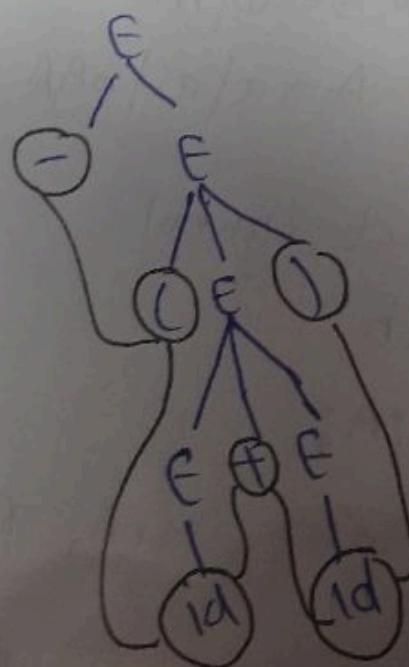
left variable is  
replaced first  
LEFTMOST DERIVATION

$$\begin{aligned} E &\rightarrow -E \\ &\rightarrow -(E) \\ &\rightarrow -(E+E) \\ &\rightarrow -(E+id) \\ &\rightarrow -(id+id) \end{aligned}$$

Right Variable is  
replaced first

### RIGHTMOST DERIVATION

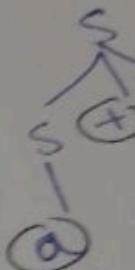
## Parse Tree



-(id+id)

Ambiguities  
grammar  
parse tree.

### Example



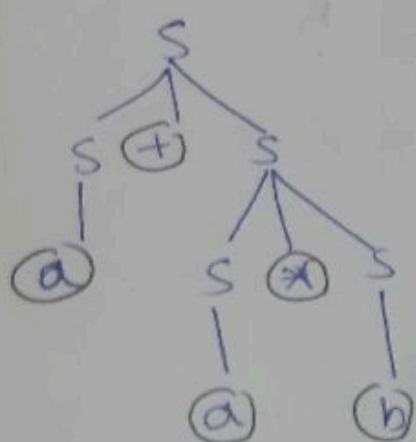
(2) Ambiguous Grammar

Grammar which constructs more than one parse tree.

Example

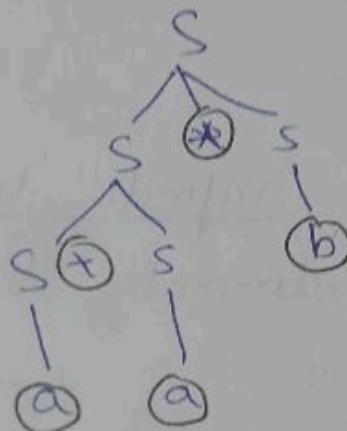
$$S \rightarrow S+S \mid S*S \mid (S) \mid ab$$

done at  $a*a*b$



$a*a*b$

Rightmost



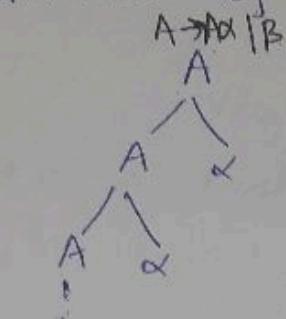
$a*a*b$

Leftmost

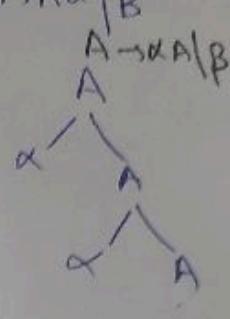
⇒ 2 different Parse Trees for same grammar.  
So it is AMBIGUOUS GRAMMAR

## \* Left Recursion

Grammar of type



$A \xrightarrow{A\alpha|B}$



Left Fact

prefix

EXAMPLE

Thus  $\alpha$  in infinite loop  
we need to eliminate it.

JUST DO IT :

$A \xrightarrow{A\alpha|B}$

↓  
replace

$$\boxed{A \xrightarrow{\beta A' | F} A' \xrightarrow{\alpha A' | \epsilon}}$$
 Removed

EXAMPLE

$E \xrightarrow{E+T | F}$

$T \xrightarrow{T * f | F}$

$f \xrightarrow{(E) | id}$

$$\begin{array}{c} A \\ \downarrow \\ E \xrightarrow{E+T | F} \end{array}$$

$$\begin{array}{c} A(\alpha) \\ \downarrow \\ E \xrightarrow{E+T | F} \end{array}$$

$$\begin{array}{c} A \\ \downarrow \\ T \xrightarrow{T * F | F} \end{array}$$

$A \xrightarrow{\beta A'}$

$E \xrightarrow{TE'}$

$T \xrightarrow{FT'}$

$A' \xrightarrow{\alpha A' | \epsilon}$

$T' \xrightarrow{*FT' | \epsilon}$

∴ Grammar after removing left  
Recursion

$$\begin{aligned} E &\xrightarrow{TE'} \\ E' &\xrightarrow{*TE' | \epsilon} \\ T &\xrightarrow{FT'} \\ T' &\xrightarrow{*FT' | \epsilon} \\ f &\xrightarrow{(E) | id} \end{aligned}$$

(4)

$A \rightarrow \alpha A \mid B$

Left Factoring

(5)

Process of factoring out the common prefixes.

EXAMPLE

$$S \rightarrow iEtses \mid iEts/a$$

$$E \rightarrow b$$

It becomes

$$S \rightarrow iEtss' \mid a$$

$$S' \rightarrow es \mid E$$

$$E \rightarrow b$$

Parsing TypesTOP DOWN

↳ Recursive Descent  
Parser.

↳ LL(1) Grammar

BOTTOM UP

↳ SLR

↳ LR(1)

↳ LALR(1)

1st & follow

is for finding first ~~of non terminals~~ -

If  $x$  is terminal, then  $\text{FIRST}(x) = \{x\}$   
if  $x$  is a non terminal, &  $x \rightarrow \alpha$ , the add  $\alpha$   
to  $\text{FIRST}(x)$ .

If  $x \rightarrow \epsilon$  the add  $\epsilon$  to  $\text{FIRST}(x)$

if  $x \rightarrow y_1 y_2 \dots y_r$  do  $y_1 y_2 \dots y_r \rightarrow$  non terminals

$\text{first}(x) = \text{first}(y_1)$

Example:

$S \rightarrow aABb$   
 $A \rightarrow c|\epsilon$   
 $B \rightarrow d|\epsilon$

\* To find first of ~~#~~ the  
non terminals add the  
first non terminals which  
we get.

$\text{first}(S) = \{a\}$   
 $\text{first}(A) = \{c, \epsilon\}$   
 $\text{first}(B) = \{d, \epsilon\}$

Example 2:

$S \rightarrow \text{if} \{ S \} S' | a$   
 $S' \rightarrow \epsilon S | \epsilon$   
 $E \rightarrow b$

First  $S = \{i, a\}$   
First  $S' = \{e, \epsilon\}$   
First  $E = \{b\}$

(6)

### Example 3

$$\begin{array}{l} E \rightarrow TE \\ E' \rightarrow +TE' \setminus \epsilon \\ T \rightarrow FT' \\ T' \rightarrow *FT'^\epsilon \\ F \rightarrow (E) \setminus id \end{array}$$

$$\text{first}(E) = \text{FIRST}(T) = \text{FIRST}(F) = \{ (, \text{id}) \}$$

$$\begin{array}{l} \text{first}(E) = \{ (, \text{id}) \} \\ \text{first}(T) = \{ (, \text{id}) \} \\ \text{first}(F) = \{ (, \text{id}) \} \end{array}$$

$$\begin{array}{l} \text{first}(E') = \{ +, \epsilon \} \\ \text{first}(T') = \{ *, \epsilon \} \end{array}$$

### Example 4

$$\begin{array}{l} S \rightarrow aAB \setminus bA \setminus \epsilon \\ A \rightarrow aAb \setminus \epsilon \\ B \rightarrow bB \setminus c \end{array}$$

$$\begin{array}{l} \text{first } S = \{ a, b, \epsilon \} \\ \text{first } A = \{ a, \epsilon \} \\ \text{first } B = \{ b, c \} \end{array}$$

(7) for

in  
for

in  
for

in  
for

in  
for

in  
for

in  
for

(7) Rules for finding follow

To find follow :-

Add  $\$$  to follow(X) where X is the starting symbol.  
 $A \rightarrow \alpha B \beta$  where  $B \neq \epsilon$ ,

If there is a production  $A \rightarrow \alpha B \beta$  where  $B = \epsilon$ ,

then first(B) is added to follow(B).

If there is a production  $A \rightarrow \alpha B \beta$  or  $A \rightarrow \alpha B \beta \gamma$  where  $B = \epsilon$ , then follow(A) is added to follow(B).

Let's elaborate :-

- ① \$ is added to follow of starting symbol.
- ② The non terminal where follow is to be found, we have to find it in the right side of productions.
- ③ Whenever the non terminal is found check the immediate next if there is something  $\rightarrow$  then first 2 add.  
 if there is nothing  $\rightarrow$  then the follow of leftmost symbol is added to follow of non terminal to be found.

$A \rightarrow \alpha B \beta$   
 $\downarrow$   
 $\rightarrow$  first of ( $B$ )  
 (except  $\epsilon$ )

$A \rightarrow \alpha B \beta$ ?  
 $\downarrow$   
 $\rightarrow$  follow A  $\rightarrow$  added to follow( $\beta$ )

(8)

Example

$$\begin{aligned} S &\rightarrow aABb \\ A &\rightarrow c \mid \epsilon \\ B &\rightarrow d \mid \epsilon \end{aligned}$$

find follow of each  
Non terminal

$$\text{Follow } S = \{ \$ \}$$

$$\text{Follow } A = \{ d, \$ \}$$

$$\text{Follow } B = \{ b \}$$

Example

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow PT'$$

$$T' \rightarrow *FT' \mid \epsilon$$

$$F \rightarrow (\epsilon) \mid id$$

$$\text{Follow}(E) = \{ \$ \}$$

$$\text{Follow}(E') = \{ \$, ) \}$$

$$\text{Follow}(T) = \{ +, \$, ) \}$$

$$\text{Follow}(T') = \{ +, \$, ) \}$$

$$\text{Follow}(F) = \{ *, +, \$, ) \}$$

Example

$$S \rightarrow ; E t S S' | a$$

$$S' \rightarrow e S | \epsilon$$

$$E \rightarrow b$$

$$\text{Follow}(S) = \{ \$, e \}$$

$$\text{Follow}(S') = \{ \$, e \}$$

$$\text{Follow}(E) = \{ t \}$$

## L(L) Grammar

10

L → left to right scanning of input

L → leftmost derivation

I → each step will consider one symbol of lookahead.

## STEPS

Convert the grammar into unambiguous.  
(Perform left factoring)

Remove left recursion.

find first & follow

construct Table.

While constructing the table, write non terminals  
on one side & terminals on other. (including  
\$)

Let's understand with example:-

Example

$$\begin{array}{l} S \rightarrow A \\ A \rightarrow ab \mid Ad \\ B \rightarrow bBc \mid f \\ C \rightarrow g \end{array}$$

Eliminate Left Recursion

$$\begin{array}{l} A \rightarrow abA' \\ A' \rightarrow dA' \mid \epsilon \end{array}$$

<u>First</u>		<u>Follow</u>
S	a	\$
A	a	\$
B	b, f	c
C	g	\$
A'	d, ε	\$

Predictive Parsing Table

	a	b	c	d	f	g	\$
S	S → A						
A	A → abA'						
B		B → bBc			B → f		
C						C → g	
A'				A' → dA'			A' → ε

\* Fill the production in the first of leftmost symbol

\* In case  $A' \rightarrow \epsilon$  (fill this production in follow of  $A'$ )

⇒ Only one entry in each cell. ∴ LL(1) Grammar

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Impl

$$E \rightarrow iACE \mid iACEeE \mid a$$

$$A \rightarrow b$$

Remove left factoring

$$E \rightarrow iACeE' \mid a$$

$$E' \rightarrow eE \mid \epsilon$$

$$A \rightarrow b$$

First

$$E \quad ia$$

$$E' \quad e, \epsilon$$

$$A \quad b$$

Follow

$$\$, ie$$

$$\$, e$$

$$c$$

### LL(1) Parsing Table

	a	b	e	i	c	\$
E	$E \rightarrow a$					
E'						$E' \rightarrow \epsilon$
A		$A \rightarrow b$				

$$E' \rightarrow eE$$

$$E' \rightarrow \epsilon$$

Multiple entries

\* Not LL(1) Grammar

## # CHOMSKY HIERARCHY

On the basis of type of productions  
 Chomsky Classify grammars in 4  
 categories :-

- ① Type - 3 Grammar R.L
- ② Type - 2 " C.F.L
- ③ Type - 1 " CSL
- ④ Type - 0 " R.E

$A \rightarrow \alpha B | \beta$  right linear  
 $A \rightarrow B \alpha | \beta$  left linear  
 $A \rightarrow \alpha B | \beta$  right linear  
 $A \rightarrow B \alpha | \beta$  left linear

### TYPE - 3

A grammar in which all the productions are of Type

$$A \rightarrow \alpha B | \beta$$

T3 RE  
 T2 CFL  
 T1 CSL

T0 RE

$$\alpha, \beta \in T^*$$
 (Terminal)

$$A, B \in V$$
 (Variable symbol)

$$A \rightarrow \alpha B | \beta$$

$$A \sim \alpha$$

1

$$\boxed{A \rightarrow B\alpha | \beta \\ A \rightarrow \alpha B | \beta}$$

Date: \_\_\_\_\_

More specifically it is called as  
right linear grammar

$$\begin{aligned} A &\rightarrow aB | b \\ B &\rightarrow cD | e \end{aligned} \quad ]$$

$$A \rightarrow aBb \quad \times$$

OR

A grammar is said to be Type 3  
if all its productions are of type

$$\boxed{A \rightarrow B\alpha | \beta}$$

more specifically it is called  
as left linear grammar

⇒ Ya k left linear ho ya right linear  
In dono ka combination haliy  
hona chahiye

7/12  
r/B/K

A → AB/P  
B → B/B

(A+B)

143 → RL  
Type 0 L.F.  
CST  
M.T.

Date:

Q

$\rightarrow A \rightarrow aA \mid bB \mid b \quad ] \text{ Right L.G.}$

$\rightarrow A \rightarrow Aa \mid Bb \mid a \quad ] \text{ LLG}$

$A \rightarrow aA \mid Ab \mid a \rightarrow \text{not Type 3}$

→ Type 2 grammar corresponds to regular languages. Or it is a generator of R.L & corresponding machine finite automata.

Type 2

If all the productions in a grammar are of type  $(A \rightarrow B)$

$A \in V$  left var. variable

$B \in (VUT)^*$  right var. variable, terminal any comb.

ex

$$A \rightarrow \underline{a} A b$$

Left hand  
 side has exactly  
 one variable!

right side kuch bhi ho.

$$S \rightarrow a S A b$$

↓

One variable      any no.

→ They are also called  
'Context free Grammar'

Means I can replace  
 a variable without  
 worrying about its  
 neighbours. (at any intermediate  
 step)

Ram goes 2 Agra }  
 too Agra } below we  
 to Agra } say same  
 noi

Context means Surrounding

→ The lang that corresponds to context free grammar in Context  
free language 2 the machine  
 that corresponds to Type 2 grammar  
 is PDA or Rush Down Automata

→ Some properties that belongs to this class of lang.

$w \in L(G)$

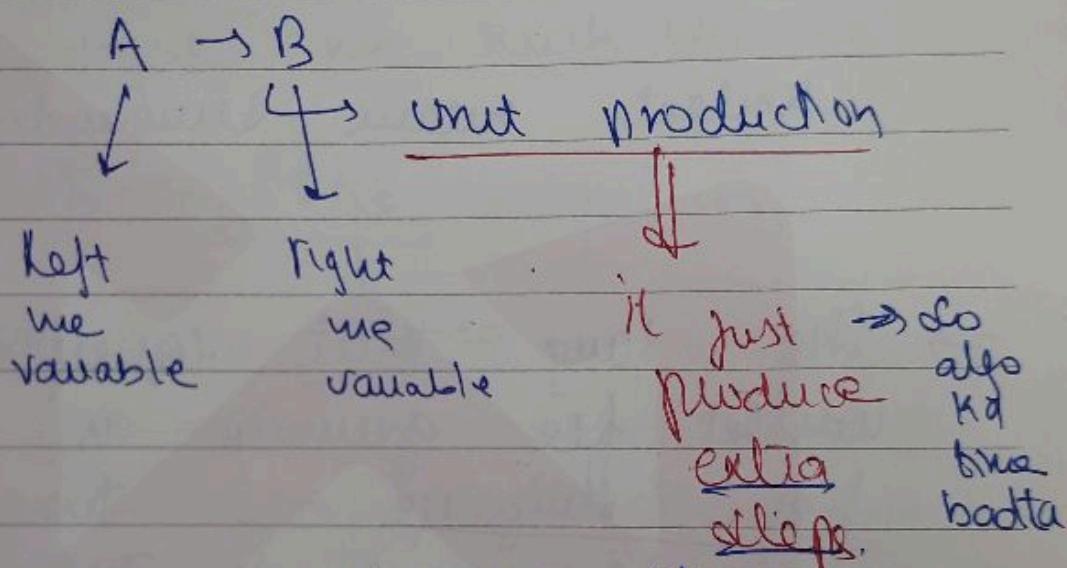
Given a string, <sup>whether</sup> belong to language generated by that grammar or not.

## Pausing

Null introduce hope of contraction

① Null production  $\rightarrow$  & we need to remove them.

②



(to know the step we note that it will take no. of steps)

③

remove unit productions

④

remove unreachable states all non reachable symbols



We remove f to speed up  
the process of reduction

Generating symbols:

Symbols which  
directly or  
indirectly  
leaves a string

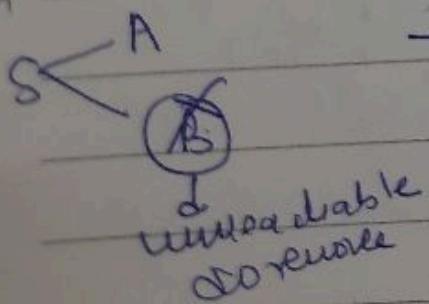
Also the symbols jo directly or  
indirectly kuch bhi strings mai  
generate karte mere kisi  
kaam ki hai hai. I.e.  
called non generating symbols

(4)

$$S \rightarrow aAS \mid bA$$

$$A \rightarrow aA \mid a$$

$$\textcircled{B} \xrightarrow{} bB \mid b$$



unreachable  $\nRightarrow$  use se  
~~non gone~~

final state  
par mat  
ajao)

remove unreachable states

### ⑤ left recursion

$$A \rightarrow A\alpha / B$$

$A \rightarrow A\alpha / B$   
 $A \rightarrow A\beta / \epsilon$   
 $A \rightarrow A\gamma / \epsilon$   
 $A \rightarrow A\delta / \epsilon$   
 $A \rightarrow A\epsilon / \epsilon$

$$w = \alpha^{100} B$$

$A \rightarrow A\alpha / B$   
 $A \rightarrow A\beta / \epsilon$   
 $A \rightarrow A\gamma / \epsilon$   
 $A \rightarrow A\delta / \epsilon$   
 $A \rightarrow A\epsilon / \epsilon$

parser will read  $\alpha \alpha \alpha \dots \beta$

$A \rightarrow B A$   
 $A \rightarrow A A / \epsilon$

problem problem of parser

In order to derive an algo we need to solve these problems first.

Context free grammar

$\downarrow$   
Simplify

$\downarrow$   
Algorithm

$A \rightarrow A\alpha B$   
 $A \rightarrow A\beta / \epsilon$   
 $A \rightarrow A\gamma / \epsilon$   
 $A \rightarrow A\delta / \epsilon$   
 $A \rightarrow A\epsilon / \epsilon$

## ① Removing null productions from the grammar.

not all can be removed.  
it is a part of theory then we can't.

Consider a grammar

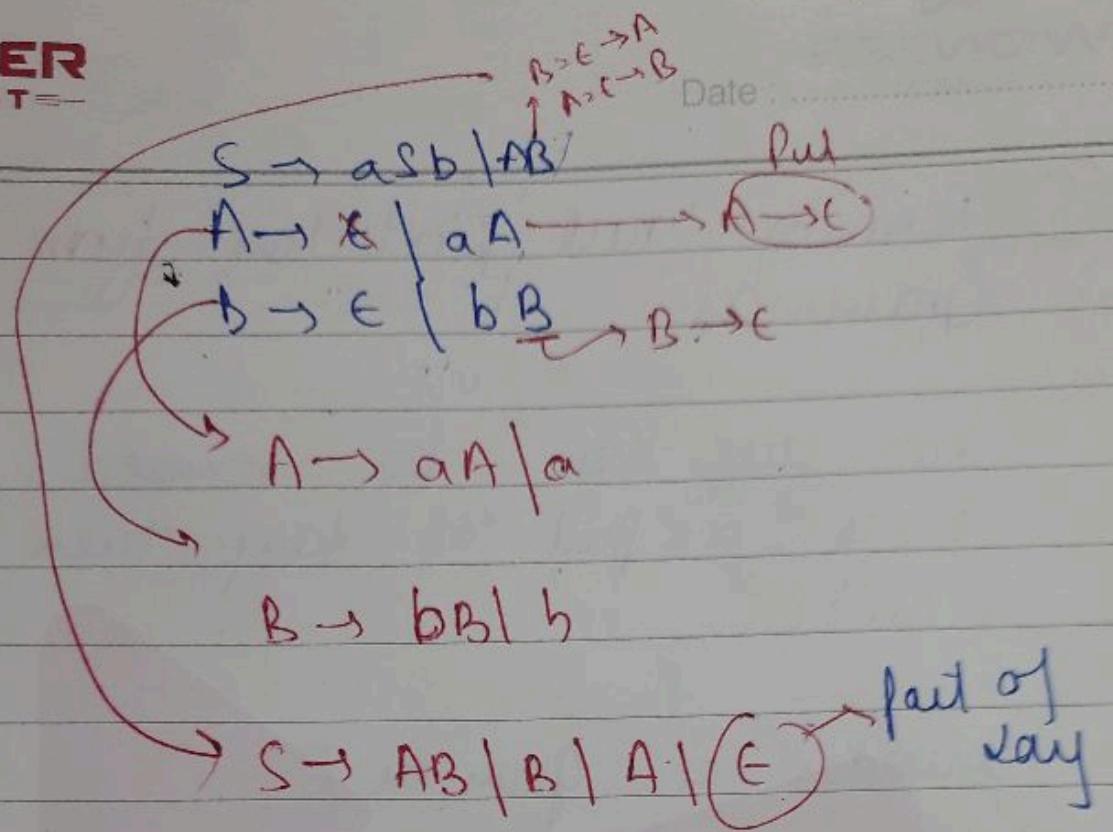
$$\begin{aligned}
 S &\rightarrow aSb \mid AB \\
 A &\rightarrow \epsilon \mid aA \\
 B &\rightarrow \epsilon \mid bB
 \end{aligned}$$

remove null productions from the given grammar.

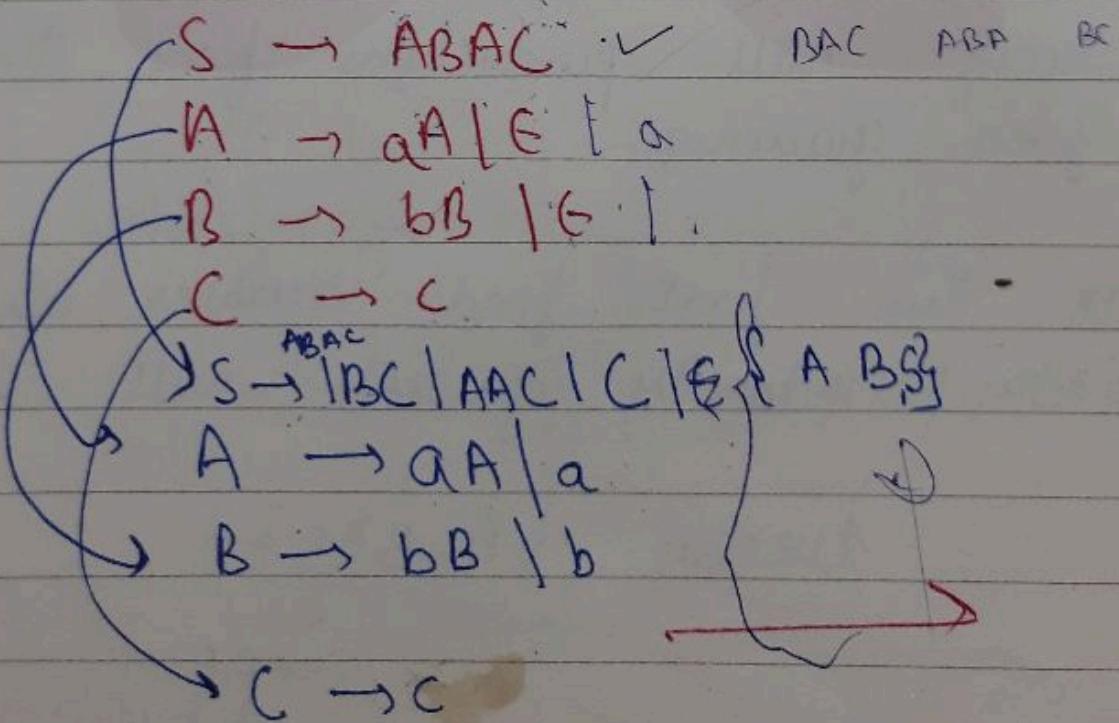
for this first find variables which directly or indirectly generate null.

directly {A, B, S}

$$\begin{array}{c}
 S \Rightarrow AB \\
 | \quad | \\
 E \quad G
 \end{array}$$



Ques remove null production from the given grammar



Ques by {A, B}  
pick one.

$S \rightarrow ABAC \mid BAC \mid ABC \mid BC \mid AAC$   
 $\qquad \qquad \qquad AC \mid \text{ } C$

A  $\rightarrow aA \mid a$

B  $\rightarrow bB \mid b$

C  $\rightarrow c$

~~Ques~~ Remove null production from the G

$S \rightarrow aS \mid B \mid aB \mid a \mid Ab$

$B \rightarrow BB \mid ab \mid aB \mid aA$

$A \rightarrow BA \mid a \mid \epsilon \quad \{A\}$

$S \rightarrow aS \mid -ab \mid a \mid b \mid Ab$

$B \rightarrow BB \mid ab \mid aB \mid aA$

$A \rightarrow B \mid a \mid \text{ } \mid BA$

now  
generating

now  
reacheable

removing useless symbols

just  
deleting  
for  
terminal  
generat  
new  
heads

Q Consider a grammar

$S \rightarrow aS \mid A \mid C$   
 $A \rightarrow a$   
 $B \rightarrow aa \quad \cancel{A} \cancel{X}$   
 ~~$C \rightarrow aCb \cancel{X}$~~  → it does not generate any terminal

$A \rightarrow a$  directly string will take has

a.

~~$B \rightarrow a$~~  string will take has

has

$\{A, B\} \cup T \rightarrow a$

ab has production & right me  
fao.

$\{A \mid B\} \cup T \rightarrow$  that generate  
string

jo directly ya indirectly  
 humne kuch generate  
 karke nahi dekhne ke  
 walo Samajh karta do.

$C \rightarrow$  non generating since  
 it does not generate  
 any string (directly)  
 (non directly)

### remove C

$$\begin{aligned}
 S &\rightarrow as | A \\
 A &\rightarrow a \\
 B &\rightarrow aa
 \end{aligned}$$

Now for non reachable (<sup>jo starting symbol se reachable na ho</sup>)  
 $S \xrightarrow{*} \alpha * \beta$

$\rightarrow$  A variable  $\alpha$  is said to be  
 reachable if  $S \xrightarrow{*} \alpha * \beta$

$$\begin{aligned}
 S \text{ se as } S \xrightarrow{*} as \\
 S \xrightarrow{*} A
 \end{aligned}$$

$S \xrightarrow{*}$  we can't reach  $B$  from  $S$   
 $B$  par  
 technique remove  $B$   
 suppose to

$B$  is unreachable

$S \xrightarrow{*} as   A$	✓
$A \rightarrow a$	

Q3 Remove useless symbols from  
given CFG.

direct  $\rightarrow S, E, f,$   
 $B$

$$\begin{array}{l}
 \boxed{S \rightarrow aSB \mid ac \mid d} \\
 \boxed{E \rightarrow aC \mid DE} \\
 \boxed{E \rightarrow cc \mid DD} \\
 \boxed{D \rightarrow CA \mid af} \\
 \boxed{F \rightarrow Sc \mid Ba \mid a.} \\
 \boxed{B \rightarrow aA \mid a} \\
 \begin{matrix} B & A \\ F & D \\ E & \end{matrix} \\
 \boxed{A \rightarrow BB \mid BC}
 \end{array}$$

$$\begin{array}{l}
 \checkmark A \rightarrow aa \mid a \mid c \mid b \\
 \cancel{B \rightarrow bb \mid clate} \\
 \cancel{C \rightarrow c \mid ale} \\
 \cancel{D \rightarrow a \mid k}
 \end{array}$$

S ~~non nullable~~

Direct production deha  
jese

$$\begin{array}{c}
 \boxed{S, E, F, B,} \\
 \Downarrow
 \end{array}$$

- ① non nullable
- ② no terminal
- ③

Ye sab direct string  
de raha hai

~~A~~  $\rightarrow$  indirect.

Unit Production :-  $A \rightarrow B \rightarrow C \rightarrow D$

A ka set {BCD}  $\rightarrow$  non BCD me  
Date:  $\rightarrow$  jo jo non  
B ka set (CD)  $\rightarrow$  terminals hai  
Date:  $\rightarrow$  A me  
Date:  $\rightarrow$  UKH do.

$$S \rightarrow aSb \mid acl \mid d$$

$$E \rightarrow ccl \mid DN$$

$$F \rightarrow sc \mid Ba \mid a \quad * \text{ye use natu hog}$$

$$B \rightarrow aA \mid a$$

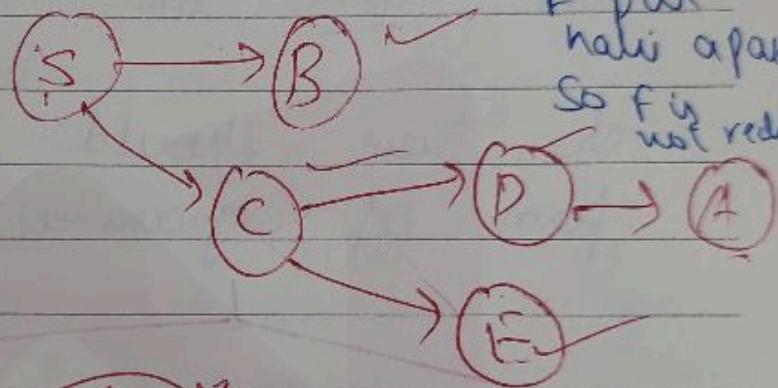
S se  
f Jane  
ka noi  
rast  
natu  
hai

$$A \rightarrow Bb \mid BC$$

$$C \rightarrow$$

$$D \rightarrow$$

S se huk  
f par  
hai apne  
so f is  
not reduce



Unit Production

$$A \xrightarrow{\text{unit}} B \xrightarrow{\text{unit}} C \xrightarrow{\text{unit}} D \xrightarrow{\text{unit}} E$$

~~$A \rightarrow aA \mid B \mid b \mid c \mid e$~~

~~$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E$~~

~~$B \rightarrow b \mid C$~~

~~$B \rightarrow a$~~

~~$C \rightarrow D \mid C$~~

~~$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E$~~

~~$D \rightarrow a \mid e$~~

~~$A = \Sigma B, C, D \}$~~

~~$B \rightarrow$~~

$$A \rightarrow B \rightarrow C \rightarrow D \rightarrow a \mid e$$

$$A \rightarrow aA \mid a \mid e \mid \text{del} \mid c \mid e$$

add now

unit production  
of all  
symbols that  
are reachable to



CNF

→ Chomsky Normal form

We could have multiple grammars that represents same language. But the way, the grammar is written has effect on the way of parsing.

So there should be a proper form of grammar.

CNF

Chomsky normal form

CINF

Ureibach Normal form

If all the productions of grammar are of type

$$\begin{array}{l} A \rightarrow BC \\ A \rightarrow a \end{array}$$

~~2w / 0 w<sup>\*</sup>~~

~~2wP~~

~~A → BC  
A → a~~

$G \rightarrow S \rightarrow aSb | aAbBb | aAa$ .  $w = aabb$   
 $A \rightarrow aAa$  derivation of  $w$   
 $B \rightarrow bBb$  will be of length  
 $|w| - 1 = 2$

then the grammar is said to be  
in CNF.

two steps me length bade  
strings me.

~~DU~~ 200 variables

$S \rightarrow BC$

$\rightarrow EFC$

$DCFC$

length of string

$|w| - 1 + |w|$  to replace  
a variable.

bcoz  
rule  
me 2

variable  
has

$|w| \rightarrow$  length ki string

will be derived in

$2|w|$

$= 2|w| - 1$  time.

$S \rightarrow BC$  ]  $|w| - 1$

$BDE$

$|w|$  [  $a^k$   
 $a^k$   
 $\underline{abc}$

$S \rightarrow BC$

$C \rightarrow DE$

$P \rightarrow$  no of producers

$|P| \rightarrow$  total no of prod.

$S \rightarrow$  asb | bsa | bsb | asa | a

~~$|P| + |P| |P|$~~  &  $S \rightarrow$  asb  $\rightarrow$  out of  
 S possibilities  
 like lie  
 bhi S possibility  
 hai

Second step k lie  
 25 poss.

$$|P| + |P| \cdot |P| + |P|^3 + \dots |P|^{2w-1}$$

$\downarrow$   
 forms a GP.

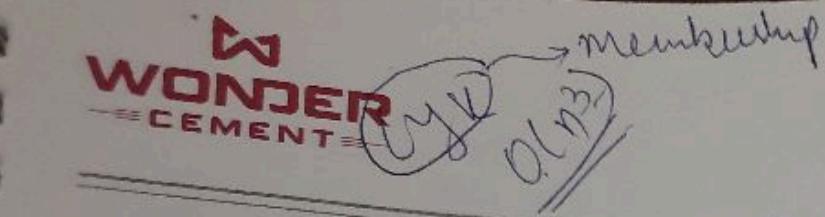
$$= 6 \left( |P|^{2w} \right) \rightarrow \text{Cnf form}$$

$O(P)^{2w}$

$O(n^3)$

$O(P^{2w})$

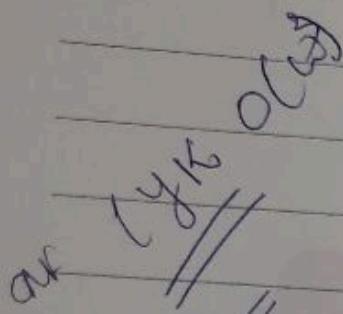
$O(P^{2w})$



$O(n^3)$

4 productions

exponential complexity



CNF (Application)

It is very less efficient  
based on dynamic programming (NFC)

It is used in CYK algorithm

$w \in L(G)$

$\hookrightarrow w^3 \rightarrow$  string length using  
CYK rule

Input of CYK algo

cube name  
par integ



grammar in the  
form of CNF

⇒ CYK algorithm is used to find the  
membership of CFG

Q: Also its complexity is  $O(n^3)$

\* If grammar is in CNF then  
parse tree will be of  
Binary tree

CNF is good.

CYK is a universal of algo,  
 kisi Par bhi laga sakte hau

$\Rightarrow \mathcal{L}(N) = \text{closure}$   
 a binary  
 $\Rightarrow O(N^2w)$ .  
 $\Rightarrow O(w^3)$  CYK.  
 Cut & Join  
 Leftmost Binary Tree  
 step

## Ambiguity

↳ Confusion

$$\begin{array}{c}
 \overline{10} \times \overline{4} \\
 \boxed{10 / 2 * 4} \\
 \xrightarrow{\hspace{2cm}} 20 \\
 | \leftarrow
 \end{array}$$

↳ Same expression,  
we get 2  
outputs

Left se complete kaise pe 20  
right " " " " 1

$a/b * c \rightarrow \text{string}$

If string has 2 O/Ps means  
it can be derived 2 to parse  
tree

⇒ If a grammar fails to provide  
unique structure of the sentence  
then grammar is ambiguous grammar

\* If there is a string  $w$ , that can be derived into <sup>one</sup> or more different ways, that string is ambiguous.  $w$  in grammar is ambiguous.

→ There is no way <sup>(algo)</sup> we could determine whether a grammar is ambiguous or not.

→ We cannot remove the ambiguity in every grammar.

→ Inherently ambiguous

If a problem cannot be solved without kitni koshish karlo to  
Ex hi rhega it cannot be solved  $\rightarrow$  then it that problem is inherently ambiguous.

It can't be solved.

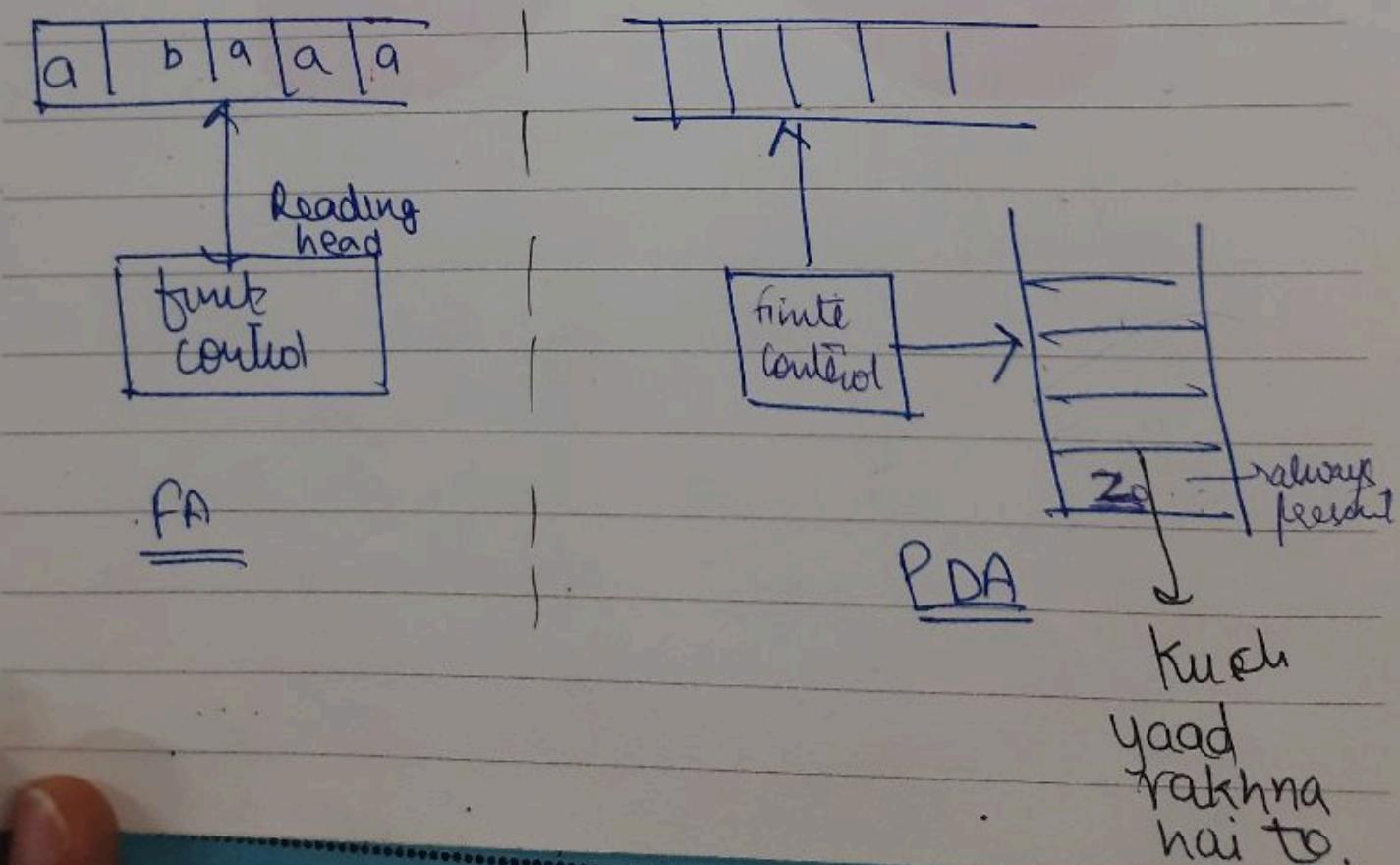
- only by ~~as~~ luck.
- but is trial.

\* Ambiguity is an undecidable problem.

## Pushdown Automata (PDA)

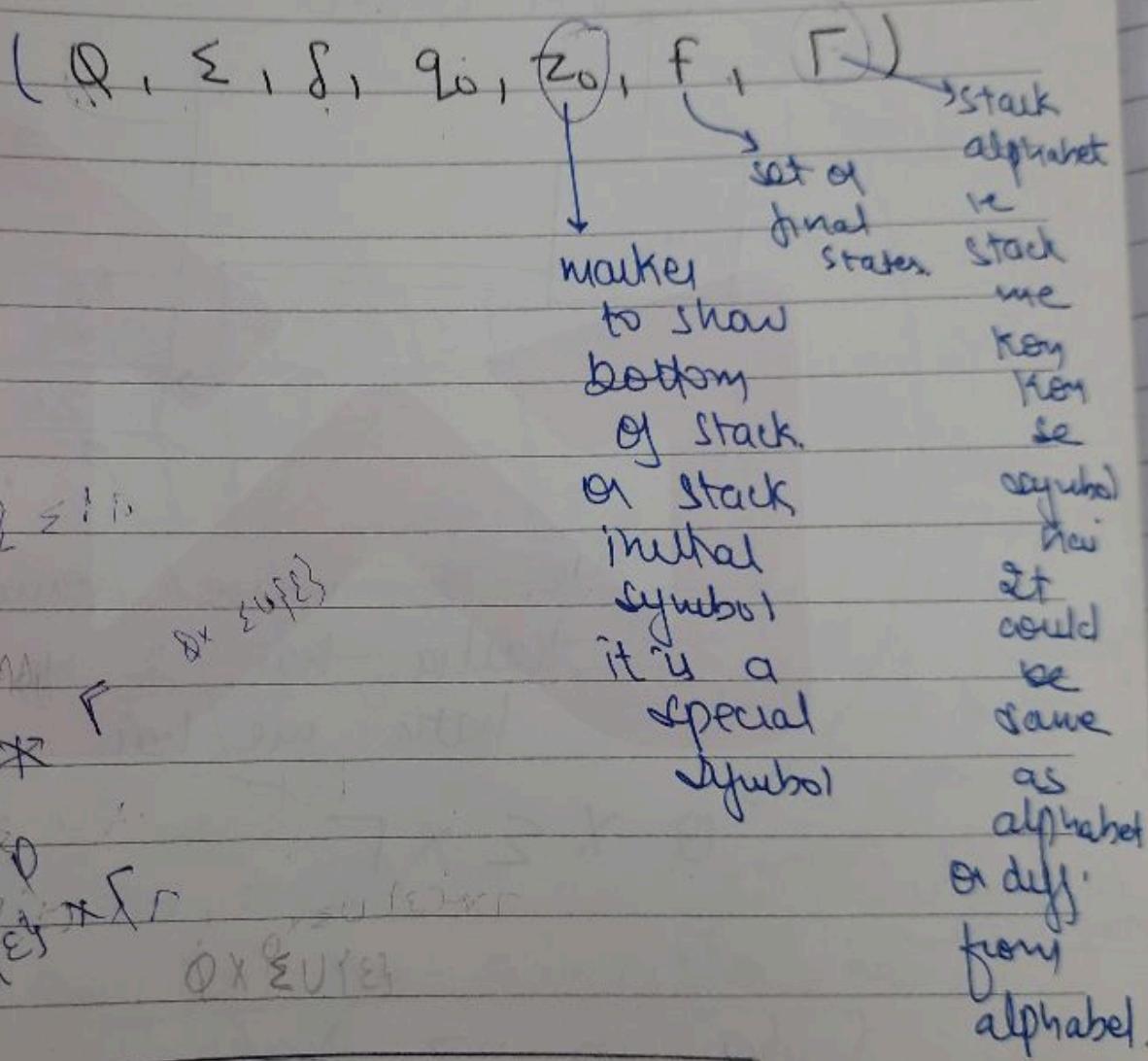
The device or machine that corresponds to context free languages is called as Push Down Automata.

Pushdown Automata = finite automata + memory element  
 ↓  
 in form of  
 ('STACK')



Stack :- Restricted Data Structure

\* PDA is a 7 tuple collection



\*  $[Q \times \Sigma \cup \{\epsilon\} \times \Gamma]$

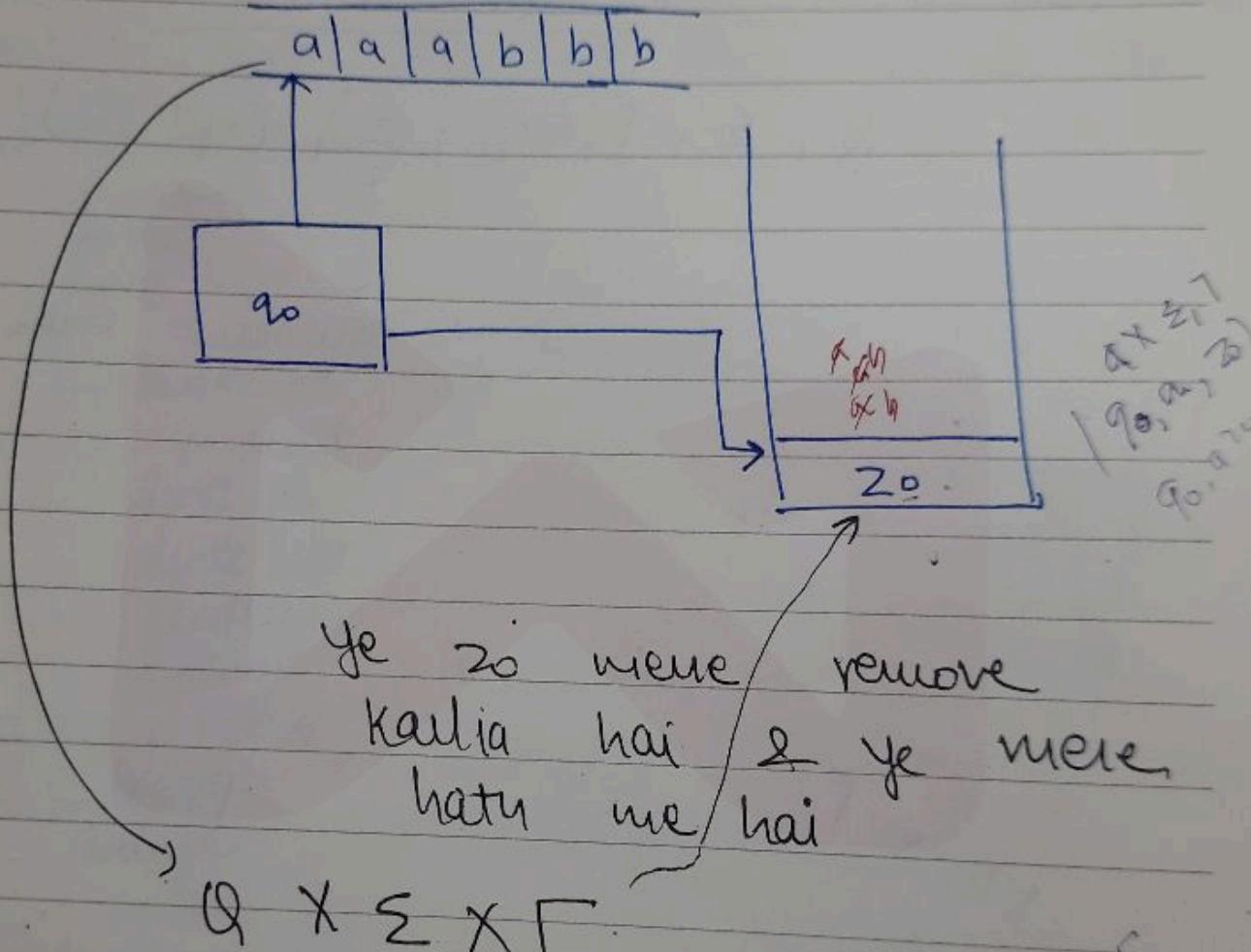
stack  $\times$  reading something

$\times$  content of stack.

Let apple mang.

$$L = \{a^n b^n \mid n \geq 1\}$$

$$a^4 b^4$$



$S(q_0, a, z_0)$

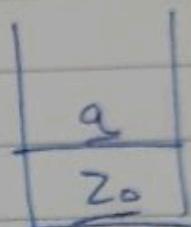
I am at state  $q_0$

reading  $a$

element on stack

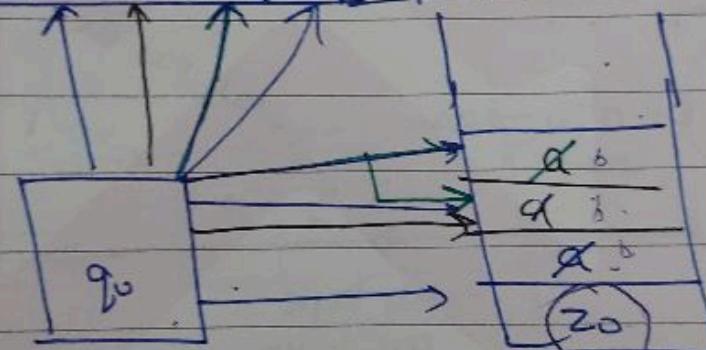
In order to push a, first push  
20 then push a.

$$= (q_0, a_{20})$$



Now

| a | a | a | b | b | b | e | c



b atta  
hi mang  
ayaja  
& purana  
a kato

pop &  
yo b  
uthaya are  
dato kato

$$\rightarrow S(q_0, a, \textcircled{a})$$

go pau keu, a atta hai  
slack pau a hai

length  
bad  
rakhi

hai now

$$\{ (q_0, \textcircled{a}) \}$$

Purana a  
ko utha lia

true,  
it is  
push

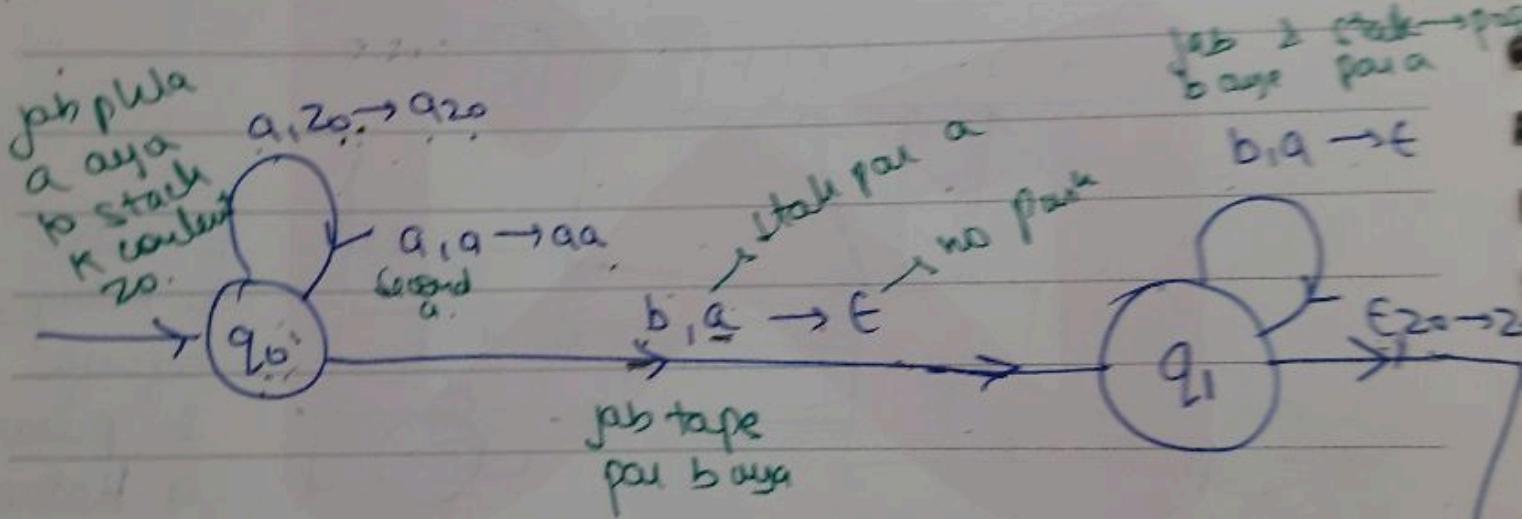
$$\delta(q_1, b, a) = (q_1, \epsilon)$$

↓      ↓  
 $q_1$  pair    b pair  
 hu      hu

click  
 me a  
 no

will  
 move (pop)

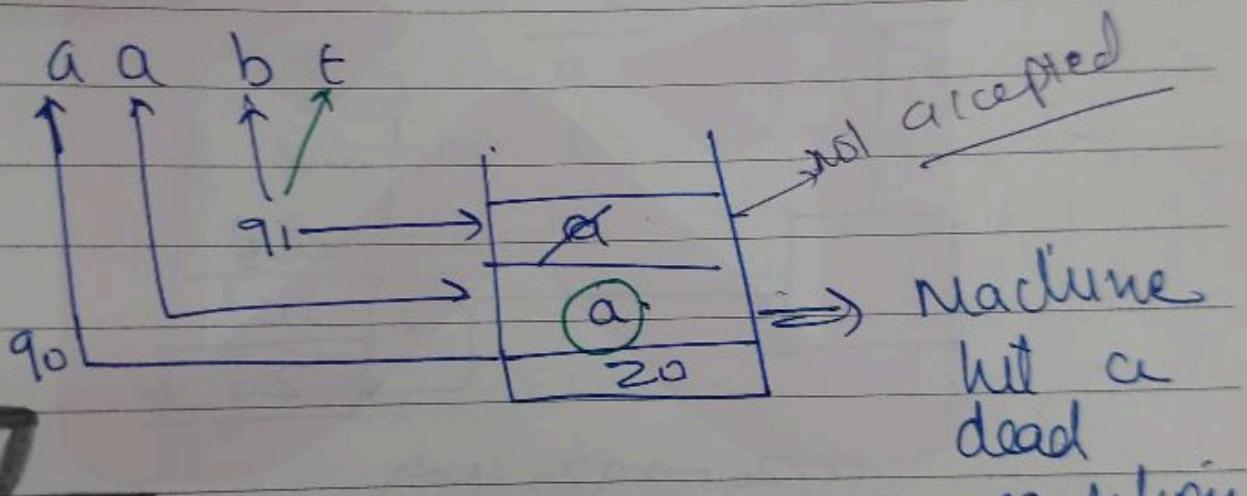
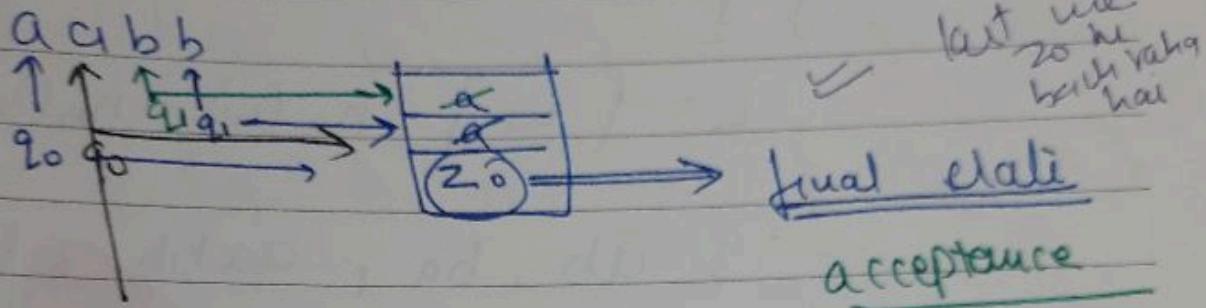
$$\delta(q_1, \epsilon, z_0) = (q_f, z_0)$$



$\epsilon z_0 \rightarrow z_0$   
 phle a hi aya ga ye to  
 confirm hai bw2  $a^n b^n, n > 1$

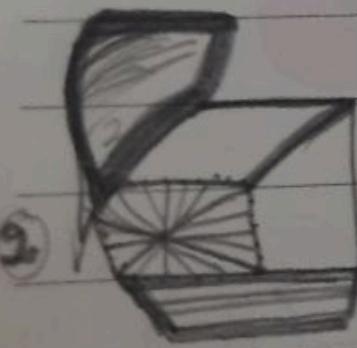
(qf)

Let's take a step qabb



Machine halts at  
a non final  
state.

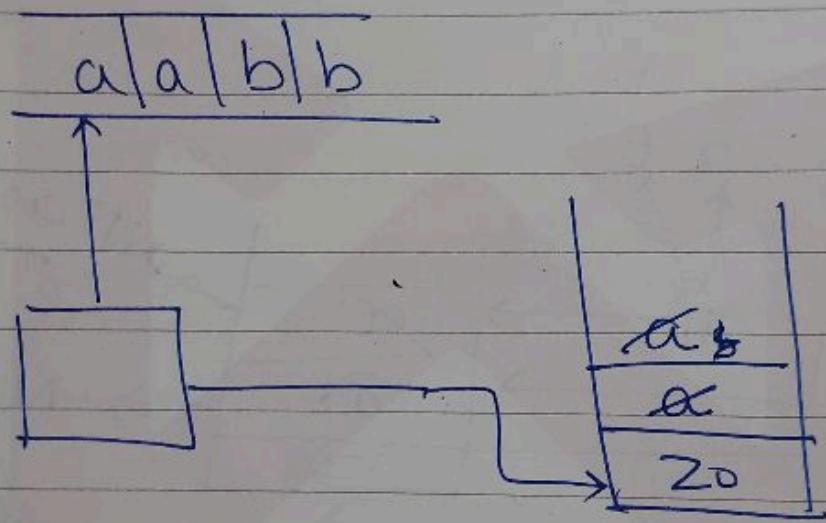
rejectance



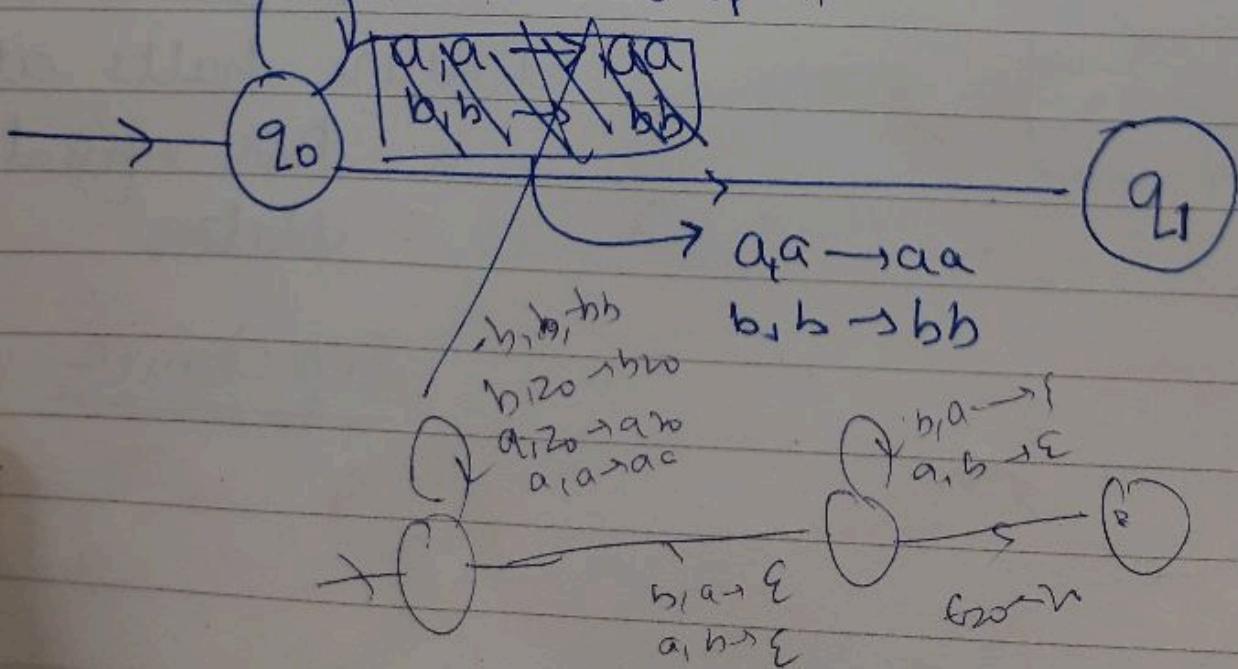
Ques Design a PDA for the language.

$$L = \{ w \mid n_a(w) = n_b(w) \}$$

$$\{ ab, ba, aabb, baab \}$$



$$a, z_0 \rightarrow a z_0 \text{ (push)} \\ b, z_0 \rightarrow b z_0 \text{ (push)}$$



$$\delta(q_0, a, z_0) \rightarrow \delta(q_0, az_0)$$

$$\delta(q_0, b, z_0) \rightarrow \delta(q_0, bz_0)$$

$$\delta(q_0, b, a) \rightarrow \delta(q_0, \epsilon)$$

$$\delta(q_0, a, b) \rightarrow \delta(q_0, \epsilon)$$

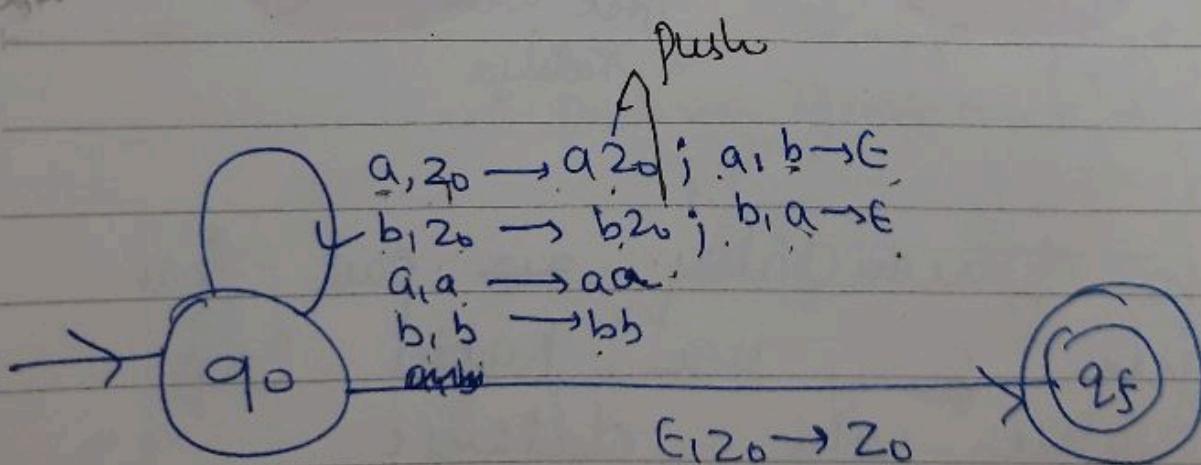
$$\delta(q_0, a, a) \rightarrow \delta(q_0, aa)$$

$$\delta(q_0, a, b) \rightarrow (q_0, \epsilon)$$

$$\delta(q_0, b, a) \rightarrow (q_0, \epsilon)$$

$$\delta(q_0, b, b) \rightarrow (q_0, bb)$$

$$\delta(q_0, \epsilon, z_0) \rightarrow (q_f, z_0)$$

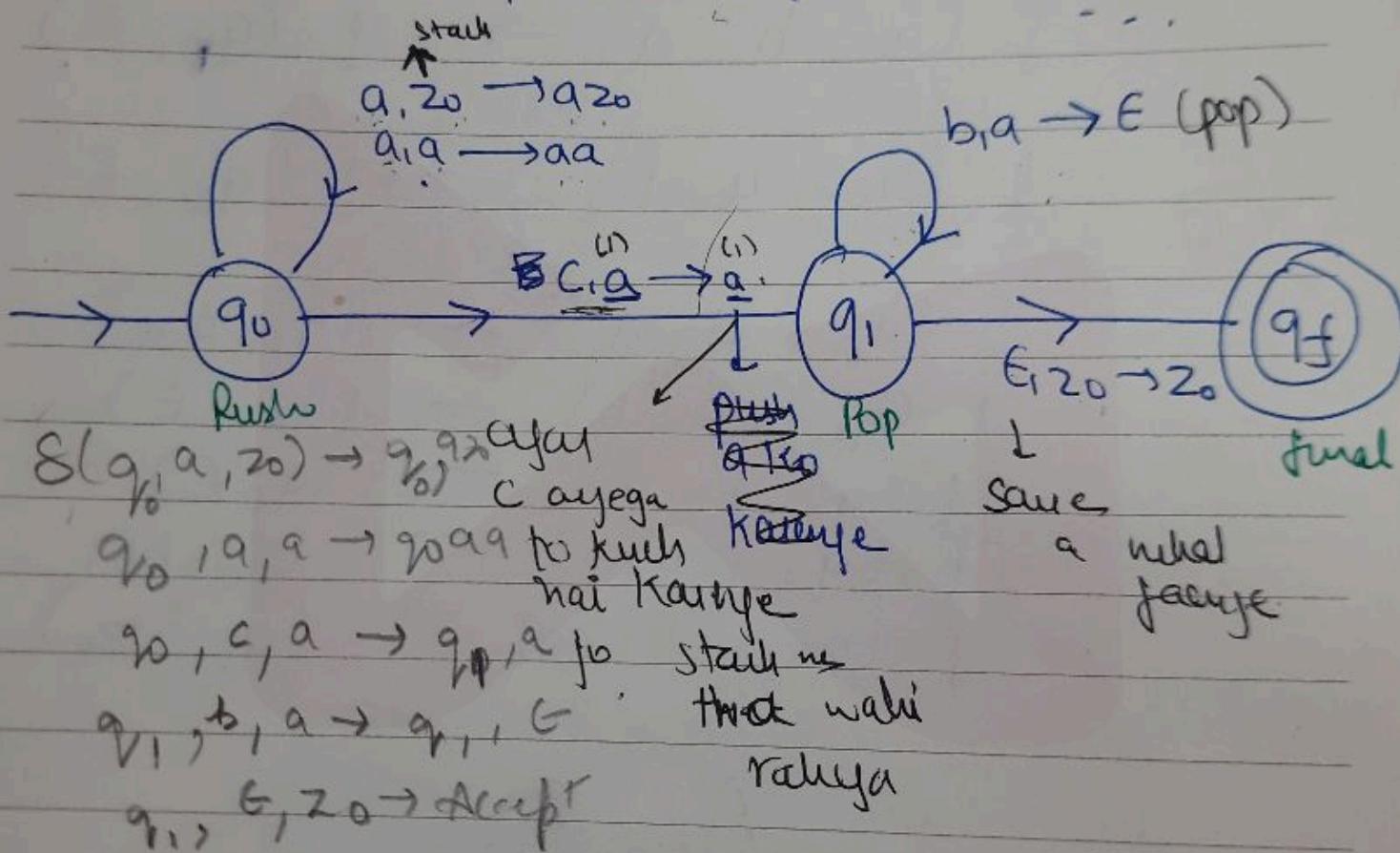


exactly 1 c

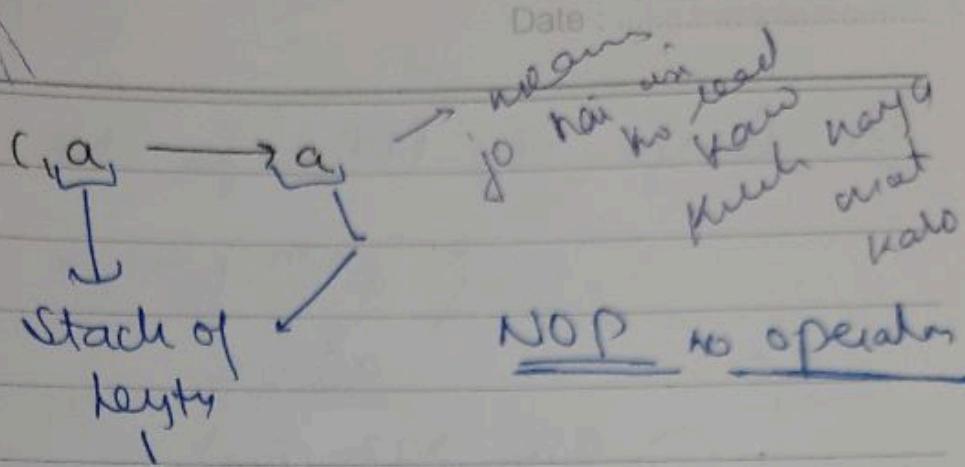
Q1 Design a PDA for the lang.

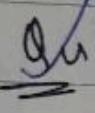
$$L = \{ \frac{a^n b^n}{a^n c^m}, \mid n \geq 1 \}$$

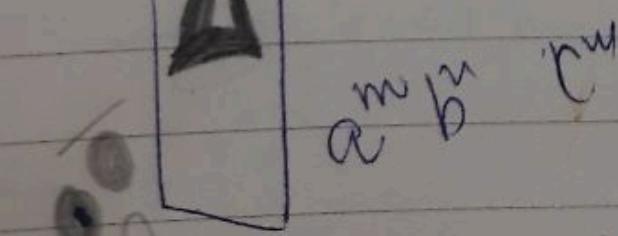
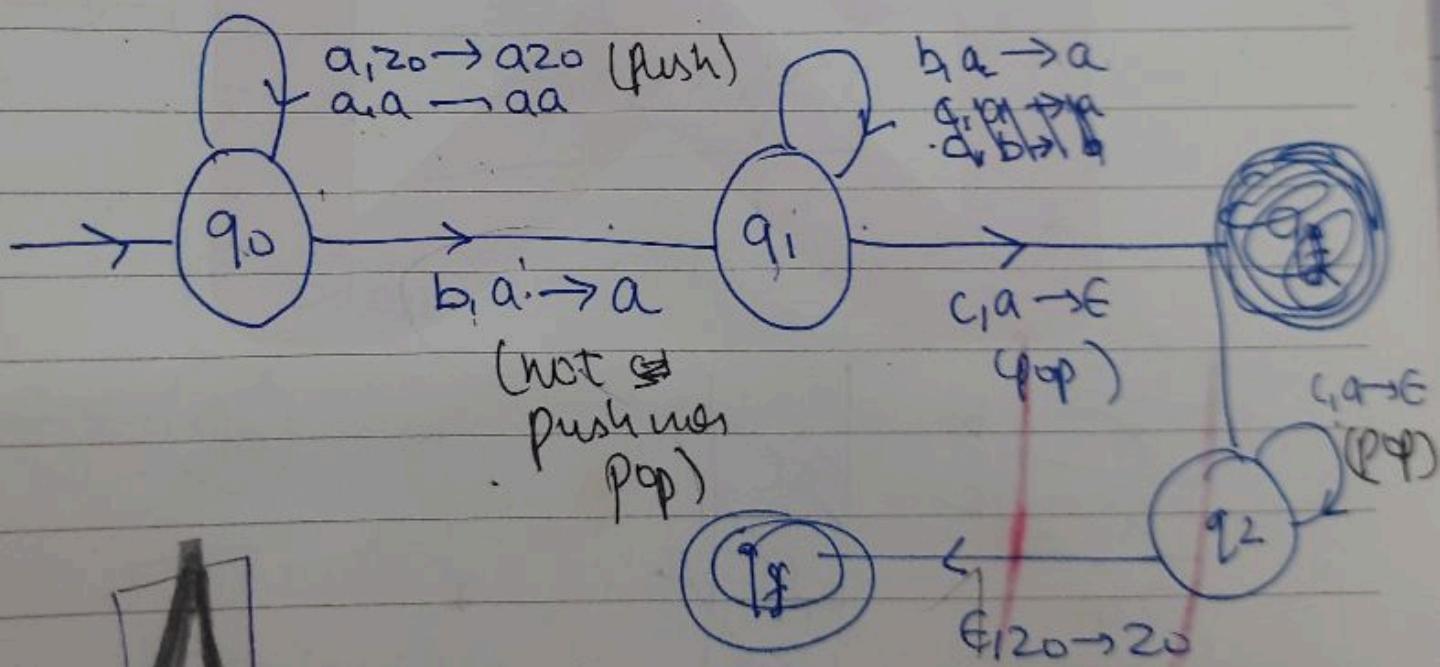
lach, aacbb, aaacbbb ~



→ jo mandatory me part hai  
jise  $\subseteq$ , me kabhi loop  
me nahi, dalenge



 Design a PDA for the lang  
 $L = \{ a^n b^m c^n \mid n \geq 1, m \geq 1 \}$



$a^m b^n c^n$

$$\delta(q_0, a, z_0) = (q_0, a z_0)$$

$$(q_1, c, a) \rightarrow q_1, t \quad \delta(q_0, q_1, a) = (q_0, a a)$$

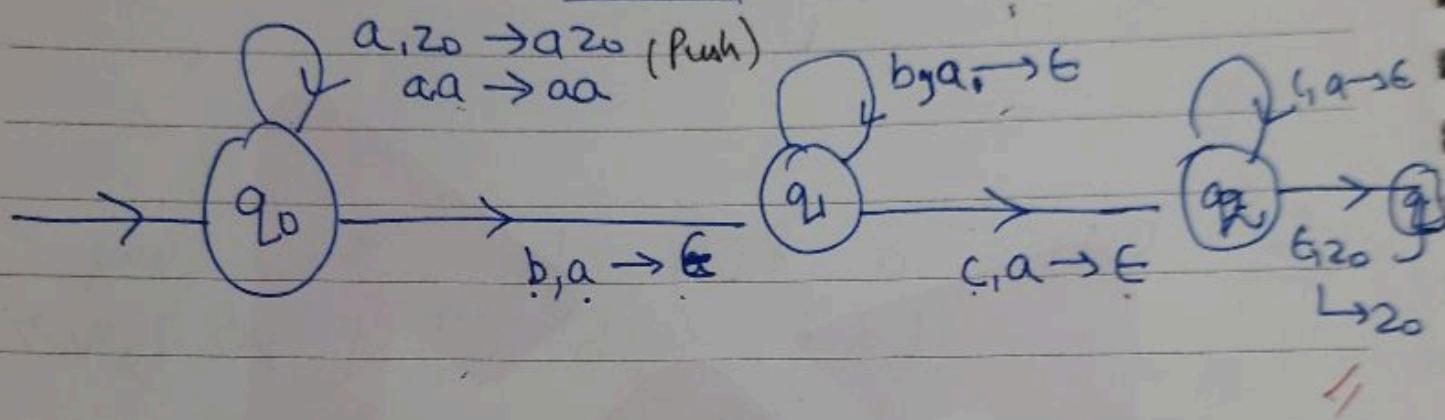
$$q_1, \epsilon, z_0 \rightarrow q_1, \lambda \quad \begin{cases} (q_0, b, a) \rightarrow q_1, a \\ (q_1, b, a) \rightarrow q_1, a \end{cases}$$

$a^m b^n c^n$

Design a PDA for the lang.

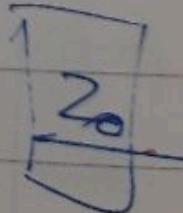
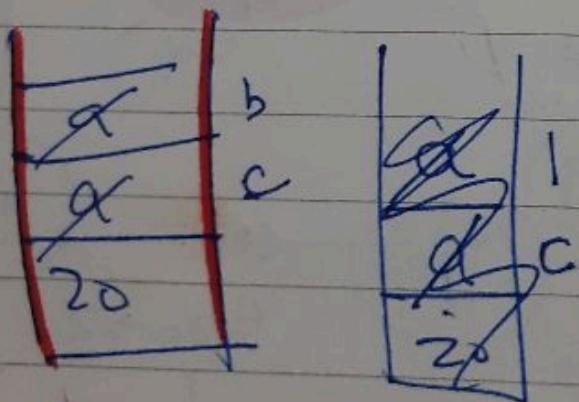
$$L = \{ a^{m+n} b^m c^n \mid m, n \geq 1 \}$$

aabc



$a^m b^n c^n$

aabc

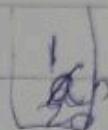


Q Design a PDA for

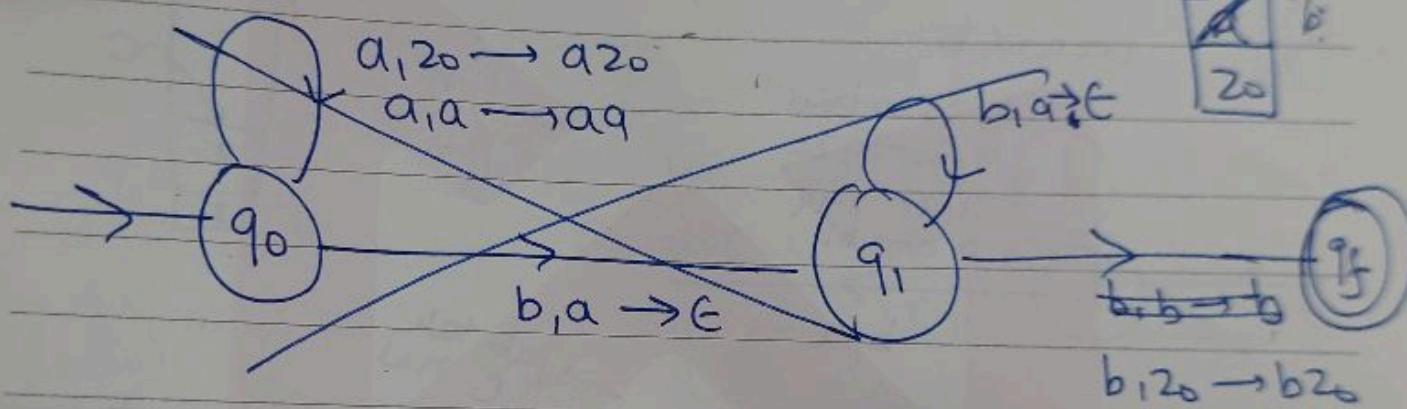
$$L = \{ a^n b^{2n} \mid n \geq 1 \}$$

abb

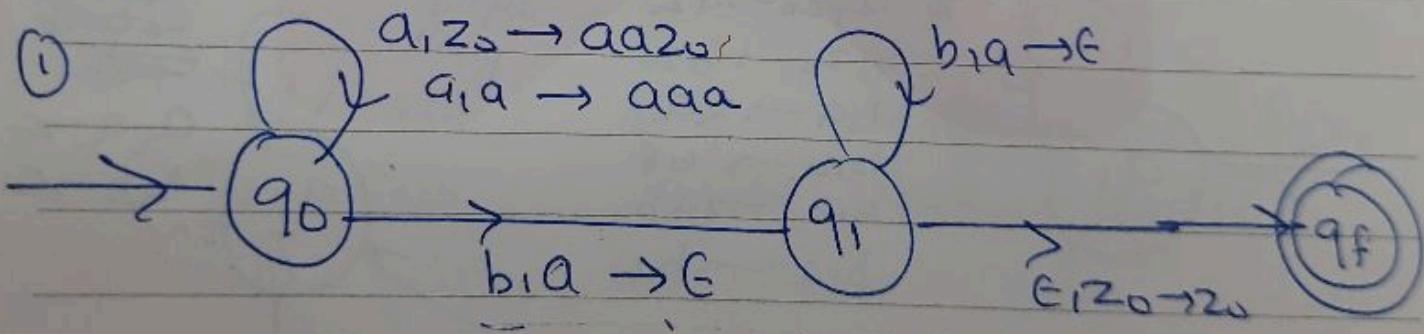
a



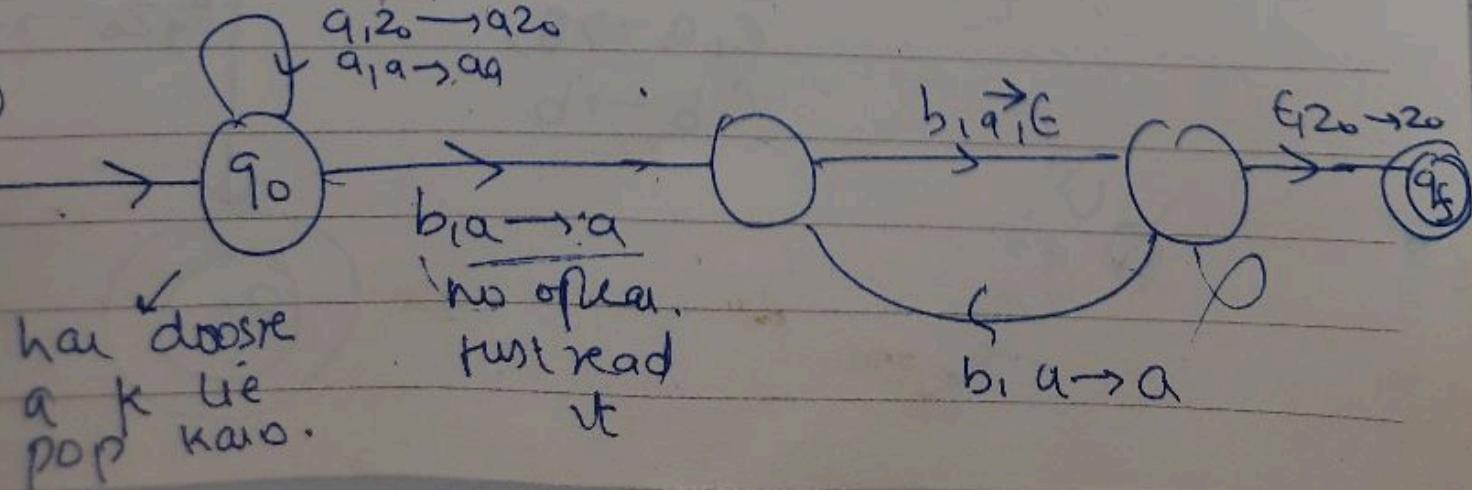
abb



①



②

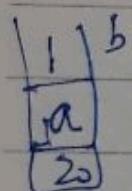




Q1 Design a PDA for a lang.

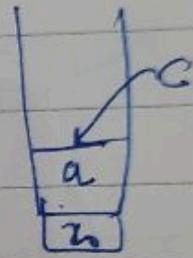
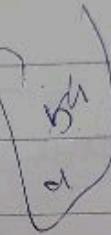
$$L = \{ w c w^R \mid w \in (a,b)^+ \}$$

↓  
stay



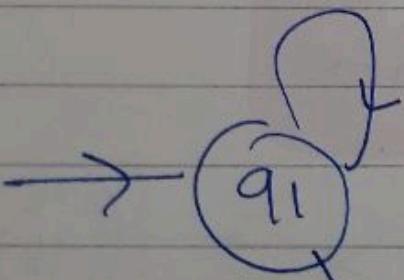
aca  
abcba  
bacab

abcba



~~b, 20 → b20  
a, 20 → a20  
a, a → aa  
b, b → bb~~

b, a

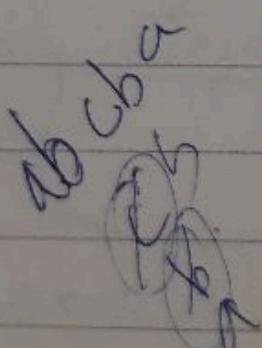
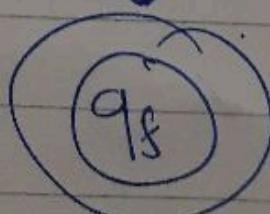


~~b, a, 20 → a20  
b, 20 → b20  
a, a → aa  
b, b → bb  
a, b → ab  
b, a → ba~~

tab tab  
c nai  
ay egg  
push

c, a → a  
G b → b

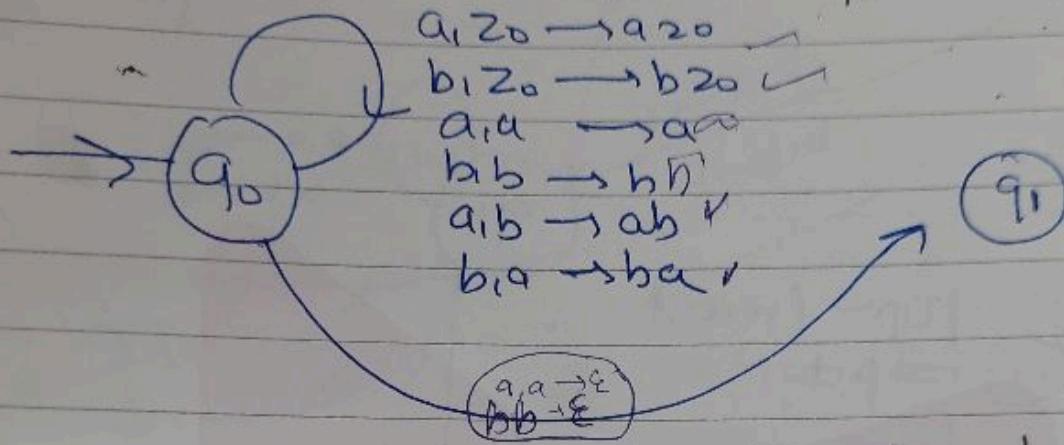
G20 → 20



Date: abba  
 Name: ~~Abhishek Patel~~  
 Class: ~~10th~~  
 Subject: ~~Computer Science~~  
 Page No.: ~~10~~  
 Date: ~~10/10/2018~~  
 Name: ~~Abhishek Patel~~  
 Class: ~~10th~~  
 Subject: ~~Computer Science~~  
 Page No.: ~~10~~

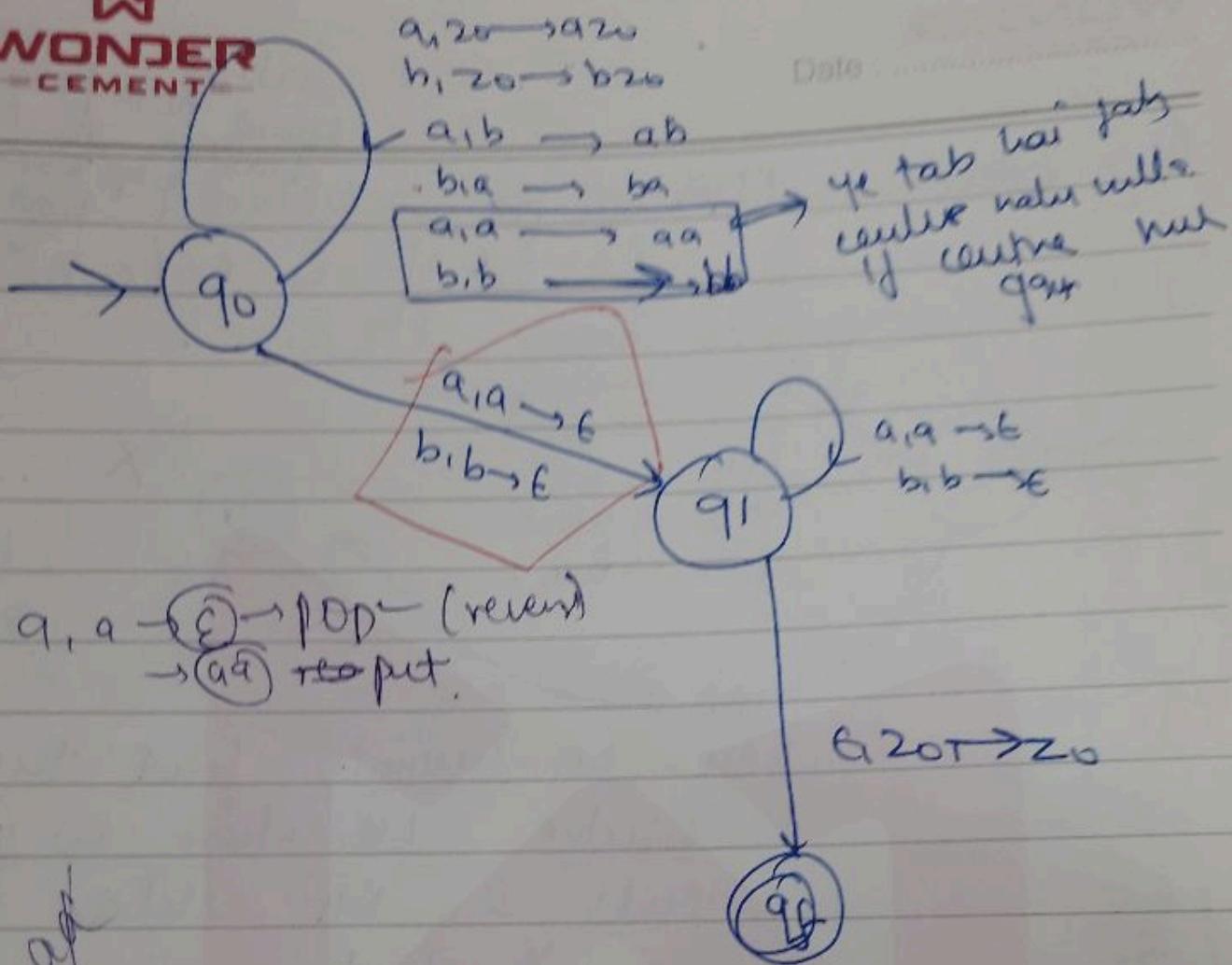
Q. Design PDA for

$$L = \{ww^R \mid w \in (a,b)^*\}$$



here we cannot find the  
centre null when w  
ends & null when  
w starts.

We can't find deterministic PDA  
for this. Kya kisi kabi咸咸  
mil jaega aur kabi咸咸 nahi mil jaega.



$a, a \xrightarrow{\epsilon} \text{PDA (revert)}$   
 $\rightarrow q_0$  re-put.

This is now deterministic  
PDA.

\*  $| \delta(q, \Sigma, \Gamma) | > 1$

PDA

\* By default it is non deterministic

stack  
↑

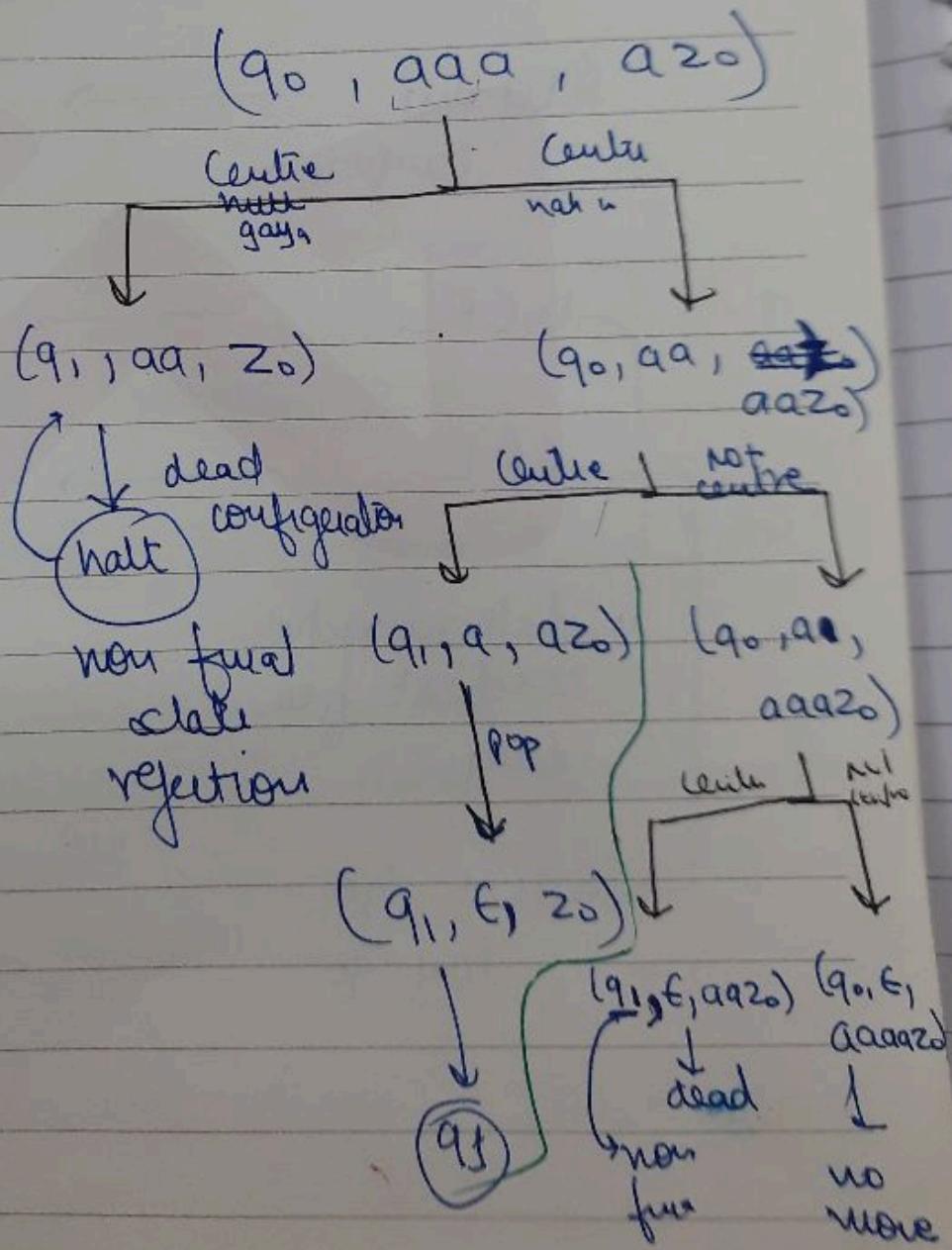
let stay is  $(q_0, \alpha\alpha\alpha, z_0)$

ID

instantaneous  
desruption of  
a PDA

$(q, w, F)$   
↓  
current state

remaining  
stay to  
be read

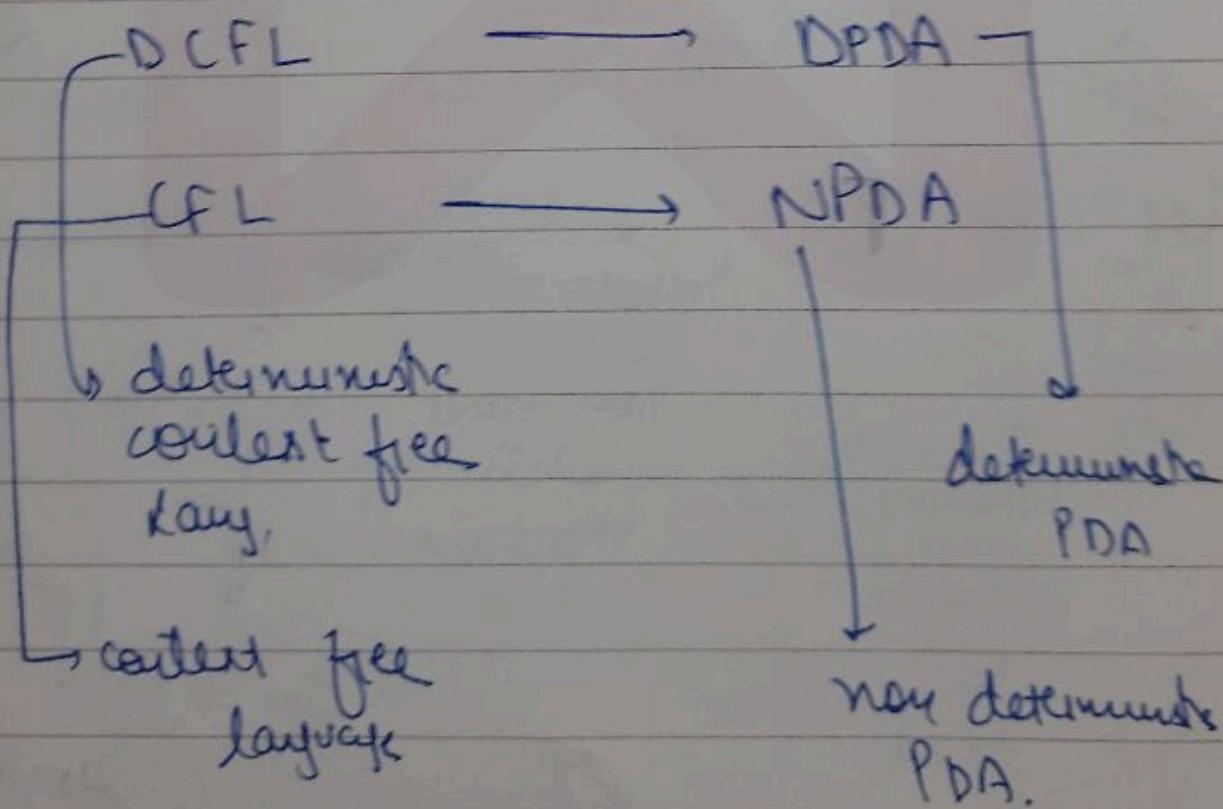


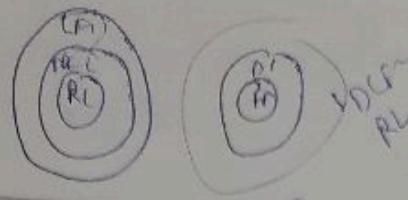
~~NO. 12~~  
~~PC~~

There are some languages to which  
we cannot make deterministic  
PDA.

Deterministic PDA is less powerful.

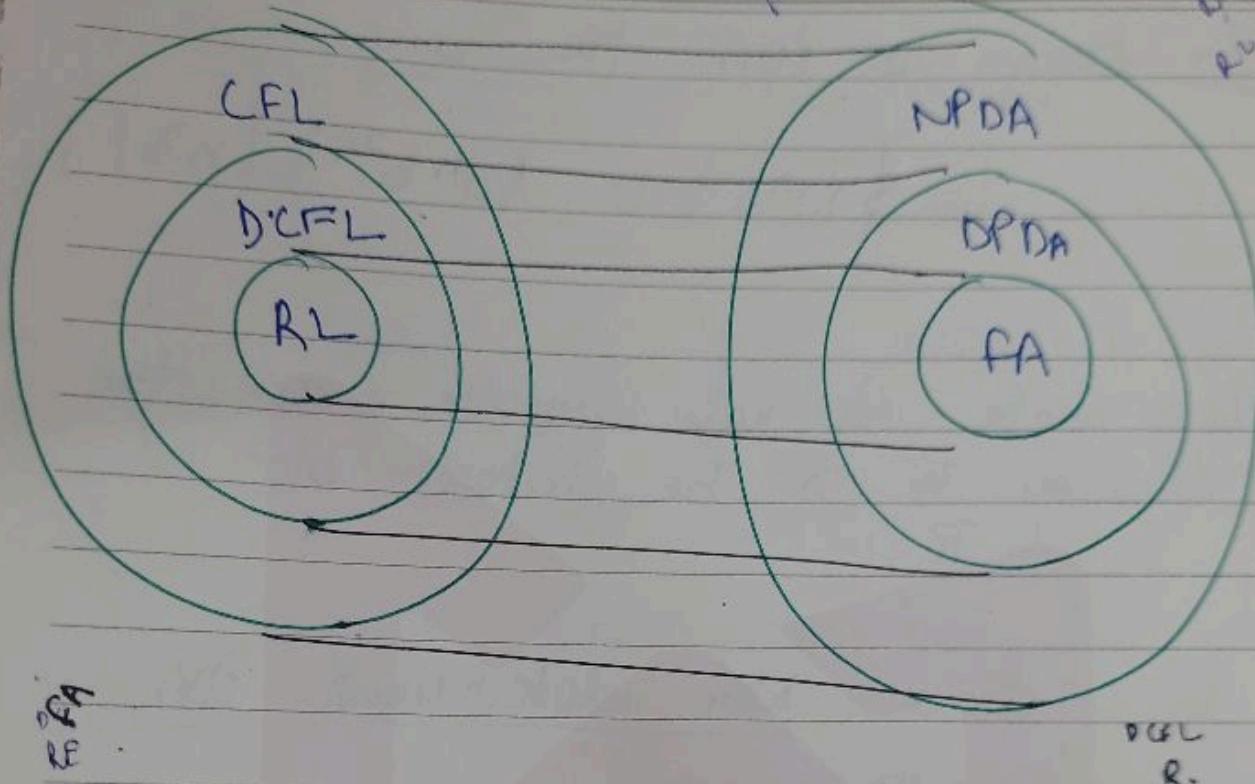
Regular language  $\longrightarrow$  finite automata.





Date: \_\_\_\_\_

$DFA \rightarrow PDA$   
 $RL \rightarrow PDA$



RE

DFA  
PDA  
RL  
PDA

P-L

NPA is more powerful than



NPDA

DPDA.

can't be converted

MD

non deterministic push down automata  
can't be converted into deterministic  
push down automata.

NPDA & PDA have diff expressive  
powers

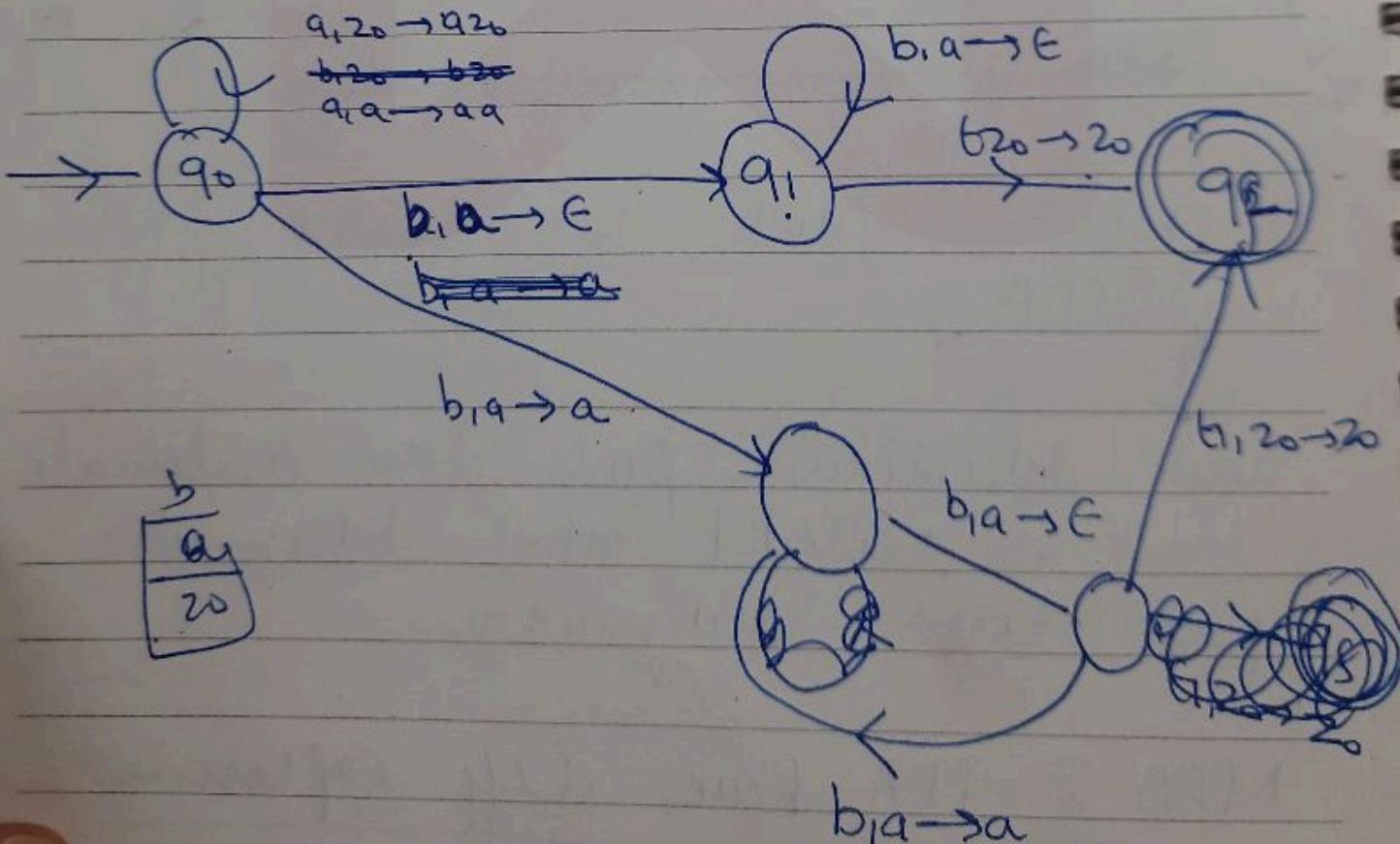


Q8 Design a PDA for lang.

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\} \quad [n \geq 1]$$

Deterministic is not possible as ~~the~~ decision is to be taken b/w I & II.

To draw Non deterministic PDA



→ A lang  $L$  satisfies the pumping lemma  
for reg. lang if & requires  $L$  is regular or  
non regular.

Date: 27/4/10

## # Pumping Lemma (Not very imp) only concept

Suppose DFA with 4 states & with  
no cycle.

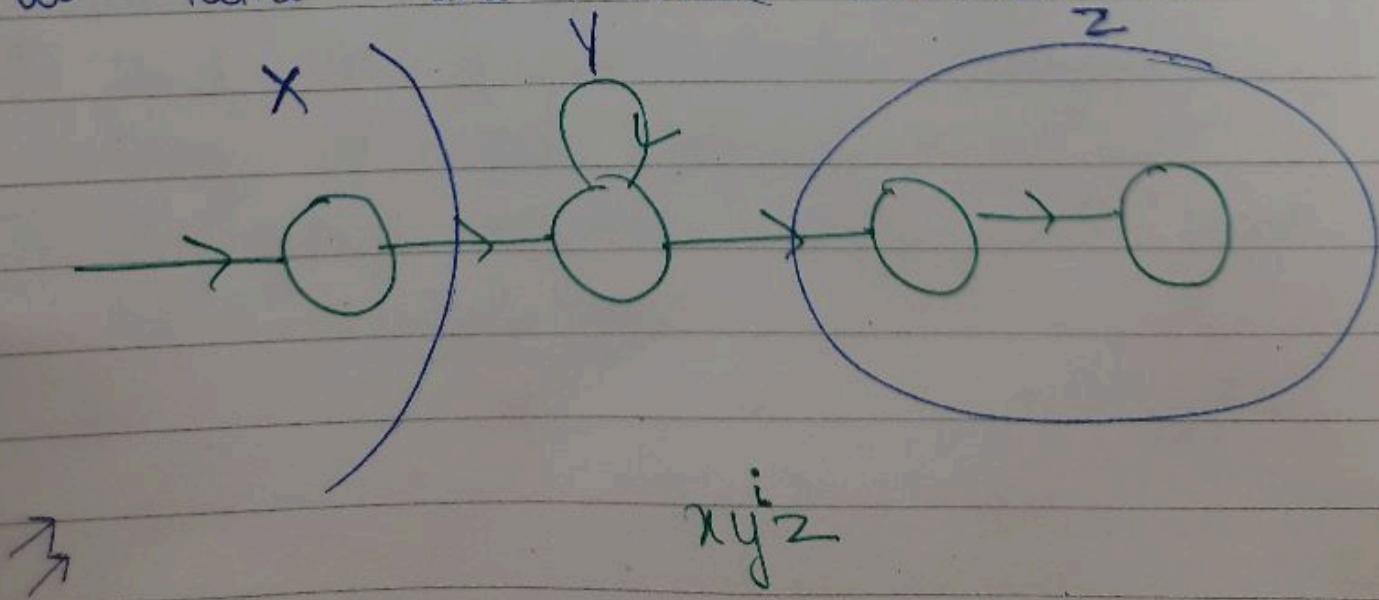
So length of string zada  $\geq 5$   
zada 3, & only finite string with  
be accepted.



2 agar ek bhi loop hai to lang.  
infinite ban jaegi

Based on Pigeon Hole Principle

also this loop is optional to  
be kahi kahi baar asakta hai



$wyz \rightarrow$  is type ku sawi slung  
ban jaeng

It is a descriptive approach used  
to proved that language is  
'not regular'.

Let L (lang) is regular  $\rightarrow$  to DFA bawega.

then slungs will be of type  
 $w = wyz$

Now we need to find exi

value of i jukka lie slung  
lang we na ho , to have  
purok kau denge it is not  
regular.

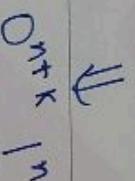
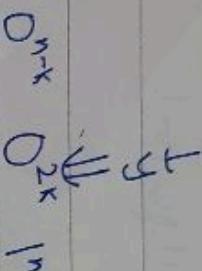
Other eq:-

$O^{n+x} \rightarrow O^{n-1^n} \rightarrow$  doping a dose

8

8

$$O^{n+x} (O^1)^2 | ^n$$



### NOTE

- ① Based on "Pigeon hole principle."
- ② Used to prove that a given lung is not regular
- ③ It gives negative result, or prove by contradiction.

## Turing Machine as The

To

D PDA  $\rightarrow$  ① either a true S/P move  
 or a null move for  
 the same state on  
 stack alphabet  $(Q, T)$

$(q_0, a, z_0) \}$  only one can  
 $(q_0, \epsilon, z_0) \}$  exist at a time

\* If S/P move is defined we  
 cannot define null move.

③  $| | (q, \Sigma, \Gamma) | \leq 1$

[ fully deterministic  $\rightarrow$  DFA  
 partially deterministic  $\rightarrow$  PDFA  
 non deterministic  $\rightarrow$  multiple  
 transitions on  
 same S/P ]

Date: 27/4/14  
Page No. 4

### DPDA

(deterministic  $Q \times \{ \sum U \cup \{\epsilon\} \} \times \Gamma \rightarrow Q \times T^*$   
 push down automata)

stack  
is clear

### NPDA

(non deterministic  $Q \times \{ \sum U \cup \{\epsilon\} \} \times \Gamma \rightarrow Q \times T^*$   
 push down automata)

### DPushdown

$Q \times \sum U^* \times \Gamma \rightarrow Q \times T^*$   
 $Q \times \sum U^* \rightarrow Q \times T^*$

$Q \times \sum U^* \times \Gamma \rightarrow Q \times T^*$

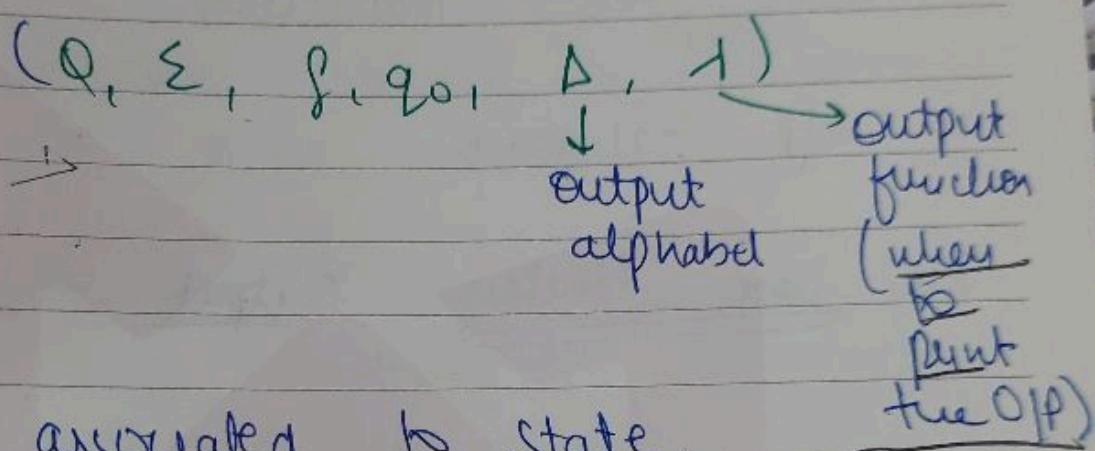
agayya b  
Transition  
flat states

Date: .....

Q = A^0\*

## Mealy & Moore

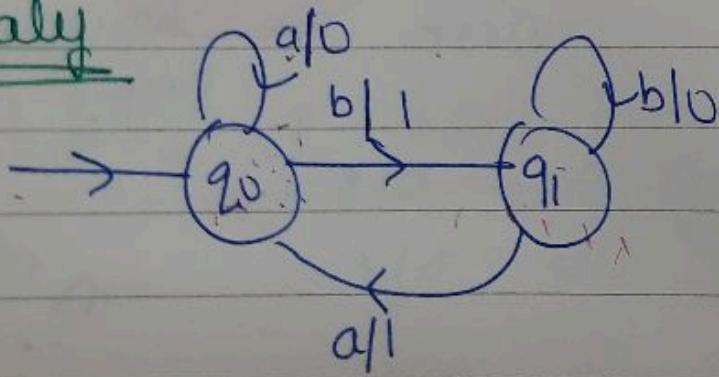
Both of them are defined by  
⑥ tuple.



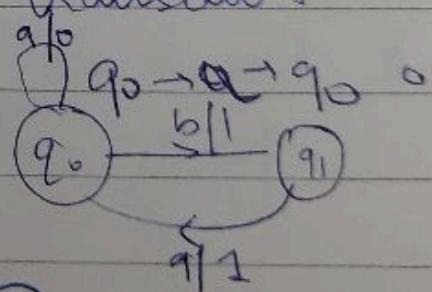
Moore      "      Mealy

Moore :  $t^n + 1 \rightarrow S$   
Mealy     $n \rightarrow T$

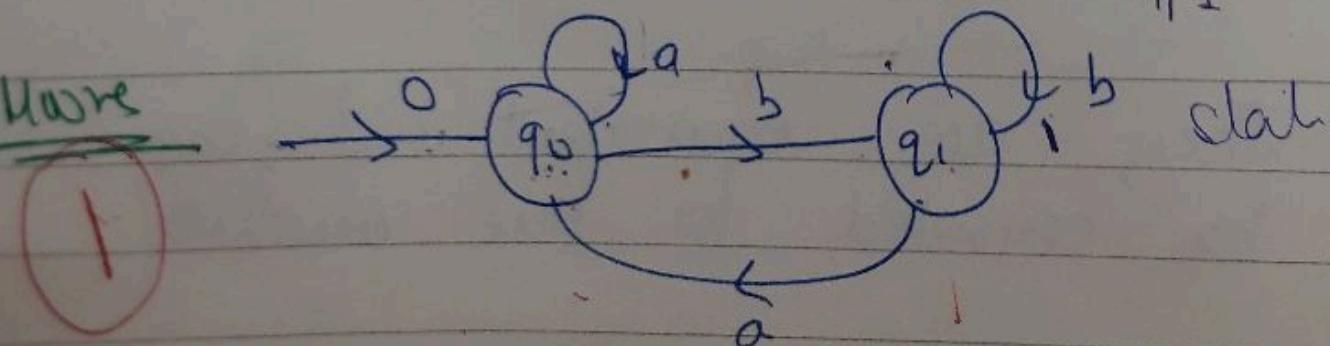
## Mealy



transition



## Moore



Moore me states ko jere  $q_0 \rightarrow q_0$   
 $q_1 \rightarrow 1$   
 assigned hota hai

Mealy me transitions.

e.g.  $\text{S/P} \rightarrow aab$

In case Mealy Output = 

0	0	
1	1	
a	a	b

In case Moore  $(n^1)$ 

0	0	0	1
	↓	↓	↓
moore	me	hmesha f	noga.

\* Moore  $\rightarrow (n+1)$  size O/P.

Mealy  $\rightarrow n$  size O/P

$\Rightarrow$  They are equivalent & can be converted to one another