(very imp )time independent wave equation.

Einstein and Fermi–Dirac statistics) only.

**Q. 6. (*b*) Define Black body radiation.** **(2)**

**Ans.** When a blakbody is placed inside a uniform temperature (isothermal) enclosure after it is in equilibrium with the enclosure it give radiation. These radiations are independent of the nature of the substance, nature of the walls of the enclosure and presence of any other body in the enclosure but depends on temperature. Such radiations in a uniform temperature enclosure are known as blackbody radiations.

form temperature enclosure are known as blackbody radiations.

**Q. 6. (c) Explain wien's and rayleigh–jeans law. (5)**

**Ans.** As per Planck's quantum hypothesis, the atoms of the wall of a blackbody behave as oscillators and each has a characteristic frequency of oscillation. Then average energy of these Planck's oscillators were calculated and finally Planck's radiation formula was derived.

Wien's law is deduced from Plank's radiation formula under the condition when the wavelength λ and temperature $T$ are very small. However, Plank's radiation formula under the condition of high temperature $T$ and wavelength λ takes the from of Rayleigh-jeans law.

Planck radiation law is as following.

$$\mu\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5}\,\frac{1}{e^{hc/\lambda KT}-1}\,d\lambda$$

When λ and $T$ are very samll then $e^{\left(\frac{hc}{\lambda KT}\right)} >> 1$

$$\therefore \qquad \mu\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5}\, e^{\frac{-hc}{\lambda KT}}\, d\lambda$$

By substituting $\qquad 8\pi hc = A$ and $\frac{hc}{K} = B$

we get $\qquad \boxed{\mu\lambda\, d\lambda = \frac{A}{\lambda^5} e^{\frac{-B}{\lambda T}}\, d\lambda}$

This is wein's law valid at low temperature and small wavelength.

When λ and $T$ are large then $e^{\frac{hc}{\lambda KT}} \approx 1 + \frac{hc}{\lambda KT}$

$$\mu\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5\left\{\left[1+\frac{hc}{\lambda KT}\right]^{-1}\right\}}\, d\lambda$$

$$\boxed{\mu\lambda\, d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda}$$

This is Rayleigh–Jeans law valid at high temperature and large wavelength.

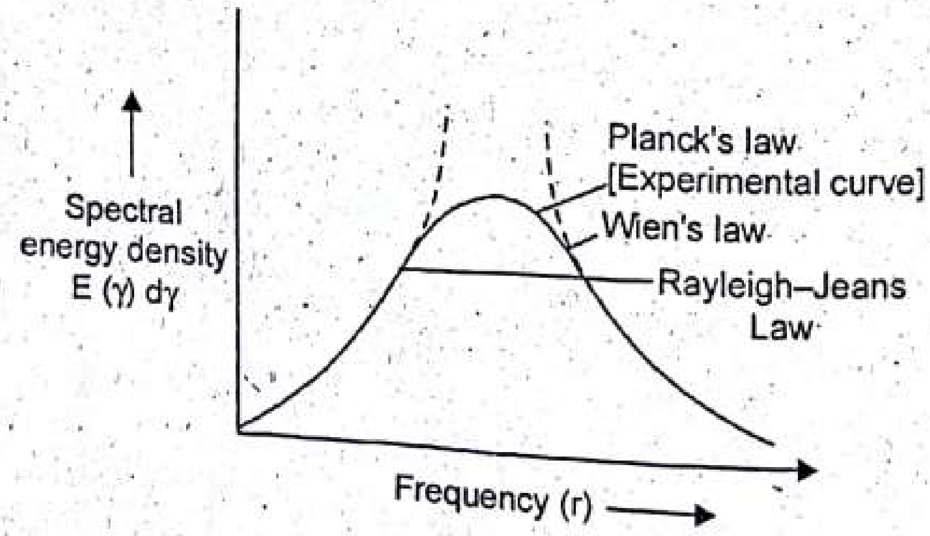


**Fig. 12.**

As shown in Fig. 12 the spectral energy density decreses with frequency at high frequency. Hence Rayleign–Jeans and wien's law are unable to explain the complete, energy distribution for blackbody radiation, while Planck's law explain it completely.

**Q.4. (*b*) Write down the Planck's formula for the distribution of energy in spectrum of blackbody. Show that Rayleigh-Jean's law and Wein's law are special cases of Planck's radiation law.** **(5)**

**Ans.** Refer to Q.4. (c) End Term Examination April 2017.

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When $\lambda$ and T are very samll then $e^{\left(\frac{hc}{\lambda KT}\right)} >> 1$

$$\therefore \quad \mu\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5}e^{\frac{-hc}{\lambda kT}}d\lambda$$

By substituting $8\pi hc = \text{A and } \frac{hc}{K} = B$

we get $\boxed{\mu\lambda\, d\lambda = \frac{A}{\lambda^5}e^{\frac{-B}{\lambda T}}d\lambda}$

This is wein' law valid at low temperature and small wavelength.

When $\lambda$ and T are large then $e^{\left(\frac{hc}{\lambda kT}\right)} >> 1$

$$\mu\lambda\, d\lambda = \frac{8\pi hc}{\lambda_5} e^{\frac{-hc}{\lambda KT}} d\lambda$$

By substituting

$$8\pi\, hc = A \text{ and } \frac{hc}{K} = B$$

we get

$$\boxed{\mu\lambda\, d\lambda = \frac{A}{\lambda^5} e^{\frac{-B}{\lambda T}} d\lambda}$$

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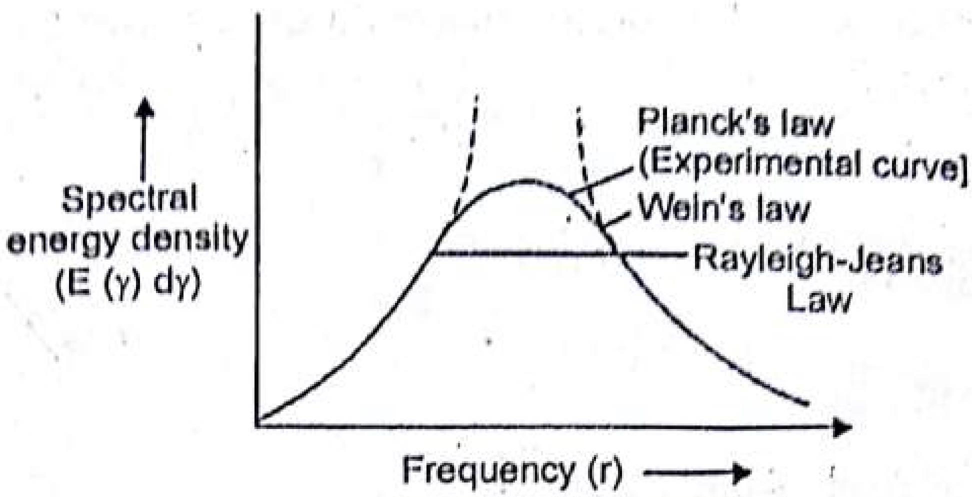
This is Rayleigh-Jeans law valid at high temperature and large wavelength.



**Q.4. (c) The eigen function of an operator $\frac{d^2}{dx^2}\psi(x) = e^{ax}$. Find the corresponding eigen value.** **(2.5)**

**Ans.** The operator and eigen value relation is

$$\hat{0}\psi = 0\psi$$

where $\hat{0}$ is operator and 0 is eigen value

Here,
$$\hat{0} = \frac{d^2}{dx^2} \text{ and } \psi = e^{ax}$$

$$\hat{0}\psi = \frac{d^2}{dx^2}e^{ax}$$

$$\hat{0}\psi = a^2e^{ax}$$

We can write this as–

$$\hat{0}\psi = a^2\psi$$

Such that
$$0 = a^2$$

$\therefore$ $a^2$ is eigen value.

**Q.5. (a) What type of statistics shall be applicable for a gas of photon? Justify your answer.** **(2.5)**

**Ans.** Bose-Einstein statistics shall be applicable for a gas of photons. Bose-Einstein statistics is obeyed by indistinguishable particles of integral spin quantum number that have symmetric wave function and does not obey Pauli exclusion principle.

**Q.5. (b) Compare the qualitative features of Maxwell Boltzmann, Bose-Einstein in and Fermi–Dirac statistics on the basis of their functions.** **(6)**

**Ans.** Refer Q.1.(b) First Term Examination 2016.

**Q.5(c) Show that Bose-Einstein, Fermi-Dirac statistics reduces to Maxwell boltzamann statistics at high temperature.** **(4)**

**Ans.** The distribution laws of three statistics are given below

$$\frac{g_i}{n_i} = e^{\alpha}e^{E_i/k_BT} \quad \text{...(1)[For M – B]}$$

$$\frac{g_i}{n_i} = e^{\alpha}e^{E_i/K_BT} - 1 \quad \text{...(2)[For B – E]}$$

and
$$\frac{g_i}{n_i} = e^{\alpha}e^{E_i/K_BT} + 1 \quad \text{...(3)[For F – D]}$$

If $\frac{g_i}{n_i} >> 1$ then $\frac{g_i}{n_i} \approx \left(\frac{g_i}{n_i}+1\right) \approx \left(\frac{g_i}{n_i}-1\right)$. In this limit both B.E. and F.D. distributions are identical with M.B. distribution. This limit $\left(\frac{g_i}{n_i} >> 1\right)$ occurs when the temperature is not too low and pressure (or density) is not too high.

UNIT III

**Q.1. (*d*) Describe the implication of Davisson-Germer experiment.** **(2.5)**

**Ans.** Davisson and Germer in 1927 designed an apparatus to determine the wavelength associated with electrons to confirm the dual nature of matter.

**Q.1. (*e*) Why is the energy of a particle trapped inside a box quantized?** **(2.5)**

**Ans.** Energy of a particle trapped inside a box is given by the following equation-

$$E_n = \frac{n^2h^2}{8ml^2}$$

where, $n = 1, 2, 3, 4....$

It is clear from the equation that particle cannot posses an arbitrary energy but can have only certain discrete energy corresponding to $n = 1, 2, 3....$

**Q.1. (g) What are the postulates of quantum theory of radiation?** **(2.5)**

**Ans.** The postulates of quantum theory of radiation are as following:—

(i) A chamber containing blackbody radiations also contains simple harmonic oscillators of molecular dimensions which can vibrate with all possible frequencies.

(ii) The frequency of radiation emitted by an oscillator is same as the frequency of its vibrations.

(iii) An oscillator cannot emit energy in a continuous manner. It can emit energy in the multiples of a small unit called quantum (photon). If an oscillator is vibrating with a frequency $\nu$, it can only radiate in quanta of magnitude $h\nu$ i.e., the oscillator can have only discrete energy $E_n$ given by

$$E_n = nh\nu = nE$$

where $h\nu = E$, $n$ is an integer and $h$ is a universal Planck constant ($6.625 \times 10^{-34}$ Js).

(iv) The oscillators can emit or absorb radiation energy in packets of magnitude $h\nu$. This implies that the exchange of energy between radiation and matter cannot take place continuously, but are limited to discrete set of values $0, h\nu, 2h\nu, 3h\nu, \ldots. nh\nu$.

(1

**Q.4. (c) Draw a neat diagram showing the energy distribution spectra of blackbody radiation. Explain how classical theory fails to explain the spectral distribution of energy. (3.5)**

**Ans.** The postulates of quantum theory of radiation are as follows-

(i) A radiation has energy. As light and heat are radiations, they are also associated with energy.

(ii) Radiant energy is not emitted or observed continously but discontinuously in the form of small packets called photons. Photon is not a material body but is considered to be a massless packet of energy.

(iii) The energy $E$ of a photon is related to the frequency of radiation, $\gamma$, the two being related as $E = h\gamma$, where h is planck's constant.

(iv) Whenever a body emits or absorbs energy it does so in whole number multiples by photons, *i.e.* $nh\gamma$, where $n = 1,2,3,4...$

Planck's radiation formula-

Let $dn$ be the number of photons distributed in frequency internal $v$ to $(v + dv)$.

$$dn = n(v)dv = g(v)f(v)dv \quad ...(1)$$

$$g(v)dv = \frac{8\pi h v^2}{c^3} dv \quad ...(2)$$

and $$f(v) = \frac{1}{(e^{hv/KBT} - 1)} \text{ [as } \alpha = 0 \text{ and } E = hv] \quad ...(3)$$

Putting these values in Eq.(1), we get

$$dn = \frac{8\pi h v^2}{c^3} \cdot \frac{1}{(e^{hv/k_BT} - 1)} dv \quad ...(4)$$

Let $dE$ be the energy distributed in frequency interval $v$ and $(v + dv)$

$$dE = Edn = hv\, dn \quad ...(5)$$

From Eqs. (4) and (5), we obtain

$$dE = E(\upsilon)d\upsilon = \frac{8\pi h\upsilon^3}{c^3} \cdot \frac{1}{(e^{h\upsilon/k_BT} - 1)} d\upsilon \quad \text{...(6)}$$

Equation (6) is known as *Planck's radiation formula* for the spectral energy density of blackbody radiation, which agrees with experimental curve (Fig. 4)

For low frequency or high wavelength

$$(e^{h\nu/k_BT} - 1) = 1 + \frac{h\upsilon}{k_BT} - 1 = \frac{h\upsilon}{k_BT}$$

Euqation (6) becomes as

$$dE = E(\nu)\, d\nu = \frac{8\pi\upsilon^2 k_BT}{c^3} d\upsilon \quad \text{...(7)}$$

Equation (7) is termed as Rayleigh-Jeans law. This laws is hold good for low frequencies and high wavelengths. Experimental agreement is shown in Fig. 5 for high frequency low frequncy:

Equation (6) can be expressed in terms of wavelength of radiation as follows:

$$E_\lambda\, d\lambda = \frac{8\pi ch}{\lambda^5}\left(\frac{1}{c[(hc/\pi k_BT) - 1]}\right) d\lambda$$

$$E_\lambda = 8\pi hc(\lambda^{-5})\left[\exp.\left(\frac{hc}{\lambda k_BT} - 1\right)\right]^{-1}$$

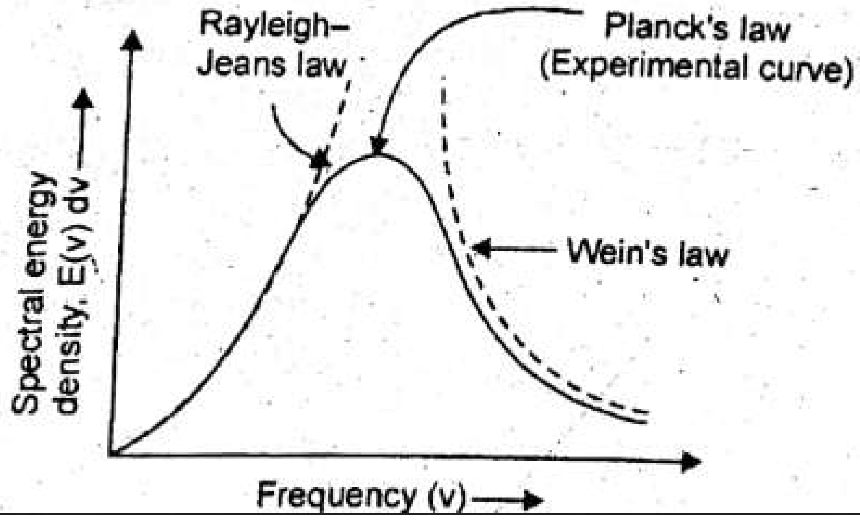


**Fig. 5. ν vs E (ν) dν- curve**

Let $\lambda_{max}$ be the wavelength whose energy density is the greatest.

Then, $$\frac{dE_\lambda}{d\lambda} = 0$$

Hence, $$\lambda_{max} T = \frac{hc}{4.965K} = 2.898 \times 10^{-3} mK$$

This is *Wein's displacement law.*

The peak in the black body spectrum shifts to progressively shorter wavelength igher frequencies) as temperature is increased.

**Q. 1. (h) What is ultraviolet catastrophe?**

**Ans.** It is observed that Wien's fifth power law $\left(E(\lambda) \propto \frac{1}{\lambda^5}\right)$ agree with experimental results only in the low wavelength ($\lambda \to 0$) region but breaks down in large wavelength ($\lambda \to \infty$) region as shown in Fig. 2.

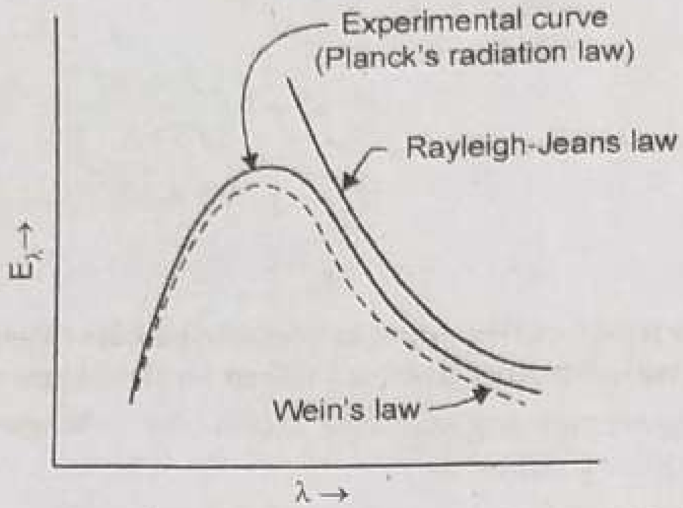



**Fig. 2. Comparison of Wien's law and Rayleigh-Jeans law with the experimental curve of blackbody radiation.**

Rayleigh-Jeans law $\left[E_\lambda \propto \frac{1}{\lambda^4}\right]$ on the other hand can account for the distribution of energy in the long wavelength region and fails completely in short wavelength. This result is said to be ultraviolet catastrophe.

**Ultraviolet Catastrophe:** According to Rayleight-Jeans law the energy density in blackbody radiation between wavelength $\lambda$ and $(\lambda + d\lambda)$ at temperature T is given as

$$E_\lambda d\lambda = \frac{8\pi k_B T}{\lambda^4} d\lambda \quad ...(1)$$

Therefore, the total energy radiation per unit volume of the enclosure for all wavelengths from $\lambda = 0$ to $\lambda = \infty$ is given by

$$E = \int_0^\infty E_\lambda d\lambda = \int_0^\infty \frac{8\pi k_B T}{\lambda^4} d\lambda = 8\pi k_B T\left[-\frac{1}{3\lambda^3}\right]_0^\infty \quad ...(2)$$

This law leads to the fact that, the energy density $E_\lambda \to \infty$ as $\lambda \to 0$, whereas experimental results show that $E_\lambda \to 0$ as $\lambda \to 0$. This implies that for a given quantity of radiant energy is finally confined in the short wavelength (ultraviolet) range.

This is serious discrepancy between theory and experiment and is known as ultraviolet catastrophe or jeans paradox.

**Q. 4. (a) State and explain Rayleigh–Jeans law. Show how its drawbacks can be overcome using Planck's radiation law.** **(3.5)**

**Ans.** As per Planck's quantum hypothesis, the atoms of the wall of a blackbody behave as oscillators and each has a characteristic frequency of oscillation. Then average energy of these Planck's oscillators were calculated and finally Planck's radiation formula was derived:

Wien's law is deduced from Planck's radiation formula under the condition when the wavelength $\lambda$ and temperature $T$ are very small. However, Plank's radiation formula under the condition of high temperature $T$ and wavelength $\lambda$ takes the from of Rayleigh-jeans law.

Planck radiation law is as following.

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When $\lambda$ and $T$ are very small then $e^{\left(\frac{hc}{\lambda KT}\right)} >> 1$

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By substituting $$8\pi hc = A \text{ and } \frac{hc}{K} = B$$

we get $$\boxed{\mu_\lambda d\lambda = \frac{8}{\lambda^5} e^{\frac{-B}{\lambda T}} d\lambda}$$

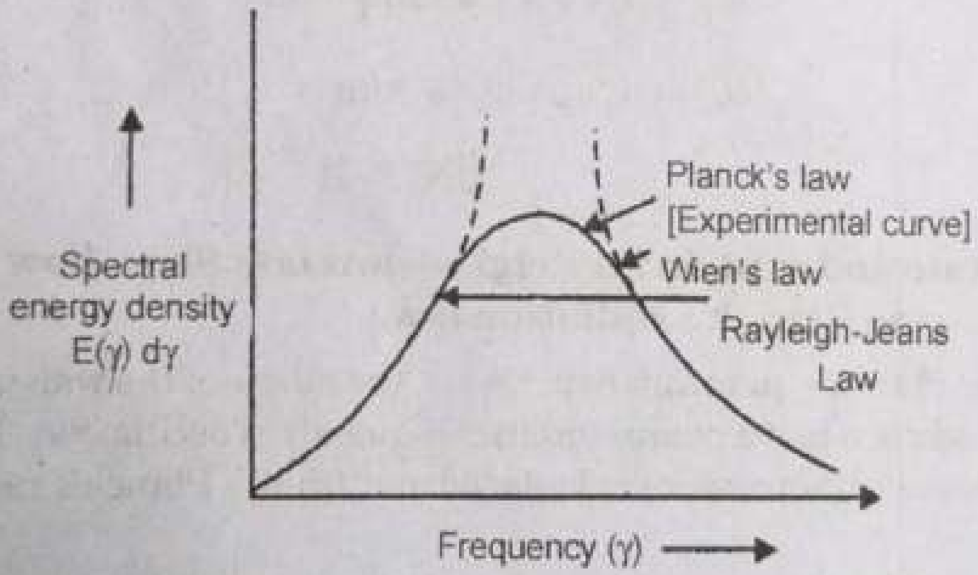
This is wien's law valid at low temperature and small wavelength.

When $\lambda$ and $T$ are large then $e^{\frac{hc}{\lambda KT}} \approx 1 + \frac{hc}{\lambda KT}$

$$\mu_\lambda d\lambda = \frac{8\pi hc}{\lambda^5 \left\{\left[1 + \frac{hc}{\lambda KT}\right]^{-1}\right\}} d\lambda$$

$$\boxed{\mu_\lambda\, d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda}$$

This is Rayleigh-Jeans law valid at high temperature and large wavelength



As shown in Fig.3. the spectral energy density decreases with frequency at high frequency. Hence Rayleigh-Jeans and Wien's law are unable to explain the complete, energy distribution for blackbody radiation, while Planck's law explain it completely.