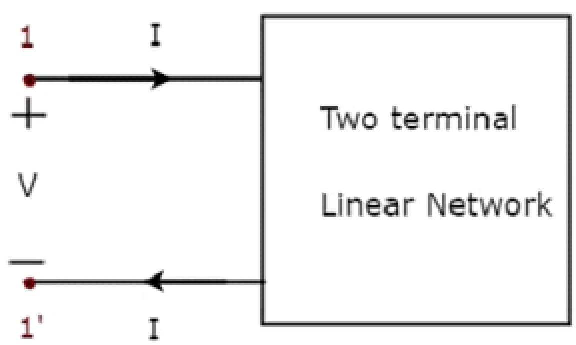
Network Theory - Two-Port Networks

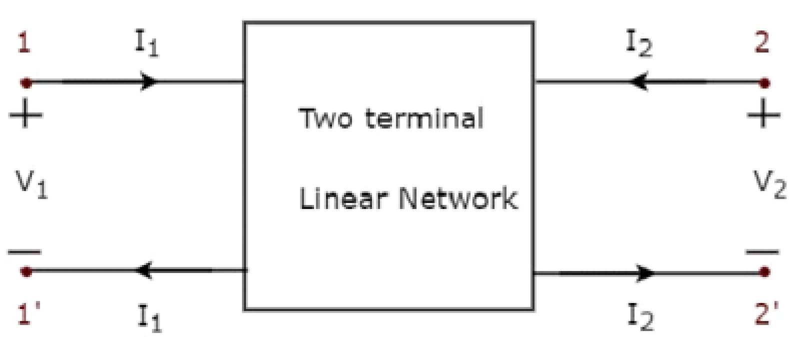
In general, it is easy to analyze any electrical network, if it is represented with an equivalent model, which gives the relation between input and output variables. For this, we can use **two port network** representations. As the name suggests, two port networks contain two ports. Among which, one port is used as an input port and the other port is used as an output port. The first and second ports are called as port1 and port2 respectively.

One port network is a two terminal electrical network in which, current enters through one terminal and leaves through another terminal. Resistors, inductors and capacitors are the examples of one port network because each one has two terminals. One port network representation is shown in the following figure.



Here, the pair of terminals, 1 & 1' represents a port. In this case, we are having only one port since it is a one port network.

Similarly, **two port network** is a pair of two terminal electrical network in which, current enters through one terminal and leaves through another terminal of each port. Two port network representation is shown in the following figure.



Here, one pair of terminals, 1 & 1' represents one port, which is called as **port1** and the other pair of terminals, 2 & 2' represents another port, which is called as **port2**.

There are **four variables** V_1, V_2, I_1 and I_2 in a two port network as shown in the figure. Out of which, we can choose two variables as independent and another two variables as dependent. So, we will get six possible pairs of equations. These equations represent the dependent variables in terms of independent variables. The coefficients of independent variables are called as **parameters**. So, each pair of equations will give a set of four parameters.

Two Port Network Parameters

The parameters of a two port network are called as **two port network parameters** or simply, two port parameters. Following are the types of two port network parameters.

- Z parameters
- Y parameters
- T parameters
- T' parameters
- h-parameters
- g-parameters

Now, let us discuss about these two port network parameters one by one.

Z parameters

We will get the following set of two equations by considering the variables V_1 & V_2 as dependent and I_1 & I_2 as independent. The coefficients of independent variables, I_1 and I_2 are called as **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

The **Z parameters** are

$$Z_{11} = \frac{V_1}{I_1}, \, when \; I_2 = 0$$

$$Z_{12} = \frac{V_1}{I_2}, \, when \; I_1 = 0$$

$$Z_{21} = \frac{V_2}{I_1}, \, when \; I_2 = 0$$

$$Z_{22} = \frac{V_2}{I_2},\ when\ I_1 = 0$$

Z parameters are called as **impedance parameters** because these are simply the ratios of voltages and currents. Units of Z parameters are Ohm (Ω).

We can calculate two Z parameters, Z_{11} and Z_{21}, by doing open circuit of port2. Similarly, we can calculate the other two Z parameters, Z_{12} and Z_{22} by doing open circuit of port1. Hence, the Z parameters are also called as **open-circuit impedance parameters**.

Y parameters

We will get the following set of two equations by considering the variables I_1 & I_2 as dependent and V_1 & V_2 as independent. The coefficients of independent variables, V_1 and V_2 are called as **Y parameters**.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

The **Y parameters** are

$$Y_{11} = \frac{I_1}{V_1},\ when\ V_2 = 0$$

$$Y_{12} = \frac{I_1}{V_2},\ when\ V_1 = 0$$

$$Y_{21} = \frac{I_2}{V_1},\ when\ V_2 = 0$$

$$Y_{22} = \frac{I_2}{V_2},\ when\ V_1 = 0$$

Y parameters are called as **admittance parameters** because these are simply, the ratios of currents and voltages. Units of Y parameters are mho.

We can calculate two Y parameters, Y_{11} and Y_{21} by doing short circuit of port2. Similarly, we can calculate the other two Y parameters, Y_{12} and Y_{22} by doing short circuit of port1. Hence, the Y parameters are also called as **short-circuit admittance parameters**.

T parameters

We will get the following set of two equations by considering the variables V_1 & I_1 as dependent and V_2 & I_2 as independent. The coefficients of V_2 and $-I_2$ are called as **T parameters**.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

The **T parameters** are

$$A = \frac{V_1}{V_2},\ when\ I_2 = 0$$

$$B = -\frac{V_1}{I_2},\ when\ V_2 = 0$$

$$C = \frac{I_1}{V_2},\ when\ I_2 = 0$$

$$D = -\frac{I_1}{I_2},\ when\ V_2 = 0$$

T parameters are called as transmission parameters or **ABCD parameters**. The parameters, A and D do not have any units, since those are dimension less. The units of parameters, B and C are ohm and mho respectively.

We can calculate two parameters, A and C by doing open circuit of port2. Similarly, we can calculate the other two parameters, B and D by doing short circuit of port2.

T ’ parameters

We will get the following set of two equations by considering the variables V_2 & I_2 as dependent and V_1 & I_1 as independent. The coefficients of V_1 and $-I_1$ are called as **T’ parameters**.

$$V_2 = A'V_1 - B'I_1$$

$$I_2 = C'V_1 - D'I_1$$

The **T' parameters** are

$$A' = \frac{V_2}{V_1},\ when\ I_1 = 0$$

$$B' = -\frac{V_2}{I_1},\ when\ V_1 = 0$$

$$C' = \frac{I_2}{V_1},\ when\ I_1 = 0$$

$$D' = -\frac{I_2}{I_1},\ when\ V_1 = 0$$

T' parameters are called as inverse transmission parameters or **A'B'C'D' parameters**. The parameters A' and D' do not have any units, since those are dimension less. The units of parameters, B' and C', are Ohm and Mho respectively.

We can calculate two parameters, A' and C', by doing an open circuit of port1. Similarly, we can calculate the other two parameters, B' and D', by doing a short circuit of port1.

h-parameters

We will get the following set of two equations by considering the variables V_1 & I_2 as dependent and I_1 & V_2 as independent. The coefficients of independent variables, I_1 and V_2, are called as **h-parameters**.

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

The h-parameters are

$$h_{11} = \frac{V_1}{I_1},\ when\ V_2 = 0$$

$$h_{12} = \frac{V_1}{V_2}, \ when \ I_1 = 0$$

$$h_{21} = \frac{I_2}{I_1}, \ when \ V_2 = 0$$

$$h_{22} = \frac{I_2}{V_2}, \ when \ I_1 = 0$$

h-parameters are called as **hybrid parameters**. The parameters, h_{12} and h_{21}, do not have any units, since those are dimension-less. The units of parameters, h_{11} and h_{22}, are Ohm and Mho respectively.

We can calculate two parameters, h_{11} and h_{21} by doing short circuit of port2. Similarly, we can calculate the other two parameters, h_{12} and h_{22} by doing open circuit of port1.

The h-parameters or hybrid parameters are useful in transistor modelling circuits (networks).

g-parameters

We will get the following set of two equations by considering the variables I_1 & V_2 as dependent and V_1 & I_2 as independent. The coefficients of independent variables, V_1 and I_2 are called as **g-parameters**.

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

The **g-parameters** are

$$g_{11} = \frac{I_1}{V_1}, \ when \ I_2 = 0$$

$$g_{12} = \frac{I_1}{I_2}, \ when \ V_1 = 0$$

$$g_{21} = \frac{V_2}{V_1}, \; when \; I_2 = 0$$

$$g_{22} = \frac{V_2}{I_2}, \; when \; V_1 = 0$$

g-parameters are called as **inverse hybrid parameters**. The parameters, g_{12} and g_{21} do not have any units, since those are dimension less. The units of parameters, g_{11} and g_{22} are mho and ohm respectively.

We can calculate two parameters, g_{11} and g_{21} by doing open circuit of port2. Similarly, we can calculate the other two parameters, g_{12} and g_{22} by doing short circuit of port1.

Two-Port Parameter Conversions

In the previous chapter, we discussed about six types of two-port network parameters. Now, let us convert one set of two-port network parameters into other set of two port network parameters. This conversion is known as two port network parameters conversion or simply, **two-port parameters conversion**.

Sometimes, it is easy to find one set of parameters of a given electrical network easily. In those situations, we can convert these parameters into the required set of parameters instead of calculating these parameters directly with more difficulty.

Now, let us discuss about some of the two port parameter conversions.

Procedure of two port parameter conversions

Follow these steps, while converting one set of two port network parameters into the other set of two port network parameters.

- **Step 1** − Write the equations of a two port network in terms of desired parameters.
- **Step 2** − Write the equations of a two port network in terms of given parameters.
- **Step 3** − Re-arrange the equations of Step2 in such a way that they should be similar to the equations of Step1.
- **Step 4** − By equating the similar equations of Step1 and Step3, we will get the desired parameters in terms of given parameters. We can represent these parameters in matrix form.

Z parameters to Y parameters

Here, we have to represent Y parameters in terms of Z parameters. So, in this case Y parameters are the desired parameters and Z parameters are the given parameters.

Step 1 − We know that the following set of two equations, which represents a two port network in terms of **Y parameters**.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

We can represent the above two equations in **matrix** form as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$ **Equation 1**

Step 2 − We know that the following set of two equations, which represents a two port network in terms of **Z parameters**.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

We can represent the above two equations in **matrix** form as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Step 3 − We can modify it as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$ **Equation 2**

Step 4 − By equating Equation 1 and Equation 2, we will get

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

$$\Rightarrow \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}}{\Delta Z}$$

Where,

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

So, just by doing the **inverse of Z parameters matrix**, we will get Y parameters matrix.

Z parameters to T parameters

Here, we have to represent T parameters in terms of Z parameters. So, in this case T parameters are the desired parameters and Z parameters are the given parameters.

Step 1 − We know that, the following set of two equations, which represents a two port network in terms of **T parameters**.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Step 2 − We know that the following set of two equations, which represents a two port network in terms of **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Step 3 − We can modify the above equation as

$$\Rightarrow V_2 - Z_{22}I_2 = Z_{21}I_1$$

$$\Rightarrow I_1 = (\frac{1}{Z_{21}})V_2 - (\frac{Z_{22}}{Z_{21}})I_2$$

Step 4 − The above equation is in the form of $I_1 = CV_2 - DI_2$. Here,

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Step 5 − Substitute I_1 value of Step 3 in V_1 equation of Step 2.

$$V_1 = Z_{11}\{(\frac{1}{Z_{12}})V_2 - (\frac{Z_{22}}{Z_{21}})I_2\} + Z_{12}I_2$$

$$\Rightarrow V_1 = (\frac{Z_{11}}{Z_{21}})V_2 - (\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}})I_2$$

Step 6 − The above equation is in the form of $V_1 = AV_2 - BI_2$. Here,

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

Step 7 − Therefore, the **T parameters matrix** is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

Y parameters to Z parameters

Here, we have to represent Z parameters in terms of Y parameters. So, in this case Z parameters are the desired parameters and Y parameters are the given parameters.

Step 1 − We know that, the following matrix equation of two port network regarding Z parameters as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equation 3

Step 2 − We know that, the following matrix equation of two port network regarding Y parameters as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Step 3 − We can modify it as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equation 4

Step 4 − By equating Equation 3 and Equation 4, we will get

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}}{\Delta Y}$$

Where,

$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

So, just by doing the **inverse of Y parameters matrix**, we will get the Z parameters matrix.

Y parameters to T parameters

Here, we have to represent T parameters in terms of Y parameters. So, in this case, T parameters are the desired parameters and Y parameters are the given parameters.

Step 1 − We know that, the following set of two equations, which represents a two port network in terms of **T parameters**.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Step 2 − We know that the following set of two equations of two port network regarding Y parameters.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Step 3 − We can modify the above equation as

$$\Rightarrow I_2 - Y_{22}V_2 = Y_{21}V_1$$

$$\Rightarrow V_1 = (\frac{-Y_{22}}{Y_{21}})V_2 - (\frac{-1}{Y_{21}})I_2$$

Step 4 − The above equation is in the form of $V_1 = AV_2 - BI_2$. Here,

$$A = \frac{-Y_{22}}{Y_{21}}$$

$$B = \frac{-1}{Y_{21}}$$

Step 5 − Substitute V_1 value of Step 3 in I_1 equation of Step 2.

$$I_1 = Y_{11}\{(\frac{-Y_{22}}{Y_{21}})V_2 - (\frac{-1}{Y_{21}})I_2\} + Y_{12}V_2$$

$$\Rightarrow I_1 = (\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}})V_2 - (\frac{-Y_{11}}{Y_{21}})I_2$$

Step 6 − The above equation is in the form of $I_1 = CV_2 - DI_2$. Here,

$$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}$$

$$D = \frac{-Y_{11}}{Y_{21}}$$

Step 7 − Therefore, the **T parameters matrix** is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{Y_{12}Y_{21}-Y_{11}Y_{22}}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{bmatrix}$$

T parameters to h-parameters

Here, we have to represent h-parameters in terms of T parameters. So, in this case hparameters are the desired parameters and T parameters are the given parameters.

Step 1 − We know that, the following **h-parameters** of a two port network.

$$h_{11} = \frac{V_1}{I_1}, \, when \, V_2 = 0$$

$$h_{12} = \frac{V_1}{V_2}, \, when \, I_1 = 0$$

$$h_{21} = \frac{I_2}{I_1}, \, when \, V_2 = 0$$

$$h_{22} = \frac{I_2}{V_2}, \, when \, I_1 = 0$$

Step 2 − We know that the following set of two equations of two port network regarding **T parameters**.

$$V_1 = AV_2 - BI_2$$ **Equation 5**

$$I_1 = CV_2 - DI_2$$ **Equation 6**

Step 3 − Substitute $V_2 = 0$ in the above equations in order to find the two h-parameters, h_{11} and h_{21} .

$$\Rightarrow V_1 = -BI_2$$

$$\Rightarrow I_1 = -DI_2$$

Substitute, V_1 and I_1 values in h-parameter, h_{11} .

$$h_{11} = \frac{-BI_2}{-DI_2}$$

$$\Rightarrow h_{11} = \frac{B}{D}$$

Substitute I_1 value in h-parameter h_{21} .

$$h_{21} = \frac{I_2}{-DI_2}$$

$$\Rightarrow h_{21} = -\frac{1}{D}$$

Step 4 − Substitute $I_1 = 0$ in the second equation of step 2 in order to find the h-parameter h_{22} .

$$0 = CV_2 - DI_2$$

$$\Rightarrow CV_2 = DI_2$$

$$\Rightarrow \frac{I_2}{V_2} = \frac{C}{D}$$

$$\Rightarrow h_{22} = \frac{C}{D}$$

Step 5 − Substitute $I_2 = (\frac{C}{D})V_2$ in the first equation of step 2 in order to find the h-parameter, h_{12} .

$$V_1 = AV_2 - B(\frac{C}{D})V_2$$

$$\Rightarrow V_1 = (\frac{AD - BC}{D})V_2$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{AD - BC}{D}$$

$$\Rightarrow h_{12} = \frac{AD - BC}{D}$$

Step 6 − Therefore, the h-parameters matrix is

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

h-parameters to Z parameters

Here, we have to represent Z parameters in terms of h-parameters. So, in this case Z parameters are the desired parameters and h-parameters are the given parameters.

Step 1 − We know that, the following set of two equations of two port network regarding **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Step 2 − We know that, the following set of two equations of two-port network regarding **h-parameters**.

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Step 3 − We can modify the above equation as

$$\Rightarrow I_2 - h_{21} I_1 = h_{22} V_2$$

$$\Rightarrow V_2 = \frac{I_2 - h_{21} I_1}{h_{22}}$$

$$\Rightarrow V_2 = (\frac{-h_{21}}{h_{22}}) I_1 + (\frac{1}{h_{22}}) I_2$$

The above equation is in the form of $V_2 = Z_{21} I_1 + Z_{22} I_2 . Here,$

$$Z_{21} = \frac{-h_{21}}{h_{22}}$$

$$Z_{22} = \frac{1}{h_{22}}$$

Step 4 − Substitute V_2 value in first equation of step 2.

$$V_1 = h_{11} I_1 + h_{21} \{ (\frac{-h_{21}}{h_{22}}) I_1 + (\frac{1}{h_{22}}) I_2 \}$$

$$\Rightarrow V_1 = (\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}) I_1 + (\frac{h_{12}}{h_{22}}) I_2$$

The above equation is in the form of $V_1 = Z_{11} I_1 + Z_{12} I_2$. Here,

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

Step 5 − Therefore, the Z parameters matrix is

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

In this way, we can convert one set of parameters into other set of parameters.