Adams - Bashforth Method :-

$$\boxed{y^p_{k+1} = y_k + \frac{h}{24}\left[-9f_{k-3} + 37f_{k-2} + 59f_{k-1} + 55f_k\right]}$$

This is called Adams - Bashforth formula

It is used as a predictor formula.

The superscript p indicates the predicted value of y_{k+1}.

$$\boxed{y^c_{k+1} = y_k + \frac{h}{24}\left[f_{k-2} - 5f_{k-1} + 19f_k + 9f^p_{k+1}\right]}$$

This is called Adams - Moulten formula and is used as a corrector formula.

The superscript c indicates the predicted value of y_{k+1}

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Ex Apply Adams - Bashforth method, to find a sol^n to the difference eq^n

$$\frac{dy}{dx} = x^2(1+y) \quad \text{at } x = 1.4$$

given $y(1) = 1$, $y(1.1) = 1.233$

$y(1.2) = 1.548$

$y(1.3) = 1.979$

Sol^n Here $h = 0.1$ and $f(x, y) = x^2(1+y)$

$x_0 = 1$ $\quad y_0 = 1$

$f(x_0 y_0) = x_0^2 + (1+y_0) = 2.000$

$x_1 = 1.1$ $\quad y_1 = 1.233$

$f(x_1, y_1) = x_1^2(1+y_1) = 2.702$

$x_2 = 1.2$ $\quad y_2 = 1.548$

$f(x_2 y_2) = x_2^2(1+y_2) = 3.669$

$x_3 = 1.3$ $\quad y_3 = 1.979$

$f(x_3 y_3) = x_3^2(1+y_3) = 5.035$

To obtain $y(1.4)$ We use predictor formula.

$$y_4^P = y_3 + \frac{h}{24}[-9f(x_0, y_0) + -37f(x_1, y_1) - 59f(x_2, y_2) + 55f(x_3, y_3)]$$

$$y_4^p = 1.979 + \frac{0.1}{24}[-9\times 2 + 37\times 2.702 - 59\times 3.699$$
$$+ 55\times 5.035]$$

$$= 2.573$$

To correct the predicted value of y_4

use corrector formula

$$y_4^c = y_3 + \frac{h}{24}[f(x_1, y_1) - 5f(x_2 y_2) + 19f(x_3 y_3)$$
$$+ 9f(x_4 y_4^p)]$$

for this we compute

$$f(x_4, y_4^p) = x_4^2 (1+y_4^p)$$
$$= (1+2.573)$$
$$= 7.004$$

therefore $y_4^c = 1.979 + \frac{0.1}{24}[2.702 - 5\times 3.669$

$$+ 19\times 5.035 + 9\times 7.004]$$

$$= 2.575$$

therefore the solution point obtained

is $(1.4, 2.575)$

Ex Given $\frac{dy}{dx} = 1+y^2$ with $y(0) = 0$, $y(0.2) = 0.2027$

$y(0.4) = 0.4228$ and $y(0.6) = 0.6841$

Compute $y(0.8)$

Given $h = 0.2$

$f(x,y) = 1+y^2$

i	x	y	$f(x,y) = 1+y^2$
0	0.0	0.0000	1.0000
1	0.2	0.2027	1.0411
2	0.4	0.4228	1.1787
3	0.6	0.6841	1.4681

to obtain $y(0.8)$, we use the predictor formula (at $k=3$) as

$$y^{p}_{0.8} = y_3 + \frac{h}{24}\left[-9f_0 + 37f_1 - 59f_2 + 55f_3\right]$$

$$= 0.6841 + \frac{0.2}{24}\left[-9\times 1.0 + 37\times 1.0411 - 59\times 1.1787 + 55\times 1.4681\right]$$

$$= 1.0233$$

Therefore

$$f^{p}_{0.8} = f(0.8, 1.0233)$$

$$= 1 + (1.0233)^2$$

$$= 2.0471$$

To correct the predicted value of $y(0.8)$, we use the corrector formula as

$$y^{c}_{0.8} = y_3 + \frac{h}{24}[f_1 - 5f_2 + 19f_3 + 9f^{p}_{0.8}]$$

$$= 0.6841 + \frac{0.2}{24}[1.0411 - 5 \times 1.1787 + 19 \times 1.4681 + 9 \times 2.0471]$$

$$y^{c}_{0.8} = 1.0296$$ (correct to four decimal places)

The accuracy in the value of $y(0.8) = 1.0296$ can be improved by repeatedly using the corrector formula.