Chapter 8

Two-Port Networks

Inside this Chapter

8.1. INTRODUCTION

Most often we have seen that the networks with terminals are connected in pairs to other networks. If the current entering one terminal of a pair is equal and opposite to the current leaving the other terminal of the pair, then this type of terminal pair is called as a "port".

A Two-port network is shown in figure 8.1. By which we observe that a two-port network is represented by a black box with four variables, namely, two voltages (V_1, V_2) and two currents (I_1, I_2) which are available for measurements and are relevant for the analysis of two port networks. Out of these four variables, which two variables may be considered 'independent' and which two 'dependent' is generally decided by the problem under consideration.

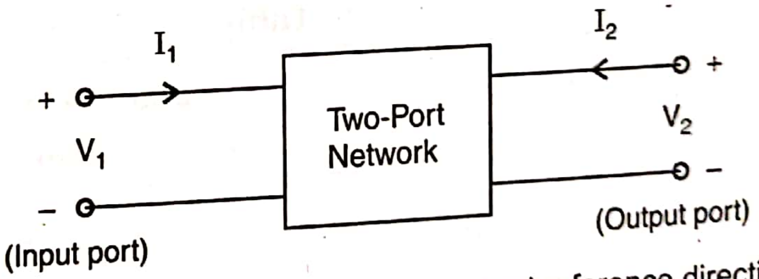


Fig. 8.1. A two-port network with standard reference directions for the voltages and currents indicated

By analogy with transmission networks, one of the ports is called the input port, while the other is termed as the output port.

Some different forms of two-port networks as shown in figure 8.2.

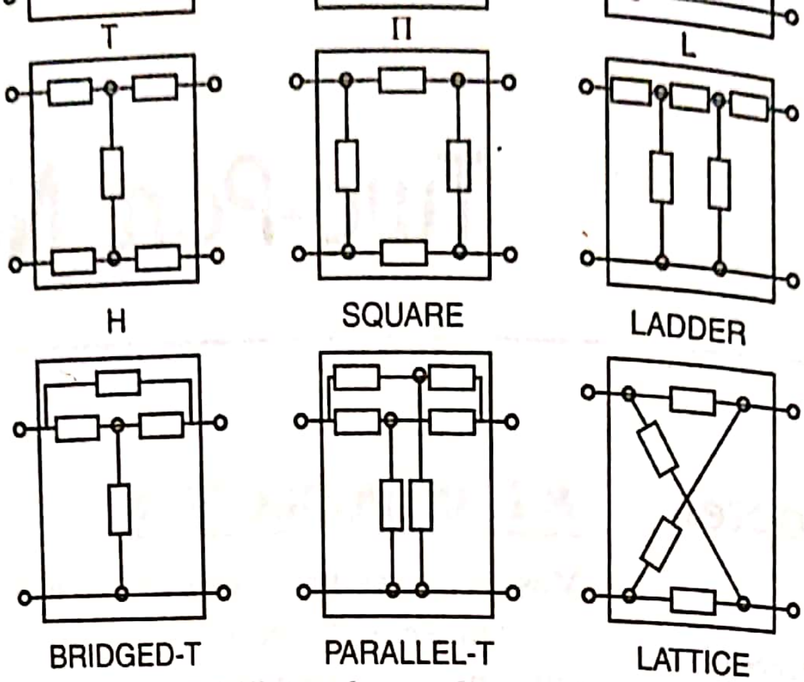


Fig. 8.2. Different forms of two-port networks

8.1.1. Characterization of Linear Time-Invariant (LTI) Two Port Networks

A two port network is a special case of multi-port network. Each port consists of two terminals, one for entering the current and the other for leaving.

In order to describe the relationships between the port voltages and port currents of a linear two port network, two linear equations are required among the four variables.

8.1.2. Relationship of Two Port Variables

The number of possible combinations generated by the four variables, taken two at a time, is six. Thus, there are total six combinations given in Table 8.1, by which we analyse any type of the two port network.

Table 8.1. Two-Port Parameters

Name	Function: Express	Function: In terms of	Matrix Equation
Open-circuit impedance $[Z]$	V_1, V_2	I_1, I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
Short-circuit admittance $[Y]$	I_1, I_2	V_1, V_2	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
Transmission or Chain $[T]$	V_1, I_1	$V_2, -I_2$	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$
Inverse transmission $[T']$	V_2, I_2	$V_1, -I_1$	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$
Hybrid (h)	V_1, I_2	I_1, V_2	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
Inverse hybrid (g)	I_1, V_2	V_1, I_2	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$

As we shall see, the names of the parameters are chosen to indicate dimensions (impedance, admittance), lack of consistent dimensions (hybrid), or the principle application of the parameters (transmission).

8.2. OPEN CIRCUIT IMPEDANCE (Z) PARAMETERS

Expressing two - port voltages in terms of two - port currents, *i.e.*,

$$(V_1, V_2) = f(I_1, I_2)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

or $$[V] = [Z][I]$$

where $[Z]$ is the open circuit impedance matrix of the two port network and impedances Z_{ij} are the *open circuit impedance (Z) parameters.*

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad ...(1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad ...(2)$$

From equations (1) and (2), we draw the two-voltage source equivalent of a two-port network may be obtained in terms of Z-parameters as shown in figure 8.3, where $Z_{12}\ I_2$ and $Z_{21}\ I_1$ are current controlled voltage sources (CCVS).

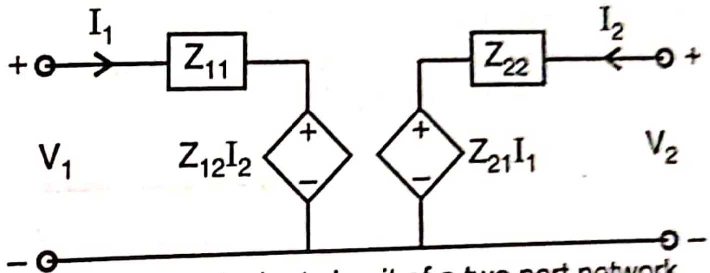


Fig. 8.3. Equivalent circuit of a two port network in terms of Z-parameters

Determination of Z-parameters

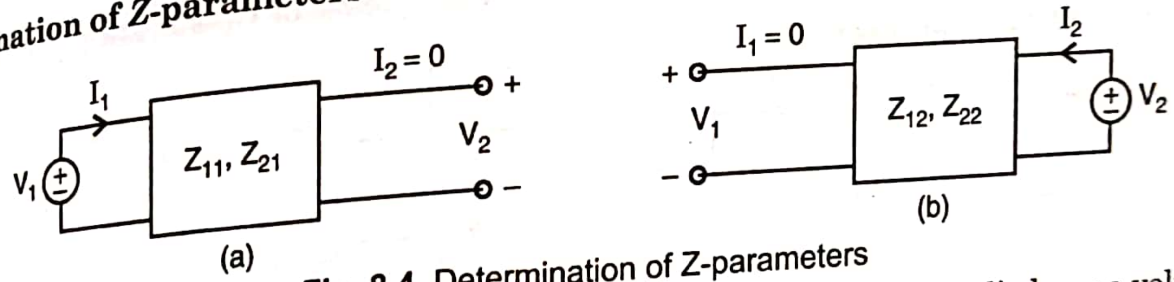


Fig. 8.4. Determination of Z-parameters

In order to determine the Z-parameters, open the output port and applied some voltage V_1 to input port as shown in figure 8.4(*a*). We determine I_1 and V_2 to obtain Z_{11} and Z_{21}. Then, the input port is open circuited and the output port is excited with the voltage V_2 as shown in figure 8.4(*b*). The ... determine I_2 and V_1, so as to obtain Z_{12} and Z_{22}. Mathematically :

... in figure 8.4(*a*)].

8.3. SHORT CIRCUIT ADMITTANCE (Y) PARAMETERS

Expressing two-port current in terms of two-port voltages. *i.e.*,

$$(I_1, I_2) = f(V_1, V_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

or

$$[I] = [Y][V]$$

Where [Y] is the short circuit admittance matrix of the two port network and admittance Y_{ij} are the *short circuit admittance (Y)-parameters.*

$$I_1 = Y_{11}V_1 + Y_{12}\,V_2 \qquad \text{...(3)}$$

$$I_2 = Y_{21}\,V_1 + Y_{22}\,V_2 \qquad \text{...(4)}$$

from equations (3) and (4), we draw the two-current source equivalent of a two-port network may be obtained in terms of Y-parameters as shown in figure 8.5, where $Y_{12}\,V_2$ and $Y_{21}\,V_1$ are voltage controlled current source (VCCS).

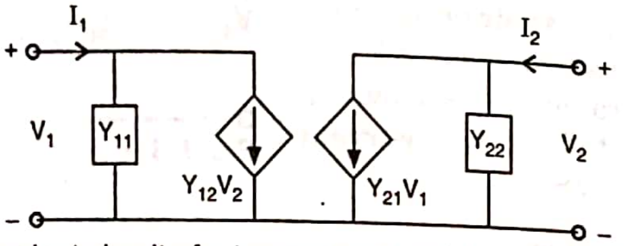


Fig. 8.5. Equivalent circuit of a two-port network in terms of Y-parameters

Determination of Y-parameters

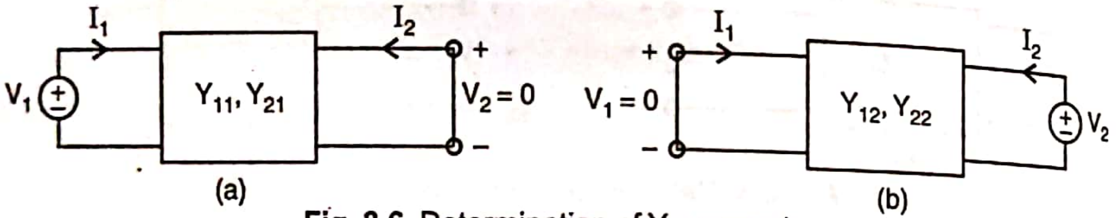


Fig. 8.6. Determination of Y-parameters

In order to determine the Y-parameters, short the output port and applied some voltage V_1 to input port as shown in figure 8.6(*a*). We determine I_1 and I_2, to obtain Y_{11} and Y_{21}. Then, the input port is short circuited and the output port is excited with the voltage V_2 as shown in figure 8.6(*b*). The circuit is analysed to determine I_2 and I_1, so as to obtain Y_{12} and Y_{22}. Mathematically.

Case I : $V_1 = V_1, I_1 = ?, I_2 = ?, V_2 = 0$ [output port short circuited as shown in figure 8.6(*a*)].

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$ (input driving point admittance with the output port short circuited)

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$ (forward transfer admittance with the output port short circuit)

Case II : $V_2 = V_2, I_2 = ?, I_1 = ?, V_1 = 0$ [input port short circuited as shown in figure 8.6(*b*)].

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$ (reverse transfer admittance with the input port short circuited)

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$ (output driving point admittance with the input port short circuited)

Note :

1. For two port passive network, Y_{12} and Y_{21} are always negative, since I_2 is negative, when output port is short circuited and similarly I_1 is negative, when input port is short circuited.
2. Short circuit admittance matrix] = [Open circuit impedance matrix]$^{-1}$

$$[Y] = [Z]^{-1}$$

or

$$Y_{ij} \neq \frac{1}{Z_{ij}} \quad i.e., \quad Y_{11} \neq \frac{1}{Z_{11}} \text{ and } Y_{12} \neq \frac{1}{Z_{12}} \text{ etc.}$$

And

8.4. TRANSMISSION (*T*) OR CHAIN OR *ABCD* PARAMETERS

Expressing one port variables in terms of the other port variables. *i.e.*,

$$(V_1, I_1) = f(V_2, -I_2)$$

T-parameters are used in the analysis of power transmission line. The input and output ports are called the sending and receiving ends respectively.

(Since in this case, output port current is considered outward, therefore negative sign arises with I_2)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

This matrix equation defines *A*, *B*, *C*, *D* parameters, where matrix is known as the transmission (*T*) or *ABCD* matrix.

$$V_1 = AV_2 + B(-I_2) \quad \text{...(5)}$$

$$I_1 = CV_2 + D(-I_2) \quad \text{...(6)}$$

Note :

The equivalent circuit of a two-port network is not possible in terms of *T*-parameters.

For passive ne...
positive.

8.5. INVERSE TRANSMISSION (T') PARAMETERS

Expressing output port variables in terms of input port variables, *i.e.*,

$$(V_2, I_2) = f(V_1, -I_1)$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Where matrix consisting of A', B', C', D', is called as inverse transmission matrix.

$$V_2 = A'\,V_1 + B'\,(-I_1) \quad \text{...(7)}$$

$$I_2 = C'\,V_1 + D'\,(-I_1) \quad \text{...(8)}$$

Note :

The equivalent circuit of a two port network is also not possible in terms of T'-parameters.

Determination of T'-parameters

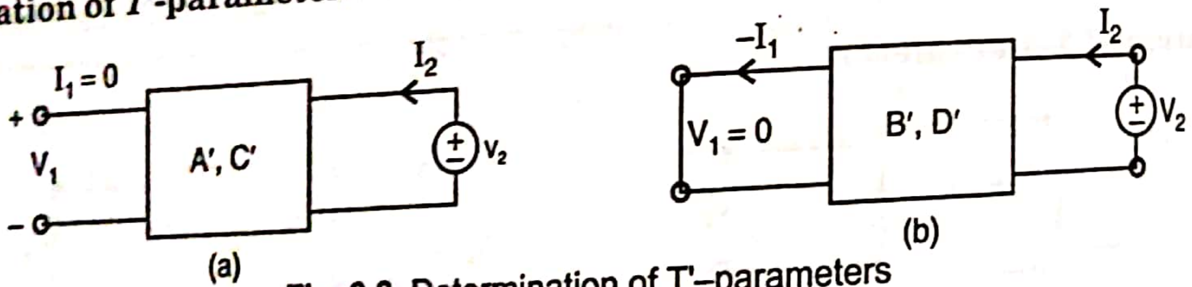


Fig. 8.8. Determination of T'–parameters

Case I : $V_2 = V_2$, $I_2 = ?$, $V_1 = ?$, $I_1 = 0$ [input port is open circuited as shown in figure 8.8(*a*)].

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0} \quad \text{(forward voltage ratio with sending end open circuited)}$$

$$C' = \left.\frac{I_2}{V_1}\right|_{I_1=0} \quad \text{(transfer admittance with sending end open circuited)}$$

Case II : $V_2 = V_2$, $I_2 = ?$ $I_1 = ?$, $V_1 = 0$ [input port is short circuited as shown in figure 8.8(*b*)].

$$B' = \left.\frac{V_2}{-I_1}\right|_{V_1=0} \quad \text{(transfer impedance with sending end short circuited)}$$

$$D = \left.\frac{I_2}{-I_1}\right|_{V_1=0} \quad \text{(forward current ratio with sending end short circuited)}$$

Note :

For passive network, in this case also all four T'-parameters are positive, as I_1 is itself negative so $-I_1$ is positive.

8.6. HYBRID (h) PARAMETERS

The hybrid parameters are wide usage in electronic circuits, especially in constructing models for transistors. In this case, voltage of the input port and the current of the output port are expressed in terms of the current of the input port and the voltage of the output port. Due to this reason, these parameters are called as "hybrid" parameters, *i.e.*,

$$(V_1, I_2) = f(I_1, V_2)$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

This matrix equation defines h-parameters, and h-parameter matrix is known as hybrid matrix.

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad \text{...(9)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad \text{...(10)}$$

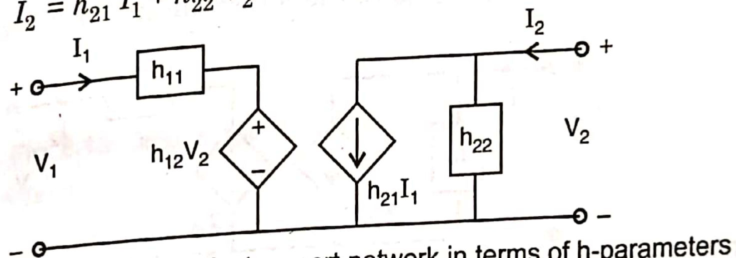


Fig. 8.9. Equivalent circuit of a two-port network in terms of h-parameters

From equations (9) and (10) we draw the one voltage and one current source equivalent of a two-port network may be obtained in terms of h-parameters as shown in figure 8.9, where $h_{12} V_2$ and $h_{21} I_1$ are voltage controlled voltage and current controlled current sources respectively.

Determination of h-parameters

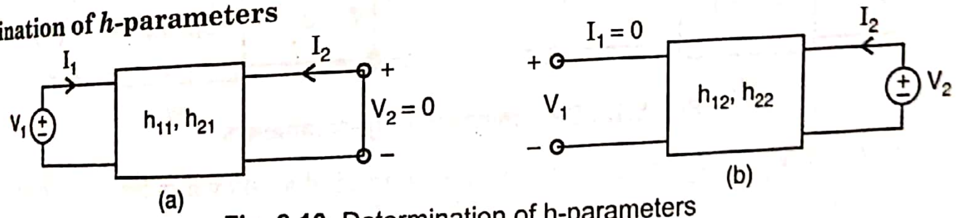


Fig. 8.10. Determination of h-parameters

Case I: $V_1 = V_1$, $I_1 = ?$, $I_2 = ?$, $V_2 = 0$ [output port short circuited as shown in figure 8.10(a)].

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{(input impedance with the output port short circuited).}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad \text{(forward current gain with the output port short circuited).}$$

Case II: $V_2 = V_2$, $I_2 = ?$, $V_1 = ?$ $I_1 = 0$ [input port open circuited as shown in figure 8.10(b)].

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{(reverse voltage gain with the input port open circuited).}$$

i.e., ... parameter and inverse hybrid parameter are duel of each other like Z and Y parameters ... this case.

$$[g] = [h]^{-1}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

This matrix equation defines g-parameters and g-parameter matrix is known as inverse hybrid matrix.

$$I_1 = g_{11} V_1 + g_{12} I_2 \quad ...(11)$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \quad ...(12)$$

From equations (11) and (12), we can draw the equivalent circuit of a two-port network as shown in figure 8.11.

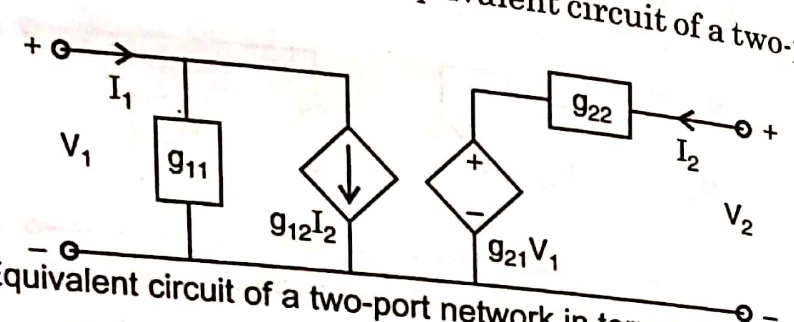
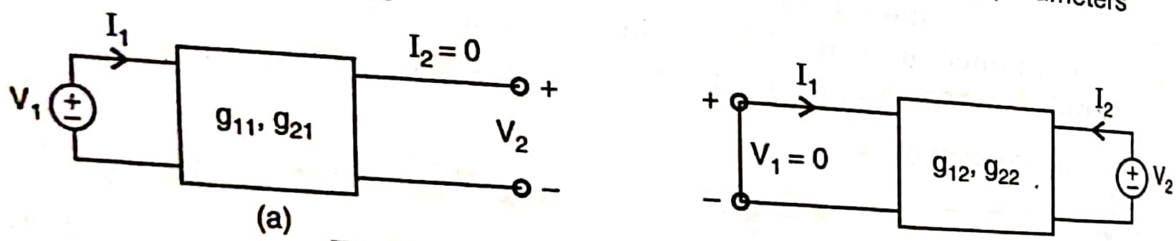


Fig. 8.11. Equivalent circuit of a two-port network in terms of g-parameters

Determination of g-parameters



(a) (b)

Fig. 8.12. Determination of g–parameters

Case I : $V_1 = V_1$, $I_1 = ?$, $V_2 = ?$, $I_2 = 0$ [output port open circuited as shown in figure 8.12(*a*)].

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \quad \text{(input admittance with output port open circuited)}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \quad \text{(forward voltage gain with output port open circuited).}$$

Case II : $V_2 = V_2$, $I_2 = ?$, $I_1 = ?$, $V_1 = 0$ [input port short circuited as shown in figure 8.12(b)].

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \quad \text{(reverse current gain with input port short circuited)}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$ (output impedance with input port short circuited)

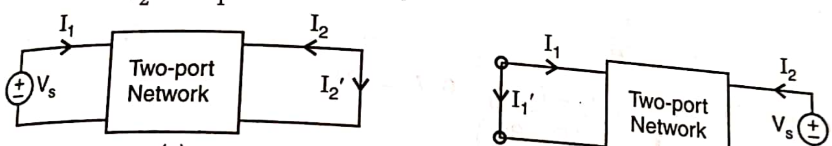
Note :

For passive network, g_{12} is always negative since I_1 is negative in this case.

8.8. CONDITION FOR RECIPROCITY

A two port network is said to be reciprocal, if the ratio of the excitation to response is invariant to an interchange of the positions of the excitation and response in the network. *Networks containing resistors, inductors, and capacitors are generally reciprocal.* Networks that additionally have dependent sources are generally non-reciprocal. Mathematically, we can say from figure 8.13 (*a*) and (*b*).

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'} \quad \text{or} \quad I_2' = I_1'$$



(a) $(V_1 = V_s, I_1 = I_1, V_2 = 0, I_2 = -I_2')$

(b) $(V_2 = V_s, I_2 = I_2, V_1 = 0, I_1 = -I_1')$

Fig. 8.13. Determination the condition for reciprocity

(i) In terms of Z-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

As in figure 8.13 (*a*); $V_1 = V_s,\ I_1 = I_1,\ V_2 = 0,\ I_2 = -I_2'$.

Therefore, $$Vs = Z_{11} I_1 - Z_{12} I_2'$$

and $$0 = Z_{21} I_1 - Z_{22} I_2'$$

Hence $$I_2' = \frac{V_s Z_{21}}{Z_{11}Z_{22} - Z_{21}Z_{12}}$$

As in figure 8.13(*b*); $V_2 = V_s,\ I_2 = I_2,\ V_1 = 0,\ I_1 = -I_1'$.

Therefore, $$0 = -Z_{11} I_1' + Z_{12} I_2$$

and $$V_s = -Z_{21} I_1' + Z_{22} I_2$$

Hence $$I_1' = \frac{V_s Z_{12}}{Z_{11}Z_{22} - Z_{21}Z_{12}}$$

Comparing I_2' and I_1', we get

$$Z_{12} = Z_{21}$$

(This is the condition of reciprocity in terms of *Z*-parameters)

(ii) In terms of Y-parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

As in figure 8.13 (*a*); $V_1 = V_s,\ I_1 = I_1,\ V_2 = 0,\ I_2 = -I_2'$.

$$I_2' = -Y_{21}\,V_s$$

As in figure 8.13 (*b*); $V_2 = V_s,\ I_2 = I_2,\ V_1 = 0,\ I_1 = -I_1'$.

$$I_1' = -Y_{12}\,V_s$$

Comparing I_2' and I_1'. Then

$$Y_{12} = Y_{21}$$

(This is the condition of reciprocity in terms of *Y*-parameters)

(iii) *In terms of T-parameters*

$$V_1 = AV_2 + B\,(-I_2)$$
$$I_1 = CV_2 + D\,(-I_2)$$

As in figure 8.13 (*a*); $V_1 = V_s,\ I_1 = I_1,\ V_2 = 0,\ I_2 = -I_2'$.

$$I_2' = \frac{V_s}{B}$$

As in figure 8.13(*b*); $V_2 = V_s,\ I_2 = I_2,\ V_1 = 0,\ I_1 = -I_1'$.

$$I_1' = V_s\left(\frac{AD - BC}{B}\right)$$

Above discussion leads to the condition of reciprocity,

$$\mathbf{AD - BC = 1 \quad or \quad \Delta T = 1}$$

or
$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$$

(iv) *In terms of T′-parameters*

Condition of reciprocity in case of *T′*-parameters is similar to as in case of *T*-parameters, *i.e.,*

$$\mathbf{A'D' - B'C' = 1 \quad or \quad \Delta T' = 1}$$

or
$$\begin{vmatrix} A' & B' \\ C' & D' \end{vmatrix} = 1$$

(v) *In terms of h-parameters*

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

As in figure 8.13(*a*) ; $V_1 = V_s,\ I_1 = I_1,\ V_2 = 0,\ I_2 = -I_2'$.

$$I_2' = -V_s\,\frac{h_{21}}{h_{11}}$$

As in figure 8.13(*b*) ;$V_2 = V_s,\ I_2 = I_2,\ V_1 = 0,\ I_1 = -I_1'$.

$$I_1' = V_s\,\frac{h_{12}}{h_{11}}$$

from the definition of reciprocity, $I_2' = I_1'$ leads to

$$\mathbf{h_{12} = -\,h_{21}}$$

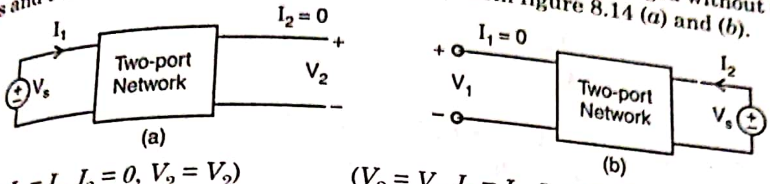
(vi) *In terms of g-parameters*

In this case, the condition of reciprocity is similar to as in case of *h*-parameters, *i.e.,*

$$\mathbf{g_{12} = -\,g_{21}}$$

8.9. CONDITION FOR SYMMETRY

A two-port network is said to be symmetrical if the ports can be interchanged without changing the port voltages and currents. Mathematically, we can say from figure 8.14 (*a*) and (*b*).



$(V_1 = V_s, I_1 = I_1, I_2 = 0, V_2 = V_2)$ $\qquad$ $(V_2 = V_s, I_2 = I_2, I_1 = 0, V_1 = V_1)$

Fig. 8.14. Determination the condition for symmetry

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$$

(i) *In terms of Z-parameters*

As in figure 8.14 (*a*); $V_1 = V_s$, $I_1 = I_1$, $I_2 = 0$, $V_2 = V_2$.

$$V_s = Z_{11}\, I_1$$

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = Z_{11}$$

As in figure 8.14 (*b*); $V_2 = V_s$, $I_2 = I_2$, $I_1 = 0$, $V_1 = V_1$.

$$V_s = Z_{22}\, I_2$$

$$\left.\frac{V_s}{I_2}\right|_{I_1=0} = Z_{22}$$

From the definition of symmetry, $\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$ leads to

$$Z_{11} = Z_{22}$$

(ii) *In terms of Y-parameters*

As in figure 8.14 (*a*) ; $V_1 = V_s$, $I_1 = I_1$, $I_2 = 0$, $V_2 = V_2$.

$$I_1 = Y_{11}\, V_s + Y_{12}\, V_2$$

$$0 = Y_{21}\, V_s + Y_{22}\, V_2$$

$$I_1 = Y_{11}\, V_s + Y_{12}\left\{\frac{-Y_{21}}{Y_{22}}\right\} V_s$$

$$\frac{V_s}{I_1} = \frac{Y_{22}}{Y_{11}\, Y_{22} - Y_{12}\, Y_{21}}$$

And as in figure 8.14(*b*); $V_2 = V_s$, $I_2 = I_2$, $I_1 = 0$, $V_1 = V_1$.

$$0 = Y_{11} V_1 + Y_{12} V_s$$

$$I_2 = Y_{21} V_1 + Y_{22} V_s$$

$$\frac{V_s}{I_2} = \frac{Y_{11}}{Y_{11} Y_{22} - Y_{12} Y_{21}}$$

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0} \text{ leads to}$$

$$Y_{11} = Y_{22}$$

(iii) In terms of T-parameters

As in figure 8.14 (*a*); $V_1 = V_s$, $I_1 = I_1$, $I_2 = 0$, $V_2 = V_2$.

$$V_s = AV_2$$

$$I_1 = CV_2$$

Then
$$\frac{V_s}{I_1} = \frac{A}{C}$$

And as in figure 8.14 (*b*); $V_2 = V_s$, $I_2 = I_2$, $I_1 = 0$, $V_1 = V_1$.

$$V_1 = AV_s - BI_2$$

$$0 = CV_s - D\,I_2 \text{ or } \frac{V_s}{I_2} = \frac{D}{C}$$

Therefore, the condition for Symmetry ; $\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$ leads to

$$\boldsymbol{A = D}$$

(iv) In terms of T′-parameters

Condition of symmetry in case of T'-parameters is similar to as in case of T-parameters, *i.e.,*

$$\boldsymbol{A' = D'}$$

(v) In terms of h-parameters

As in figure 8.14(*a*); $V_1 = V_s$, $I_1 = I_1$, $I_2 = 0$, $V_2 = V_2$.

$$V_s = h_{11}\, I_1 + h_{12}\, V_2$$

$$0 = h_{21}\, I_1 + h_{22}\, V_2$$

$$\frac{V_s}{I_1} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}$$

And as in figure 8.14(*b*); $V_2 = V_s$, $I_2 = I_2$, $I_1 = 0$, $V_1 = V_1$.

$$V_1 = h_{12}\, V_s$$

$$I_2 = h_{22}\, V_s \text{ or } \frac{V_s}{I_2} = \frac{1}{h_{22}}$$

Therefore, the condition for symmetry ; $\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$ leads to

$$\boldsymbol{h_{11}\, h_{22} - h_{12}\, h_{21} = 1 \text{ or } \Delta h = 1}$$

or
$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$

(vi) In terms of g-parameters

Condition of symmetry in case of g-parameters is similar to as in case of h-parameters, *i.e.*

$$\boldsymbol{g_{11}\, g_{22} - g_{12}\, g_{21} = 1 \text{ or } \Delta g = 1}$$

or
$$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$$

Parameters		
$[Z]$	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
$[Y]$	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
$[T]$ or $[ABCD]$	$\Delta T = AD - BC = 1$	$A = D$
$[T']$ or $[A'B'C'D']$	$\Delta T' = A'D' - B'C' = 1$	$A' = D'$
$[h]$	$h_{12} = -h_{21}$	$\Delta h = h_{11} h_{22} - h_{12} h_{21} = 1$
$[g]$	$g_{12} = -g_{21}$	$\Delta g = g_{11} g_{22} - g_{12} g_{21} = 1$

.10. RELATIONSHIPS BETWEEN PARAMETER SETS

introducing the six sets of parameters in the previous sections of this chapter, we have suggested applications for each of the parameter sets. We cannot say, however, that all transistor problems are solved by using h-parameters, and we frequently find it necessary to convert from one set of parameters to another. It is a simple matter to find the relationships of the sets of parameters as follows :

"If we want to express α-parameters in terms of β-parameters, we have to write β-parameter equations and then, by algebraic manipulation, rewrite the equations as needed for α-parameters".

Note :

The equivalences involve a factor

$$\Delta x = x_{11} x_{22} - x_{12} x_{21}$$

where x is either Z, Y, T, T', h or g.

.10.1. Z-parameters in Terms of Other Parameters

(i) *Z-parameters in terms of Y-parameters*

As in this case we know,

$$[I] = [Y][V]$$

and

$$[V] = [Z][I]$$

Therefore,

$$[Z] = [Y]^{-1}$$

or

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta Y}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

or

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \; ; \; Z_{12} = \frac{-Y_{12}}{\Delta Y} \; ; Z_{21} = \frac{-Y_{21}}{\Delta Y} \text{ and } Z_{22} = \frac{Y_{11}}{\Delta Y}$$

(ii) *Z-parameters in terms of T-parameters*

The transmission parameter equations (5) and (6) are

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

Rewriting the equation (6), *i.e.*,

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

Comparing with equation (2) ; $Z_{21} = \dfrac{1}{C}$ and $Z_{22} = \dfrac{D}{C}$

Again from equaion (5) ;

$$V_1 = A\left[\frac{1}{C}\cdot I_1 + \frac{D}{C}\cdot I_2\right] + B(-I_2) = \frac{A}{C}\cdot I_1 + \left(\frac{AD}{C} - B\right)\cdot I_2$$

Comparing with equation (1) ;

$$Z_{11} = \frac{A}{C}$$

and

$$Z_{12} = \frac{AD - BC}{C} \quad \text{or} \quad Z_{12} = \frac{\Delta T}{C}$$

(iii) Z-parameters in terms of T'-parameters

As similar to above case :

$$Z_{11} = \frac{D'}{C'}, Z_{12} = \frac{1}{C'}, Z_{21} = \frac{\Delta T'}{C'} \text{ and } Z_{22} = \frac{A'}{C'}$$

(iv) Z-parameters in terms of h-parameters

The h-parameter equations (9) and (10) are

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting the equation (10), *i.e.*,

$$V_2 = -\frac{h_{21}}{h_{11}} I_1 + \frac{1}{h_{22}} I_2$$

Comparing with equation (2);

$$Z_{21} = -\frac{h_{21}}{h_{22}} \text{ and } Z_{22} = \frac{1}{h_{22}}$$

Again from equation (9);

$$V_1 = h_{11} I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right] = \left[h_{11} - \frac{h_{21}h_{12}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

Comparing with equation (1) ;

$$Z_{11} = \frac{\Delta h}{h_{22}} \text{ and } Z_{12} = \frac{h_{12}}{h_{22}}$$

(v) Z-parameters in terms of g-parameters

As similar to above case :

$$Z_{11} = \frac{1}{g_{11}}, Z_{12} = -\frac{g_{12}}{g_{11}}, Z_{21} = \frac{g_{21}}{g_{11}} \text{ and } Z_{22} = \frac{\Delta g}{g_{11}}$$

8.10.2. Y-parameters in Terms of Other Parameters

(i) Y-parameters in terms of Z-parameters

As we know,

$$[Y] = [Z]^{-1}$$

Therefore,

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, Y_{12} = -\frac{Z_{12}}{\Delta Z}, Y_{21} = -\frac{Z_{21}}{\Delta Z} \text{ and } Y_{22} = \frac{Z_{11}}{\Delta Z}$$

(ii) Y-parameters in terms of T-parameters

The transmission parameter equations (5) and (6) are

$$V_1 = A V_2 + B(-I_2)$$
$$I_1 = C V_2 + D(-I_2)$$

Rewriting the equation (5), *i.e.*,

$$I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2$$

Comparing with equation (4) ;

$$Y_{21} = -\frac{1}{B}, \text{ and } Y_{22} = \frac{A}{B}$$

Again from equation (6);

$$I_1 = CV_2 + D\left\{\frac{1}{B}V_1 - \frac{A}{B}V_2\right\} = \frac{D}{B}V_1 + \left(C - \frac{AD}{B}\right)V_2$$

Comparing with equation (3) ;

$$Y_{11} = \frac{D}{B} \text{ and } Y_{12} = -\frac{\Delta T}{B}$$

(iii) *Y-parameter in terms of T'-parameters*

As similar to above case :

$$Y_{11} = \frac{A'}{B'}, Y_{12} = -\frac{1}{B'}, Y_{21} = -\frac{\Delta T'}{B'} \text{ and } Y_{22} = \frac{D'}{B'}$$

(iv) *Y-parameters in terms of h-parameters*

The *h*-parameter equations (9) and (10) are

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting the equation (9), *i.e.*,

$$I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2$$

Comparing with equation (3) ;

$$Y_{11} = \frac{1}{h_{11}} \text{ and } Y_{12} = -\frac{h_{12}}{h_{11}}$$

Again from equation (10);

$$I_2 = h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2 = \frac{h_{21}}{h_{11}}V_1 + \frac{\Delta h}{h_{11}}V_2$$

Comparing with equation (4) ;

$$Y_{21} = \frac{h_{21}}{h_{11}} \text{ and } Y_{22} = \frac{\Delta h}{h_{11}}$$

(v) *Y-parameters in terms of g-parameters*

As similar to above case :

$$Y_{11} = \frac{\Delta g}{g_{22}}, Y_{12} = \frac{g_{12}}{g_{22}}, Y_{21} = -\frac{g_{21}}{g_{22}} \text{ and } Y_{22} = \frac{1}{g_{22}}$$

8.10.3. T-parameters in Terms of Other Parameters

(i) *T-parameters in terms of Z-parameters*

The *Z*-parameter equation (1) and (2) are

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Rewriting the equation (2), *i.e.*,

$$I_1 = \frac{1}{Z_{21}}V_2 + \frac{Z_{22}}{Z_{21}}(-I_2)$$

Comparing with equation (6) ;

$$C = \frac{1}{Z_{21}} \text{ and } D = \frac{Z_{22}}{Z_{21}}$$

Again from equation (1);

$$V_1 = Z_{11}\left(\frac{1}{Z_{21}}V_2 + \frac{Z_{22}}{Z_{21}}(-I_2)\right) + Z_{12}I_2 = \frac{Z_{11}}{Z_{21}}V_2 + \left(\frac{Z_{11}Z_{22}}{Z_{21}} - Z_{12}\right)(-I_2)$$

Comparing with equation (5) ;

$$A = \frac{Z_{11}}{Z_{21}} \text{ and } B = \frac{\Delta Z}{Z_{21}}$$

(ii) T-parameters in terms of Y-parameters

As similar to above case :

$$A = -\frac{Y_{22}}{Y_{21}},\ B = -\frac{1}{Y_{21}},\ C = -\frac{\Delta Y}{Y_{21}} \text{ and } D = -\frac{Y_{11}}{Y_{21}}$$

(iii) T-parameters in terms of T'-parameters

As we know, the T'-parameter equation in matrix form,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Rewritting the above equation, *i.e.*,

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A' & -B' \\ -C' & D' \end{bmatrix}\begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

or

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & -B' \\ -C' & D' \end{bmatrix}^{-1}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Therefore,

$$[T] = \begin{bmatrix} A' & -B' \\ -C' & D' \end{bmatrix}^{-1} = \frac{1}{\Delta T'}\begin{bmatrix} D' & B' \\ C' & A' \end{bmatrix}$$

or $A = \dfrac{D'}{\Delta T'}, B = \dfrac{B'}{\Delta T'}, C = \dfrac{C'}{\Delta T'}$ and $D = \dfrac{A'}{\Delta T'}$

Note :

From above discussion, we see that $[Z] = [Y]^{-1}$ and $[h] = [g]^{-1}$ while $[T] \neq [T']^{-1}$.

(iv) T-parameters in terms of h-parameters

The h-parameter equations (9) and (10) are

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Rewriting the equation (10), *i.e.*,

$$I_1 = \frac{-h_{22}}{h_{21}}V_2 + \left(\frac{-1}{h_{21}}\right)(-I_2)$$

Comparing with equation (6) ;

$$C = \frac{-h_{22}}{h_{21}} \text{ and } D = \frac{-1}{h_{21}}$$

Again from equation (9);

$$V_1 = h_{11}\left[-\frac{h_{22}}{h_{21}}V_2 + \left(-\frac{1}{h_{21}}\right)(-I_2)\right] + h_{12}V_2 = -\frac{\Delta h}{h_{21}}V_2 + \left(-\frac{h_{11}}{h_{21}}\right)(-I_2)$$

Comparing with equation (5) ;

$$A = -\frac{\Delta h}{h_{21}} \text{ and } B = -\frac{h_{11}}{h_{21}}$$

(v) *T-parameters in terms of g-parameters*

As similar to above case :

$$A = \frac{1}{g_{21}},\ B = \frac{g_{22}}{g_{21}},\ C = \frac{g_{11}}{g_{21}} \text{ and } D = \frac{\Delta g}{g_{21}}$$

8.10.4. T'-parameters in Terms of Other Parameters

(i) ***T'-parameters in terms of Z-parameters***

The Z-parameter equations (1) and (2) are

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Rewriting the equation (1), *i.e.*,

$$I_2 = \frac{1}{Z_{12}}V_1 + \frac{Z_{11}}{Z_{12}}(-I_1)$$

Comparing with equation (8) ;

$$C' = \frac{1}{Z_{12}} \text{ and } D' = \frac{Z_{11}}{Z_{12}}$$

Again from equation (2) ;

$$V_2 = Z_{21}I_1 + Z_{22}\left\{\frac{1}{Z_{12}}V_1 + \frac{Z_{11}}{Z_{12}}(-I_1)\right\} = \left(-Z_{21} + \frac{Z_{11}Z_{22}}{Z_{12}}\right)(-I_1) + \frac{Z_{22}}{Z_{12}}V_1$$

Comparing with equation (7) ;

$$A' = \frac{Z_{22}}{Z_{12}} \text{ and } B' = \frac{\Delta Z}{Z_{12}}$$

(ii) ***T'-parameters in terms of Y-parameters***

As similar to above case :

$$A' = -\frac{Y_{11}}{Y_{12}},\ B' = -\frac{1}{Y_{12}},\ C' = -\frac{\Delta Y}{Y_{12}} \text{ and } D' = -\frac{Y_{22}}{Y_{12}}$$

(iii) ***T'-parameters in terms of T-parameters***

As we know, the *T*-parameter equation is matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Rewriting the above equation, *i.e.*

$$\begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix}\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

or

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix}^{-1}\begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Therefore,

$$[T'] = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix}^{-1}$$

or

$$A' = \frac{D}{\Delta T},\ B' = \frac{B}{\Delta T},\ C' = \frac{C}{\Delta T} \text{ and } D' = \frac{A}{\Delta T}$$

Note :

From above discussion, we see that $[Y] = [Z]^{-1}$ and $[g] = [h]^{-1}$ while $[T'] \neq [T]^{-1}$.

(iv) T'-parameters in terms of h-parameters

The *h*-parameter equations (9) and (10) are :

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting the equation (9), *i.e.*,

$$V_2 = -\frac{h_{11}}{h_{12}} I_1 + \frac{1}{h_{12}} V_1$$

Comparing with equation (7),

$$A' = \frac{1}{h_{12}} \text{ and } B' = \frac{h_{11}}{h_{12}}$$

Again from equation (10) ;

$$I_2 = h_{21} I_1 + h_{22}\left\{-\frac{h_{11}}{h_{12}} I_1 + \frac{1}{h_{12}} V_1\right\} = \frac{h_{22)}}{h_{12}} V_1 + \left(\frac{h_{11} h_{22}}{h_{12}} - h_{21}\right)(-I_1)$$

Comparing with equation (8) ;

$$C' = \frac{h_{22}}{h_{12}} \text{ and } D' = \frac{\Delta h}{h_{12}}$$

(v) T'-parameters in terms of g-parameters

As similar to above case :

$$A' = -\frac{\Delta g}{g_{12}},\ B' = -\frac{g_{22}}{g_{12}},\ C' = -\frac{g_{11}}{g_{12}} \text{ and } D' = -\frac{1}{g_{12}}$$

8.10.5. *h*-parameters in Terms of Other Parameters

(i) h-parameters in terms of Z-parameters

The *Z*-parameter equations (1) and (2) are

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{11} I_2$$

Rewriting the equation (2), *i.e.*,

$$I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2$$

Comparing with equation (10);

$$h_{21} = -\frac{Z_{21}}{Z_{22}} \text{ and } h_{22} = \frac{1}{Z_{22}}$$

Again from equation (1) ;

$$V_1 = Z_{11}I_1 + Z_{12}\left\{-\frac{Z_{21}}{Z_{22}}I_1 + \frac{1}{Z_{22}}V_2\right\}$$

$$V_1 = \left(Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}\right)I_1 + \frac{Z_{12}}{Z_{22}}V_2$$

Comparing with equation (9);

$$h_{11} = \frac{\Delta Z}{Z_{22}} \text{ and } h_{12} = \frac{Z_{12}}{Z_{22}}$$

(ii) *h-parameters in terms of Y-parameters*

As similar to above case :

$$h_{11} = \frac{1}{Y_{11}},\ h_{12} = \frac{-Y_{12}}{Y_{11}},\ h_{21} = \frac{Y_{21}}{Y_{11}} \text{ and } h_{22} = \frac{\Delta Y}{Y_{11}}$$

(iii) *h-parameters in terms of T-parameters*

The T-parameter equations (5) and (6) are

$$V_1 = A\,V_2 + B(-I_2)$$

$$I_1 = C\,V_2 + D(-I_2)$$

Rewriting the equation (6), *i.e.*,

$$I_2 = -\frac{1}{D}I_1 + \frac{C}{D}V_2$$

Comparing with equation (10),

$$h_{21} = -\frac{1}{D} \text{ and } h_{22} = \frac{C}{D}$$

Again from equation (5) ;

$$V_1 = AV_2 + B\left\{\frac{1}{D}I_1 - \frac{C}{D}V_2\right\} = \frac{B}{D}I_1 + \left(A - \frac{BC}{D}\right)V_2$$

Comparing with equation (9);

$$h_{11} = \frac{B}{D} \text{ and } h_{12} = \frac{\Delta T}{D}$$

(iv) *h-parameters in terms of T'-parameters*

As similar to above case :

$$h_{11} = \frac{B'}{A'},\ h_{12} = \frac{1}{A'},\ h_{21} = -\frac{\Delta T'}{A'} \text{ and } h_{22} = \frac{C'}{A'}$$

(v) *h-parameters in terms of g-parameters*

As we know, $[h] = [g]^{-1}$

Therefore,

$$h_{11} = \frac{g_{22}}{\Delta g},\ h_{12} = -\frac{g_{12}}{\Delta g},\ h_{21} = \frac{-g_{21}}{\Delta g} \text{ and } h_{22} = \frac{g_{11}}{\Delta g}$$

0.6. g-parameters in Terms of Other Parameters

parameters in terms ...

The Z...

Rewriting the equation (1), *i.e.*,

$$I_1 = \frac{1}{Z_{11}} V_1 - \frac{Z_{12}}{Z_{11}} I_2$$

Comparing with equation (11);

$$g_{11} = \frac{1}{Z_{11}} \quad \text{and} \quad g_{12} = -\frac{Z_{12}}{Z_{11}}$$

Again from equation (2);

$$V_2 = Z_{21}\left\{\frac{1}{Z_{11}} V_1 - \frac{Z_{12}}{Z_{11}} I_2\right\} + Z_{22} I_2 = \frac{Z_{21}}{Z_{11}} V_1 + \left(Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11}}\right) I_2$$

Comparing with equation (12);

$$g_{21} = \frac{Z_{21}}{Z_{11}} \quad \text{and} \quad g_{22} = \frac{\Delta Z}{Z_{11}}$$

(ii) *g-parameters in terms of Y-parameters*

As similar to above case :

$$g_{11} = \frac{\Delta Y}{Y_{22}}, \; g_{12} = \frac{Y_{12}}{Y_{22}}, \; g_{21} = \frac{-Y_{21}}{Y_{22}} \quad \text{and} \quad g_{22} = \frac{1}{Y_{22}}$$

(iii) *g-parameters in terms of T-parameters*

The *T*-parameter equations (5) and (6) are

$$V_1 = AV_2 + B(-I_2)$$
$$I_1 = CV_2 + D(-I_2)$$

Rewriting the equation (5), *i.e.*,

$$V_2 = \frac{1}{A} V_1 + \frac{B}{A} I_2$$

Comparing with equation (12);

$$g_{21} = \frac{1}{A} \quad \text{and} \quad g_{22} = \frac{B}{A}$$

Again from equation (6);

$$I_1 = C\left[\frac{1}{A} V_1 + \frac{B}{A} I_2\right] + D(-I_2) = \frac{C}{A} V_1 + \left(\frac{BC}{A} - D\right) I_2$$

Comparing with equation (11);

$$g_{11} = \frac{C}{A} \quad \text{and} \quad g_{12} = -\frac{\Delta T}{A}$$

(iv) *g-parameters in terms of T'-parameters*

As similar to above case :

$$g_{11} = \frac{C'}{D'}, \; g_{12} = -\frac{1}{D'}, \; g_{21} = \frac{\Delta T'}{D'} \quad \text{and} \quad g_{22} = \frac{B'}{D'}$$

(v) *g-parameters in terms of h-parameters*

As we know, $[g] = [h]^{-1}$

Therefore, $g_{11} = \frac{h_{22}}{\Delta h}, \; g_{12} = -\frac{h_{12}}{\Delta h}, \; g_{21} = \frac{-h_{21}}{\Delta h} \quad \text{and} \quad g_{22} = \frac{h_{11}}{\Delta h}$

8.11.1. Series Connection

Figure 8.15 shows a series connection of two two-port networks N_a and N_b with open circuit Z-parameters Z_a and Z_b respectively, i.e., for network N_a,

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

Similarly, for network N_b,

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

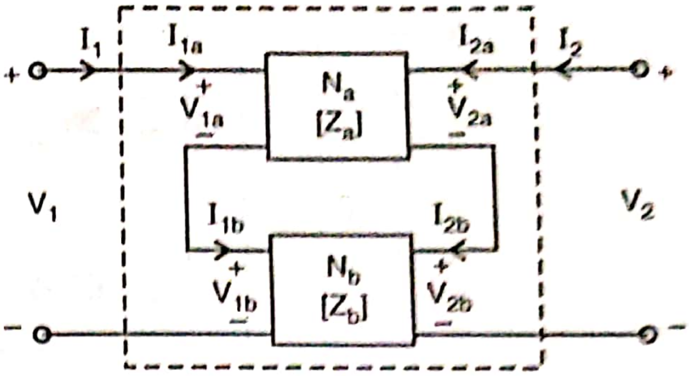


Fig. 8.15. Series connection of two two-port networks

Then, their series connection requires that

$$I_1 = I_{1a} = I_{1b},\ I_2 = I_{2a} = I_{2b}$$

$$V_1 = V_{1a} + V_{1b}\ ,\ V_2 = V_{2a} + V_{2b}$$

Now, $V_1 = V_{1a} + V_{1b} = (Z_{11a} I_{1a} + Z_{12a} I_{2a}) + (Z_{11b} I_{1b} + Z_{12b} I_{2b})$

$$= (Z_{11a} + Z_{11b})I_1 + (Z_{12a} + Z_{12b})I_2$$

$$(\text{since } I_1 = I_{1a} = I_{1b} \text{ and } I_2 = I_{2a} = I_{2b})$$

And similarly, $V_2 = V_{2a} + V_{2b} = (Z_{21a} + Z_{21b})I_1 + (Z_{22a} + Z_{22b})I_2$

So, in matrix form the Z-parameters of the series-connected combined network can be written as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where

$$Z_{11} = Z_{11a} + Z_{11b}$$
$$Z_{12} = Z_{12a} + Z_{12b}$$
$$Z_{21} = Z_{21a} + Z_{21b}$$
$$Z_{22} = Z_{22a} + Z_{22b}$$

or in the matrix form

$$[Z] = [Z_a] + [Z_b]$$

This result may be generalised for any number of two-port networks connected in series.

"The overall Z-parameter matrix for series connected two-port networks is simply the sum of Z-parameter matrices of each individual two port network connected in series".

Note :

The series connection of two-port networks can only be achieved if port property of the individual network does not violate. More precisely, the current entering one terminal is equal to the current leaving the other terminal. The series connection is also called as series-series connection, since both input and output ports are series connected.

8.11.2. Parallel Con...

[Z]	Z_{11}	Z_{12}	$\frac{Y_{22}}{\Delta Y}$	$-\frac{Y_{12}}{\Delta Y}$	$\frac{A}{C}$	$\frac{\Delta T}{C}$	$\frac{D'}{C'}$	$\frac{1}{C'}$	$\frac{\Delta h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$
	Z_{21}	Z_{22}	$-\frac{Y_{21}}{\Delta Y}$	$\frac{Y_{11}}{\Delta Y}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{\Delta T'}{C'}$	$\frac{A'}{C'}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta g}{g_{11}}$
[Y]	$\frac{Z_{22}}{\Delta Z}$	$-\frac{Z_{12}}{\Delta Z}$	Y_{11}	Y_{12}	$\frac{D}{B}$	$-\frac{\Delta T}{B}$	$\frac{A'}{B'}$	$-\frac{1}{B'}$	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$
	$-\frac{Z_{21}}{\Delta Z}$	$\frac{Z_{11}}{\Delta Z}$	Y_{21}	Y_{22}	$-\frac{1}{B}$	$\frac{A}{B}$	$-\frac{\Delta T'}{B'}$	$\frac{D'}{B'}$	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$
[T]	$\frac{Z_{11}}{Z_{21}}$	$\frac{\Delta Z}{Z_{21}}$	$-\frac{Y_{22}}{Y_{21}}$	$-\frac{1}{Y_{21}}$	A	B	$\frac{D'}{\Delta T'}$	$\frac{B'}{\Delta T'}$	$-\frac{\Delta h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$
	$\frac{1}{Z_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$-\frac{\Delta Y}{Y_{21}}$	$-\frac{Y_{11}}{Y_{21}}$	C	D	$\frac{C'}{\Delta T'}$	$\frac{A'}{\Delta T'}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta g}{g_{21}}$
[T']	$\frac{Z_{22}}{Z_{12}}$	$\frac{\Delta Z}{Z_{12}}$	$-\frac{Y_{11}}{Y_{12}}$	$-\frac{1}{Y_{12}}$	$\frac{D}{\Delta T}$	$\frac{B}{\Delta T}$	A'	B'	$\frac{1}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$-\frac{\Delta g}{g_{12}}$	$-\frac{g_{22}}{g_{12}}$
	$\frac{1}{Z_{12}}$	$\frac{Z_{11}}{Z_{12}}$	$-\frac{\Delta Y}{Y_{12}}$	$-\frac{Y_{22}}{Y_{12}}$	$\frac{C}{\Delta T}$	$\frac{A}{\Delta T}$	C'	D'	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$	$-\frac{1}{g_{12}}$

(4)		(5)		(6)	
$\frac{B}{D}$	$\frac{\Delta T}{D}$	$\frac{B'}{A'}$	$\frac{1}{A'}$	h_{11}	h_{12}
$-\frac{1}{D}$	$\frac{C}{D}$	$-\frac{\Delta T'}{A'}$	$\frac{C'}{A'}$	h_{21}	h_{22}
$\frac{C}{A}$	$-\frac{\Delta T}{A}$	$\frac{C'}{D'}$	$-\frac{1}{D'}$	$\frac{h_{22}}{\Delta h}$	$-\frac{h_{12}}{\Delta h}$
$\frac{1}{A}$	$\frac{B}{A}$	$\frac{\Delta T'}{D'}$	$\frac{B'}{D'}$	$-\frac{h_{21}}{\Delta h}$	$\frac{h_{11}}{\Delta h}$

Similarly, for network N_b,

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$V_1 = V_{1a} = V_{1b},\ V_2 = V_{2a} = V_{2b}$$

$$I_1 = I_{1a} + I_{1b},\ I_2 = I_{2a} + I_{2b}$$

Now, $I_1 = I_{1a} + I_{1b}$

$$= (Y_{11a} V_{1a} + Y_{12a} V_{2a}) + (Y_{11b} V_{1b} + Y_{12b} V_{2b})$$

$$= (Y_{11a} + Y_{11b})V_1 + (Y_{12a} + Y_{12b})V_2$$

(since $V_1 = V_{1a} = V_{1b}$

and $V_2 = V_{2a} = V_{2b}$)

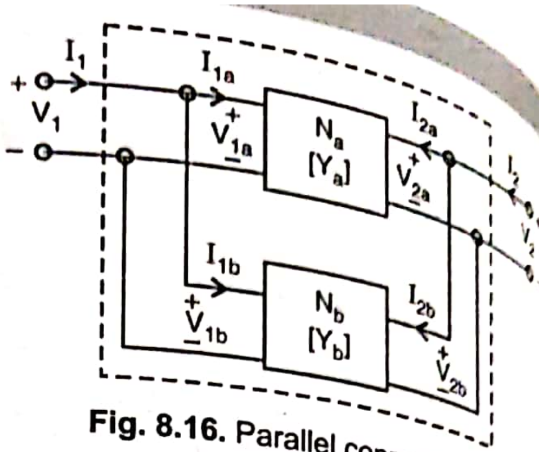


Fig. 8.16. Parallel connection of two two-port networks

And Similarly, $I_2 = I_{2a} + I_{2b} = (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$

So, in matrix form the Y-parameters of the parallel connected combined network can be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where

$$Y_{11} = Y_{11a} + Y_{11b}$$
$$Y_{12} = Y_{12a} + Y_{12b}$$
$$Y_{21} = Y_{21a} + Y_{21b}$$
$$Y_{22} = Y_{22a} + Y_{22b}$$

or in the matrix form

$$[Y] = [Y_a] + [Y_b]$$

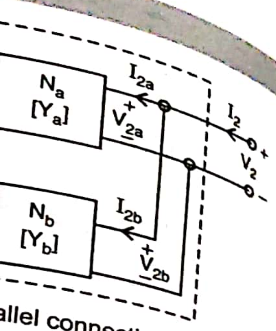
This result may be generalized for any number of two-port networks connected in parallel. "The overall Y-parameter matrix for parallel connected two-port networks is simply the sum of Y-parameter matrices of each individual two port network connected in parallel".

Note :

Parallel connection is also called as parallel-parallel connection, since both input and output ports are parallel connected.

8.11.3. Cascade Connection

The simplest possible interconnection of two-port networks is cascade or tandem-connection. Two two-port networks are said to be connected in cascade if the output port of the first becomes the input port of the second as shown in figure 8.17.



allel connection of two
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input and output

m-connection. Two
becomes the input

V_2

, for network N_a,

Similarly, for network N_b,

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

Then their cascade connection requires that

$$I_1 = I_{1a} \qquad -I_{2a} = I_{1b} \qquad I_{2b} = I_2$$
$$V_1 = V_{1a} \qquad V_{2a} = V_{1b} \qquad V_{2b} = V_2$$

Now,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

So, in matrix form the T-parameters of the cascade connected combined network can be written as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

or in the equation form

$$A = A_a A_b + B_a C_b$$
$$B = A_a B_b + B_a D_b$$
$$C = C_a A_b + D_a C_b$$
$$D = C_a B_b + D_a D_b$$

or in the matrix form

$$[T] = [T_a] \cdot [T_b]$$

This result may be generalised for any number of two-port networks connected in cascade.

"The overall T-parameter matrix for cascade connected two port networks is simply the matrix product of the T-parameter matrices of each individual two port network in cascade".

Note :

Similarly, we can derive in terms of inverse transmission (T')-parameters. The overall T'-parameter matrix for cascaded two-port network is simply the matrix product of the T'-parameter matrices for each individual two-port network in reverse order. For example : for two two-port networks

$$[T'] = [T'_b] \cdot [T'_a]$$

8.11.4. Series-Parallel Connection

Two two-port networks are said to be connected in series-parallel if the input ports are connected in series while the output ports are connected in parallel as shown in figure 8.18. Then, the connections require that

$$V_1 = V_{1a} + V_{1b}$$
$$I_1 = I_{1a} = I_{1b}$$
$$V_2 = V_{2a} = V_{2b}$$
$$I_2 = I_{2a} + I_{2b}$$

For network N_a,

$$\begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} h_{11a} & h_{12a} \\ h_{21a} & h_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix}$$

Similarly, for network N_b,

$$\begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} h_{11b} & h_{12b} \\ h_{21b} & h_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

Now

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix}$$

$$= \begin{bmatrix} h_{11a} + h_{11b} & h_{12a} + h_{12b} \\ h_{21a} + h_{21b} & h_{22a} + h_{22b} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

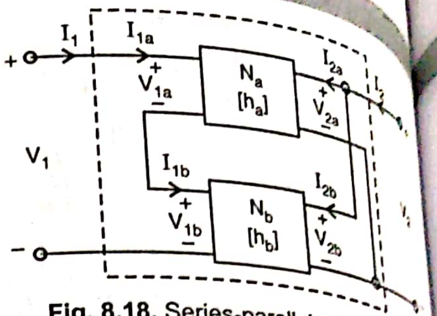


Fig. 8.18. Series-parallel connection of two two-port networks

So, in matrix form the h-parameters of the series-parallel connected network can be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where

$$h_{11} = h_{11a} + h_{11b}$$
$$h_{12} = h_{12a} + h_{12b}$$
$$h_{21} = h_{21a} + h_{21b}$$
$$h_{22} = h_{22a} + h_{22b}$$

or in the matrix form

$$[h] = [h_a] + [h_b]$$

This result may be generalised for any number of two-port networks connected in series-parallel. "The overall h-parameter matrix for series-parallel connected two-port networks is simply the sum of h-parameter matrices of each individual two-port network connected in series-parallel".

8.11.5. Parallel-Series Connection

Figure 8.19 shows a parallel-series connection of two two-port networks N_a and N_b with g-parameters g_a and g_b respectively. In this case input ports are connected in parallel while the output ports are connected in series. Then, the connections require that

$$V_1 = V_{1a} = V_{1b}$$
$$I_1 = I_{1a} + I_{1b}$$
$$V_2 = V_{2a} + V_{2b}$$
$$I_2 = I_{2a} = I_{2b}$$

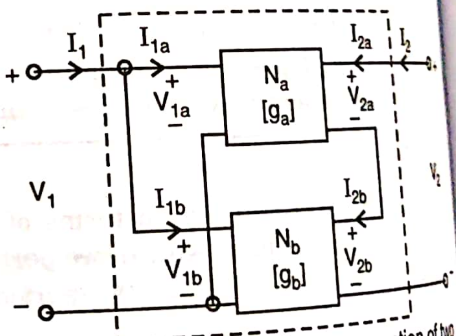


Fig. 8.19. Parallel-series connection of two two-port networks

As similar to previous case,

$$g_{11} = g_{11a} + g_{11b} \qquad g_{12} = g_{12a} + g_{12b}$$
$$g_{21} = g_{21a} + g_{21b} \qquad g_{22} = g_{22a} + g_{22b}$$

or in the matrix form

$$[g] = [g_a] + [g_b]$$

This result may be generalised for any number of two-port networks connected in parallel-series.

"The overall g-parameters matrix for parallel-series connected two-port networks is simply the sum of g-parameter matrices of each individual two-port network connected in parallel-series".

Table 8.4 : Overall Parameter Matrix for Interconnections

Interconnection	Individual Parameter Matrix	Overall Parameter Matrix
Series-Series or Series	$[Z_a]$, $[Z_b]$	$[Z] = [Z_a] + [Z_b]$
Parallel-Parallel or Parallel	$[Y_a]$, $[Y_b]$	$[Y] = [Y_a] + [Y_b]$
Cascade or Tandem	$[T_a]$, $[T_b]$	$[T] = [T_a] \cdot [T_b]$
Series-Parallel	$[h_a]$, $[h_b]$	$[h] = [h_a] + [h_b]$
Parallel-Series	$[g_a]$, $[g_b]$	$[g] = [g_a] + [g_b]$

EXAMPLE 8.1 **For the network shown in figure 8.20. Calculate (i) Z, (ii) Y, (iii) T, (iv) T', (v) h, and (vi) g-parameters.**

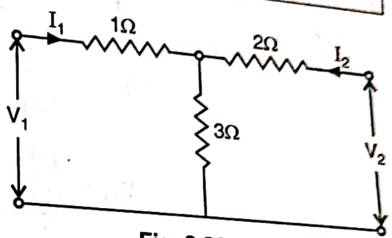


Fig. 8.20.

Solution : The loop equations become

$$V_1 = 1\cdot I_1 + 3\cdot(I_1 + I_2)$$

or $$V_1 = 4\,I_1 + 3\,I_2 \qquad \ldots(A)$$

and $$V_2 = 2I_2 + 3.\,(I_1 + I_2)$$

or $$V_2 = 3I_1 + 5\,I_2 \qquad \ldots(B)$$

(i) Z-parameters :

$$(V_1, V_2) = f(I_1, I_2)$$

Case I : When $I_2 = 0$, From equations (A) and (B)

$$V_1 = 4I_1, \quad \text{and } V_2 = 3I_1$$

Therefore, $$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{V_1}{\left(\frac{V_1}{4}\right)} = 4\,\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 3\,\Omega$$

Case II : When $I_1 = 0$, from equations (A) and (B)

$$V_1 = 3I_2 \text{ and } V_2 = 5I_2,$$

Therefore, $$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 3\,\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 5\,\Omega$$

(ii) Y-parameters :

$$(I_1, I_2) = f(V_1, V_2)$$

Case I : When $V_2 = 0$, from equations (A) and (B)

and $$3I_1 = -5I_2$$

$$V_1 = 4I_1 + 3I_2,$$

then $$V_1 = \frac{11}{5} I_1 \quad \text{and} \quad V_1 = -\frac{11}{3} I_2$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{5}{11}\ \mho$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{3}{11}\ \mho$$

Case II : When $V_1 = 0$, from equations (*A*) and (*B*)

$$4I_1 = -3I_2$$
$$V_2 = 3I_1 + 5I_2,$$

and

$$V_2 = -\frac{11}{3} I_1 \text{ and } V_2 = \frac{11}{4} I_2$$

then

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{3}{11}\ \mho$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{4}{11}\ \mho$$

(iii) ***T*-parameters :**

$$(V_1, I_1) = f(V_2, -I_2)$$

Case I : When $I_2 = 0$, from equations (*A*) and (*B*)

$$V_1 = 4I_1$$

and

$$V_2 = 3I_1$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{4}{3}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{3}\ \mho$$

Case II : When $V_2 = 0$, from equations (A) and (B)

$$3I_1 = -5\,I_2,\quad V_1 = 4\left(\frac{-5}{3}I_2\right) + 3\,I_2 = -\frac{11}{3}I_2$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{11}{3}\ \Omega$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{5}{3}$$

(iv) ***T'*-parameters :**

$$(V_2, I_2) = f(V_1, -I_1)$$

Case I : When $I_1 = 0$, from equation (*A*) and (*B*)

$$V_1 = 3\,I_2$$
$$V_2 = 5\,I_2$$

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0} = \frac{5}{3}$$

$$C' = \left.\frac{I_2}{V_1}\right|_{I_1=0} = \frac{1}{3}\ \mho$$

Case II : When $V_1 = 0$, from equations (*A*) and (*B*)

$$4\,I_1 = -3\,I_2$$

$$V_2 = 3\,I_1 + 5\left(-\frac{4}{3}I_1\right) = -\frac{11}{3}\,I_1$$

$$B' = \left.\frac{V_2}{-I_1}\right|_{V_1=0} = \frac{11}{3}\ \Omega$$

$$D' = \left.\frac{I_2}{-I_1}\right|_{V_1=0} = \frac{4}{3}$$

(*v*) ***h*-parameters :**

$$(V_1, I_2) = f(I_1, V_2)$$

Case I : When $V_2 = 0$, from equations (*A*) and (*B*)

$$3\,I_1 = -5\,I_2$$

$$V_1 = 4\,I_1 + 3\left(-\frac{3}{5}I_1\right) = \frac{11}{5}\,I_1$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{11}{5}\ \Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{3}{5}$$

Case II : when $I_1 = 0$, from equations (*A*) and (*B*)

$$V_1 = 3\,I_2,\ V_2 = 5\,I_2$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{3}{5}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{5}\mho$$

(*vi*) ***g*-parameters :** $(I_1, V_2) = f(V_1, I_2)$

Case I : when $I_2 = 0$, from equations (*A*) and (*B*)

$$V_1 = 4\,I_1,\ V_2 = 3\,I_1$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \frac{1}{4}\mho$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{3}{4}$$

Case II : When $V_1 = 0$, from equations (*A*) and (*B*)

$$4\,I_1 = -3\,I_2,\ V_2 = 3\left(-\frac{3}{4}I_2\right) + 5\,I_2 = \frac{11}{4}\,I_2$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = -\frac{3}{4}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{11}{4}\ \Omega$$

EXAMPLE 8.2 **Verify the above passive two port network as shown in figure 8.20 is reciprocal and not symmetrical in terms of all sets of parameters.**

Solution : *(i) For Reciprocity*

Since,

$$Z_{12} = Z_{21} = 3\ \Omega$$

$$Y_{12} = Y_{21} = -\frac{3}{11}\ \mho$$

$$\Delta T = AD - BC = \frac{4}{3}\cdot\frac{5}{3} - \frac{1}{3}\cdot\frac{11}{3} = 1$$

$$\Delta T' = A'D' - B'C' = \frac{5}{3}\cdot\frac{4}{3} - \frac{1}{3}\cdot\frac{11}{3} = 1$$

$$h_{12} = -h_{21} = \frac{3}{5}$$

$$g_{12} = -g_{21} = -\frac{3}{4}$$

Therefore, given network is *reciprocal.*

(ii) For Symmetry

Since,

$$Z_{11} \neq Z_{22} \quad \text{as} \quad 4 \neq 5$$

$$Y_{11} \neq Y_{22} \quad \text{as} \quad \frac{5}{11} \neq \frac{4}{11}$$

$$A \neq D \quad \text{as} \quad \frac{4}{3} \neq \frac{5}{3}$$

$$A' \neq D' \quad \text{as} \quad \frac{5}{3} \neq \frac{4}{3}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21} = \frac{11}{5}\cdot\frac{1}{5} - \frac{3}{5}\left(-\frac{3}{5}\right) = \frac{11}{25} + \frac{9}{25} \neq 1$$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21} = \frac{1}{4}\cdot\frac{11}{4} - \left(-\frac{3}{4}\right)\cdot\frac{3}{4} = \frac{11}{16} + \frac{9}{16} \neq 1$$

Therefore, given network is not *symmetrical.*

EXAMPLE 8.3 **For the T network shown in figure 8.21, obtain the Z-parameters.**

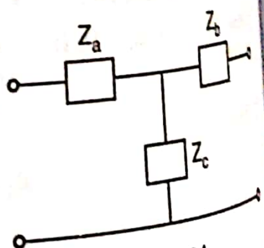


Fig. 8.21.

Solution : From example 8.1,

The *Z*-parameter matrix is given by

$$Z = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix}$$

EXAMPLE 8.4 **For the π-network of figure 8.22, obtain the *Y*-parameters (where Y_a, Y_b and Y_c are admittances).**

Solution : For *Y*-parameters :

$$(I_1, I_2) = f(V_1, V_2)$$

i.e.,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}, \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0},$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}, \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

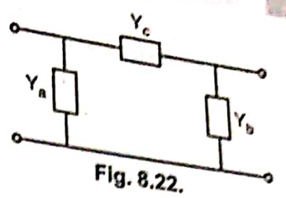


Fig. 8.22.

Case I: When $V_2 = 0$ [as shown in figure 8.23(*a*)].

$$V_1 = (I_1 + I_2)\frac{1}{Y_a}$$

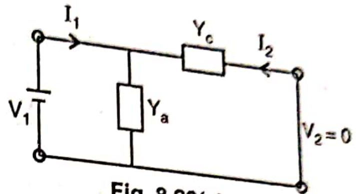


Fig. 8.23(a).

and, also $V_1 = \dfrac{-I_2}{Y_c}$

then, $V_1 = [I_1 + V_1(-Y_c)] \cdot \dfrac{1}{Y_a}$

or $(Y_a + Y_c)\, V_1 = I_1$

Therefore,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_a + Y_c$$

and

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\,Y_c$$

Case II: $V_1 = 0$ [as shown in figure 8.23(*b*)].

$$V_2 = (I_1 + I_2).\frac{1}{Y_b}$$

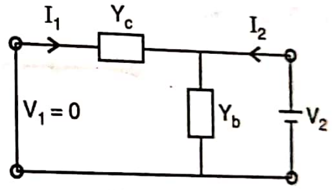


Fig. 8.23(b).

and also,

$$V_2 = -\frac{I_1}{Y_c}$$

then,

$$V_2 = (-V_2\, Y_c + I_2).\frac{1}{Y_b}$$

or

$$V_2\,(Y_b + Y_c) = I_2$$

Therefore, $Y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1=0} = -\,Y_c$

and $Y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1=0} = Y_b + Y_c$

Therefore, the Y-parameter matrix is given by

$$Y = \begin{bmatrix} Y_a + Y_c & -Y_c \\ -Y_c & Y_b + Y_c \end{bmatrix}$$

EXAMPLE 8.5 **Determine the Y-parameters for the network shown in figure 8.24.**

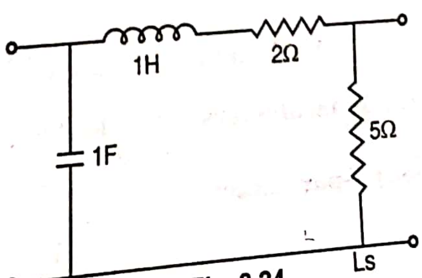


Fig. 8.24.

Solution : As we know Impedance; $Z \equiv R \equiv Ls \equiv \dfrac{1}{Cs}$

and Admittance; $\quad Y \equiv \dfrac{1}{R} \equiv \dfrac{1}{Ls} \equiv Cs$

Redrawing the network in s-domain in terms of admittance as shown in figure 8.25.

As compared to example 8.4,

$$Y_a = s$$

$$Y_b = \frac{1}{5}$$

$$Y_c = \frac{1}{s+2} \qquad \text{(Since } Z = s+2\text{)}$$

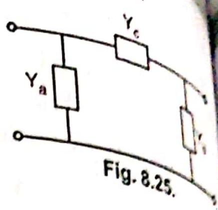


Fig. 8.25.

$$Y_{11} = Y_a + Y_c = s + \frac{1}{s+2} = \frac{s^2+2s+1}{s+2}$$

$$Y_{12} = Y_{21} = -Y_c = -\frac{1}{s+2}$$

$$Y_{22} = Y_b + Y_c = \frac{1}{5} + \frac{1}{s+2} = \frac{s+7}{5(s+2)}$$

EXAMPLE 8.6 **Calculate the Z-parameters for the network shown in figure 8.26.**

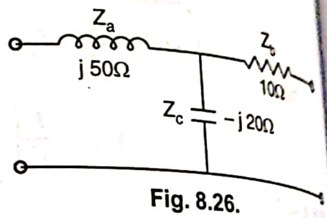


Fig. 8.26.

Solution : As we know, from example 8.3,

$$Z_{11} = Z_a + Z_c = j\,30\ \Omega$$

$$Z_{12} = Z_{21} = Z_c = -j\,20\ \Omega$$

and $\quad Z_{22} = Z_b + Z_c = (10 - j\,20)\ \Omega$

EXAMPLE 8.7 **Determine the (*i*) Z, (*ii*) Y, (*iii*) T, and (*iv*) h parameters of the network shown in figure 8.27.**

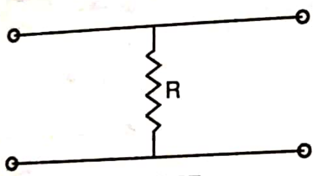


Fig. 8.27.

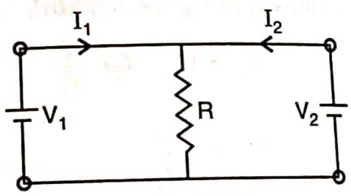


Fig. 8.28.

Solution : The loop equation becomes (from figure 8.28),

$$V_1 = (I_1 + I_2)\,R$$

$$V_2 = (I_1 + I_2)\,R$$

i.e., $\quad V_1 = V_2$

(*i*) Z-parameters :

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = R \ ; \ Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = R$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = R \ ; \ Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = R$$

So, the Z-parameters matrix is, $[Z] = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$

(*ii*) Y-parameters : They don't exist, since denominater of all Y-parameters is

$$V_1 = V_2 = 0$$

(*iii*) T-parameters :

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \ ; \ C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{R}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 0 \;;\; D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$$

So, the T-parameter matrix is,

$$[T] = \begin{bmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{bmatrix}$$

(iv) h-parameters :

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 0 \;;\; h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -1$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 1 \;;\; h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R}$$

So, the h-parameter matrix is,

$$[h] = \begin{bmatrix} 0 & 1 \\ -1 & 1/R \end{bmatrix}$$

EXAMPLE 8.8 Determine the (*i*) *Z*, (*ii*) *Y*, (*iii*) *T*, and (*iv*) *h*-parameters of the network shown in figure 8.29.

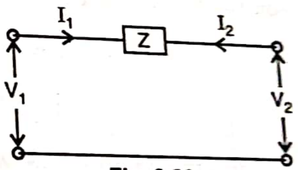


Fig. 8.29.

Solution : The loop equation becomes

$$V_1 = V_2 + I_1 Z$$

while, $I_1 = -I_2$

(i) Z-parameters : They don't exist, since currents I_1 and I_2 are not indepen-dent, *i.e.*, when $I_2 = 0$, I_1 is also equal to zero and vice-versa.

(ii) Y-parameters :

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{Z} \;;\; Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{Z}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{1}{Z} \;;\; Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{Z}$$

So, the Y-parameter matrix is,

$$[Y] = \begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix}$$

(iii) T-parameters :

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \;;\; C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = Z \;;\; D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$$

So, the T-parameter matrix is,

$$[T] = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

(iv) h-parameters :

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = Z \quad ; \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -1$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 1 \quad ; \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = 0$$

So, the h-parameter matrix is,

$$[h] = \begin{bmatrix} Z & 1 \\ -1 & 0 \end{bmatrix}$$

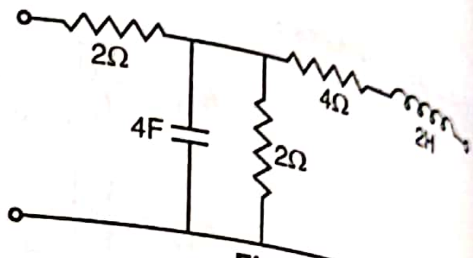


Fig. 8.30.

EXAMPLE 8.9 **Obtain the Z-parameters of the network shown in figure 8.30.**

Solution : First we convert given network into s-domain as shown in figure 8.31. Since $\frac{1}{4s}$ and 2Ω are in parallel, so equivalent impedance

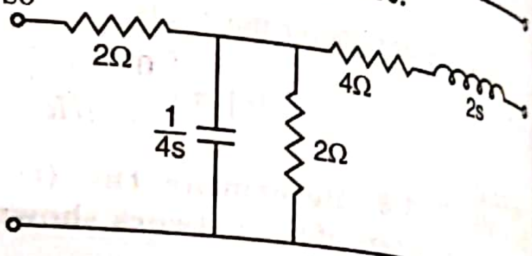


Fig. 8.31.

$$= \frac{\frac{1}{4s}\cdot 2}{\frac{1}{4s}+2} = \frac{2}{1+8s}$$

Now, redrawing the network as given in figure 8.32.

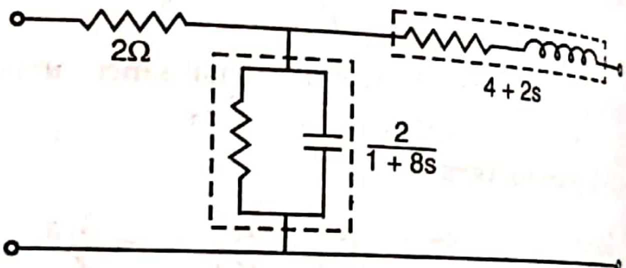


Fig. 8.32.

As, $Z_a = 2\,\Omega$

$Z_b = 4 + 2s$

$$Z_c = \frac{2}{1+8s}$$

Therefore,

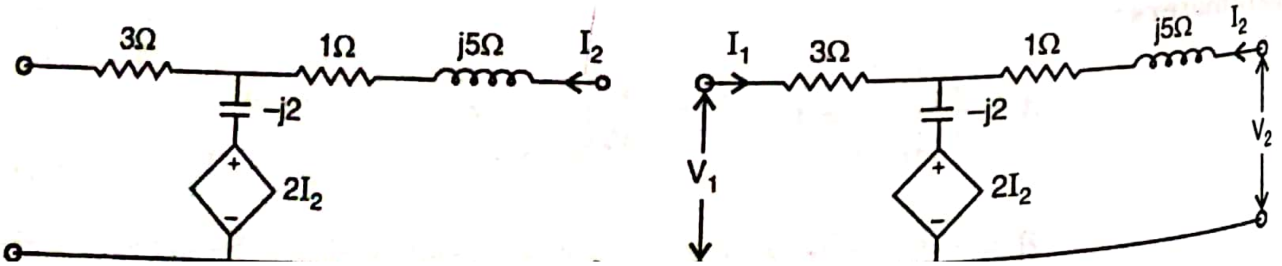
$$Z_{11} = Z_a + Z_c$$

$$= 2 + \frac{2}{1+8s} = \frac{4+16s}{1+8s}$$

$$Z_{12} = Z_{21} = Z_c = \frac{2}{1+8s}$$

$$Z_{22} = Z_b + Z_c = (4+2s) + \frac{2}{1+8s} = \frac{6+34s+16s^2}{1+8s}$$

EXAMPLE 8.10 **Calculate Z-parameters of the network shown in figure 8.33 and show that the network is neither reciprocal nor symmetrical.**



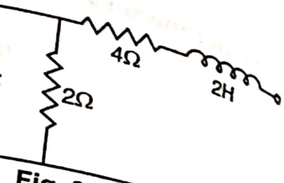


Fig. 8.30.

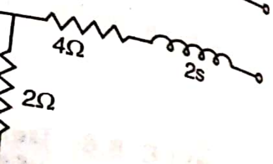
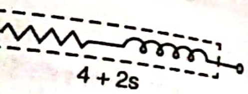
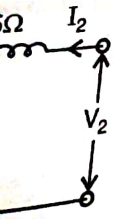


Fig. 8.31.



32.

33 and show that



...(A)

and, $V_2 = (1 + j5)\, I_2 - j2\cdot(I_1 + I_2) + 2I_2$

or $V_2 = -j2\cdot I_1 + (3 + j3)\, I_2$ ...(B)

Therefore, from equations (A) and (B);

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = (3 - j2)\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = (-j2)\ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = (2 - j2)\ \Omega$$

and,

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = (3 + j3)\ \Omega$$

Clearly, $Z_{12} \neq Z_{21}$, network is not reciprocal, and

Since $Z_{11} \neq Z_{22}$, network is not symmetrical.

EXAMPLE 8.11 **Determine the Z-parameters of network shown in figure 8.35.**

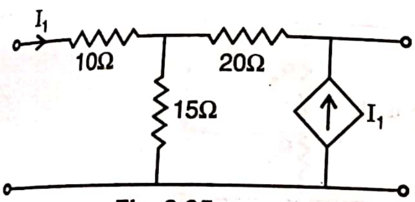


Fig. 8.35.

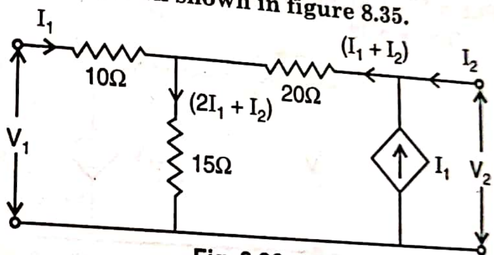


Fig. 8.36.

Solution : Redrawing the network as shown in figure 8.36. Then the loop equations become

$V_1 = 10\, I_1 + 15\, (2\, I_1 + I_2)$

or $V_1 = 40\, I_1 + 15\, I_2$ ...(A)

and $V_2 = 20\, (I_1 + I_2) + 15(2I_1 + I_2)$

or $V_2 = 50\, I_1 + 35 I_2$ ...(B)

Therefore, from equations (A) and (B);

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 40\ \Omega\ ;\ Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 50\ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 15\ \Omega\ ;\ Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 35\ \Omega$$

EXAMPLE 8.12 **Calculate the Z-parameters for the network shown in figure 8.37.**

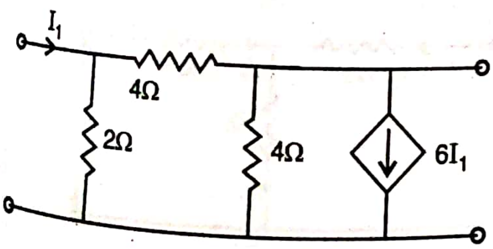


Fig. 8.37.

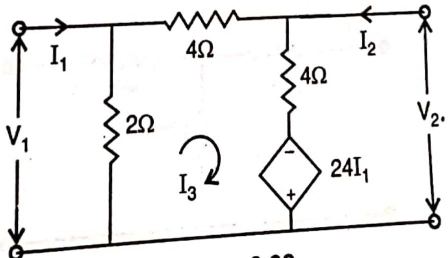


Fig. 8.38.

Solution : First, we convert current source in equivalent voltage source as shown in figure 8.38.

Then the loop equations are

$$V_1 = 2\,(I_1 - I_3)$$

$$24\,I_1 = 2\,(I_3 - I_1) + 4\,I_3 + 4\,(I_3 + I_2)$$

or $$26\,I_1 - 4\,I_2 - 10\,I_3 = 0 \qquad ...(A)$$

$$V_2 = 4\,(I_2 + I_3) - 24\,I_1$$

$$V_2 = -\,24\,I_1 + 4\,I_2 + 4\,I_3 \qquad ...(B)$$

from equation (*B*); $$I_3 = 2.6\,I_1 - 0.4\,I_2 \qquad ...(C)$$

putting the value of I_3 in equations (*A*) and (*C*)

$$V_1 = 2\,I_1 - 2\,(2.6\,I_1 - 0.4\,I_2)$$

or $$V_1 = -\,3.2\,I_1 + 0.8\,I_2 \qquad ...(i)$$

And $$V_2 = -\,24\,I_1 + 4\,I_2 + 4\,(2.6\,I_1 - 0.4\,I_2)$$

or $$V_2 = -\,13.6\,I_1 + 2.4\,I_2 \qquad ...(ii)$$

Therefore, from equations (*i*) and (*ii*);

$$Z_{11} = -\,3.2\,\Omega\ ;\ Z_{21} = -13.6\,\Omega$$

$$Z_{12} = 0.8\,\Omega\ ;\ Z_{22} = 2.4\,\Omega$$

EXAMPLE 8.13 Calculate h-parameters for a modal of a common–emitter connected transistor circuit of figure 8.39.

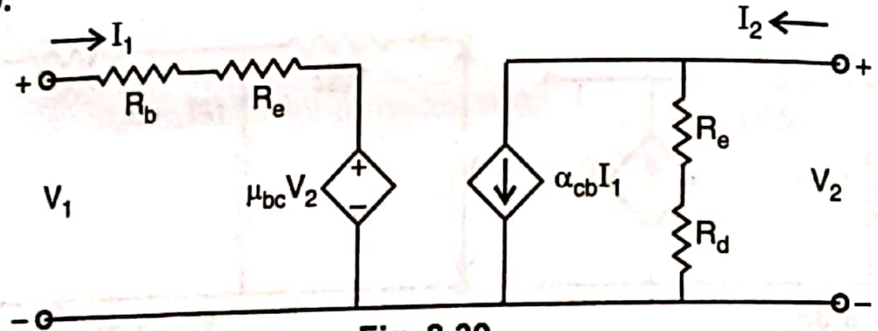


Fig. 8.39.

Solution : For the input loop :

$$V_1 = (R_b + R_e)\,I_1 + \mu_{bc}\,V_2 \qquad ...(a)$$

And for the output node :

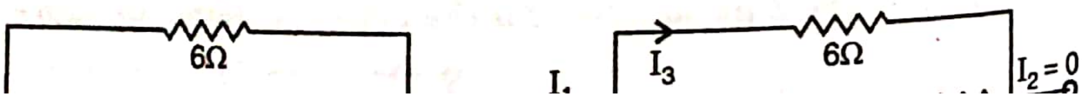
$$I_2 = \alpha_{cb}\,I_1 + \frac{V_2}{R_e + R_d} \qquad ...(b)$$

Comparing equations (*a*) and (*b*) with the *h*-parameters equations (9) and (10), we get

$$h_{11} = R_b + R_e\ ;\ h_{12} = \mu_{bc}$$

$$h_{21} = \alpha_{cb},\ h_{22} = \frac{1}{R_e + R_d}$$

EXAMPLE 8.14 Obtain the open circuit Z-parameters of the network shown in figure 8.40.



Solution : Case I : When output is open circuited (*i.e.*, $I_2 = 0$) as shown in figure 8.41.

$$V_1 = 2\,(I_1 - I_3) + 10\,I_1 = 12\,I_1 - 2\,I_3$$
$$V_2 = 4\,I_3 + 10\,I_1$$

and
$$I_3 = I_1 \times \frac{2}{2+(6+4)} = \frac{I_1}{6}$$

Therefore,
$$V_1 = 12\,I_1 - 2\cdot\frac{I_1}{6} = \frac{35}{3}\,I_1$$

and
$$V_2 = 4\cdot\frac{I_1}{6} + 10\,I_1 = \frac{32}{3}\,I_1$$

Hence,
$$Z_{11} = \frac{35}{3}\,\Omega$$

and
$$Z_{21} = \frac{32}{3}\,\Omega$$

Case II : When input is open circuited (*i.e.*, $I_1 = 0$) as shown in figure 8.42.

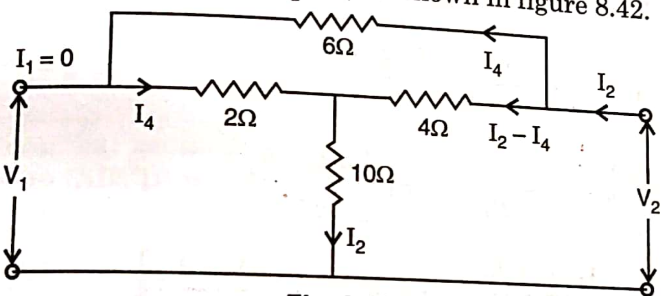


Fig. 8.42.

$$V_1 = 2\,I_4 + 10\,I_2$$
$$V_2 = 4\,(I_2 - I_4) + 10\,I_2 = 14\,I_2 - 4\,I_4$$

and
$$I_4 = I_2 \times \frac{4}{4+(6+2)} = \frac{I_2}{3}$$

Therefore,
$$V_1 = 2\cdot\frac{I_2}{3} + 10\,I_2 = \frac{32}{3}\,I_2$$

and
$$V_2 = 14\,I_2 - 4\,.\,\frac{I_2}{3} = \frac{38}{3}\,I_2$$

Hence,
$$Z_{12} = \frac{32}{3}\,\Omega \quad \text{and} \quad Z_{22} = \frac{38}{3}\,\Omega$$

EXAMPLE 8.15 Find the hybrid parameters for the network of figure 8.43 (which represents a transistor). *(U.P.T.U., 2002)*

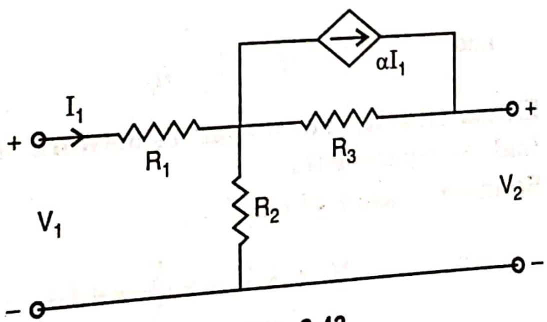


Fig. 8.43.

Solution : Case I : As we know

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

and
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

Therefore, short circuiting ... shown in figure 8.44. Let v be the node voltage at the junction of R_1, R_2 and R_3. Then

Applying KCL at that node;

$$I_1 = \frac{v}{R_2} + \frac{v}{R_3} + \alpha I_1$$

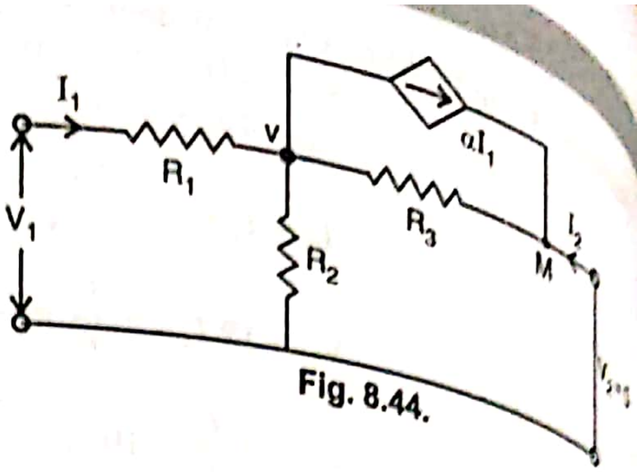


Fig. 8.44.

This gives

$$v = \frac{(1-\alpha)I_1R_2R_3}{R_2+R_3}$$

Applying KVL; $\quad V_1 = I_1\, R_1 + v$

or $$V_1 = I_1\left[R_1 + \frac{(1-\alpha)R_2R_3}{R_2+R_3}\right]$$

Now, again applying KCL at node M;

$$\frac{v}{R_3} + \alpha\, I_1 + I_2 = 0$$

$$I_2 = -\alpha\, I_1 - \frac{1}{R_3}\left[\frac{(1-\alpha)I_1R_2R_3}{R_2+R_3}\right]$$

or $$I_2 = I_1\left[-\alpha - \frac{(1-\alpha)R_2}{R_2+R_3}\right] = -\, I_1\left[\frac{\alpha R_3 + R_2}{R_2+R_3}\right]$$

So, $$h_{11} = \left[R_1 + \frac{(1-\alpha)R_2R_3}{R_2+R_3}\right] \text{ and } h_{21} = -\left[\frac{\alpha R_3 + R_2}{R_2+R_3}\right]$$

Case II : $$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Therefore, open circuiting the input port as shown in figure 8.45 (current source is open circuited since controlled current source is dependent on input current I_1 which is zero).

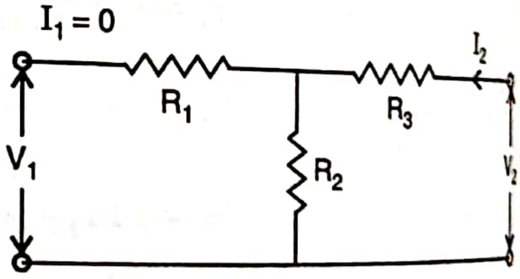


Fig. 8.45.

and $$V_2 = I_2\,(R_2 + R_3)$$
$$V_1 = I_2\, R_2$$

So $$h_{12} = \frac{R_2}{R_2+R_3}$$

and $$h_{22} = \frac{1}{R_2+R_3}$$

EXAMPLE 8.16 **For the lattice network of figure 8.46, find the Z-param...**

Case II : *When* $I_1 = 0$,

$$V_2 = 8 \cdot \frac{I_2}{2} + 4 \cdot \frac{I_2}{2} = 6\, I_2$$

$$V_1 = 8 \cdot \frac{I_2}{2} - 4 \cdot \frac{I_2}{2} = 2\, I_2$$

Therefore,

$$Z_{11} = Z_{22} = 6\ \Omega$$

and

$$Z_{12} = Z_{21} = 2\ \Omega$$

EXAMPLE 8.17 **Find the *T*-parameters for the above network of figure 8.46.**

Solution :

$$\Delta Z = Z_{11}\,Z_{22} - Z_{12}\,Z_{21} = 36 - 4 = 32$$

$$A = \frac{Z_{11}}{Z_{21}} = \frac{6}{2} = 3$$

$$B = \frac{\Delta Z}{Z_{21}} = \frac{32}{2} = 16\ \Omega$$

$$C = \frac{1}{Z_{21}} = \frac{1}{2}\ \Omega$$

$$D = \frac{Z_{22}}{Z_{21}} = 3$$

EXAMPLE 8.18 **Two identical sections of the network shown in figure 8.47 are cascaded. Calculate the transmission (ABCD) parameters of the resulting network.** (*I.P. Univ., 2001*)

Solution : From the figure 8.47.

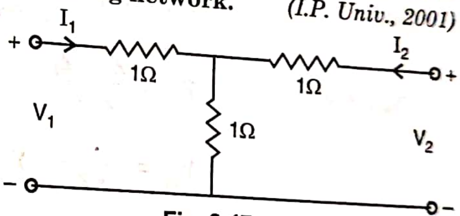


Fig. 8.47.

Loop equations are

$$V_1 = 1 \cdot I_1 + 1 \cdot (I_1 + I_2)$$

or

$$V_1 = 2\,I_1 + I_2 \qquad ...(A)$$

and

$$V_2 = 1 \cdot I_2 + 1 \cdot (I_2 + I_1)$$

or

$$V_2 = I_1 + 2\,I_2 \qquad ...(B)$$

Therefore,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 2, \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 1$$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = 3, \quad D = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = 2$$

Therefore, overall *T*-parameters of the cascaded network is given by

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

EXAMPLE 8.19 **For the network shown in figure 8.48, find**

(a) Impedance and admittance parameters

(b) Transmission and inverse transmission parameters

(c) Hybrid and inverse hybrid parameters. (*I.P. Univ., 2001*)

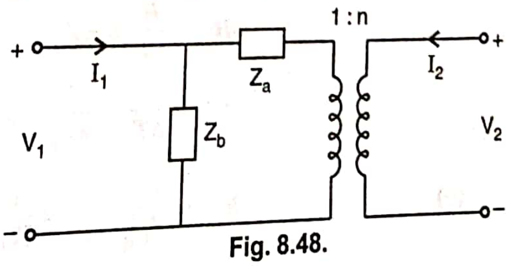


Fig. 8.48.

Solution : Let the V'_2 and I'_2 be the input voltage and input current of the transformer respectively, as shown in figure 8.49. Then the loop equations become.

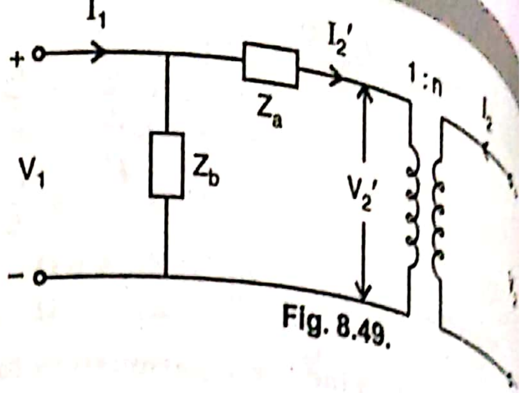


Fig. 8.49.

$$V_1 = (I_1 - I_2')Z_b$$
$$V_2 = nV_2' = n(V_1 - I_2' Z_a)$$

and $$I_2 = -\frac{1}{n} I_2'$$

or $$I_2' = -nI_2$$

Therefore, $$V_1 = (I_1 + nI_2) Z_b$$

or $$V_1 = Z_b I_1 + nZ_b I_2$$

and $$V_2 = n(V_1 + nI_2 Z_a)$$

or $$V_2 = n(I_1 Z_b + nI_2 Z_b + nI_2 Z_a)$$

or $$V_2 = nZ_b I_1 + n^2 (Z_a + Z_b) I_2$$

from equations (A) and (B) ;

(a) $$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_b \quad ; \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = nZ_b$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = nZ_b \quad ; \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = n^2 (Z_a + Z_b)$$

And, $$[Y] = [Z]^{-1} = \begin{bmatrix} Z_b & nZ_b \\ nZ_b & n^2(Z_a + Z_b) \end{bmatrix}^{-1}$$

Therefore, $$Y_{11} = \frac{Z_a + Z_b}{Z_a Z_b} \quad ; \quad Y_{12} = -\frac{1}{nZ_a}$$

$$Y_{21} = -\frac{1}{nZ_a}; \quad Y_{22} = \frac{1}{n^2 Z_a}$$

(b) $$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{n}, \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{nZ_b}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = nZ_a\,, \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = n\left(\frac{Z_a + Z_b}{Z_b}\right)$$

$(\Delta T = AD - BC = 1)$

And, $$A' = \frac{D}{\Delta T} = n\frac{(Z_a + Z_b)}{Z_b}$$

$$B' = \frac{B}{\Delta T} = nZ_a$$

$$C' = \frac{C}{\Delta T} = \frac{1}{nZ_b} \quad \text{and} \quad D' = \frac{A}{\Delta T} = \frac{1}{n}$$

(c) $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \left(\equiv \frac{1}{Y_{11}}\right) = \frac{Z_a Z_b}{Z_a + Z_b}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \left(\equiv -\frac{1}{D}\right) = -\frac{1}{n}\left(\frac{Z_b}{Z_a + Z_b}\right)$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \equiv \left(\frac{1}{A'}\right) = \frac{Z_b}{n(Z_a+Z_b)}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \equiv \left(\frac{1}{Z_{22}}\right) = \frac{1}{n^2(Z_a+Z_b)}$$

And,

$$[g] = [h]^{-1}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$= \frac{Z_aZ_b}{n^2(Z_a+Z_b)^2} + \frac{Z_b^2}{n^2(Z_a+Z_b)^2} = \frac{Z_b}{n^2(Z_a+Z_b)}$$

Therefore,

$$g_{11} = \frac{h_{22}}{\Delta h} = \frac{1}{Z_b} \quad ; \quad g_{21} = -\frac{h_{12}}{\Delta h} = n$$

$$g_{12} = -\frac{h_{21}}{\Delta h} = -n \quad ; \quad g_{22} = \frac{h_{11}}{\Delta h} = n^2 Z_a$$

8.12. OPEN CIRCUIT AND SHORT CIRCUIT IMPEDANCES

Open-circuit impedances can be written as

$$Z_{1o} = \left.\frac{V_1}{I_1}\right|_{I_2=0} (\equiv Z_{11}) \text{ and } Z_{2o} = \left.\frac{V_2}{I_2}\right|_{I_1=0} (\equiv Z_{22})$$

And short-circuit impedances can be written as

$$Z_{1s} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \left(\equiv \frac{1}{Y_{11}}\right)$$

and

$$Z_{2s} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \left(\equiv \frac{1}{Y_{22}}\right)$$

The first subscript of Z denotes the port at which the measurment is made while the second subscript denotes the condition at the other port. Thus Z_{1o} is an impedance at port 1 with port 2 open-circuited. Similarly, Z_{1s} denotes the impedance at port 1 with port 2 short-circuited.

8.12.1. Open Circuit and Short Circuit Impedances in Terms of T-Parameters

The T-parameter equations (5) and (6) for the two-port network are

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2 \quad \text{...(i)}$$

Here,

$$Z_{1o} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{A}{C} \quad \text{...(ii)}$$

$$Z_{2o} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{D}{C} \quad \text{...(iii)}$$

$$Z_{1s} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{B}{D} \quad \text{...(iv)}$$

and

$$Z_{2s} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{B}{A}$$

8.12.2. T-Parameters in Terms of Open Circuit and Short Circuit Impedances

Solving equations (*i*) to (*iv*) for *ABCD*, we have

$$A = \pm\sqrt{\frac{Z_{1o}}{Z_{2o}-Z_{2s}}}$$

$$B = \pm Z_{2s}\sqrt{\frac{Z_{1o}}{Z_{2o}-Z_{2s}}}$$

$$C = \pm\sqrt{\frac{1}{Z_{1o}(Z_{2o}-Z_{2s})}}$$

and

$$D = \pm\frac{Z_{2o}}{\sqrt{Z_{1o}(Z_{2o}-Z_{2s})}}$$

8.13. INPUT AND OUTPUT IMPEDANCES

8.13.1. Input Impedance

If a load impedance Z_L is connected to output port in figure 8.50, then $Z_{ip}\left(=\frac{V_1}{I_1}\right)$ is called the input impedance. The input impedance can be determined in terms of the two-port parameters and load impedance Z_L.

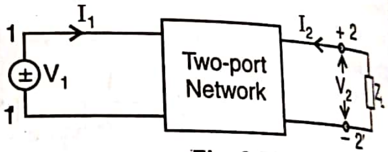


Fig. 8.50.

(i) Input impedance in terms of Z-parameters

The *Z*-parameter equations (1) and (2) are

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Also $\quad V_2 = -I_2 Z_L$

Hence, $\quad -I_2 Z_L = Z_{21}I_1 + Z_{22}I_2$

or $\quad I_2 = \dfrac{-Z_{21}}{Z_{22}+Z_L}I_1$

Now, $\quad V_1 = Z_{11}I_1 + Z_{12}\left[-\dfrac{Z_{21}}{Z_{22}+Z_L}\right]I_1$

Therefore, $\quad Z_{ip} = \dfrac{V_1}{I_1} = \dfrac{Z_{11}Z_{22}-Z_{12}Z_{21}+Z_{11}Z_L}{Z_{22}+Z_L}$

(ii) Input impedance in terms of T-parameters

The *T*-parameter equations (5) and (6) are

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Again, $\quad V_2 = -I_2 Z_L$

Hence, $\quad V_1 = A(-I_2 Z_L) - BI_2 = -(AZ_L+B)I_2$

and $\quad I_1 = C(-I_2 Z_L) - DI_2 = -(CZ_L+D)I_2$

Therefore,

$$Z_{ip} = \frac{V_1}{I_1} = \frac{AZ_L+B}{CZ_L+D}$$

(ii) Input impedance in terms of h-parameters

The h-parameter equations (9) and (10) are

$$V_1 = h_{11}\, I_1 + h_{12}\, V_2$$

$$I_2 = h_{21}\, I_1 + h_{22}\, V_2$$

Again,

$$V_2 = -\, I_2\, Z_L$$

or

$$V_2 = -\, Z_L(h_{21}\, I_1 + h_{22}\, V_2)$$

$$V_2 = -\frac{h_{21}Z_L}{1+h_{22}Z_L}\, I_1$$

Hence,

$$V_1 = h_{11}I_1 + h_{12}\left[-\frac{h_{21}Z_L}{1+h_{22}Z_L}\right] I_1$$

Therefore,

$$Z_{ip} = \frac{V_1}{I_1} = h_{11} - \frac{h_{12}h_{21}Z_L}{1+h_{22}Z_L}$$

8.13.2. Output Impedance

As in this case of input impedance, if a load impedance Z_L is connected to input port in figure 8.51, then, $Z_{op}\left(=\frac{V_2}{I_2}\right)$ is termed the output impedance. The output impedance can also be determined in terms of the two-port parameters and load impedance Z_L.

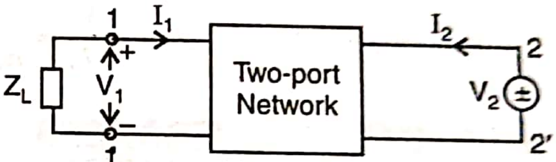


Fig. 8.51.

(i) Output impedance in terms of Z-parameters

The Z-parameter equations (1) and (2) are

$$V_1 = Z_{11}\, I_1 + Z_{12}\, I_2$$

$$V_2 = Z_{21}\, I_1 + Z_{22}\, I_2$$

Also,

$$V_1 = -\, I_1\, Z_L$$

or,

$$I_1 = -\frac{(Z_{11}I_1 + Z_{12}I_2)}{Z_L}$$

or

$$I_1 = -\frac{Z_{12}}{Z_{11}+Z_L}\, I_2$$

Hence,

$$V_2 = Z_{21}\left[-\frac{Z_{12}}{Z_{11}+Z_L}\right] I_2 + Z_{22}I_2$$

After simplification,

$$Z_{op} = \frac{V_2}{I_2} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21} + Z_{22}Z_L}{\ldots}$$

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Therefore, $$Z_{op} = \frac{V_2}{I_2} = \frac{DZ_L + B}{CZ_L + A}$$

(iii) Output impedance in terms of h-parameters

The h-parameter equations (9) and (10) are

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

And again, $$V_1 = -I_1 Z_L$$

or, $$h_{11} I_1 + h_{12} V_2 = -I_1 Z_L$$

or, $$I_1 = -\frac{h_{12}}{(h_{11} + Z_L)} V_2$$

Hence, $$I_2 = h_{21}\left[-\frac{h_{12}}{h_{11} + Z_L}\right] V_2 + h_{22} V_2$$

Therefore, $$Z_{op} = \frac{V_2}{I_2} = \frac{h_{11} + Z_L}{h_{11}h_{22} - h_{12}h_{21} + h_{22}Z_L}$$

8.14. IMAGE IMPEDANCES

In a two-port network, if the impedance at input port with impedance Z_{i2} connected across output port be Z_{i1} as shown in figure 8.52(*a*) and the impedance at output port with impedance Z_{i1} connected across input port be Z_{i2}, as shown in figure 8.52(*b*) then Z_{i1} and Z_{i2}, are termed as the *image impedances* of the network.

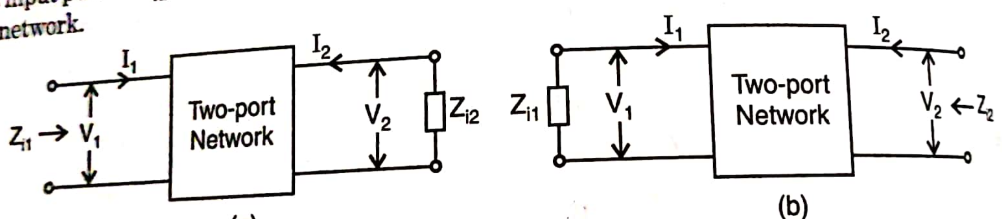


Fig. 8.52. Image impedances

For symmetrical network, image impedances are equal to each other, i.e. $Z_{i1} = Z_{i2}$ and is called the *characteristic* or *iterative impedance* Z_c.

$Z_{i1} = \frac{V_1}{I_1}$ (driving point impedance at input port)

Similarly, $Z_{i2} = \frac{V_2}{I_2}$ (driving point impedance at output port)

8.14.1. Image Impedances in Terms of Input and Output Impedances

$$Z_{i1} = Z_{ip} \text{ and } Z_{i2} = Z_{op}$$

8.14.2. Image Impedances in Terms of T-parameters

$$Z_{i1} = Z_{ip}$$

or, $$Z_{i1} = \frac{AZ_{i2} + B}{CZ_{i2} + D}$$

And, $$Z_{i2} = Z_{op}$$

or, $$Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + A}$$

Solving equations (*A*) and (*B*

$Z_{i1} =$

8.14.3. Image Impedances

$Z_{i1} = \sqrt{\ldots}$

And, $Z_{i2} = \sqrt{\ldots}$

Note :

The image impedance done ... which we shall get from th...

The *T*-parameter equatio...

$V_1 = \ldots$

$I_1 = \ldots$

and

Since $Z_{i2} = \ldots$

$V_1 = \ldots$

and $I_1 = \ldots$

Now, $Z_{i2} = \ldots$

Therefore, $\frac{V_1}{V_2} \ldots$

and $\frac{I_1}{I_2} \ldots$

Now, $-\frac{V_1}{V_2} \cdot \frac{I_1}{I_2} \ldots$

or $\sqrt{-\frac{V_1 I_1}{V_2 I_2}} \ldots$

Let $\sqrt{AD \ldots}$

then $\sqrt{AD - \ldots}$

Therefore, $\sqrt{-\frac{V_1}{V_2} \cdot \ldots}$

Solving equations (A) and (B) we have

$$Z_{i1} = \sqrt{\frac{AB}{CD}} \text{ and } Z_{i2} = \sqrt{\frac{BD}{AC}}$$

8.14.3. Image Impedances in Terms of Open-circuit and Short-circuit Impedances

$$Z_{i1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{A}{C}\cdot\frac{B}{D}} = \sqrt{Z_{1o}\cdot Z_{1s}}$$

And,

$$Z_{i2} = \sqrt{\frac{BD}{AC}} = \sqrt{\frac{D}{C}\cdot\frac{B}{A}} = \sqrt{Z_{2o}\cdot Z_{2s}}$$

Note :

The image impedance donot completely define a two-port network. We need another parameter which we shall get from the voltage and current ratios as follows:

The *T*-parameter equations (5) and (6) are

$$V_1 = AV_2 - BI_2$$

and

$$I_1 = CV_2 - DI_2$$

Since

$$Z_{i2} = -\frac{V_2}{I_2}, \text{ then above equations become}$$

$$V_1 = \left(A + \frac{B}{Z_{i2}}\right)V_2 \quad \text{or} \quad \frac{V_1}{V_2} = A + \frac{B}{Z_{i2}}$$

and

$$I_1 = -(CZ_{i2} + D)I_2 \quad \text{or} \quad \frac{I_1}{I_2} = -(CZ_{i2} + D)$$

Now,

$$Z_{i2} = \sqrt{\frac{BD}{AC}}$$

Therefore,

$$\frac{V_1}{V_2} = A + B\sqrt{\frac{AC}{BD}} = A + \frac{\sqrt{ABCD}}{D}$$

and

$$\frac{I_1}{I_2} = -\left(C\sqrt{\frac{BD}{AC}} + D\right) = -\left(D + \frac{\sqrt{ABCD}}{A}\right)$$

Now,

$$-\frac{V_1}{V_2}\cdot\frac{I_1}{I_2} = \left(A + \frac{\sqrt{ABCD}}{D}\right)\left(D + \frac{\sqrt{ABCD}}{A}\right)$$

$$= AD + 2\sqrt{ABCD} + BC = \left(\sqrt{AD} + \sqrt{BC}\right)^2$$

or

$$\sqrt{-\frac{V_1I_1}{V_2I_2}} = \sqrt{AD} + \sqrt{BC} = \sqrt{AD} + \sqrt{AD-1} \quad [\text{Since } AD - BC = 1]$$

Let

$$\sqrt{AD} = \cos h\theta$$

then

$$\sqrt{AD-1} = \sin h\theta$$

Therefore,

$$\sqrt{-\frac{V_1}{V_2}\cdot\frac{I_1}{I_2}} = \cos h\theta + \sin h\theta = e^{\theta}$$

or $$\theta = \ln\sqrt{\frac{-V_1 I_1}{V_2 I_2}} = \frac{1}{2}\ln\left(\frac{-V_1 I_1}{V_2 I_2}\right)$$

where θ is called the image transfer constant.

Since $V_1 = Z_{i1}I_1$ and $V_2 = -Z_{i2}I_2$, the image transfer constant may be written as

$$\theta = \frac{1}{2}\ln\left[\frac{-(Z_{i1}I_1)I_1}{(-Z_{i2}\,I_2)I_2}\right] = \frac{1}{2}\ln\left[\frac{Z_{i1}\,I_1^2}{Z_{i2}\,I_2^2}\right]$$

$$= \frac{1}{2}\left[\ln\left(\frac{Z_{i1}}{Z_{i2}}\right) + \ln\left(\frac{I_1}{I_2}\right)^2\right] = \frac{1}{2}\ln\left(\frac{Z_{i1}}{Z_{i2}}\right) + \ln\frac{I_1}{I_2}$$

From above, other forms are

$$\theta = \cos h^{-1}\sqrt{AD}$$

$$\theta = \sin h^{-1}\sqrt{BC} = \sin h^{-1}\sqrt{AD-1}$$

$$\theta = \tan h^{-1}\sqrt{\frac{BC}{AD}} = \tan h^{-1}\sqrt{\frac{Z_{1s}}{Z_{1o}}} = \tan h^{-1}\sqrt{\frac{Z_{2s}}{Z_{2o}}}$$

Let $$m = \tanh\,\theta = \sqrt{\frac{Z_{1s}}{Z_{1o}}} = \frac{e^{\theta}-e^{-\theta}}{e^{\theta}+e^{-\theta}} = \frac{e^{2\theta}-1}{e^{2\theta}+1}$$

So that $$e^{2\theta} = \frac{1+m}{1-m} = re^{j\phi}$$

where r and ϕ may be computed from the knowledge of Z_{1s} and Z_{1o}.

Hence, $$\theta = \frac{1}{2}\ln(r) + j\left(\frac{\phi}{2}+n\,\pi\right)\ ;\ n = 1,2,3,\ldots = \alpha + j\beta$$

It may thus be seen that, in general, θ is a complex quantity. The real part of θ, *i.e.*, $\alpha = \frac{1}{2}\ln(r)$, is always positive and termed the *image attenuation constant*, while the imaginary part, *i.e.*, $\beta = \left(\frac{\phi}{2}+n\pi\right)$, is termed the *phase constant*.

EXAMPLE 8.20 **Determine the open-circuit and short-circuit impedances of the network shown in figure 8.53(a).**

Rewriting in matrix form,

$$\begin{bmatrix} 6 & 4 & -2 \\ 4 & 10 & 6 \\ -2 & 6 & 16 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix}$$

Open-circuiting output port, *i.e.*, $I_2 = 0$, by Cramer's rule,

$$Z_{1o} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{23}{4}\,\Omega$$

Open-circuiting input port, *i.e.*, $I_1 = 0$, by Cramer's rule,

$$Z_{2o} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{31}{4}\,\Omega$$

Similarly short-circuiting output and input ports, we have

$$Z_{1s} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{88}{31}\,\Omega$$

and

$$Z_{2s} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{88}{23}\,\Omega$$

.15. T-π TRANSFORMATION

nd π networks are the two networks which are used to represent e equivalence of transmission lines, filters etc.

(i) The *T*-network is represented by the model shown in figure 54(a). For which the *Z*-parameters are

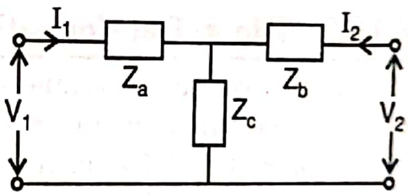


Fig. 8.54(a).

$$Z_{11} = Z_a + Z_c \qquad \text{...(A)}$$

$$Z_{12} = Z_{21} = Z_c \qquad \text{...(B)}$$

and

$$Z_{22} = Z_b + Z_c \qquad \text{...(C)}$$

From equations (*A*) to (*C*) the elements of *T*-network, Z_a, Z_b and Z_c can be expressed in terms of parameters.

$$\boldsymbol{Z_a = Z_{11} - Z_{12}}$$

$$\boldsymbol{Z_b = Z_{22} - Z_{12}}$$

$$\boldsymbol{Z_c = Z_{12} = Z_{21}}$$

Therefore, if *Z*-parameters of a network are given, then the equivalent *T*-network model can be structed.

(ii) The π-network is represented by the model shown in figure 54(b). For which the *Y*-parameter are

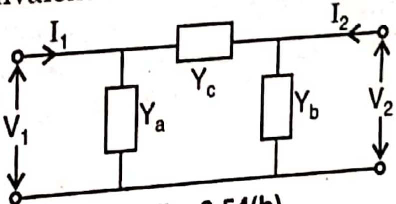


Fig. 8.54(b).

$$Y_{11} = Y_a + Y_c \qquad \text{...(D)}$$

$$Y_{12} = Y_{21} = -Y_c \qquad \text{...(E)}$$

$$Y_{22} = Y_b + Y_c \qquad \text{...(F)}$$

Using equation (*D*) to (*E*), we can express Y_a, Y_b and Y_c in terms of *Y*-parameters. Thus

$$\boldsymbol{Y_a = Y_{11} + Y_{12}}$$

$$\boldsymbol{Y_b = Y_{22} + Y_{12}}$$

$$\boldsymbol{Y_c = -Y_{12} = -Y_{21}}$$

Therefore, if Y-parameters of a network are given then it is convenient to construct an equivalent π model rather than an equivalent T-model.

8.15.1. π to T Transformation

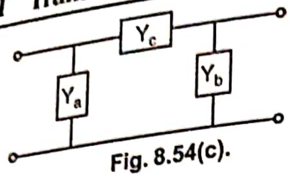


Fig. 8.54(c).

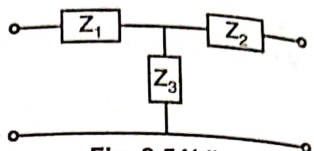


Fig. 8.54(d).

Admittances Y_a, Y_b and Y_c of the figure 8.54(c) are known and it is required to calculate equivalent impedances Z_1, Z_2 and Z_3 of the figure 8.54(d).

Using delta to star conversion from article 7.2.1, we have

$$Z_1 = \frac{\frac{1}{Y_a} \cdot \frac{1}{Y_c}}{\frac{1}{Y_a} + \frac{1}{Y_b} + \frac{1}{Y_c}} = \frac{Y_b}{Y_a Y_b + Y_b Y_c + Y_c Y_a}$$

Similarly,
$$Z_2 = \frac{Y_a}{Y_a Y_b + Y_b Y_c + Y_c Y_a}$$

and
$$Z_3 = \frac{Y_c}{Y_a Y_b + Y_b Y_c + Y_c Y_a}$$

8.15.2. T to π Transformation

In this case, the impedances Z_1, Z_2 and Z_3 of the figure 8.54(d) are known and it is required to calculate equivalent admittances Y_a, Y_b and Y_c of the figure 8.54(c).

Using star to delta conversion from article 7.2.2, we have

$$\frac{1}{Y_a} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

or
$$Y_a = \frac{Z_2}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

Similarly,
$$Y_b = \frac{Z_1}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

and
$$Y_c = \frac{Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

EXAMPLE 8.21 **The Z-parameters of a two-port network are $Z_{11} = 10\Omega$, $Z_{22} = 20\Omega$, $Z_{12} = Z_{21} = 5\Omega$. Find its equivalent T-network.**

Solution : Using relationships developed in article 8.15, we have

$$Z_a = Z_{11} - Z_{12} = 5\Omega$$
$$Z_c = Z_{12} = 5\Omega$$
$$Z_b = Z_{22} - Z_{12} = 15\Omega$$

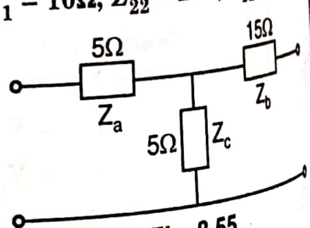


Fig. 8.55.

The equivalent T-network is shown in figure 8.55.

EXAMPLE 8.22 **A two port network has the following impedances :**

$$Z_{1o} = (250 + j100)\Omega$$
$$Z_{2o} = 200\Omega$$
$$Z_{1s} = (400 + j300)\Omega.$$

Determine the equivalent T-network parameters.

...tion : Referring to figure 8.56, we observe

$$Z_{1o} = Z_a + Z_c = 250 + j100 \quad ...(i)$$

$$Z_{2o} = Z_b + Z_c = 200 \quad ...(ii)$$

and

$$Z_{1s} = Z_a + \frac{Z_b \cdot Z_c}{Z_b + Z_c} = 400 + j300 \quad ...(iii)$$

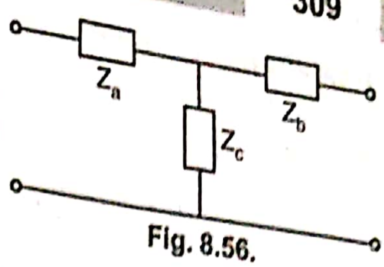


Fig. 8.56.

Substraction (i) from (iii),

$$\frac{Z_b \cdot Z_c}{Z_b + Z_c} - Z_c = 150 + j200$$

or

$$Z_b Z_c - Z_b Z_c - Z_c^2 = (Z_b + Z_c)(150 + j200)$$

or

$$Z_c^2 = -200(150 + j200) \text{ (from equation ii)}$$

$$= 10^4(-3 - j4) = 10^4(1 - j2)^2$$

$$Z_c = \mathbf{(100 - j200)\Omega}$$

Therefore,

$$Z_a = \mathbf{(150 + j300)\Omega}$$

and

$$Z_b = \mathbf{(100 + j200)\Omega}$$

...AMPLE 8.23 Obtain the image impedance for a *T*-network as shown in figure 8.57, for ...ich the resistance of three arms are equal to 3 Ω.

...tion : We obtain the Z-parameters of a T-network as.

$$Z_{11} = Z_{22} = 6\,\Omega \text{ and } Z_{12} = Z_{21} = 3\,\Omega.$$

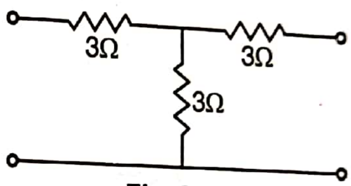


Fig. 8.57.

Hence,

$$A = \frac{Z_{11}}{Z_{21}} = 2$$

$$B = \frac{\Delta Z}{Z_{21}} = \frac{36-9}{3} = 9\Omega$$

$$C = \frac{1}{Z_{21}} = \frac{1}{3}\mho$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{6}{3} = 2$$

The image impedance is then

$$Z_{i1} = \sqrt{\frac{AB}{CD}}$$

or

$$Z_{i2} = \sqrt{\frac{BD}{AC}} = \sqrt{\frac{18}{(2/3)}} = \sqrt{27} = 5.2\Omega$$

To check.

Let the terminating impedance at output be 5.2 Ω. Then

$$Z_{i1} = 3 + [3 \,||\, (3 + 5.2)]$$

$$= 3 + \frac{3 \times 8.2}{11.2} = 3 + \frac{24.6}{11.2} = 5.2\Omega$$

...AMPLE 8.24 Starting from the definition of the parameters h_{11} and h_{21}, establish their ...ation to Z and Y parameters.

(I.P. Univ., 2000)

Specifically show that $h_{11} = \dfrac{1}{Y_{11}}$ and $h_{22} = \dfrac{1}{Z_{22}}$

...tion : Refer articles 8.10.5 (*i*) and 8.10.5 (*ii*).

EXAMPLE 8.25 Find short circuit parameters of the circuit shown in figure 8.58. (U.P.T.U., 20..)

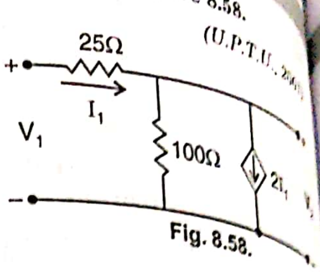


Fig. 8.58.

Solution : Applying KCL,

$$I_1 + I_2 = \frac{V_2}{100} + 2I_1 \qquad \text{....(1)}$$

or

$$I_1 = -\frac{V_2}{100} + I_2 \qquad \text{...(2)}$$

Also

$$I_1 = \frac{V_1 - V_2}{25}$$

or

$$I_1 = \frac{1}{25}V_1 - \frac{1}{25}V_2 \qquad \text{...(2a)}$$

From equations (1) and (2), we have

$$\frac{V_1 - V_2}{25} = -\frac{V_2}{100} + I_2$$

or

$$I_2 = \frac{1}{25}V_1 - \frac{3}{100}V_2 \qquad \text{...(3)}$$

Comparing equations (2*a*) and (3) with the *Y*-parameter equations (3) and (4), we get

$$Y_{11} = \frac{1}{25}\,℧ \quad ; \quad Y_{21} = \frac{1}{25}\,℧$$

$$Y_{12} = -\frac{1}{25}\,℧ \quad ; \quad Y_{22} = -\frac{3}{100}\,℧$$

EXAMPLE 8.26 Determine transmission parameters of a T-network shown in figure 8.59, considering three sections as shown in the figure 8.59, assuming connected in cascade manner.(*U.P.T.U., 2001*)

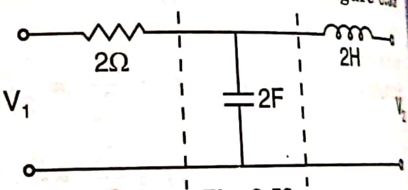


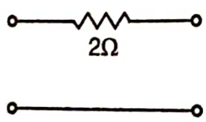
Fig. 8.59.

Solution : From figure 8.59, we can say that the three different two port networks are cascaded. So the overall transmission parameters of the network shown in figure 8.59 is given as

$$T = T_a \cdot T_b \cdot T_c$$

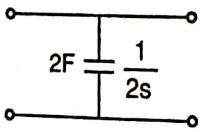
where T_a, T_b and T_c are the *T*-parameter matrices of the networks N_a, N_b and N_c respectively.

For N_a



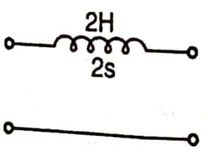
$$T_a = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

For N_a



$$T_b = \begin{bmatrix} 1 & 0 \\ 2s & 1 \end{bmatrix}$$

ForN_b



$$T_c = \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix}$$

Therefore, the required transmission parameters of the given network are

$$T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2s & 1 \end{bmatrix}\begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+4s & 2 \\ 2s & 1 \end{bmatrix}\begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4s & 2s(1+4s)+2 \\ 2s & 2s(2s)+1 \end{bmatrix} = \begin{bmatrix} 4s+1 & 2(4s^2+s+1) \\ 2s & 4s^2+1 \end{bmatrix}$$

EXAMPLE 8.27 **Find hybrid parameters of the network shown in figure 8.60.** *(U.P.T.U., 2001)*

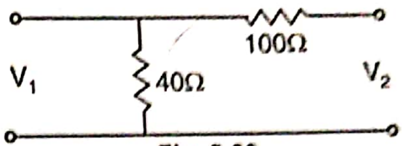


Fig. 8.60.

Solution : The loop equations become

$$V_1 = 40(I_1 + I_2) \quad \text{...(1)}$$

$$V_2 = 100I_2 + 40(I_1 + I_2) \quad \text{...(2)}$$

or

$$V_2 = 40\,I_1 + 140\,I_2$$

and

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{40\left(I_1 - \frac{2}{7}I_1\right)}{I_1} = \frac{200}{7}\,\Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-\frac{2}{7}I_1}{I_1} = -\frac{2}{7}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{40I_2}{140\,I_2} = \frac{2}{7}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{I_2}{140I_2} = \frac{1}{140}\,\mho$$

EXAMPLE 8.28 **Determine transmission parameters of the network shown in figure 8.61, using the concept of interconnection of two two-port networks N_1 and N_2 in cascade :** *(U.P.T.U., 2002)*

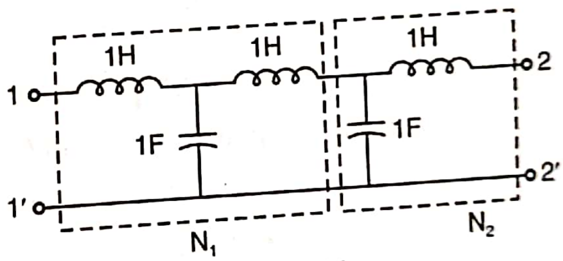


Fig. 8.61.

Solution : For network N_1, as shown in figure 8.62(*a*),

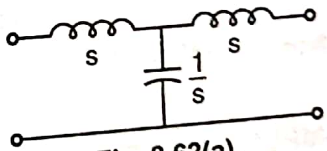


Fig. 8.62(a).

$$A_a = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 + s^2$$

$$B_a = \left.\frac{V_1}{-I_2}\right|_{I_2=0} = 2\,s + s^3$$

$$C_a = \left.\frac{I_1}{V_2}\right|_{I_2=0} = s$$

$$D_a = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1 + s^2$$

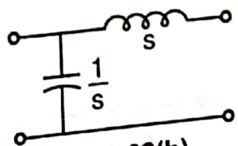


Fig. 8.62(b).

For network N_2, as shown in figure 8.62(*b*),

$$A_b = 1$$
$$B_b = s$$
$$C_b = s$$
$$D_b = 1 + s^2$$

Therefore, transmission parameters of the network shown in figure 8.61 is given by

$$[T] = [T_a]\,[T_b]$$

$$\text{or} \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+s^2 & 2s+s^3 \\ s & 1+s^2 \end{bmatrix} \begin{bmatrix} 1 & s \\ s & 1+s^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+s^2+2s^2+s^4 & s+s^3+2s+2s^3+s^3+s^5 \\ s+s+s^3 & s^2+1+2s^2+s^4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3s^2+s^4 & 3s+4s^3+s^5 \\ 2s+s^3 & 1+3s^2+s^4 \end{bmatrix}$$

EXAMPLE 8.29 **Find the Y and Z parameters for the network of figure 8.63.**

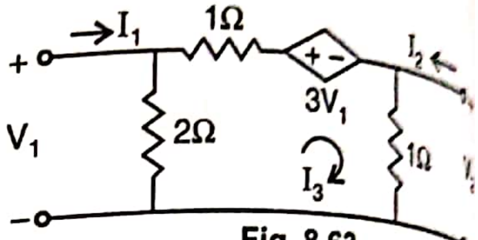


Fig. 8.63.

Solution : Let I_3 be the current in middle Loop.

Applying KVL,

$$V_1 = 2(I_1 - I_3) \quad \text{...(i)}$$

$$2(I_3 - I_1) + 1.I_3 + 3V_1 + 1.(I_3 + I_2) = 0 \quad \text{...(ii)}$$

or $$4I_3 = 2I_1 - I_2 - 3V_1 \quad \text{...(iii)}$$

and $$V_2 = 1.(I_2 + I_3)$$

From equations (*i*), (*ii*) and (*iii*), we have

$$V_1 = 2I_1 - 2\left(\frac{I_1}{2} - \frac{I_2}{4} - \frac{3V_1}{4}\right) = -2I_1 - I_2 \quad \text{...(iv)}$$

and $$V_2 = I_2 + \frac{1}{4}(2I_1 - I_2 - 3V_1)$$

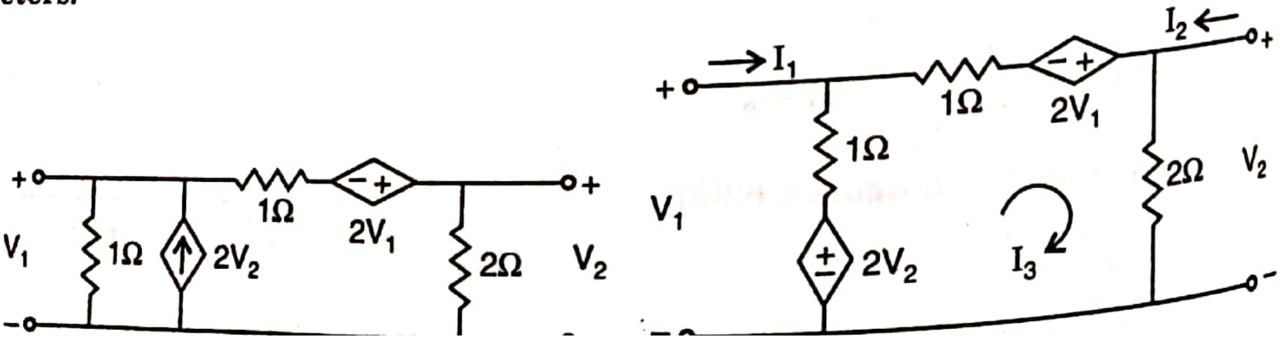
$$V_2 = I_2 + \frac{1}{4}(2I_1 - I_2 + 6I_1 + 3I_2) = 2I_1 + \frac{3}{2} I_2 \quad \text{...(v)}$$

Therefore, from equations (*iv*) and (*v*), we have

$$Z = \begin{bmatrix} -2 & -1 \\ 2 & \frac{3}{2} \end{bmatrix}$$

Now, $$[Y] = [Z]^{-1} = \begin{bmatrix} -2 & -1 \\ 2 & \frac{3}{2} \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{3}{2} & -1 \\ 2 & 2 \end{bmatrix}$$

EXAMPLE 8.30 **The network of figure 8.64(*a*) contains both a dependent current source and a dependent voltage source. For the element values given, determine the Y and Z parameters.**



Applying KVL,

$$V_1 = 1.(I_1 - I_3) + 2V_2$$

or $$V_1 = I_1 - I_3 + 2V_2$$

or $$I_3 = I_1 + 2V_2 - V_1 \qquad ...(i)$$

And $$1.I_3 - 2V_1 + 2(I_3 + I_2) - 2V_2 + 1\,(I_3 - I_1) = 0$$

or $$2V_1 + 2V_2 = -I_1 + 2I_2 + 4I_3 \qquad ...(ii)$$

or $$V_2 = 2(I_2 + I_3) \qquad ...(iii)$$

Putting the value of I_3 from equation (*i*) into equations (*ii*) and (*iii*), we have

$$2V_1 + 2V_2 = -I_1 + 2I_2 + 4(I_1 + 2V_2 - V_1)$$

or $$6V_1 - 6V_2 = 3I_1 + 2I_2 \qquad ...(iv)$$

And $$V_2 = 2(I_2 + I_1 + 2V_2 - V1)$$

or $$2V_1 - 3V_2 = 2I_1 + 2I_2 \qquad ...(v)$$

For Y-Parameters :

Subtracting equation (*v*) from equation (*vi*), we have

$$I_1 = 4V_1 - 3V_2 \qquad ...(vi)$$

Now, from equations (*v*) and (*vi*)

$$2V_1 - 3V_2 = 2(4V_1 - 3V_2) + 2I_2$$

or $$I_2 = -3V_1 + \frac{3}{2}V_2 \qquad ...(vii)$$

Therefore, from equations (*vi*) and (*vii*), the required *Y*-parameters are

$$Y = \begin{bmatrix} 4 & -3 \\ -3 & \frac{3}{2} \end{bmatrix}$$

For Z-parameters :

From equations (*iv*) and (*v*)

$$6V_1 - 2(2V_1 - 2I_1 - 2I_2) = 3I_1 + 2I_2$$

or $$2V_1 = -I_1 - 2I_2$$

or $$V_1 = -\frac{1}{2}I_1 - I_2 \qquad ...(viii)$$

And $$3(3V_2 + 2I_1 + 2I_2) - 6V_2 = 3I_1 + 2I_2$$

or $$3V_2 = -3I_1 - 4I_2$$

or $$V_2 = -I_1 - \frac{4}{3}I_2 \qquad ...(ix)$$

Therefore, from equations (*viii*) and (*ix*), we have

$$Z = \begin{bmatrix} -\frac{1}{2} & -1 \\ -1 & -\frac{4}{3} \end{bmatrix}$$

Alternatively :

$$Z = [Y]^{-1} = \begin{bmatrix} 4 & -3 \\ -3 & \frac{3}{2} \end{bmatrix}^{-1} = -\frac{1}{3}\begin{bmatrix} \frac{3}{2} & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -1 \\ -1 & -\frac{4}{3} \end{bmatrix}$$

EXAMPLE 8.31 ***The accompanying network of figure 8.65, contains a voltage-controlled source and a current-controlled source. For the element values specified, determine Z and Y parameters.***

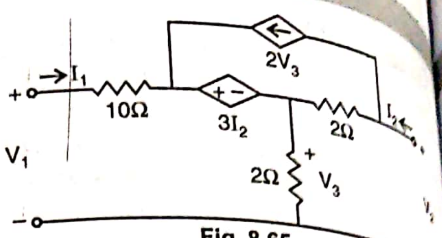


Fig. 8.65.

Solution : Applying KVL,

$$V_1 = 10I_1 + 3I_2 + 2(I_1 + I_2)$$

or $$V_1 = 12I_1 + 5I_2 \qquad ...(i)$$

$$V_2 = 2(I_2 - 2V_3) + 2(I_1 + I_2)$$

or $$V_2 = 2I_1 + 4I_2 - 4V_3 \qquad ...(ii)$$

And $$V_3 = 2(I_1 + I_2) \qquad ...(iii)$$

From equations (*ii*) and (*iii*), we have

$$V_2 = 2I_1 + 4I_2 - 4(2I_1 + 2I_2)$$

$$V_2 = -6I_1 - 4I_2 \qquad ...(iv)$$

From equations (*i*) and (*iv*), the required *Z*-parameters of the given network are

$$Z = \begin{bmatrix} 12 & 5 \\ -6 & -4 \end{bmatrix}$$

Hence, $$Y = [Z]^{-1} = \begin{bmatrix} \frac{2}{9} & \frac{5}{18} \\ -\frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

EXAMPLE 8.32 **The network shown in the figure 8.66, consists of a resistive *T*-and a resistive *π*-network connected in parallel. For the element values given, determine the *Y*-parameters.**

Solution : Let top network is *A* and bottom network is *B*. Then

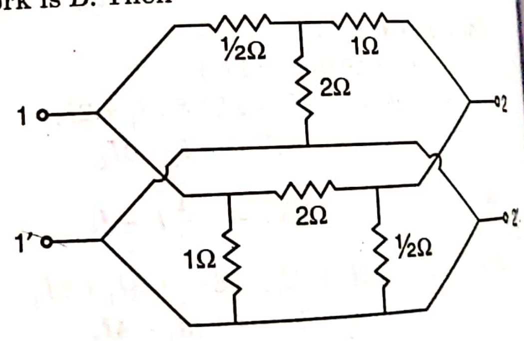


Fig. 8.66.

$$Y_A = [Z_A]^{-1} = \begin{bmatrix} \frac{1}{2}+2 & 2 \\ 2 & 2+1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{5}{2} & 2 \\ 2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{6}{7} & -\frac{4}{7} \\ \frac{-4}{7} & \frac{5}{7} \end{bmatrix}$$

And $$Y_B = \begin{bmatrix} \frac{1}{1}+\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}+2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

Therefore, the required Y-parameters are

$$Y = Y_A + Y_B = \begin{bmatrix} \frac{33}{14} & -\frac{15}{14} \\ -\frac{15}{14} & \frac{45}{14} \end{bmatrix}$$

EXAMPLE 8.33 Apply the Y-Δ transformation to the network of the figure 8.67(a) to obtain an equivalent (a) T-network, (b) π-network.

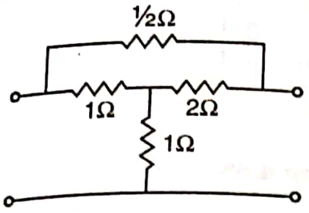


Fig.8.67(a).

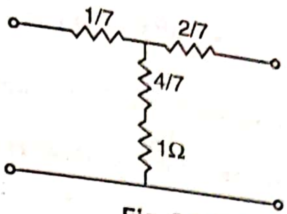


Fig. 8.67(b).

Solution : (a) Applying $\Delta - Y$ conversion, we have the network as shown in figure 8.67(b). Now,

$$Z_a = \frac{1}{7}\,\Omega \quad ; \quad Z_b = \frac{2}{7}\,\Omega$$

And

$$Z_c = \frac{4}{7} + 1 = \frac{11}{7}\,\Omega$$

Hence, the equivalent T-network is as shown in figure 8.67(c).

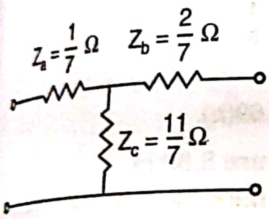


Fig. 8.67(c).

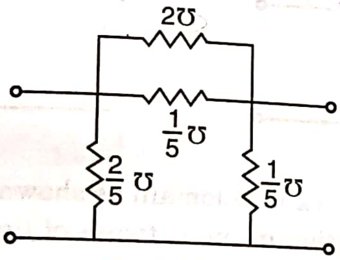


Fig. 8.67(d).

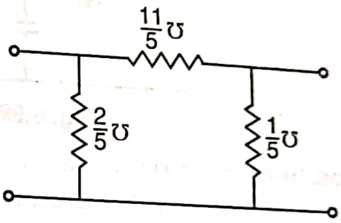


Fig. 8.67(e).

(b) Applying $Y - \Delta$ conversion, we have the network as shown in figure 8.67(d). Now,

$$Y_a = \frac{2}{5}\ \mho \quad ; \quad Y_b = \frac{1}{5}\ \mho$$

$$Y_c = 2 + \frac{1}{5} = \frac{11}{5}\ \mho$$

Hence, the equivalent π-network is as shown in figure 8.67(e).

EXAMPLE 8.34 Determine the Y-parameters for the twin-T network shown in figure 8.68(a), if all the resistances are of 1Ω.

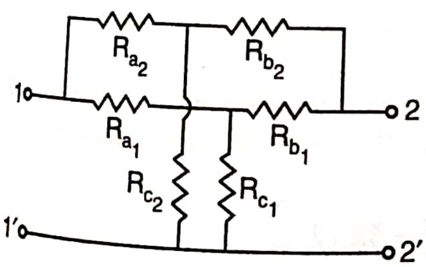


Fig. 8.68(a).

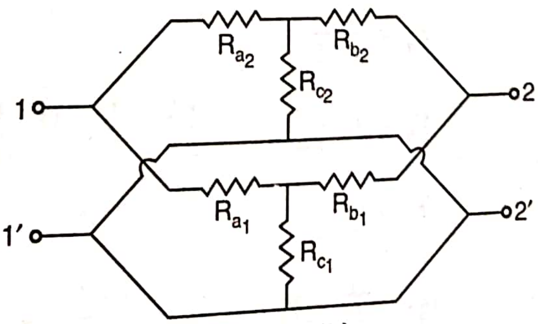


Fig. 8.68(b).

Solution : We can easily see that, the network of figure 8.68(a) is topologically equivalent to the network shown in figure 8.68(b) which is the parallel connection of two T-networks. So the individual Y-parameters are added to determine the overall Y-parameters. To illustrate, call the top network A and the bottom network B. Then by inspection, we have

$$Y_A = [Z_A]^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

Since both the networks A and B are same. Hence

$$Y_A = Y_B$$

Therefore, Y-parameters for the network of figure 8.68(a) are

$$Y = 2Y_A = \begin{bmatrix} \frac{4}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{4}{3} \end{bmatrix}$$

EXAMPLE 8.35 For a bridged-T RC network as shown in figure 8.69(a), calculate Y-parameters.

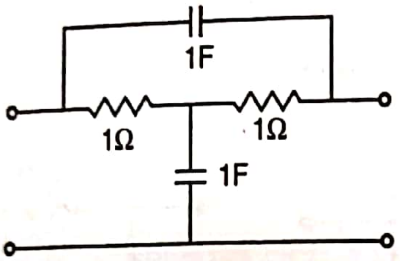


Fig. 8.69(a).

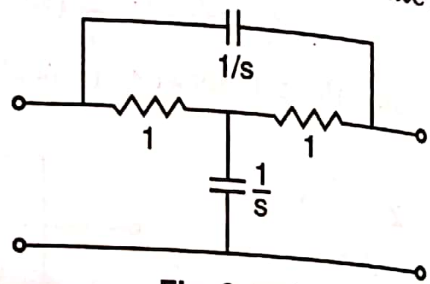


Fig. 8.69(b).

Solution : First redrawing the network in s-domain as shown in figure 8.69(b).

Now, applying $Y-\Delta$ transformation in the network of figure 8.69(b), we get the network of figure 8.69(c).

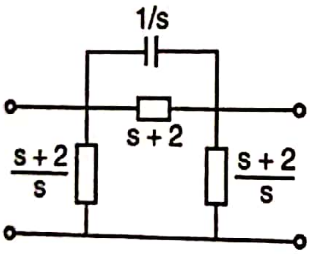


Fig. 8.69(c).

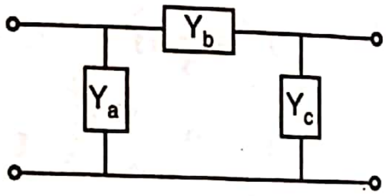


Fig. 8.69(d).

Now, converting, in the admittance form the network of figure 8.69(c) becomes the network of figure 8.69(d), where

$$Y_a = \frac{s}{s+2}$$

$$Y_b = s + \frac{1}{s+2} = \frac{s^2+2s+1}{s+2} = \frac{(s+1)^2}{s+2}$$

$$Y_c = \frac{s}{s+2}$$

Therefore,

$$Y_{11} = Y_a + Y_b = \frac{s}{s+2} + \frac{(s+1)^2}{s+2} = \frac{s^2+3s+1}{s+2}$$

$$Y_{12} = Y_{21} = -Y_b = -\frac{(s+1)^2}{s+2}$$

$$Y_{22} = Y_c + Y_b = \frac{s^2+3s+1}{s+2}$$

Alternatively : The bridged-*T* *RC* network of figure 8.69(*a*) is equivalent to the parallel connection of the two networks shown in figure 8.69(*e*).

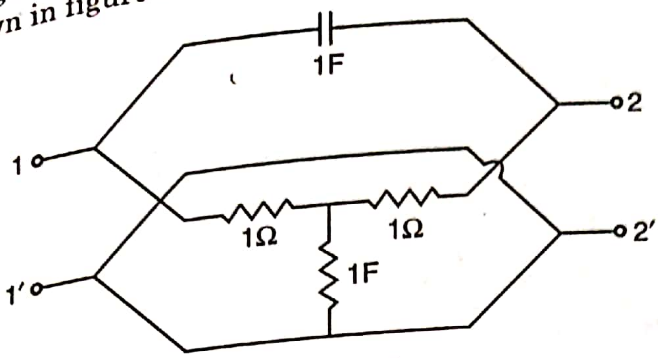


Fig. 8.69(e).

Y-parameters of the top network (refer to example 8.8) are

$$Y_A = \begin{bmatrix} s & -s \\ -s & s \end{bmatrix}$$

And Y-parameters of the bottom network are

$$Y_B = \begin{bmatrix} 1+\frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & 1+\frac{1}{s} \end{bmatrix}^{-1}$$

$$Y_B = \begin{bmatrix} \frac{s+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{s+1}{s} \end{bmatrix}$$

$$Y_B = \begin{bmatrix} \frac{s+1}{s+2} & -\frac{1}{s+2} \\ -\frac{1}{s+2} & \frac{s+1}{s+2} \end{bmatrix}$$

Therefore, the required *Y*-parameters are

$$Y = Y_A + Y_B = \begin{bmatrix} \frac{s^2+3s+1}{s+2} & -\frac{(s+1)^2}{s+2} \\ -\frac{(s+1)^2}{s+2} & \frac{s^2+3s+1}{s+2} \end{bmatrix}$$

EXAMPLE 8.36 **Find open circuit transfer impedance V_2/I_1 and open circuit voltage ratio V_2/V_1 for the ladder network shown in figure 8.70(*a*).** *(U.P.T.U., 2003)*

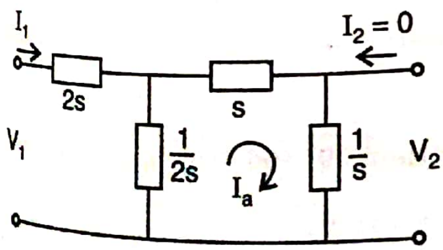


Fig. 8.70(a).

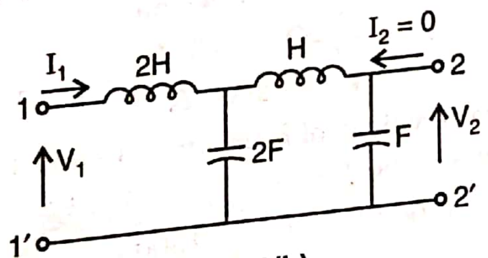


Fig. 8.70(b).

Solution : (By taking H is 1H and F is 1F). The network in transform-domain is as shown in figure 8.70(*b*).

Applying KVL,

$$V_1 = 2s.I_1 + \frac{1}{2s}(I_1 - I_a)$$

$$V_2 = \frac{1}{s}I_a$$

And
$$I_a = \frac{I_1.\frac{1}{2s}}{\frac{1}{2s}+s+\frac{1}{s}} = \frac{I_1}{2s^2+3}$$

From equations (*ii*) and (*iii*), we have

$$\frac{V_2}{I_1} = \frac{1}{s(2s^2+3)}$$

From equations (*i*) and (*iii*),

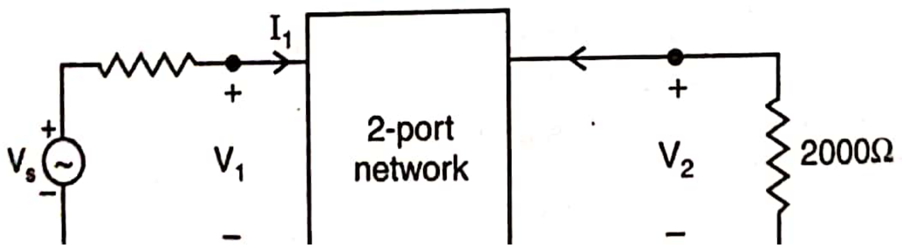
$$V_1 = \left[\left(2s+\frac{1}{2s}\right)(2s^2+3) - \frac{1}{2s}\right]I_a$$

$$= \left[\frac{(4s^2+1)(2s^2+3)-1}{2s}\right]I_a = \left(\frac{4s^4+7s^2+1}{s}\right)I_a$$

From equations (*ii*) and (*iv*),

$$\frac{V_2}{V_1} = \frac{1}{4s^4+7s^2+1}$$

EXAMPLE 8.37 The h-parameters of a two port network shown in figure 8.71 are $h_{11} = 1000\,\Omega$, $h_{12} = 0.003$, $h_{21} = 100$ and $h_{22} = 50 \times 10^{-6}$ ℧. Find V_2 and Z-parameters of the network, if $V_s = 10^{-2}\angle 0°V$.



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Now, from equations (10) and (b),

$$\frac{V_2}{-2000} = 100\, I_1 + 50 \times 10^{-6}\, V_2 \qquad \text{...(d)}$$

or $$I_1 = -5.5 \times 10^{-6}\, V_2$$

Therefore, from equations (c) and (d), we get

$$0.003\, V_2 = 10^{-2} + 1500\,(5.5 \times 10^{-6})\, V_2$$

or $$V_2 = \mathbf{-1.905\ V}$$

Z-parameters are :

$$Z_{11} = \frac{\Delta h}{h_{22}} = \frac{1000\times 50\times 10^{-6} - 0.003\times 100}{50\times 10^{-6}} = \mathbf{-5000\ \Omega}$$

$$Z_{12} = \frac{h_{12}}{h_{22}} = \mathbf{60\ \Omega}$$

Similarly,

$$Z_{21} = -\frac{h_{21}}{h_{22}} = \mathbf{-2 \times 10^6\ \Omega}$$

and $$Z_{22} = \frac{1}{h_{22}} = \mathbf{20 \times 10^3\ \Omega}$$

EXAMPLE 8.38 Determine the transmission parameters of the network shown in figure 8.72, using the concept of interconnection of four two-port networks N_1, N_2, N_3 and N_4 in cascade.

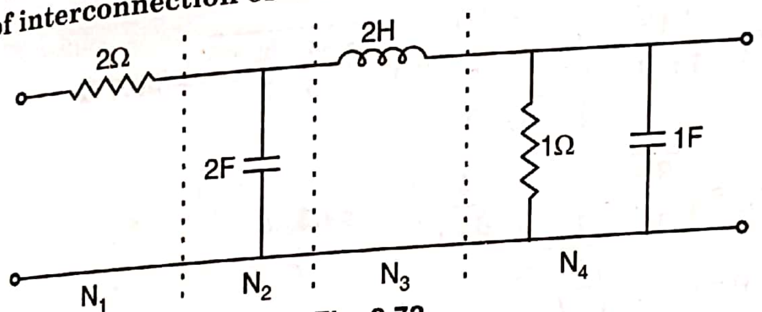


Fig. 8.72.

Solution : With the results of examples 8.7 and 8.8, we can directly write :

$$T_1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 \\ 2s & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix}$$

$$T_4 = \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

Therefore, the required transmission parameters of the given network are

$$T = T_1\, T_2\, T_3\, T_4 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2s & 1 \end{bmatrix} \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4s & 2 \\ 2s & 1 \end{bmatrix} \begin{bmatrix} 1+2s^2+2s & 2s \\ s+1 & 1 \end{bmatrix} = \begin{bmatrix} 8s^3+10s^2+8s+3 & 8s^2+2s+2 \\ 4s^3+4s^2+3s+1 & 4s^2+1 \end{bmatrix}$$

EXAMPLE 8.39 **Determine the image parameters of the T-network shown in figure 8.73.**

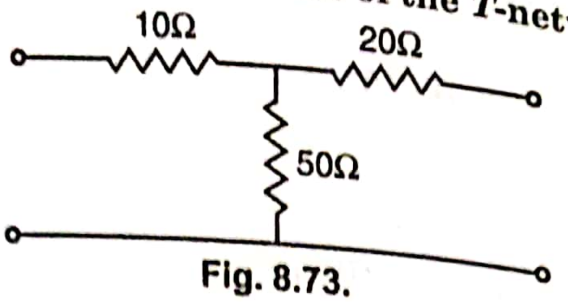


Fig. 8.73.

Solution :

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{6}{5}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{50}\,℧$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 34\,\Omega$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{7}{5}$$

Since the network is not symmetrical, therefore, Z_{i1}, Z_{i2} and θ are to be calculated to describe the network.

$$Z_{i1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{\left(\frac{6}{5}\right)(34)}{\left(\frac{1}{50}\right)\left(\frac{7}{5}\right)}} = \sqrt{\frac{6\times 34\times 50\times 5}{5\times 7}} = \mathbf{38.7\ \Omega}$$

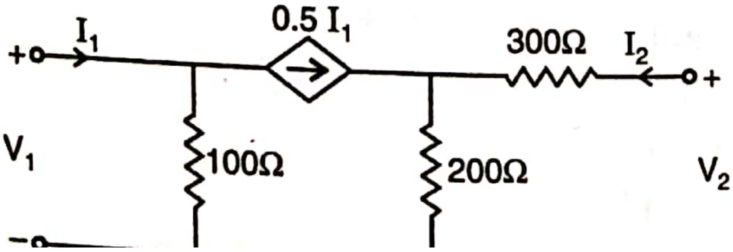
$$Z_{i2} = \sqrt{\frac{BD}{AC}} = \sqrt{\frac{34\times 7\times 5\times 50}{5\times 6\times 1}} = \mathbf{44.5\ \Omega}$$

and
$$\theta = \tanh^{-1}\sqrt{\frac{BC}{AD}}$$

$$= \tanh^{-1}\sqrt{\frac{34\times 1\times 5\times 5}{50\times 6\times 7}} = \tanh^{-1}\sqrt{0.405} = \mathbf{0.75}$$

(Alternatively : $\theta = \cosh^{-1}\sqrt{AD} = \sinh^{-1}\sqrt{BC} = 0.75$)

EXAMPLE 8.40 **Determine Y-parameters for the network shown in figure 8.74.**



From equations (A) and (B)

$$I_1 = \frac{1}{50} V_1$$

and

$$I_2 = -\frac{1}{250} V_1 + \frac{1}{500} V_2 \quad \text{...(C)}$$

$$\text{...(D)}$$

Comparing equations (C) and (D) with equations (3) and (4) respectively, we have

$$Y_{11} = \frac{1}{50} \text{℧}, \; Y_{12} = 0, \; Y_{21} = -\frac{1}{250} \text{℧}, \; Y_{22} = \frac{1}{500} \text{℧}.$$

EXAMPLE 8.41 **Determine the transmission parameters of the circuit shown in figure 8.75(a).**

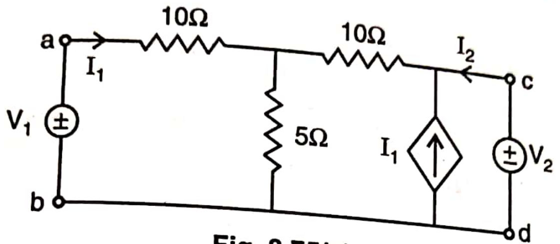


Fig. 8.75(a).

Solution : Applying KVL, in the circuit of figure 8.75(*b*),

$$V_1 = 10I_1 + 5\,(2I_1 + I_2)$$

or

$$V_1 = 20\,I_1 + 5I_2 \quad \text{...(1)}$$

and

$$V_2 = 10(I_1 + I_2) + 5(2I_1 + I_2)$$

or

$$V_2 = 20\,I_1 + 15I_2 \quad \text{...(2)}$$

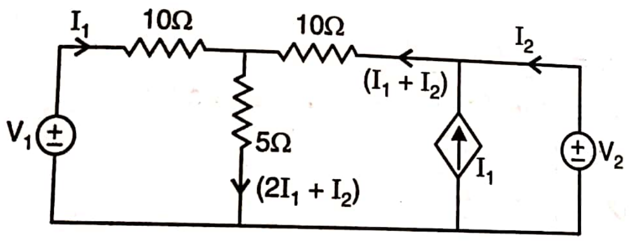


Fig. 8.75(b).

Case I : When $I_2 = 0$; $\quad A = \dfrac{V_1}{V_2} = 1$

$$C = \frac{I_1}{V_2} = \frac{1}{20} \text{℧}$$

Case II : When $V_2 = 0$;

$$B = \frac{V_1}{-I_2} = \frac{20I_1 + 5I_2}{-I_2} = \frac{-10I_2}{-I_2} = 10\;\Omega$$

$$D = \frac{I_1}{-I_2} = \frac{15}{20} = \frac{3}{4}$$

EXAMPLE 8.42 **Obtain the *Y*-parameters of the circuit shown in figure 8.76(*a*). Find its equivalent circuit using *Y*-parameter and find whether the network is (*i*) reciprocal (*ii*) symmetrical.** *(U.P.T.U., 2005)*

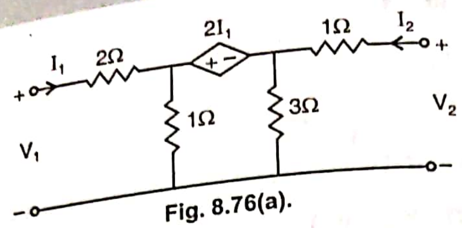


Fig. 8.76(a).

Solution : Applying KVL, in the circuit of figure 8.76(*b*),

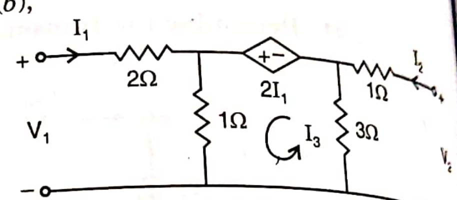


Fig. 8.76(b).

$$V_1 = 2 . I_1 + 1 . (I_1 + I_3) \qquad ...(1)$$

or
$$V_1 = 3I_1 + I_3$$
$$V_2 = 1 . I_2 + 3(I_2 - I_3) \qquad ...(2)$$

or
$$V_2 = 4I_2 - 3I_3$$
$$2I_1 = 1 . (I_1 + I_3) + 3(I_3 - I_2) \qquad ...(3)$$

or
$$0 = -I_1 - 3I_2 + 4I_3$$

From equation (3),
$$I_3 = \frac{1}{4}(I_1 + 3I_2) \qquad ...(4)$$

Now from equation (1), (2) and (4),
$$V_1 = 3I_1 + \frac{1}{4}(I_1 + 3I_2) = \frac{13}{4} I_1 + \frac{3}{4} I_2 \qquad ...(5)$$

and
$$V_2 = 4I_2 - \frac{3}{4}(I_1 + 3I_2)$$

or
$$V_2 = -\frac{3}{4} I_1 + \frac{7}{4} I_2 \qquad ...(6)$$

From equations (5) and (6), the Z-parameters are given by
$$[Z] = \begin{bmatrix} \frac{13}{4} & \frac{3}{4} \\ -\frac{3}{4} & \frac{7}{4} \end{bmatrix}$$

Therefore, Y-parameters are given by
$$[Y] = [Z]^{-1} = \frac{1}{\left(\frac{91}{16} + \frac{9}{16}\right)} \begin{bmatrix} \frac{7}{4} & -\frac{3}{4} \\ \frac{3}{4} & \frac{13}{4} \end{bmatrix} = \begin{bmatrix} \frac{7}{25} & -\frac{3}{25} \\ \frac{3}{25} & \frac{13}{25} \end{bmatrix}$$

The equivalent circuit is terms of Y-parameters is shown in figure 8.76(*c*).

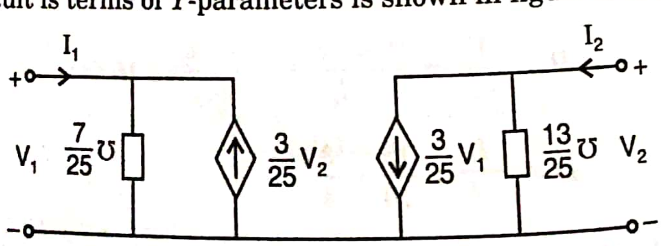


Fig. 8.76(c).

(*i*) Since $Y_{12} \neq Y_{21}$, hence network is not reciprocal,
(*ii*) Since $Y_{11} \neq Y_{22}$, hence network is not symmetrical.

EXAMPLE 8.43 Find the short ... impedances of the network sho... hence obtain its π equivalent.

Solution : Let I_3 be the current in t... 8.77(*b*).

Applying KVL, mesh equations...
$$V_1 = 3I_1 + I_2 - 2I_3$$
$$V_2 = 3I_1 + 3I_2 + 2I_3$$
$$0 = -2I_1 + 2I_2 + 5I_3$$

From equation (3), $I_3 = \frac{2}{5}(I_1 ...$

From equations (1), (2) and (4...

$V_1 = 3I_1 + I_2 - \frac{4}{5}(I ...$

or $V_1 = \frac{11}{5} I_1 + \frac{9}{5} I_2 ...$

$V_2 = I_1 + 3I_2 + ...$

or $V_2 = \frac{9}{5} I_1 + \frac{11}{5} ...$

From equations (5) and (6...
Short-circuited impedanc...

$Z_{1s} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}$

$Z_{2s} = \left.\frac{V_2}{I_2}\right|_{V_1 = ...}$

Open-circuited impeda...

$Z_{1o} = \left.\frac{V_1}{I_1}\right|_{I_2 = ...}$

Let π-equivalent of t... 8.77(c). Since $Z_{1s} = Z_{2s}$ ... symmetrical, *i.e.*,

When output port is...

$Z_{1s} = Z_a \| Z_b = ...$

When output port ...

or $\frac{Z_a(Z ...}{2Z_a ...}$

EXAMPLE 8.43 **Find the short circuit and open circuit impedances of the network shown in figure 8.77(a) and hence obtain its π equivalent.** *(U.P.T.U., 2005)*

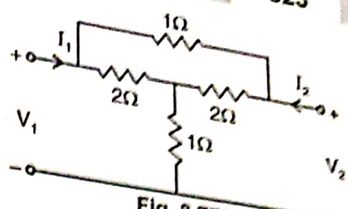


Fig. 8.77(a).

Solution : Let I_3 be the current in the circuit as shown in figure 8.77(b).

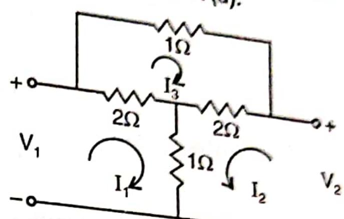


Fig. 8.77(b).

Applying KVL, mesh equations become

$$V_1 = 3I_1 + I_2 - 2I_3 \quad \text{... (1)}$$

$$V_2 = 3I_1 + 3I_2 + 2I_3 \quad \text{... (2)}$$

$$0 = -2I_1 + 2I_2 + 5I_3 \quad \text{... (3)}$$

From equation (3), $I_3 = \frac{2}{5}(I_1 - I_2)$ ... (4)

From equations (1), (2) and (4),

$$V_1 = 3I_1 + I_2 - \frac{4}{5}(I_1 - I_2)$$

or

$$V_1 = \frac{11}{5}I_1 + \frac{9}{5}I_2 \quad \text{... (5)}$$

$$V_2 = I_1 + 3I_2 + \frac{4}{5}(I_1 - I_2)$$

or

$$V_2 = \frac{9}{5}I_1 + \frac{11}{5}I_2 \quad \text{... (6)}$$

From equations (5) and (6),

Short-circuited impedances :

$$Z_{1s} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{\left\{\frac{11}{5} + \frac{9}{5}\left(-\frac{9}{11}\right)\right\} I_1}{I_1} = \frac{11}{5} - \frac{81}{55} = \frac{40}{55} = \frac{8}{11}\,\Omega$$

$$Z_{2s} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{\left\{\frac{9}{5}\left(-\frac{9}{11}\right) + \frac{11}{5}\right\} I_2}{I_2} = \frac{8}{11}\,\Omega$$

Open-circuited impedances :

$$Z_{1o} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{11}{5}\,\Omega$$

$$Z_{2o} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{11}{5}\,\Omega$$

Let π-equivalent of this two port network is as shown in figure 8.77(c). Since $Z_{1s} = Z_{2s}$ and also $Z_{1o} = Z_{2o}$. Hence network must be symmetrical, *i.e.,*

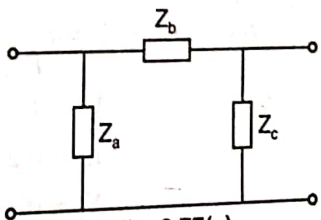


Fig. 8.77(c).

$$Z_a = Z_c.$$

When output port is short-circuit,

$$Z_{1s} = Z_a \parallel Z_b = \frac{Z_a Z_b}{Z_a + Z_b} = \frac{8}{11} \quad \text{... (7)}$$

When output port is open-circuit,

$$Z_{1o} = Z_a \parallel (Z_b + Z_c) = Z_a \parallel (Z_b + Z_a)$$

or

$$\frac{Z_a(Z_b + Z_a)}{2Z_a + Z_b} = \frac{11}{5} \quad \text{... (8)}$$

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From equations (7) and (8),

$$\frac{1}{Z_a} + \frac{1}{Z_b} = \frac{11}{8} \quad \text{...(9)}$$

$$\frac{1}{Z_a} + \frac{1}{Z_a + Z_b} = \frac{5}{11} \quad \text{...(10)}$$

From equations (9) and (10),

$$\frac{1}{Z_b} - \frac{1}{Z_a + Z_b} = \frac{11}{8} - \frac{5}{11} = \frac{81}{88}$$

or

$$\frac{Z_a}{Z_b(Z_a + Z_b)} = \frac{81}{88} \quad \text{...(11)}$$

From equation (7),

$$Z_a + Z_b = \frac{11}{8} Z_a Z_b$$

Now equation (11) becomes,

$$\frac{Z_a}{Z_b\left(\frac{11}{8} Z_a Z_b\right)} = \frac{81}{88}$$

or

$$\frac{8}{11 Z_b^2} = \frac{81}{88}$$

or

$$Z_b^{\,2} = \frac{64}{81}$$

or

$$Z_b = \frac{8}{9}\ \Omega.$$

From equation (9),

$$\frac{1}{Z_a} = \frac{11}{8} - \frac{9}{8}$$

or

$$Z_a = 4\ \Omega.$$

EXAMPLE 8.44 **A 2-port network made up of passive linear resistors is fed at port 1 by an ideal voltage source of V volts. It is loaded at port 2 by a resistance R:**

(i) With V = 10 volts and $R = 6\ \Omega$, currents at ports 1 and 2 were 1.44 A and 0.2 A respectively.

(ii) With V = 15 volt and $R = 8\ \Omega$, the current at port 2 was 0.25 A. Determine π equivalent circuit of 2-port network. *(U.P.T.U., May, 2006)*

Solution : The two port network with output port (port 2) is loaded by a resistance R is shown in figure 8.78(a).

From figure 8.78(a),

$$V_2 = RI_2' = -RI_2$$

(i) $V_1 = 10$ V, $I_1 = 1.44$ A,

$I_2' = 0.2$ A, $I_2 = -0.2$ A

$V_2 = (6)(0.2) = 1.2$ V

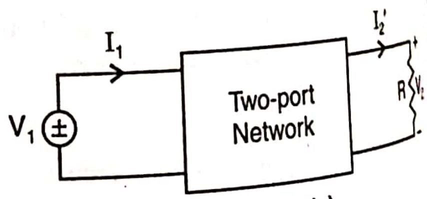

Fig. 8.78(a).

(ii) $V_1 = 15$ V, $I_2' = 0.25$ A, $I_2 = -0.25$ A

$V_2 = (8)(0.25) = 2$ V

Since in both the cases, V_1 and V_2 are available and also π-equivalent circuit can be easily determined in terms of Y-parameters. Hence we consider Y-parameter equations.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Case I:

$$1.44 = Y_{11}(10) + Y_{12}(1.2) \quad \text{...(1)}$$

$$-0.2 = Y_{21}(10) + Y_{22}(1.2) \quad \text{...(2)}$$

Case II:

$$-0.25 = Y_{21}(15) + Y_{22}(2) \quad \text{...(3)}$$

And we know that for a possible two-port network

$$Y_{12} = Y_{21} \quad \text{...(4)}$$

Hence, from equations (1), (2), (3) and (4)

$$10Y_{11} + 1.2Y_{12} = 1.44 \quad \text{...(5)}$$

$$10Y_{12} - 1.2Y_{22} = -0.2 \quad \text{...(6)}$$

$$15Y_{12} + 2Y_{22} = -0.25 \quad \text{...(7)}$$

Solving equations (5), (6) and (7), we have

$$Y_{11} = 0.204 \ \mho$$

$$Y_{12} = -0.05 \ \mho$$

$$Y_{22} = 0.25 \ \mho$$

Let π-equivalent of this network is as shown in figure 8.78(*b*).

Hence,

$$Y_a = Y_{11} + Y_{12} = 0.154 \ \mho$$

$$Y_b = -Y_{12} = 0.05 \ \mho$$

$$Y_c = Y_{22} + Y_{12} = 0.2 \ \mho$$

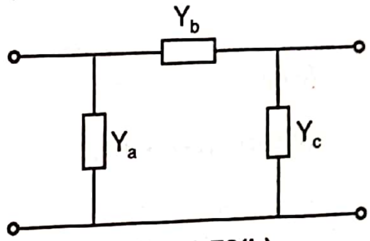

Fig. 8.78(b).

MPLE 8.45 Determine the *h*-parameters of the network given in figure 8.79(*a*).
(U.P.T.U., 2006)(U.P.T.U., May 2007)

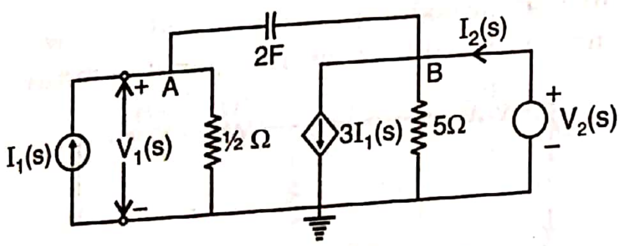

Fig. 8.79(a).

tion: Redraw the given network in s-domain as shown in figure 8.79(*b*).

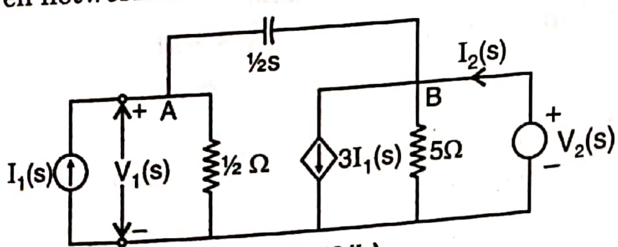

Fig. 8.79(b).

pply KCL at point '*A*';

$$I_1(s) = \frac{V_1(s)}{1/2} + \frac{V_1(s) - V_2(s)}{1/2s}$$

$$I_1(s) = 2(s+1)\, V_1(s) - 2\,sV_2(s) \quad \text{...(i)}$$

$$V_1(s) = \frac{1}{2(s+1)} I_1(s) + \frac{s}{s+1} V_2(s)$$

Apply KCL at point 'B';

$$I_2(s) = \frac{V_2(s)}{5} + 3I_1(s) + \frac{V_2(s) - V_1(s)}{1/2s}$$

$$I_2(s) = 3\,I_1(s) + \left(2s + \frac{1}{5}\right)V_2(s) - 2s\,V_1(s)$$

Putting the value of $V_1(s)$ in the above equation from (i), we get

$$I_2(s) = 3I_1(s) + \left(\frac{10s+1}{5}\right)V_2(s) - \frac{s}{s+1}\,I_1(s) - \frac{2s^2}{s+1}\,V_2(s)$$

or

$$I_2(s) = \frac{(2s+3)}{s+1}\,I_1(s) + \frac{11s+1}{5(s+1)}\,V_2(s)$$

When $V_2(s) = 0$:

From (i),

$$h_{11} = \frac{V_1(s)}{I_1(s)} \Rightarrow h_{11} = \frac{1}{2(s+1)}$$

From (ii),

$$h_{21} = \frac{I_2(s)}{I_1(s)} \Rightarrow h_{21} = \frac{2s+3}{s+1}$$

When $I_1(s) = 0$:

From (i),

$$h_{12} = \frac{V_1(s)}{V_2(s)} \Rightarrow h_{12} = \frac{s}{s+1}$$

From (ii),

$$h_{22} = \frac{I_2(s)}{V_2(s)} \Rightarrow h_{22} = \frac{11s+1}{5(s+1)}$$

EXAMPLE 8.46 Determine transmission parameters of a network given in figure 8.80(a), considering sections as shown are connected in cascade manner. *(U.P.T.U., 2006)*

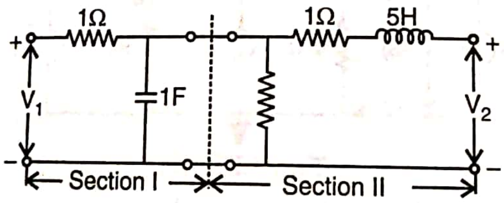

Fig. 8.80(a).

Ans. ***T*-Parameters of Section**—I as shown in figure 8.80(b).

Apply KVL in loop ①,

$$V_1 = \left(1 + \frac{1}{s}\right)I_1 + \frac{1}{s}\,I_2 \qquad \ldots(i)$$

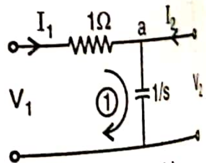

Fig. 8.80(b).

Apply KCL at 'a',

$$I_1 + I_2 = s\,V_2$$

or

$$I_1 = s\,V_2 - I_2 \qquad \ldots(ii)$$

From equations (i) and (ii)

$$V_1 = \left(1 + \frac{1}{s}\right)\cdot[sV_2 - I_2] + \frac{1}{s}\,I_2$$

or

$$V_1 = (s+1)V_2 - I_2 \qquad \ldots(iii)$$

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T-Parameters of Section—II

On applying KVL in loops ① and ②, we get

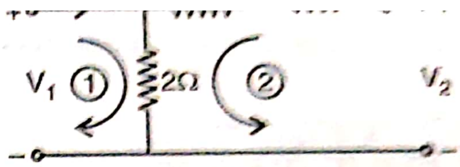


Fig. 8.30(c).

$$V_1 = 2\,I_1 + 2\,I_2 \qquad \text{...(i)}$$

$$V_2 = 2\,I_1 + (3 + 5s)\,I_2 \qquad \text{...(ii)}$$

From equation (ii),

$$I_1 = \frac{1}{2}V_2 - \frac{5s+3}{2}I_2 \qquad \text{...(iii)}$$

From equation (i), $V_1 = V_2 - (5s + 1)\,I_2$ ...(iv)

From (iv) and (iii),

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 5s+1 \\ 1/2 & \dfrac{5s+3}{2} \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

T-Parameters of section–II are $[T_2] = \begin{bmatrix} 1 & 5s+1 \\ \dfrac{1}{2} & \dfrac{5s+3}{2} \end{bmatrix}$

Overall T-parameters of cascaded networks,

$$[T] = [T_1]\cdot[T_2]$$

$$= \begin{bmatrix} (s+1) & 1 \\ s & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 5s+1 \\ \dfrac{1}{2} & \dfrac{3+5s}{2} \end{bmatrix}$$

or $$[T] = \begin{bmatrix} \dfrac{s+3}{2} & (s+1)\left(\dfrac{s+5}{2}\right) \\ \left(\dfrac{s+2}{2}\right) & \dfrac{s^2+5s+2}{2} \end{bmatrix}$$

EXERCISES

8.1. For the network shown in figure *P.8.1*. Calculate

(*i*) *Z*-parameters

(*ii*) *Y*-parameters

(*iii*) *T*-parameters

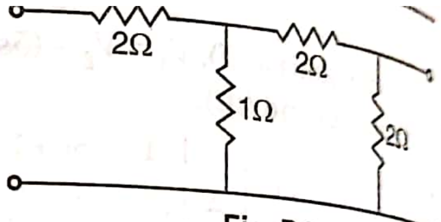


Fig. P.8.1.

8.2. Determine the Transmission parameters of the network shown in figure P.8.2.

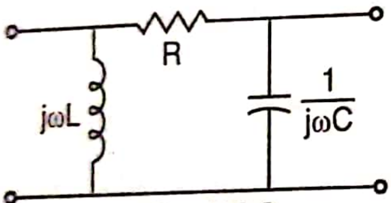


Fig. P.8.2.

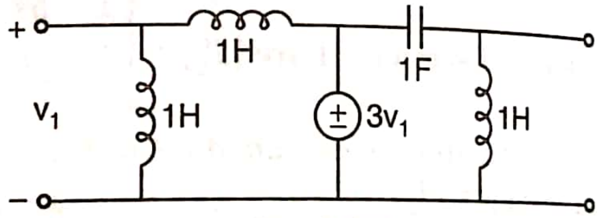


Fig. P.8.3.

8.3. Find the *Z*-parameters for the network shown in figure P.8.3.

8.4. Find the *T*-parameters for network given in figure P.8.4.

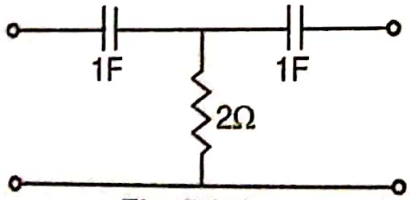


Fig. P.8.4.

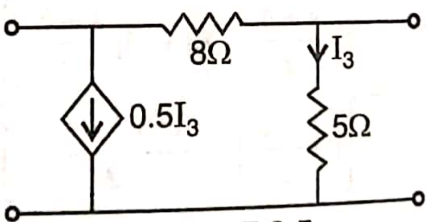


Fig. P.8.5.

8.5. Obtain the *Z*-parameters for the π-circuit model of the network shown in figure P.8.5.

8.6. Find the *Y*-parameters of the network of figure P.8.6.

The two-port network shown in figure P.8.9, containing a current controlled voltage source. Find its Z-parameters.

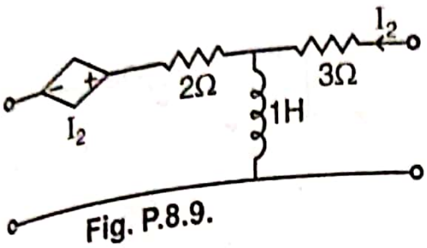


Fig. P.8.9.

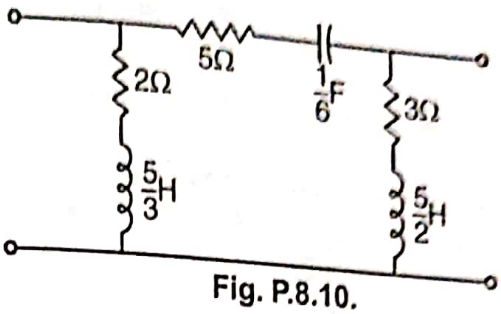


Fig. P.8.10.

8.10. Find the Y-parameters of the circuit in figure P.8.10.

8.11. Find the h-parameters of figure P.8.11.

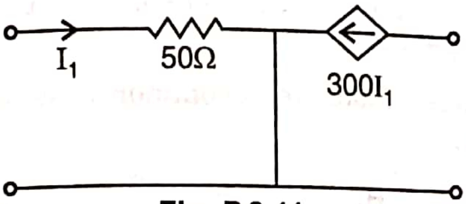


Fig. P.8.11.

8.12. Obtain the T-parameters of the cascaded connection of three networks, as shown in figure P.8.12(a) to (c) and verifty the result with that of figure P.8.12(d).

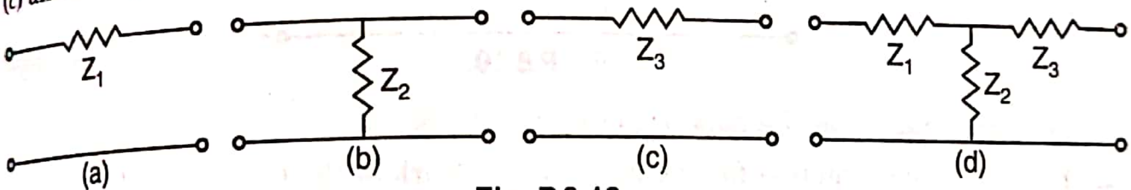


Fig. P.8.12.

8.13. Determine the Z and Y-parameters of the network shown in figure P.8.13.

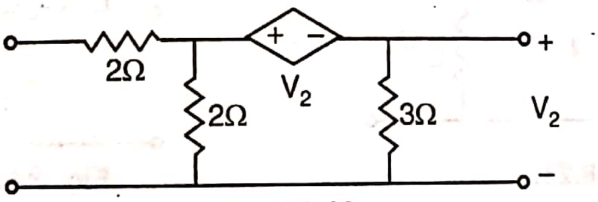


Fig. P.8.13.

8.14. The currents I_1 and I_2 at input and output ports respectively of a two port network can be expressed as

$$I_1 = 5V_1 - V_2 \quad \text{and} \quad I_2 = -V_1 + V_2$$

(a) Find the Y-parameters.

(b) Find the equivalent π-network parameters.

(c) Find the input impedance when a load of $(3 + j5)\Omega$ is connected across the output port.

8.15. A two-port T-network has open circuit impedances $Z_{1o} = 900\Omega$, $Z_{2o} = 900\Omega$, and short circuit impedance $Z_{1s} = 650\Omega$. Determine the parameters of the T-network.

8.16. Find out the ABCD parameters of the network shown in figure P.8.16. Also find the image impedances for the network.

8.17. A symmetrical *T* network as shown in figure P.8.17 has the following open-circuit and short-circuit impedances.

$$Z_{1o} = Z_{2o} = 800\ \Omega$$
$$Z_{1s} = Z_{2s} = 600\ \Omega$$

Determine the parameters of the *T*-network.

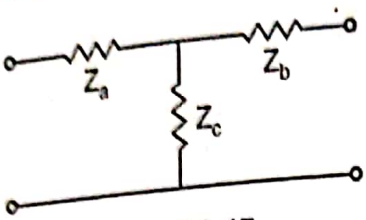


Fig. P.8.17.

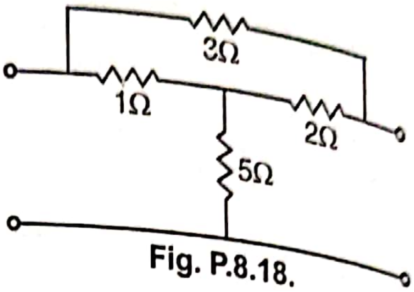


Fig. P.8.18.

8.18. Obtain *T* and *π* equivalent circuits for the network of figure P.8.18, by using impedance and admittance parameters respectively.

8.19. The network of figure P.8.19 is a model for a common-base connected transistor, calculate the *Z* parameters for the network.

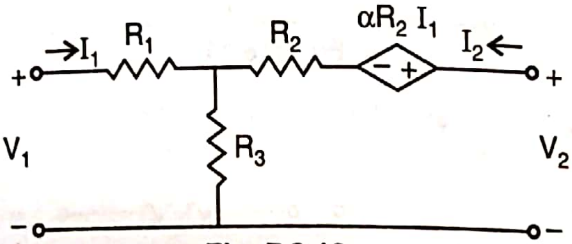


Fig. P.8.19.

8.20. Find the *Z*-parameters of the network of figure P.8.6.

8.21. Find the *Z* and *Y*-parameters for the resistive network of the figure P.8.21.

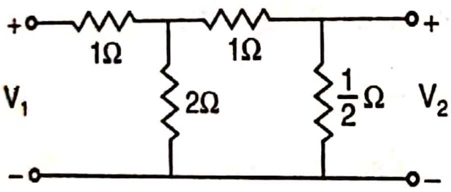


Fig. P.8.21.

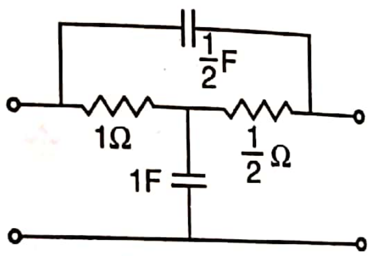


Fig. P.8.22.

8.22. The network of the figure P.8.22 is a bridged-*T RC* network. For the values given, find the *Y*-parameters.

8.23. Show that the two networks of figures P.8.23(*a*) and (*b*) are equivalent if

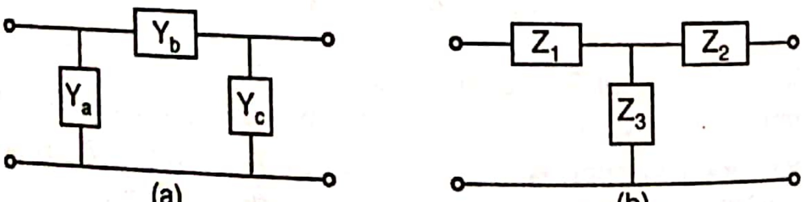


Fig. P.8.23.

(*a*) $Y_a = \dfrac{Z_2}{D_1}$, $Y_b = \dfrac{Z_3}{D_1}$ and $Y_c = \dfrac{Z_1}{D_1}$ where $D_1 = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$

(*b*) $Z_1 = \dfrac{Y_c}{D_2}$, $Z_2 = \dfrac{Y_a}{D_2}$ and $Z_3 = \dfrac{Y_b}{D_2}$ where $D_2 = Y_aY_b + Y_bY_c + Y_cY_a$

8.24. Obtain the equivalent π-network for the capacitive network given in figure P.8.24.

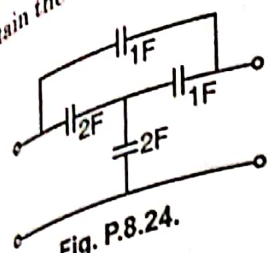


Fig. P.8.24.

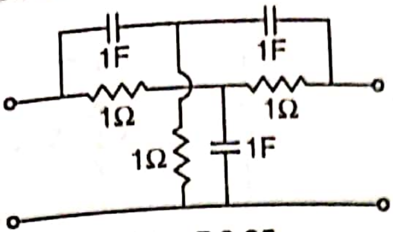


Fig. P.8.25.

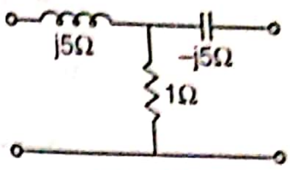


Fig. P.8.26.

8.25. Obtain the Y-parameters of the twin-T network as shown in figure P.8.25.

8.26. Transform the network of the figure P.8.26 into π form.

8.27. Determine the ABCD parameters of the networks shown in figures P.8.27 (a) and (b).

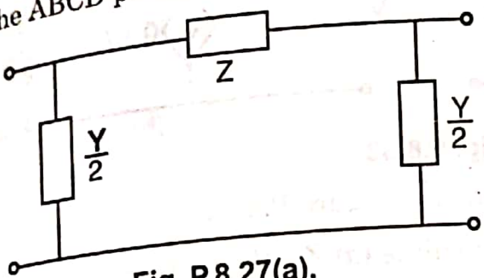


Fig. P.8.27(a).

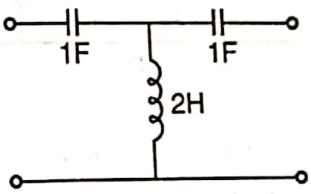


Fig. P.8.27(b).

8.28. Determine the ABCD parameters of the two networks connected in cascade as shown in figure P.8.28.

8.29. Two identical sections of the network shown in figure P.8.29 are connected in parallel. Find the Y-parameters of the resulting network. Also verify the result by direct calculation.

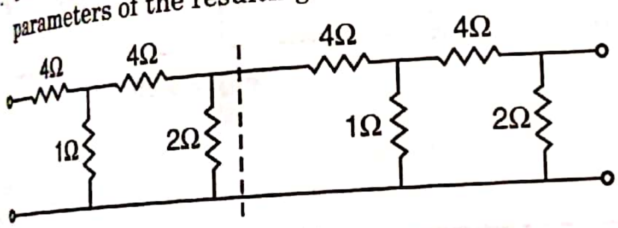


Fig. P.8.28.

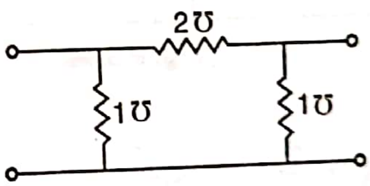
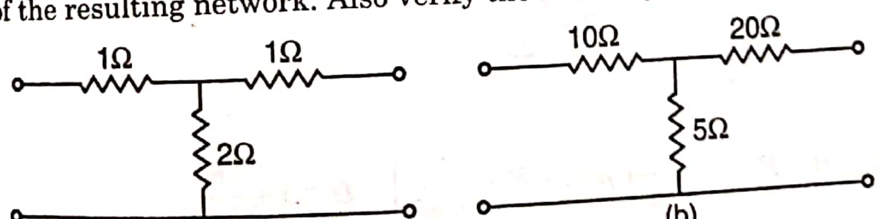


Fig. P.8.29.

8.30. Two two-port networks shown in figures P.8.30(a) and (b) are connected in series. Obtain the Z-parameters of the resulting network. Also verify the result by direct calculation.



(a) (b)

Fig. P.8.30.

8.31. For a two port network, the readings are obtained experimentally as follows :

(a) With output short circuited

$V_1 = 25$ V, $I_1 = 5$ mA, $I_2 = -0.3$ mA

(b) With input short circuited

$V_2 = 30$ V, $I_2 = 10$ mA, $I_1 = 5$ mA

Determine Y-parameters of the network.

8.32. Find the Y-parameters for the network shown in figure P.8.32.

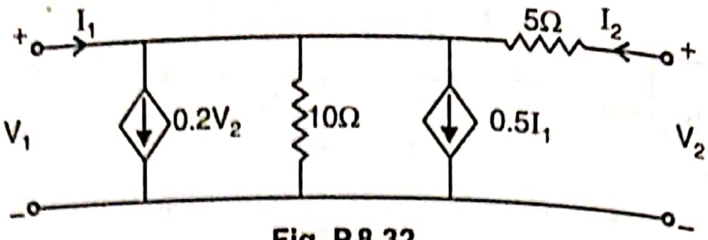


Fig. P.8.32.

8.33. Find the T-parameters for the network shown in figures P.8.33(a) and (b).

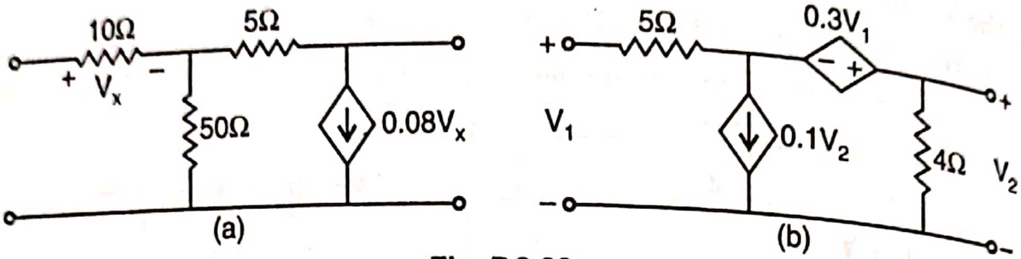


Fig. P.8.33.

8.34. Find the Z-parameters for the network shown in figure P.8.34.

8.35. For the network shown in figure P.8.35, calculate (a) Z_{12} (b) Y_{12} (c) h_{12}.

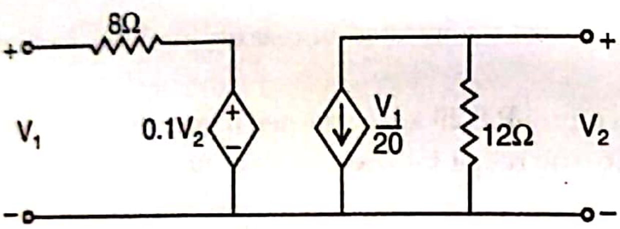


Fig. P.8.34.

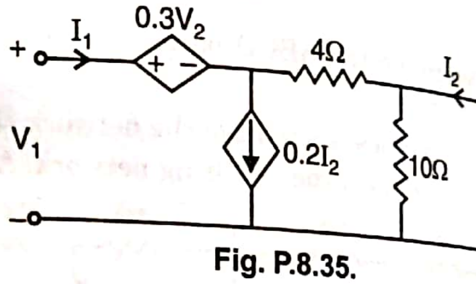


Fig. P.8.35.

ANSWERS

8.1. $[Z]=\begin{bmatrix}\frac{14}{5} & \frac{2}{5}\\ \frac{2}{5} & \frac{6}{5}\end{bmatrix}$, $Y=\begin{bmatrix}\frac{3}{8} & -\frac{1}{8}\\ -\frac{1}{8} & \frac{7}{8}\end{bmatrix}$, $T=\begin{bmatrix}7 & 8\\ 2.5 & 3\end{bmatrix}$

8.2. $A=1+j\omega CR$; $B=R$; $C=\frac{C}{L}\left(j\omega L+R+\frac{1}{j\omega C}\right)$; $D=1+\frac{R}{j\omega L}$

8.3. $Z_{11}=-s,\ Z_{12}=0,\ Z_{21}=-\frac{3s^3}{1+s^2},\ Z_{22}=\frac{s}{1+s^2}$

8.4. $A=1+\frac{1}{2s},\ B=\frac{4s+1}{2s^2},\ C=\frac{1}{2},\ D=1+\frac{1}{2s}$

8.5. $Z_{11}=8.67\ \Omega,\ Z_{12}=0.67\ \Omega,\ Z_{21}=3.33\ \Omega,\ Z_{22}=3.33\ \Omega$

8.6. $Y_{11}=\frac{3}{2}\ ℧,\ Y_{12}=Y_{22}=-\frac{1}{2}\ ℧,\ Y_{21}=4\ ℧$

8.7. $Z_{11}=\frac{4}{15}\Omega,\ Z_{12}=\frac{1}{15}\Omega,\ Z_{21}=-\frac{1}{5}\Omega,\ Z_{22}=\frac{7}{10}\Omega$

$g_{11} = \frac{1}{R_1 + R_2}$, $g_{12} = -\frac{R_2}{R_1 + R_2}$

$Z_{11} = s + 2$, $Z_{12} = s - 1$, $Z_{21} = s$, $Z_{22} = s + 3$

$Y_{11} = \frac{s+3}{5s+6}$, $Y_{12} = Y_{21} = \frac{-s}{5s+6}$, $Y_{22} = \frac{s+2}{5s+6}$

$h_{11} = 50$, $h_{12} = 0$, $h_{21} = 300$, $h_{22} = 0$

$$[T] = \begin{bmatrix} 1+\frac{Z_1}{Z_2} & Z_3 + \frac{Z_1 Z_3}{Z_2} + Z_1 \\ \frac{1}{Z_2} & \frac{Z_3}{Z_2} + 1 \end{bmatrix}$$

$$[Z] = \begin{bmatrix} \frac{7}{2} & \frac{3}{2} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix}, [Y] = \begin{bmatrix} 0.5 & -1 \\ -0.5 & 7/3 \end{bmatrix}$$

(a) $Y_{11} = 5\mho$, $Y_{12} = Y_{21} = -1\mho$, $Y_{22} = 1\mho$

(b) $Y_a = 4\mho$, $Y_b = 0$, $Y_c = 1\mho$

(c) $Z_{ip} = 0.243 \angle 1.7^\circ \Omega$

$Z_a = Z_b = 425.66\Omega$, $Z_c = 474.34\Omega$

$A = 2.5$, $B = 4\Omega$, $C = 1\mho$, $D = 2$

$Z_{i1} = 2.24\Omega$, $Z_{i2} = 1.79\Omega$

$Z_a = Z_b = 400\Omega$, $Z_c = 400\Omega$

$Z_a = \frac{1}{2}\Omega$; $Z_b = 1\,\Omega$; $Z_c = \frac{16}{3}\Omega$; $Y_a = \frac{2}{17}\mho$; $Y_b = \frac{1}{17}\mho$; $Y_c = \frac{32}{51}\mho$

$$Z = \begin{bmatrix} R_1 + R_3 & R_3 \\ \alpha R_2 + R_3 & R_2 + R_3 \end{bmatrix}$$

$$Z = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$$

$$Z = \begin{bmatrix} \frac{13}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} \end{bmatrix}; \; Y = \begin{bmatrix} 3/5 & -2/5 \\ -2/5 & 13/5 \end{bmatrix}$$

$$Y = \frac{1}{2s+6}\begin{bmatrix} s^2 + 5s + 4 & -(s^2 + 3s + 4) \\ -(s^2 + 3s + 4) & s^2 + 7s + 4 \end{bmatrix}$$

$C_a = \frac{4}{5}F$; $C_b = \frac{7}{5}F$; $C_c = \frac{2}{5}F$

$$Y_{11} = Y_{22} = \frac{s^3 + 5s^2 + 5s + 1}{(2s+1)(s+2)}$$

$$Y_{12} = Y_{21} = -\frac{s^3 + 2s^2 + 2s + 1}{(2s+1)(s+2)}$$

8.26.

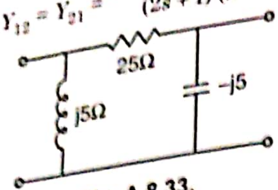


Fig. A.8.33.

8.27. (a) $A = D = 1 + \frac{YZ}{2}$

$B = Z$

$C = Y + \frac{Y^2 Z}{4}$

(b) $A = D = 1 + \frac{1}{2s^2}$

$B = \frac{4s^2 + 1}{2s^3}$

$C = \frac{1}{2s}$

8.28. $T = \begin{bmatrix} 373 & 408 \\ 77 & 109 \end{bmatrix}$

8.29. $Y_{11} = 6℧$

$Y_{12} = Y_{21} = -4℧$

$Y_{22} = 6℧.$

8.30. $Z_{11} = 18\ \Omega$

$Z_{12} = Z_{21} = 7\ \Omega$

$Z_{22} = 28\ \Omega.$

8.31 $Y_{11} = 0.2$ m℧, $Y_{21} = -0.012$ m℧, $Y_{22} = 0.33$ m℧, $Y_{12} = 0.167$ m℧.

8.32. $\begin{bmatrix} 0.6 & 0 \\ -0.2 & 0.2 \end{bmatrix}.$

8.33. (a) $\begin{bmatrix} 10/3 & 400/3 \\ 1/6 & 55/6 \end{bmatrix}$ (b) $\begin{bmatrix} 2.12 & 3.85 \\ 0.35 & 1 \end{bmatrix}.$

8.34. $\begin{bmatrix} 7.55 & 1.13 \\ -4.53 & 11.32 \end{bmatrix}.$

8.35 (a) 9.6 Ω (b) – 0.24 ℧ (c) 1.2

Chapter 9

Inside this

- Introduction
- Concept of Co
  Frequency
- Network Func
- Poles and Zer
  Functions
- Stability
- Natural Re
  Natural Freq
- Time-domai
  From Pole-ze
- Frequency-
  ponse From
- Relation B
  Response a
  Response
- Convolution
- Frequency
- Angle and
  Function
- Bode Plot
- Standard
- Bode Plots
  of $H(j\omega)$
- Steps to
  Plot
- Measure
  lity