

END TERM EXAMINATION
SECOND SEMESTER (B.TECH)-MAY 2014
APPLIED PHYSICS-II-[ETPH-104]

M.M.: 75

Time : 3 hrs.

Note: Attempt any six questions including Q.No. 1 which is compulsory.

Q.1. (a) Find the constant a , the vector $\vec{A} = (x+3y)\hat{i} + (2y+3z)\hat{j} + (x+az)\hat{k}$ is a solenoidal vector. (2)

Ans. We know that

$$\begin{aligned}\vec{A} &= A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \\ &= (x+3y)\hat{i} + (2y+3z)\hat{j} + (x+az)\hat{k} \\ A_x &= x+3y \\ A_y &= 2y+3z \\ A_z &= x+az\end{aligned}$$

Fun Solenoidal vector $\nabla \cdot \vec{A} = 0$

$$\begin{aligned}&= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \\ &= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot ((x+3y)\hat{i} + (2y+3z)\hat{j} + (x+az)\hat{k}) \\ &= \frac{\partial}{\partial x}(x+3y) + \frac{\partial}{\partial y}(2y+3z) + \frac{\partial}{\partial z}(x+az) \\ &= 1 + 2 + a = 0\end{aligned}$$

$a = -3$

Q.1.(b) State Gauss's law for electric field. Express it in its integral form. (2)

Ans. The total outward flux of an electric field own a closed surface is equal to $\frac{1}{\epsilon}$ times the total net charge contained in a volume enclosed by the surface. In integral form,

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon}$$

where,

E – Electric field intensity
 q – Charge enclosed by surface.
 $d\vec{s}$ – surface element

ϵ – permittivity of the medium

Q.1. (c) State the Poynting theorem. Explain the term poynting vector. (2)
 Ans. Poynting Theorem: The time rate of change of electromagnetic energy with certain volume plus time rate of energy flowing out through the boundary surface is equal to the power transferred into the electromagnetic field.

Poynting vector –

$$\vec{S} = \vec{E} \times \vec{H}$$

\vec{S} is known as poynting vector and is interpreted as the power flux, i.e., amount of energy crossing unit area placed perpendicular to the vector, per unit time.

18-2014

Second Semester, Applied Physics-II

$$\lambda_m T = 2.89 \times 10^{-3} \text{ mK}$$

$$\lambda_m = \frac{2.89 \times 10^{-3}}{1370.45} \text{ m}$$

$$\lambda_m = 2.109 \times 10^{-6} \text{ m}$$

$$\boxed{\lambda_m = 2109 \text{ nm}}$$

Q.1.(g) Set up a relationship between group and phase velocity. (2)

Ans. Phase velocity $\Rightarrow V_p = \frac{w}{k}$

Group Velocity $\Rightarrow V_g = \frac{dw}{dx}$

Where, w is the angular frequency of the wave and k is the wave vector.
But

$$w = v_p k$$

$$V_g = \frac{d}{dk}(v_p k)$$

$$V_g = v_p + k \frac{dV_p}{dx}$$

$$V_g = V_p + k \frac{dV_p}{d\lambda} \frac{d\lambda}{dx}$$

We know that

$$k = \frac{2\pi}{\lambda}$$

$$\frac{d\lambda}{dk} = \frac{d}{dx}\left(\frac{2\pi}{x}\right) = \frac{-2\pi}{k^2}$$

But

$$V_g = V_p + x \frac{dV_p}{d\lambda} \left(\frac{-2\pi}{k^2} \right)$$

$$V_g = V_p - \frac{2\pi}{k} \frac{dv_p}{d\lambda}$$

$$\boxed{V_g = V_p - \frac{\lambda dV_p}{d\lambda}}$$

Q.1.(h) A particle limited to the x-axis has the wave function $\psi = ax$, between $x = 0$ and $x = 1$ and $\psi = 0$ elsewhere. Find (a) the probability that particle can be found between $x = 0.45$ and $x = 0.55$ (b) The expectation value $\langle x \rangle$ of the particle's position. (3)

Ans. Given-

$$\begin{aligned} \psi &= ax \quad \text{when } x = 0 \text{ and } x = 1 \\ \psi &= 0 \quad \text{elsewhere} \end{aligned}$$

$$(a) \quad \text{Probability} = \int_{x_1}^{x_2} |\psi|^2 dx = a^2 \int_{0.45}^{0.55} x^2 dx$$

$$= a^2 \left[\frac{x^3}{3} \right]_{0.45}^{0.55} = 0.0251 a^2$$

$$\boxed{P = 0.0251 a^2}$$

(b) Expectation value of $\langle x \rangle$ is

$$\begin{aligned}
 \langle x \rangle &= \int_0^1 \psi^* x \psi dx \\
 (2) &= \int_0^1 x |\psi|^2 dx = a^2 \int_0^1 x^3 dx \\
 &= a^2 \left[\frac{x^4}{4} \right]_0^1 = \frac{a^2}{4} \\
 \boxed{\langle x \rangle = \frac{a^2}{4}}
 \end{aligned}$$

Q.1.(i) Copper has FCC structure and the atomic radius is 1.278 \AA . Calculate the density of copper crystal (Atomic weight of copper = 63.5, Avogadro's number 6.023×10^{23}). (2)

Ans. We know that—

$$\begin{aligned}
 \rho &= \frac{n M}{a^3 N} = \frac{nM}{\left(\frac{4r}{\sqrt{2}} \right)^3 N} \\
 &= \frac{4 \times 63.54}{\left(\frac{4 \times 1.278 \times 10^{-8}}{\sqrt{2}} \right)^3 \times (6.02 \times 10^{23})} \\
 \boxed{\rho = 8.98 \text{ gm/cc}}
 \end{aligned}$$

Q.1.(j) What is difference between schottky and Frenkel's defects? (2)

Ans. Schotky Defect: In real crystals, there are irregularities of the atomic array in which atoms are missing at some lattice points. Such a point is called vacancy. This vacancy is known as schottky defect.

Frenkel defect: The extra interstitial atoms are squeezed between normal atom but when an interstitial is caused by transferring an atom from a lattice site to an interstitial position, a vacancy is created. The associated vacancy and interstitial atom is called frenkel defect. (2)

Q.1. (k) Draw the planes (101), (100) and (200)

Ans. The planes are as following—

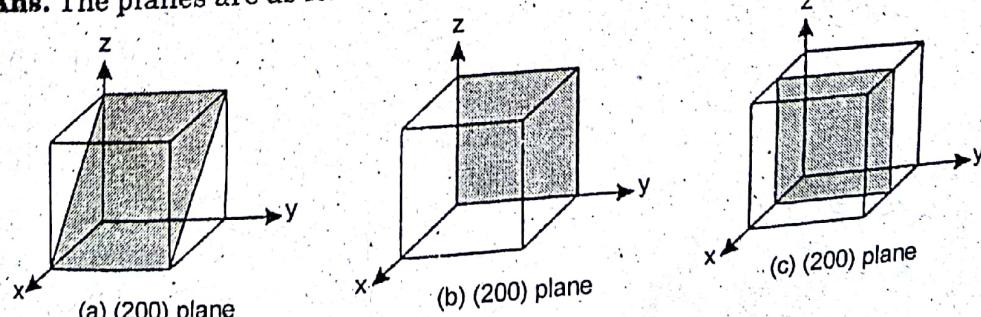


Fig. 2

$$J = \sigma E$$

This is actually another form of Ohm's law. Now, from Eqs. (8) and (9) we have ... (1)

$$\vec{\nabla} \cdot (\sigma \vec{E}) = 0$$

$$\text{or } \sigma \vec{\nabla} \cdot \vec{E} + \vec{E} \cdot \vec{\nabla} \sigma = 0 \quad \dots(12)$$

For a homogeneous medium $\vec{\nabla} \sigma = 0$ and Eq. (12) becomes

$$\vec{\nabla} \cdot \vec{E} = 0$$

Substituting $E = -\vec{\nabla} V$, where V is the potential at a point within a conductor, we can write the preceding equation as

$$\vec{\nabla}^2 V = 0 \quad \dots(13)$$

Equation (13) asserts that the potential distribution within a conducting medium satisfies Laplace's equation as long as the medium is homogeneous and the current distribution is time invariant or steady with time.

Physical Significance of Equation of Continuity

The equation of continuity implies that, in absence of source or sink of charge the rate of decrease of charge in a certain volume of the conductor is equal to the rate of flow of charge into the volume through the surface enclosing the volume. The continuity equation represents conservation of charge.

Q.2.(b) A long cylinder carries a charge density (ρ) which is proportional to the distance from the axis $\rho = kr$, for some constant k . Find electric field inside the cylinder.

Ans. Let us consider a Gaussian cylinder of length l and radius r . For this Gauss's surface, Gauss's law is as

$$\oint \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} (q_{\text{enclosed}}) \quad \dots(1)$$

Now enclosed charge in the Gaussian surface

$$q_{\text{enclosed}} = \int \rho d\tau = \int (kr') (r' dr' d\phi dz)$$

\Rightarrow

$$q_{\text{enclosed}} = 2\pi k l \int_0^r r'^2 dr' = \frac{2}{3} k l r^3 \quad \dots(2)$$

Here the volume element $d\tau = r' dr' d\phi dz$ in appropriate cylindrical coordinates. Here the limit of integration $\phi \rightarrow$ from 0 to 2π ; $dz \rightarrow$ from 0 to l ; r is chosen to distinguish it from r of the Gaussian surface.

Here E is directed radially outward so for the curved surface portion of the Gaussian cylinder, we have net—

$$\phi_E = \int \vec{E} \cdot d\vec{S} = \int |E| dS = E(2\pi rl) \quad \dots(3)$$

The plane surface contributes no field as $\theta = 90^\circ$ between the normal to the plane surface and the direction of E .

Applying Gauss's law,

$$\phi_E = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$E(2\pi rl) = \frac{2}{3} \pi k l r^3$$

$$E = \frac{1}{3\epsilon_0} kr^2 \hat{r} \text{ N/C}$$

Q.2. (c) What is displacement current?

(2.5)

Ans. In one of the Maxwell's equations, the changing electric field term, is known as the displacement current D. This was an analogy with a dielectric material. If a dielectric material is placed in an electric field, the molecules are distorted, their positive charges moving slightly to the right, say, the negative charges slightly to the left. Now consider, what happens to dielectric in an increasing electric field. The positive charges will be displaced to the right by a continuous increasing distance, so, as long as the electric field is increasing in strength, these charges are moving; there is actually a displacement current. Meanwhile, the negative charges are moving the other way, so adds to the effect of the positive charges motion.

Q.3. (a) Discuss the propagation of E.M. waves in dielectric. Derive an expression for phase velocity and show that it is less than the speed of light. Also find wave impedance in Dielectric media.

(7)

Ans. Maxwell's equations are:

$$\left. \begin{array}{l} \text{div } \vec{D} = \vec{\nabla} \cdot \vec{D} = \rho \quad \dots(a) \\ \text{div } \vec{B} = \vec{\nabla} \cdot \vec{B} = 0 \quad \dots(b) \\ \text{curl } \vec{E} = \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \dots(c) \\ \text{curl } \vec{H} = \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \dots(d) \end{array} \right\} \quad \dots(1)$$

and

In an isotropic dielectric (or non-conducting isotropic medium)

$$\vec{D} = \epsilon \vec{E}, \vec{B} = \mu \vec{H}, \vec{J} = \sigma \vec{E} = 0 \text{ and } \rho = 0 \quad \dots(2)$$

Therefore, Maxwell's equations in this case take the form

$$\text{div } \vec{E} = \vec{\nabla} \cdot \vec{E} = 0 \quad \dots(a)$$

$$\text{div } \vec{B} = \vec{\nabla} \cdot \vec{B} = 0 \quad \dots(b)$$

$$\text{curl } \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \dots(c)$$

$$\text{curl } \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \quad \dots(d) \quad \dots(3)$$

and

Taking curl of Eq. 3 (c), we get

$$\text{curl curl } \vec{E} = -\mu \frac{\partial}{\partial t} (\text{curl } \vec{H})$$

Substituting curl H from Eq. 3(d) in above equation.

$$\text{curl curl } \vec{E} = -\mu \frac{\partial}{\partial t} \left(\epsilon \frac{\partial \vec{E}}{\partial t} \right)$$

$$\text{curl curl } \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \dots(4)$$

i.e.,

$$\text{curl curl } \vec{H} = -\mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \dots(5)$$

Similarly, if we take curl of Eq.3 (d) and substitute curl \vec{E} from Eq. 3. (c), we get

$$\text{curl curl } \vec{H} = -\mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \dots(5)$$

and keeping in mind Eqs.3(a) and 3. (b) i.e., $\text{div } \vec{E} = 0$ and $\text{div } \vec{H} = 0$ Eqs. (4) and (5) give

The unit of Z comes out to be Ohm, since

$$Z = \left| \frac{E}{H} \right| = \frac{\text{Volt/m}}{\text{Amp/m}} = \frac{\text{Volt}}{\text{Amp}} = \text{Ohm}$$

Hence the value Z is referred to as wave impedance of isotropic dielectric medium. The wave of impedance of medium is related to that of free space by the relation.

$$Z = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} = \sqrt{\frac{\mu_r}{\epsilon_r}} Z_0 \quad \dots(26)$$

where $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ is called the wave impedance of free space.

Q. (b) Show that equation of continuity $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$ is contained in Maxwell's equation.

Ans. From Maxwell's IVth equation—

$$\nabla \times \vec{B} = \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$

Taking divergence on both sides.

$$\nabla \cdot (\nabla \times \vec{B}) = \mu (\nabla \cdot \vec{J}) + \mu \epsilon \frac{\partial}{\partial t} (\nabla \cdot \vec{E})$$

But

$$\nabla \cdot (\nabla \times \vec{B}) = 0$$

$$\mu \left(\nabla \cdot \vec{J} + \epsilon \frac{\partial}{\partial t} (\nabla \cdot \vec{E}) \right) = 0$$

$$\nabla \cdot \vec{J} + \epsilon \frac{\partial}{\partial t} (\nabla \cdot \vec{E}) = 0$$

Also, from Maxwell's Ist equation—

$$\nabla \cdot \vec{E} = \rho / \epsilon$$

$$\nabla \cdot \vec{J} + \epsilon \times \frac{1}{\epsilon} \frac{\partial \rho}{\partial t} = 0$$

$$\boxed{\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

This is the continuity equation.

Q.3. (c) Find skin depth at a frequency 2 MHz in copper where—

$$\sigma = 6 \times 10^7 \frac{\text{mho}}{\text{m}}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ W/A-m}$$

Ans. Skin depth is—

$$\delta = \sqrt{\frac{2}{\mu \sigma \omega}} = \sqrt{\frac{2}{\mu \sigma 2\pi f}}$$

Given

$$\delta = \sqrt{\frac{1}{\pi \mu \sigma f}}$$

$$\mu = 4\pi \times 10^{-7}; \sigma = 6 \times 10^7; f = 2 \times 10^6$$

$$\delta = \sqrt{\frac{1}{3.14 \times 4\pi \times 10^{-7} \times 6 \times 10^7 \times 2 \times 10^6}}$$

$$\boxed{\delta = 45.9 \mu m}$$

UNIT-II

Q.4. (a) Derive Planck's formula for the distribution of energy in the spectrum of a black body, Deduce from it Wein's law and Rayleigh Jean's law.

(6)

(3) **Ans.** If N be the total number of Planck's oscillators and E is their energy, then the average energy per Planck's oscillator is given by-

$$\bar{E} = \frac{E}{N} \quad \dots(1)$$

Let, there be $N_0, N_1, N_2, N_3, \dots, N_r$ oscillators having energy $0, E, 2E, \dots, rE$ respectively.

Now, we have $N = N_0 + N_1 + N_2 + N_3 + \dots + N_r$

and $E = 0 + EN_1 + 2EN_2 + 3EN_3 + \dots + rEN_r$... (3)

According to Maxwell's distribution formula, the number of oscillators having energy rE is given by

$$N_r = N_0 e^{-rE/k_B T} = N_0 \exp\left[-\frac{rE}{k_B T}\right] \quad \dots(4)$$

where k_B = Boltzmann's constant $= 1.3807 \times 10^{-23} \text{ J/K}$.

Substituting the values of N_1, N_2, N_3, \dots from Eq. (4) in Eqs. (2) and (3) we have

$$N = N_0 + N_0 \exp\left(-\frac{E}{k_B T}\right) + N_0 \exp\left(-\frac{2E}{k_B T}\right) + \dots + N_0 \exp\left(\frac{-rE}{k_B T}\right)$$

$$= N_0 \left[1 + \exp\left(-\frac{E}{k_B T}\right) + \exp\left(-\frac{2E}{k_B T}\right) + \exp\left(-\frac{3E}{k_B T}\right) + \dots + \exp\left(\frac{-rE}{k_B T}\right) \right]$$

$$\text{Now let } \exp\left[\frac{-E}{k_B T}\right] = x$$

$$\text{Then } N = N_0 [1 + x + x^2 + x^3 + \dots]$$

$$\Rightarrow N = N_0 \times \frac{1}{1-x}$$

Particular	M.B. Statistics	B.E. Statistics	F.D. Statistics
Nature of particles	Particles are identical but distinguishable	Particles are identical but indistinguishable	Particles are identical but indistinguishable.
Size of the phase cell	The available volume of the phase space cell can be small as we like and can even approach zero.	Minimum size of the available phase space volume is of the order of \hbar^3 , where \hbar is the Planck's constant.	Minimum size of the available phase space volume is of the order of \hbar^3 .
Number of phase cells	The phase space can be divided into any large number of cells of our choice.	The phase space can be divided into number of cells whose number is limited as the phase space volume of each cell cannot be less than \hbar^3 .	The phase space can be divided into the number of cells, whose number is limited as the phase space volume of each cell cannot be less than \hbar^3 .
Number of particles in a cell	Any number of particles can occupy a single phase space cell.	Any number of particles can occupy a single phase cell	A phase space cell cannot accommodate more than one particle.
Macrostates and Microstates	A macrostate can have more than one microstates	Each macrostate has one microstate.	Each macrostate has one microstate.
Energy	Continuous distribution energy	Energy is quantized.	Energy is quantized.
Spin	The particles are spinless.	The particles have half integral spin.	The particles have half integral spin.
Occupation index	$\frac{1}{e^{\alpha+\beta E_i} - 1}$	$\frac{1}{e^{\alpha+\beta E_i} + 1}$	$\frac{3}{2} N k_B T \left(1 + \frac{1}{z^{5/2} e^{-\alpha}} \right)$
Total energy	$\frac{3}{2} N k_B T$	$\frac{3}{2} N k_B T \left(1 - \frac{1}{z^{5/2} e^{-\alpha}} \right)$	Symmetric under interchange of the coordinates of any two Bosons.
Wave function			Antisymmetric on interchange of the coordinates of any two Fermions.

Q.4. (c) Using exchange symmetry of wave function. Show that the Boson do not obey Pauli Exclusion Principle. (2.5)

Ans. The particles whose spin angular momentum are integral multiple of \hbar are called Bosons. They have integral spins $0, \hbar, 2\hbar, 3\hbar, \dots$

Let there be N particles described by the wave function $\psi(1, 2, 3, \dots, r, \dots, s, \dots, N)$, where 1 stands for all coordinates (position and spin) of particle 1 and so on. If the wave function is operated by an exchange operation \hat{P} then

$$\hat{P}_{r,s} \psi(1, 2, 3, \dots, r, \dots, s, \dots, N) = P \psi(1, 2, 3, \dots, s, \dots, r, \dots, N) \quad \dots(1)$$

The effect of the operation \hat{P}_{rs} is to interchange the particles r and s i.e., formerly called particle ' r ' is now called ' s ' and vice versa. For two particles system, we write.

$$\hat{P}_{12} \psi(1, 2) = P \psi(2, 1) \quad \dots(2)$$

Equation (2) is the eigenvalue equation of the operator $\hat{P}_{r,s}$ and P stands for the eigenvalue of the operator $\hat{P}_{r,s}$.

There are two kinds of wave function ψ , depending on value of \hat{P} -symmetric wave function and antisymmetric wave functions.

(i) Symmetric Wave Function (ψ_S)

A wave function is said to be symmetric if the interchange of any pair of particles among its arguments, leave the wave function unchanged.

i.e.,

$$\hat{P}_{1,2} \psi_S(1, 2) = +\psi_S(2, 1) \quad (\text{here } P = 1) \quad \dots(3)$$

(ii) Antisymmetric Wave Function (ψ_A)

The wave function is antisymmetric if the interchange of any pair of particles among its arguments, changes the sign of the wave function.

$$\hat{P}_{1,2} \psi_A(1, 2) = -\psi_A(2, 1); \quad (\text{here } P = -1)$$

The exchange symmetry of the wave functions has close relation with the intrinsic angular momentum of the particle. The relationship is listed as follows:

(a) The identical particles having an integral quantum number of their intrinsic spins are described by symmetric wave functions, such as

$$\psi_S(1, 2) = +\psi(2, 1) \quad \dots(5)$$

The type of particles described by symmetric wave functions are known as **BOSONS** as they obey Bose-Einstein statistics.

(b) The particles which have half integral quantum number of their intrinsic spins are described by the antisymmetric wave functions, such as

$$\psi_A(1, 2) = -\psi(2, 1) \quad \dots(6)$$

The type of particles described by antisymmetric wave functions are called **FERMIONS** as they obey Fermi-Dirac statistics.

Q.5. (a) Set up the Schrodinger equation for a particle in an infinite well (one-dimensional) Solve it for eigenvalues and eigen functions and plot the first three eigen functions ψ_1, ψ_2, ψ_3 and also plot its probability. (3 + 4 + 3 = 10)

Ans. Consider a particle moving inside a box along the x -direction. The particle is bouncing back and forth between the walls of the box. The width of the box is shown in Fig. 3.

$$V = 0 \text{ for } 0 < x < l$$

$$V = \infty \text{ for } x \leq 0 \text{ and } x \geq l$$

The particle cannot exist outside the box, so its wave functions ψ is 0 for $x \leq 0$ and $x \geq l$.

$$\Rightarrow \frac{A^2}{2} \left[\int_0^l dx - \int_0^l \cos \frac{2n\pi x}{l} dx \right] = 1$$

$$\Rightarrow \frac{A^2}{2} [l] = 1$$

$$\Rightarrow A = \sqrt{\frac{2}{l}}$$

$$\therefore \text{Eigen function } \psi_n = \sqrt{\frac{2}{l}} \sin \frac{n\pi x}{l}$$

where

$$n = 1, 2, 3, \dots \quad \dots(5)$$

Although ψ_n may be negative as well as positive, $|\psi_n|^2$ is always positive and since ψ_n is normalised, its square value at a given x is equal to the probability of finding the particle. The first three eigen functions ψ_1, ψ_2, ψ_3 together with probability densities $|\psi_1|^2, |\psi_2|^2, |\psi_3|^2$ are shown in Figs. 4(a) and (b) respectively.

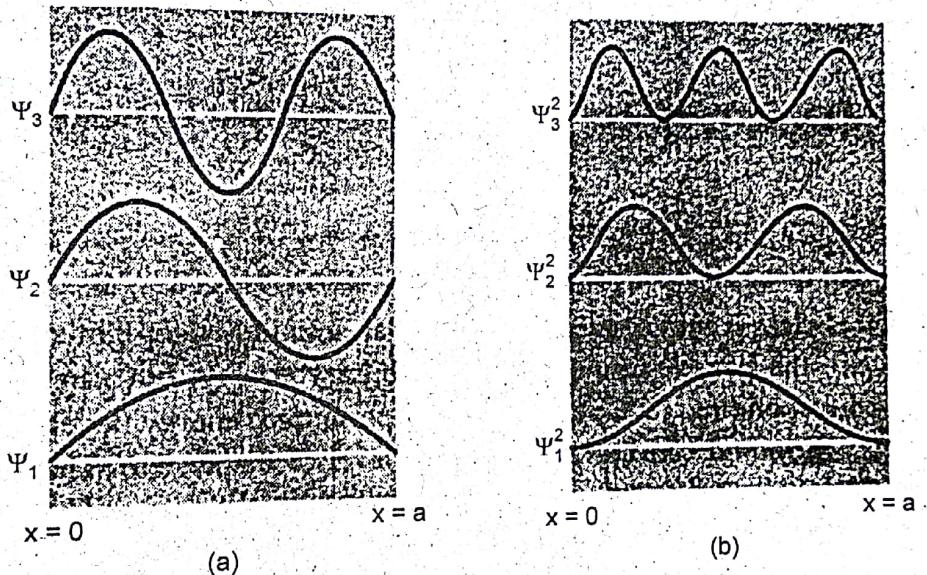


Fig. Wave-functions and probability densities of a particle confined to a box with rigid walls.

Q.5.(b) By applying uncertainty principle explain non-existence of electron in atomic nucleus. (2.5)

Ans. According to theory of relativity, energy of a particle is given by the relation

$$E^2 = p^2 c^2 + m_0^2 c^4 \quad \dots(1)$$

where p = momentum of particle and m_0 = rest mass of the particle

According to Heisenberg's uncertainty principle,

$$\Delta x \Delta p_x = \frac{h}{2\pi} \quad \dots(2)$$

The diameter of the nucleus is of the order of 10^{-14} m. If the electron exists in the nucleus, it can be anywhere within the diameter of the nucleus. Therefore, the maximum uncertainty Δx in the position of electron is the same as the diameter of the nucleus.
i.e., $\Delta x = 10^{-14}$ m

∴ The minimum uncertainty in the momentum is given by

$$\Delta p_x = \frac{h}{2\pi\Delta x} = \frac{6.63 \times 10^{-34}}{2\pi \times 10^{-14}} = \frac{6.63 \times 10^{-20}}{2\pi}$$

$$= 1.055 \times 10^{-20} \text{ kg m/s}$$

It means that if electron exists in the nucleus, its minimum momentum must be $p_{\min} = 1.055 \times 10^{-20} \text{ kg m/s}$

For the electron of the minimum momentum, the minimum energy is given by

$$E_{\min}^2 = p_{\min}^2 c^2 + m_0^2 c^4$$

$$= (1.055 \times 10^{-20} \times 3 \times 10^8)^2 + (9.1 \times 10^{-31})^2 \times (3 \times 10^8)^4$$

$$= (3 \times 10^8)^2 [1.113 \times 10^{-40} + 7.4692 \times 10^{-40}]$$

Since the second term in the bracket is much smaller than the first, it can be neglected, then,

$$E_{\min} = 3 \times 10^8 \sqrt{1.113 \times 10^{-40}} \text{ J}$$

$$= 3 \times 10^8 \times 1.055 \times 10^{-20} \text{ J} = 3.1649 \times 10^{-12} \text{ J}$$

or $E_{\min} = \frac{3.1649 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} \approx 20 \text{ MeV.}$

Thus, if a free electron exists in the nucleus it must have a minimum energy of about 20 MeV.

The maximum K.E. which a β -particle, emitted from radioactive nuclei is of the order of 4 MeV. Therefore, this clearly shows that electrons cannot be present within the nucleus.

UNIT-III

Q.6. (a) Deduce atomic radius, co-ordination number and packing fraction of body centered cubic and closed packed structure. (5)

Ans. Body Centered Cubic (bcc) Structure

A body centered cubic structure has eight corner atoms and one body centered atom. The atom at centre touches all eight corner atoms. The bcc structure is shown in Fig. 5.

In bcc unit cell, each and every corner atom is shared by eight adjacent unit cells.

So, the total number of atoms contributed by the corner atoms is

$$\frac{1}{8} \times 8 = 1$$

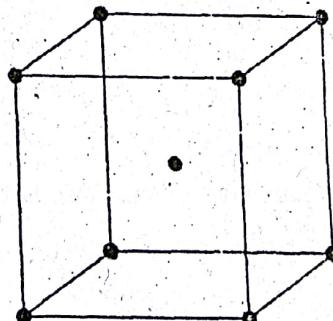


Fig. 5. Body Centered Cubic Structure.

A bcc unit cell has one full atom at the centre of the unit cell.

∴ Total number of atoms present in bcc unit cell

$$= 1 + 1 = 2$$

Coordination Number (CN): Let us consider a body centered atom. The nearest neighbour for a body centered atom is a corner atom. A body centered atom is surrounded by eight corner atoms.

Therefore, the coordination number of a bcc unit cell = 8.
(Figure 6 shows 8 atoms in bcc unit cell.)

Atomic Radius (r): For a body centered cubic unit cell, the atomic radius can be calculated by the following way:

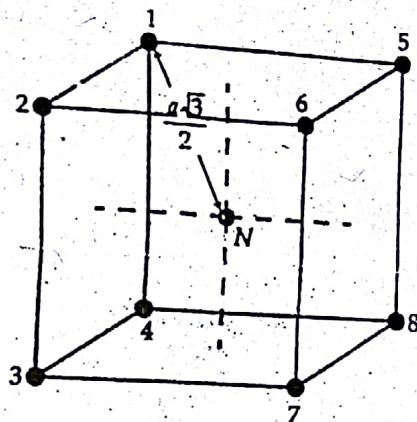


Fig. 6. Eight atoms in bcc crystal.

From Fig. 7.

$$AG = 4r \text{ and } DG = a$$

$$\text{From } \Delta AGD, \quad AD^2 + DG^2 = AG^2$$

For AD , from ΔABD

$$AB^2 + BD^2 = AD^2 \quad \dots(2)$$

or

$$a^2 + a^2 = AD^2$$

i.e.,

$$AD^2 = 2a^2$$

Therefore,

$$AD = \sqrt{2}a$$

Substituting the values of AD , AG and DG in Eq. (1) we get.

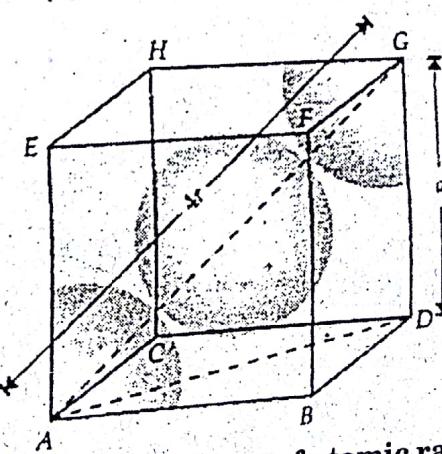


Fig. 7. Calculation of atomic radius

$$AD^2 + DG^2 = AG^2$$

$$2a^2 + a^2 = (4r)^2$$

$$16r^2 = 3a^2$$

$$r^2 = \frac{3}{16}a^2$$

or

i.e.,

$$r = \frac{\sqrt{3}}{4}a \quad \dots(3)$$

Therefore, the atomic radius

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Atomic Packing Factor (APF)/Packing Density

$$APF = \frac{v}{V}$$

The number of atoms present in an unit cell = 2

$$v = 2 \times \frac{4}{3} \times \pi r^3; V = a^3$$

$$APF = \frac{2 \times \frac{4}{3} \times \pi r^3}{a^3}$$

Substituting $r = \frac{\sqrt{3}a}{4}$, we get

$$APF = \frac{2 \times \frac{4}{3} \pi \left(\frac{\sqrt{3}a}{4} \right)^3}{a^3}$$

$$\Rightarrow APF = \frac{\sqrt{3}\pi}{8} = 0.68 \quad \dots(5)$$

Thus, 68% of the volume of the bcc unit cell is occupied by atoms and remaining 32% volume of the unit cell is vacant or void space.

One of closed packed structures is Face-Centred cubic crystal (FCC). Its details are given below.

Face Centered Cubic (fcc) structure

A face centered cubic cell consists of eight corner atoms and six face centered atoms. A face centered cubic unit cell is shown in Fig.8.

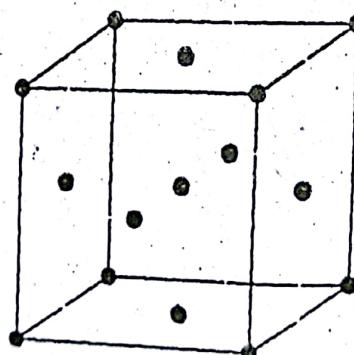


Fig. 8. Face centred cubic unit cell

The atom in a fcc unit cell touches along the face diagonal. Each and every corner atom is shared by eight adjacent unit cells. Therefore each and every corner atom contributes $\frac{1}{8}$ of its part to one unit cell.

So, the number of atoms contributed by the corner atoms = $\frac{1}{8} \times 8 = 1$.

Two unit cells share each and every face centered atom. Therefore, the contribution of a face centered atom to unit cell is $\frac{1}{2}$. So the total number of atoms contributed by face centered atoms = $\frac{1}{2} \times 6 = 3$

\therefore Total number of atoms present in a fcc unit cell = $1 + 3 = 4$.

Coordination Number (CN)

Let us consider a corner atom. In its own plan it touches four face centered atoms. These face centered atoms are its nearest neighbours. In a plane, which lies just above this corner atom, it has four more face centered atoms as nearest neighbours. Similarly, in a plane, which lies just below this corner atom, it has yet four more face centered atoms as its nearest neighbours.

Therefore, the number of nearest neighbour i.e., coordination number for fcc unit cell = $4 + 4 + 4 = 12$. These CN are shown in Fig. 9.

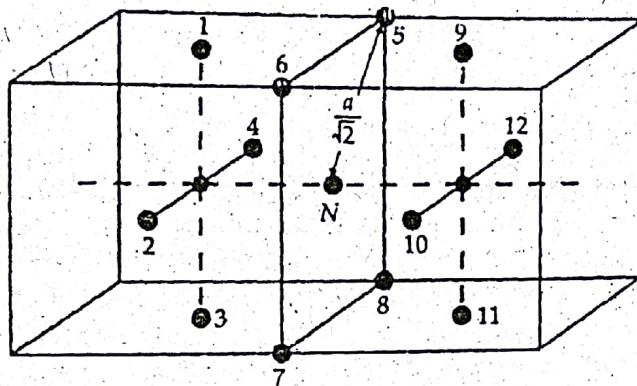


Fig. 9. Atoms in fcc crystal.

Atomic Radius (r)

The atomic radius can be calculated from Fig. 10 as follows:

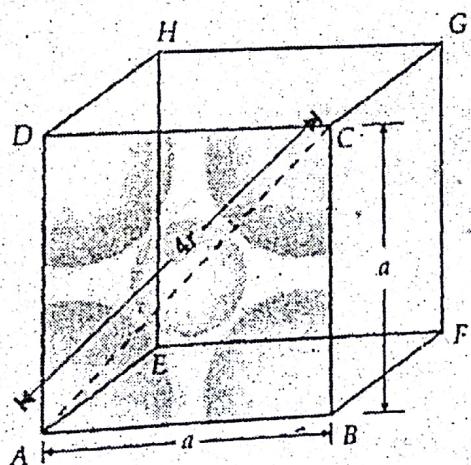


Fig. 10. Calculation of atomic radius of fcc.

Consider $\triangle ABC$,

$$AC^2 = AB^2 + BC^2$$

$$(4r)^2 = a^2 + a^2$$

$$16r^2 = 2a^2$$

$$r = \frac{a}{2\sqrt{2}} \quad \dots(6)$$

The atomic radius

Atomic Packing Factor (APF)/Packing Density

$$APF = \frac{v}{V}$$

$$v = \frac{4}{3} \times 4\pi r^3; V = a^3$$

$$APF = \frac{4 \times 4\pi r^3}{3a^3}$$

Substituting

$$r = \frac{2a}{4}, \text{ we get}$$

$$\text{APF} = \frac{\pi}{3\sqrt{2}} = 0.74$$

..(7)

Thus 74% of the volume of the fcc unit cell is occupied by atoms and the remaining 26% volume of the unit cell is vacant or void space.

Q. (b) What are Miller indices? Show that spacing d of plane (hkl) in a simple cubic lattice of side a is $d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

struc
maxi
A

Ans. The orientation of planes or faces in a crystal can be described in terms of their intercepts on the three axes. Miller introduced a system to designate a plane in a crystal. He introduced a set of three numbers to specify a plane in a crystal. This set of three numbers is known as Miller indices of the plane.

Let us consider a cube with edge a and a plane IJK as shown in Fig. 11.

Let the Miller indices of the plane be $[h k l]$. Now, draw HL perpendicular from origin H of the cube to the plane IJK .

Let a be the lattice parameter i.e., cube edge and d be the perpendicular distance between the origin H and the plane i.e., HL .

HI , HJ and HK be the intercepts of the plane along x , y and z -axes respectively.

We know that the Miller indices of a plane are the smallest integers of the reciprocals of its intercepts. Therefore the intercepts may also be expressed as reciprocals of Miller indices or in other words.

$$HI : HJ : HK = \frac{1}{h} : \frac{1}{k} : \frac{1}{l} = \frac{a}{h} : \frac{a}{k} : \frac{a}{l}$$

From the geometry of Fig. 11 (right angles HIL , HJL and HKL) we know that

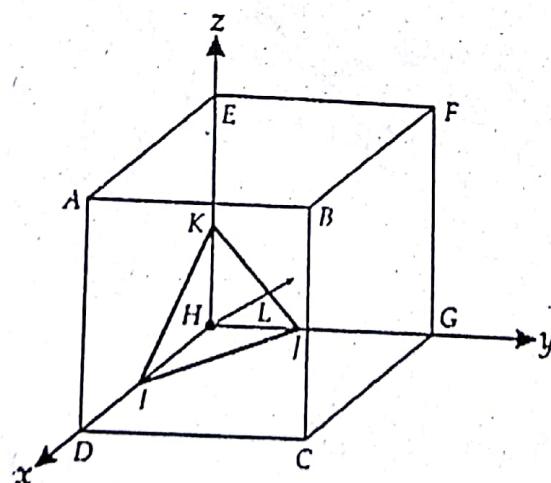


Fig. 11. Spacing of the Plane IJK.

$$\cos \alpha = \frac{HL}{HI} = \frac{hd}{a}$$

$$\cos \beta = \frac{HL}{HJ} = \frac{kd}{a}$$

$$\cos \gamma = \frac{HL}{HK} = \frac{ld}{a}$$

and

Disc
atom
path

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\frac{d^2}{a^2} (h^2 + k^2 + l^2) = 1$$

Or

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Q.6.(c) Calculating the glancing angle of the (111) plane of simple cubic structure atomic radius $r = 1.404 \text{ \AA}$ corresponding to second order diffraction maxima for the x-rays of wavelength $\lambda = 1 \text{ \AA}$ (3.5)

Ans. We know that-

$$2d \sin \theta = n\lambda$$

and

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$= \frac{2.808 \text{ \AA}}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{2.808}{\sqrt{3}} \text{ \AA}$$

$$\sin \theta = \frac{n\lambda}{2d}$$

$$\theta = \sin^{-1} \left(\frac{n\lambda}{2d} \right)$$

$$\theta = \sin^{-1} \left(\frac{2 \times 1 \times 10^{-1} \times \sqrt{3}}{2.808 \times 10^{-10}} \right)$$

$$\theta = \sin^{-1}(1.23)$$

This is not possible and it is only possible at $n = 1$.

Q.7. (a) Derive Bragg's law of crystal diffraction and give its significance.

Discuss briefly the Laue method of crystal structure determination. (6)

Ans. Let us consider two parallel rays LMN and PQR , which are reflected by two atoms M and Q as shown in Fig. 12. The atom Q is vertically below M . The length of the path PQR is greater than

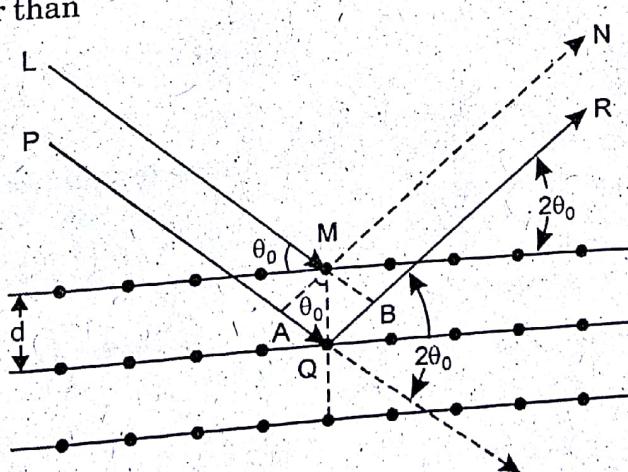


Fig. 12 Illustration for the derivation of the Bragg's law.
The length of the path LMN . The path difference is $(AQ + QB)$ and according to the condition of reflection, we have.
 $(AQ + QB) = n\lambda$... (1)

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But from Fig. 12

$$AQ = BQ = d \sin \theta_0$$

$$2d \sin \theta_0 = n\lambda$$

Equation (2) is known as *Bragg's law* and gives the condition for reflection of X-rays from series of atomic layers in a given plane. For a given set up with monochromatic radiation, the wavelength λ is fixed and hence Eq. (2) has only a particular set of solutions namely.

For

$$n = 1; \theta_0 = \sin^{-1} \left(\frac{\lambda}{2d} \right)$$

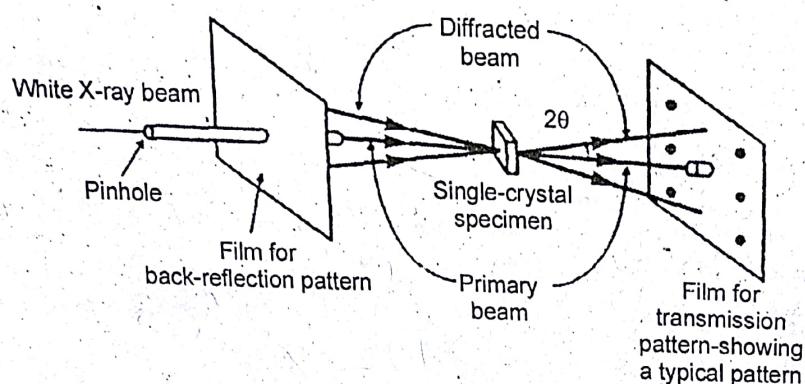
$$n = 2; \theta_0 = \sin^{-1} 2 \left(\frac{\lambda}{2d} \right)$$

$$n = 3; \theta_0 = \sin^{-1} 3 \left(\frac{\lambda}{2d} \right) \dots \text{etc}$$

These are known as the first, second, third,... reflection according to n is 1, 2, 3,... This shows that crystal cannot give rise to reflections at any angle but only at those discrete angles indicated by Eq. (3). Bragg's reflection can occur only for wavelength $\lambda \leq 2d$. Due to this fact the visible light wavelength cannot be used in diffraction.

Laue Method of crystal structure determination

A single crystal is mounted on a goniometer, which enables the crystal to be rotated through known angles in two perpendicular planes, and maintained stationary in a beam of X-rays ranging in wavelength from about 0.2 to 2.0 Å. The crystal selects out and diffracts those values of λ for which planes exist, of spacing d and glancing angle θ , satisfying the Bragg's equation. A flat photographic film is placed to receive either the transmitted diffracted beam or the reflected diffracted beams as illustrated in Fig. 13.

**Fig. 13. Illustration for Laue technique.**

As shown in Fig. 3 the resulting Laue pattern consists of a series of spots. Sharp well-defined spots on the film are good evidence of a perfect crystal structure, whereas diffuse, broken or extended spots indicate lattice distortion, defects or other departures from the perfect crystal lattice.

The Laue pattern reveals the symmetry of the crystal structure in the orientation used; for example, if a cubic crystal is oriented with a cube edge, e.g., [100] axis, parallel to the incident beam, the Laue pattern will show the four fold symmetry appropriate to this axis.

Q.7.(b) Show that the number of frankel defects in equilibrium at a given temperature is proportional to $(NN_i)^{1/2}$, where N and N_i are number of atoms and interstitial atoms respectively. (6.5)

Ans. We know that interstitial atoms are the extra atoms that are squeezed in between normal atoms sites. When an interstitial is caused by transferring an atom from a lattice site to an interstitial position, a vacancy is created. The associated vacancy and interstitial atom is called Frankel defect. This is shown in Fig. 14.

Proceeding in the same ways as in the case of schottky defect, we can calculate number of frankel defects in equilibrium at a temperature T .

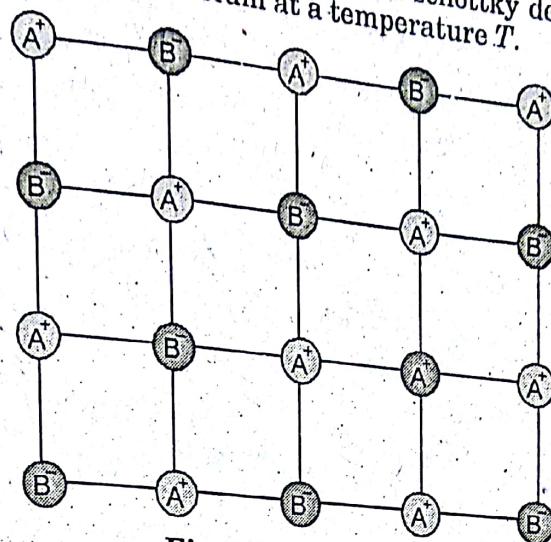


Fig. 14 Frenkel Defect.

Let in a perfect crystal,

E_i = the energy required to displace an atom from regular lattice site to an interstitial position;

N_i = the interstitial atoms and

N = the atoms

Then the total number of ways in which n Frenkel defects can be formed, will be given by

$$\omega = \frac{N!}{(N-n)!} \times \frac{N_i!}{(N_i-n)!n!} \quad \dots(1)$$

The corresponding increase in entropy due to creation of Frenkel defect is

$$S = k_B \ln \omega = k_B T \ln \left[\frac{N!}{(N-n)!} \times \frac{N_i!}{(N_i-n)!n!} \right]$$

which in turn produces a changes in free energy

$$F = U - TS$$

$$= nE_i - k_B T \ln \left[\frac{N_i!}{(N-n)!} \times \frac{N!}{(N_i-n)!n!} \right] \quad \dots(3)$$

Using the Stirling's approximation for logarithmic term, we get

$$\begin{aligned} \ln \left[\frac{N!}{(N-n)!} \times \frac{N_i!}{(N_i-n)!n!} \right] &= \ln \frac{N!}{(N-n)!n!} + \ln \frac{N_i!}{(N_i-n)!n!} \\ &\approx N \ln N + N_i \ln N_i - (N-n) \ln (N-n) - (N_i-n) \ln (N_i-n) - 2n \ln n \\ &\approx N \ln N + N_i \ln N_i - (N-n) \ln (N-n) - (N_i-n) \ln (N_i-n) \end{aligned}$$

Putting this value of logarithmic term in the expression for free energy and then differentiating with respect to n , we get

$$\left(\frac{\partial F}{\partial n} \right)_T = E_i - k_B T \ln \frac{(N-n)(N_i-n)}{n^2} \quad \dots(4)$$

At equilibrium, the free energy is constant, so that.

$$\begin{aligned} \left(\frac{\partial F}{\partial n} \right)_T &= 0 \\ E_i &= k_B T \ln \frac{(N-n)(N_i-n)}{n^2} \end{aligned}$$

$$\equiv k_B T \ln \frac{NN_i}{n^2} \quad \dots(5)$$

Taking $N \gg n$ and $NN_i \gg n$.

Thus,

$$E_i = k_B T [\ln (NN_i) - 2 \ln n],$$

or

$$2 \ln n - \ln NN_i = \frac{E_i}{k_B T}$$

or

$$\ln \frac{NN_i}{n^2} = + \frac{E_i}{k_B T}$$

$$n^2 = NN_i \exp\left(\frac{E_i}{k_B T}\right)$$

or

$$n = (NN_i)^{1/2} \exp\left(-\frac{E_i}{2k_B T}\right) \quad \dots(6)$$

Showing that n should be proportional to $(NN_i)^{1/2}$.

UNIT-IV

Q.8. (a) Explain Kroning-Penny model for the motion of electron in a periodic potential. What do you mean by negative mass. Explain. (8)

Ans. The Kronig-Penny model (named after Ralph Kronig and William Penney) is a simple, idealized quantum mechanical system that consists of an infinite array of rectangular potential barriers. Kronig and Penney in 1931 solved the Schrödinger's equation for electrons in a simple idealized periodic field. The model is of considerable importance because it interprets the main features of the band structure of metals. The periodic field was assumed in the form of potential wells of zero potential energy of width a separated by rectangular barriers of width b and height V_0 as shown in Fig. 15. For the potential of Fig. 15. The Schrödinger equation has to be solved in two regions:

(a) Region I, $0 < x < a$, potential well region in which $V = 0$ and

(b) Region II, $-b < x < 0$, barrier region in which $V = V_0$.

In order to find the allowed energies, we solve the Schrödinger's equation in these two regions and apply the appropriate boundary condition.

In region I, the Schrödinger's equation is

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0$$

or

$$\frac{d^2\psi_1}{dx^2} + \beta^2 \psi_1 = 0 \quad \left[\text{Here, } \beta^2 = \frac{2mE}{\hbar^2} \right] \quad \dots(1)$$

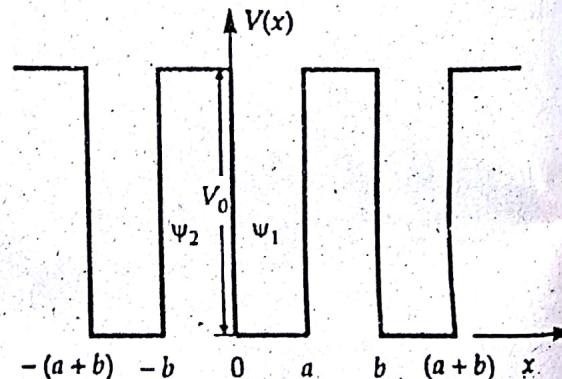


Fig. 15 One dimensional chain of potential wells.

where A and B are constants.

$$\psi_1 = Ae^{i\beta x} + Be^{-i\beta x}$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$$

or

$$\frac{d^2\psi_2}{dx^2} - \alpha^2\psi_2 = 0$$

Solution of this equation, for $E < V_0$ is

$$\left[\text{Here, } \alpha^2 = \frac{2m(V_0 - E)}{\hbar^2} \right] \dots(3)$$

where C and D are constants. For continuity at $x = 0$

$$\psi_1(0) = \psi_2(0)$$

$$\left. \frac{\partial\psi_1}{\partial x} \right|_{x=0} = \left. \frac{\partial\psi_2}{\partial x} \right|_{x=0} \dots(5)$$

$$\left. \frac{\partial\psi_1}{\partial x} \right|_{x=0} = \left. \frac{\partial\psi_2}{\partial x} \right|_{x=0} \dots(6)$$

In a periodic lattice $V(x+a) = V(x)$, it is expected that the wavefunction will also exhibit this periodicity. Since the wave function must be a Bloch wave function, therefore Eq. (1) gives

$$\psi_k(x+a+b) = e^{-ik(a+b)}\psi_k(x) \dots(7)$$

We incorporate this requirement at $x = -b$ and $x = a$, then we obtain the continuity boundary condition is

$$\psi_2|_{x=-b} = e^{-ik(a+b)}\psi_1|_{x=a} \dots(8)$$

$$\left. \frac{\partial\psi_1}{\partial x} \right|_{x=-b} = C^{-ik(a+b)} \left. \frac{\partial\psi_1}{\partial x} \right|_{x=a} \dots(9)$$

When we apply the boundary conditions, Eqs. (5), (6), (7) and (8), we obtain four equations for four unknowns.

$$A + B = C + D \dots(10)$$

$$i\beta(A - B) = \alpha(C - D) \dots(11)$$

$$Ce^{-ab} + De^{ab} = e^{-ik(ab)}[Ae^{i\beta a} + B^{-i\beta a}] \dots(12)$$

$$\alpha Ce^{-ab} - \alpha De^{ab} = e^{-ik(a+b)i\beta}[Ae^{i\beta a} - Be^{-i\beta a}] \dots(13)$$

By solving these equations simultaneously e.g., by requiring that the determinants of the coefficients of A, B, C and D vanish, we get

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ i\beta & -i\beta & -\alpha & \alpha \\ -e^{-ik(a+b)+i\beta a} & -e^{-ik(a+b)-i\beta a} & e^{-ab} & e^{ab} \\ -i\beta e^{-ik(a+b)+\beta a} & i\beta e^{-ik(a+b)-i\beta a} & -ae^{-ab} & -ae^{ab} \end{vmatrix} = 0 \dots(14)$$

On solving the determinant Eq. (14) and after simplification, we get

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta} \sinh ab \sin \beta a + \cosh ab \cos \beta a = \cos(a+b)k \dots(15)$$

To simplify Eq. (15), Kronig and Penney considered the case when $V_0 \rightarrow \infty$ and $b \rightarrow 0$, but the product $V_0 b$ has a finite value i.e., the potential barriers become delta functions. Under these conditions, the model is modified in such a way that represent a

series of well separated by infinitely thin potential barriers of infinitely large potential. The limiting value of $V_0 b$ for $V_0 \rightarrow \infty$ and $b \rightarrow 0$ is known as barrier strength.

As $b \rightarrow 0$, $\sin h \alpha b \rightarrow \alpha b$ and $\cosh \alpha b \rightarrow 1$

Also from Eqs. (1) and (3)

$$\alpha^2 = \frac{-2m(V_0 - E)}{\hbar^2}$$

and

$$\beta^2 = \frac{2mE}{\hbar^2}$$

Then

$$\beta^2 - \alpha^2 = \frac{2mV_0}{\hbar^2}$$

or,

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta} = \frac{mV_0}{\alpha\beta\hbar^2} \quad \dots(16)$$

Putting the value $\frac{\beta^2 - \alpha^2}{2\alpha\beta}$ from Eq. (16) in Eq. (15) we get

$$\frac{mV_0}{\alpha\beta\hbar^2} \alpha b \sin \beta a + \cos \beta a = \cos ka$$

$$\frac{mV_0 b}{\beta\hbar^2} \sin \beta a + \cos \beta a = \cos ka \quad \dots(17)$$

Let us define a quantity $p = \frac{mV_0 ba}{\hbar}$, which is measure of the area $V_0 b$ of the potential barrier, then

$$p \frac{\sin \beta a}{\alpha\beta} + \cos \beta a = \cos ka \quad \dots(18)$$

The physical significance of the quantity p is that if p is increased and the given vector is bound more strongly to a particular potential well.

When $p \rightarrow 0$, then potential barrier becomes very weak which means that electrons are free electrons. In this, we obtain from Eq. (18).

$$\beta a = ka \text{ or } \beta = k$$

Now

$$\beta^2 = \frac{2mE}{\hbar^2} = k^2 \quad \dots(19)$$

Hence

$$E = \frac{k^2 \hbar^2}{2m} = \frac{h^2 k^2}{8\pi^2 m} \quad \dots(20)$$

This result is similar as obtained by free electron theory. Equation (18) also gives the condition, which must be satisfied so that solutions of the wave equation may exist. Since cosines can have values between +1 and -1. Such values of βa , therefore, represent wave like solutions of the form

$$\psi(x) = e^{ikx} U_k(x)$$

and are allowed values. The other values of βa are not allowed.

If we plot a graph between $\left(\frac{p \sin \beta a}{\beta a} + \cos \beta a \right)$ and βa for the value of $p = \frac{3a}{2}$, we get the curve as shown in Fig. 16.

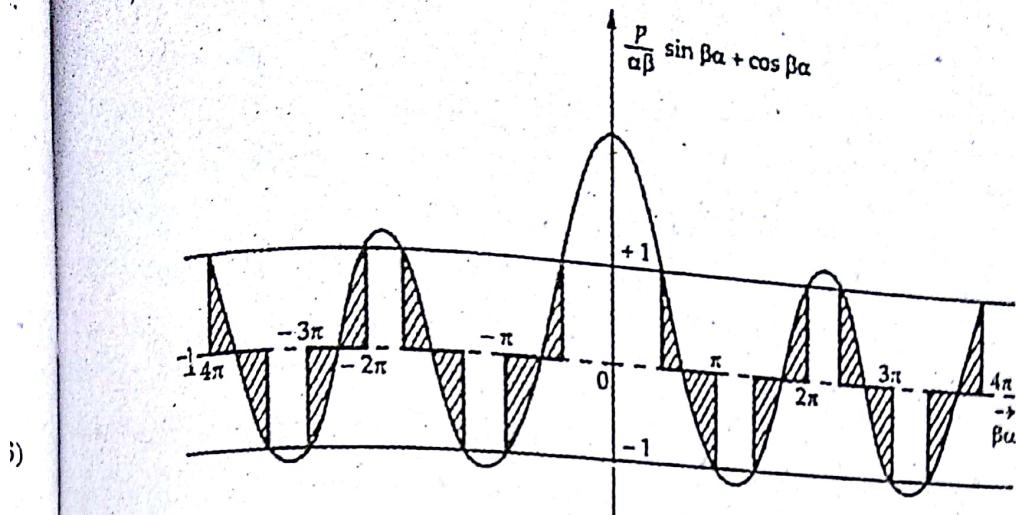


Fig. 16. Plot of $\frac{P}{\beta a} \sin \beta a + \cos \beta a$ vs. to show allowed and for bidden band and the K.P. appoximation.

The electrons in a crystal are not completely free but interact with the periodic potential of the crystal lattice. As a result their wave particle motion is different from that of an electron in free space. Thus while using the equations of electrodynamics for charge carriers in a crystal, we must use alter value of particle mass or effective mass after taking into account the effect of the crystal lattice so that the electron (and the hole) can be treated as almost free carriers of charge for the calculations.

The large effective mass can be interpreted as being due to strong binding force between the electron and the lattice for some value of k . The negative mass may be

interpreted in terms of Bragg reflection when k is close to $\pm \frac{\pi}{a}, \pm \frac{2\pi}{a}$ on account of which of force in one direction leads to a gain in momentum in opposite direction, which results in negative effective mass. The results of free electron theory can be applied in the band theory by simply replacing m by m^* . Thus Fermi energy in a metal is given by.

$$E_F = \frac{\hbar^2}{2m^*} \left[3\pi^2 \frac{N}{V} \right]^{2/3} = \frac{\hbar^2}{2m^*} \left[\frac{3N}{8\pi V} \right] \quad \dots(21)$$

Physical Significance of Effective Mass

In most conductors $m^* = m$ because the valence band is only partially filled. But in semiconductors and insulators having almost filled valence bands the effective mass plays an important role in conduction process and provides a satisfactory description of the charge carriers in crystals. In crystals, such as alkali metals which have partially filled energy band, the conduction takes place mainly through electrons. But in crystals which have energy bands nearly full except for a few electron vacancies near the top of the band, these negative charge and negative mass vacancies may be considered as positive charge and positive mass particle called holes which act as positive charge carriers to bring out conduction.

The concept of effective mass is able to account for many observed effects like high electronic specific heat of transition metals and their high paramagnetic susceptibility.

Q.8.(b) At what temperature we can expect 10% probability that electron in silver have an energy which is 1% above the fermi energy? The fermi energy of silver is 5.5 ev. (4.5)

Ans. Given that

We know that,

$$\begin{aligned}f(E) &= 10\% \\E &= E_F + 1\% \text{ of } E_F \\E_F &= 5.5 \text{ eV}\end{aligned}$$

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$

$$\begin{aligned}E &= 5.5 + \frac{5.5}{100} = 5.5 + 0.555 \\E - E_F &= 0.555\end{aligned}$$

$$0.1 = \frac{1}{0.555 \times 1.6 \times 10^{-19} e^{1.38 \times 10^{-23} T} + 1}$$

$$0.1 = \frac{1}{e^{\frac{637.7}{T}} + 1}$$

$$T = 290.2K$$

Q.9. (a) What is Hall effect? Derive an expression for Hall Coefficient. Discuss some of its important applications.

Ans. The phenomenon of Hall Effect is a consequence of motion of charge in both magnetic and electric fields. If a piece of conductor (metal or semiconductor) carrying current is placed in a transverse magnetic field, then an electric field or a potential difference is produced inside the conductor in a direction normal to both current and magnetic field. This phenomenon is known as *Hall effect*. The developed electric field and potential are called *Hall field* and *Hall voltage*.

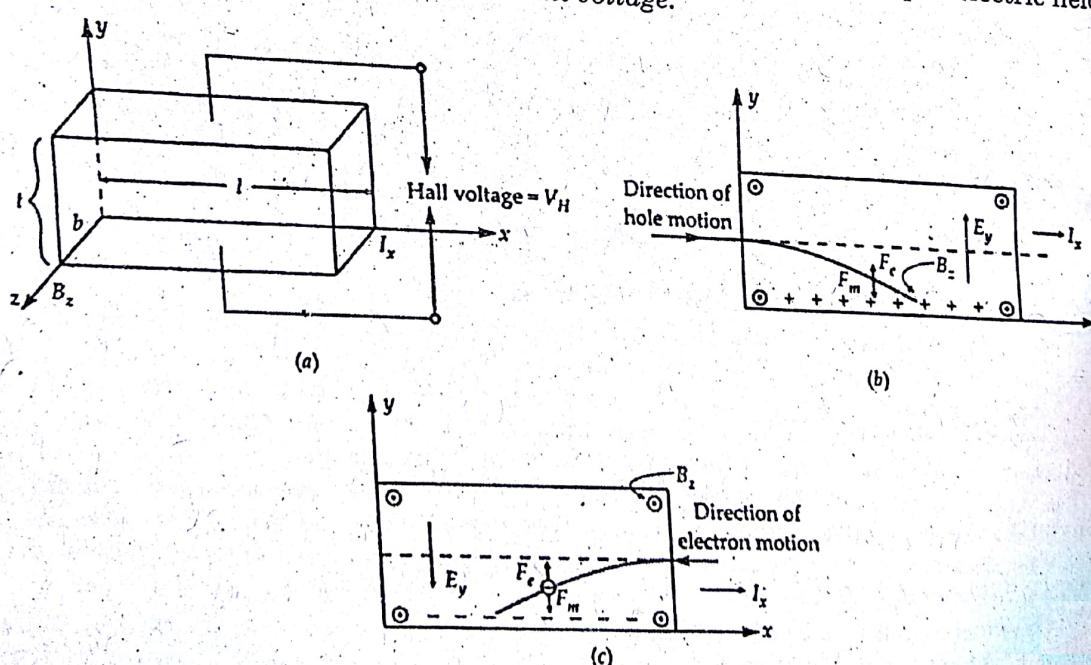


Fig. 17. (a) Illustration of Hall effect (b) motion of hole in P-type semiconductor specimen (c) motion of electron of N-type semiconductor or metal specimen of Hall experiment.

Consider a specimen in the form of a rectangular cross-section carrying current I_x in the x -direction. A uniform magnetic field B_z is applied along z -axis as shown in Fig. 17(a). If specimen is of conducting material or N-type semiconductor, then current flows due to motion of electron, but in P-type semiconductor and intrinsic semiconductor, current is carried by motion of holes and electrons-holes respectively.

A moving charge in transverse magnetic field experiences a force normal to both current and magnetic field. Thus moving charges slightly shift towards y direction due to magnetic force $ev_x B_z$ [Figs. 17(a) and (c)]. This shift of charges produces a potential difference or an electric field E_y along y -direction. The produced electric field also applies a force $e E_y$ on charge. When both the forces become equal, the equilibrium occurs, i.e., upto this condition charges shift along y direction.

In equilibrium

$$\begin{aligned} F_e &= F_m \\ eE_y &= ev_x B_z \\ E_y &= v_x B_z \end{aligned} \quad \dots(1)$$

If J_x is the current density in x direction, then

$$J_x = \frac{I_x}{A} = nev_x \Rightarrow V_x = \frac{J_x}{ne} \quad \dots(2)$$

Here n is density of charge carrier (electron or hole or both) and A (= breadth \times thickness) is the cross-sectional area of specimen perpendicular to direction of J_x .

From Eqs. (1) and (2) we can write.

$$E_y = \frac{B_z J_x}{ne} \quad \dots(3)$$

This is the expression for Hall field. The Hall effect is described by means of Hall coefficient R_H , defined in terms of J_x by the following relation:

$$R_H = \frac{E_y}{J_x B_z} \quad \dots(4)$$

From Eqs. (3) and (4), we have—

$$R_H = \frac{1}{ne} \quad \dots(5)$$

For the conductor or N-type semiconductor, the electric field is developed in negative y direction. Hence from Eqs. (4) and (5), we can write.

$$R_H = -\frac{E_y}{J_x B_z} = -\frac{1}{ne} \quad \dots(6)$$

In P-type semiconductor, the produced Hall field is along y direction, thus we have

$$R_H = \frac{E_y}{J_x B_z} = \frac{1}{n_h e} \quad \dots(7)$$

Here n_h is the hole density. Equations (6) and (7) indicate that the sign of R_H is same as the sign of carrier. Most of the metals have negative Hall coefficient or resistance, but some metals have positive Hall coefficient (i.e., charge carriers are holes). If both holes and electrons contribute to current, then R_H can be positive or negative depending upon the relative density and mobility of carriers.

Application of Hall Effect

- The sign of Hall coefficient can be used to determine whether a given semiconductor is N or P-type.
- The carrier concentration can be evaluated when R_H is known (as $n = 1/R_H e$).

3. Since $\mu = R_H \sigma$, thus mobility of charge carrier can also be well determined with the help of R_H .

4. Since Hall voltage is proportional to magnetic flux density B_z and current I through a sample or specimen. The Hall effect can be used as the basis for design of a magnetic flux density meter.

5. Hall effect can be used to determine the power flow in an electromagnetic wave because V_H is proportional to product of E_y and B_z , i.e., magnitude of Poynting vector in e.m. wave.

Q.9.(b) The Hall coefficient of certain silicon specimen was found to be $-7.35 \times 10^{-5} \text{ m}^3 \text{ c}^{-1}$ from 100 to 400 K. Determine the nature of semiconductor. If the conductivity was found to be $200 \text{ m}^{-1} \Omega$. Calculate the density and mobility of charge carrier. (4.5)

Ans. The negative sign of the Hall coefficient indicates that the nature of semiconductor is N-type.

$$\text{We know that, } R_H = -\frac{1}{ne}$$

$$\text{or, } n = \frac{1}{eR_H} = \frac{1}{1.6 \times 10^{-19} \times 7.35 \times 10^{-5}}$$

$$n = 8.503 \times 10^{22} \text{ m}^{-3}$$

and

$$\sigma = ne\mu$$

$$\text{or, } \mu = \frac{\sigma}{ne} = \frac{200}{8.503 \times 10^{22} \times 1.6 \times 10^{-19}}$$

$$\mu = 1.47 \times 10^{-2} \text{ m}^2 \text{ V}^{-1} \text{ S}^{-1}$$

Q.9.(c) Explain the effect of temperature on the Fermi level of the n-type of semiconductor. (2)

Ans.

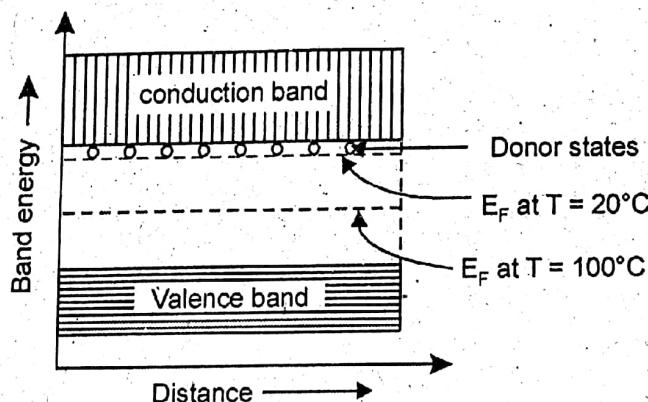


Fig. 18. Position of Fermi level (E_F) for N-type semiconductor.

N-type material the electrons are coming from two different sources. Some electrons come from donor state, which are easily separated from their parent atom and they do not vary much as the temperature is increased. The other electrons in the conduction band are present because of the breaking of a covalent bond. Such intrinsic effect will increase in number as temperature raised. So proportion of the total number of conduction electrons will increase from valence band. Thus as the temperature rises the material becomes more and more intrinsic and Fermi level moves closer to the intrinsic position, i.e., at the centre of the forbidden gap.

END TERM EXAMINATION [MAY-2015]

SECOND SEMESTER [B. TECH]

APPLIED PHYSICS-II [ETPH-104]

Time: 3 Hrs.

MM : 75

Q.1. Write brief notes on any five of the following.

(a) Wave packet

(3 × 5 = 15)

Ans. When plane waves of slightly different wavelengths travel simultaneously in the same direction along a straight line, through a dispersive medium; (*i.e.* a medium in which the phase velocity $v_p = \frac{\omega}{k}$ a wave depends on its wavelength) successive group of the waves are produced as shown in fig. 1.

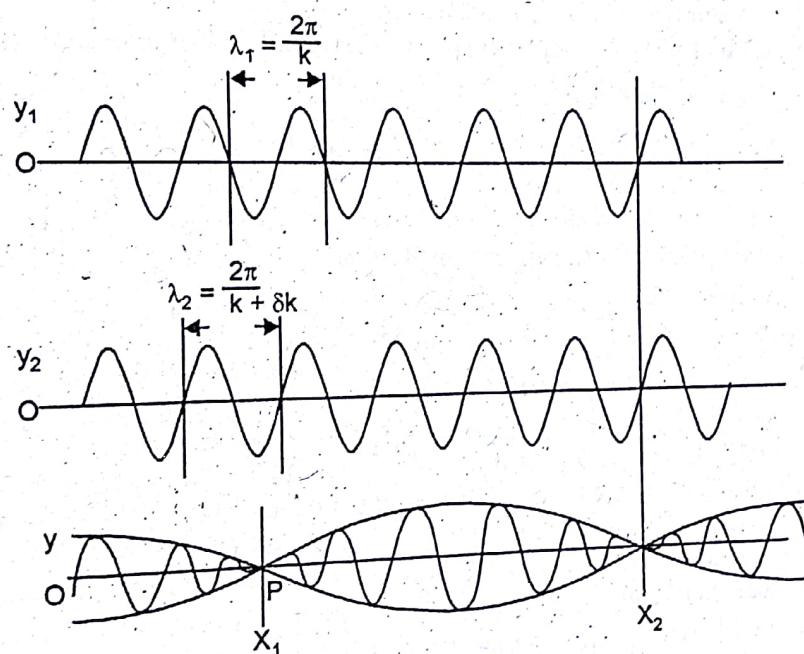


Fig.1 Successive groups of wave.

There wave group are called wave packets. Each wave group travels with a velocity is called **group velocity**. "The velocity with which the resultant envelope of the group of waves travels is called group velocity", denoted by v_g . The velocity of the group is different from that of the individual components of the wave. The group velocity is different from the phase velocity of a wave.

The velocity with which resultant envelope moves is called group velocity and the velocity with which a point like P on the wave moves is called phase velocity.

Q.1. (b) Schottky and frankel deyets.

Ans. Refer Q.4. (b) of First Term Examination 2014.

Q.1. (c) Define Poynting vetror and state Poynting theorem,

Ans. $\vec{S} = \vec{E} \times \vec{H}$, is known as Poynting vector and is interpreted as the power flux i.e., amount of energy crossing unit area placed perpendicular to the vector, per unit time. The conception of energy of the electromagnetic field as residing in the medium in

the medium is very fundamental one and has great advantage in the development of the theory. Maxwell thought of the medium as resembling as elastic solid the electrical energy representing the potential energy of strain of the medium, the magnetic energy the kinetic energy of motion. Though such a mechanical view no longer exists, still the energy is regarded as being localised in space and as travelling in the manner indicating by poynting vector. In a light wave there is certain energy per unit volume, proportional to the square of the amplitude (E or H). This energy travels along it and poynting vector is the vector that measures the rate of flow or the intensity of the wave. In a plane electromagnetic wave E and H are at right angle to each other and at right angle to the direction of flow. thus $E \times H$ must be along the direction of flow. In more complicated waves as well, poynting vector point along the directions of flow of radiation. For example, if we have a source of light and we wish to find at what rate it is emitting energy, we surround it by a close surface and integrate the normal component of poynting vector over the surface. The whole conception of energy being transported in the medium is fundamental to the electromagnetic theory of light.

"Poynting theorem states that the net power flowing out of a given volume V is equal to the time rate of decrease in energy stored within V minus the ohmic losses."

$$\text{i.e.} \quad \oint \operatorname{div}(E \times H) dV = -\frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right] dV - \int_V \sigma E^2 dV$$

Total power leaving the volume = Rate of decrease in energy stored in electric and magnetic field – Ohmin power dissipated

Q. 1. (d) Bragg's law.

Ans. Refer Q.2(a) of Important Questions Page No. 10.

Q.1. (e) Effective mass of an electron.

Ans. The electrons in a crystal are not completely free but interact with the periodic potential of the crystal lattice. As a result their wave particle motion is different from that of an electron in free space. Thus while using the equations of electrodynamics for charge carriers in a crystal, we must use alter value of particle mass of effective mass after taking into account the effect of the crystal lattice so that the electron (and the hole) can be treated as almost free carriers of charge for the calculations.

Let us now consider the motion of an electron in a crystal in the presence of an external applied electric field ξ . Suppose the electron is initially in a state k , when the external electric field ξ is applied the magnitude of the force acting on the electron is $e\xi$.

If the electron moves a distance dx in a small time dt , under the action of this force, then

$$\text{Work done} = \text{Energy gained} \quad dE = e\xi dx \quad \dots(1)$$

If v is the velocity gained by the electron, then

$$v = \frac{dx}{dt} \text{ or } dx = v dt \quad \dots(2)$$

$$dE = e\xi v dt \quad \dots(3)$$

$$v = \frac{1}{h} \frac{dE}{dk} \quad \dots(4)$$

$$dE = \frac{e\xi}{h} \frac{dE}{dk} dt$$

Now

Development of the electrical energy till the energy is proportional to wave. In a plane wave, Poynting vector is right angle to the wave. For example, if poynting vector in the medium is

volume V is equal to "losses."

$$V = \int_V \sigma E^2 dV$$

Energy stored in

or

Now

Differentiating Eq. (3) with respect to t , we get

$$\frac{dk}{dt} = \frac{e\xi}{h} \quad \dots (5)$$

$$hk = p \text{ and } h \frac{dk}{dt} = \frac{dp}{dt} = F$$

$$h \frac{dk}{dt} = e\xi = F \quad \dots (6)$$

From Eq. (6),

$$\frac{dk}{dt} = \frac{F}{h}$$

$$\frac{dv}{dt} = a \text{ (the acceleration)}$$

Also

From Eq. (7),

$$a = \frac{1}{h} \frac{d^2 E}{dk^2} F$$

$$\frac{a}{F} = \frac{1}{h^2} \frac{d^2 E}{dk}$$

or

Taking $m^* = \frac{F}{a}$ as the effective mass of electron in the crystal, we have

$$m^* = \frac{h^2}{d^2 E / dk^2} \quad \dots (9)$$

In some cases, a factor f_k is also used. It gives the ratio of the rest mass of a free electron m to its effective mass in the crystal in the k -state

$$f_k = \frac{m}{m^*} = \frac{m}{h^2} \left(\frac{d^2 E}{dk^2} \right) \quad \dots (10)$$

This expression determines the extent to which the electron in the k -state behaves as a free electron.

If $f_k = 1$, $m^* = m$, the electron in the crystal in the k -state behaves as a free electron.

Q. 1. (f) Zero point energy.

Ans. Zero point energy is the lowest possible energy that a quantum mechanical physical system may have; it is the energy of its ground state.

All quantum mechanical systems undergo fluctuations even in their ground state and have an associated zero-point energy, a consequence of their wave-like nature. The uncertainty principle requires every physical system to have a zero point energy greater than the minimum of its classical potential well. For example, liquid helium does not freeze under atmospheric pressure at any temperature because of its zero point energy.

Also,

The possible energy of a particle in box of length 'a' is given by

$$E_n = \frac{n^2 h^2}{8ma^2} \quad (\text{Where } n = 1, 2, 3, \dots)$$

If $n = 1$, then

$$E_1 = \frac{h^2}{8ma^3}$$

This is the energy of the ground state of particle. since the particle in a box cannot be at rest, its minimum energy is positive and is after called the zero point energy.

Q. 2. (a) Apply Gauss's Law. calculate-

(i) The electric yield at any point due to two parallel sheets of charge. (3)

(ii) Calculate the intensity of electric yield at a point between oppositely charged parallel plates. (3)

Ans. (i) Consider two infinitely parallel sheets A and B of charge carrying densities $+\sigma$ and $-\sigma$ respectively. The intensities of electric yield at each point near the sheet is

$$E = \frac{\sigma}{2\epsilon_0}$$

Between the sheets- If point p is between the sheets A and B, the electric field

strength due to sheet A - $E_1 = \frac{\sigma}{2\epsilon_0}$ (away from sheet A)

Electric field strength due to sheet B -

$$E_2 = \frac{\sigma}{2\epsilon_0} \quad (\text{towards sheet B})$$

E_1 and E_2 are in same direction therefore net electric field strength

at p.

$$E = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

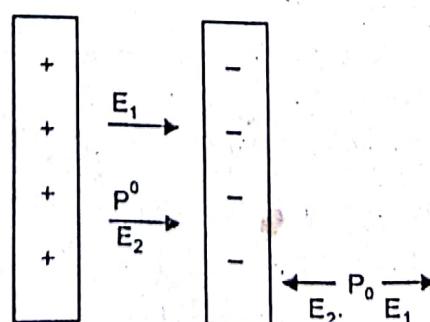


Fig. 5

Outside the sheets- Let point p outside the sheets electric field strength at p due

to sheet A - $E_1 = \frac{\sigma}{2\epsilon_0}$ (away from sheet A)

Electric field strength at p due to sheet B - $E_2 = \frac{\sigma}{2\epsilon_0}$ (towards sheet B)

E_1 and E_2 are in opposite direction, therefore net electric field at p is -

$$E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

$$\boxed{E = 0}$$

Consider two oppositely charged plates placed parallel to each other. Let these plates are separated by a small distance as compared to their size.

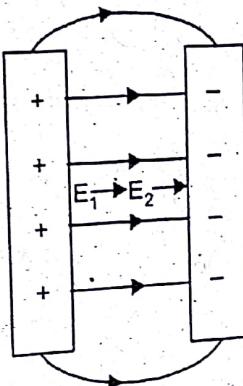


Fig. 6.

Surface density of charge on each plate is " σ ". Since the electric lines of force are parallel except near the edges, each plate may be regarded as a sheet of charges.

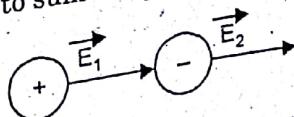
Electric intensity at a point between the plates due to positive plate is -

$$E_1 = \frac{\sigma}{2\epsilon_0}$$

Electric intensity at a point between the plates due to negative plate is -

$$E_2 = \frac{\sigma}{2\epsilon_0}$$

Since both intensities are directed from the +ve to -ve plate hence total intensity between the plates will be equal to sum of E_1 and E_2 .



$$E = E_1 + E_2 \Rightarrow \frac{\sigma}{2\epsilon_0} + E_1 = \frac{\sigma}{2\epsilon_0} = E_1 = \frac{\sigma}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

Q. 2. (b) If the earth receives $2 \text{ cal min}^{-1} \text{ cm}^{-2}$ solar energy, what would be the amplitudes of electric and magnetic field radiation? (5)

Ans. Here, solar energy which the earth receives is $2 \text{ cal min}^{-1} \text{ cm}^{-2}$

$$\therefore \frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{4\pi \times 4\pi \times 9 \times 10^9 \times 10^{-7}} = \sqrt{9 \times 4\pi \times 4\pi \times 10^2}$$

$$= 4\pi \times 3 \times 10 = 120\pi$$

$$= 120 \times 3.14$$

$$= 376.8 \approx 377$$

Poynting vector,

$$\vec{P} = \vec{E} \times \vec{H}$$

$$= EH J m^{-2} \text{ sec}^{-1}$$

$$= \frac{2 \times 4.2}{60 \times 10^{-4}} J m^{-2} s^{-1} = 140 J m^{-2} \text{ sec}^{-1}$$

$$EH = 1400$$

$$\frac{E_0}{H_0} = 377 = \frac{E}{H}$$

$$E_2 = 1400 \times 377$$

or

$$E = 10 \times \sqrt{14 \times 377} = 726.5 \text{ V/m}$$

$$H = \frac{E}{377} = \frac{726.5}{377} = 1.927$$

Amplitudes of electric and magnetic field vector are

$$E_0 = E \sqrt{2} = 1024.3 \text{ V/m}$$

$$H_0 = H \sqrt{2} = 2.717 \text{ A/m}$$

Q. 2.(c) Write Maxwell's equations in both forms. State the physical Significance of each. (4)

Ans. 1. Maxwell's first equation: $(\nabla \cdot D = \rho)$

Integrating this over an arbitrary volume V , we get

$$\int_V \nabla \cdot D dV = \int_V \rho dV$$

Charging volume integral into surface integral in L.H.S. of above equation by Gauss divergence theorem, we get

$$\int_S D \cdot dS = \int_V \rho dV \quad \dots(1)$$

Where S is the surface, which bounds volume V , Eq. (1) represents Maxwell's first equation ($\nabla \cdot D = \rho$) in integral form. Since $\int_V \rho dV = q$, the net charge contained in volume V , therefore, Maxwell's first equation signifies that:

The net outward flux of electric displacement vector through the surface enclosing a volume is equal to the net charge contained within that volume.

2. Maxwell's second equation: $\nabla \cdot B = 0$

Integrating this over an arbitrary volume V , we get

$$\int_V \nabla \cdot B dV = 0$$

Using Gauss divergence theorem to change volume integral into surface integral, we get

$$\int_S B \cdot dS = 0 \quad \dots(2)$$

Where S is the surface, which bounds volume V . Equation (2) represents Maxwell's second equation in integral form and signifies that:

The net outward flux of magnetic induction B through any close surface is equal to zero.

3. Maxwell's Third equation: $\nabla \times E = -\frac{\partial B}{\partial t}$

Integrating above equation over a surface S bounded by a curve C , we get

$$\int_S (\nabla \times E) \cdot dS = - \int_S \frac{\partial B}{\partial t} \cdot dS$$

Using stoke's theorem to convert surface integral on L.H.S. of above equation into line integral along the boundary C , we get

$$\int_C E \cdot dl = - \frac{\partial}{\partial t} \int_S B \cdot dS \quad \dots(3)$$

Equation (3) represents Maxwell's third equation in integral form and signifies that:

The electromotive force $\left(\text{emf. } e = \int_C E \cdot dl \right)$ around a close path is equal to negative

rate of change of magnetic flux linked with the path $\left(\because \text{magnetic flux } \phi = \int_S B \cdot dS \right)$.

4. Maxwell's fourth equation:

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Taking surface integral over surface S bounded by curve C , we obtain

$$\int_S (\nabla \times H) \cdot dS = \int_S \left(J + \frac{\partial D}{\partial t} \right) \cdot dS$$

Using stoke's theorem to convert surface integral on L.H.S of above equation into line integral, we get

$$\oint_C H \cdot dl = \int_S \left(J + \frac{\partial D}{\partial t} \right) \cdot dS \quad \dots(4)$$

Equation (4) represents Maxwell's 4th equation in integral form and signifies that, The magnetomotive force ($m.mf. = \oint H \cdot dl$) around a close path is equal to the conduction current plus displacement current through any surface bounded by the path.

Q.3. (a) An electron is constrained to move in a one dimensional box of length 0.1 nm . Find the first three energy eigen values and the corresponding de-Broglie wave lengths. (4)

Ans. Given

$$L = 1.0 \times 10^{-10}\text{ m.}$$

Formulae used are

$$E_n = \frac{n^2 h^2}{8mL^2} \text{ and } p_n = \frac{nh}{2L}$$

for $n = 1$

$$\begin{aligned} E_1 &= \frac{h^2}{8mL^2} \frac{(6.63 \times 10^{-34})^2}{8 \times (9.1 \times 10^{-31}) \times (10^{-10})^2} \\ &= 6.04 \times 10^{-18}\text{ J} \end{aligned}$$

similarly, for $n = 2$, is equal to

$$E_2 = (2)^2 E_1 = 24.16 \times 10^{-18}\text{ J}$$

and for $n = 3$,

$$E_3 = (3)^2 E_1 = 54.36 \times 10^{-18}\text{ J}$$

as we know

$$\lambda_n = \frac{h}{p_n} \text{ and } p_n = \frac{nh}{2L}$$

or

$$\lambda_n = \frac{2L}{n}$$

For $n = 1$

$$\lambda_1 = 2L = 2.0 \times 10^{-10}\text{ m} = 2\text{ A}$$

For $n = 2$,

$$\lambda_2 = \frac{2L}{2} = l = 1.0 \times 10^{-10}\text{ m} = 1.0\text{ A}$$

For $n = 3$,

$$\lambda_3 = \frac{2L}{3} = 0.667 \times 10^{-10}\text{ m} = 0.66\text{ A}$$

Q. 3. (b) Find the probability that a particle trapped in a box 'L' wide can be found between $0.45 L$ and $0.55 L$ for the first excited state. (4)

Ans.

$$\Psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

For $n = 1$

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

$$\begin{aligned}
 \rho &= \int_{0.45L}^{0.55L} |\psi(x)|^2 dx = \frac{2}{L} \int_{0.45L}^{0.55L} \frac{\sin^2 \pi x}{L} \\
 \rho &= \frac{0.55L}{0.45L} \cdot \frac{2}{2\pi L} \left(1 - \cos \frac{2\pi x}{L} \right) dx = \frac{1}{L} \int_{0.45L}^{0.55L} \left(1 - \cos \frac{2\pi x}{L} \right) dx \\
 \rho &= \frac{1}{L} \left[x - \left[\sin \left(\frac{2\pi x}{L} \right) \right] \times \left(\frac{L}{2\pi} \right) \right]_{0.45L}^{0.55L} \\
 \rho &= \frac{1}{L} [0.55L - 0.45L] - \frac{1}{L} \times \frac{L}{2\pi} \left[\sin \left(\frac{2\pi \times 0.55L}{L} \right) - \sin \left(\frac{2\pi \times 0.45L}{L} \right) \right] \\
 \rho &= 0.10 + \frac{0.617}{2\pi} \\
 \boxed{\rho = \frac{0.20\pi + 0.617}{2\pi}}
 \end{aligned}$$

Q. 3. (c) Explain the postulates of quantum mechanics.

(4)

Ans. The postulates of quantum mechanics are as following.

(i) A wave function may be associated with any particle moving in a conservative field of force and it determines every thing that can be known about the system in consistence of uncertainty principle.

(ii) The wave function of a system evolves in time according to time dependent schrodinger equation.

$$H\psi = i\hbar \frac{\partial \psi}{\partial t}$$

(iii) The total wave function must be antisymmetric with respect to the inter change of all co-ordinates of one fermion with those of another. The Pauli's exclusion principle is direct result of this antisymmetry principle.

(iv) Corresponding to every observable in classical mechanics there is linear operator in quantum mechanics.

(v) In any measurement of the observable associated with operator A , the observed values are eigen values as satisfying the equation.

$$A\psi = a\psi$$

(vi) If a system is in a state described by a normalised wave function ψ , then the average value of the observable correspond to \hat{A} is given by

$$\langle a \rangle = \int_{-\alpha}^{+\alpha} \psi^* \hat{A} \psi dv \quad (3)$$

Q. 3. (d) Explain phase and group velocities.

Ans. Refer Q.1. (g) of End Term Examination 2014.

Second Semester, Applied Physics-II

26-2015

Q. 4. (a) Discuss Kroning-Penney model. Using the model show the energy spectrum of electron consisting of a number of allowed energy bands separated by forbidden bands. (8)

Ans. Free electron model ignore the effects those arise when the electrons interact with crystal lattice. However, now we consider this by making some general remarks about the effect of the periodic variation in the potential. Due to this periodicity in potential for an infinitely long lattice, the wave function does not remain sinusoidal travelling waves of constant amplitude but now they include the lattice periodicity in their amplitudes, and electrons may be scattered by the lattice. When the deBroglie wavelength of the electron corresponds to a periodicity in the spacing of the ions, the electrons strongly with the lattice. This situation is the same as an electromagnetic wave suffers Bragg's reflection, when the Bragg's condition is satisfied.

In order to find the allowed energies of electrons in solids, we consider the effect of formation of a solid when the individual constituent atoms are brought. We solve the Schrödinger equation for periodic potential seen by an electron in a crystal lattice. We also consider that the periodic potential is a succession of rectangular wells and barriers. The solution of Schrödinger equation is a sinusoidal wave in certain energy ranges i.e. allowed states, and real decaying exponential wave in the other range i.e. the forbidden bands. For this purpose, here we present only qualitative approach.

It is found that the potential is not constant but varies periodically. The effect of periodicity is to change the free particle travelling wave eigen function. Therefore, the travelling wave eigen function has a varying amplitude which changes with the period of the lattice. If we consider that the space periodicity is as (fig. 7), then according to block, the eigen function for non-dimensional system has the form

$$\psi(x) = u_k(x)e^{ikx}$$

As is clear, this is different from the free travelling wave function $\psi(x) = Ae^{ikx}$. $u_k(x)$ is the periodic function with the periodicity a of the periodic potential i.e.

$$u_k(x) = u_k(x+a)$$

In general,

$$u_k(x) = u_k(x+na)$$

Where n is an integer. Hence, with the effect of periodicity the complete wave function is

$$\psi(x, t) = u_k(x)e^{i(kx-wt)}$$

In the above equation, the exponential term indicates a wave of wavelength $\lambda = \frac{2\pi}{k}$ which travels along $+x$ direction if k is positive and it moves along $-x$ direction if the value of k is negative.

The exact form of the function $u_k(x)$ depends on the particular potential assumed and the value of k .

In 1930, Kronig and Penny, proposed a one-dimensional model for the shape of rectangular potential well and barriers having the lattice periodicity, as shown in fig. 7. Each well represents an approximation to the potential produced by one ion. In the region such as $0 < x < a$, the potential energy is assumed to be zero while, in the region $-b < x < 0$ or $a < x < (a+b)$, the potential energy is taken as V_0 . The relevant Schrödinger equations for these two regions are

$$\frac{d^2\psi}{dx^2} + \left[\frac{2m}{h^2} E \right] \psi = 0 \quad (ii) \quad [0 < x < a]$$

$$\frac{d^2\psi}{dx^2} + \left[\frac{2m}{\hbar^2} (E - V_0) \right] \psi = 0 \quad (iii) \quad [-b < x < 0]$$

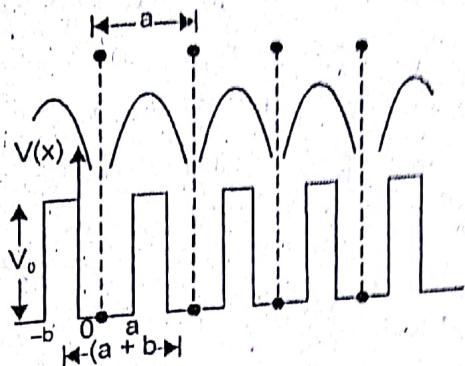


Fig. 7

The electron of not too high energy high energy is practically bound within one of the wells that are deep and widely spaced. So the lower energy eigen value are those of a single well. However, for the wells those are closer together the eigen function can penetrate the potential barriers more easily. Because of this, spreading of previously single energy level into a band energy levels takes place. The band becomes wider with the decrease in the separation of the wells. Under the limit of zero barrier thickness, we obtain an infinitely wide single well in which all energies are allowed. So the present case is reduced to the free electron model. The comparison between the allowed energies of a single well and an array of wells (Kronig-penney model) is shown in Fig. 8. In this figure, we have assumed $b = a/16$ and the well strength as $2m V_0 a^2 / \hbar^2 = 121$. It is clear from the figure that each band corresponds to a single energy level of the single well. The forbidden bands appear even for energies $E > V_0$.

Here we will solve the Schrodinger wave equation for electron for Kronig-penny potential under the condition that ψ and $\frac{d\psi}{dx}$ are continuous at the boundaries of the well. A complicated expression for the allowed energies in terms of k shows that gaps in energy are obtained at values such that

$$k = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}, \dots \quad \dots (iv)$$

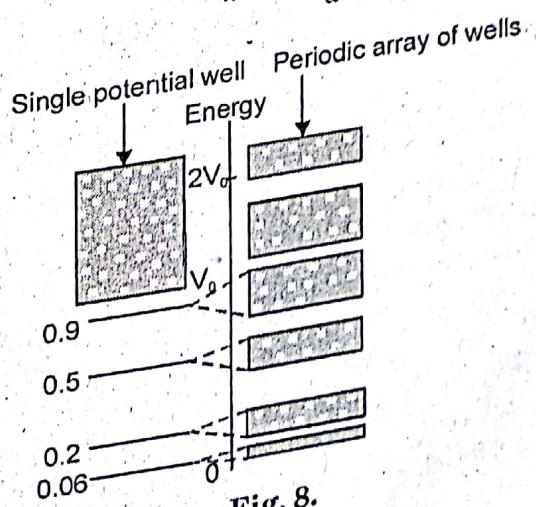


Fig. 8.

The solution of the schroedinger wave equation for free-electrons results in the energy value given by

$$E = \frac{h^2 k^2}{8\pi^2 m} = \frac{h^2 k^2}{2m} \quad \dots(v)$$

or
and

Energy E Versus wave Number k Diagram

From Eq. (v), it is clear the relation between E and k is parabolic. The parabolic relation between E and k , valid in case of free electrons, is therefore, interrupted at different value of k , as shown in fig. 9. It means the energies corresponding to the value of k given by Eq. (iv) are not permitted for electrons in the crystal. Thus, the energies of electrons are divided into forbidden and allowed bands (Fig. 9)

The occurrence of the gaps can be understood on the basic of Bragg's condition for the diffraction, given as

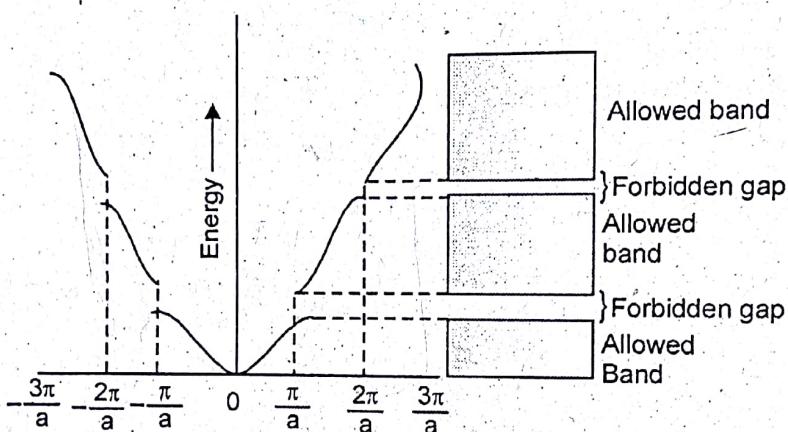


Fig. 9.

$$2a \sin \theta = n\lambda, n = 1, 2, 3, \dots(vi)$$

Where a is spacing between the ions of the lattice and θ is the angle of incidence.

Eq. (vi) can be written as

$$2a = n\lambda \quad (\text{for } \theta = 90^\circ)$$

or

$$2a = n \frac{2\pi}{k}$$

or

$$k = \frac{n\pi}{a}$$

or

$$k = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}, \dots$$

We have put \pm signs because the incident wave can travel along $+x$ -axis as well as along $-x$ -axis. At all these values of k the gaps in energy occur, as shown in fig. 9.

The waves corresponding to value of k satisfying the Bragg's condition are reflected and resulted in standing waves. On each subsequent Bragg reflection, the direction in which the wave is travelling is reversed again. The eigen function and corresponding

reflected waves for $k = \pm n \frac{\pi}{a}$ are therefore $e^{i(n\pi/a)x}$, and $e^{-i(n\pi/a)x}$. These two eigen functions can be combined in two different ways to give total eigen function

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$$\begin{aligned}\psi_1 &= e^{i(\pi/a)x} + e^{-i(\pi/a)x} = \cos(\pi/a)x \\ \psi_2 &= e^{i(\pi/a)x} - e^{-i(\pi/a)x} = \sin(\pi/a)x\end{aligned}$$

... (viii)
... (ix)

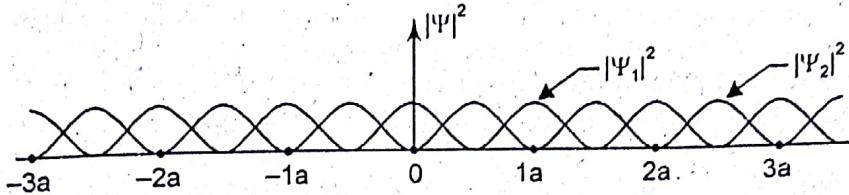


Fig. 10

Hence, the two standing waves are obtained. The probability density curve for these two stationary waves, i.e. $|\psi_1|^2$ and $|\psi_2|^2$, are shown in fig. 10. From this figure and Eq. (viii) it is clear that the value of $|\psi_1|^2$, is maximum in between the positive of positive ions. From fig. 7 it is evident that the potential energy of an electron is maximum between the ions and minimum at the positions of the ions. So an electron can have two different

value of energies, i.e. E_1 and E_2 for $k = \frac{\pi}{a}$ corresponding to the standing waves ψ_1 and ψ_2 .

Hence, no electron can have any energy between E_1 and E_2 . This phenomenon creates a difference in energy ($E_1 - E_2$) which is known as energy gap.

Q.4. (b) For an intrinsic semiconductor having band gap $E_g = 0.7$ ev, calculate the density of holes and electrons at room temperature ($= 27^\circ\text{C}$). (4)

Ans. Given $E_g = 0.7\text{ eV}$.

In intrinsic semiconductor, the concentration of electrons and holes are same. So

$$n_e = n_h = 2 \left[\frac{2\pi k T m}{h^2} \right]^{3/2} e^{(E_F - E_C - kT)}$$

The fermi level lies exactly in the middle of conduction and valence band.

$$\text{i.e. } E_F = \frac{E_C + E_V}{2}$$

$$\therefore E_F - E_C = \frac{E_C + E_V}{2} - E_C = - \frac{(E_C - E_V)}{2} = \frac{-E_g}{2}$$

$$\therefore n_e = n_h = 2 \left[\frac{2\pi k T m}{h^2} \right]^{3/2} e^{-E_g/2kT}$$

$$= 2 \times \left[\frac{2 \times 3.14 \times 1.38 \times 10^{-23} \times 300 \times 9.1 \times 10^{-31}}{6.62 \times 10^{-34}} \right] e^{\frac{0.7}{2 \times 0.026}}$$

$$= 3.6 \times 10^{19} \text{ per } m^3$$

Q.4. (c) Assuming that there are 5×10^{28} atoms/m³ in copper, find the Hall coefficient. (1)

Ans. Given $n = 5 \times 10^{28} \text{ atoms/m}^3$.

Formula used is

$$\begin{aligned}R_H &= -\frac{1}{ne} = \frac{1}{5 \times 10^{28} \times 1.6 \times 10^{-19}} \\ &= -0.125 \times 10^{-9} \text{ m}^3/\text{C}\end{aligned}$$

30-2015

Second Semester, Applied Physics-II

Q.4. (d) What do you mean by hall effect and hall voltage.

Ans. If a current carrying conductor is placed in a transverse magnetic field, a potential is developed in the conductor in the direction perpendicular to both the current and magnetic field. This phenomenon is known as hall effect.

Under Hall effect, with the application of magnetic field B , the accumulation of charge on the surfaces of the specimen continues until the force on moving charges due to electric field associated with the accumulated charge itself is large enough to cancel the force exerted by the magnetic field. So a steady state condition is achieved. In this state, the value of electric field is called Hall electric field and the potential difference so

developed is called Hall voltage. The Hall voltage V_H is given by $V_H = -\frac{JBD}{ne}$, where J is the current density, d is the width of the specimen (strip), n is the number of electrons per unit volume and e is the electronic charge. The coefficient $-\frac{1}{ne}$ is called the Hall coefficient R_H .

Q. 5. (a) Deduce the miller indices of a set of parallel which make intercepts in the ratio of $a : 2b$ on the x and y axis are parallel to z axis, $\bar{a}, \bar{b}, \bar{c}$ being primitive vectors of lattice. Also calculate the interplanar distance d of the plane taking the lattice to the cube with $a = b = c = 5\text{A}^\circ$ (5)

Ans. The parallel planes are parallel to z axis. It means that their intercepts on the z -axis are infinite. Thus, the intercepts are $a, 2b$ and ∞ . And, lattice constant $a = b = c = 5\text{A}^\circ$.

Then,

$$a : 2b : \infty = \frac{a}{h} : \frac{b}{k} : \frac{\infty}{l}$$

or

$$\frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 1 : 2 : \infty$$

or

$$h : k : l = 1 : \frac{1}{2} : \frac{1}{\infty} = 2 : 1 : 0$$

Therefore, the miller indices are (210).

∴ Interplanar distance

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{5 \times 10^{-10}}{\sqrt{2^2 + 1^2 + 0^2}}$$

$$= \frac{5 \times 10^{-10}}{\sqrt{5}} = \sqrt{5} \text{ A}^\circ$$

Q. 5. (b) In a simple cubic crystal

- (i) find the ratio of intercepts of three axes by (123) plane and
- (ii) find the ratio of spacing of {1103} and {111} planes. (4)

Ans. (i) Given $(h k l)$ of the plane as (1 2 3). Intercepts on the axes of a simple cubic crystal are given as $a/h, a/k, a/l$.

∴ The ratio of intercept are

$$\frac{a}{1} : \frac{a}{2} : \frac{a}{3} = 1 : \frac{1}{2} : \frac{1}{3}$$

(ii) The spacings d of plane (h, k, l) in a simple cubic crystal of side a

$$= \frac{1}{\sqrt{h^2 + k^2 + l^2}}$$

\therefore For plane (110),

$$d_{110} = \frac{a}{\sqrt{1^2 + 1^2 + 0}} = \frac{a}{\sqrt{2}}$$

and for plane (1 1 1),

$$d_{111} = \frac{a}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{a}{\sqrt{3}}$$

Therefore the ratio of spacing between these two plane is $d_{110}/d_{111} = \sqrt{3}/2 = 1.225$

Q.5. (c) Describe in detail laue method and also describe the usefulness of this method (6)

Ans. Refer Q.7. (a) of End Term Examination 2014.

Q. 6. (a) Give the salient postulates of statistical mechanics on the basis of which the statistical distribution functions are defined. Also state that clearly the difference in evaluation of the thermodynamic probability for each distribution function. (8)

Ans. Refer Q.3(a) of Second Term Examination 2015.

Every solid, liquid or gas in an assembly of an enormous number of microscopic particles. Likewise, radiation is an assembly of photons. Obviously the actual motions or interactions of individual particles cannot be investigated. However, the macroscopic properties of such assemblies can be explained in terms of the statistical distribution of the individuals among different possible states and their most probable behaviour.

For example, from maxwell distribution of speeds among the molecules we can calculate mean speed (which is related to the momentum carried by the molecules), mean square speed (which is related to the energy of the molecules) and so on, from the average quantities we calculate observable properties like pressure and temperature of the gas.

Usually we consider how a fixed amount of energy is distributed among the various identical particles of an assembly.

Now there are three kinds of identical particles

(i) Identical particles of any spin which are so much separated in the assembly that they can be distinguished from one another. The molecules of a gas are particles of this kind.

(ii) Identical particles of zero or integral spin which cannot be distinguished from one another. These are called *Bose particles* (or bosons) and do not obey pauli's exclusion principle. Phonons and α -particle are of this kind.

(iii) Identical particles of half integral spin which cannot be distinguished from one another. These are called *Fermi particles* (or fermions) and do obey pauli's exclusion principle. Electrons, protons, neutrons are particles of this kind.

The first kind of the particles are the classical particles and obey the Maxwell-Boltzmann energy distribution law. The second kind of particles are quantum particles and energy distribution laws for them can be derived by methods of quantum statistics (Bose-Einstein and Fermi-Dirac statistics) only.

Q. 6. (b) Define Black body radiation. (2)

Ans. When a blackbody is placed inside a uniform temperature (isothermal) enclosure after it is in equilibrium with the enclosure it gives radiation. These radiations are independent of the nature of the substance, nature of the walls of the enclosure and presence of any other body in the enclosure but depends on temperature. Such radiations in a uniform temperature enclosure are known as blackbody radiations.

Q. 6. (c) Explain wien's and rayleigh-jeans law. (5)

Ans. As per Planck's quantum hypothesis, the atoms of the wall of a blackbody behave as oscillators and each has a characteristic frequency of oscillation. Then average energy of these Planck's oscillators were calculated and finally Planck's radiation formula was derived.

Wien's law is deduced from Plank's radiation formula under the condition when the wavelength λ and temperature T are very small. However, Plank's radiation formula under the condition of high temperature T and wavelength λ takes the form of Rayleigh-Jeans law.

Planck radiation law is as following.

$$\mu\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1} d\lambda$$

When λ and T are very small then $e^{\left(\frac{hc}{\lambda KT}\right)} \gg 1$

$$\therefore \mu\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda KT}} d\lambda$$

By substituting

$$8\pi hc = A \text{ and } \frac{hc}{K} = B$$

we get

$$\mu\lambda d\lambda = \frac{A}{\lambda^5} e^{-\frac{B}{\lambda T}} d\lambda$$

This is Wein's law valid at low temperature and small wavelength.

When λ and T are large then $e^{\frac{hc}{\lambda KT}} \approx 1 + \frac{hc}{\lambda KT}$

$$\mu\lambda d\lambda = \frac{8\pi hc}{\lambda^5 \left\{ \left[1 + \frac{hc}{\lambda KT} \right]^{-1} \right\}} d\lambda$$

$$\mu\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda$$

This is Rayleigh-Jeans law valid at high temperature and large wavelength.

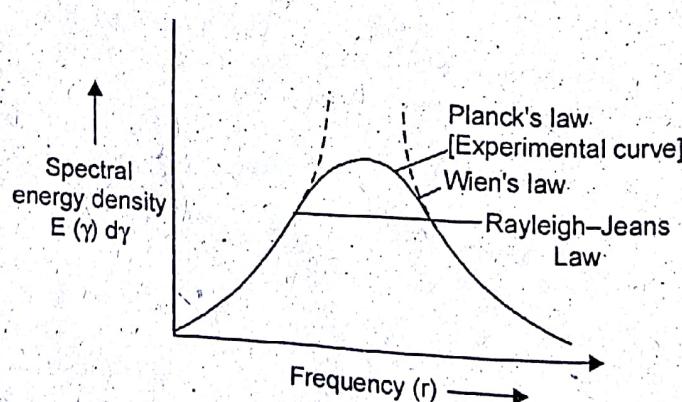


Fig. 12.

As shown in Fig. 12 the spectral energy density decreases with frequency at high frequency. Hence Rayleigh-Jeans and Wien's law are unable to explain the complete energy distribution for blackbody radiation, while Planck's law explains it completely.

**END TERM EXAMINATION [MAY-2016]
SIXTH SEMESTER [B.TECH]
APPLIED PHYSICS-II [ETPH-104]**

Time : 3 Hrs.

M.M. : 75

Note: Attempt any five questions including Q. No. 1 which is compulsory. Select one question from each unit.

Q.1 Attempt all the parts.

(a) Find the gradient of a vector -

$$\mathbf{A} = (x^2 - xy + z) \hat{i} + (x^3 - xz + x) \hat{j} + (y^2 - y + z) \hat{k}$$

Ans.

$$\vec{\mathbf{A}} = (x^2 - xy + z) \hat{i} + (x^3 - xz + x) \hat{j} + (y^2 - y + z) \hat{k}$$

Taking A as scalar:

Gradient

$$\begin{aligned}\vec{\nabla}A &= \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) [(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)] \\ &= \frac{\partial}{\partial x} [(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)] \hat{i} + \\ &\quad \frac{\partial}{\partial y} [(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)] \hat{j} + \\ &\quad \frac{\partial}{\partial z} [(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)] \hat{k} \\ &= [2x - x + 3x^2 - x + 1 + 0] \hat{i} + [-x + 0 + 2y - 1] \hat{j} + \\ &\quad [1 - x + 1] \hat{k} \\ &= [3x^2 + 1] \hat{i} + [2y - x - 1] \hat{j} + [2 - x] \hat{k}\end{aligned}$$

At point (2, 1, 1)

$$\vec{\nabla}A = [3 \times 4 + 1] \hat{i} + [2 \times 1 - 2 - 1] \hat{j} + [2 - 2] \hat{k}$$

$$\boxed{\vec{\nabla}A = 13\hat{i} - \hat{j}}$$

Q.1. (b) Write the expression of gradient in spherical coordinates. (2)

Ans. In spherical co-ordinate gradient is given by following-

$$x = r \sin \theta \cos \phi; y = r \sin \theta \sin \phi; z = r \cos \theta$$

$$\vec{\nabla}f = \hat{r} \frac{\partial f}{\partial r} + \hat{\theta} \frac{\partial f}{r \partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi}$$

Q.1. (c) Give the expression of Gauss's law for magnetic field in both differential and integral form. (2)

Ans. Gauss's law for magnetic field in differential form is-

$$\vec{\nabla} \cdot \vec{B} = 0$$

Gauss's law for magnetic field in integral form is-

$$\int_s \vec{B} \cdot d\vec{S} = 0$$

Q.1. (d) How will the equation for Poynting theorem change for static fields? (2)

Ans. Poynting theorem for static field is $\int_s (\vec{E} \times \vec{H}) d\vec{s} = 0$

Q.1. (e) Write the expression for general wave equation show how it changes in case of dielectric medium. (2)

Ans. The general wave equation is— $\nabla^2\phi - \frac{1}{V^2} \frac{\partial^2\phi}{\partial t^2} = 0$

For dielectric medium, the wave equation is— $\nabla^2\phi - \mu \epsilon \frac{\partial^2\phi}{\partial t^2} = 0$

Q.1. (f) Calculate the value of wave impedance in vacuum. (2)

Ans.

$$Z_0 = \left| \frac{E}{H} \right| = \left| \frac{E_0}{H_0} \right| = \mu_0 c \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \left[\because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \right]$$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = 376.6 \text{ Ohms}$$

where the units of Z_0 are most easily seen from the fact that it measures a ratio of E in V/m to H in A/m and these must equal V/A or Ohms. Because the units of E/H are the same as those of impedance, the value of Z_0 is often referred to as the wave impedance of free space. Further, since the ratio $Z_0 = \left| \frac{E}{H} \right|$ is real and positive; this implies that field vectors E and H are in the same phase i.e., they have same relative magnitude at all points at all times.

Q.1. (g) State the significance of normalization of wave function. (2)

Ans. The normalized wave equation is— $\int_{-\infty}^{+\infty} \psi^* \psi dv = 1$

This normalized wave gives the physically applicable wave function or probability amplitude. All wave function, representing the real particles must be normalized. This help in discarding solutions of Schrodinger's equation which do not have a finite integral in a given interval. This integral cannot be infinite. The integral must be real quantity greater than or equal to zero if ψ has to describe a real body properly. Also the wave function should have single value and its first derivative and second derivative should be finite and continuous.

Q.1. (h) Are energy levels of a particle in a box equally spaced? Show with appropriate equation. (2)

Ans. The energy of particle in a box is given by following equation-

$$E_n = \frac{n^2 h^2}{8ml^2}$$

$$n = 1, 2, 3, 4, \dots$$

where

and l is the length of the box

Now, for $n = 1$

$$E_1 = \frac{h^2}{8ml^2}$$

for $n = 2$

$$E_2 = \frac{4h^2}{8ml^2}$$

for $n = 3$

$$E_3 = \frac{9h^2}{8ml^2}$$

As, $E_n \propto n^2$, the energy levels are not equally spaced.

Q.1. (i) What thermodynamic statistics do electron follow, Bose-Einstein or Fermi-Dirac?

Ans. Electron follow Fermi-Dirac statistic as electron is indistinguishable and has half integral spin. Also, it has anti symmetric wave function and obey Pauli exclusion principle.

Q.1. (j) Draw the plane in a unit cube represented by the miller indices (110).

Ans:

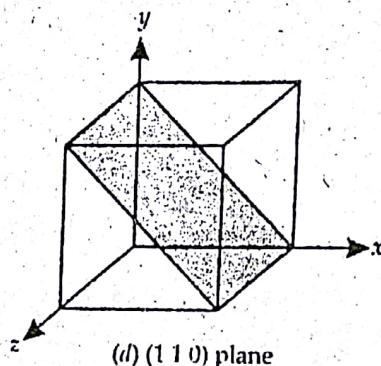


Fig. 1.

Q.1. (k) A p-type semiconductor has a Hall coefficient equal to $0.0125 \text{ m}^3/\text{C}$. Find the density of charge carrier in it.

Ans. Given

$$R_H = 0.0125 \text{ m}^3/\text{C}$$

$$R_H = \frac{1}{Ne}$$

$$N = \frac{1}{R_H e}$$

$$= \frac{1}{0.0125 \times 1.6 \times 10^{-19}} = 50 \times 10^{19} \text{ per m}^3$$

$$N = 50 \times 10^{20} \text{ per m}^3$$

Q.1. (l) State Bloch Theorem.

Ans. The Bloch theorem is a mathematical statement regarding the form of the one electron wave functions for a perfectly periodic potential.

UNIT-I

Q.2. (a) Show that the trajectory of motion of a charged particle in crossed electric and magnetic fields (constant) is a cycloid.

Ans. If a charged particle (mass = m , charge = q) enters with velocity v in such a region where an electric field E and a magnetic field B simultaneously acts, then the force acting on the particle will be:

$$\mathbf{F} = q\vec{E} + q\vec{v} \times \vec{B} - q[\vec{E} + (\vec{v} \times \vec{B})] \quad \dots(1)$$

Now if electric field E and magnetic field B are in mutually perpendicular Y and Z directions respectively, then

$$\mathbf{E} = E\hat{j} \text{ and } \mathbf{B} = B\hat{k}$$

Then force

$$\mathbf{F} = q[\hat{E}\hat{j} + (v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) \times B\hat{k}]$$

$$= qE\hat{j} + qB(v_y\hat{i} - v_x\hat{j}) \quad \dots(2)$$

But

$$\mathbf{F} = m \left(\hat{i} \frac{dv_x}{dt} + \hat{j} \frac{dv_y}{dt} + \hat{k} \frac{dv_z}{dt} \right)$$

Hence the cartesian components of acceleration of the particle will be

$$\frac{dv_x}{dt} = \frac{qBv_y}{m}, \frac{dv_y}{dt} = \frac{qE}{m} - \frac{qBv_x}{m}$$

$$\frac{dv_z}{dt} = 0 \quad \dots(8)$$

and

It is clear that the electric and magnetic fields have no effect on the component of velocity of the particle along the magnetic field (i.e., in Z-direction). It initially $v_z = 0$, then the motion of particle will be in X-Y plane.

From Eq. (3), we have

$$v_x = \frac{E}{B} - \frac{m}{qB} \frac{dv_y}{dt}$$

$$\therefore \frac{d}{dt} \left[\frac{E}{B} - \frac{m}{qB} \frac{dv_y}{dt} \right] = \frac{qBv_y}{m}$$

$$\text{or } -\frac{m}{qB} \frac{d^2v_y}{dt^2} = \frac{qB}{m} v_y \quad \text{or } \frac{d^2v_y}{dt^2} + \left(\frac{qB}{m} \right)^2 v_y = 0$$

Assuming $\left(\frac{qB}{m} \right)^2 = \omega^2$, then we get

$$\frac{d^2v_y}{dt^2} + \omega^2 v_y = 0 \quad \dots(4)$$

The above expression (4) represents the *simple harmonic motion of the particle*, whose solution is

$$v_y = A \sin(\omega t + \phi)$$

$$\text{where } \omega = \text{angular frequency} = \frac{qB}{m}$$

Again from Eq. (3), we have

$$v_x = \frac{E}{B} - \frac{m}{qB} \frac{d}{dt} [A \sin(\omega t + \phi)]$$

$$= \frac{E}{B} - A \cos(\omega t + \phi) \quad \dots(5)$$

Now for calculating the constant A and ϕ , let the particle be initially stationary i.e. at $t = 0$, $v_x = 0$ and $v_y \neq 0$

Then

$$\phi = 0 \text{ and } A = \frac{E}{B}$$

$$\text{Hence } v_y = \frac{dy}{dt} = \frac{E}{B} \sin \omega t, v_x = \frac{dx}{dt} = \frac{E}{B} [1 - \cos \omega t] \text{ and } v_z = \frac{dz}{dt} = 0$$

Integrating above equations, we get

$$x = \frac{E}{B} \left[t - \frac{\sin \omega t}{\omega} \right] + C_1, y = -\frac{E}{B} \cos \omega t + C_2 \text{ and } z = C_3$$

Now if $t = 0$, $x = y = z = 0$ i.e., the particle at origin, then

$$C_1 = 0, C_2 = \frac{E}{B\omega} \text{ and } C_3 = 0$$

Then

$$\begin{aligned}x &= \frac{E}{B\omega}(\omega t - \sin \omega t) \\y &= \frac{E}{B\omega}(1 - \cos \omega t)\end{aligned}\quad \dots(6)$$

and

$$z = 0$$

It is clear from Eq. (6) that the path of motion of the particle is **cycloidal** as shown in Fig. 2.

By putting

$$R = \frac{E}{\omega B}$$

We can get

$$(x - R\omega t)^2 + (y - R)^2 = R^2 \quad \dots(7)$$

This is the equation of a **circle** having radius R and centre which is moving in X-direction with the constant speed

$$v = \omega R = \frac{E}{B}$$

Suppose the charged particle is at rest at the origin. The magnetic force is zero; but the electric force accelerates the particle in Y-direction. When it starts moving, a magnetic force is developed. This magnetic force pulls the particle in a circular path in the downward direction. The motion is now against the electric force, therefore, it shows down. The magnetic force also goes on decreasing. The particle comes at rest at point (Fig. 2). The entire process starts again which takes the particle to point 2 and it continues. This trajectory is called as **Cycloid**. The particle moves as if a point on the rim of wheel rolling down along x-axis.

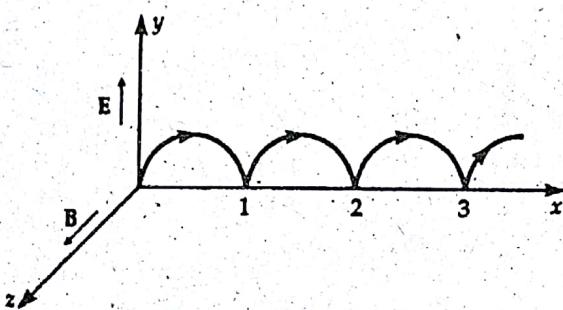


Fig. 2. Cycloid motion of charged particle in mutually perpendicular E and B fields.

Q.2. (b) What is skin depth in electromagnetic? Does it depend upon the frequency of the electromagnetic radiation? (2.5)

Ans. The distance an electromagnetic wave takes to reduce its amplitude by a factor of

$\frac{1}{e} = 0.369$ i.e., about one third its value at the surface, is called **skin depth** and is denoted by d . The skin depth also called **penetration depth**, is a measure of the distance of penetration of an electromagnetic wave into a conductor. It is expressed as

$$d = \frac{1}{\beta} = \frac{1}{\omega \sqrt{\mu \epsilon}} \left[\frac{-1 + \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2}}{2} \right]^{1/2}$$

(i) A material is called poor conductor if $\sigma \ll \epsilon \omega$ or $\sigma/\mu \omega \ll 1$, then

$$\alpha = \sqrt{\frac{\mu \epsilon}{2}} \omega \left[1 + \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} \right]^{1/2}$$

becomes

$$\alpha \approx \omega \sqrt{\mu \epsilon}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[-1 + \sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} \right]^{1/2}$$

becomes

$$\beta \approx \frac{\sigma}{2} \sqrt{\mu/\epsilon}$$

Thus skin depth

$$d = \frac{1}{\beta} = \frac{2}{\sigma} \sqrt{\mu/\epsilon}$$

Here the skin depth is independent of frequency.

(ii) For a good conductor, $\sigma \gg \omega\epsilon$ or $\sigma/\omega\epsilon \gg 1$. In this case, α and β are approximately equal i.e.

$$\alpha \approx \beta = \sqrt{\frac{\mu\sigma\omega}{2}}$$

thus the skin depth

$$d = \frac{1}{\beta} \approx \sqrt{\frac{2}{\mu\sigma\omega}}$$

It is shows that the skin depth decreases with increasing frequency. This is why in high frequency circuits, the current flows only on the surface of the conductor and a thin sheet of conducting material can act as a low pass filter for electromagnetic waves. The major importance of skin depth is that it measures the depth to which an electromagnetic wave can penetrate a conducting medium. Thus the same substance can be a good conductor at low frequency and poor conductor at high frequency. The conducting sheets which are used as electromagnetic shields must be thicker than the skin depth.

Q.2. (c) Write the Maxwell equations in differential form and state their significance. (4)

Ans. Maxwell's First Equation:

$$(\vec{\nabla} \cdot \vec{D} = \rho)$$

⇒ The net outward flux of electric displacement vector through the surface enclosing volume is equal to the net charge contained within that volume.

2. Maxwell's Second Equation: $(\vec{\nabla} \cdot \vec{B} = 0)$

⇒ The net outward flux of magnetic induction \vec{B} through any closed surface is equal to zero.

3. Maxwell's Third Equation: $(\vec{\nabla} \cdot \vec{E} = -\frac{\partial \vec{B}}{\partial t})$

⇒ The electromotive force ($emf = e = \int_C \vec{E} d\vec{l}$) around a closed path is equal to negative rate of change of magnetic flux with the path (\because magnetic flux $\phi = \int_S \vec{B} \cdot d\vec{s}$).

4. Maxwell's Fourth Equation: $(\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t})$

⇒ The magnetomotive force ($m.m.f = \oint \vec{H} \cdot d\vec{l}$) around a closed path is equal to the induction current plus displacement current through any surface bounded by the path.

Q.3. (a) An electric field in a region is given by $\vec{E} = 3\hat{i} + 4\hat{j} - 5\hat{k}$. Calculate the electric flux through the source. $\vec{S} = 2.0 \times 10^{-5} \text{ m}^2$

Ans.

$$\phi_E = \vec{E} \cdot \vec{S} = (3\hat{i} + 4\hat{j} - 5\hat{k}) \cdot (2.0 \times 10^{-5} \hat{k}) \\ = 0 + 0 - 10 \times 10^{-5} = 1.0 \times 10^{-4} \text{ Vm.}$$

$$\boxed{\phi_E = 1.0 \times 10^{-4} \text{ Vm}}$$

Q.3. (b) Discuss the continuity equation. Distinguish between conduction current and displacement current.

Ans. An equation, which expresses the equality of incoming and outgoing charges in a system and follows the law of conservation of charge, is known as the equation of continuity.

The current density J and the charge density ρ are related at each point through a differential equation. This relation is based on the fact that *electric charge can neither be created nor be destroyed and rate of increase of the total charge inside any arbitrary volume must be equal to the net flow of charge into this volume.*

$$\therefore I = \int_S \mathbf{J} \cdot d\mathbf{S} \quad \dots(1)$$

Again, considering charge leaving a volume V per second

$$I = -\frac{\partial}{\partial t} \int_V \rho dV \quad \dots(2)$$

The negative sign comes here because the current is positive when the net charge is from the outside of V to within.

Since, we are dealing with a fixed volume V , hence we may write,

$$-\frac{\partial}{\partial t} \int_V \rho dV = -\int_V \frac{\partial \rho}{\partial t} dV \quad \dots(3)$$

Again, using divergence theorem, we can write

$$\int_S \mathbf{J} \cdot d\mathbf{S} = -\int_V (\nabla \cdot \mathbf{J}) dV \quad \dots(4)$$

Therefore, from Eqs. (2), (3) and (4), we have

$$I = \int_V (\nabla \cdot \mathbf{J}) dV = -\int_V \frac{\partial \rho}{\partial t} dV \quad \dots(5)$$

$$\int_V (\nabla \cdot \mathbf{J}) dV = -\int_V \frac{\partial \rho}{\partial t} dV$$

$$\text{or } \int_V \left((\nabla \cdot \mathbf{J}) + \frac{\partial \rho}{\partial t} \right) dV = 0 \quad \dots(6)$$

This integral must be zero for any arbitrary volume. It is only possible when integrand is zero, i.e.,

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0 \quad \dots(7)$$

This differential equation is known as the equation of continuity. If the region does not contain a source or sink of current, $\frac{\partial \rho}{\partial t} = 0$ and hence for steady current, we have

$$\nabla \cdot \mathbf{J} = 0$$

Difference between conduction current and displacement current is given below:

S. No.	Conduction current	Displacement current
1.	Actually flows through in conductivity medium and obeys Ohm's law.	Set up in a dielectric medium due to changing electric field across the dielectric, which leads to variation of induced displacement of charge.
2.	$J_c = \sigma E, I = \frac{V}{R}$	$J_d = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t}$
3.	For constant $E, J_c \neq 0$	For constant $E, J_d = 0$

Q.3. (c) If the earth receives 20 cal/min/sq. cm. solar energy, what are the amplitudes of electric and magnetic fields of radiation. (4)

Ans. Poynting vector is—

$$\vec{S} = \vec{E} \times \vec{H} = EH \sin 90^\circ = EH$$

$$\text{Solar energy} = 20 \text{ Cal/min/cm}^2$$

$$= \frac{20 \times 4.18 \times 10^4}{60} \text{ Jm}^{-2}\text{s}^{-1}$$

$$\therefore EH = \frac{20 \times 4.18 \times 10^4}{60} = 14000$$

But

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377$$

$$EH \times \frac{E}{H} = 14000 \times 377$$

$$E^2 = \sqrt{14000 \times 377} = \sqrt{5278000} = 2297.4 \frac{\text{V}}{\text{m}}$$

$$\text{and } H = \frac{E}{377} = \frac{2297.4}{377} = 6.094 \approx 6.1 \frac{\text{A}}{\text{m}}$$

Amplitudes of electric and magnetic fields of radiation are—

$$E_0 = E\sqrt{2} = 2297.4\sqrt{2} = 3249.01 \text{ V/m}$$

$$H_0 = H\sqrt{2} = 6.1\sqrt{2} = 8.63 \frac{\text{A}}{\text{m}}$$

Q.3.(d) A 2 KW laser beam is concentrated by a lens into cross-sectional area about 10^{-6} cm^2 . Find the poynting vector. (2)

Ans. Poynting vector is $\frac{\text{Power}}{\text{Area}}$

$$\vec{S} = \frac{P}{\text{Area}}$$

Given

$$P = 2 \text{ kW} = 2 \times 10^3 \text{ W}$$

$$\text{Area} = 10^{-6} \text{ cm}^2 = 10^{-10} \text{ m}^2$$

$$\therefore \vec{S} = \frac{2 \times 10^3}{10^{-10}} = 2 \times 10^{13} \frac{\text{W}}{\text{m}^2}$$

UNIT-II

Q.4. (a) Using the uncertainty principle show that an electron does not exist inside a nucleus. (2)

Ans. According to Heisenberg's uncertainty principle,

$$\Delta x \Delta p_x = \frac{h}{2\pi}$$

The diameter of the nucleus is of the order of 10^{-14} m. If the electron exists in the nucleus, it can be anywhere within the diameter of the nucleus. Therefore, the maximum uncertainty Δx in the position of electron is the same as the diameter of the nucleus.

i.e. $\Delta x = 10^{-14}$ m

∴ The minimum uncertainty in the momentum is given by

$$\begin{aligned} \Delta p_x &= \frac{h}{2\pi\Delta x} \\ &= \frac{6.63 \times 10^{-34}}{2\pi \times 10^{-14}} = \frac{6.63 \times 10^{-20}}{2\pi} \\ &= 1.055 \times 10^{-20} \text{ kg m/sec.} \end{aligned}$$

It means that if electron exists in the nucleus, its minimum momentum must be

$$p_{\min} = 1.055 \times 10^{-20} \text{ kg m/s}$$

For the electron of the minimum momentum, the minimum energy is given by

$$\begin{aligned} E_{\min}^2 &= p_{\min}^2 c^2 + m^2 c^4 \\ &= (1.055 \times 10^{-20} \times 3 \times 10^8)^2 + (9.1 \times 10^{-31})^2 (3 \times 10^8)^4 \\ &= (3 \times 10^8)^2 [1.113 \times 10^{-40} + 7.4692 \times 10^{-44}] \end{aligned}$$

Since the second term in the bracket is much smaller than the first, it can be neglected,

then,

$$\begin{aligned} E_{\min} &= 3 \times 10^8 \sqrt{1.113 \times 10^{-40}} \text{ J} \\ &= 3 \times 10^8 \times 1.055 \times 10^{-20} \text{ J} = 3.1649 \times 10^{-12} \text{ J} \end{aligned}$$

or

$$E_{\min} = \frac{3.1649 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} \approx 20 \text{ MeV.}$$

Thus, if a free electron exists in the nucleus it must have a minimum energy of about 20 MeV.

The maximum K.E. which a β -particle, emitted from radioactive nuclei is of the order of 4 MeV. Therefore, this clearly shows that electrons can not be present within the nucleus.

Q.4. (b) Describe Davisson-Germer experiment. Find the lowest energy in eV, for an electron in one dimensional box of length $a = 0.2$ nm. (6)

Ans. The first proof of the existence of "matter waves" was obtained in 1927 by Davisson and Germer the two American physicists. They succeeded in measuring the de Broglie wavelength for slow electrons, accelerated by a low potential difference by diffraction methods. The experimental arrangement is shown in the Fig. 3. The electron beam is produced from an electron gun consisting of tungsten filament F heated by low tension battery. The electrons excited by the filament are accelerated in an electric field of known potential difference from a high tension battery. The electrons are collimated to a fine beam and made to strike a Nickel target

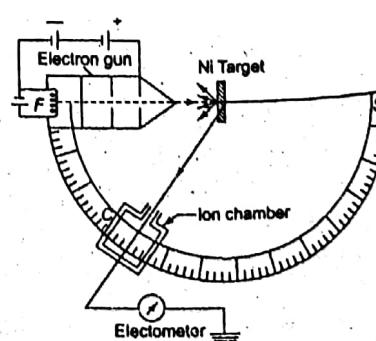


Fig.3. Davisson and Germer electron Diffraction apparatus

which is capable of rotation about an axis parallel to the axis of the incident beam. The electrons are scattered in all directions by the atoms of the crystal. The intensity of the electron beam scattered in a given direction is measured by allowing it to enter in a Faraday cylinder called collector C which can be moved along a graduated circular scale S, so that it is able to receive the reflected electrons at all angles between 20° and 90° .

Davisson and Germer noticed that the strongest of the scattered electron beams corresponded accurately to diffraction maxima that would be expected in the diffraction of X-rays by the same crystal. The angular distribution of scattered electrons was analogous to optical diffraction patterns from a plane diffraction grating whose lines consisted of the rows of nickel atoms in the surface of the target crystal. The wavelength associated with the diffraction pattern can be obtained according to Bragg's law:

$$n\lambda = 2d \sin \theta$$

where d is the distance between the rows of atoms.

It was observed by Davisson and Germer that when an electron beam accelerated by a potential of 54 volts was directed upon a Nickel target, a sharp diffraction maxima appeared in the electron currents. The incident and the scattered beams in this case make an angle of 65° with the family of Bragg's planes. The spacing of planes in this family which can be determined by X-rays diffraction is 0.91 \AA . From above Bragg's equation, taking $n = 1$, we have,

$$2 \times 0.91 \times \sin 65^\circ = 1 \cdot \lambda \text{ or } \lambda = 1.65 \text{ \AA}$$

The wavelength of electrons accelerated through potential of 54 volts can be calculated as

$$\frac{1}{2}mv^2 = eV,$$

m is the mass of the electron.

$$m^2v^2 = 2meV$$

or

$$mv = \sqrt{2meV}$$

The de-Broglie wavelength of the electron will be given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

Putting $h = 6.6 \times 10^{-34}$ joule sec, $m = 9 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ coulomb, we obtain

$$\lambda = \frac{12.28}{\sqrt{V}}, \text{ with } V = 54 \text{ volts}$$

$$\lambda = 1.66 \text{ \AA}$$

There is an excellent agreement between the two results. Thus Davisson Germer experiment is a definite evidence that the electron beams do behave as wave and the wavelength of these beams is thus given by the de-Broglie equation.

Energy of particle in a box is given by

$$E_n = \frac{h^2 n^2}{8ml^2}$$

Given,

$$n = 1, h = 6.62 \times 10^{-34} \text{ J-S}, m = 9.1 \times 10^{-31} \text{ kg}$$

$$l = 0.2 \times 10^{-9} \text{ m}$$

Lowest energy will be

$$E = \frac{1 \times (6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.2 \times 10^{-9})^2}$$

$$= \frac{43.83}{2.912} \times 10^{-68} \times 10^{+31} \times 10^{+18} = 15.05 \times 10^{-19} \text{ J}$$

$$E = 15.05 \times 10^{-19} \text{ J}$$

$$E = \frac{15.05 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19}} = 9.4 \text{ eV}$$

Q.4. (c) The eigen function of an operator $\frac{d^2}{dx^2}\psi(x) = e^{ax}$. Find the corresponding eigen value.

Ans. The operator and eigen value relation is

$$\hat{O}\psi = 0\psi$$

where \hat{O} is operator and 0 is eigen value

Here,

$$\hat{O} = \frac{d^2}{dx^2} \text{ and } \psi = e^{ax}$$

$$\hat{O}\psi = \frac{d^2}{dx^2} e^{ax}$$

$$\hat{O}\psi = a^2 e^{ax}$$

We can write this as—

$$\hat{O}\psi = a^2 \psi$$

Such that

$$0 = a^2$$

$\therefore a^2$ is eigen value.

Q.5. (a) What type of statistics shall be applicable for a gas of photon? Justify your answer.

Ans. Bose-Einstein statistics shall be applicable for a gas of photons. Bose-Einstein statistics is obeyed by indistinguishable particles of integral spin quantum number that have symmetric wave function and does not obey Pauli exclusion principle.

Q.5. (b) Compare the qualitative features of Maxwell Boltzmann, Bose-Einstein in and Fermi-Dirac statistics on the basis of their functions.

Ans. Refer Q.1.(b) First Term Examination 2016.

Q.5(c) Show that Bose-Einstein, Fermi-Dirac statistics reduces to Maxwell boltzamann statistics at high temperature.

Ans. The distribution laws of three statistics are given below.

$$\frac{g_i}{n_i} = e^{\alpha} e^{E_i/k_B T} \quad \dots(1) [\text{For M - B}]$$

$$\frac{g_i}{n_i} = e^{\alpha} e^{E_i/K_B T} - 1 \quad \dots(2) [\text{For B - E}]$$

$$\frac{g_i}{n_i} = e^{\alpha} e^{E_i/K_B T} + 1 \quad \dots(3) [\text{For F - D}]$$

If $\frac{g_i}{n_i} \gg 1$ then $\frac{g_i}{n_i} \approx \left(\frac{g_i}{n_i} + 1 \right) \approx \left(\frac{g_i}{n_i} - 1 \right)$. In this limit both B.E. and F.D.

distributions are identical with M.B. distribution. This limit $\left(\frac{g_i}{n_i} \gg 1 \right)$ occurs when the temperature is not too low and pressure (or density) is not too high.

UNIT-III

Q.6. (a) Chromium has structure. It has atomic radius 0.125 nm. Calculate the free volume/unit cell.

Ans. Given—

$$r = 0.125 \text{ nm} = 1.25 \times 10^{-10} \text{ m}$$

Since chromium has bcc unit cell, so

$$a = \frac{4}{\sqrt{3}}r = \frac{4}{\sqrt{3}} \times 1.25 \times 10^{-10} = 2.885 \times 10^{-10} \text{ m}$$

Volume of unit cell -

$$V = a^3 = (2.885 \times 10^{-10})^3 = 2.402 \times 10^{-29} \text{ m}^3$$

No. of atoms in unit cell = 2

$$\text{Volume of atoms in unit cell} = v = \frac{4}{3}\pi r^3 \times 2$$

$$V = \frac{4}{3} \times 3.14 \times (1.25 \times 10^{-10})^3 \times 2 = 1.633 \times 10^{-29} \text{ m}^3$$

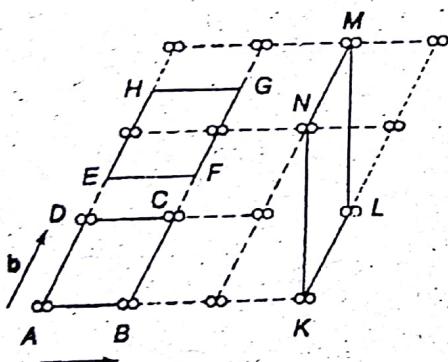
$$\text{Free volume per cell} = V - v = 2.402 \times 10^{-29} - 1.633 \times 10^{-29} = 7.67 \times 10^{-30} \text{ m}^3$$

Q.6. (b) Describe with proper diagram the following terms.

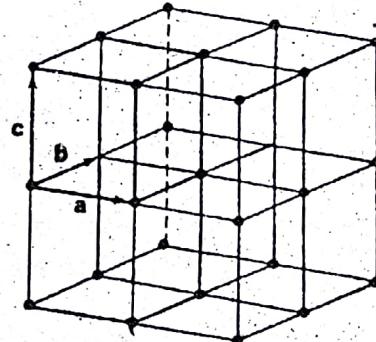
(a) Unit cell (b) Packing factor (c) Coordination number (d) fcc (4)

Ans. (a) Unit cell: In a space lattice an array of lines divides the space into identical volumes. These volumes fill the space completely by translation along the respective sets of axes and are known as unit cells.

Consider part of a two dimensional crystal, the atoms of which are arranged in a pattern shown in Fig. 4. (a). Each cluster of atoms consists of a dot two open circles and called pattern unit. It is observed that when a parallelogram as ABCD is repeatedly translated by vector \mathbf{a} and \mathbf{b} corresponding to AB and AD respectively, the pattern may be reproduced. (the dotted parallelograms). The region ABCD is called



(a) Unit cells in two dimensional lattice



(b) Primitive tive cells of the space lattice in three dimensions

Fig.4.

a unit cell and the vectors \mathbf{a} and \mathbf{b} as basis vectors. Unit cells can also be defined in any of the possible ways $EFGH$ or $KLMN$. The unit cell may thus be defined as "a region of the crystal that will cover all space under action of the translation operations defined by fundamental translation vectors." In two dimensions \mathbf{a} , \mathbf{b} are translation vectors of the lattice and area of unit cell is $a \cdot b$ and in three dimensions volume of the unit cell is $|a \times b \cdot c|$.

(b) **Packing factor** – It is the ratio of volume occupied by the atoms molecules or ions in a unit cell (v) by the total volume of unit cell (V).

$$PF = \frac{\text{Volume occupied by the atoms in an unit cell}}{\text{Volume of unit cell}}$$

$$= \frac{(\text{Number of atoms present}) \times (\text{Volume of an atom})}{\text{Volume of unit cell}}$$

$$PF = \frac{v}{V}$$

(c) **Coordination Number** – It is the number of nearest neighbouring atoms, molecules or ions to a particular atom.

(d) Face Centered Cubic (fcc) structure— A face centered cubic cell consists of eight corner atoms and six face centered atoms. A face centered cubic unit cell is shown in Fig.4. (c)

The atom in a fcc unit cell touches along the face diagonal. Each and every corner atom is shared by eight adjacent unit cells. Therefore each and every corner atoms contributes $\frac{1}{8}$ of its part to one unit cell.

So, the number of atoms contributed by

$$\text{the corner atoms} = \frac{1}{8} \times 8 = 1.$$

Two unit cells share each and every face centered atom. Therefore, the contribution of a face centered atom to unit cell is $\frac{1}{2}$. So the total number of atoms contributed by face centered atoms

$$= \frac{1}{2} \times 6 = 3.$$

\therefore Total number of atoms present in a fcc unit cell $= 1 + 3 = 4$.

Q.6.(c) Deduce the Miller indices of a plane which cuts off intercepts in the ratio $1a:3b:-2c$ along the three axis. (2)

Ans. From the law of rational indices

$$\frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 1 : 3 : -2; \quad 1a : 3b : -2c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

$$\therefore h : k : l = 1 : \frac{1}{3} : \frac{1}{2} = 6 : 2 : -3$$

Thus

$$h = 6, k = 2, l = -3$$

Hence the plane is $(62\bar{3})$

Q.6. (d) What is the difference between (111) and $\langle 111 \rangle$ for Miller indices. (2.5)

Ans. (111) —It represent $h = 1, k = 1, l = 1$ – miller indices. It represent family of planes orthogonal to $ha + kb + lc$ where a, b, c are the basis of reciprocal lattice vectors.

$\langle 111 \rangle$ —It represent the set of all directions that are equivalent to $[111]$ by symmetry. $[111]$ denotes a direction in the basis of the direct lattice vectors instead of the reciprocal lattice.

Q.7. (a) Define the following: [4]

(a) Unit cell (b) Primitive cell (c) Primitive Lattice (d) Bravis Lattice

Ans. (a) Unit cell— A unit cell is defined as a fundamental building block of a crystal structure which can generate the complete crystal by repeating its own dimensions in various directions.

(b) Primitive cell— It is the smallest unit cell in volume constructed by primitives. It consists of only one full atom.

(c) Primitive lattice—It is a lattice having lattice point only at the corners of the unit cell.

(d) Bravis lattice— It is a three dimensional lattice. A bravis lattice tiles space without any gaps or holes. There are 14 ways in which this can be accomplished. Lattices are characterized by translational symmetry. Bravis lattices contain seven crystal systems and four lattice centering types.

Q.7. (b) Germanium crystallizes in diamond form structure with 8 atoms per unit cell. If lattice constant is 5.62 Angstrom, calculate the density of germanium. (2.5)

Ans. We know,

$$a^3 = \frac{nM}{\rho N}$$

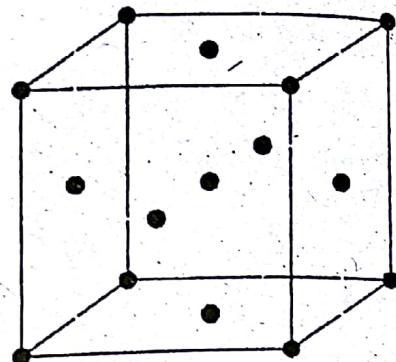


Fig. 4. (c) Face centered cubic cell

or

$$\rho = \frac{nM}{Na^3} = \frac{8 \times 72.59}{(5.62 \times 10^{-10})^3 \times 6.023 \times 10^{26}}$$

$$\rho = 5434.5 \text{ kg/m}^3 = 5.435 \text{ g/cc.}$$

Q.7. (c) Write short notes on-

(a) Point defect (b) Schottky defect (c) Frenkel defect

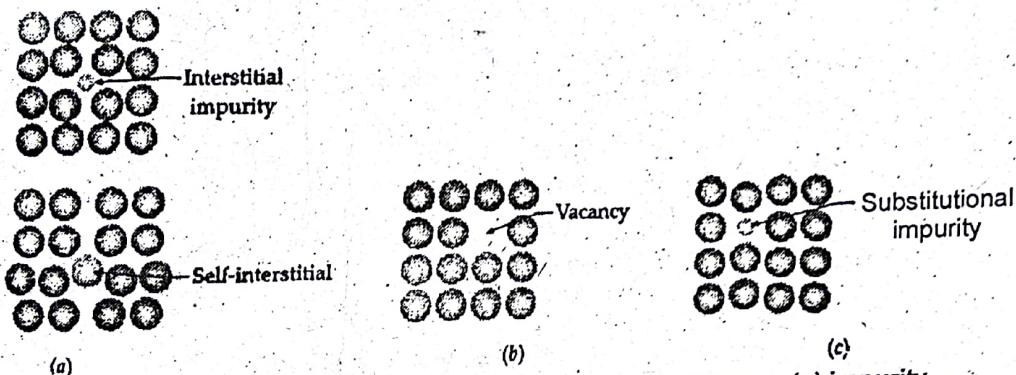
(6)

Ans. (a) Point defect: A lattice defect which spreads out very little in all the three dimensions is called a point defect. There are following types :

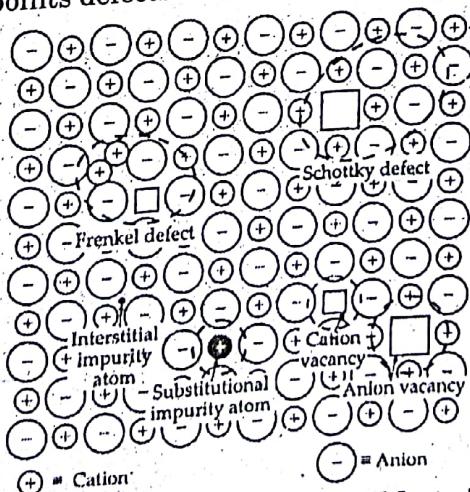
(i) Interstitial atom (ii) Vacancies (iii) Impurity atom.

(i) Interstitial atom: This is an atom inserted into the voids (called interstice of the lattice) between the regularly occupied sites. Thus such an atom does not occupy regular lattice sites. This extra atom may be an impurity atom or an atom of the same type as on the regular sites. Fig. 5(a).**(ii) Vacancies:** These are the lattice sites from which the atoms are missing [Fig. 5(b).] Such a vacancy is also called *Schottky defect*.But if a vacancy is created by transferring an atom from a regular lattice site to an interstitial position then it is called *Frenkel defect*.

In this case, therefore, two imperfections are created vacancy as well as an interstitial atom.

(iii) Impurity atom: This is a defect in which a foreign atom occupies a regular lattice site (Fig. 5 (c))**Fig. 5. Three common point defects. (a) interstitial, (b) vacancy, (c) impurity.**

A point imperfection is localised near a point or atom in the structure or in other words point defects are lattice errors at isolated lattice points. Point defects occur in metallic as well as ionic lattices. These types of point defects are shown in Fig. 5 (d)

**Fig. 5(d): Illustrating a number of possible point defects in a two-dimensional ionic crystal.**

(b) Schottky defect

In real crystals, there are irregularities of the atomic array in which atoms are missing at some lattice points. Such a point is called a *vacancy* (also called Schottky defect). This is shown in Fig 5. (e).

In spite of care taken in the preparation of crystals, vacancies are always present in all crystals. In fact, as a result of thermal fluctuations, vacancies are produced and destroyed constantly in the crystal. Formally such a defect might be produced by plucking an interior atom out of its regular lattice site and placing it on the surface. This act requires energy. Moreover, the disorder increases

resulting in an increase in the entropy. In thermal equilibrium, a certain number of lattice vacancies are always present which can be estimated as follows :

If E_v is the energy required to take an atom from a lattice site inside the crystal to lattice site on the surface, $n E_v$ is the increase in internal energy associated with the production of n isolated vacant sites. The total number of ways in which we can pick up n atoms from the crystal consisting N atom is

$$\omega = \frac{N!}{(N-n)!n!} \quad \dots(1)$$

Since disorder increases due to creation of n vacancies the corresponding increase in entropy is given by

$$S = K_B \ln \omega = K_B \ln \frac{N!}{(N-n)!n!} \quad \dots(2)$$

This in turn produces a change in free energy F

$$F = U - TS \quad \dots(3)$$

$$F = nE_v - K_B T \ln \frac{N!}{(N-n)!n!} \quad \dots(4)$$

The second term on right hand side can be simplified by the use of *Stirling approximation*:

$$\ln x! = x \ln x - x \quad \dots(5)$$

$$F = nE_v - K_B T [N \ln N - N - (N-n) \ln (N-n) + N - n - n \ln n + n] \quad \dots(6)$$

$$F = nE_v - K_B T [N \ln N - (N-n) \ln (N-n) - n \ln n] \quad \dots(6)$$

Free energy in thermal equilibrium at constant volume must be minimum with respect to changes in n i.e.

$$\left(\frac{\partial F}{\partial n} \right)_T = 0 = E_v - K_B T \ln \frac{N-n}{n} \quad \dots(7)$$

or

$$0 = E_v + K_B T \ln \frac{n}{N-n} \quad \dots(8)$$

or

$$n = (N-n) \exp \left(\frac{-E_v}{K_B T} \right) \quad \dots(8)$$

If $n \ll N$, we can neglect n in $(N-n)$ so that

$$n \approx N \exp \left(\frac{-E_v}{K_B T} \right) \quad \dots(9)$$

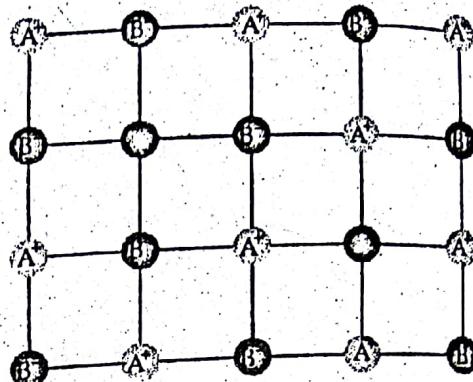


Fig. 5 (e) Schottky defect

If

$$E_v = 1 \text{ eV} \text{ and } T = 1000 \text{ K}, \text{ then}$$

$$n \approx e^{-12} \approx 10^{-5} / \text{cm}^3 \quad \dots(10)$$

The equilibrium concentration of vacancies decreases as the temperature decreases.

Figure 5(f) shows an alkali halide crystal with vacancies. Fig. 5(f) displays two separate vacancies and a vacancy pair in alkali halide crystal. The pairs are formed because the cation and anion vacancies in alkali halides are electrically charged and they attract each other electrostatically.

Due to formation of the pair a dipole is also produced.

rium, a certain number d as follows:
ce site inside the crys
energy associated with
ays in which we can pi

In ionic crystals, formation of paired vacancies is most favoured, i.e., in ionic crystals, equal number of positive and negative ion vacancies are produced. The formation of pairs makes it possible to keep the surface of the crystal electrostatically neutral. The number of pairs can be related to the total number of atoms present in the crystal on following the same procedure as adopted above.

The different ways in which n separated pairs can be formed are:

$$\omega = \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(11)$$

So that increase in entropy will be

$$S = K_B \ln \omega \\ = K_B \ln \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(12)$$

with corresponding change in free energy

$$F = U - TS$$

$$F = nE_p - K_B T \ln \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(13)$$

where E_p = Energy of formation of a pair.

Applying Stirling's approximation to simplify the factorial terms i.e.,

$$\begin{aligned} \ln \left[\frac{N!}{(N-n)!n!} \right]^2 &\equiv 2[\ln N! - \ln(N-n)! - \ln n!] \\ &= 2[N \ln N - N(N-n) \ln(N-n) + (N-n) - n \ln n + n] \\ &= 2[N \ln N - (N-n) \ln(N-n) - n \ln n] \end{aligned} \quad \dots(14)$$

Putting it in Eq. (13), we get

$$F = nE_p - 2K_B T [N \ln N - (N-n) \ln(N-n) - n \ln n]$$

Differentiating the above equation with respect to n , we get

$$\begin{aligned} \left(\frac{\partial F}{\partial n} \right)_T &= E_p - 2k_B T [0 + \ln(N-n) + 1 - \ln n - 1] \\ &= E_p - 2k_B T \ln \frac{(N-n)}{n} \end{aligned} \quad \dots(16)$$

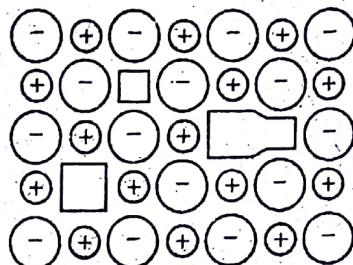


Fig. 5 (f) A plane of a pure alkali halide crystal, showing a vacant positive ion site, a vacant negative ion site, and a coupled pair of vacant sites of opposite sign (vacancies in alkali halide)

the corresponding

$(N-n)!$
ified by the use of S_{min}
 $n \ln n$
ume must be minimum

$-n$

i

Free energy in thermal equilibrium attained at temperature T is constant, so that

$$E_p - 2K_B T \ln \frac{(N-n)}{n} = 0$$

or

$$\ln \frac{N-n}{n} = \frac{E_p}{2K_B T}$$

or

$$\frac{N-n}{n} = \exp \left[\frac{E_p}{2K_B T} \right]$$

or

$$n = N \exp \left(\frac{-E_p}{2K_B T} \right)$$

...(17)

provided $n \ll N$. In NaCl crystal $E_p = 2.02 \text{ eV}$ and at room temperature $n = 10^6 \text{ cm}^{-3}$

(c) FRENKEL DEFECT

We know that interstitial atoms are the extra atoms that are squeezed in between normal atom sites. When an interstitial is caused by transferring an atom from a lattice site to an interstitial position, a vacancy is created. The associated vacancy and interstitial atom is called *Frenkel defect*. This is shown in Fig. 5(g)

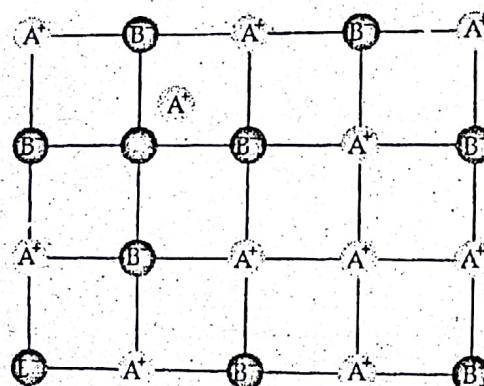


Fig. 5(g) Frenkel defect

Proceeding in the same way as in the case of schottky defect, we can calculate number of Frenkel in equilibrium at a temperature T .

Let in a perfect crystal

E_i = the energy required to displace an atom from regular lattice site to an interstitial position;

N_i = the interstitial atoms and

N = the atoms

Then the total number of ways in which n Frenkel defects can be formed, will be given by

$$\omega = \frac{N!}{(N-n)!n!} \times \frac{N_i!}{(N_i-n)!n!} \quad \dots(18)$$

The corresponding increase in entropy due to creation of Frenkel defect is

$$S = K_B \ln \omega = K_B T \ln \left[\frac{N!}{(N-n)!n!} \times \frac{N_i!}{(N_i-n)!n!} \right] \quad \dots(19)$$

which in turn produces a change in free energy

$$F = U - TS$$

$$= nE_i - K_B T \ln \left[\frac{N!}{(N-n)!} \times \frac{N!}{(N-n)!n!} \right] \quad \dots(20)$$

Using the Stirling's approximation for logarithmic term, we get

$$\begin{aligned} \ln \left[\frac{N!}{(N-n)!n!} \times \frac{N_i!}{(N_i-n)!n!} \right] &= \ln \frac{N!}{(N-n)!n!} + \ln \frac{N_i!}{(N_i-n)!n!} \\ &\approx N \ln N + N_i \ln N_i - (N-n) \ln(N-n) - (N_i-n) \ln(N_i-n) - 2n \ln n \end{aligned} \quad \dots(21)$$

Putting this value of logarithmic term in the expression for free energy and then differentiating with respect to n , we get

$$\left(\frac{\partial F}{\partial n} \right)_T = E_i - K_B T \ln \frac{(N-n)(N_i-n)}{n^2} \quad \dots(22)$$

At equilibrium, the energy is constant, so that

$$\begin{aligned} \left(\frac{\partial F}{\partial n} \right)_T &= 0 \\ E_i &= K_B T \ln \frac{(N-n)(N_i-n)}{n^2} \\ &\approx K_B T \ln \frac{NN_i}{n^2} \end{aligned} \quad \dots(23)$$

Taking $N \gg n$ and $N_i \gg n$

$$\text{Thus, } E_i = K_B T [\ln(NN_i) - 2 \ln n]$$

$$\text{or } 2 \ln n - \ln NN_i = \frac{E_i}{K_B T}$$

$$\text{or } \ln \frac{NN_i}{n^2} = + \frac{E_i}{K_B T}$$

$$n_2 = NN_i \exp \left(- \frac{E_i}{K_B T} \right)$$

$$n = (NN_i)^{1/2} \exp \left(- \frac{E_i}{2K_B T} \right) \quad \dots(24)$$

Showing that n should be proportional to $(NN_i)^{1/2}$

In an *ionic crystal* Schottky and Frenkel defects are shown in Fig. 5(h). The arrows indicate the direction of the displacement of the ions. In Schottky defect the ion ends up on the surface of the crystal, whereas in *Frenkel defect* ion is removed to an interstitial position. From the study ionic conductivity and the density measurements, it is concluded that in pure alkali halides Schottky vacancies are most common, whereas in pure silver halides Frenkel vacancies are most common. Due to Schottky defects, volume of the crystal is increased without any change in the mass and consequently, production of this defect, lowers the density of the crystal. On the other hand, the production of

Γ is constant.

Frenkel defects does not change the volume of the crystal so that the density of the crystal remains constant.

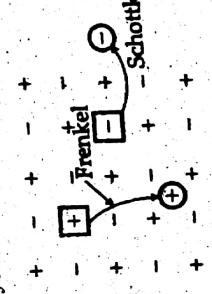


Fig.5.(b) Schottky and Frenkel defects in an ionic crystal.
The arrows indicate the direction of displacement of the ions.

In alkali halide crystal doped with divalent impurities like Ca^{2+} , each impurity ion replaces two host cations to keep the crystal electrically neutral e.g., if KCl crystal is taken then impurity ion Ca^{++} will replace two K^+ ions, one K^+ ion in place will be filled with Ca^{++} ion and the second will remain vacant (Fig. 5(i)). Though Ca^{++} ion is heavier and smaller than K^+ ion but due to production of vacancies, Density of KCl crystal decreases as predicted by experimental results.

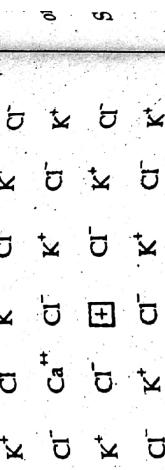


Fig.5.(i) Production of cation lattice vacancy by the solution of CaCl_2 in KCl :

to ensure electrical neutrality a cation vacancy is introduced into the lattice with each divalent cation Ca^{++} .

In alkali halide crystal doped with divalent impurities like Ca^{2+} , each impurity ion replaces two host cations to keep the crystal electrically neutral e.g., if KCl crystal is taken then impurity ion Ca^{++} will replace two K^+ ions, one K^+ ion in place will be filled with Ca^{++} ion and the second will remain vacant (Fig. 5(i)). Though Ca^{++} ion is heavier and smaller than K^+ ion but due to production of vacancies, Density of KCl crystal decreases as predicted by experimental results.

UNIT-IV

Q.8. (a) Describe Kronig-Penney Model.

Ans. The Kronig-Penney model is a simple, idealized quantum mechanical system that consists of an infinite array of rectangular potential barriers.

Kronig-and Penney in 1931 solved the Schrodinger's equation for electrons in a simple idealized periodic field. The model is of considerable importance because it interprets the main features of the band structure of metals. The periodic field was assumed in the form of potential wells of zero potential energy of width a separated by rectangular barriers of width b and height V_0 as shown in Fig. 6. For the potential of Fig. 6, the Schrodinger equation has to be solved in two regions:

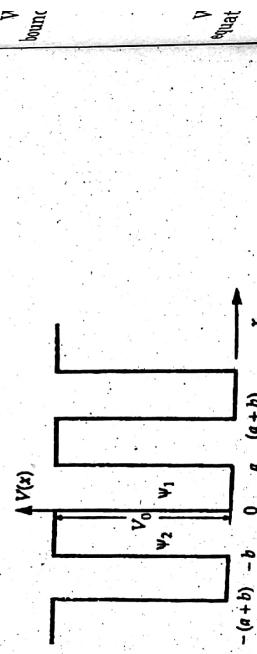


Fig. 6. One dimensional chain of potential wells.

(a) Region I, $0 < x < a$, potential well region in which $V = 0$ and

(b) Region II, $-b < x < 0$, barrier region in which $V = V_0$

In order to find the allowed energies, we solve the Schrodinger's equation in these two regions and apply the appropriate boundary condition.

In region I, the Schrodinger's equation is

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

or $\frac{d^2\psi_1}{dx^2} + \beta^2\psi_1 = 0$ [Here, $\beta^2 = \frac{2mE}{\hbar^2}$] ... (1)

Solution of this equation is

$$\psi_1 = Ae^{i\beta x} + Be^{-i\beta x}, \text{ where } A \text{ and } B \text{ are constants.} \quad \dots(2)$$

In region II, the Schrodinger's equation is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$$

or $\frac{d^2\psi_2}{dx^2} - \alpha^2\psi_2 = 0$ [Here, $\alpha^2 = \frac{2m(E - V_0)}{\hbar^2}$] ... (3)

Solution of this equation, for $E < V_0$ is

$$\psi_2 = Ce^{\alpha x} + De^{-\alpha x} \quad \dots(4)$$

where C and D are constants.

For continuity at $x = 0$

$$\psi_1(0) = \psi_2(0) \quad \dots(5)$$

$$\left. \frac{\partial \psi_1}{\partial x} \right|_{x=0} = \left. \frac{\partial \psi_2}{\partial x} \right|_{x=0} \quad \dots(6)$$

In a periodic lattice $V(x+a) = V(x)$ it is expected that the wavefunction will also exhibit this periodicity. Since the wave function must be a Bloch wave function, therefore Eq. (1) gives

$$\psi_k(x+a+b) = e^{-ik(a+b)}\psi_k(x) \quad \dots(7)$$

We incorporate this requirement at $x = -b$ and $x = a$, then we obtain the continuity boundary condition as

$$\psi_2 \Big|_{x=-b} = e^{-ik(a+b)}\psi_1 \Big|_{x=a} \quad \dots(8)$$

$$\left. \frac{\partial \psi_1}{\partial x} \right|_{x=-b} = e^{-ik(a+b)} \left. \frac{\partial \psi_1}{\partial x} \right|_{x=a} \quad \dots(9)$$

When we apply the boundary conditions, Eqn. (5), (6), (7) and (8), we obtain four equations for four unknowns

$$A + B = C + D \quad \dots(10)$$

$$i\beta(A - B) = \alpha(C - D) \quad \dots(11)$$

$$Ce^{ab} + De^{ab} = e^{-ik(a+b)}[Ae^{i\beta a} + Be^{-i\beta a}] \quad \dots(12)$$

$$\alpha Ce^{ab} - \alpha De^{ab} = e^{-ik(a+b)}i\beta[Ae^{i\beta a} - Be^{-i\beta a}] \quad \dots(13)$$

By solving these equations simultaneously e.g. by requiring that the determinants of the coefficients of A, B, C and D vanish, we get

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ i\beta & -i\beta & -\alpha & \alpha \\ -e^{-ik(a+b)+i\beta a} & -e^{-ik(a+b)-i\beta a} & e^{-ab} & e^{ab} \\ -i\beta e^{-ik(a+b)+i\beta a} & i\beta e^{-ik(a+b)-i\beta a} & \alpha e^{-ab} & -\alpha e^{ab} \end{vmatrix} = 0 \quad \dots(14)$$

On solving the determinant Eq. (14) and after simplification, we get

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta} \sin h ab \sin \beta a + \cosh ab \cos \beta a = \cos(a+b)k \quad \dots(15)$$

To simplify Eq. (15), Kronig and Penney considered the case when $V_0 \rightarrow \infty$ and $b \rightarrow 0$, but product $V_0 b$ has a finite value i.e., the potential barriers become *delta functions*.

Under these conditions, the model is modified in such a way that represent a series of well separated by infinitely thin potential barriers of infinitely large potential. The limiting value of $V_0 b$ for $V_0 \rightarrow \infty$ and $b \rightarrow 0$ is known as *barrier strength*.

Also from Eqs. (9) and (3)

$$\alpha^2 = \frac{2m(E - V_0)}{\hbar^2} \text{ and } \beta^2 = \frac{2mE}{\hbar^2}$$

Then

$$\beta^2 - \alpha^2 = \frac{2mV_0}{\hbar^2}$$

or

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta} = \frac{mV_0}{\alpha\beta\hbar^2} \quad \dots(16)$$

Putting the value $\frac{\beta^2 - \alpha^2}{2\alpha\beta}$ from Eq. (16) in Eq. (15) we get

$$\frac{mV_0}{\alpha\beta\hbar^2} ab \sin \beta a + \cos \beta a = \cos ka$$

$$\frac{mV_0 b}{\beta\hbar^2} \sin \beta a + \cos \beta a = \cos ka$$

Let us define a quantity $p = \frac{mV_0 ba}{\hbar^2}$, which is measure of the area $V_0 b$ of the potential barrier,

$$\text{then } p \frac{\sin \beta a}{\beta a} + \cos \beta a = \cos ka \quad \dots(18)$$

The physical significance of the quantity p is that if p is increased and the given vector is bound more strongly to a particular potential well.

When $p \rightarrow 0$, then potential barrier becomes very weak which means that electrons are free electrons. In this, we obtain from Eq. (18),

$$\beta a = ka \text{ or } \beta = k \quad \dots(19)$$

Hence

$$\beta^2 = \frac{2mE}{\hbar} = k^2$$

This result is similar as obtained by free electron theory. Equation (18) also given the condition, which must be satisfied so that solutions of the wave equation may exist. Since $\cos ka$ can have values between +1 and -1. Such values of βa , therefore, represent wave like solutions of the form

$$\psi(x) = e^{ikx} U_k(x)$$

and are *allowed values*. The other values of βa are not allowed.

If we plot a graph between $\left(\frac{p \sin \beta a}{\beta a} + \cos \beta a \right)$ and βa for the value of $p = \frac{3a}{2}$, we get the curve as shown in Fig. 7.

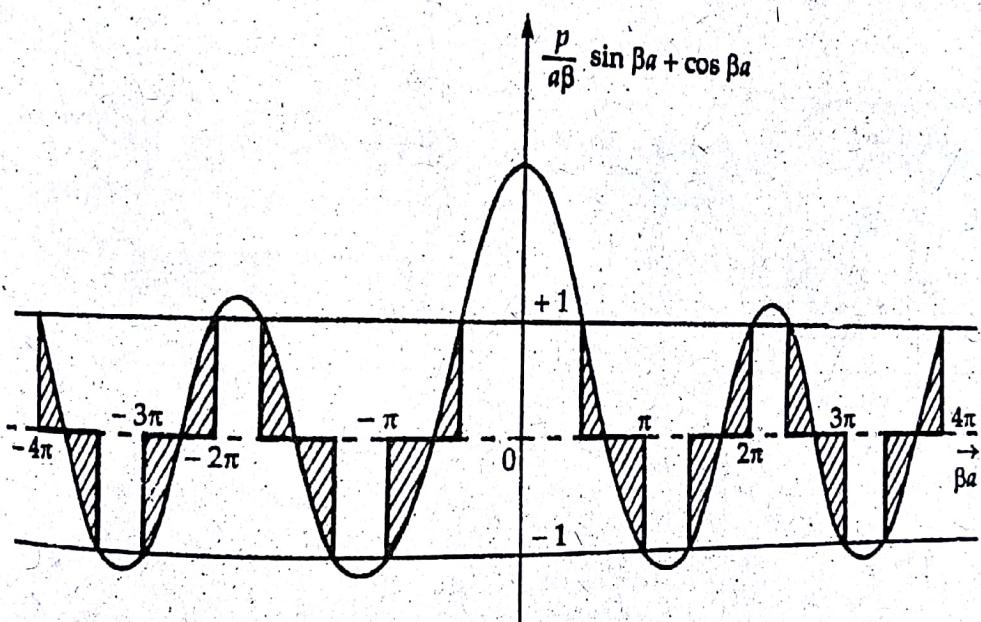


Fig. 7. Plot of $\frac{p}{\beta a} \sin \beta a + \cos \beta a$ vs. βa to show allowed and forbidden band for the K.P. approximation.

Q.8. (b) Define law of mass action for a semiconductor. (2.5)

Ans. The law of mass action states that the product of the electron concentration in the conduction band and hole concentration in the valence band is equal to the square of intrinsic carrier concentration of the material. Let n_i , n_e and n_h are the intrinsic concentration of the material, the electron concentration in the conduction band and the hole concentration in the valance band respectively, then

$$n_i^2 = n_e n_h$$

This equation is known as *law of mass action*.

Q.8. (c) How does the location of Fermi level change with doping of a semiconductor? (6)

Ans. In an intrinsic semiconductor the number of electrons is equal to number of holes ($n_i = p_i$). But in N-type extrinsic semiconductor number of electrons are increased

due to doping of pentavalent atom and number of holes are decreased than which would be available for intrinsic semiconductor. The number of holes are decreased because of the large number of electrons present which increase the rate of recombination of electrons and holes. With similar argument it can be shown that in P-type extrinsic semiconductor the number of holes are increased due to doping of trivalent impurity atom ($n_h > n_i$) and number of electrons decreased ($n_e < n_i$) than which would be available for intrinsic semiconductor. Now since the Fermi level E_F is a measure of the probability of occupancy of the allowed energy states by the electron, so for an intrinsic semiconductor when $n_i = p_i$ Fermi level is at the centre of the forbidden gap [Fig. 11.(a)]. But for N-type semiconductor, since $n_e > n_h$, it is clear that E_F must move closer to conduction band [Fig. 11. (b)] to indicate that many of the energy states in the band are filled by the donor electrons, and of few holes exist in the valence band. Similarly, for P-type semiconductor, since $n_h > n_e$ so E_F must move from the centre of the forbidden gap closer to the valence band [Fig. (11.1 (c))]

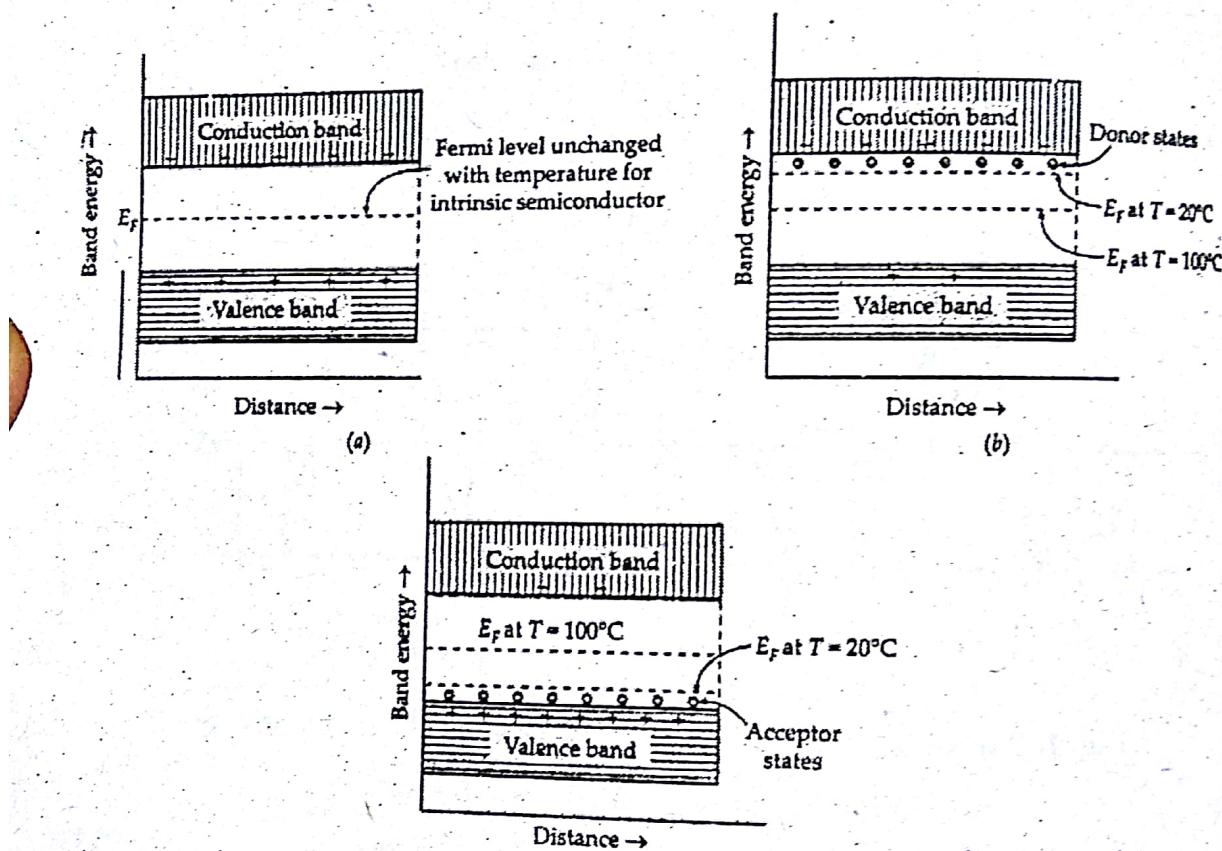


Fig. 11. Position of Fermi level (E_F) for (a) intrinsic semiconductor; (b) N-type semiconductor; (c) P-type semiconductor.

For an intrinsic semiconductor $n_i = p_i$ and as temperature increases both n_i and p_i will increase. Thus the Fermi level (E_F) will remain approximately at the centre of the forbidden gap. Thus for intrinsic semiconductor Fermi level is independent of temperature [Fig. 11.(a)].

But in an extrinsic semiconductor it is different. Say for N-type material the electrons are coming from two different sources. Some electrons come from donor state, which are easily separated from their parent atom and they do not vary much as the temperature is increased. The other electrons in the conduction band are present because of the breaking of a covalent bond. Such intrinsic effect will increase in number as temperature raised. So proportion of the total number of conduction electrons will

re decreased than which holes are decreased because the rate of recombination shown that in P-type semiconductor than which would be a gap [Fig. 11.(a)]. But for the band are filled by the bidden gap closer to the

increase from valence band. Thus as the temperature rises the material becomes more and more intrinsic and Fermi level moves closer to the intrinsic position, i.e., at the centre of the gap [Fig. 11.(b)].

Similarly, for P-type semiconductor as temperature increases the material also becomes increasingly intrinsic and the Fermi-level rises until it approaches the centre of the gap as in the intrinsic semiconductor [Fig. 11(c)]. Thus both N-type and P-type materials become more and more like intrinsic material at high temperature. This places a limit on the operating temperature of an extrinsic semiconductor device.

Q.9. (a) Derive the expression for hall coefficient. (5)

Ans. Consider a specimen in the form of a rectangular cross-section carrying current I_x in the x direction A uniform magnetic field B_z is applied along z -axis as shown in Fig. 12(a).

A moving charge in transverse magnetic field experiences a force normal to both current and magnetic field. Thus moving charges slightly shift towards y direction

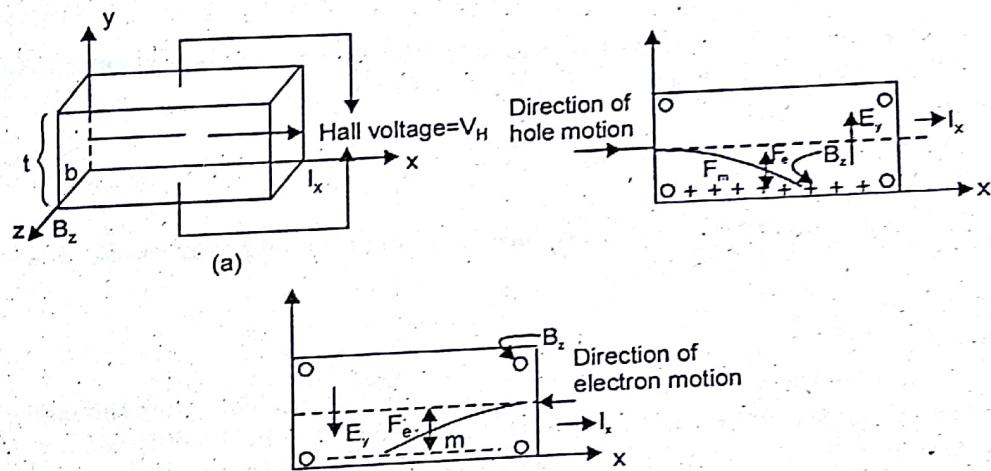
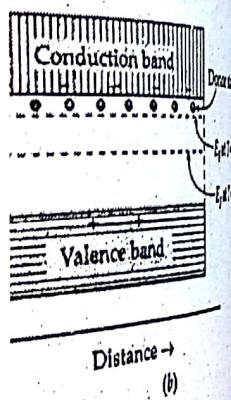


Fig.(12)

Fig. 12.(a) Illustration of Hall effect 12(b) motion of hole in p-type semiconductor specimen (c) motion of electron of N-type semiconductor or metal specimen of Hall experiment.

Force $ev_x B_z$ [Fig. 12. (a) and (c)] This shift of charges produces a potential difference or an electric field E_y along y direction. The produced electric field also applies a force eE_y on charge. When both the forces become equal, the equilibrium occurs, i.e., upto this condition charges shift along y direction.

In equilibrium,

$$F_e = F_m$$

$$eE_y = ev_x B_z$$

$$E_y = v_x B_z$$

...(1)

If J_x is the current density in x direction, then

$$J_x = \frac{I_x}{A} = nev_x$$

...(2)

Here n is density of charge carrier (electron or hole-or both) and $A(=breadth \times thickness)$ is the cross-sectional area of specimen perpendicular to direction of J_x .

From Eqs. (1) and (2), we can write

$$E_y = \frac{B_z J_x}{ne} \quad \dots(3)$$

This is the expression for Hallfield. The Hall effect is described by means of Hall coefficient R_H , defined in terms of J_x by the following relation :

$$R_H = \frac{E_y}{J_x B_z} \quad \dots(4)$$

From Eqs. (3) and (4) we have

$$R_H = \frac{1}{ne} \quad \dots(5)$$

For the conductor or N-type semiconductor, the electric field is developed in negative y direction. Hence from Eqs. (4) and (5), we can write

$$R_H = \frac{E_y}{J_x B_z} = -\frac{1}{ne} \quad \dots(5)$$

In P-type semiconductor, the produced Hall field is along y direction, thus we have

$$R_H = \frac{E_y}{J_x B_z} = \frac{1}{n_h e} \quad \dots(7)$$

Here n_h is the hole density. Equations (6) and (7) indicate that the sign of R_H is same as the sign of carrier. Most of the metals have negative Hall coefficient or resistance, but some metals have positive Hall coefficient (i.e., charge carriers are holes). If both holes and electrons contribute to current, then R_H can be positive or negative depending upon the relative density and mobility of carriers.

Q.9. (b) What are Brillouin zones? (2.5)

Ans. If schrodinger wave equation for electron energies is solved with a periodic function $u(k)$ to give the energies of an electrons in a solid, the solutions falls into permitted energy bonds. If the solutions are plotted in the reciprocal lattice of the crystal being considered, the zones enclosing the solutions for $k = 1, 2, 3, \dots, n$ are called *Brillouin zones*.

Q.9. (c) Show that the Fermi energy lies midway between the conduction band and valence band for an intrinsic semiconductor. (6)

Ans. From the law of mass action $n_i^2 = n_e n_h$

we put the value of n_e and n_h .

$$n_i^2 = N_C N_V \exp \left[\frac{E_F - E_C}{k_B T} \right] \exp \left[\frac{E_V - E_F}{k_B T} \right]$$

or

$$n_i^2 = N_C N_V \exp \left[\frac{-(E_C - E_V)}{k_B T} \right] \quad \dots(1)$$

Since $E_C - E_V = E_g$, the band gap of the semiconducting material, Eq. (1) may be written as

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{k_B T}\right)$$

Substituting the value of N_C and N_V in Eq. (2) we get

$$\begin{aligned} n_i^2 &= 2 \left[\frac{2\pi m_e * k_B T}{h^2} \right]^{3/2} \times 2 \left[\frac{2\pi m_h * k_B T}{h^2} \right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \\ &= 4 \left[\frac{2\pi k_B T}{h^2} \right]^3 \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \end{aligned} \quad \dots(3)$$

Multiplying the numerator and denominator of Eq. (3) by m^3 , we get

$$n_i^2 = 4 \left[\frac{2\pi k_B m T}{h^2} \right]^3 \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \quad \dots(4)$$

The value of n_i can be written as

$$\begin{aligned} n_i &= 2 \left[\frac{2\pi k_B m T}{h^2} \right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4} \exp\left(\frac{-E_g}{2k_B T}\right) \\ &= 2 \left[\frac{2\pi k_B m}{h^2} \right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4} T^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right) \end{aligned} \quad \dots(5)$$

Equation (5) may also be written as

$$n_i = AT^{3/2} \exp\left[\frac{-E_g}{2k_B T}\right] \quad \dots(6)$$

$$A = 2 \left[\frac{2\pi k_B m}{h^2} \right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4} = 4.82 \times 10^{21} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4}$$

where Equation (5) gives the intrinsic carrier concentration in semiconductor.

Value of Fermi energy

In intrinsic semiconductor $n_e = n_h$

$$\begin{aligned} \therefore 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/2} \times e^{(E_F - E_C)k_B T} &= 2 \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/2} \times e^{(E_V - E_F)k_B T} \\ &= (m_e^*)^{3/2} \times e^{(E_F - E_C)k_B T} = (m_h^*)^{3/2} \times e^{(E_V - E_F)k_B T} \\ \Rightarrow e^{(2E_F - EC - EV)k_B T} &= \left(\frac{m_h^*}{m_e^*} \right)^{3/2} \end{aligned} \quad \dots(7)$$

field is along y direction, thus

nd (7) indicate that the sign of negative Hall coefficient that the sign of .e., charge carriers are holes, it's can be positive or negative depending s.

tron energies is solved with a few ns in a solid, the solutions for k plotted in the reciprocal lattice. Solutions for k = 1, 2, 3, ... are as s midway between the conduction band and valence band.

$$h \left[\frac{E_v - E_F}{k_B T} \right] \exp\left[\frac{E_F - E_C}{k_B T} \right]$$

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$$\Rightarrow \left\{ \frac{2E_F - E_C - E_V}{k_B T} \right\} = \frac{3}{2} \log \left(\frac{m_h^*}{m_c^*} \right)$$
$$\Rightarrow E_F = \frac{E_C + E_V}{2} + \frac{3}{4} k_B T \log \left(\frac{m_h^*}{m_c^*} \right) \quad \dots(8)$$

$\left[\text{for } m_e^* = m_h^*, \log \frac{m_h^*}{m_e^*} = \log 1 = 0 \right]$

$$\Rightarrow E_F = \frac{E_C + E_V}{2} \quad \dots(9)$$

So Fermi level lies exactly in the middle of the top valence band and the bottom of the conduction band.

Q.1.
Ans
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Q.1.

END TERM EXAMINATION [MAY-JUNE 2017]

SECOND SEMESTER [B.TECH.]

APPLIED PHYSICS-II

(ETPH-104)

M.M. : 75

Time : 3 hrs.

Note: Attempt any five questions including Q.no.1 which is compulsory. Select one question from each unit.

Q.1. Attempt each one of the following:

(2.5)

Q.1. (a) Define divergence and curl of a vector.

Ans. Divergence: The divergence of a vector field at any point is defined as the amount of flux per unit volume diverging from that point.

Curl: The curl of a vector field is defined as the maximum line integral of the vector per unit area.

Q.1. (b) Distinguish between displacement current and conventional current.

(2.5)

Ans. Refer Q.1(b) of First Term Examination Feb. 2017.

Q.1. (c) Find constant a , b and c so that

$\vec{V} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

(2.5)

Ans. For irrotational vector-

$$\vec{\nabla} \times \vec{A} = 0$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

Here,

$$A_x = x + 2y + az$$

$$A_y = bx - 3y - z$$

$$A_z = 4x + cy + 2z$$

Solving for $\vec{\nabla} \times \vec{A}$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x + 2y + az & bx - 3y - z & 4x + cy + 2z \end{vmatrix}$$

$$= \hat{i} \left[\frac{\partial}{\partial y} (4x + cy + 2z) - \frac{\partial}{\partial z} (bx - 3y - z) \right] +$$

$$+ \hat{j} \left[\frac{\partial}{\partial z} (x + 2y + az) - \frac{\partial}{\partial x} (4x + cy + 2z) \right] +$$

$$+ \hat{k} \left[\frac{\partial}{\partial x} (bx - 3y - z) - \frac{\partial}{\partial y} (x + 2y + az) \right]$$

$$= \hat{i}(c + 1) + \hat{j}(a - 4) + \hat{k}(b - 2) = 0$$

$$\begin{array}{l} c = 1 \\ a = 4 \\ b = 2 \end{array}$$

Q.1. (d) Describe the implication of Davisson-Germer experiment. (2.5)

Ans. Davisson and Germer in 1927 designed an apparatus to determine the wavelength associated with electrons to confirm the dual nature of matter.

Q.1. (e) Why is the energy of a particle trapped inside a box quantized?

Ans. Energy of a particle trapped inside a box is given by the following equation- (2.5)

$$E_n = \frac{n^2 h^2}{8ml^2}$$

where,

$$n = 1, 2, 3, 4, \dots$$

It is clear from the equation that particle cannot possess an arbitrary energy but can have only certain discrete energy corresponding to $n = 1, 2, 3, \dots$

Q.1. (f) An electron is confined to move between two rigid walls separated by 2×10^{-9} m. Find the deBroglie wavelengths representing the first three allowed energy status of the electron. (2.5)

Ans.

$$E_n = \frac{n^2 h^2}{8ml^2}$$

Given that

$$l = 2 \times 10^{-9} \text{ m}$$

and

$$m = 9 \times 10^{-31} \text{ kg}$$

$$h = 6.63 \times 10^{-34} \text{ J-S}$$

For $n = 1$ -

$$E_1 = \frac{(1)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2} \times \frac{1}{1.6 \times 10^{-19}}$$

and

$$E_1 = \frac{hc}{\lambda_1} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_1} \times \frac{1}{1.6 \times 10^{-19}}$$

Solving for λ_1 -

$$\lambda_1 = 1.31 \times 10^{-5} \text{ m}$$

For $n = 2$ -

$$E_2 = \frac{(2)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV}$$

and

$$E_2 = \frac{hc}{\lambda_2} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_2} \times \frac{1}{1.6 \times 10^{-19}}$$

Solving for λ_2 -

$$\lambda_2 = 3.2 \times 10^{-6} \text{ m}$$

For $n = 3$

$$E_3 = \frac{(3)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV}$$

and $E_3 = \frac{hc}{\lambda_3} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_3} \times \frac{1}{1.6 \times 10^{-19}}$

Solving for λ_3

$$\lambda_3 = 1.4 \times 10^{-6} \text{ m}$$

Q.1. (g) What are the postulates of quantum theory of radiation? (2.5)

Ans. The postulates of quantum theory of radiation are as following:—

(i) A chamber containing blackbody radiations also contains simple harmonic oscillators of molecular dimensions which can vibrate with all possible frequencies.

(ii) The frequency of radiation emitted by an oscillator is same as the frequency of its vibrations.

(iii) An oscillator cannot emit energy in a continuous manner. It can emit energy in the multiples of a small unit called quantum (photon). If an oscillator is vibrating with a frequency v , it can only radiate in quanta of magnitude hv i.e., the oscillator can have only discrete energy E_n given by (110)

$$E_n = nhv = nE$$

where $hv = E$, n is an integer and h is a universal Planck constant ($6.625 \times 10^{-34} \text{ Js}$).

(iv) The oscillators can emit or absorb radiation energy in packets of magnitude hv . This implies that the exchange of energy between radiation and matter cannot take place continuously, but are limited to discrete set of values $0, hv, 2hv, 3hv, \dots nhv$.

Q.1. (h) Copper has fcc structure and the atomic radius is 0.127 nm. Find the ratio of the spacing of (111) and (123) planes. (2.5)

Ans. For (111) plane:-

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$d_{111} = \frac{2\sqrt{2} \times 0.127 \times 10^{-9} \text{ m}}{\sqrt{1^2 + 1^2 + 1^2}}$$

$$= \frac{2 \times 0.127 \times \sqrt{2} \times 10^{-9}}{\sqrt{3}} \text{ m}$$

$$d_{111} = 0.207 \text{ nm}$$

For (123) plane:-

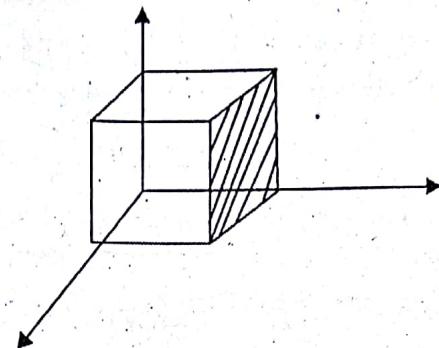
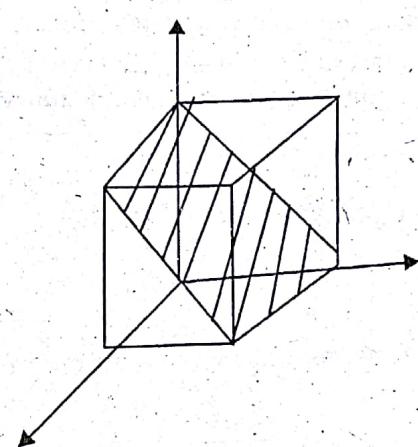
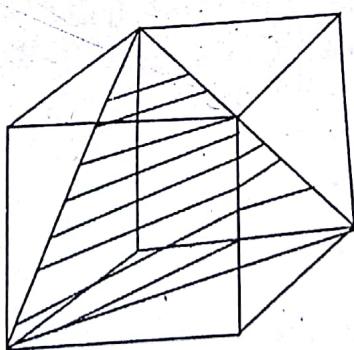
$$d_{123} = \frac{2\sqrt{2} \times 0.127 \times 10^{-9}}{\sqrt{1^2 + 2^2 + 3^2}}$$

$$= \frac{2 \times 0.127 \times \sqrt{2} \times 10^{-9}}{\sqrt{14}} \text{ m}$$

$$d_{123} = 0.096 \text{ nm}$$

Q.1. (i) Draw the planes (100), (110) and (111)
Ans. (100) plane

(2.5)

**Fig. 1.****(110) plane-****Fig. 2.****(111) plane-****Fig. 3.**

Q.1. (j) Define Fermi energy. What is its physical significance? (2.5)

Ans. Fermi energy is a quantum phenomenon which translates as the difference in energy state occupied by the lowest level electron to the highest level. It can also be measured at the top of the collection of electron energy level at absolute zero. The maximum energy that electrons may possess at 0 kelvin is the Fermi energy.

UNIT-I

Q.2. (a) Derive Maxwell's equation and express it in integral form. Give the physical significance of each. (8)

Ans. 1. Maxwell's first equation: $(\nabla \cdot D = \rho)$

Integrating this over an arbitrary volume V, we get

$$\int_V \bar{\nabla} \cdot \bar{D} dV = \int_V \rho dV$$

Charging volume integral into surface integral in L.H.S. of above equation by Gauss divergence theorem, we get

$$\int_S \bar{D} \cdot d\bar{S} = \int_V \rho dV \quad \dots(1)$$

Where S is the surface, which bounds volume V. Eq. (1) represents Maxwell's first equation ($\nabla \cdot D = \rho$) in integral form. Since $\int \rho dV = q$, the net charge contained in volume V, therefore, Maxwell's first equation signifies that:

The net outward flux of electric displacement vector through the surface enclosing a volume is equal to the net charge contained within that volume.

2. Maxwell's second equation: $(\bar{\nabla} \cdot \bar{B} = 0)$

Integrating this over an arbitrary volume V, we get

$$\int_V \bar{\nabla} \cdot \bar{B} dV = 0$$

Using Gauss divergence theorem to change volume integral into surface integral, we get

$$\int_S \bar{B} \cdot d\bar{S} = 0 \quad \dots(2)$$

Where S is the surface, which bounds volume V. Equation (2) represents Maxwell's second equation in integral form and signifies that:

The net outward flux of magnetic induction B through any close surface is equal to zero.

3. Maxwell's Third equation: $(\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t})$

Integrating above equation over a surface S bounded by a curve C, we get

$$\int_S (\bar{\nabla} \times \bar{E}) \cdot d\bar{S} = - \int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$$

Using stoke's theorem to convert surface integral on L.H.S. of above equation into line integral along the boundary C, we get

$$\int_C \bar{E} \cdot d\bar{l} = - \frac{\partial}{\partial t} \int_S \bar{B} \cdot d\bar{S} \quad \dots(3)$$

Equation (3) represents Maxwell's third equation in integral form and signifies that:

The electromotive force $\left(\text{emf.e} = \int_C \vec{E} \cdot \vec{l} \right)$ around a close path is equal to negative rate of change of magnetic flux linked with the path $\left(\therefore \text{magnetic flux } \phi = \int_S \vec{B} ds \right)$

4. Maxwell's fourth equation:

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking surface integral over surface S bounded by curve C, we obtain

$$\int_s (\vec{\nabla} \times \vec{H}) d\vec{S} = \int_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$$

Using stoke's theorem to convert surface integral on L.H.S of above equation into line integral, we get

$$\oint \vec{H} \cdot d\vec{l} = \int_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S} \quad \dots(4)$$

Equation (4) represents Maxwell's 4th equation in integral form and signifies that.

The magnetomotive force $m.mf = \oint \vec{H} \cdot d\vec{l}$ around a close path is equal to the conduction current plus displacement current through any surface bounded by the path.

Q.2. (b) Write down the equation of continuity and mention its physical significance. (2)

Ans. An equation, which expresses the equality of incoming and outgoing charges in a system and follows the law of conservation of charge, is known as the equation of continuity.

The current density J and the charge density ρ are related at each point through a differential equation. This relation is based on the fact that *electric charge can neither be created nor be destroyed and rate of increase of the total charge inside any arbitrary volume must be equal to the net flow of charge into this volume.*

$$I = \int_S J dS \quad \dots(1)$$

Again, considering charge leaving a volume V per second

$$I = - \frac{\partial}{\partial t} \int_V \rho dV \quad \dots(2)$$

The negative sign comes here because the current is positive when the net charge is from the outside of V to within.

Since, we are dealing with a fixed volume V, hence we may write,

$$-\frac{\partial}{\partial t} \int_V \rho dV = -\int_V \frac{\partial \rho}{\partial t} dV \quad \dots(3)$$

Again, using divergence theorem, we can write

$$\int_S J \cdot dS = - \int_V (\nabla \cdot J) dV \quad \dots(4)$$

Therefore, from Eqs. (2), (3) and (4), we have

$$I = \int_V (\nabla \cdot J) dV = - \int_V \frac{\partial \rho}{\partial t} dV \quad \dots(5)$$

$$\int_V (\nabla \cdot J) dV = - \int_V \frac{\partial \rho}{\partial t} dV$$

or $\int \left\{ (\nabla \cdot J) + \frac{\partial \rho}{\partial t} \right\} dV = 0 \quad \dots(6)$

This integral must be zero for any arbitrary volume. It is only possible when integrand is zero, i.e.,

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0 \quad \dots(7)$$

This differential equation is known as the equation of continuity. If the region does not contain a source or sink of current, $\frac{\partial \rho}{\partial t} = 0$ and hence for steady current, we have

$$\nabla \cdot J = 0$$

Q.2. (c) A plane electromagnetic wave is travelling in an unbounded lossless dielectric medium with relative permeability $\mu_r = 1$ and permittivity $\epsilon_r = 3$. Find the velocity of the wave and the impedance of the medium. (2.5)

Ans. Velocity of the wave is-

$$V = \frac{1}{\sqrt{\mu \epsilon}} = \frac{c}{\sqrt{\mu_r \epsilon_r}}$$

Given

$$\mu_r = 1, \epsilon_r = 3 \text{ and } c = 3 \times 10^8 \frac{m}{s}$$

$$V = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8 = 1.732 \times 10^8 \frac{m}{s}$$

$V = 1.732 \times 10^8 \frac{m}{s}$

Impedance of the wave is-

$$Z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r \mu_0}{\epsilon_0 \epsilon_r}}$$

$$Z = \sqrt{\frac{1 \times 4\pi \times 10^{-7}}{3 \times 8.86 \times 10^{-12}}} = 2.17 \times 10^2 \Omega$$

$$Z = 2.17 \times 10^2 \Omega$$

Q.3. (a) Derive Pointing theorem from Maxwell's equations and given its interpretation. (6)

Ans. The electromagnetic waves carry energy when they propagate and there is an energy density associated with both electric and magnetic field. The amount of energy flowing through unit area, perpendicular to the direction of energy propagation per unit time, i.e. the rate of energy transport per unit area is called poynting vector. It is also called instantaneous energy flux density and is represented by $\vec{S} = \vec{E} \times \vec{H}$

Pointing theorem:

When em wave propagates through space from source to receiver there exist a simple and direct relationship between power transferred and amplitude of electric and magnetic field strength. The relationship may be obtained through maxwell equation.

from Maxwell IVth eqn.

$$\vec{\nabla} \times \vec{B} = \mu \left[\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

$$\vec{B} = \mu \vec{H} \Rightarrow \frac{\vec{B}}{\mu} = \vec{H}$$

But, as

$$\epsilon \vec{E} = \vec{D}$$

and

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \epsilon \frac{d \vec{E}}{dt}$$

$$\vec{J} = \vec{\nabla} \times \vec{H} - \epsilon \frac{\partial \vec{E}}{\partial t}$$

This is equation of current density, when multiplied by E, this will result in a relation between the quantities which have the dimension of power.

$$\vec{E} \cdot \vec{J} = \vec{E} \cdot (\vec{\nabla} \times \vec{H}) - \epsilon \vec{E} \frac{\partial \vec{E}}{\partial t}$$

from vector identity-

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{H})$$

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

$$\vec{E} \cdot \vec{J} = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) - \epsilon \vec{E} \frac{\partial \vec{E}}{\partial t}$$

From Maxwell's IIIrd equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

we have,

$$= \vec{E} \cdot \vec{J} = -\mu \left(\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \right) - \epsilon \left(\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \right) - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

$$\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} H^2$$

$$\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} E^2$$

$$\vec{E} \cdot \vec{J} = -\frac{\mu}{2} \frac{\partial}{\partial t} H^2 - \frac{\epsilon}{2} \frac{\partial}{\partial t} E^2 - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

Integrating over volume, v –

$$\int_v \vec{E} \cdot \vec{J} dv = -\frac{\partial}{\partial t} \int_v \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) dv - \int_v \vec{\nabla} \cdot (\vec{E} \times \vec{H}) dv$$

Using divergence theorem-

$$\int_v \vec{\nabla} \cdot (\vec{E} \times \vec{H}) dv = \oint_s (\vec{E} \times \vec{H}) \cdot ds$$

$$\boxed{\oint_s (\vec{E} \times \vec{H}) \cdot ds = -\frac{\partial}{\partial t} \int_s \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) dv - \int_v \vec{E} \cdot \vec{J} dv}$$

$$\text{or } -\frac{\partial}{\partial t} \left\{ \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) \right\} dv = \int_s (\vec{E} \times \vec{H}) \cdot ds + \int_v \vec{E} \cdot \vec{J} dv$$

i.e. Rate of energy flow = rate of decrease of stored energy + total instantaneous ohmic power dissipated within the volume:

This is called the "Poynting theorem" and

$\vec{S} = \vec{E} \times \vec{H}$ is called "poynting vector".

$-\int_v \vec{E} \cdot \vec{J} dv$ – Rate of energy transferred into em field through the motion of free charge in Volume V.

$-\frac{\partial}{\partial t} \int_v \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) dv$ – Rate of decrease of em energy stored in volume V.

$\oint_s (\vec{E} \times \vec{H}) \cdot ds$ – Amount of em energy crossing the closed surface per second.

Physical meaning of equation- The time rate of change of em energy with certain volume plus time rate of energy flowing out through the boundary surface is equal to power transferred into em field.

Q.3. (b) Show that in free space the direction of flow of electromagnetic energy is along the direction of wave propagation. (4)

Ans. The Poynting vector (i.e., energy flow per unit area per unit time) for a plane electromagnetic wave is given by

$$\begin{aligned} \vec{S} &= \vec{E} \times \vec{H} = \vec{E} \times \frac{\hat{n} \times \vec{E}}{\mu_0 c} \\ &= \frac{1}{\mu_0 c} \vec{E} \times (\hat{n} \times \vec{E}) \\ &= \frac{1}{\mu_0 c} E^2 \hat{n} \quad [\vec{E} \cdot \hat{n} = 0; E \text{ being } \perp \text{ to } \hat{n}] \\ &= \frac{E^2}{Z_0} \hat{n} \end{aligned}$$

For a plane electromagnetic wave of angular frequency ω , the average value of S over a complete cycle is given by

$$\begin{aligned} \langle S \rangle &= \frac{1}{Z_0} \langle E^2 \rangle \hat{n} = \frac{1}{Z_0} \left\langle (E_0 e^{ik.r-i\omega t})^2 \right\rangle_{\text{real}} \hat{n} \\ &= \frac{1}{Z_0} E_0^2 \langle \cos^2(k.r - \omega t) \rangle \hat{n} \\ &= \frac{1}{Z_0} \frac{E_0^2}{2} \hat{n} \quad [\text{Average value of } \cos^2(k.r - \omega t) = \frac{1}{2}] \\ &= \frac{1}{Z_0} E_{\text{rms}}^2 \hat{n} \quad [\because E_{\text{rms}} = E_0 / \sqrt{2}] \end{aligned}$$

It is obvious that the direction of Poynting vector is along the direction of propagation of electromagnetic wave. This means that the flow of energy in a plane electromagnetic wave in free space is along the direction of wave.

Q.3. (c) Find the skin depth at a frequency of 90 Hz in aluminium where $\sigma = 3.54 \times 10^7$ mho/m. Also find the wave velocity. (2.5)

Ans. Skin depth is-

Given-

$$\mu = \mu_0 \mu_r$$

$$\mu = 4\pi \times 10^{-7} \text{ (taking } \mu_r = 1)$$

$$\omega = 2\pi f$$

$$f = 90 \text{ Hz}$$

$$\delta = \sqrt{\frac{2}{w\mu\sigma}} = \sqrt{\frac{2}{2\pi \times 90 \times 4\pi \times 10^{-7} \times 3.54 \times 10^7}}$$

$$\delta = 8.92 \times 10^{-3} \text{ m}$$

Wave velocity is-

$$V = \frac{w}{\alpha} = \frac{w}{\beta} \quad (\alpha = \beta \text{ in good conductor})$$

$$\text{But } \delta = \frac{1}{\beta}$$

$$\therefore V = \omega\delta = 2\pi \times 90 \times 8.92 \times 10^{-3}$$

$$V = 5.041 \text{ m/s}$$

UNIT-II

Q.4. (a) Distinguish between a boson and a fermion. Give one example each.

(4)

Ans.

Bose-Einstein statistics (Quantum statistics)	Fermi-Dirac statistics (Quantum (statistics))
<p>Apply to systems of indistinguishable particles not obeying Pauli's exclusion principle, such as photons, phonons and liquid helium at low temperature</p> <p>The distribution law is</p> $n_i = \frac{g_i}{(e^{\alpha e^{E_i/k_B T}} - 1)}$ <p>There is no restriction on the number of particles in a given state.</p> <p>The volume of phase cell is of order of h^3.</p> <p>For $E_i \gg k_B T$, exponential $E_i \ll k_B T$, lies above M.B.</p> <p>The energy may be zero at absolute zero.</p>	<p>Apply to systems of indistinguishable particle obeying Pauli exclusion principle, such as free electrons in metal electrons in a star (white dwarf star)</p> <p>The distribution law is</p> $n_i = \frac{g_i}{(e^{\alpha e^{E_i/k_B T}} + 1)}$ <p>Only one particle in a given quantum state is allowed.</p> <p>The volume of phase cell is of the order of h^3</p> <p>For $E_i \gg k_B T$, exponential where $E_i \gg E_F$. If $E_F \gg k_B T$ decreases abruptly near E_F</p> <p>The energy at absolute zero, cannot be zero because all particles cannot come down to ground state due to Pauli's exclusion principle.</p>

Q.4. (b) Write down the Planck's formula for the distribution of energy in spectrum of blackbody. Show that Rayleigh-Jean's law and Wein's law are special cases of Planck's radiation law.

(5)

Ans. Refer to Q.4. (c) End Term Examination April 2017.

As per Planck's quantum hypothesis, the atoms of the wall of a blackbody behave as oscillators and each has a characteristic frequency of oscillation. Then average energy

of these Planck's oscillators were calculated and finally Planck's radiation formula was derived.

Wein's law is deduced from Plank's radiation formula under the condition when the wavelength λ and temperature T are very small. However, Plank's radiation formula under the condition of high temperature T and wavelength λ , takes the form of Rayleigh-Jeans law.

Planck radiation law is as following.

$$\mu\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1} d\lambda$$

When λ and T are very small then $e^{\left(\frac{hc}{\lambda KT}\right)} \gg 1$

$$\mu\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda KT}} d\lambda$$

By substituting $8\pi hc = A$ and $\frac{hc}{K} = B$

we get

$$\boxed{\mu\lambda d\lambda = \frac{A}{\lambda^5} e^{-\frac{B}{\lambda T}} d\lambda}$$

This is Wein's law valid at low temperature and small wavelength.

When λ and T are large then $e^{\left(\frac{hc}{\lambda KT}\right)} \gg 1$

$$\mu\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda KT}} d\lambda$$

By substituting $8\pi hc = A$ and $\frac{hc}{K} = B$

we get

$$\boxed{\mu\lambda d\lambda = \frac{A}{\lambda^5} e^{-\frac{B}{\lambda T}} d\lambda}$$

This is Wein's law valid at low temperature and small wavelength.

When λ and T are large then $e^{\frac{hc}{\lambda KT}} \approx 1 + \frac{hc}{\lambda KT}$

$$\mu\lambda d\lambda = \frac{8\pi hc}{\lambda^5 \left\{ \left[1 + \frac{hc}{\lambda KT} \right]^{-1} \right\}} d\lambda$$

$$\boxed{\mu\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda}$$

This is Rayleigh-Jeans law valid at high temperature and large wavelength.

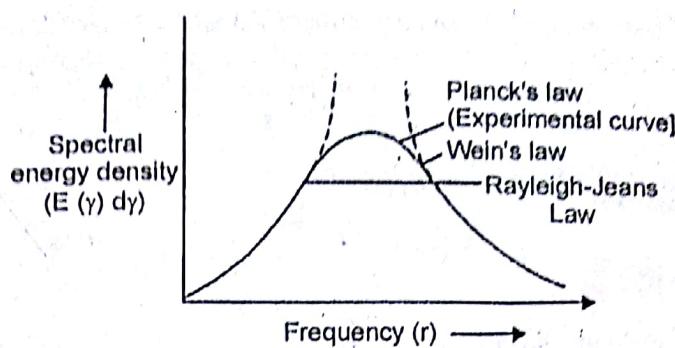


Fig. 4.

As shown in Fig. 4 the spectral energy density decreases with frequency at high frequency. Hence Rayleigh-Jeans and Wien's law are unable to explain the complete, energy distribution for blackbody radiation, while Planck's law explain it completely.

Q.4. (c) Draw a neat diagram showing the energy distribution spectra of blackbody radiation. Explain how classical theory fails to explain the spectral distribution of energy. (3.5)

Ans. The postulates of quantum theory of radiation are as follows-

(i) A radiation has energy. As light and heat are radiations, they are also associated with energy.

(ii) Radiant energy is not emitted or observed continuously but discontinuously in the form of small packets called photons. Photon is not a material body but is considered to be a massless packet of energy.

(iii) The energy E of a photon is related to the frequency of radiation, γ , the two being related as $E = h\gamma$, where h is Planck's constant.

(iv) Whenever a body emits or absorbs energy it does so in whole number multiples by photons, i.e. $n h\gamma$, where $n = 1, 2, 3, 4, \dots$

Planck's radiation formula-

Let dn be the number of photons distributed in frequency interval v to $(v + dv)$.

$$dn = n(v)dv = g(v)f(v)dv \quad \dots(1)$$

$$g(v)dv = \frac{8\pi h v^2}{c^3} dv \quad \dots(2)$$

and

$$f(v) = \frac{1}{(e^{hv/k_B T} - 1)} \quad [\text{as } \alpha = 0 \text{ and } E = hv] \quad \dots(3)$$

Putting these values in Eq.(1), we get

$$dn = \frac{8\pi h v^2}{c^3} \cdot \frac{1}{(e^{hv/k_B T} - 1)} dv \quad \dots(4)$$

Let dE be the energy distributed in frequency interval v and $(v + dv)$

$$dE = Edn = hv dn \quad \dots(5)$$

$$dE = E(v)dv = \frac{8\pi h v^3}{c^3} \cdot \frac{1}{(e^{hv/k_B T} - 1)} dv \quad \dots(6)$$

Equation (6) is known as *Planck's radiation formula* for the spectral energy density of blackbody radiation, which agrees with experimental curve (Fig. 4)

For low frequency or high wavelength

$$(e^{hv/k_B T} - 1) = 1 + \frac{hv}{k_B T} - 1 = \frac{hv}{k_B T}$$

Equation (6) becomes as

$$dE = E(v)dv = \frac{8\pi v^2 k_B T}{c^3} dv \quad \dots(7)$$

Equation (7) is termed as Rayleigh-Jeans law. This law holds good for low frequencies and high wavelengths. Experimental agreement is shown in Fig. 5 for high frequency low frequency:

Equation (6) can be expressed in terms of wavelength of radiation as follows:

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \left(\frac{1}{c[(hc/\pi k_B T) - 1]} \right) d\lambda$$

$$E_\lambda = 8\pi hc(\lambda^{-5}) \left[\exp\left(\frac{hc}{\lambda k_B T} - 1\right) \right]^{-1}$$

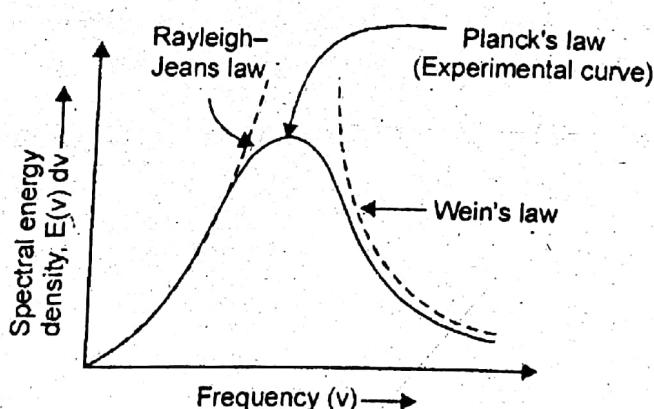


Fig. 5. v vs $E(v) dv$ -curve

Let λ_{\max} be the wavelength whose energy density is the greatest.

$$\text{Then, } \frac{dE_\lambda}{d\lambda} = 0$$

$$\text{Hence, } \lambda_{\max} T = \frac{hc}{4.965 K} = 2.898 \times 10^{-3} mK$$

This is *Wein's displacement law*.

The peak in the black body spectrum shifts to progressively shorter wavelength (higher frequencies) as temperature is increased.

Q.5. (a) Derive schrodinger time dependent wave equation for non-relativistic particle. Give the physical interpretation of Hamilton operator H .

(6)

Ans. Let us assume that ψ for a particle moving freely in positive x -direction is

$$\psi = Ae^{-i\omega(t-\frac{x}{v})} \quad \dots(1)$$

as

$$\omega = 2\pi v \quad v = v\lambda$$

$$\psi = Ae^{-2\pi i(\frac{vt-x}{\lambda})} \quad \dots(2)$$

As

$$E = hv = 2n\pi\hbar v \text{ and } \lambda = \frac{\hbar}{p} = \frac{2\pi\hbar}{p}$$

\therefore For a free particle wave equation becomes

$$\psi = Ae^{-\frac{i}{\hbar}(Et-px)} \quad \dots(3)$$

But

$$E\psi = i\hbar \frac{\partial \psi}{\partial t} \text{ and } p\psi = \frac{\hbar}{i} \frac{\partial \psi}{\partial x} \dots(a)$$

As total energy, $E = \text{Kinetic energy (K)} + \text{Potential energy (V)}$

Now,

$$\text{K.E.} = \frac{p^2}{2m}$$

\therefore Equation (4) in term of wave function ψ can be written as

$$E\psi = \left(\frac{p^2}{2m} \right) \psi + V\psi \quad \dots(5)$$

Putting the values of $E\psi$ and $p\psi$ from Eq. (a) in Eq. (5) we have

$$i\hbar \frac{\partial \psi}{\partial t} = \left(\frac{\hbar}{i} \frac{\partial}{\partial x} \right)^2 \frac{1}{2m} \psi + V\psi$$

$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$

Equation (6) is Schrödinger's time dependent wave equation in one-dimension.

The time dependent Schrödinger's equation in three-dimensional form,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right] + V\psi \quad \dots(7)$$

or

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$$

$\left[\because \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right]$

or
$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots(9)$$

or
$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots(10)$$

Equation (10) contains time and hence is called time dependent Schrodinger equation.

The operator $\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right)$ is called Hamiltonian and is represented by H , while operator $i\hbar \frac{\partial}{\partial t}$, operated on ψ , given E which may be seen by Eq. (7) Eq. (10) may be written as

$$H\psi = E\psi \quad \dots(11)$$

The above forms of the Schrodinger's equation describe the motion of a non-relativistic material particle.

Q.5. (b) State the properties of wave function in a quantum mechanical system. Give one example each for an acceptable and non-acceptable wave function. (4)

Ans. Properties of Wave Function are—

- $\psi(x)$ must be single valued, finite and continuous for all values of x .
- $\frac{d\psi(x)}{dt}$ must be finite and continuous for all values of x , except at those point where $V \rightarrow \infty$. At these points $\frac{d\psi(x)}{dt}$ has a finite discontinuity but ψ remains continuous.

• For bound state and probability of finding the particle between x and $(x + dx)$, i.e., $|\psi|^2 dx$ must vanish as $|x| \rightarrow \infty$.

Hence $|\psi(x)| \rightarrow 0$ as $|x| \rightarrow \infty$ i.e., $\psi(x)$ is a square integrable wave function.

Examples of not acceptable wave functions:

(i) $\psi = \frac{1}{x}$ as $x \rightarrow 0$, $\psi \rightarrow \infty$ (ii) $\psi = x^n$ as $a \rightarrow \pm \infty$, $\psi \rightarrow \pm \infty$

Examples of acceptable wave functions:

(i) $\psi = \frac{1}{(1+x^2)}$ as $x \rightarrow \infty$, $\psi \rightarrow 0$ (ii) $\psi = \sin x$ as ψ oscillates between +1 and -1.

Q.5. (c) An electron has a speed of $2 \times 10^4 \frac{m}{s}$ within the accuracy of 0.01%.

Calculate the uncertainty in the position of the electron. (2.5)

Ans. Given-

$$\Delta V = \frac{0.01}{100} \times 2 \times 10^4 = \frac{2m}{s}$$

From uncertainty principle-

$$\Delta x \cdot \Delta p_x \geq \frac{h}{2\pi}$$

Uncertainty P_n position-

$$\Delta x = \frac{h}{2\pi p_x} = \frac{h}{2\pi m \Delta V}$$

$$\begin{aligned}\Delta x &= \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 2} \\ &= \frac{6.63 \times 10^{-34}}{114.296 \times 10^{-31}} \\ &= 0.058 \times 10^{-3} \text{ m}\end{aligned}$$

$$\boxed{\Delta x = 58 \times 10^{-6} \text{ m}}$$

UNIT - III

Q.6. (a) What do you understand by packing factor in crystals? Calculate the packing factor in the case of bcc and fcc lattices. (4.5)

Ans. Atomic packing factor (APF) or density of packing: It is the ratio of volume occupied by the atoms, molecules or ions in a unit cell (v) to the total value of the unit cell (V).

$$\text{APF} = \frac{\text{Volume occupied by the atoms in an unit cell}}{\text{Volume of the unit cell}}$$

$$\Rightarrow \text{APF} = \frac{\text{Number of atoms present in an unit cell} \times \text{Volume of an atom}}{\text{Volume of the unit cell}}$$

i.e.,

$$\boxed{\text{APF} = \frac{v}{V}}$$

BCC-

$$\boxed{\text{APF} = \frac{v}{V}}$$

The number of atoms present in an unit cell = 2

$$v = 2 \times \frac{4}{3} \times \pi r^3; \quad V = a^3$$

$$\text{APF} = \frac{2 \times \frac{4}{3} \times \pi r^3}{a^3}$$

Substituting $r = \frac{\sqrt{3}a}{4}$, we get

$$APF = \frac{2 \times \frac{4}{3} \pi \left(\frac{\sqrt{3}a}{4} \right)^3}{a^3}$$

 \Rightarrow

$$APE = \frac{\sqrt{3}\pi}{8} = 0.68$$

Thus, 68% of the volume of the bcc unit cell is occupied by atoms and remaining 32% volume of the unit cell is vacant or void space.

FCC-

Atomic Packing Factor (APF)/ Packing Density

$$APF = \frac{v}{V}$$

$$v = 4 \times \frac{4}{3} \times \pi r^3; \quad V = a^3$$

$$APF = \frac{4 \times 4\pi r^3}{3a^3}$$

Substituting $r = \frac{\sqrt{2}a}{4}$, we get

$$APF = \frac{\pi}{3\sqrt{2}} = 0.74$$

Thus 74% of the volume of the fcc unit cell is occupied by atoms and the remaining 26% volume of the unit cell is vacant or void space.

Q.6. (b) What do you mean by lattice constant? Obtain a relation between the lattice constant of a cubic crystal and the density of the crystal material.

(4)

Ans. In a lattice a unit cell has the length of the edges and the angles between the edges are of definite values. Each edge of the unit cell is a distance between the atoms of the same kind and is known as the lattice constant.

Let us consider a cubic lattice constant a . If ρ is the density of the crystal under consideration, then

$$\text{Volume of unit cell} = a^3$$

$$\text{Mass in each unit cell} = a^3 \rho \quad \dots(1)$$

If M is the molecular weight and N the Avogadro's number (i.e., the number of molecules per kg mole of the substance), then

$$\text{Mass of each molecule} = \frac{M}{N}$$

Further if n is the number of molecules (lattice points) per unit cell, then

$$\text{the mass in each unit cell} = n \frac{M}{N} \quad \dots(2)$$

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From Eqs. (1) and (2)

$$a^3 \rho = \frac{nM}{N}$$

$$\Rightarrow a = \left(\frac{nM}{N\rho} \right)^{1/3} \quad \dots(3)$$

From the relation the values of the lattice constant 'a' can be calculated.

Q.6. (c) Define Miller indices. If a , b and c are principle vectors of the unit cell a plane of miller indices (310) cut the crystal lattice, find the intercepts of the plane along the three axes. (4)

Ans. Miller indices is defined as the reciprocals of the intercepts made by the plane on the three axes.

Given- Miller indices - (3 1 0)

The formula is-

$$a : b : c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

$$a, b, c \Rightarrow (3 1 0)$$

3a, 1b, 0c are the intercepts

\therefore 3a and 1b are the intercepts

Q.7. (a) What are Schottky and Frenkel defects? Obtain an expression for concentration of schottky defects in crystal. (6)

Ans. Refer End Term Examination Q.7. (c) May 2016.

Q.7. (b) Why do we use X-rays to study crystal diffraction? Is it possible to observe diffraction pattern if radiation of wavelength 2×10^{-6} m is incident on a crystal with interplanas separation of 10^{-8} cm. Justify. (4)

Ans. A crystal is nothing but a three dimensional grating of periodic array of atoms. The typical interatomic spacing in crystal is of the order of 1\AA . Therefore to observe diffraction in crystal, wavelength of electromagnetic radiation used should be of the order of interatomic spacing. Since this wavelength range corresponds to X-rays, so X-rays are used for diffraction studied in crystals.

Given

$$\lambda = 2 \times 10^{-6} \text{ m}$$

$$d = 10^{-8} \text{ cm} = 10^{-10} \text{ m}$$

For $n = 1 \Rightarrow$ from Bragg's law-

$$2d \sin \theta = n\lambda$$

$$\sin \theta = \frac{1 \times 2 \times 10^{-6}}{2 \times 10^{-10}}$$

$$\sin \theta = 10^4$$

But $\sin \theta > 1$ which is not possible.

So, it is not possible to observe diffraction pattern with $\lambda = 2 \times 10^{-6}$ m.

Q.7. (c) Lead has fcc structure and its body diagonal is 0.86 nm. When X-rays of wavelength 0.7 Å undergoes diffraction from (110) plane to produce second order maxima, calculate the glancing angle. (2.5)

Ans. We know that-

$$2d \sin \theta = n\lambda$$

and

Given

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$h = 1, k = 1, l = 0$$

$$n = 2, \lambda = 0.71 \times 10^{-10} \text{ m}$$

$$d_{110} = \frac{a}{\sqrt{1+1+0}} = \frac{a}{\sqrt{2}}$$

$$\therefore 2 \frac{a}{\sqrt{2}} \sin \theta = 2 \times 0.71 \times 10^{-10}$$

For FCC-

Also,

$$a = 2 \times \sqrt{2} \times 0.86 \times 10^{-9} \text{ m}$$

$$a = 2.98 \times 10^{-9} \text{ m}$$

$$\frac{2 \times 2.98 \times 10^{-9}}{\sqrt{2}} \sin \theta = 2 \times 0.71 \times 10^{-10}$$

$$3.44 \times 10^{-9} \sin \theta = 1.42 \times 10^{-10}$$

$$\sin \theta = \frac{1.42 \times 10^{-10}}{3.44 \times 10^{-9}}$$

$$= 0.413 \times 10^{-1}$$

$$\sin \theta = 0.0413$$

$$\theta = \sin^{-1}(0.0413)$$

$$\boxed{\theta = 2.37^\circ}$$

UNIT-IV

Q.8. (a) What are Brillouin zones? How are they related to the energy levels of an electron in a metal? (4)

Ans. If Schrodinger wave equation for electron energies is solved with a periodic function $u(k)$ to give the energies of electrons in a solid, the solutions fall into permitted energy bands. If the solutions are plotted in the reciprocal lattice of the crystal being considered, the zones enclosing the solutions for $k = 1, 2, 3, \dots, n$ are called **Brillouin zones**.

BRILLOUIN ZONES

Linear Lattice-

According to Kronig and Penney, the discontinuities in a one dimensional lattice

occur when $k = \frac{n\pi}{a}$, where n is a positive or negative integer. In a one dimensional mono-

atomic lattice a line representing the value of k is divided by energy discontinuities into segments of length $\frac{2\pi}{a}$. These line segments are known as **Brillouin Zones**.

The first reflection and the first energy gap occurs at $k = \pm \frac{\pi}{a}$. The reflection at

$k = \pm \frac{\pi}{a}$ arises because the wave reflected from one atom in a linear lattice interfaces constructively with the wave from a nearest neighbour atom, the phase difference between the two reflected waves for the values of $k = \pm \frac{\pi}{a}$ and $k = -\frac{\pi}{a}$ being $\frac{2\pi}{a}$. The

region in k-space between $-\frac{\pi}{a}$ and $+\frac{\pi}{a}$, i.e., the segment $-\frac{\pi}{a} \leq k \leq +\frac{\pi}{a}$ is called the first Brillouin Zone.

The second Brillouin zone contains electrons with $k > \frac{\pi}{a}$, i.e., electrons having

k-values between $\frac{\pi}{a}$ and $\frac{2\pi}{a}$ for electrons moving in the $\pm x$ direction. The second reflection

and second energy gap occurs at $k = \pm \frac{2\pi}{a}$ and therefore, the region in the k space

between $-\frac{2\pi}{a}$ to $\frac{-\pi}{a}$ and $+\frac{\pi}{a}$ to $+\frac{2\pi}{a}$, i.e., the segment $-\frac{2\pi}{a} \leq k \leq -\frac{\pi}{a}$ and $+\frac{\pi}{a} \leq k \leq \frac{2\pi}{a}$ forms the second Brillouin zone of the lattice.

Similarly, we can discuss the formation of third Brillouin zone; ...etc.

The first and second Brillouin zones and the energy gaps are shown in Fig. by plotting value of E in eV corresponding to various values to k .

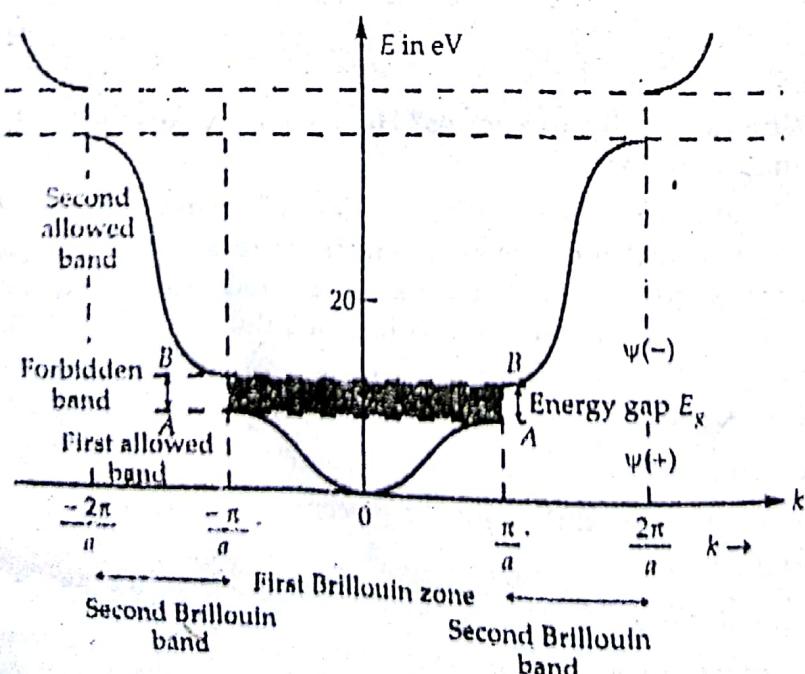


Fig. 6. The first and second Brillouin Zones and energy gap

Two Dimensional Lattice: Proceeding as in the case of a linear lattice, the first zone for a two dimensional lattice in the X-Y plane is the square ABCD the boundaries of which are given by the relation

$$k_x = +\frac{\pi}{a} \text{ and } k_x = -\frac{\pi}{a} \text{ and } k_y = +\frac{\pi}{a} \text{ and }$$

$$k_y = -\frac{\pi}{a} \text{ as shown in Fig.}$$

The second Brillouin zone for a two dimensional lattice in the X-Y plane is the square EFGH the boundary of which are given by the relation

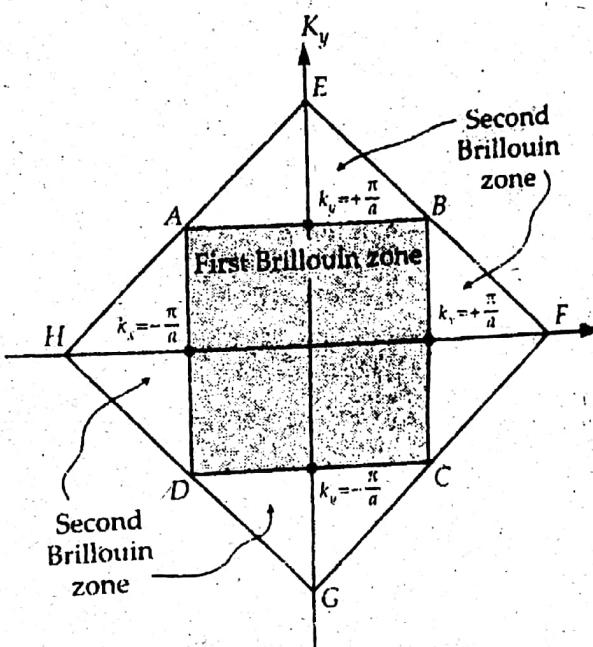


Fig. 7. First and Second Brillouin Zones in 2D lattice

$$k_x = \pm \frac{2\pi}{a}; \quad k_y = \pm \frac{2\pi}{a}$$

Q.8. (b) Define effective mass of electron. Obtain an expression for effective mass of an electron moving in a periodic potential (5)

Ans. The electrons in a crystal are not completely free but interact with the periodic potential of the crystal lattice. As a result their wave particle motion is different from that of an electron in free space. Thus while using the equations of electrodynamics for charge carriers in a crystal, we must use alternate value of particle mass or effective mass after taking into account the effect of the crystal lattice so that the electron (and the hole) can be treated as almost free carriers of charge for the calculations.

The motion of an electron in a crystal in the presence of an external applied electric field ξ . Suppose the electron is initially in a state k , when the external electric field is applied the magnitude of the force acting on the electron is $e\xi$.

If the electron moves a distance dx in a small time dt , under the action of this force, Then

$$\text{Work done} = \text{Energy gained } dE = e\xi dx \quad \dots(1)$$

If v is the velocity gained by the electron, then

$$v = \frac{dx}{dt} \text{ or } dx = v dt$$

$$\therefore dE = e\xi v dt \quad \dots(2)$$

Now

$$v = \frac{1}{\hbar} \frac{dE}{dk} \quad \dots(3)$$

$$\therefore dE = \frac{e\xi}{\hbar} \frac{dE}{dk} dt \quad \dots(4)$$

or

$$\frac{dk}{dt} = \frac{e\xi}{\hbar} \quad \dots(5)$$

Now $\hbar k = p$ and

$$\hbar \frac{dk}{dt} = \frac{dp}{dt} = F$$

$$\therefore \hbar \frac{dk}{dt} = e\xi = F \quad \dots(6)$$

Differentiating Eq. (3) with respect to t , we get

$$\frac{dv}{dt} = \frac{1}{\hbar} \frac{d^2 E}{dk dt} = \frac{1}{\hbar} \frac{d^2 E}{dk^2} \frac{dk}{dt} \quad \dots(7)$$

From Eq. (6)

$$\frac{dk}{dt} = \frac{F}{\hbar}$$

Also

$$\frac{dv}{dt} = a \text{ (the acceleration)}$$

From Eq. (7)

$$a = \frac{1}{\hbar} \frac{d^2 E}{dk^2} \cdot \frac{F}{\hbar}$$

or

$$\frac{a}{F} = \frac{1}{\hbar^2} \frac{d^2 E}{dk^2} \quad \dots(8)$$

Taking $m^* = \frac{F}{a}$ as the effective mass of electron in the crystal, we have

$$m^* = \frac{\hbar^2}{d^2 E / dk^2} \quad \dots(9)$$

In some cases, a factor f_k is also used. It gives the ratio of the rest mass of a free electron m to its effective mass in the crystal in the k -state

$$f_k = \frac{m}{m^*} = \frac{m}{\hbar^2} \left(\frac{d^2 E}{dk^2} \right) \quad \dots(10)$$

This expression determines the extent to which the electron in the k -state behaves as a free electron.

If $f_k = 1$; $m^* = m$, the electron in the crystal in the k -state behaves as a free electron.

Q.8. (c) Where does Fermi level lie in an n type semiconductor? Discuss the effect of temperature and doping concentration on the position of the Fermi level in an n type semiconductor. (3.5)

Ans. For N-type semiconductor, since $n_e > n_h$, E_F must move closer to conduction band to indicate that many of the energy states in the band are filled by the donor electrons, and of few holes exist in the valence band.

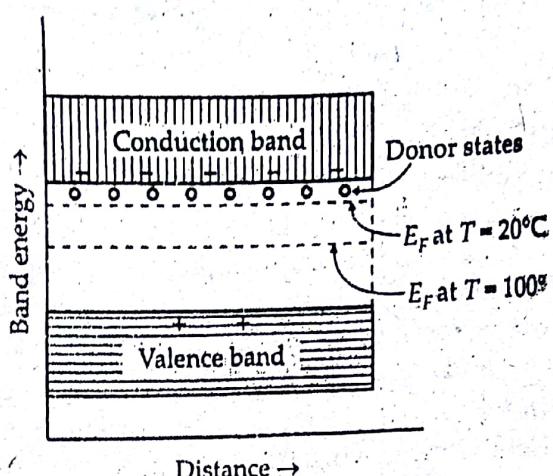


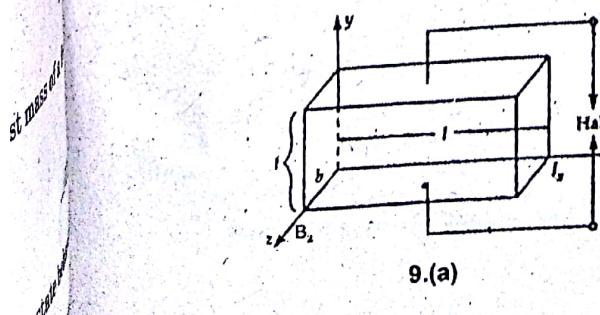
Fig. 8.

For an intrinsic semiconductor $n_i = p_i$ and as temperature increases both n_i and p_i will increase. Thus the Fermi level (E_F) will remain approximately at the centre of the forbidden gap. Thus for intrinsic semiconductor fermi level is independent of temperature.

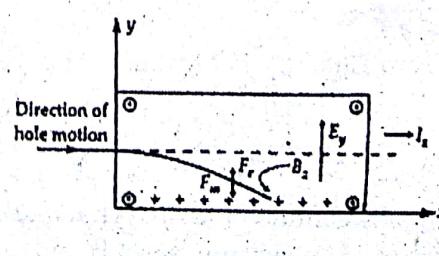
But in an extrinsic semiconductor it is different. Say for N-type material the electrons are coming from two different sources. Some electrons come from donor state, which are easily separated from their parent atom and they do not vary much as the temperature is increased. The other electrons in the conduction band are present because of the breaking of a covalent bond. Such intrinsic effect will increase in number as temperature raised. So proportion of the total number of conduction electrons will increase from valence band. Thus as the temperature rises the material becomes more and more intrinsic and Fermi level moves closer to the intrinsic position, i.e., at the centre of the forbidden gap.

Q.9. (a) What is Hall effect? Derive an expression for Hall coefficient. Explain how the measurement of hall coefficient helps to determine the mobility of electrons in metals. (7)

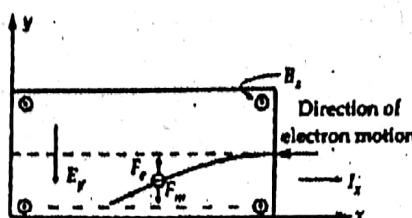
Ans. The phenomenon of Hall Effect is a consequence of motion of charge in both magnetic and electric fields. If a piece of conductor (metal or semiconductor) carrying current is placed in a transverse magnetic field, then an electric field or a potential difference is produced inside the conductor in a direction normal to both current and magnetic field. This phenomenon is known as *Hall effect*. The developed electric field and potential are called *Hall field* and *Hall voltage*.



9.(a)



9.(b)



9.(c)

Fig. 9. (a) Illustration of Hall effect, 9.(b) Motion of hole in P-type semiconductor specimen 9.(c) Motion of electron of N-type semiconductor

Consider a specimen in the form of a rectangular cross-section carrying current I_x in the x -direction. A uniform magnetic field B_z is applied along z -axis as shown in Fig. 9. If specimen is of conducting material or N-type semiconductor, then current flows due to motion of electron, but in P-type semiconductor and intrinsic semiconductor, current is carried by motion of holes and electrons-holes respectively.

A moving charge in transverse magnetic field experiences a force normal to both current and magnetic field. Thus moving charges slightly shift towards y direction due to magnetic force $e v_x B_z$ Fig. (a) & (c). This shift of charges produces a potential difference or an electric field E_y along y -direction. The produced electric field also applies a force $e E_y$ on charge. When both the forces become equal, the equilibrium occurs, i.e., upto this condition charges shift along y direction.

In equilibrium

$$\begin{aligned} F_e &= F_m \\ eE_y &= ev_x B_z \\ E_y &= v_x B_z \end{aligned} \quad \dots(1)$$

If J_x is the current density in x direction, then

$$J_x = \frac{I_x}{A} = nev_x \Rightarrow V_x = \frac{J_x}{ne} \quad \dots(2)$$

Here n is density of charge carrier (electron or hole or both) and A (breadth \times thickness) is the cross-sectional area of specimen perpendicular to direction of J_x . From Eqs. (1) and (2) we can write.

$$E_y = \frac{B_z J_x}{ne} \quad \dots(3)$$

This is the expression for Hall field. The Hall effect is described by means of Hall coefficient R_H , defined in terms of J_x by the following relation:

$$R_H = \frac{E_y}{J_x B_z} \quad \dots(4)$$

From Eqs. (3) and (4), we have-

$$R_H = \frac{1}{ne} \quad \dots(5)$$

For the conductor or N-type semiconductor, the electric field is developed in negative y direction. Hence from Eqs. (4) and (5), we can write.

$$R_H = -\frac{E_y}{J_x B_z} = -\frac{1}{ne} \quad \dots(6)$$

In P-type semiconductor, the produced Hall field is along y direction, thus we have

$$R_H = \frac{E_y}{J_x B_z} = \frac{1}{n_h e} \quad \dots(7)$$

Here n_h is the hole density. Equations (6) and (7) indicate that the sign of R_H is same as the sign of carrier. Most of the metals have negative Hall coefficient or resistance, but some metals have positive Hall coefficient (i.e., charge carriers are holes). If both holes and electrons contribute to current, then R_H can be positive or negative depending upon the relative density and mobility of carriers.

Application of Hall Effect

1. The sign of Hall coefficient can be used to determine whether a given semiconductor is N or P-type.
2. The carrier concentration can be evaluated when R_H is known (as $n = 1/R_H e$)
3. Since $u = R_H \sigma$, thus mobility of charge carrier can also be well determined with the help of R_H .
4. Since Hall voltage is proportional to magnetic flux density B_z and current I_x through a sample or specimen. The Hall effect can be used as the basis for design of a magnetic flux density meter.
5. Hall effect can be used to determine the power flow in an electromagnetic wave because V_H is proportional to product of E_y and B_z , i.e., magnitude of Poynting vector in e.m. wave,

Q.9. (b) The Hall voltage for the metal sodium is 0.001 mV, measured at

$I = 100 \text{ mA}$, $B = \frac{2\omega b}{m^2}$ and width of specimen is 0.05 mm. Calculate the number of carriers per cubic meter in sodium. (5)

Ans. Given

$$V_H = 0.001 \text{ mV} = 0.001 \times 10^{-3} \text{ V}$$

$$I = 100 \text{ mA} = 100 \times 10^{-3} \text{ A}$$

$$B = 2\omega b/\text{m}^2$$

$$d = 0.05 \text{ mm} = 0.05 \times 10^{-3} \text{ m}$$

The number of carriers per cubic meter in sodium is as following-

$$n = \frac{IB}{V_H d}$$

$$n = \frac{100 \times 10^{-3} \times 2}{0.001 \times 10^{-3} \times 0.05 \times 10^{-3} \times 1.6 \times 10^{-19}}$$

$$= \frac{0.2}{0.8 \times 10^{-29}} = 0.25 \times 10^{29}$$

$$\boxed{n = 2.5 \times 10^{28} / \text{m}^3}$$

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Q.9. (c) Distinguish between intrinsic and extrinsic semiconductors. (2.5)

Ans. The difference between intrinsic and extrinsic semiconductors are as following-

Intrinsic Semiconductor	Extrinsic Semiconductor
<ol style="list-style-type: none">1. It is a semiconductor in its pure form.2. These semiconductors have low electrical conductivity.3. The operating temperature is low for this type of semiconductors.4. Charge carriers are produced due to thermal excitation. Examples : Ge, Si	<p>It is semiconductor doped with impurities.</p> <p>These semiconductors have high electrical conductivity.</p> <p>The operating temperature is high for this type of semiconductors.</p> <p>Additional charge carriers are also produced due to added impurities. Examples: Ge and Si doped with P, As, Bi, Sb etc.</p>

**END TERM EXAMINATION [MAY-JUNE 2018]
SECOND SEMESTER [B.TECH]
APPLIED PHYSICS-II [ETPH-104]**

Time : 3 hrs.

M.M. : 75

Note: Attempt five questions in all including Q. No. 1 which is compulsory. Select one question from each unit. Symbols have their usual meaning.

Q. 1. (a) Show that curl of gradient of a scalar function is always zero. (2)

Ans. Let the scalar function be $\phi(x, y, z)$

Solving for $\vec{\nabla} \times \vec{\nabla}\phi$

$$\begin{aligned} \vec{\nabla} \times \vec{\nabla}\phi &= \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \times \left(\hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z} \right) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{vmatrix} = \hat{i} \left(\frac{\partial^2 \phi}{\partial y \partial z} - \frac{\partial^2 \phi}{\partial z \partial y} \right) + \hat{j} \left(\frac{\partial^2 \phi}{\partial z \partial x} - \frac{\partial^2 \phi}{\partial x \partial z} \right) + \hat{k} \left(\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial y \partial x} \right) \\ &= 0 \quad \left[\because \frac{\partial^2 \phi}{\partial y \partial z} = \frac{\partial^2 \phi}{\partial z \partial y} \text{ and so on.} \right] \end{aligned}$$

Q. 1. (b) Distinguish between conduction current density and displacement current density. (2)

Ans. Refer Q. 1. (b) First Term Examination, Feb, 2018.

Q. 1. (c) Find the skin depth at a frequency of 10^5 Hz for silver. Given

$$\sigma = 2 \times 10^7 \frac{\text{S}}{\text{m}} \text{ and } \mu = 4 \times 10^{-7} \frac{\text{H}}{\text{m}}. \quad (2)$$

Ans. Skin depth is given by $\delta = \sqrt{\frac{2}{\mu \sigma \omega}}$

Given, $\mu = 4 \times 10^{-7}$; $\sigma = 2 \times 10^7$; $\omega = 2\pi \times 10^5$

$$\begin{aligned} \delta &= \sqrt{\frac{2}{4 \times 10^{-7} \times 2 \times 10^7 \times 2 \times 3.14 \times 10^5}} \\ &= \sqrt{\frac{10^{-6}}{2.512}} = 0.63087 \times 10^{-3} \text{ m} \\ \boxed{\delta = 630.87 \mu\text{m}} \end{aligned}$$

Q. 1. (d) Write down the normalization condition of a wave function. Why is it necessary for a wave function to be normalized? (3)

Ans. ψ cannot be interpreted in terms of an experiment. The probability that something be in certain place at a given time must lie between 0 (the object is not definitely there) and 1 (the object is definitely there). An intermediate probability, say 0.2 means, there is a 20% chance of finding the object. But the amplitude of wave can be negative as well as positive and negative probability is meaningless. Hence ψ by itself cannot be an observed quantity.

The probability of experimentally finding the body described by the wave function ψ at the point (x, y, z) at the time t is proportional to the value $|\psi|^2$ there at t . A large value of $|\psi|^2$ means the strong possibility of the body's presence, while a small value of $|\psi|^2$ is not actually zero somewhere, there is a definite change however small, of detecting it there.

A wave function is a mathematical tool in quantum mechanics describing the quantum state of a particle or system of particles. Mathematically, it is a function from a space that maps the possible states of the system. The laws of quantum mechanics described how the wave function evolves over the time.

In quantum mechanics, probability amplitude is a complex number whose modulus squared represents a probability density. The principal use of probability amplitudes is as the physical meaning of the wave function.

Before going through actual calculation of ψ , it must satisfy certain requirements. In quantum mechanics, wave function describing real particle must be normalizable, i.e., the integral of $|\psi|^2$ over all space must be finite-after all the body is located somewhere. Mathematically, it is expressed as:

$$\int \psi^* \psi dV = 1$$

or

$$\int |\psi|^2 dV = 1$$

A wave function that obeys above equation is said to be normalized. The normalization of wave function is done to obtain the physically applicable wave function or probability amplitudes. All wave function representing real particles must be normalisable. This helps in discarding solutions of Schrodinger's equation which do not have a finite integral in a given interval.

Q. 1. (e) Why is the wave nature of matter not more apparent in our daily observation? (2)

Ans. Since de-Broglie wavelength is given by $\lambda = \frac{h}{mv}$ and the value of Planck's constant is very small, so the wavelength associated with ordinary object is small and difficult to observe. In our daily observations we deal with the objects having larger mass and smaller velocity, that is why the wave nature of such objects is not more apparent in our daily life. But for smaller objects like electrons and neutrons the wave behaviour of particle is dominant as the mass of electron and neutron are smaller than the mass of an ordinary tennis ball.

Q. 1. (f) What is energy quantization? How is it possible for particle in a rigid box? (3)

Ans. Energy quantization means that the system or an atom can have only certain energies and not a continuum of energies. This discretization of energy values is known as quantization, meaning that the allowed energies are separated by discrete energy differences known as "quanta".

For particle in a box of length " l " has following energy.

$$E_n = \frac{n^2 h^2}{8ml^2}$$

where $n = 1, 2, 3, \dots$; m = mass of particle; h = Planck's constant

Particle cannot possess an arbitrary energy but can have only certain discrete energy corresponding to $n = 1, 2, 3$. Each permitted energy level energy is called eigen value of the particle and constitute the energy level of the system.

Q. 1. (g) Based on Fermi-Dirac statistics, state the nature of Fermi distribution function. How does it vary with temperatures? (2)

Ans. The Fermi-Dirac distribution is plotted in Fig. 1. for three different values of T and α . In the distribution the occupation index never goes above 1. This signifies that we cannot have more than 1 particle per quantum state as required by Pauli's exclusion principle which applies in this case.

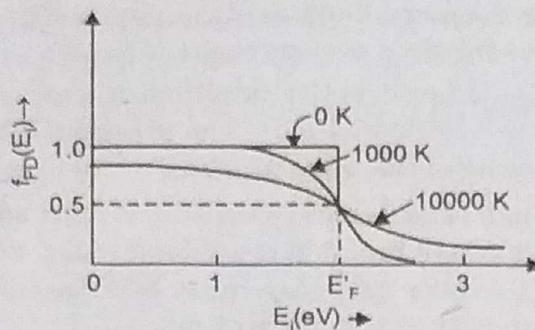


Fig. 1. F.D. distribution curve.

Further, in this distribution the parameter α is strongly dependent on temperature T, and we write

$$\alpha = \frac{-E_F}{k_B T} \quad \dots(1)$$

So that the Fermi-Dirac occupation index becomes.

$$f_{FD}(E_i) = \frac{1}{(e^{(E_i - E_F)k_B T} + 1)} \quad \dots(2)$$

where E_F is called Fermi energy.

Let us consider the situation at the absolute zero of temperature.

At $T = 0$, $(E_i - E_F)/k_B T = -\infty$ (for $E_i < E_F$) and $(E_i - E_F)/k_B T = +\infty$ (for $E_i > E_F$).

Therefore

$$\text{for } E_i < E_F \quad f_{FD}(E_i) = \frac{1}{e^{-\infty} + 1} = 1 \quad (\because e^{-\infty} = 0) \quad \dots(3)$$

$$\text{and for } E_i > E_F \quad f_{FD}(E_i) = \frac{1}{e^{\infty} + 1} = 0 \quad (\because e^{\infty} = 0) \quad \dots(4)$$

Thus at $T = 0$, all energy states from $E_i = 0$ to $E_i = E_F$ are occupied because $f_{FD}(E_i) = 1$, while all states above E_F are vacant.

As the temperature rises, some of the states just below E_F become vacant, while some just above E_F are occupied. The higher the temperature, the more is the spreading in $f_{FD}(E_i)$.

At

$$E_i = E_F, \text{ we have}$$

$$f_{FD}(E_i) = \frac{1}{e^0 + 1} = \frac{1}{2}, \text{ at all temperatures.}$$

That is, the average number of particles per quantum state is exactly $\frac{1}{2}$. In other words, the probability of finding an electron with energy equal to the Fermi energy in a metal is $\frac{1}{2}$ at any temperature.

Q. 1. (h) What is ultraviolet catastrophe?

(2)

Ans. It is observed that Wien's fifth power law $\left(E(\lambda) \propto \frac{1}{\lambda^5} \right)$ agree with experimental results only in the low wavelength ($\lambda \rightarrow 0$) region but breaks down in large wavelength ($\lambda \rightarrow \infty$) region as shown in Fig. 2.

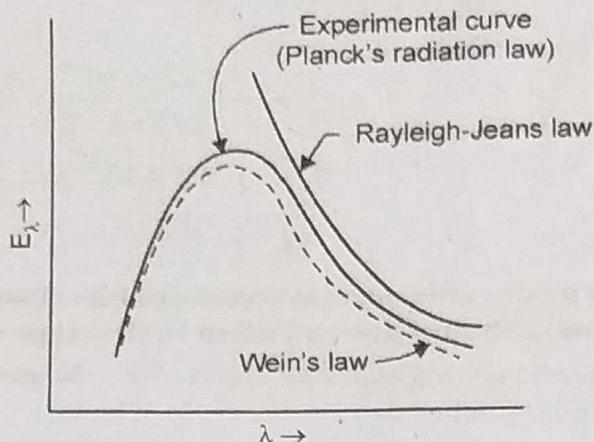


Fig. 2. Comparison of Wien's law and Rayleigh-Jeans law with the experimental curve of blackbody radiation.

Rayleigh-Jeans law $\left[E_\lambda \propto \frac{1}{\lambda^4} \right]$ on the other hand can account for the distribution of energy in the long wavelength region and fails completely in short wavelength. This result is said to be ultraviolet catastrophe.

Ultraviolet Catastrophe: According to Rayleigh-Jeans law the energy density in blackbody radiation between wavelength λ and $(\lambda + d\lambda)$ at temperature T is given as

$$E_\lambda d\lambda = \frac{8\pi k_B T}{\lambda^4} d\lambda \quad \dots(1)$$

Therefore, the total energy radiation per unit volume of the enclosure for all wavelengths from $\lambda = 0$ to $\lambda = \infty$ is given by

$$\begin{aligned} E &= \int_0^\infty E_\lambda d\lambda = \int_0^\infty \frac{8\pi k_B T}{\lambda^4} d\lambda = 8\pi k_B T \left[-\frac{1}{3\lambda^3} \right]_0^\infty \\ &= 8\pi k_B T \left[\frac{1}{0} - \frac{1}{\infty} \right] = \infty \end{aligned} \quad \dots(2)$$

This law leads to the fact that, the energy density $E_\lambda \rightarrow \infty$ as $\lambda \rightarrow 0$, whereas experimental results show that $E_\lambda \rightarrow 0$ as $\lambda \rightarrow 0$. This implies that for a given quantity of radiant energy is finally confined in the short wavelength (ultraviolet) range.

This is serious discrepancy between theory and experiment and is known as ultraviolet catastrophe or jeans paradox.

Q. 1. (i) Sodium Crystallies is bcc structure. If the radius of the sodium atom is 1.55 nm, compute the spacing between (111) planes. (2)

Ans. Interplanar spacing is given by

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Given

$$a = \frac{4r}{\sqrt{3}} = \frac{4 \times 1.55 \times 10^{-9}}{\sqrt{3}} \left(r = \frac{\sqrt{3}}{4} a \right)$$

$$h = 1, k = 1, l = 1$$

$$\sqrt{h^2 + k^2 + l^2} = \sqrt{3}$$

$$d = \frac{4 \times 1.55 \times 10^{-9}}{\sqrt{3} \times 3} = \frac{6.2 \times 10^{-9}}{5.196}$$

$$d = 1.193 \times 10^{-9} \text{ m}$$

$$d = 1.193 \text{ nm}$$

Q. 1. (j) Which type of cubic crystal structure has closest packing of atoms? How many nearest neighbours does an atom in this type of crystal have? (2)

Ans. FCC has closest packing of atoms, that is 74%. Nearest neighbours in FCC is given by its coordination Number as—

$$CN = 4 + 4 + 4 = 12$$

Therefore it has 12 nearest neighbours.

Q. 1. (k) What do you understand by effective mass of an electron? Explain its significance. (3)

Ans. The electrons in a crystal are not completely free but interact with the periodic potential of the crystal lattice. As a result their wave particle motion is different from that of an electron in free space. Thus while using the equations of electrodynamics for charge carriers in a crystal, we must use alter value of particle mass of effective mass after taking into account the effect of the crystal lattice so that the electron (and the, hole) can be treated as almost free carriers of charge for the calculations.

Let us now consider the motion of an electron in a crystal in the presence of an external applied electric field E . Suppose the electron is initially in a state k , when the external electric field E is applied the magnitude of the force acting on the electron is eE .

If the electron moves a distance dx in a small time dt , under the action of this force, then

$$\text{Work done} = \text{Energy gained} \quad dE = eEdx \quad \dots(1)$$

If v is the velocity gained by the electron, then

$$v = \frac{dx}{dt} \text{ or } dx = v dt$$

$$dE = eEv dt \quad \dots(2)$$

Now

$$v = \frac{1}{h} \frac{dE}{dk} \quad \dots(3)$$

$$dE = \frac{eE}{h} \frac{dE}{dk} dt \quad \dots(4)$$

or

$$\frac{dk}{dt} = \frac{eE}{h} \quad \dots(5)$$

Now

$$hk = p \text{ and } h \frac{dk}{dt} = \frac{dp}{dt} = F$$

$$h \frac{dk}{dt} = eE = F \quad \dots(6)$$

Differentiating Eq. (3) with respect to t , we get

$$\frac{dv}{dt} = \frac{1}{h} \frac{d^2E}{dk dt} = \frac{1}{h} \frac{d^2E}{dk^2} \frac{dk}{dt} \quad \dots(7)$$

From Eq. (6),

$$\frac{dk}{dt} = \frac{F}{h}$$

Also

$$\frac{dv}{dt} = a \text{ (the acceleration)}$$

From Eq. (7),

$$a = \frac{1}{h} \frac{d^2E}{dk^2} \frac{F}{h}$$

or

$$\frac{a}{F} = \frac{1}{h^2} \frac{d^2E}{dk} \quad \dots(8)$$

Taking $m^* = \frac{F}{a}$ as the effective mass of electron in the crystal, we have

$$f_k = \frac{h^2}{d^2E/dk^2} \quad \dots(9)$$

In some cases, a factor f_k is also used. It gives the ratio of the rest mass of a free electron m to its effective mass in the crystal in the k -state

$$m^* = \frac{m}{m^*} = \frac{m}{h^2} \left(\frac{d^2E}{dk^2} \right) \quad \dots(10)$$

This expression determines the extent to which the electron in the k -state behaves as a free electron.

If $f_k = 1$, $m^* = m$, the electron in the crystal in the k -state behaves as a free electron.

UNIT-I

Q. 2. (a) A conducting sphere of radius 5cm has an unknown charge. If the electric field 10 cm from the centre of the sphere is $1.5 \times 10^3 \frac{N}{C}$ and points inwards, what is the charge on the sphere? (2.5)

Ans. For conducting sphere at a point outside the charged sphere is given by

$$E = \frac{q}{4\pi r^2 \epsilon_0 \epsilon_r}$$

Given:

Radius of sphere = $a = 5 \text{ cm}$; $E = 1.5 \times 10^3 \frac{N}{C}$; $r = 10 \text{ cm} = 10^{-1} \text{ m} = 0.1 \text{ m}$

For air $\epsilon_r = 1$

and $\epsilon_0 = 8.854 \times 10^{-12} \frac{F}{m}$

$$1.5 \times 10^3 = \frac{q}{4 \times 3.14 \times (10^{-1})^2 \times 1 \times 8.854 \times 10^{-12}}$$

$$q = 1.5 \times 10^3 \times 4 \times 3.14 \times 10^{-2} \times 8.854 \times 10^{-12} = 166.80 \times 10^{-11} \text{ C}$$

$$q = 0.1668 \times 10^{-8} \text{ C}$$

Q. 2. (b) Show that Ampere's Law is inconsistent in time varying fields. (3)

Ans. Refer Q. 1. (c) of First Term Examination 2018.

Q. 2. (c) Write down the significance of Pointing vector. Derive an expression for Pointing Theorem and give interpretation of each term. (2 + 5 = 7)

Ans. Refer Q. 2. (a) First Term Examination 2018.

Q. 3. (a) Deduce the equation for propagation of electromagnetic wave in free space and obtain an expression for the velocity. Show that electric and magnetic field vectors are normal to each other and to the direction of propagation of waves.

Ans. Maxwell's equations are: (8)

$$\operatorname{div} D = \nabla \cdot D = \rho \quad \dots(a)$$

$$\operatorname{div} B = \nabla \cdot B = 0 \quad \dots(b)$$

$$\operatorname{curl} E = -\frac{\partial B}{\partial t} \quad \dots(c)$$

and

$$\operatorname{curl} H = J + \frac{\partial D}{\partial t} \quad \dots(d)$$

Free space is characterised by,

$$\rho = 0, \sigma = 0, \mu = \mu_0 \text{ and } \epsilon = \epsilon_0 \quad \dots(2)$$

Therefore, Maxwell's Eqs. (1) reduce to

$$\operatorname{div} E = 0 \quad \dots 3(a)$$

$$\operatorname{div} H = 0 \quad \dots 3(b)$$

$$\operatorname{curl} E = -\mu_0 \frac{\partial H}{\partial t} \quad \dots 3(c)$$

and

$$\operatorname{curl} H = \epsilon_0 \frac{\partial E}{\partial t} \quad \dots 3(d)$$

Taking curl of Eq. 3. (c), we get

$$\operatorname{curl} \operatorname{curl} E = -\mu_0 \frac{\partial}{\partial t} (\operatorname{curl} H)$$

Substituting curl H from Eq. 3. (d) we get

$$\operatorname{curl} \operatorname{curl} E = -\mu_0 \frac{\partial}{\partial t} \left(\epsilon_0 \frac{\partial E}{\partial t} \right)$$

i.e., $\operatorname{curl} \operatorname{curl} E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \dots(4)$

Now, $\operatorname{curl} \operatorname{curl} E = \operatorname{grad} \operatorname{div} E - \nabla^2 E = -\nabla^2 E$

[$\because \operatorname{div} E = 0$ from Eq. 3 (a)]

Making substitution Eq. (4), becomes

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \quad \dots(5)$$

Similarly, on taking curl of Eq. 3. (d) and putting the value of curl E from Eq. 3. (c) we have

$$\begin{aligned} \text{curl curl } H &= \epsilon_0 \frac{\partial}{\partial t} \left(-\mu_0 \frac{\partial H}{\partial t} \right) \\ &= -\mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} \end{aligned} \quad \dots(6)$$

Again using identity $\text{curl curl } H = \text{grad div } H - \nabla^2 H$ and noting that $\text{div } H = 0$ from Eq. 3 (b), we obtain

$$\text{curl curl } H = -\nabla^2 H$$

Making this substitution in Eq. (6), we get

$$\nabla^2 H - \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} = 0 \quad \dots(7)$$

Equations (5) and (7) represent wave equation governing electric and magnetic fields (E and H) in free space. It may be noted that these equations may be obtained by using Eq. (2) in Eqs.(5) and (7) are vector equations of identical form, which means that each of six components of E and H separately satisfies the same scalar wave equation of the form

$$\nabla^2 \phi - \mu_0 \epsilon_0 \frac{\partial^2 \phi}{\partial t^2} = 0 \quad \dots(8)$$

where ϕ is a scalar and cannot stand for one of the components of E and H . It is obvious that Eq. (8) resembles with general wave equation

$$\nabla^2 \phi = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} \quad \dots(9)$$

where v is the velocity of wave.

Comparing Eqs. (8) and (9), we see that the field vectors E and H are propagated in free space as waves at a speed equal to

$$\begin{aligned} v &= \frac{1}{\sqrt{\mu_0 \epsilon_0}} \\ &= \sqrt{\frac{4\pi}{\mu_0 4\pi \epsilon_0}} = \sqrt{\frac{4\pi}{4\pi \times 10^{-7} \times 9 \times 10^9}} \\ &= 3 \times 10^8 \text{ m/s} \\ &= c = \text{speed of light in vacuum.} \end{aligned} \quad \dots(10)$$

Therefore, it is reasonable to write c the speed of light in place of $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$, so Eqs. (5) and (7) take the form

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \quad \dots(11)$$

$$\nabla^2 H - \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} = 0 \quad \dots(12)$$

and

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0 \quad \dots(13)$$

The plane wave solution of above equations in well known form may be written as

$$E(r, t) = E_0 e^{ik \cdot r - i\omega t} \quad \dots(14)$$

$$H(r, t) = H_0 e^{ik \cdot r - i\omega t} \quad \dots(15)$$

$$\phi(r, t) = \phi_0 e^{ik \cdot r - i\omega t} \quad \dots(16)$$

where E_0, H_0 and ϕ_0 are complex amplitudes which are constant in space and time while k is a wave propagation vector denoted as

$$k = k\hat{n} = \frac{2\pi}{\lambda} \hat{n} = \frac{2\pi v}{c} \hat{n} = \frac{\omega}{c} \hat{n} \quad \dots(17)$$

Here \hat{n} is a unit vector in the direction of wave propagation. Now in order to apply the condition $\nabla \cdot E = 0$ and $\nabla \cdot H = 0$. Let us first find $\nabla \cdot E$ and $\nabla \cdot H$.

$$\begin{aligned} \nabla \cdot E &= \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot E_0 e^{ik \cdot r - i\omega t} \\ &= \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot [(iE_{0x} + jE_{0y} + kE_{0z}) e^{i(k_x x + k_y y + k_z z) - i\omega t}] \\ &[\because \hat{k} \cdot \hat{r} = (ik_x + jk_y + kk_z) \cdot (ix + jy + kz) = k_x x + k_y y + k_z z] \end{aligned}$$

$$\begin{aligned} \nabla \cdot E &= (E_{0x} \hat{i} k_x + E_{0y} \hat{j} k_y + E_{0z} \hat{k} k_z) e^{ik \cdot r - i\omega t} \\ &= \hat{i} (k_x E_{0x} + k_y E_{0y} + k_z E_{0z}) e^{ik \cdot r - i\omega t} \\ &= \hat{i} (\hat{i} k_x + \hat{j} k_y + \hat{k} k_z) \cdot (\hat{i} E_{0x} + \hat{j} E_{0y} + \hat{k} E_{0z}) e^{ik \cdot r - i\omega t} \\ &= \hat{i} k \cdot E_0 e^{ik \cdot r - i\omega t} = \hat{i} k \cdot E \end{aligned}$$

Similarly, $\nabla \cdot H = \hat{i} k \cdot H$

Thus the requirement $\nabla \cdot E = 0$ and $\nabla \cdot H = 0$ demands that

$$k \cdot E = 0 \quad \text{and} \quad k \cdot H = 0 \quad \dots(18)$$

This means that electromagnetic field vectors E and H are both perpendicular to the direction of propagation vector k . This implies that electromagnetic waves are transverse in character.

Further restrictions are provided by curl. From Eqs. 3. (c) and 3. (d).

$$\text{curl } E = -\mu_0 \frac{\partial H}{\partial t} \quad \text{and} \quad \text{curl } H = \epsilon_0 \frac{\partial E}{\partial t}$$

Using Eqs. (14), (15) and (17), above equation yields

$$ik \cdot E = -\mu_0 \cdot (-i\omega H) \quad \text{or} \quad k \cdot E = \mu_0 \omega H \quad \dots(19)$$

$$\text{and} \quad ik \cdot H = \epsilon_0 \cdot (-i\omega E) \quad \text{or} \quad k \cdot H = -\epsilon_0 \omega E \quad \dots(20)$$

From Eq. (17), it is obvious that field vector H is perpendicular to both k and E and according to Eq. (18), E perpendicular to both k and H . This simply means that field vectors E and H are mutually perpendicular and they are also perpendicular to the direction of propagation of wave.

Q. 3. (b) If the earth receives 4 cal/min/cm² solar energy. What are the amplitudes of electric and magnetic fields of radiation? (4.5)

Ans. Here, solar energy which the earth receives is 2 cal/min/cm².

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377$$

Poynting vector

$$\vec{S} = \vec{E} \times \vec{H} = EH \text{ J m}^{-2} \text{ sec}^{-1}$$

$$EH = \frac{4 \times 4.2}{60 \times 10^{-4}} = 2800 \text{ J m}^{-2} \text{ sec}^{-1} \quad \dots(1)$$

Also, $\frac{E_0}{H_0} = 377 = \frac{E}{H}$... (2)

Multiply eqn. (1) and (2)

$$E^2 = 2800 \times 377 = 1055600$$

$$E = 1027.42 \frac{\text{V}}{\text{m}}$$

or, $H = \frac{E}{377} = \frac{1027.42}{377} = 2.73 \frac{\text{A}}{\text{m}}$

Amplitudes of electric and magnetic field vectors are

$$E_0 = E\sqrt{2} = 1.452.78 \text{ V/m}$$

$$H_0 = H\sqrt{2} = 3.85 \text{ A/m}$$

UNIT-II

Q. 4. (a) State and explain Rayleigh-Jeans law. Show how its drawbacks can be overcome using Planck's radiation law. (3.5)

Ans. As per Planck's quantum hypothesis, the atoms of the wall of a blackbody behave as oscillators and each has a characteristic frequency of oscillation. Then average energy of these Planck's oscillators were calculated and finally Planck's radiation formula was derived:

Wien's law is deduced from Planck's radiation formula under the condition when the wavelength λ and temperature T are very small. However, Plank's radiation formula under the condition of high temperature T and wavelength λ takes the form of Rayleigh-Jeans law.

Planck radiation law is as following.

$$\mu_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1} d\lambda$$

When λ and T are very small then $e^{\left(\frac{hc}{\lambda KT}\right)} \gg 1$

$$\mu_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda KT}} d\lambda$$

By substituting

$$8\pi hc = A \text{ and } \frac{hc}{K} = B$$

we get

$$\mu_\lambda d\lambda = \frac{8}{\lambda^5} e^{-\frac{B}{\lambda T}} d\lambda$$

This is Wien's law valid at low temperature and small wavelength.

When λ and T are large then $e^{-\frac{hc}{\lambda KT}} \approx 1 + \frac{hc}{\lambda KT}$

$$\mu_\lambda d\lambda = \frac{8\pi hc}{\lambda^5 \left[1 + \frac{hc}{\lambda KT} \right]^{-1}} d\lambda$$

$$\mu_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda$$

This is Rayleigh-Jeans law valid at high temperature and large wavelength

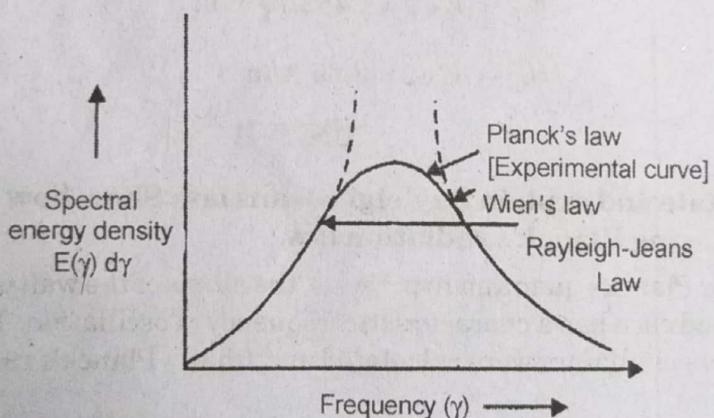


Fig. 3

As shown in Fig.3. the spectral energy density decreases with frequency at high frequency. Hence Rayleigh-Jeans and Wien's law are unable to explain the complete energy distribution for blackbody radiation, while Planck's law explain it completely.

Q. 4. (b) Distinguish between Maxwell-Boltzmann (MB), Bose-Einstein (BE) and Fermi-Dirac (FD) statistics on the basis of their distribution function and show that the BE and FD distributions at very high temperature goes to MB distribution. (5)

Ans. Difference between Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac staticstics and as following-

Maxwell-Boltzmann statistics (Classical statistics)	Bose-Einstein statistics (Quantum statistics)	Fermi-Dirac statistics (Quantum (statistics))
1. Apply to systems of distinguishable particles, such as gases.	Apply to systems of indistinguishable particles not obeying Pauli's exclusion principle, such as photons, phonons and liquid helium at low temperature	Apply to systems of indistinguishable particle obeying Pauli exclusion principle, such as free electrons in metal electrons in a star (white dwarf star)
2. The distribution law is $S_i = \frac{g_i}{e^{\alpha} e^{E_i/k_B T}}$	The distribution law is $n_i = \frac{g_i}{(e^{\alpha} e^{E_i/k_B T}) - 1}$	The distribution law is $n_i = \frac{g_i}{(e^{\alpha} e^{E_i/k_B T}) + 1}$
3. There is no restriction on the number of particles, in the given state.	There is no restriction on the number of particles in a given state.	Only one particle in a given quantum state is allowed.
4. In the phase space the volume of the phase cell is not fixed.	The volume of phase cell is of order of \hbar^3 .	The volume of phase cell is of the order of \hbar^3
5. The behaviour of distribution function $f(E_i)$ against E_i is exponential.	For $E_i \gg k_B T$, exponential where $E_i \ll k_B T$, lies above M.B.	For $E_i \gg k_B T$; exponential where $E_i \gg E_F$. If $E_F \gg k_B T$ decreases abruptly near E_F .
6. The energy of M.B. system, such as in ideal monoatomic gas, may be zero at absolute zero.	The energy may be zero at absolute zero.	The energy at absolute zero, cannot be zero because all particles cannot come down to ground state due to Pauli's exclusion principle.

For N-type semiconductor, since $n_e > n_h$, E_F must move closer to conduction band to indicate that many of the energy states in the band are filled by the donor electrons, and of few holes exist in the valence band.

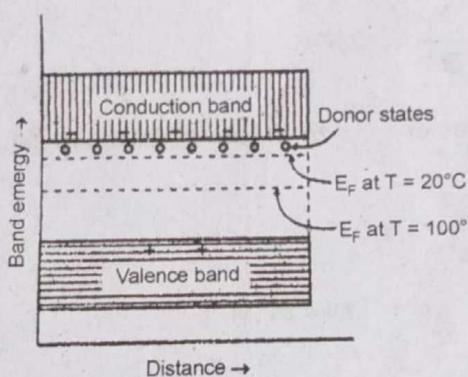


Fig.

For an intrinsic semiconductor $n_i = p_i$ and as temperature increases both n_i and p_i will increase. Thus the Fermi level (E_F) will remain approximately at the centre of the forbidden gap. Thus for intrinsic semiconductor fermi level is independent of temperature.

But in an extrinsic semiconductor it is different. Say for N-type material the electrons are coming from two different sources. Some electrons come from donor state, which are easily separated from their parent atom and they do not vary much as the temperature is increased. The other electrons in the conduction band are present because of the breaking of a covalent bond. Such intrinsic effect will increase in number as temperature raised. So proportion of the total number of conduction electrons will increase from valence band. Thus as the temperature rises the material becomes more and more intrinsic and Fermi level moves closer to the intrinsic position, i.e., at the centre of the forbidden gap.

Q. 4. (c) Distinguish between Bosons and Fermions. Give one example each.

(4)

Ans. Refer Q. 4. (a) End Term Examination May-June 2017.

Q. 5. (a) Derive Schrodinger's time independent wave equation. (5)

Ans. Consider a system of stationary waves associated with a moving particle. If the position coordinates of the particle are (x, y, z) and Ψ be the periodic displacement for the matter waves at any instant of time t , then we can represent the motion of the wave by a differential equation as follows.

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \quad \dots(1)$$

where v is the velocity of wave associated with the particle. The solution of Eq. (1) given Ψ as a periodic displacement in terms of time i.e.

$$\Psi(x, y, z, t) = \Psi_0(x, y, z)e^{-i\omega t} \quad \dots(2)$$

where Ψ_0 is the amplitude of the particle wave at the point (x, y, z) which is independent of time (t) . It is a function of (x, y, z) . i.e., the position r and not of time t , Here.

$$r = x\hat{i} + y\hat{j} + z\hat{k} \quad \dots(3)$$

Eq. (2) may be expressed as

$$\Psi(r, t) = \Psi_0(r)e^{-i\omega t} \quad \dots(4)$$

Differentiating Eq. (4) twice with respect to t , we get

$$\frac{\partial^2 \Psi}{\partial t^2} = -\omega^2 \Psi_0(r)e^{-i\omega t}$$

$$\text{or} \quad \frac{\partial^2 \Psi}{\partial t^2} = -\omega^2 \Psi \quad \dots(5)$$

Substituting the value of $\frac{\partial^2 \Psi}{\partial t^2}$ from this equation in Eq. (1), we get

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{\omega^2}{v^2} \Psi = 0 \quad \dots(6)$$

where

$$\omega = 2\pi v = 2\pi(v/\lambda)$$

[as $u = \lambda v$]

so that

$$\frac{\omega}{u} = \frac{2\pi}{\lambda} \quad \dots(7)$$

$$\text{Also} \quad \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \nabla^2 \Psi \quad \dots(8)$$

where ∇^2 is known as Laplacian operator. Using Eqs. (6), (7) and (8), we have

$$\nabla^2 \Psi + \frac{4\pi^2}{\lambda^2} \Psi = 0 \quad \dots(9)$$

Also from the de-Broglie wave concept

$$\lambda = \frac{h}{mv}$$

Using this relation in Eq. (9) gives:-

$$\nabla^2 \Psi + \frac{4\pi^2 m^2 v^2}{\hbar^2} \Psi = 0 \quad \dots(10)$$

If E and V are respectively the total energy and potential energy of the particle then its kinetic energy is given by

$$\begin{aligned} \frac{1}{2}mv^2 &= E - V \\ m^2v^2 &= 2m(E - V) \end{aligned} \quad \dots(11)$$

The use of Eq. (11) in Eq. (10) gives

$$\begin{aligned} \nabla^2 \Psi + \frac{8\pi^2 m}{\hbar^2}(E - V)\Psi &= 0 \\ \text{or} \quad \nabla^2 \Psi + \frac{2m}{\hbar^2}(E - V)\Psi &= 0 \end{aligned} \quad \dots(12)$$

This is the time independent Schroedinger equation, where the quantity Ψ is known as wave function.

For a freely moving or free particle $V = 0$. Therefore, Eq. (12) becomes

$$\nabla^2 \Psi + \frac{2mE}{\hbar^2} \Psi = 0 \quad \dots(13)$$

This is called time independent Schrödinger equation for a free particle.

Q. 5. (b) What is a wave packet? Show that phase velocity of de-Broglie wave is greater than the velocity of light. (1 + 3)

Ans. When plane waves of slightly different wavelength travel simultaneously in the same direction along a straight line, through a dispersive medium then successive groups of waves are produced. These waves are called wave packets. Each wave group travels with a velocity called group velocity.

$$\text{Phase velocity} = \frac{w}{K} \quad \dots(1)$$

$$\text{Also, phase velocity} = V_p = \gamma \lambda \quad \dots(2)$$

For an electromagnetic wave

$$\begin{aligned} E &= h\nu \\ \text{or} \quad \gamma &= \frac{E}{h} \end{aligned} \quad \dots(3)$$

According to de-Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \dots(4)$$

From eqn. (2), (3) and (4), we get

$$\begin{aligned} V_p &= \gamma \lambda \\ &= \frac{E}{h} \times \frac{h}{mv} = \frac{E}{mv} = \frac{mc^2}{mv} = \frac{c^2}{v} \\ \boxed{V_p = \frac{C^2}{V}} & \quad \dots(5) \end{aligned}$$

Since $C \gg v$, Eqn (5) implies that the phase velocity of de-Broglie wave is greater than the velocity of light.

Q. 5. (c) A particle is confined to an one-dimensional infinite potential well of width 0.2×10^{-9} m. It is found that when the energy of the particle is 230 eV, its eigenfunction has 5 antiodes. Find the mass of the particle and show that it can never have energy equal to 1 Kev. (3.5)

Ans. Energy of particle of mass "m" in a box of length "l" is given by

$$E_n = \frac{n^2 h^2}{8ml^2}$$

Given

$$l = 0.2 \times 10^{-9} \text{ m} = 2 \times 10^{-10} \text{ m}$$

$$h = 6.63 \times 10^{-34} \text{ J-S}$$

$$n = 5$$

$$E = 230 \text{ eV} = 368 \times 10^{-19} \text{ J}$$

$$m = ?$$

$$368 \times 10^{-19} = \frac{25 \times (6.63 \times 10^{-34})^2}{8 \times m \times (2 \times 10^{-10})^2}$$

$$m = \frac{25 \times (6.63 \times 10^{-34})^2}{8 \times (2 \times 10^{-10})^2 \times 368 \times 10^{-19}}$$

$$= \frac{1098.93 \times 10^{-68}}{11776 \times 10^{-39}}$$

$$m = 0.0933 \times 10^{-29} \text{ kg} = 9.33 \times 10^{-31} \text{ kg}$$

$$\boxed{m = 9.33 \times 10^{-31} \text{ kg}}$$

Solving for energy at $n = 1$

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.33 \times 10^{-31} \times (2 \times 10^{-10})^2} = 0.147 \times 10^{-17} \text{ J}$$

$$E_1 = \frac{0.147 \times 10^{-17}}{1.6 \times 10^{-19}} = 9.187 \text{ eV}$$

For

$$n = 2 \Rightarrow E_2 = 36.75 \text{ eV}$$

For

$$n = 3 \Rightarrow E_3 = 82.683 \text{ eV}$$

For

$$n = 4 \Rightarrow E_4 = 146.992 \text{ eV}$$

For

$$n = 5 \Rightarrow E_5 = 229.675 \text{ eV}$$

$$\therefore E_n < 1 \text{ KeV Or } E_n < 1000 \text{ eV}$$

UNIT-III

Q. 6. (a) Establish the relation between the lattice constant of a cubic crystal and the density of the crystal material. (2.5)

Ans. In a lattice a unit cell has the length of the edges and the angles between the edges are of definite values. Each edge of the unit cell is a distance between the atoms of the same kind and is known as the lattice constant.

Let us consider a cubic lattice constant a . If ρ is the density of the crystal under consideration, then

$$\text{Volume of unit cell} = a^3$$

$$\text{Mass in each unit cell} = a^3 \rho$$

...(1)

If M is the molecular weight and N the Avogadro's number (i.e., the number of molecules per kg mole of the substance), then

$$\text{Mass of each molecule} = \frac{M}{N}$$

Further if n is the number of molecules (lattice points) per unit cell, then

$$\text{the mass in each unit cell} = n \frac{M}{N} \quad \dots(2)$$

From Eqs. (1) and (2)

$$\begin{aligned} a^3 \rho &= \frac{nM}{N} \\ \Rightarrow a &= \left(\frac{nM}{N\rho} \right)^{1/3} \end{aligned} \quad \dots(3)$$

From the relation the values of the lattice constant 'a' can be calculated.

Q. 6. (b) Copper has fcc structure and its atomic radius is 0.1278 nm. If the atomic weight of copper is 63.5. Calculate its density. (2.5)

Ans. Density is given by

$$\rho = \frac{nM}{a^3 N}$$

Given that

$$n = 4; M = 63.5; N = 6.02 \times 10^{23}$$

$$a = \frac{4r}{\sqrt{2}} = \frac{4 \times 0.1278 \times 10^{-9}}{\sqrt{2}}$$

$$\rho = \frac{4 \times 63.5 \times (\sqrt{2})^3}{(4 \times 0.1278 \times 10^{-9})^3 \times 6.02 \times 10^{23}} = 8.98 \frac{\text{gm}}{\text{CC}}$$

$$\boxed{\rho = 8.98 \frac{\text{gm}}{\text{CC}}}$$

Q. 6. (c) What are the different types of point defect in crystal lattice? Show that the number of Schottky defects at a given temperature is proportional to N , the number of atoms present in the crystal. (7.5)

Ans. (a) Point defect: A lattice defect which spreads out very little in all the three dimensions is called a point defect. There are following types :

- (i) Interstitial atom (ii) Vacancies (iii) Impurity atom.

(i) Interstitial atom: This is an atom inserted into the voids (called interstice of the lattice) between the regularly occupied sites. Thus such an atom does not occupy regular lattice sites. This extra atom may be an impurity atom or an atom of the same type as on the regular sites. Fig. 5(a).

(ii) Vacancies: These are the lattice sites from which the atoms are missing [Fig. 5 b.)] Such a vacancy is also called *Schottky defect*.

But if vacancy is created by transferring an atom from a regular lattice site to an interstitial position then it is called *Frenkel defect*.

In this case, therefore, two imperfections are created vacancy as well as an interstitial atom.

(iii) **Impurity atom:** This is a defect in which a foreign atom occupies a regular lattice site (Fig. 5 (c)]

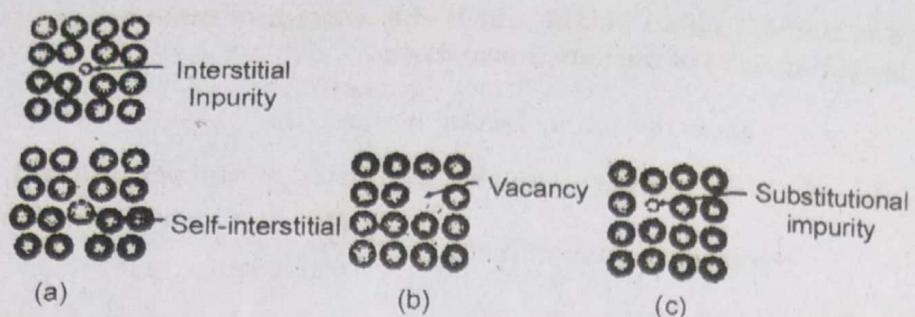


Fig. 5. Three common point defects, (a) interstitial, (b) vacancy, (c) impurity.

A point imperfection is localised near a point or atom in the structure or in other words point defects are lattice errors at isolated lattice points. Point defects are lattice errors at isolated lattice points. Point defects occurs in metallic as well as ionic lattices. These types of points defects are shown in Fig. 5 (d)

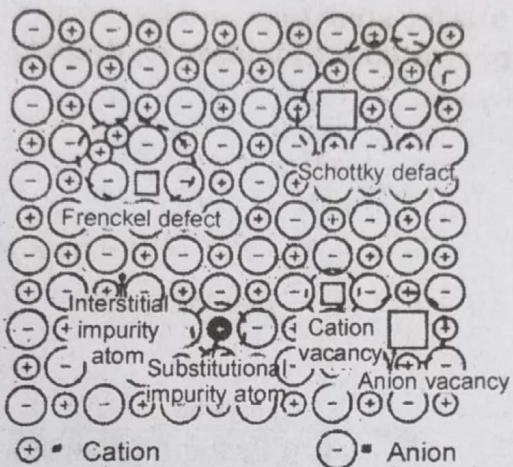


Fig. 5. (d): Illustrating a number of possible point defects in a two-dimensional ionic crystal.

(b) Schottky defect

In real crystals, there are irregularities of the atomic array in which atoms are missing at some lattice points. Such a points is called a *vacancy* (also called Schottky defect). This is showsn in Fig 5. (e).

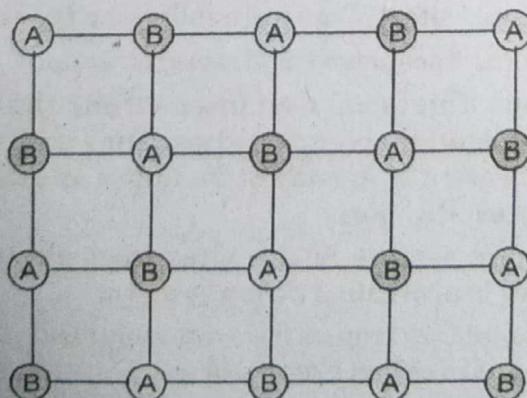


Fig. 5(e) Schottky defect

In spite of care taken in the preparation of crystals, vacancies are always present in all crystals. In fact, as a result of thermal fluctuations, vacancies are produced and destroyed constantly in the crystal. Formally such a defect might be produced by plucking an interior atom out of its regular lattice site and placing it on the surface. This act requires energy. Moreover, the disorder increases resulting in an increase in the entropy. In thermal equilibrium, a certain number of lattice vacancies are always present which can be estimated as follows :

If E_v is the energy required to take an atom from a lattice site inside the crystal to lattice site on the surface, $n E_v$ is the increase in internal energy associated with the production of isolated vacant sites. The total number of ways in which we can pick up n atoms from the crystal consisting N atoms is

$$\omega = \frac{N!}{(N-n)!n!}$$

Since disorder increases due to creation of n vacancies the corresponding increase in entropy is given by

$$S = K_B \ln \omega$$

$$= K_B \ln \frac{N!}{(N-n)!n!}$$

This in turn produces a change in free energy F

$$F = U - TS$$

$$\Rightarrow F = nE_v - K_B T \ln \frac{N!}{(N-n)!n!}$$

The second term on right hand side can be simplified by the use of *Stirling approximation*.

$$\ln x_i = x \ln x - x \quad \dots(5)$$

$$F = nE_v - K_B T [N \ln N - N - (N-n) \ln(N-n) + N - n - n \ln n + n]$$

$$F = nE_v - k_B T [N \ln N - (N-n) \ln(N-n) - n \ln n] \quad \dots(6)$$

Free energy in thermal equilibrium at constant volume must be minimum with respect to changes in n i.e.

$$\left(\frac{\partial F}{\partial n} \right)_T = 0 = E_v - k_B T \ln \frac{N-n}{n} \quad \dots(7)$$

$$0 = E_v + k_B T \ln \frac{n}{N-n}$$

$$n = (N-n) \exp \left(\frac{-E_v}{K_B T} \right)$$

If $n \ll N$, we can neglect n in $(N-n)$ so that

$$n \approx N \exp \left(\frac{-E_v}{K_B T} \right)$$

If

$$E_v = 1 \text{ eV and } T = 1000 \text{ K, then}$$

$$n \approx e^{-12} \approx 10^{-5} / \text{cm}^3$$

The equilibrium concentration of vacancies (decreases as the temperature decreases).

Figure 5(f) shows an alkali halide crystal with f vacancies Fig. 5(f) displays two separate vacancies and a vacancy pair in alkali halide crystal. The pairs are formed because the cation and anion vacancies in alkali halides are electrically charged and they attract each other electrostatically.

Due to formation of the pair a dipole is also produced.

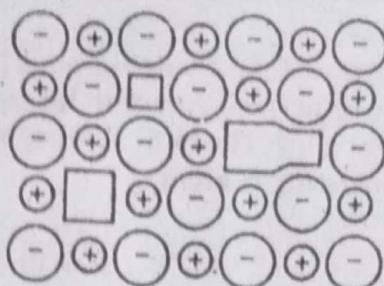


Fig. 5 (f) A plane of a pure alkali halide crystal, showing a vacant positive ion site, a vacant negative ion site, and a coupled pair of vacant sites of opposite sign (vacancies in alkali halide)

In ionic crystals, formation of paired vacancies is most favoured, i.e., in ionic crystals, equal number of positive and negative ion vacancies are produced. The formation of pairs makes it possible to keep the surface of the crystal electrostatically neutral. The number of pairs can be related to the total number of atoms present in the crystal on following the same procedure as adopted above.

The different ways in which n separated pairs can be formed are.

$$\omega = \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(11)$$

So that increase in entropy will be

$$\begin{aligned} S &= k_B \ln \omega \\ &= k_B \ln \left[\frac{N!}{(N-n)!n!} \right]^2 \end{aligned} \quad \dots(12)$$

with corresponding change in free energy

$$\begin{aligned} F &= U - TS \\ \Rightarrow F &= nE_p - K_B T \ln \left[\frac{N!}{(N-n)!n!} \right]^2 \end{aligned} \quad \dots(13)$$

where E_p = Energy of formation of a pair.

Applying Stirling's approximation to simplify the factorial terms i.e.,

$$\begin{aligned} \ln \left[\frac{N!}{(N-n)!n!} \right]^2 &\approx 2[\ln N! - \ln(N-n)! - \ln n!] \\ &2[N \ln N - (N-n) \ln(N-n) - n \ln n] \end{aligned} \quad \dots(14)$$

Putting it in Eq. (13), we get

$$F = nE_p - 2k_B T [N \ln N - (N-n) \ln (N-n) - n \ln n]$$

Differentiating the above equation with respect to n , we get

$$\begin{aligned} \left(\frac{\partial F}{\partial n} \right)_T &= E_p - 2k_B T \ln (N-n) + 1 - \ln n - 1 \\ &= E_p - 2k_B T \ln \frac{(N-n)}{n} \end{aligned} \quad \dots(16)$$

Free energy in thermal equilibrium attained at temperature T is constant, so that

$$E_p - 2K_B T \ln \frac{(N-n)}{n} = 0$$

or $\ln \frac{N-n}{n} = \frac{E_p}{2K_B T}$

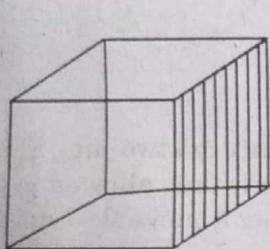
or $\frac{N-n}{n} = \exp \left[\frac{E_p}{2K_B T} \right]$

or $n = N \exp \left(\frac{-E_p}{2K_B T} \right)$

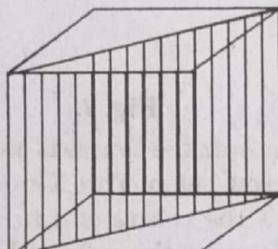
provided $n \ll N$. In NaCl crystal $E_p = 2.02 \text{ eV}$ and at room temperature $n = 10^6 \text{ cm}^{-3}$

Q. 7. (a) Draw (010), (110) and (222) planes in a cubic crystal. (4.5)

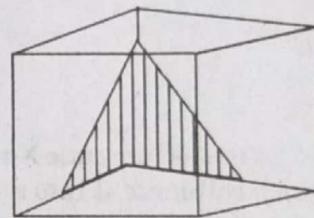
Ans.



(010) Plane
(a)



(110) Plane
(b)



(222) Plane
(c)

Fig. (6)

Q. 7. (b) A certain crystal has lattice constant of 4.24 \AA , 10 \AA and 3.66 \AA on the x , y , z axis respectively. Determine the miller indices of the plane of this crystal having 4.24 \AA , 5 \AA and 1.83 \AA as its x , y , z intercepts respectively. (3)

Ans. Given $a = 4.24 \text{ \AA}$; $b = 10 \text{ \AA}$; $c = 3.66 \text{ \AA}$

Intercepts at 4.24 \AA , 5 \AA and 1.83 \AA respectively

\therefore we can say that intercepts are in the ratio $a : \frac{b}{2} : \frac{c}{2}$

From the law of rational indices

$$a : \frac{b}{2} : \frac{c}{2} = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

where h, k, l are the miller indices.

$$\frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 1 : \frac{1}{2} : \frac{1}{2}$$

$$h : k : l = 1 : 2 : 2$$

Thus

$$h = 1, k = 2, l = 2$$

Hence, the plane is (122)

Q. 7. (c) Describe Bragg's spectrometer and explain with the help of an example how it is used to study the crystal structure?

Ans. Bragg's X-ray Spectrometer

Bragg devised an apparatus used to study the glancing angle θ and the intensities of diffracted X-rays for a given sample. The essential parts of Bragg's spectrometer are shown in Fig. 7.

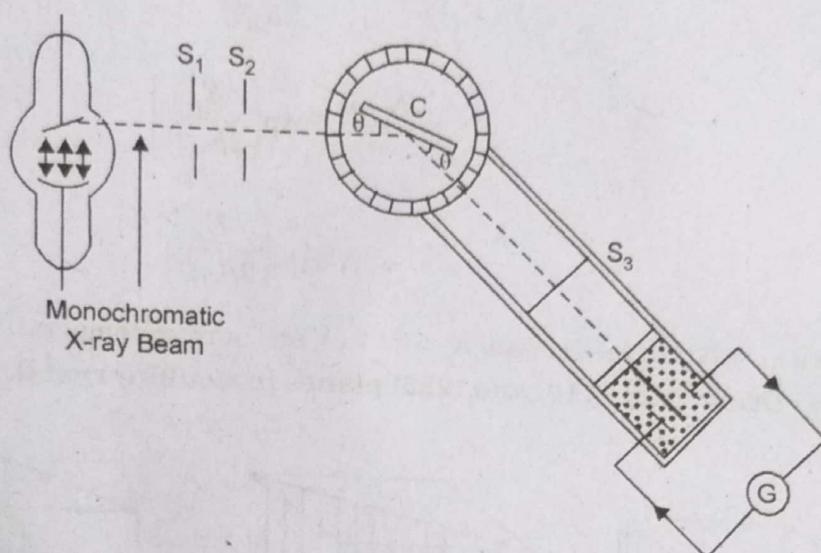


Fig. 7.

A monochromatic X-ray beam from the source is passed through two slits, S_1 and S_2 , which collimate it into a fine narrow beam. This X-ray beam is then allowed to fall on the crystal sample C mounted at the centre of a turn table. A movable arm is also attached to the turn table for detecting the reflected X-ray beam from the crystal C. This turn table is capable of rotation about a vertical axis and the angle of rotation can be measured on the circular scale. The rates of rotation of the turn table and the detector (ionisation chamber) arm are such that the ionisation chamber always receives the diffracted beam. When the turn table rotates through an angle θ , the ionisation chamber arm automatically rotates through an angle 2θ with the direction of incident ray. This way the measurements of different diffracted X-ray beams' intensities and angles are recorded. Then using Bragg's condition, we get the interplanar spacing and the structure of the crystal.

UNIT-IV

Q. 8. (a) Derive an expression for the density of holes in an intrinsic semiconductor.

(6)

Ans. Since a hole signifies a vacancy created by removal of an electron i.e. an empty energy level, the Fermi function for a hole is $1 - f(E)$. Here $f(E)$ represents the probability that the level is occupied by an electron. Along with the expression of $f(E)$, the factor $1 - f(E)$ becomes

$$\begin{aligned}
 1 - f(E) &= 1 - \frac{1}{1 + e^{(E-E_F)/kT}} \\
 &= 1 - [1 + e^{(E-E_F)/kT}]^{-1} \\
 &= 1 - [1 - e^{(E-E_F)/kT}] \quad \text{for } (E - E_F)/kT \ll 1 \\
 &= 1 - 1 + e^{(E-E_F)/kT} \\
 1 - f(E) &= e^{(E-E_F)/kT}
 \end{aligned}$$

For the top of the valence band (the maximum energy), the density of states is given by

$$N(E) = \frac{4\pi}{h^3} (2m_h)^{3/2} (E_V - E)^{1/2}$$

Here m_h is the effective mass of holes near the top of the valence band, where the energy is E_V . With the above relation, the density of holes in the valence band is calculated as

$$\begin{aligned}
 n_h &= \frac{4\pi}{h^3} \int_{-\infty}^{E_V} N(E)[1-f(E)]dE \\
 &= \frac{4\pi}{h^3} (2m_h)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{[E-E_F]kT} dE \\
 &= \frac{4\pi}{h^3} (2m_h)^{3/2} e^{[E-E_F]kT} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{(E-E_V)kT} dE \\
 &= \frac{4\pi}{h^3} (2m_h)^{3/2} e^{[E_V-E_F]kT} \int_{-\infty}^0 x^{1/2} (kT)^{1/2} e^{-x} kT dx
 \end{aligned}$$

where we have substituted $\frac{E_V - E}{kT} = x$ so that $dE = -kT dx$. Now

$$\begin{aligned}
 n_h &= \frac{4\pi}{h^3} (2m_h)^{3/2} (kT)^{3/2} e^{(E_V-E_F)/kT} \int_{-\infty}^0 x^{1/2} e^{-x} dx. \\
 n_h &= 2 \left[\frac{2\pi m_h kT}{h^2} \right]^{3/2} e^{(E_V-E_F)/kT}
 \end{aligned}$$

Q. 8. (b) Show that the Fermi Energy level in an intrinsic semiconductor lies approximately half way between the top of the valence band and the bottom of the conduction band.

Ans. From the law of mass action $n_i^2 = n_e \cdot n_h$

we put the value of n_e and n_h .

$$n_i^2 = N_C N_V \exp\left(\frac{E_F - E_C}{k_B T}\right) \exp\left[\frac{E_V - E_F}{k_B T}\right] \quad \dots(1)$$

or

$$n_i^2 = N_C N_V \exp\left[\frac{-(E_C - E_V)}{k_B T}\right]$$

Since $E_C - E_V = E_g$, the band gap of the semiconducting material, Eq. (1) may be written as

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{k_B T}\right) \quad \dots(2)$$

Substituting the value of N_C and N_V in Eq. (2) we get

$$\begin{aligned} n_i^2 &= 2 \left[\frac{2\pi m_e^* k_B T}{h^2} \right]^{3/2} \times 2 \left[\frac{2\pi m_h^* k_B T}{h^2} \right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \\ &= 4 \left[\frac{2\pi k_B T}{h^2} \right]^3 \times [m_e^* m_h^*]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \end{aligned} \quad \dots(3)$$

Multiplying the numerator and denominator of Eq. (3) by m^3 , we get

$$n_i^2 = 4 \left[\frac{2\pi k_B m T}{h^2} \right]^3 \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \quad \dots(4)$$

The value of n_i can be written as

$$\begin{aligned} n_i &= 2 \left[\frac{2\pi k_B m T}{h^2} \right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4} \exp\left(\frac{-E_g}{2k_B T}\right) \\ &= 2 \left[\frac{2\pi k_B m T}{h^2} \right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4} T^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right) \end{aligned} \quad \dots(5)$$

Equation (5) may also be written as

$$n_i = A T^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right) \quad \dots(6)$$

where

$$A = 2 \left[\frac{2\pi k_B m}{h^2} \right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4} = 4.82 \times 10^{21} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4}$$

Equation (5) gives the intrinsic carrier concentration in semiconductor.

Value of Fermi energy-

In intrinsic semiconductor $n_e = n_h$

$$\begin{aligned} \therefore 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/2} \times e^{(E_F - E_C)k_B T} &= 2 \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/2} \times e^{(E_V - E_F)k_B T} \\ &\Rightarrow (m_e^*)^{3/2} \times e^{(E_F - E_C)k_B T} = (m_h^*)^{3/2} \times e^{(E_V - E_F)k_B T} \end{aligned}$$

$$\begin{aligned}
 \Rightarrow e^{(2E_F - E_C - E_V)/k_B T} &= \left(\frac{m_h^*}{m_e^*}\right)^{3/2} \\
 \Rightarrow \left\{\frac{2E_F - E_C - E_V}{k_B T}\right\} &= \frac{3}{2} \log\left(\frac{m_h^*}{m_e^*}\right) \\
 \Rightarrow E_F &= \frac{E_C + E_V}{2} + \frac{3}{4} k_B T \log\left(\frac{m_h^*}{m_e^*}\right) \\
 &\quad \left[\text{for } m_e^* = m_h^*, \log \frac{m_h^*}{m_e^*} = \log 1 = 0 \right] \\
 \Rightarrow E_F &= \frac{E_C + E_V}{2}
 \end{aligned} \tag{7}$$

So Fermi level lies exactly in the middle of the top valence band and the bottom of the conduction band.

Q. 8. (c) Hall voltage of 0.001 mV is found to be developed when a sample carrying a current of 100 mA is placed in a transverse magnetic field of 2T. Calculate the charge carrier concentration of the sample, given the thickness of the sample along the direction of magnetic field is 0.05 mm. (2.5)

Ans. Hall coefficient

$$R_H = \frac{V_H d}{I_x B}$$

Also,

$$R_H = \frac{1}{ne}$$

$$n = \frac{I_x B}{V_H d e}$$

Given that: $V_H = 0.001 \times 10^{-3}$ V ; $d = 0.05 \times 10^{-3}$ m ; $e = 1.6 \times 10^{-19}$; $I_x = 100 \times 10^{-3}$ Ampere.

$$B = 2T$$

$$\begin{aligned}
 n &= \frac{100 \times 10^{-3} \times 2}{(0.001 \times 10^{-3}) \times (0.05 \times 10^{-3}) \times 1.6 \times 10^{-19}} \\
 &= 25 \times 10^{27} \text{ m}^{-3}
 \end{aligned}$$

$$n = 2.5 \times 10^{28} \text{ m}^{-3}$$

Q. 9. (a) What is the effect of periodic potential of the energy of electrons in a metal? Explain it on the basis of Kronig-Penny model and also explain the formation of energy bands. (8.5)

Ans. Free electron model ignore the effects those arise when the electrons interact with crystal lattice. However, now we consider this by making some general remarks about the effect of the periodic variation in the potential. Due to this periodicity in

potential, for an infinitely long lattice, the wave function does not remain sinusoidal travelling waves of constant amplitude but now they include the lattice periodicity in their amplitudes, and electrons may be scattered by the lattice.

In order to find the allowed energies of electrons in solids, we consider the effect of formation of a solid when the individual constituent atoms are brought. We solve the Schrödinger equation for periodic potential seen by an electron in a crystal lattice. We also consider that the periodic potential is a succession of rectangular wells and barriers. The solution of Schrödinger equation is a sinusoidal wave in certain energy ranges i.e. allowed states, and real decaying exponential wave in the other range i.e. the forbidden bands. For this purpose, here we present only qualitative approach.

It is found that the potential is not constant but varies periodically. The effect of periodicity is to change the free particle travelling wave eigen function. Therefore, the travelling wave eigen function has a varying amplitude which changes with the period of the lattice. If we consider that the space periodicity as shown in fig.8 then according to Bloch, the eigen function for non-dimensional system has this form

As is clear, this is different from the free travelling wave function $\psi(x) = Ae^{ikx}$. $u_k(x)$ is the periodic function with the periodicity a of the periodic potential i.e.

$$u_k(x) = u_k(x+a) \quad \dots(i)$$

In general,

$$u_k(x) = u_k(x+na) \quad \dots(ii)$$

Where n is an integer. Hence, with the effect of periodicity the complete wave function is

$$\psi(x, t) = u_k(x)e^{i(kx - wt)} \quad \dots(iii)$$

The exact form of the function $u_k(x)$ depends on the particular potential assumed and the value of k .

In 1930, Kronig and Penney, proposed a one-dimensional model for the shape of rectangular potential well and barriers having the lattice periodicity, as shown in fig. 8. Each well represents an approximation to the potential produced by one ion. In the region such as $0 < x < a$, the potential energy is assumed to be zero while, in the region $-b < x < 0$ or $a < x < (a+b)$, the potential energy is taken as V_0 . The relevant Schrödinger equations for these two regions are

$$\frac{d^2\psi}{dx^2} + \left[\frac{2m}{h^2} \right] E \psi = 0 [0 < x < a] \quad (iv)$$

$$\frac{d^2\psi}{dx^2} + \left[\frac{2m}{h^2} \right] (E - V_0) \psi = 0 [-b < x < 0] \quad (v)$$

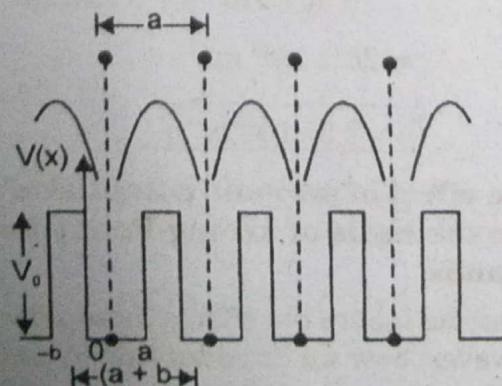


Fig. 8

The electron of not too high energy is practically bound within one of the wells that are deep and widely spaced. So the lower energy eigen value are those of a single well. However, for the wells those are closer together the eigen function can penetrate the potential barriers more easily. Because of this, spreading of previously single energy level into a band energy levels takes place. The band becomes wider with the decrease in the separation of the wells. Under the limit of zero barrier thickness, we obtain an infinitely wide single well in which all energies are allowed. So the present case is reduced to the free electron model. The comparison between the allowed energies of a single well and an array of wells (Kronig-penney model) is shown in the Fig. 9. In this figure, we have assumed $b = a/16$ and the well strength as $2mV_0a^2/h^2$. It is clear from the figure that each band corresponds to a single energy level of the single well. The forbidden bands appear even for energies $E > V_0$.

Here we will solve the Schrodinger wave equation for electron for Kronig-Penny potential under the condition that ψ and $\frac{d\psi}{dx}$ are continuous at the boundaries of the well. A complicated expression for the allowed energies in terms of k shows that gaps in energy are obtained at values such that

$$k = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}, \dots$$

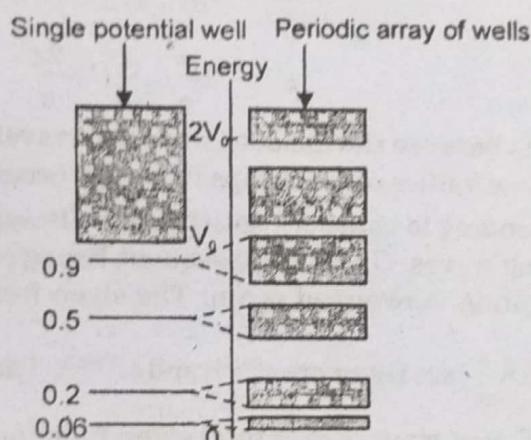


Fig. 9

The solution of the Schrödinger wave equation for free-electrons results in the energy value given by

$$E = \frac{h^2 k^2}{8\pi^2 m} = \frac{h^2 k^2}{2m} \quad \dots(vi)$$

Energy E Versus wave Number k Diagram

From Eq. (v), it is clear the relation between E and k is parabolic. The parabolic relation between E and k , valid in case of free electrons, is therefore, interrupted at different values of k , as shown in fig. 10. It means the energies corresponding to the values of k given by Eq. (iv) are not permitted for electrons in the crystal. Thus, the energies of electrons are divided into forbidden and allowed bands (Fig. 9).

The occurrence of the gaps can be understood on the basis of Bragg's condition for the diffraction, given as

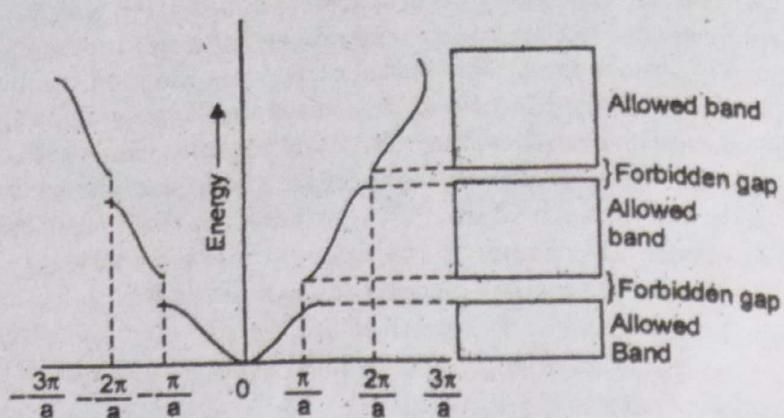


Fig. 10

$$2a \sin \theta = n\lambda \quad (\text{where, } n = 1, 2, 3, \dots)$$

Where a is spacing between the ions of the lattice and θ is the angle of incidence.

$$2a = n\lambda \quad (\text{for } \theta = 90^\circ)$$

or

$$2a = n \frac{2\pi}{k}$$

or

$$k = \frac{n\pi}{a}$$

or

$$k = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}, \dots \quad \dots(\text{vii})$$

We have put \pm signs because the incident wave can travel along $+x$ -axis as well as along $-x$ -axis. At all these values of k the gaps in energy occur, as shown in fig. 10.

The waves corresponding to value of k satisfying the Bragg's condition are reflected and resulted in standing waves. On each subsequent Bragg reflection, the direction in which the wave is travelling is reversed again. The eigen function and corresponding

reflected waves for $k = \pm n \frac{\pi}{a}$ are therefore $e^{i(\pi/a)x}$ and $e^{-i(\pi/a)x}$. These two eigen function can

be combined in two different ways to give total eigen function

$$\omega_1 = e^{i(\pi/a)x} + e^{-i(\pi/a)x} = \cos(\pi/a)x$$

$$\omega_2 = e^{i(\pi/a)x} - e^{-i(\pi/a)x} = \sin(\pi/a)x$$

or

$$\psi_1 \propto \cos \frac{\pi}{a} x \quad \dots(\text{viii})$$

and

$$\psi_2 \propto \sin \frac{\pi}{a} x$$

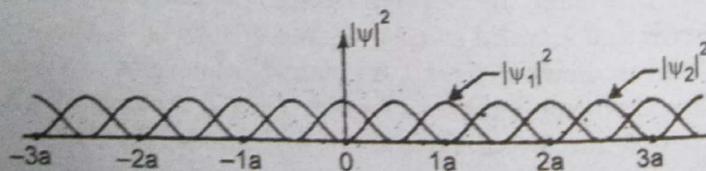


Fig. 11

Hence, the two standing waves are obtained. The probability density curve for these two stationary waves, i.e. $|\psi_1|^2$ and $|\psi_2|^2$, are shown in fig. 11. From this figure and Eq. (viii) it is clear that the value of $|\psi_1|^2$, is maximum in between the positive of positive ions. From fig. 8 it is evident that the potential energy of an electron is maximum between the ions and minimum at the positions of the ions. So an electron can have two different value of energies, i.e. E_1 and E_2 for $k = \frac{\pi}{a}$ corresponding to the standing waves ψ_1 and ψ_2

Hence, no electron can have any energy between E_1 and E_2 . This phenomenon create a difference in energy ($E_1 - E_2$) which is known as energy gap.

Q. 9. (b) Show that the Hall coefficient is independent of the applied magnetic field and is inversely proportional to the current density and electronic charge. (4)

Ans. Consider a specimen in the form of a rectangular cross-section carrying current I_x in the x direction. A Uniform magnetic field B_z is applied along z -axis as shown in Fig. 12.

A moving charge in transverse magnetic field experiences a force normal to both current and magnetic field. Thus moving charges slightly shift towards y direction

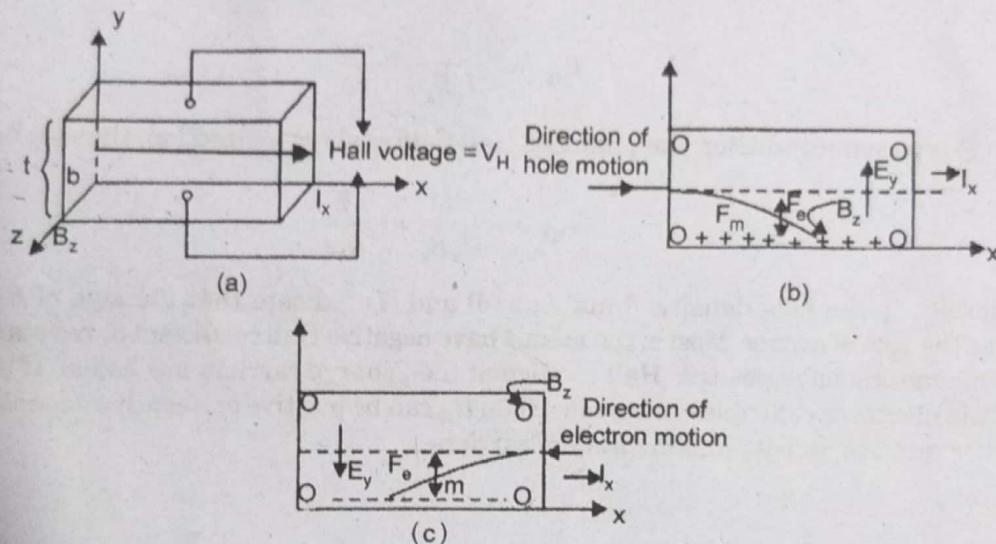


Fig. 12

Fig. 12. (a) Illustration of Hall effect 12 (b) motion of hole in p-type semiconductor specimen (c) motion of electron of N-type semiconductor or metal specimen of Hall experiment.

Force $ev_x B_z$ [Fig. 12. (a) and (c)] This shift of charges produces a potential difference or an electric field E_y along y direction. The produced electric field also applies a force $e E_y$ on charge. When both the forces become equal, the equilibrium occurs, i.e., upto this condition charges shift along y direction.

In equilibrium,

$$\begin{aligned} F_e &= F_m \\ eE_y &= ev_x B_z \\ E_y &= v_x B_z \end{aligned} \quad \dots(1)$$

If J_x is the current density in x direction, then

$$J_x = \frac{I_x}{A} = nev_x \quad \dots(2)$$

Here n is density of charge carrier (electron or hole-or both) and A (=breadth x thickness) is the cross-sectional area of specimen perpendicular to direction of J_x .

From Eqs. (1) and (2), we can write

$$E_y = \frac{B_z J_x}{ne} \quad \dots(3)$$

This is the expression for Hallfield. The Hall effect is described by means of Hall coefficient R_H , defined in terms of J_x by the following relation :

$$R_H = \frac{E_y}{J_x B_z} \quad \dots(4)$$

From Eqs. (3) and (4) we have

$$R_H = \frac{1}{ne} \quad \dots(5)$$

For the conductor or N-type semiconductor, the electric field is developed in negative y direction. Hence from Eqs. (4) and (5), we can write

$$R_H = \frac{E_y}{J_x B_z} = -\frac{1}{ne} \quad \dots(6)$$

In P-type semiconductor, the produced Hall field is along y direction, thus we have

$$R_H = \frac{E_y}{J_x B_z} = -\frac{1}{n_h e} \quad \dots(7)$$

Here n_h is the hole density. Equations (6) and (7) indicate that the sign of R_H is same as the sign of carrier. Most of the metals have negative Hall coefficient or resistance, but some metals have positive Hall coefficient (i.e., charge carriers are holes). If both holes and electrons contribute to current, then R_H can be positive or negative depending upon the relative density and mobility of carriers.