Picard's Method :-

$$\boxed{y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\, dx}$$

Solve $\frac{dy}{dx} = x + y^2$, it is given that $y = 1$ when $x = 0$

Sol^n We know

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y)\, dx$$

$$y_1 = y_0 + \int_{x_0}^{x} (x + y^2)\, dx.$$

Here $y_0 = 1$ and $x_0 = 0$, So we get $f(x, y_0) = x + 1$

$$y_1 = 1 + \int_{x_0 = 0}^{x} (x+1)\, dx.$$

$$= 1 + \left[\frac{x^2}{2} + x\right]_0^x = 1 + \frac{x^2}{2} + x.$$

$$y_2 = y_0 + \int_0^x x + \left(1 + \frac{x^2}{2} + x\right)^2 dx.$$

$$y_2 = 1 + x + \frac{3}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{20}x^5$$

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Ex find the value of y, when $x = 0.25, 0.5, 1.0$

and $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial condition

$y=0$ when $x=0$

Sol^n

$$f(x,y) = \frac{x^2}{y^2+1} \quad ; \quad x_0 = 0,\ y_0 = 0$$

so $f(x, y_0) = x^2$

$$y_1 = y_0 + \int_{x_0}^{x} (x, y_0)\, dx = 0 + \int_0^x x^2 dx = \frac{x^3}{3}$$

$$y_2 = y_0 + \int_0^x f(x, y_1)\, dx$$

$$= 0 + \int_0^x \frac{x^2}{\left(\frac{x^3}{3}\right)^2 + 1} dx$$

$$= \tan^{-1}\left(\frac{x^3}{3}\right)$$

$$y_2 = \frac{1}{3}x^3 - \frac{1}{81}x^9 + \cdots$$

$$y(0.25) = \frac{1}{3}(0.25)^3 - \frac{1}{81}(0.25)^9$$

$$\simeq 0.0520833$$

$$y(0.5) = \frac{1}{3}(0.05)^3 - \frac{1}{81}(0.5)^9 = 0.0416666$$

$$y(1.0) = \frac{1}{3}(1.0)^3 - \frac{1}{81}(1.0)^9 \simeq 0.321$$

Ex Solve $\frac{dy}{dx} = 2x+z$, $\frac{dz}{dx} = 3xy + x^2z$

given $y=2$, $z=0$ when $x=0$

Solⁿ

$y_0 = 2$ $\quad z_0 = 0$ and $x_0 = 0$

$$y_1 = y_0 + \int_{x_0}^{x} (2x + z_0)\, dx$$

$$= 2 + \int_0^x 2x\, dx = 2 + x^2$$

$$\boxed{y_1 = 2 + x^2}$$

$$z_1 = z_0 + \int_{x_0}^{x} (3xy_0 + x^2 z_0)\, dx$$

$$= 0 + \int_0^x (3x \times 2 + x^2 \times 0)\, dx$$

$$\boxed{z_1 = 3x^2}$$

$$y_2 = y_0 + \int_0^x (2x + z_1)\, dx$$

$$= 2 + \int_0^x (2x + 3x^2)\, dx$$

$$\boxed{y_2 = 2 + x^2 + x^3}$$

$$z_2 = z_0 + \int_0^x (3xy_1 + x^2 z_1)\, dx$$

$$= \int_0^x [3x(2+x^2) + x^2 \cdot 3x^2]\, dx$$

$$\boxed{z_2 = 3x^2 + \frac{3}{4}x^4 + \frac{3}{5}x^5}$$

we get

$$y_3 = 2 + x^2 + x^3 + \frac{3x^5}{20} + \frac{x^6}{10}$$

$$z_3 = 3x^2 + \frac{3x^4}{4} + \frac{3}{5}x^5 + \frac{3}{28}x^4 + \frac{3}{40}x^8$$

se Picard Method to approximate the value when $x = 0.1, 0.2, 0.3, 0.4$ and 0.5 given that $y = 1$ at $x = 0$ and $y' = 1 + xy$ correct to three decimal places.

Use Picard's Method to approximate the value of y when $x = 0.1$ given that $y = 1$ when $x = 0$ and $\frac{dy}{dx} = \frac{y - x}{y + x}$

Approximate y and z at $x = 0.1$ using Picard's method for the solution to the equation $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y + z)$, given that $y(0) = 1$ and $z(0) = \frac{1}{2}$.

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Ex Use Picard Method to approximate the value y when $x = 0.1, 0.2, 0.3, 0.4$ and 0.5 given that $y = 1$ at $x = 0$ and $y = 1 + xy$ correct to three decimal places.

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