Hypothesis Testing

Statistical Hypothesis-

A statistical hypothesis is a statement about the parameters of one or more populations.

Null and Alternative Hypothesis-

The null hypothesis, denoted by H_0, is the claim that is initially assumed to be true, and is usually denoted by H_0.

The alternative hypothesis, denoted by H_a, is the assertion that is contradictory to H_0.

For example-

The population has a mean μ_0

Then, Null hypothesis $H_0 : \mu = \mu_0$

$$H_a : \mu \neq \mu_0 \qquad -(1)$$

or

$$H_a : \mu > \mu_0 \qquad -(2)$$

or

$$H_a : \mu < \mu_0 \qquad -(3)$$

The alternative hypothesis in (1) is known as a two-tailed alternative and (ii) & (3) are known as right-tailed and left-tailed alternative respectively.

the null hypothesis is accepted. i.e, if the observed test Statistic is in the confidence interval then we accept the null hypothesis and reject the alternative Hypothesis.

Critical Region -

A critical region, also known as the rejection region, is a set of values for the test statistic for which the null hypothesis is rejected.

Critical Values -

The critical value at a certain significance level can be thought of as a cut-off point. If ~~at~~ a test statistic on one side of the critical value results in accepting the null hypothesis, a test statistic on the other side will result in rejecting the null hypothesis.

Type I Error–

Rejecting the null hypothesis H_0 when it is true is defined as type-I error.

Type II Error–

Failing to reject the null hypothesis when it is false is defined as a type II error.

		Decision From Sample	
		Reject H_0	Accept H_0
True State	H_0 True	Wrong (Type I Error)	Correct
	H_0 False (H_1 True)	Correct	Wrong (Type II Error)

Level of Significance–

The probability of type-I error is called the significance level or α-error.

$$\alpha = P(\text{type-I error}) = P(\text{Reject } H_0 \text{ when } H_0 \text{ is true})$$

Confidence Interval–

A confidence interval, also known as the acceptance region, is a set of values for the test statistic for which the

One-tailed Test-

A test of any statistical hypothesis where the alternative hypothesis is one tailed (right-tailed or left-tailed) is called a one-tailed test.

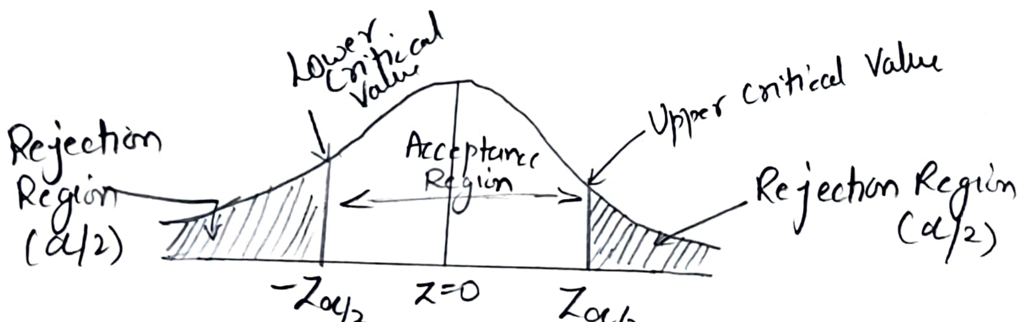
For example- A test for testing the mean of a population $H_0: \mu = \mu_0$ against the alternative hypothesis:

$H_1: \mu > \mu_0$ (Right-tailed) or $H_1: \mu < \mu_0$ (Left-tailed)

is a ~~signis~~ single tailed test.

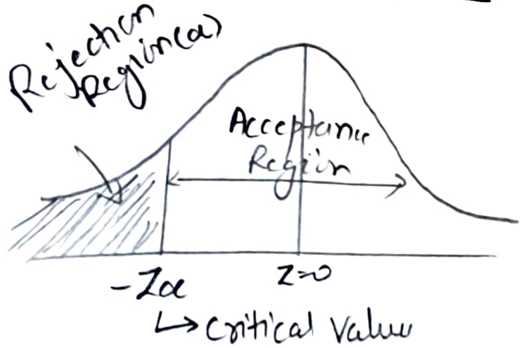
Two-tailed Test-

A test of statistical hypothesis where the alternative hypothesis is two-tailed such as $H_0: \mu = \mu_0$ against the alternative hypothesis $H_1: \mu \neq \mu_0$ ($\mu > \mu_0$ and $\mu < \mu_0$) is known as two tailed test.

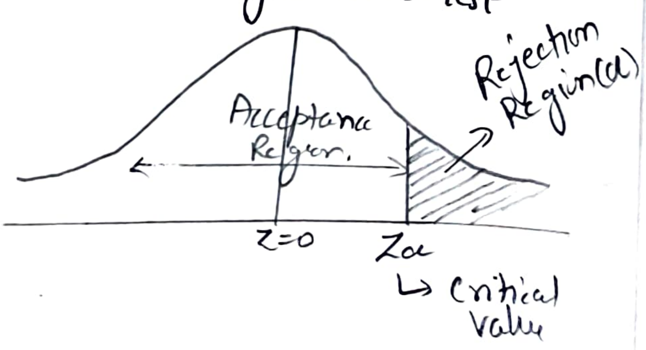


Two-tailed Test

Left-Tailed Test



Right-Tailed Test



→ Critical Value of Z at commonly used level of significance for both two-tailed and ~~sign~~ single-tailed test.

Critical Value (Z_α)	Level of Significance (α) 1%	5%	10%
Two-tailed Test	2.58	1.96	1.645
Right-tailed Test	2.33	1.645	1.28
Left-tailed Test	-2.33	-1.645	-1.28

Single Sample : Test a single Mean with known Variance -

Q A random sample of 100 recorded deaths in the united State during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seems to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance

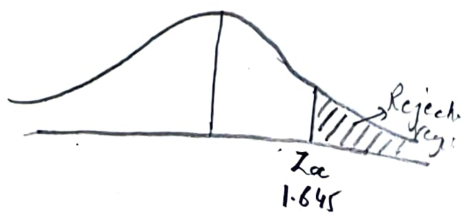
Soln

Null Hypothesis $H_0 : \mu = 70$ years

Alternative Hypothesis $H_1 : \mu > 70$ years

Test statistics :

$$Z_{cal} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$



$$Z_{cal} = \frac{71.8 - 70}{8.9/\sqrt{100}} = 2.02$$

Since $Z_{cal} > Z_\alpha$. Reject H_0 and conclude that the mean life span today is greater than 70 years

Q A manufacturer of sports equipment has developed a new Synthetic fishing line that he claims has a mean breaking Strength of 8 kilograms with a standard deviation of 0.5 kg. Test the hypothesis that $\mu = 8$ kilograms against the alternative that $\mu \neq 8$ kilogram if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 Kilograms Use a 0.01 level of significance.

Soln

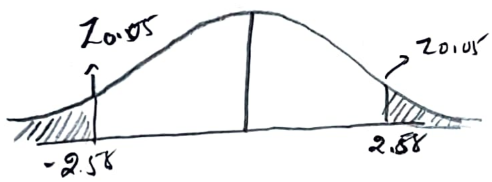
Null Hypothesis $H_0 : \mu = 8$ kilograms.

Alternative Hypothesis $H_1 : \mu \neq 8$ kilograms.

Test Statistics $$Z_{cal} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

$$= \frac{7.8 - 8}{0.5/\sqrt{50}}$$

$$= -2.83$$



Since Z_{cal} $-2.83 < -2.58$ i.e $Z_{cal} < Z_{\alpha/2}$. Reject the H_0 and Conclude that the average breaking strength is not equal to 8 but is, in fact less than 8 kilograms.

Single Sample: Test on a mean with Variance Unknown -

Q The Edison Electric Institute has published figures on the annual number of kilowatt hours expended by various home appliances. It is claimed that a vaccum cleaner expends on average of 46 kilowatt hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners expend an average of 42 kilowatt hours per year with a standard deviation deviation of 11.9 kilowatt hours, does this suggest at the 0.05 level of significance that vacuum cleaners expend, on the average, less than 46 kilowatt hours annually? Assume the population of kilowatt hours to be normal.

Soln Null Hypothesis H_0: $\mu = 46$ kilowatt hours

Alternative Hypothesis H_1: $\mu < 46$ kilowatt hours

Test Statistics : $t = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}}$ with 11 degree of freedom.

$$t_{cal} = \frac{42-46}{11.9/\sqrt{12}}$$

$$t_{cal} = -1.16$$

The tabular value of $t_{\alpha, n-1}$ i.e $t_{0.05, 11} = 1.796$

Note :-

If $|t_{cal}| \ngtr t_{\alpha}$ (tabulated value), null hypothesis is rejected and if $|t_{cal}| < t_{\alpha}$, H_0 may be accepted at the level of significance.

Since $|t_{cal}| < t_{0.05, 11}$, Then H_0 is accepted and conclude that the average number of kilowatt hours expended annually by home vacuum cleaner is not significantly less than 46.

One Sample Tests on the Variance –

Q. A manufacturer of car batteries claims that the life of his batteries is approximately normally distributed with a standard deviation equal to 0.9 years. If a random sample of 10 of these batteries has a standard deviation of 1.2 years, do you think that $\sigma > 0.9$ years? Use a 0.05 level of significance.

Soln Null Hypothesis : $H_0 : \sigma^2 = 0.81$

Alternative Hypothesis $H_1 : \sigma^2 > 0.81$

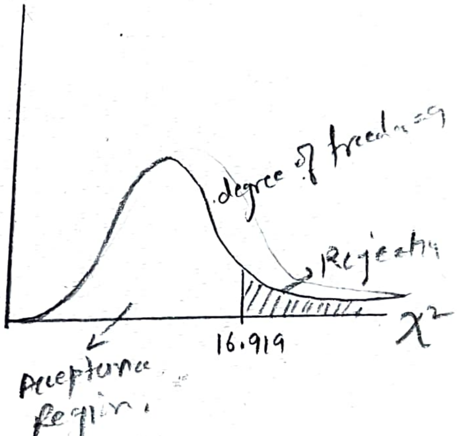
Test statistic :

$$\chi^2_{cal} = \frac{(n-1)s^2}{\sigma_0^2}$$

$$= \frac{(10-1)(1.2)^2}{(0.9)^2}$$

$$= \frac{9 \times 1.44}{0.81}$$

$$\chi^2_{cal} = 16.0$$



Since $\chi^2_{cal} = 16.0 < \chi^2_{0.05,9}$ (16.919). We conclude that χ_0^2 is not significant at the 0.05 level. So, we will ~~reject~~ accept the null Hypothesis H_0.

Note :–

Null Hypothesis $H_0 : \sigma^2 = \sigma_0^2$

Test statistics : $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

Alternative Hypothesis	Rejection Region
$H_1 : \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2, n-1}$ or $\chi_0^2 < -\chi^2_{\alpha/2, n-1}$
$H_1 : \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha, n-1}$
$H_1 : \sigma^2 < \sigma_0^2$	$\chi_0^2 < -\chi^2_{\alpha, n-1}$

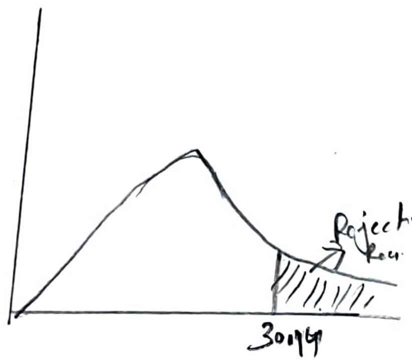
Q An automated filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $s^2 = 0.0153$ (fluid ounces)^2. If the variance of fill volume exceeds 0.01 (fluid ounces)^2, an unacceptable proportion of bottles will be underfilled or overfilled. Is there evidence in the sample data to suggest that the manufacturer has a problem with underfilled bottles? Use $\alpha = 0.05$ and assume that fill volume has a normal distribution.

Soln

Null Hypothesis $H_0: \sigma^2 = 0.01$

Alternative Hypothesis $H_1: \sigma^2 > 0.01$

Test statistic :



$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$= \frac{(20-1)(0.0153)}{0.01}$$

$$= 29.07$$

Conclusion: Since $\chi_0^2 = 29.07 < \chi^2_{0.05,19} = 30.14$, we conclude that there is no strong evidence that the variance of fill volume exceeds 0.01 (fluid ounces)^2. So, there is no strong evidence of a problem with incorrectly filled bottles.

One Sample : Test on a Single Proportion..

Test of hypothesis Concerning proportions are required in many areas. The ~~probability~~ politician is certainly interested in knowing what fraction of the voters will favor him in the next election. All manufacturing firms are concerned about the proportion of defective items when a shipment is made.

Q A builder claims that heat pumps are installed in 70% of all homes being constructed today in the city Richmond, Virginia. Would you agree with this claim if a random survey of new homes in the city shows that 8 out of 15 had heat pumps installed? Use a 0.10 level of significance

Soln

Null Hypothesis $H_0 : p = 0.7$

Alternative Hypothesis $H_1 : p \neq 0.7$

Test statistic :

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$$

$$= \frac{+0.53 - 0.7}{\sqrt{(0.3 \times 0.7)/15}}$$

$$= \frac{-0.17}{\sqrt{0.21/15}}$$

$$= \frac{-0.17}{\sqrt{0.014}}$$

$$= -1.44$$

Since $|Z_{cal}| < Z_{tab}$ (1.96). We do not reject H_0. Conclude that there is not sufficient reason to doubt the builders claim.

Q A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance.

Soln

Null Hypothesis $H_0 : p = 0.6$

Alternative Hypothesis $H_1 : p > 0.6$

Test statistic

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$$

$$= \frac{0.7 - 0.6}{\sqrt{(0.6)(0.4)/100}}$$

$$= \frac{0.1}{\sqrt{0.0024}}$$

$$= \frac{0.1}{0.0489}$$

1.645

$$= 2.04$$

Since $Z_{cal} > Z_{\alpha}$ (1.645). Reject H_0 and conclude that the new drug is superior.

Two Samples: Test on two means with known Variance

Q A product developer is interested in reducing the drying time of a primer paint. Two formulations of the paint are tested: formulation 1 is the standard chemistry, and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another 10 specimens are painted in random order. The two sample average drying times are $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes, respectively. What conclusions can the product developer draw about the effectiveness of the new ingrediant, using $\alpha = 0.05$?

Soln Null Hypothesis $H_0: \mu_1 - \mu_2 = 0$ or $H_0 = \mu_1 = \mu_2$

Alternative Hypothesis $H_1: \mu_1 > \mu_2$ (we reject H_0 if the new ingredient reduces mean drying time)

Test Statistic -
$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where, $\sigma_1^2 = \sigma_2^2 = (8)^2 = 64$ and $n_1 = n_2 = 10$

Since $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes

$$z = \frac{(121 - 112) - (0)}{\sqrt{\frac{(8)^2}{10} + \frac{(8)^2}{10}}}$$

$$= \frac{9}{\sqrt{6.4 + 6.4}}$$

$$= \frac{9}{\sqrt{12.8}}$$

$$= \cancel{\frac{9}{\sqrt{3.578}}} \; \frac{9}{3.578}$$

$$= 2.51$$

Conclusion: Since $Z = 2.51 > Z_\alpha (1.645)$. We reject H_0. It Conclude that the adding the new ingredient to the ~~painter~~ paint significantly reduces the drying time.

Two Sample : Test on Two Means with Unknown but Equal Variance

Two-Sample T-Test

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where, $d_0 = \mu_1 - \mu_2$, $\quad s_p = \frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}$

Q) An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve piece of material 1 were tested by exposing each piece to a machine measuring wear. Ten pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The sample of material 1 gave an average (coded) wear of 85 units with a sample standard deviation of 4, while the samples of material 2 gave an average of 81 and a sample standard deviation of 5. Can we conclude at the 0.05 level of significance that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units? Assume the population to be approximately normal with equal variances.

Soln Let μ_1 and μ_2 represent the population means of the abrasive wear for material 1 and material 2, respectively.

Null Hypothesis $H_0 : \mu_1 - \mu_2 = 2$

Alternative Hypothesis $H_1 : \mu_1 - \mu_2 > 2$

Computation: $\bar{x}_1 = 85$, $s_1 = 4$, $n_1 = 12$

$\bar{x}_2 = 81$, $s_2 = 5$, $n_2 = 10$

$$s_p^2 = \frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}$$

$$s_p = \sqrt{\frac{(16 \times 11) + (25 \times 9)}{12+10-2}}$$

$$s_p = 4.478$$

Test statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{(85-81) - 2}{4.478\sqrt{\frac{1}{12} + \frac{1}{10}}}$$

$$= 1.04$$

Conclusion: Since $t \doteq 1.04 < t_{0.05, 20}$ (1.725). So we will no reject the H_0. We are unable ~~#~~ to conclude that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units.

Two Sample : Tests on Two Means with Unknown but Unequal Variances

test statistic -

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

where degree of freedom

$$V = \frac{(S_1^2/n_1 + S_2^2/n_2)^2}{\frac{(S_1^2/n_1)^2}{(n_1-1)} + \frac{(S_2^2/n_2^2)}{n_2-1}}$$

Q Arsenic concentration in public drinking water supplies is a potential health risk. An article in the Arizona Republic reported drinking water arsenic concentration in parts per billion for 10 metropolitan Phoenix communities and 10 communities in rural Arizona. The $\bar{x}_1 = 12.5$, $s_1 = 7.63$ (for Phoneix) and $\bar{x}_2 = 27.5$, $S_2 = 15.3$ (for Rural Arizona) We wish to determine if there is any difference in mean arsenic concentration between metropolitan Phoenix communities and communities in rural Arizona. Use 0.05 level of significance.

Soln Null Hypothesis $H_0 : \mu_1 - \mu_2 = 0$

Alternative Hypthesis $H_1 : \mu_1 \neq \mu_2$

Test statistic -

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$= \frac{(12.5 - 27.8) - 0}{\sqrt{\frac{(7.63)^2}{10} + \frac{(15.3)^2}{10}}}$$

$$= -2.77$$

The degree of freedom

$$v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2/n_1)^2}{n_1 - 1} + \frac{(S_2^2/n_2)^2}{n_2 - 1}}$$

$$= \frac{\left[\frac{(7.63)^2}{10} + \frac{(15.3)^2}{10}\right]^2}{\frac{[(7.63)^2/10]^2}{9} + \frac{[(15.3)^2/10]^2}{9}}$$

$$= 13.2$$

$$\simeq 13$$

Therefore, using $\alpha = 0.05$ and a fixed significance level test we would reject $H_0: \mu_1 = \mu_2$ if $t > t_{0.025, 13} = 2.160$ or if $t_0 < -t_{0.025, 13} = -2.160$

Conclusion :- Since $t \simeq -2.77 < t_{0.025, 13} = -2.160$ we reject the null hypothesis

Two Sample Tests On the Variance -

Null Hypothesis : $H_0 \quad \sigma_1^2 = \sigma_2^2$

Test Statistic -

$$F_0 = \frac{S_1^2}{S_2^2}$$

Q Oxide layers on semiconductor wafers are etched in a mixture of gases to achieve the proper thickness. The variability in the thickness of these oxides layer is a critical characteristic of the wafer, and low variability is desirable for subsequent processing steps. Two different mixtures of gases are being studied to determine whether one is superior in reducing the variability of the oxide thickness. Sixteen wafers are etched in each gas. The sample standard deviation of oxide thickness are $S_1 = 1.96$ angstroms and $S_2 = 2.13$ angstroms, respectively. Is there any evidence to indicate that either gas is preferable? Use a fixed-level test with $\alpha = 0.05$.

Soln

Null Hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative Hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$

Test statistic

$$f_0 = \frac{S_1^2}{S_2^2} = \frac{3.84}{4.54} = 0.85$$

Conclusion! Since $f_0 = 0.85 > f_{0.025,15,15}\,(0.35)$ and $f_0 = 0.85 < f_{0.975,15,15}\,(2.86)$, We Cannot reject the null hypothesis $H_0 .\ \sigma_1^2 = \sigma_2^2$ at the 0.05 level of significance.

Two Samples: Tests on Two Proportions -

Null Hypothesis $H_0: p_1 = p_2$

Test statistic
$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Q A vote is to be taken among the residents of a town and the surrounding country to determine whether a proposed chemical plant should be constructed. The construction site is within the town limits, and for this reason many voters in the country feel that the proposal will pass because of the large proportion of town voters who favour the construction. To determine if there is a significant difference in the proportion of two town voters and county voters favoring the proposal, a poll is taken. If 120 of 200 town voters favour the proposal and 240 of 500 county residents favor it, would you agree that the proportion of town voters favoring the propsal is higher than the proportion of county voters? Use an $\alpha = 0.05$ level of significance.

Soln Let p_1 and p_2 be the true proportion of voters in the town and country, respectively favoring the proposal.

Null Hypothesis $H_0: p_1 = p_2$

Alternative Hypothesis $H_1: p_1 > p_2$

Test statistic -
$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

where $\hat{p}_1 = \frac{x_1}{n_1} = \frac{120}{200} = 0.60$, $\hat{p}_2 = \frac{x_2}{n_2} = \frac{240}{500} = 0.48$

~~where~~ $\hat{p} = \frac{x_1+x_2}{n_1+n_2} = \frac{120+240}{200+500} = 0.51$

$\hat{q} = 1 - \hat{p} = 1 - 0.51 = 0.49$

Therefore,

$$Z_0 = \frac{0.60 - 0.48}{\sqrt{(0.51)(0.49)\left(\frac{1}{200} + \frac{1}{500}\right)}}$$

$$= 2.9$$

Conclusion: $Z_0 = 2.9 > Z_{0.05}$ (1.645). Reject H_0 and agree that the proportion of town voters favoring the proposal is higher than the proportion of country voters.

Table: Tests Concerning Means

H_0	Value of Test statistic	H_1	Critical Region
$\mu = \mu_0$	$Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ σ known	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$Z < -Z_\alpha$ $Z > Z_\alpha$ $Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$
$\mu = \mu_0$	$t = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}}$; $\nu = n-1$ σ unknown	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$t < -t_{\alpha,\nu}$ $t > t_{\alpha,\nu}$ $t < -t_{\alpha/2,\nu}$ or $t > t_{\alpha/2,\nu}$
$\mu_1 - \mu_2 = d_0$	$Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ σ_1 and σ_2 known	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$Z < -Z_\alpha$ $Z > Z_\alpha$ $Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$
$\mu_1 - \mu_2 = d_0$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ $\sigma_1 = \sigma_2$ but unknown $S_p^2 = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$ $\nu = n_1 + n_2 - 2$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t < -t_{\alpha,\nu}$ $t > t_{\alpha,\nu}$ $t < -t_{\alpha/2,\nu}$ or $t > t_{\alpha/2,\nu}$
$\mu_1 - \mu_2 = d_0$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$ $\nu = \dfrac{(S_1^2/n_1 + S_2^2/n_2)^2}{\dfrac{(S_1^2/n_1)^2}{n_1-1} + \dfrac{(S_2^2/n_2)^2}{n_2-1}}$ $\sigma_1 \neq \sigma_2$ and unknown	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t < -t_{\alpha,\nu}$ $t > t_{\alpha,\nu}$ $t < -t_{\alpha/2,\nu}$ or $t > t_{\alpha/2,\nu}$

Table : Tests Concerning on Variance

H_0	Value of Test statistic	H_1	Critical Region
$\sigma^2 = \sigma_0^2$	$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$\sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2, n-1}$ or $\chi_0^2 < -\chi^2_{\alpha/2, n-1}$
		$\sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha, n-1}$
		$\sigma^2 < \sigma_0^2$	$\chi_0^2 < -\chi^2_{\alpha, n-1}$
$\sigma_1^2 = \sigma_2^2$	$F = \frac{S_1^2}{S_2^2}$	$\sigma_1^2 \neq \sigma_2^2$	$F < F_{1-\alpha/2}(\nu_1, \nu_2)$ or $F > F_{\alpha/2}(\nu_1, \nu_2)$
		$\sigma_1^2 > \sigma_2^2$	$F > F_{\alpha}(\nu_1, \nu_2)$
		$\sigma_1^2 < \sigma_2^2$	$F < F_{1-\alpha}(\nu_1, \nu_2)$ where ν_1 & ν_2 are degree of freedom.

Table : Tests Concerning on Proportion

H_0	Value of Test statistic	H_1	Critical Region
$p = p_0$	$Z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$	$p \neq p_0$	$Z_0 > Z_{\alpha/2}$ or $Z_0 < -Z_{\alpha/2}$
		$p > p_0$	$Z_0 > Z_\alpha$
		$p < p_0$	$Z_0 < -Z_\alpha$
$p_1 = p_2$	$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$	$p_1 \neq p_2$	$Z > Z_{\alpha/2}$ or $Z < Z_{\alpha/2}$
		$p_1 > p_2$	$Z > Z_\alpha$
		$p_1 < p_2$	$Z < -Z_\alpha$