Important Topics Marked by ma'am while teaching

UNIT I

1) Derivation of phase velocity

2) Applications of HUP (Zero point energy, non existance of electron in nucleus)

3) (Very imp) Wave function and its physical significance.

4)(Very imp) Particle in a box or infinite square potential well

UNIT II

5)Phase space

6) (Very imp) Diff b/w Maxwell boltzman statistics, Bose Einstien statistics and Fermi Dirac statistics

7) diff b/w bosons and fermions

8) black body Radiation, spectrum

Q.1.(g) Set up a relationship between group and phase velocity.

Ans. Phase velocity ⇒ $$V_p = \frac{w}{k}$$

Group Velocity ⇒ $$V_g = \frac{dw}{dx}$$

Where, w is the angular frequency of the wave and k is the wave vector.
But

$$w = v_p k$$

$$V_g = \frac{d}{dk}(v_p k)$$

$$V_g = v_p + k\frac{dV_p}{dx}$$

$$V_g = V_p + k\frac{dV_p}{d\lambda}\frac{d\lambda}{dx}$$

We know that $$k = \frac{2\pi}{\lambda}$$

$$\frac{d\lambda}{dk} = \frac{d}{dx}\left(\frac{2\pi}{x}\right) = \frac{-2\pi}{k^2}$$

But $$V_g = V_p + x\frac{dV_p}{d\lambda}\left(\frac{-2\pi}{k^2}\right)$$

$$V_g = V_p - \frac{2\pi}{k}\frac{dv_p}{d\lambda}$$

$$\boxed{V_g = V_p - \frac{\lambda dV_p}{d\lambda}}$$

$d\lambda$

Q.1.(h) A particle limited to the x-axis has the wave function $\psi = ax$, between $x = 0$ and $x = 1$ and $\psi = 0$ elsewhere. Find (a) the probability that particle can found between $x = 0.45$ and $x = 0.55$ (b) The expectation value $<x>$ of the particle's position. (3)

Ans. Given–

$$\psi = ax \quad \text{when } x = 0 \text{ and } x = 1$$
$$\psi = 0 \quad \text{elsewhere}$$

(a)

$$\text{Probability} = \int_{x_1}^{x_2} |\psi|^2 \, dx = a^2 \int_{0.45}^{0.55} x^2 dx$$

$$= a^2 \left[\frac{x^3}{3}\right]_{0.45}^{0.55} = 0.0251\, a^2$$

$$\boxed{P = 0.0251 a^2}$$

(b) **Expectation value of $<x>$ is**

$$<x> = \int_0^1 \psi^* x \psi dx$$

$$= \int_0^1 x |\psi|^2 dx = a^2 \int_0^1 x^3 dx$$

$$= a^2 \left|\frac{x^4}{4}\right|_0^1 = \frac{a^2}{4}$$

$$\boxed{<x> = \frac{a^2}{4}}$$

Particular	M.B. Statistics	B.E. Statistics	F.D. Statistics
Nature of particles	Particles are identical but distinguishable	Particles are identical but indis tinguishable	Particles are identical but indis tinguishable.
Size of the phase cell	The available volume of the phase space cell can be small as we like and can even approach zero.	Minimum size of the available phase space volume is of the order of h^3, where h is the Plank's constant.	Minimum size of the available phase space volume is of the order of h^3.
Number of phase cells	The phase space can be divided into any large number of cells of our choice.	The phase space can be divided into number of cells whose number is limited as the phase space volume of each cell cannot be less than h^3.	The phase space can be divided into the number of cells, whose number is limited as the phase space volume of each cell cannot be less than h^3.
Number of particles in a cell	Any number of particles can occupy a single phase space cell.	Any number of particles can occupy a single phase cell	A phase space cell cannot accommodate more than one particle.
Macrostates and	A macrostate can have more than one microstates	Each macrostate has one microstate.	Each macrostate has one microstate.
Energy	Continuous distribution energy	Energy is quantized.	Energy is quantized.
Spin	The particles are spinless.	The particles have half integral spin.	The particles have half integral spin.
Occupation index	$\frac{1}{e^{\alpha+\beta E_i}}$	$\frac{1}{e^{\alpha+\beta E_i}-1}$	$\frac{1}{e^{\alpha+\beta E_i}+1}$
Total energy	$\frac{3}{2}Nk_BT$	$\frac{3}{2}Nk_BT\left(1-\frac{1}{z^{5/2}e^{-\alpha}}\right)$	$\frac{3}{2}Nk_BT\left(1+\frac{1}{z^{5/2}e^{-\alpha}}\right)$
Wave function		Symmetric under interchange of the coordinates of any two Bosons.	Antisymmetric on interchange of the coordinates of any two Fermions.

Q.4. (c) Using exchange symmetry of wave function. Show that the Boson do not obey Pauli Exclusion Principle. **(2.5)**

Ans. The particles whose spin angular momentum are integral multiple of to $\hbar$-are called Bosons. They have integral spins 0, $\hbar$, $2\hbar$, $3\hbar$.....

Let there be N particles described by the wave function ψ (1, 2, 3,..., r, ..., s, ..., N), where 1 stands for all coordinates (position and spin) of particle 1 and so on. If the wave function is operated by an exchange operation $\hat{P}$ then

$$\hat{P}_{r,s}\psi(1,2,3,...,r,...,s,...,N) \;=\; P\psi(1,2,3,....,s,...,r,...,N) \qquad ...(1)$$

The effect of the operation $\hat{P}_{rs}$ is to interchange the particles r and s i.e., formerly called particle 'r' is now called 's' and vice versa. For two particles system, we write.

$$\hat{P}_{12}\psi(1,2) \;=\; P\psi(2,1) \qquad ...(2)$$

Equation (2) is the eigenvalue equation of the operator $\hat{P}_{r,s}$ and P stands for the eigenvalue of the operator $\hat{P}_{r,s}$.

There are two kinds of wave function ψ, depending on value of P-symmetric wave function and antisymmetric wave functions.

(*i*) Symmetric Wave Function (ψ_S)

A wave function is said to be symmetric if the interchange of any pair of particles among its arguments, leave the wave function unchanged.

i.e.,

$$\hat{P}_{1,2}\psi_S(1,2) \;=\; +\psi_S(2,1) \qquad \text{(here } P = 1) \qquad ...(3)$$

(*ii*) Antisymmetric Wave Function (ψ_A)

The wave function is antisymmetric if the interchange of any pair of particles among its arguments, changes the sign of the wave function.

$$\hat{P}_{1,2}\psi_A(1,2) \;=\; -\psi_A(2,1); \qquad \text{(here } P = -1)$$

The exchange symmetry of the wave functions has close relation with the intrinsic angular momentum of the particle. The relationship is listed as follows:

(*a*) The identical particles having an integral quantum number of their intrinsic spins are described by symmetric wave functions, such as

$$\psi_S(1, 2) \;=\; +\psi(2,1) \qquad ...(5)$$

The type of particles described by symmetric wave functions are known as *BOSONS* as they obey Bose-Einstein statistics.

(*b*) The particles which have half integral quantum number of their intrinsic spins are described by the antisymmetric wave functions, such as

$$\psi_A(1, 2) \;=\; -\psi(2, 1) \qquad ...(6)$$

The type of particles described by antisymmetric wave functions are called FERMIONS as they obey Fermi-Dirac statistics.

...equation for a particle in an infinite well ...and plot the first

The type of particles described by [illegible]
FERMIONS as they obey Fermi-Dirac statistics.

Q.5. (*a*) Set up the Schrodinger equation for a particle in an infinite well (one-dimensional) Solve it for eigenvalues and eigen functions and plot the first three eigen functions ψ_1, ψ_2, ψ_3 and also plot its probability. **(3 + 4 + 3 = 10)**

Ans. Consider a particle moving inside a box along the x-direction. The particle is bouncing back and forth between the walls of the box l is the width of the box as shown in Fig. 3.

$$V = 0 \text{ for } 0 < x < l$$
$$V = \infty \text{ for } x \leq 0 \text{ and } x \geq l$$

The particle cannot exist outside the box, so its wave functions ψ is 0 for $x \leq 0$ and $x \geq l$.

$$\Rightarrow \quad \frac{A^2}{2}\left[\int_0^l dx - \int_0^l \cos\frac{2n\pi x}{l}\,dx\right] = 1$$

$$\Rightarrow \quad \frac{A^2}{2}[l] = 1$$

$$\Rightarrow \quad A = \sqrt{\frac{2}{l}}$$

$$\therefore \quad \text{Eigen function } \psi_n = \sqrt{\frac{2}{l}}\sin\frac{n\pi x}{l}$$

where $\qquad n = 1, 2, 3, \ldots \qquad \ldots(5)$

Although ψ_n may be negative as well as positive, $|\psi_n|^2$ is always positive and since ψ_n is normalised, its square value at a given x is equal to the probability of finding the particle. The first three eigen functions ψ_1, ψ_2, ψ_3 together with probability densities $|\psi_1|^2$, $|\psi_2|^2$, $|\psi_3|^2$ are shown in Figs. 4(a) and (b) respectively.

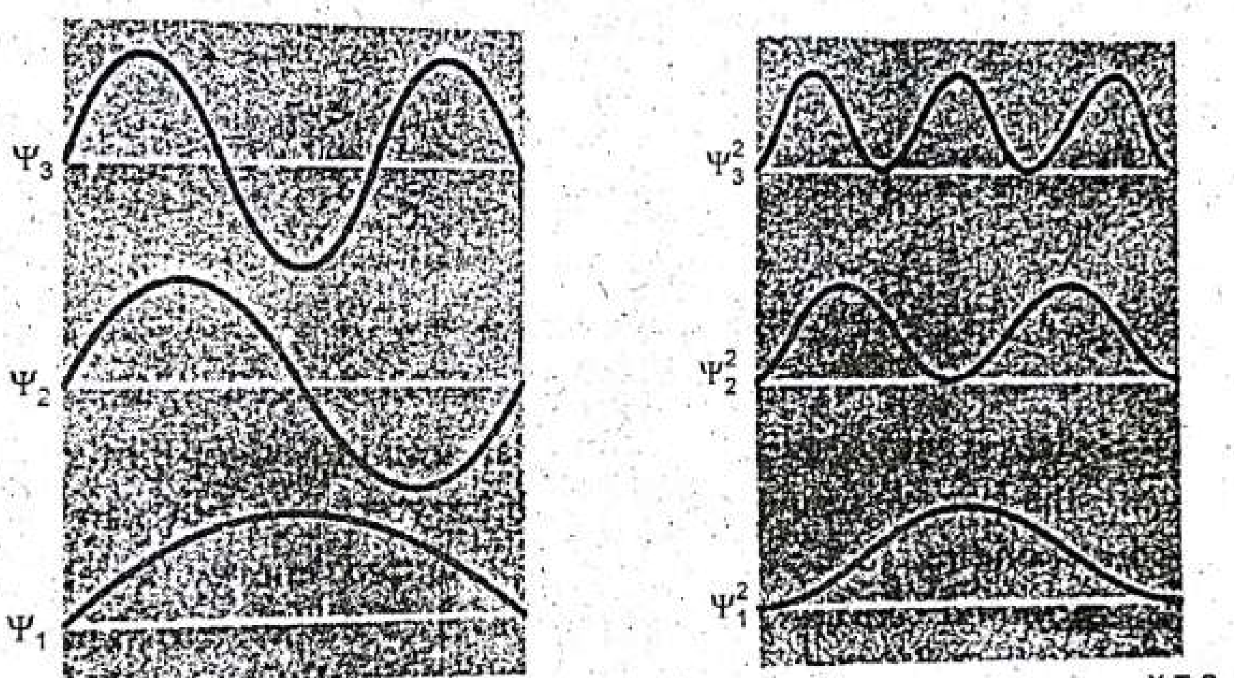

Fig. Wave-functions and probability densities of a particle confined to a box with rigid walls.

Q.5.(*b*) By applying uncertainity principle explain non-existence of electron in atomic nucleus. **(2.5)**

Ans. According to theory of relativity, energy of a particle is given by the relation

$$E^2 = p^2c^2 + m_0^2c^4 \quad ...(1)$$

where p = momentum of particle and m_0 = rest mass of the particle

According to Heisenberg's uncertainty principle,

$$\Delta x \, \Delta p_x = \frac{h}{2\pi} \quad ...(2)$$

The diameter of the nucleus is of the order of 10^{-14} m. If the electron exists in the nucleus, it can be anywhere within the diameter of the nucleus. Therefore, the maximum uncertainity Δx in the position of electron is the same as the diameter of the nucleus.

i.e.,

$$\Delta x = 10^{-14} \text{ m}$$

∴ The minimum uncertainty in the momentum is given by

$$\Delta p_x = \frac{h}{2\pi\Delta x} = \frac{6.63\times10^{-34}}{2\pi\times10^{-14}} = \frac{6.63\times10^{-20}}{2\pi}$$

$$= 1.055\times10^{-20} \text{ kg m/s}$$

It means that if electron exists in the nucleus, its minimum momentum must be

$$p_{\min} = 1.055\times10^{-20} \text{ kg m/s}$$

For the electron of the minimum momentum, the minimum energy is given by

$$E_{\min}^2 = p_{\min}^2 c^2 + m_0^2 c^4$$

$$= (1.055\times10^{-20}\times3\times10^8)^2 + (9.1\times10^{-31})^2\times(3\times10^8)^4$$

$$= (3\times10^8)^2\,[1.113\times10^{-40} + 7.4692\times10^{-40}]$$

Since the second term in the bracket is much smaller than the first, it can be neglected, then,

$$E_{\min} = 3\times10^8\sqrt{1.113\times10^{-40}}\,J$$

$$= 3\times10^8\times1.055\times10^{-20}\text{ J} = 3.1649\times10^{-12}\text{ J}$$

or

$$E_{\min} = \frac{3.1649\times10^{-12}}{1.6\times10^{-19}}\,eV \cong 20 \text{ MeV.}$$

Thus, if a free electron exists in the nucleus it must have a minimum energy of about 20 MeV.

The maximum K.E. which a β-particle, emitted from radioactive nuclei is of the order of 4 MeV. Therefore, this clearly shows that electrons cannot be present within the nucleus.

... the path.

Q.3. (*a*) An electron is constrained to move in a one dimensional box of length 0.1 *nm*. Find the first three energy eigen values and the coresponding de-Broglie wave lengths. (4)

Ans. Given

$$L = 1.0 \times 10^{-10} \text{ m.}$$

Formulae used are

$$E_n = \frac{n^2h^2}{8mL^2} \text{ and } p_n = \frac{nh}{2L}$$

$$\text{for } n = 1$$

$$E_1 = \frac{h^2}{8mL^2}\ \frac{(6.63 \times 10^{-34})^2}{8 \times (9.1 \times 10^{-31}) \times (10^{-10})^2}$$

$$= 6.04 \times 10^{-18} J$$

similaly, for $n = 2$, is equal to

$$E_2 = (2)^2 E_1 = 24.16 \times 10^{-15} J$$

and for $n = 3$,

$$E_3 = (3)^2 E_1 = 54.36 \times 10^{-18} J$$

as we know

$$\lambda_n = \frac{h}{p_n} \text{ and } p_n = \frac{nh}{2L}$$

or

$$\lambda_n = \frac{2L}{n}$$

For $n = 1$

$$\lambda_n = 2L = 2.0 \times 10^{-10} m = 2A$$

For $n = 2$,

$$\lambda_n = \frac{2L}{2} = l = 1.0 \times 10^{-10} m = 1.0 A$$

For $n = 3$,

$$\lambda_n = \frac{2L}{3} = 0.667 \times 10^{-10} m = \mathbf{0.66 A}$$

Q. 3. (b) Find the probability that a particle trapped in a box 'L' wide can be found between 0.45 L and 0.55 L for the first excited state. (4)

Ans. $$\Psi n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

For $n = 1$ $$\Psi(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

$$\rho = \int_{0.45L}^{0.55L} |\psi(x)|^2 dx = \frac{2}{L} \int_{0.45L}^{0.55L} \frac{\sin^2 \pi x}{L}$$

$$\rho = \int_{0.45L}^{0.55L} \frac{2}{2\times L}\left(1-\cos\frac{2\pi x}{L}\right)dx = \frac{1}{L}\int_{0.45L}^{0.55L}\left(+1-\cos\frac{2\pi x}{L}\right)dx$$

$$\rho = \frac{1}{L}\left[x-\left[\sin\left(\frac{2\pi x}{L}\right)\right]\times\left(\frac{L}{2\pi}\right)\right]_{0.45L}^{0.55L}$$

$$\rho = \frac{1}{L}[0.55L - 0.45L] - \frac{1}{L}\times\frac{L}{2\pi}\left[\sin\left(\frac{2\pi\times 0.55L}{L}\right)-\sin\left(\frac{2\pi\times 0.45L}{L}\right)\right]$$

$$\rho = 0.10 + \frac{0.617}{2\pi}$$

$$\boxed{\rho = \frac{0.20\pi + 0.617}{2\pi}}$$

Q. 3. (c) Explain the postulates of quantum mechanics.

Ans. The postulates of quantum mechanics are as following.

(*i*) A wave function may be associated with any particle moving in a conservative field of force and it determines every thing that can be known about the system in consistence of uncertainity principle.

(*ii*) The wave function of a system evolves in time according to time dependent schrodinger equation.

$$H\psi = i h \frac{\partial \psi}{\partial t}$$

(*iii*) The total wave function must be antisymmetric with respect to the inter change of all co-ordinates of one fermion with those of another. The Pauli's exclusion principle is direct result of this antisymmetry principle.

(*iv*) Corresponding to every observable in classical mechanics there is linear operator in quantum mechanics.

(*v*) In any measurment of the observable associated with operator A, the observed values are eigen values as satisfying the equation.

$$A\psi = a\psi$$

(*vi*) If a system is in a state described by a normalised wave function ψ, then the average value of the observable correspond to $\hat{A}$ is given by

$$<a> = \int_{-\alpha}^{+\alpha} \psi^* \hat{A} \psi \, dv$$

(3)

Q. 3. (*d*) Explain phase and group velocities.

Ans. Refer Q.1. (*g*) of End Term Examination 2014.

... of the following. (3 × 5 = 15)

(a) Wave packet

Ans. When plane waves of slightly different wavelenghths travel simutaneously in the same direction along a straight line, through a dispersive medium, (*i.e.* a medium in which the phase velocity $v_p = \frac{\omega}{k}$ a wave depends on its wavelength) successive group of the waves are produced as shown in fig. 1.

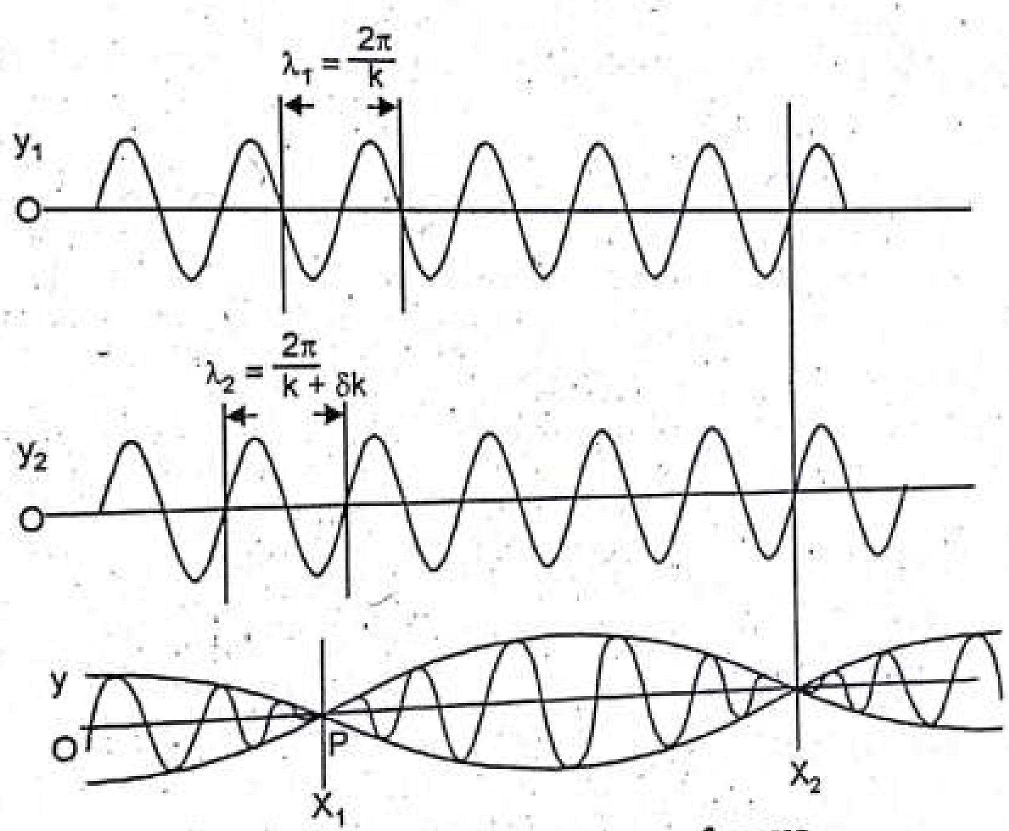

Fig.1 Successive groups of wave.

There wave group are called **wave packets.** Each wave group travels with a velocity is called **group velocity.** "The velocity with which the resultant envelope of the group of waves travels is called group velocity", denoted by v_g. The velocity of the group is different from that of the indigvidant components of the wave. The group velocity is different from the phase velocity of a wave.

The velocity with which resultant envelope moves is called group velocity and the velocity with which a point like *P* on the wave moves is called phase velocity.

... and frankel deyects.

... 2014.

as a free electron.

If $f_k = 1$, $m^* = m$, the elctron in the crystal in the ...

Q. 1. (f) Zero point energy.

Ans. Zero point energy is the lowest possible energy that a quantum mechanical physical system may have, it is the energy of its ground state.

All quantam mechanical systems undergo fluctuations even in their ground state and have an associated zero-point energy, a consequence of their wave-like nature. The uncertainity priniciple requires every physical system to have a zero point energy greater than the minimum of its classical potential well. For example, liquid helium does not freeze under atmospheric pressure at any temperature because of its zero point energy.

Also, ...

The possible energy of a particle in box of length 'a' is given by

$$E_n = \frac{n^2h^2}{8ma^2} \text{ (Where } n = 1,2,3,...)$$

If $n = 1$, then

$$E_1 = \frac{h^2}{8ma^3}$$

This is the energy of the ground state of particle. since the particle in a box cannot be at rest, its minimum energy is positive and is after called the zero point energy.

Q. 2. (a) Apply Gauss's Law, calculate–

Q. 6. (*a*) Give the salient postulates of statistical mechanics on the basis of which the statistical distribution functions are defined. Also state that clearly the difference in evaluation of the thermodynamic probability for each distribution function. **(8)**

Ans. Refer Q.3(a) of Second Term Examination 2015.

Every solid, liquid or gas in an assembly of an enormous number of microscopic particles. Likewise, radiation is an assembly of photons. Obviously the actual motions or interactions of individual particles cannot be investigated. However, the macroscopic properties of such assemblies can be explained in terms of the statistical distribution of the individuals among different possible states and their most probable behaviour.

For example, from maxwell distribution of speeds among the molecules we can calculate mean speed (which is related to the momentum carried by the molecules), mean square spped (which is related to the energy of the molecules) and so on, from the average quantities we calculate observable properties like pressure and temperature of the gas.

Usually we consider how a fixed amount of energy is distributed amount the various indentical particles of an assembly.

Now there are three kinds of identical particles

(*i*) Identical particles of any spin which are so much separated in the assembly that they can be distinguished from one another. The molecules of a gas are particles of this kind.

(*ii*) Identical particles of zero or integral spin which cannot be distinguished from one another. These ar called *Bose particles* (or bosons) and do not obey pauli's exclusion principle. Phonons and α–particle are of this kind.

(*iii*) Identical particles of half integral spin which cannot be distinguished from one another. These are called *Fermi particles* (or fermions) and do obey pauli's exclusion principle. Electrons, protons, neutrons are particles of this kind.

The first kind of the particles are the classical particles and obey the Maxwell-Boltzmann energy distribution law. The second kind of particles are quantum particles and energy distribution laws for then can be derived by methods of quantum statistics (Bose–Einstein and Fermi–Dirac statistics) only.

vectors E and H are

points at all times.

Q.1. (g) State the significance of normalization of wave function. **(2)**

Ans. The normalized wave equation is– $\int_{-\infty}^{+\infty} \psi^* \psi dv = 1$

This normalized wave gives the physically applicable wave function or probability amplitude. All wave function, representing the real particles must be normalized. This help in discarding solutions of Schrodinger's equation which do not have a finite integral in a given interval. This integral cannot be infinite. The integral must be real quantity greater than or equal to zero if ψ has to describe a real body properly. Also the wave function should have single value and its first derivative and second derivate should be finite and continuous.

levels of a particle in a box equally spaced? Show with **(2)**

function should have single

finite and continuous.

Q.1. (*h*) Are energy levels of a particle in a box equally spaced? Show with appropriate equation. **(2)**

Ans. The energy of particle in a box is given by following equation-

$$E_n = \frac{n^2h^2}{8ml^2}$$

where $\quad n = 1, 2, 3, 4....$

and l is the lenght of the box

Now, for $n = 1$ $\quad E_1 = \frac{h^2}{8ml^2}$

for $n = 2$ $\quad E_2 = \frac{4h^2}{8ml^2}$

for $n = 3$ $\quad E_3 = \frac{9h^2}{8ml^2}$

As, $E_n \alpha\ n^2$, the energy levels are not equally spaced.

Q.1. (*i*) What thermodynamic statics do electron follow, Bose– Einstein or Fermi; Dirac" (2)

Ans. Electron follow Fermi-Dirac statistic as electron is indistinguishable and has half integral spin. Also, it has anti symmetric wave function and obey Pauli exclusion principle.

Q.1. (*ii*) Draw the plane in a unit cube represented by the miller indices

Q.1. (*l*) State Bloch Theorem. **(2)**

Ans. The Bloch theorem is a mathematical statement regarding the form of the one electron wave functions for a perfectly periodic potential.

Q.4. (a) Using the uncertainity principle show that an electron does not exist inside a nucleus. (2)

Ans. According to Heisenberg's uncertainty principle,

$$\Delta x \, \Delta p_x = \frac{h}{2\pi}$$

The diameter of the nucleus is of the order of 10^{-14} m. If the electron exists in the nucleus, it can be anywhere within the diameter of the nucleus. Therefore, the maximum uncertainty Δx in the position of electron is the same as the diameter of the nucleus.

i.e. $\qquad \Delta x = 10^{-14}$m

$\therefore$ The minimum uncertainty in the momentum is given by

$$\Delta p_x = \frac{h}{2\pi \Delta x}$$

$$= \frac{6.63\times10^{-34}}{2\pi\times10^{-14}} = \frac{6.63\times10^{-20}}{2\pi}$$

$$= 1.055\times10^{-20} \text{kg m/sec}$$

It means that if electron exists in the nucleus, its minimum momentum must be

$$p_{min} = 1.055 \times 10^{-20} \text{ kg m/s}$$

For the electron of the minimum momentum, the minimum energy is given by

$$E^2_{min} = p^2_{min} c^2 + m^2_0 c^4$$

$$= (1.055\times10^{-20} \times 3\times10^8)^2 + (9.1\times10^{-31})^2 \, (3\times10^8)^4$$

$$= (3\times10^8)^2 \, [1.113\times10^{-40} + 7.4692\times10^{-44}]$$

Since the second term in the bracket is much smaller than the first, it can be neglected,

then,

$$E_{min} = 3\times10^8 \sqrt{1.113\times10^{-40}} J$$

$$= 3\times10^8 \times 1.055\times10^{-20} \text{ J} = 3.1649\times10^{-12} \text{ J}$$

or $\qquad E_{min} = \frac{3.1649\times10^{-12}}{1.6\times10^{-19}} \text{eV} \cong 20 \text{ MeV}.$

Thus, if a free electron exists in the nucleus it must have a minimum energy of about 20 MeV.

The maximum K.E. which a β-particle, emitted from radioactive nuclei is of the order of 4 MeV. Therefore, this clearly shows that electrons can not be present within the nucleus.

Q.4. (*b*) Describe Davission-Germer experiment. Find the lowest energy in eV, for an electron in one dimensional box of length a = 0.2 nm. (6)

Ans. The first proof of the existence of "matter waves" was obtained in 1927 by Davisson and Germer the two American physicists. They succeeded in measuring the de Broglie wavelength for slow electrons, accelerated by a low potential difference by diffraction methods. The experimental arrangement is shown in the Fig. 3. The electron beam is produced from an electron gun consisting of tungsten filament F heated by low tension battery. The electrons excited by the filament are accelerated in an electric field of known potential difference from a high tension battery. The electrons are collimated to a fine beam and made to strike a Nickel target

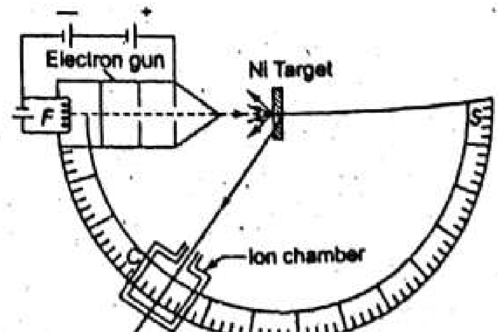

Fig.3. Davisson and Germer electron Diffraction apparatus

which is capable of rotation about an axis parallel to the axis of the incident beam. The
electrons are scattered in all directions by the atoms of the crystal. The intensity of the
electron beam scattered in a given direction is measured by allowing it to enter in a
Faraday cylinder called collector C which can be moved along a graduated circular scale
S, so that it is able to receive the reflected electrons at all angles between 20° and 90°.

Davisson and Germer noticed that the strongest of the scattered electron beams
corresponded accurately to diffraction maxima that would be expected in the diffraction
of X-rays by the same crystal. The angular distribution of scattered electrons was
analogous to optical diffraction patterns from a plane diffraction grating whose lines
consisted of the rows of nickel atoms in the surface of the target crystal. The wavelength
associated with the diffraction pattern can be obtained according to Bragg's law:

$$n\lambda = 2d \sin \theta$$

where d is the distance between the rows of atoms.

It was observed by Davisson and Germer that when an electorn beam accelerated by a potential of 54 volts was directed upon a Nickel target, a sharp diffraction maxima appeared in the electron currents. The incident and the scattered beams in this case make an angle of 65° with the family of Bragg's planes. The spacing of planes in this family which can be determined by X-rays diffraction is 0.91 Å. From above Bragg's equation, taking $n = 1$, we have,

$2 \times 0.91 \times \sin 65 = 1.\lambda$ or $\lambda = 1.65$ Å

The wavelength of electrons accelerated through potential of 54 volts can be calculated as

$$\frac{1}{2}mv^2 = \text{eV},$$

m is the mass of the electron.

$$m^2v^2 = 2\ me\ \text{V}$$

or

$$mv = \sqrt{2meV}$$

The de-Broglie wavelength of the electron will be given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

$mv \quad \sqrt{2meV}$

Putting $h = 6.6 \times 10^{-34}$ joule sec, $m = 9 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ coulomb, we obtain

$$\lambda = \frac{12.28}{\sqrt{V}}, \text{ with V} = 54 \text{ volts}$$

$$\lambda = 1.66\,\text{Å}$$

There is an excellent agreement between the two results. Thus Davisson Germer experiment is a definite evidence that the electron beams do behave as wave and the wavelength of these beams is thus given by the de-Broglie equation.

Energy of particle in a box is given by

$$E_n = \frac{h^2 h^2}{8ml^2}$$

Given, $n = 1, h = 6.62 \times 10^{-34}\, J\text{–}S, m = 9.1 \times 10^{-31}$ kg

$l = 0.2 \times 10^{-9}$ m

Lowest energy will be

$$E = \frac{1 \times (6.6.2 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-3} \times (0.2 \times 10^{-9})^2}$$

$$= \frac{43.83}{2.912} \times 10^{-68} \times 10^{+31} \times 10^{+18} = 15.05 \times 10^{-19}\text{J}$$

$$E = 15.05 \times 10^{-19}\text{J}$$

$$E = \frac{15.05 \times 10^{-19}\text{J}}{1.6 \times 10^{-19}} = 9.4\text{eV}$$

Q.4. (c) The eigen function of an operator $\frac{d^2}{dx^2}\psi(x) = e^{ax}$. Find the corresponding eigen value. **(2.5)**

Ans. The operator and eigen value relation is

$$\hat{0}\psi = 0\psi$$

where $\hat{0}$ is operator and 0 is eigen value

Here,
$$\hat{0} = \frac{d^2}{dx^2} \text{ and } \psi = e^{ax}$$

$$\hat{0}\psi = \frac{d^2}{dx^2}e^{ax}$$

$$\hat{0}\psi = a^2e^{ax}$$

We can write this as–

$$\hat{0}\psi = a^2\psi$$

Such that
$$0 = a^2$$

$\therefore$ a^2 is eigen value.

Q.5. (a) What type of statistics shall be applicable for a gas of photon? Justify your answer. **(2.5)**

Ans. Bose-Einstein statistics shall be applicable for a gas of photons. Bose-Einstein statistics is obeyed by indistinguishable particles of integral spin quantum number that have symmetric wave function and does not obey Pauli exclusion principle.

Q.5. (b) Compare the qualitative features of Maxwell Boltzmann, Bose-Einstein in and Fermi-Dirac statistics on the basis of their functions. **(6)**

Ans. Refer Q.1.(b) First Term Examination 2016.

Q.5(c) Show that Bose-Einstein, Fermi-Dirac statistics reduces to Maxwell boltzamann statistics at high temperature. **(4)**

Ans. The distribution laws of three statistics are given below

$$\frac{g_i}{n_i} = e^{\alpha} e^{E_i/k_B T} \qquad \text{...(1)[For M – B]}$$

$$\frac{g_i}{n_i} = e^{\alpha} e^{E_i/K_B T} - 1 \qquad \text{...(2)[For B – E]}$$

and

$$\frac{g_i}{n_i} = e^{\alpha} e^{E_i/K_B T} + 1 \qquad \text{...(3)[For F – D]}$$

If $\frac{g_i}{n_i} >> 1$ then $\frac{g_i}{n_i} \approx \left(\frac{g_i}{n_i}+1\right) \approx \left(\frac{g_i}{n_i}-1\right)$. In this limit both B.E. and F.D. distributions are identical with M.B. distribution. This limit $\left(\frac{g_i}{n_i} >> 1\right)$ occurs when the temperature is not too low and pressure (or density) is not too high.

Q.8. (a) Describe Kronig - Penney Model. **(4)**

Ans. The Kronig-Penney model is a simple, idealized quantum mechanical system that consists of an infinite array of rectangular potential barriers.

Kronig~and Penney in 1931 solved the Schrodinger's equation for electrons in a simple idealized periodic field. The model is of considerable importance because it interprets the main features of the band structure of metals. The periodic field was assumed in the form of potential wells of zero potential energy of width a separated by rectangular barriers of width b and height V_0 as shown in Fig. 6. For the potential of Fig. 6. the Schrodinger equation has to be solved in two regions:

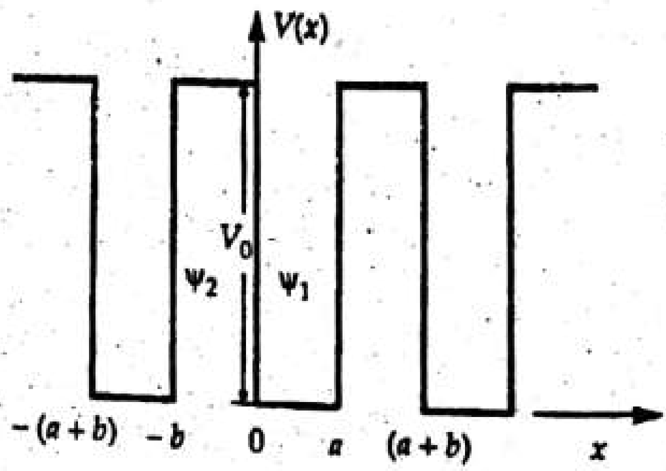

Fig. 6. One dimensional chain of potential wells.

(a) **Region I,** $0 < x < a$, potential well region in which $V = 0$ and

(b) **Region II,** $-b < x < 0$, barrier region in which $V = V_0$.

In order to find the allowed energies, we solve the Schrodinger's equation in these two regions and apply the appropriate boundary condition.

In *region I*, the Schrodinger's equation is

$$\frac{d^2\psi}{dx^2}+\frac{2mE}{\hbar^2}\psi=0$$

or
$$\frac{d^2\psi_1}{dx^2}+\beta^2\psi_1=0 \quad \left[\text{Here, } \beta^2=\frac{2mE}{\hbar^2}\right] \qquad (1)$$

Solution of this equation is

$$\psi_1 = Ae^{i\beta x}+Be^{-i\beta x}, \text{ where A and B are constants.} \qquad ...(2)$$

In region II, the Schrodinger's equation is

$$\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}(E-V_0)\psi = 0$$

or
$$\frac{d^2\psi_2}{dx^2}-\alpha^2\psi_2 = 0 \quad \left[\text{Here, } \alpha^2=\frac{2m(E-V_0)}{\hbar^2}\right] \qquad ...(3)$$

Solution of this equation, for $E < V_0$ is

$$\psi_2 = Ce^{\alpha x}+De^{-\alpha x} \qquad ...(4)$$

where C and D are constants.

For continuity at $x = 0$

$$\psi_1(0) = \psi_2(0) \qquad ...(5)$$

$$\left.\frac{\partial\psi_1}{\partial x}\right|_{x=0}=\left.\frac{\partial\psi_2}{\partial x}\right|_{x=0} \qquad ...(6)$$

In a periodic lattice $V(x + a) = V(x)$ it is expected that the wavefunction will also exhibit this periodicity. Since the wave function must be a Bloch wave function, therefore Eq. (1) gives

$$\psi_k(x+a+b) = e^{-ik(a+b)}\psi_k(x) \qquad ...(7)$$

We incorporate this requirement at $x = -b$ and $x = a$, then we obtain the continuity boundary condition as

$$\psi_2\big|_{x=-b}=e^{-ik(a+b)}\psi_1\big|_{x=a} \qquad ...(8)$$

$$\left.\frac{\partial\psi_1}{\partial x}\right|_{x=-b}=e^{-ik(a+b)}\left.\frac{\partial\psi_1}{\partial x}\right|_{x=a} \qquad ...(9)$$

When we apply the boundary conditions, Eqn. (5), (6), (7) and (8), we obtain four equations for four unknowns

$$A+B=C+D \qquad (10)$$

$$i\beta(A-B)=\alpha(C-D) \qquad ...(11)$$

$$Ce^{\alpha b}+De^{\alpha b}=e^{-ik(a+b)}\left[Ae^{i\beta a}+Be^{-i\beta a}\right] \qquad (12)$$

$$\alpha Ce^{\alpha b}-\alpha De^{\alpha b} = e^{-ik(a+b)}i\beta\left[Ae^{i\beta a}-Be^{-i\beta a}\right] \qquad (13)$$

By solving these equations simultaneously e.g. by requiring that the determinants of the coefficients of A, B, C and D vanish, we get

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ i\beta & -i\beta & -\alpha & \alpha \\ -e^{-ik(a+b)+i\beta a} & -e^{-ik(a+b)-i\beta a} & e^{-\alpha b} & e^{\alpha b} \\ -i\beta e^{-ik(a+b)+i\beta a} & i\beta e^{-ik(a+b)-i\beta a} & \alpha e^{-\alpha b} & -\alpha e^{\alpha b} \end{vmatrix} = 0 \quad ...(14)$$

On solving the determinant Eq. (14) and after simplification, we get

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta} \sin h\, \alpha b \sin \beta a + \cosh \alpha b \cos \beta a = \cos(a+b)k \quad ...(15)$$

To simplify Eq. (15), Kronig and Penney considered the case when $V_0 \to \infty$ and $b \to 0$, but product $V_0 b$ has a finite value *i.e.*, the potential barriers become *delta functions.*

Under these conditions, the model is modified in such a way that represent a series of well separated by infinitely thin potential barriers of infinitely large potential. The limiting value of $V_0 b$ for $V_0 \to \infty$ and $b \to 0$ is known as *barrier strength.*

Also from Eqs. (9) and (3)

$$\alpha^2 = \frac{2m(E - V_0)}{\hbar^2} \text{ and } \beta^2 = \frac{2mE}{\hbar^2}$$

Then $$\beta^2 - \alpha^2 = \frac{2mV_0}{\hbar^2}$$

or $$\frac{\beta^2 - \alpha^2}{2\alpha\beta} = \frac{mV_0}{\alpha\beta\hbar^2} \quad ...(16)$$

Putting the value $\frac{\beta^2 - \alpha^2}{2\alpha\beta}$ from Eq. (16) in Eq. (15) we get

$$\frac{mV_0}{\alpha\beta\hbar^2} \alpha b \sin \beta a + \cos \beta a = \cos ka$$

$$\frac{mV_0 b}{\beta\hbar^2} \sin \beta a + \cos \beta a = \cos ka$$

Let us define a quantity $p = \frac{mV_0 ba}{\hbar^2}$, which is measure of the area $V_0 b$ of the potential barrier,

then $$p \frac{\sin \beta a}{\beta a} + \cos \beta a = \cos ka \quad ...(18)$$

The physical significance of the quantity p is that if p is increased and the given vector is bound more strongly to a particular potential well.

When $p \to 0$, then potential barrier becomes very weak which means that electrons are free electrons. In this, we obtain from Eq. (18),

$$\beta a = ka \quad \text{or} \quad \beta = k \quad ...(19)$$

Hence $$\beta^2 = \frac{2mE}{\hbar} = k^2$$

This result is similar as obtained by free electron theory. Equation (18) also given the condition, which must be satisfied so that solutions of the wave equation may exist. Since $\cos ka$ can have values between + 1 and –1. Such values of βa, therefore, represent wave like solutions of the form

$$\psi(x) = e^{ikx} U_k(x)$$

and are *allowed values*. The other values of βa are not allowed.

If we plot a graph between $\left(\frac{p \sin \beta a}{\beta a} + \cos \beta a\right)$ and βa for the value of $p = \frac{3a}{2}$, we get the curve as shown in Fig. 7.

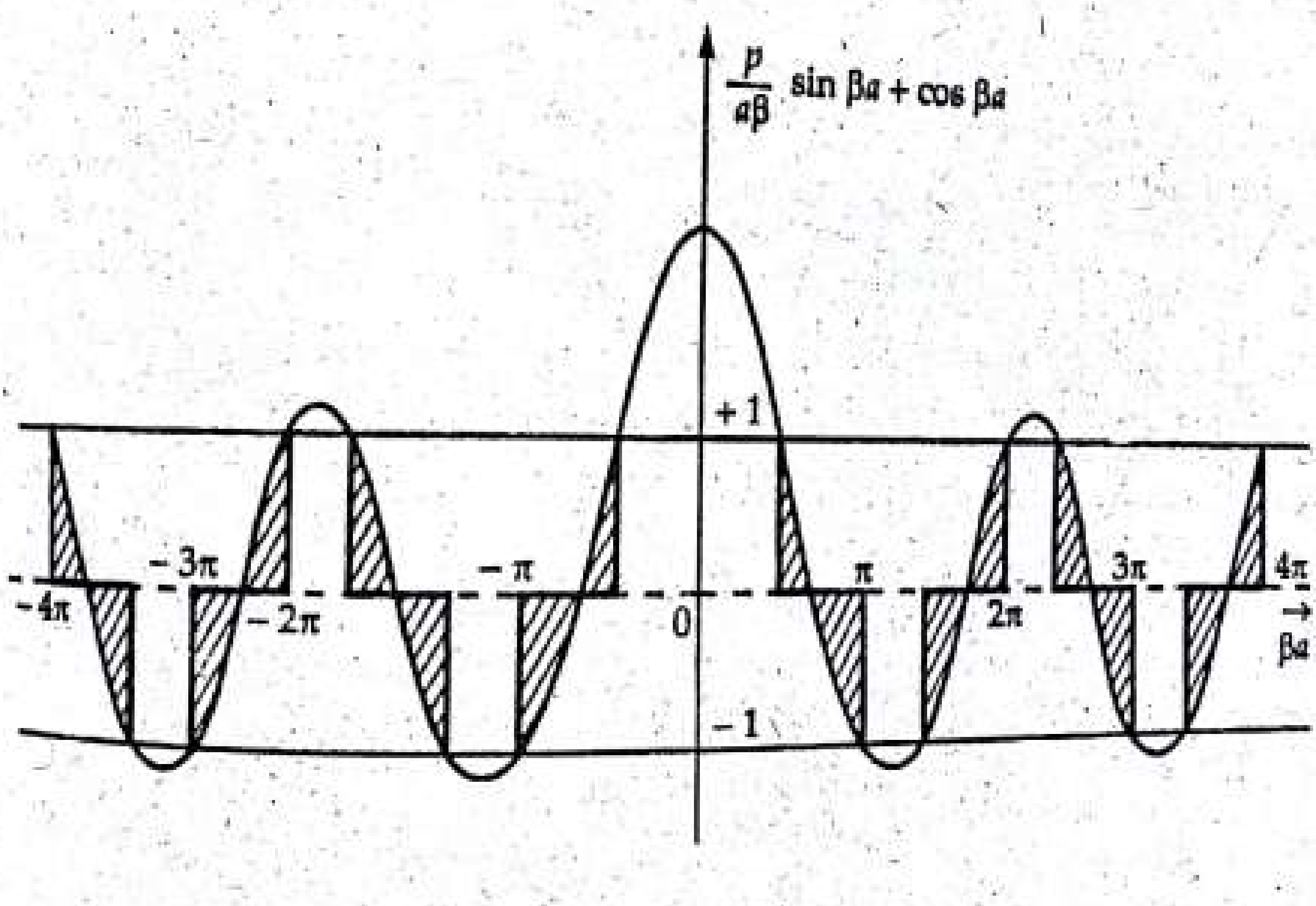

Fig. 7. Flot of $\frac{p}{\beta a} \sin \beta a + \cos \beta a$ vs. βa to show allowed and forbidden band for the K.P. approximation.

Q.1. (*d*) Describe the implication of Davisson-Germer experiment. (2.5)

Ans. Davisson and Germer in 1927 designed an apparatus to determine the wavelength associated with electrons to confirm the dual nature of matter.

Q.1. (*e*) Why is the energy of a particle trapped inside a box quantized? (2.5)

Ans. Energy of a particle trapped inside a box is given by the following equation-

$$E_n = \frac{n^2h^2}{8ml^2}$$

where, $n = 1, 2, 3, 4....$

It is clear from the equation that particle cannot posses an arbitrary energy but can have only certain discrete energy corresponding to $n = 1, 2, 3....$

have only certain discrete energy corresponding to $n = 1, 2, 3....$

Q.1. (f) An electron is confined to move between two rigid walls separated by 2×10^{-9} m. Find the deBrogile wavelengths representing the first three allowed energy status of the electron. **(2.5)**

Ans. $$E_n = \frac{n^2h^2}{8ml^2}$$

Given that $l = 2 \times 10^{-9}$ m

and $m = 9 \times 10^{-31}$ kg

$h = 6.63 \times 10^{-34}$ J–S

For $n = 1$–

$$E_1 = \frac{(1)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2} \times \frac{1}{1.6 \times 10^{-19}}$$

and $$E_1 = \frac{hc}{\lambda_1} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_1} \times \frac{1}{1.6 \times 10^{-19}}$$

Solving for λ_1–

$$\lambda_1 = 1.31 \times 10^{-5} \text{ m}$$

For $n = 2$–

$$E_2 = \frac{(2)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{eV}$$

and $$E_2 = \frac{hc}{\lambda_2} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_2} \times \frac{1}{1.6 \times 10^{-19}}$$

Solving for λ_2

$$\lambda_2 = 3.2 \times 10^{-6} \text{ m}$$

For $n = 3$

$$E_3 = \frac{(3)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{eV}$$

and $$E_3 = \frac{hc}{\lambda_3} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_3} \times \frac{1}{1.6 \times 10^{-19}}$$

Solving for λ_3

$$\lambda_3 = 1.4 \times 10^{-6} \text{ m}$$

$$\lambda_3 = 1.4 \times 10^{-6}\ \text{m}$$

Q.1. (g) What are the postulates of quantum theory of radiation? **(2.5)**

Ans. The postulates of quantum theory of radiation are as following:—

(i) A chamber containing blackbody radiations also contains simple harmonic oscillators of molecular dimensions which can vibrate with all possible frequencies.

(ii) The frequency of radiation emitted by an oscillator is same as the frequency of its vibrations.

(iii) An oscillator cannot emit energy in a continuous manner. It can emit energy in the multiples of a small unit called quantum (photon). If an oscillator is vibrating with a frequency ν, it can only radiate in quanta of magnitude $h\nu$ i.e., the oscillator can have only discrete energy E_n given by

$$E_n = nh\nu = nE$$

where $hv = E$, n is an integer and h is a universal Planck constant $(6.625 \times 10^{-34}\ \text{Js})$.

(iv) The oscillators can emit or absorb radiation energy in packets of magnitude hv. This implies that the exchange of energy between radiation and matter cannot take place continuously, but are limited to discrete set of values $0, hv, 2hv, 3hv, \ldots. nhv$.

(2.5)

Q.1. (*j*) Define Fermi energy. What is its physical significance?

Ans. Fermi energy is a quantum phenomenon which translates as the difference in energy state occupied by the lowest level electron to the highest level. It can also be measured at the top of the collection of electron energy level at absolute zero. The maximum energy that electrons may possess at 0 kelvin is the Fermi energy.

Q.4. (a) Distinguish between a boson and a fermion. Give one example each. **(4)**

Ans.

Bose-Einstein statistics (Quantum statistics)	**Fermi-Dirac statistics (Quantum (statistics)**
Apply to systems of indistinguishable particles not obeying Pauli's exclusion principle, such as photons, phonons and liquid helium at low temperature	Apply to systems of indistinguishable particle obeying Pauli exclusion principle, such as free electrons in metal electrons in a star (white dwarf star)
The distribution law is $n_i = \frac{g_i}{(e^{\alpha} e^{E_i / k_B T} - 1)}$	The distribution law is $n_i = \frac{g_i}{(e^{\alpha} e^{E_i / k_B T} + 1)}$
There is no restriction on the number of particles in a given state.	Only one particle in a given quantum state is allowed.
The volume of phase cell is of order of h^3.	The volume of phase cell is of the order of h^3
For $E_i >> k_B T$, exponential $E_i << k_B T$, lies above M.B.	For $E_i >> k_B T$, exponential where $E_i >> E_F$ If $E_F >> k_B T$ decreases abruptly near E_F
The energy may be zero at absolute zero.	The energy at absolute zero, cannot be zero because all particles cannot come down to ground state due to Pauli's exclusion principle.

...of energy in

Q.5. (a) Derive schrodinger time dependent wave equation for non-relativistic particle. Give the physical interpretation of Hamilton operator H. **(6)**

Ans. Let us assume that Ψ for a particle moving freely in positive x-direction is

$$\psi = Ae^{-i\omega\left(t-\frac{x}{v}\right)} \qquad ...(1)$$

as

$$\omega = 2\pi\nu \quad v = \nu\lambda$$

$$\psi = Ae^{-2\pi i\left(\nu t-\frac{x}{\lambda}\right)} \qquad ...(2)$$

As

$$E = h\nu = 2n\pi\hbar\nu \text{ and } \lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$$

$\therefore$ For a free particle wave equation becomes

$$\psi = Ae^{-\frac{i}{\hbar}(Et-px)} \qquad ...(3)$$

But

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \text{ and } p\psi = \frac{\hbar}{i}\frac{\partial\psi}{\partial x} \; ... (a) \qquad ...(4)$$

As total energy, E = Kinetic energy (K) + Potential energy (V)

Now,

$$\text{K.E.} = \frac{p^2}{2m}$$

$\therefore$ Equation (4) in term of wave function ψ can be written as

$$E\psi = \left(\frac{p^2}{2m}\right)\psi + V\psi \qquad ...(5)$$

Putting the values of $E\psi$ and $p\psi$ from Eq. (a) in Eq. (5) we have

$$i\hbar\frac{\partial\psi}{\partial t} = \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2 \frac{1}{2m}\psi + V\psi$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi} \qquad ...(6)$$

Equation (6) is Schrödinger's time dependent wave equation in one-dimension. The time dependent Schrödinger's equation in three-dimensional form,

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left[\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right] + V\psi \qquad ...(7)$$

or

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

$$\left[\because \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] \qquad ...(8)$$

or $$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial\tau} \quad ...(9)$$

or $$\boxed{\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}} \quad ...(10)$$

Equation (10) contains time and hence is called time dependent Schrodinger equation.

The operator $\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$ is called Hamiltonian and is represented by H, while operator $i\hbar\frac{\partial}{\partial t}$, operated on ψ, given E which may be seen by Eq. (7) Eq. (10) may be written as

$$\boxed{H\psi = E\psi} \quad ...(11)$$

The above forms of the Schrodinger's equation describe the motion of a non-relativistic material particle.

relativistic material particle.

Q.5. (b) State the properties of wave function in a quantum mechanical system. Give one example each for an acceptable and non-acceptable wave function. (4)

Ans. Properties of Wave Function are—

- $\psi(x)$ must be single valued, finite and continuous for all values of x.
- $\frac{d\psi(x)}{dt}$ must be finite and continuous for all values of x, except at those point

where $V \to \infty$. At these points $\frac{d\psi(x)}{dt}$ has a finite discontinuity but ψ remains continuous.

- For bound state and probability of finding the particle between x and $(x + dx)$, i.e., $|\psi|^2 dx$ must vanish as $|x| \to \infty$.

Hence $|\psi(x)| \to 0$ as $|x| \to \infty$ i.e., $\psi(x)$ is a square integrable wave function.

Examples of not acceptable wave functions:

(i) $\psi = \frac{1}{x}$ as $x \to 0, \psi \to \infty$ (ii) $\psi = x^n$ as $a \to \pm\infty, \psi \to \pm\infty$

Examples of acceptable wave functions:

(i) $\psi = \frac{1}{(1+x^2)}$ as $x \to \infty, \psi \to 0$ (ii) $\psi = \sin x$ as ψ oscillates between $+1$ and -1.

Q.5. (c) An electron has a speed of $2 \times 10^4 \frac{m}{s}$ within the accuracy of 0.01%. Calculate the uncertainity in the position of the electron. **(2.5)**

Ans. Given-

$$\Delta V = \frac{0.01}{100} \times 2 \times 10^4 = \frac{2m}{s}$$

From uncertainity principle-

$$\Delta x.\Delta p_x \geq \frac{h}{2\pi}$$

Uncertainity P_n position-

$$\therefore \quad \Delta x = \frac{h}{2\pi p_x} = \frac{h}{2\pi m\Delta V}$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 2}$$

$$= \frac{6.63 \times 10^{-34}}{114.296 \times 10^{-31}}$$

$$= 0.058 \times 10^{-3} \text{ m}$$

$$\boxed{\Delta x = 58 \times 10^{-6} m}$$

Q. 1. (d) Write down the normalization condition of a wave function. Why is it necessary for a wave function to be normalized? **(3)**

Ans. ψ cannot be interpreted in terms of an experiment. The probability that something be in certain place at a given time must lie between 0 (the object is not definitely there) and 1 (the object is definitely there). An intermediate probability, say 0.2 means, there is a 20% chance of finding the object. But the amplitude of wave can be negative as well as positive and negative probability is measingless. Hence ψ by itself cannot be an observed quantity.

The probability of experimentally finding the body described by the wave function ψ at the point (x, y, z) at the time t is proportional to the value $|\psi|^2$ there at t. A large value of $|\psi|^2$ means the strong possibility of the body's presence, while a small value of $|\psi|^2$ is not actually zero somewhere, there is *a* definite change however small, of detecting it there.

A wave function is a mathematical tool in quantum mechanics describing the quantum state of a particle or system of particles. Mathematically, its is a function from a space that maps the possible states of the system. The laws of quantum mechanics described how the wave function evolves over the time.

In quantum mechanics, probability amplitude is a complex number whose modulus squared represents a probability density. The principal use of probability amplitudes is as the physical meaning of the wave function.

Before going through actual calculation of ψ, it must satisfy certain requirements. In quantum mechanics, wave function describing real particle must be normalizable, *i.e.*, the integral of $|\psi|^2$ over all space must be finite-after all the body is located somewhere. Mathematically, it is expressed as:

$$\int_{-\infty}^{\infty} \psi^* \psi \, dV = 1$$

or

$$\int_{-\infty}^{\infty} |\psi|^2 \, dV = 1$$

A wave function that obeys above equation is said to be normalized. The normalization of wave function is done to obtain the physically applicable wave function or probability amplitudes. All wave function representing real particles must be normalisable. This helps in discarding solutions of Schrodinger's equation which do not have a finite integral in a given interval.

Q. 1. (e) Why is the wave nature of matter not more apparent in our daily observation? **(2)**

Ans. Since de-Broglie wavelength is given by $\lambda = \frac{h}{mv}$ and the value of Planck's constant is very small, so the wavelength associated with ordinary object is small and difficult to observe. In our daily observations we deal with the objects having larger mass and smaller velocity, that is why the wave nature of such objects is not more apparent in our daily life. But for smaller objects like electrons and neutrons the wave behaviour of particle is dominant as the mass of electron and neutron are smaller than the mass of an ordinary tennis ball.

Q. 1. (f) What is energy quantization? How is it possible for particle in a rigid box? **(3)**

Ans. Energy quantization means that the system or an atom can have only certain energies and not a continuum of energies. This discretization of energy values is known as quantization, meaning that the allowed energies are separated by discrete energy differences known as "quanta".

For particle in a box of length "l" has following energy.

$$E_n = \frac{n^2h^2}{8ml^2}$$

where $n = 1, 2, 3....$; m = mass of particle ; h = Planck's constant

Particle cannot posses an arbitrary energy but can have only certain discrete energy corresponding to $n = 1, 2, 3$. Each permitted energy level energy is called eigen value of the particle and constitute the energy level of the system.

Q. 1. (g) Based on Fermi-Dirac statistics, state the nature of Fermi distribution function. How does it vary with temperatures? **(2)**

Ans. The Fermi-Dirac distribution is plotted in Fig. 1. for three different values of T and α. In the distribution the occupation index never goes above 1. This signifies that we cannot have more than 1 particle per quantum state as required by Pauli's exclusion principle which applies in this case.

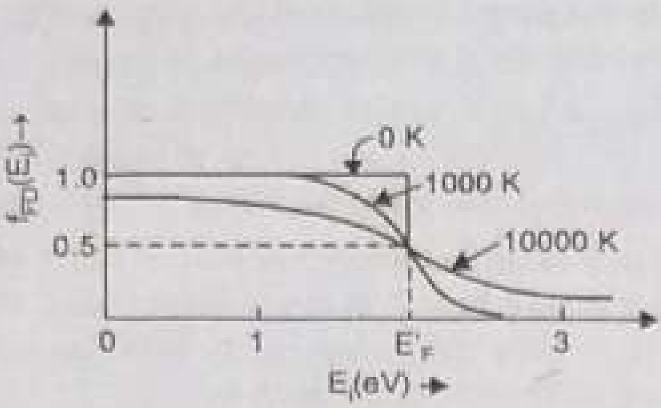

Fig. 1. F.D. distribution curve.

Further, in this distribution the parameter α is strongly dependent on temperature T, and we write

$$\alpha = \frac{-E_F}{k_B T} \qquad ...(1)$$

So that the Fermi-Dirac occupation index becomes.

$$f_{FD}(E_i) = \frac{1}{(e^{(E_i - E_F)k_BT} + 1)} \qquad ...(2)$$

where E_F is called Fermi energy.

Let us consider the situation at the absolute zero of temperature.

At T = 0, $(E_i - E_F)/k_BT = -\infty$ (for $E_i < E_F$) and $(E_i - E_F)/k_BT = +\infty$ (for $E > E_F$).

Therefore

for $\quad E_i < E_F \;\; f_{FD}(E_i) = \dfrac{1}{e^{-\infty}+1} = 1 \;\; (\because e^{-\infty} = 0) \qquad ...(3)$

and for $\quad E_i > E_F \;\; f_{FD}(E_i) = \dfrac{1}{e^{\infty}+1} = 0 \;\; (\because e^{\infty} = 0) \qquad ...(4)$

Thus at T = 0, all energy states from $E_i = 0$ to $E_i = E_F$ are occupied because $f_{FD}(E_i) = 1$, while all states above E_F are vacant.

As the temperature rises, some of the states just below E_F become vacant, while some just above E_F are occupied. The higher the temperature, the more in the spreading in $f_{FD}(E_i)$.

At $E_i = E_F$, we have

$$f_{FD}(E_i) = \frac{1}{e^0 + 1} = \frac{1}{2}, \text{ at all temperatures.}$$

That is, the average number of particles per quantum state is exactly $\frac{1}{2}$. In other words, the probability of finding an electron with energy equal to the Fermi energy in a metal is $\frac{1}{2}$ at any temperature.

Maxwell-Boltzmann statistics (Classical statistics)	Bose-Einstein statistics (Quantum statistics)	Fermi-Dirac statistics (Quantum (statistics)
1. Apply to systems of distinguishable particles, such as gases.	Apply to systems of indistinguishable particles not obeying Pauli's exclusion principle, such as photons, phonons and liquid helium at low temperature	Apply to systems of indistinguishable particle obeying Pauli exclusion principle, such as free electrons in metal electrons in a star (white dwarf star)
2. The distribution law is S, $n_i = \frac{g_i}{e^{\alpha} e^{E_i / k_B T}}$	The distribution law is $n_i = \frac{g_i}{(e^{\alpha} e^{E_i / k_B T}) - 1}$	The distribution law is $n_i = \frac{g_i}{(e^{\alpha} e^{E_i / k_B T}) + 1}$
3. There is no restriction on the number of particles, in the given state.	There is no restriction on the number of particles in a given state.	Only ene particle in a given quantum state is allowed.
4. In the phase space the volume of the phase cell is not fixed.	The volume of phase cell is of order of h^3.	The volume of phase cell is of the order of h^3
5. The behaviour of distribution function $f(E_i)$ against E_i is exponential.	For $E_i >> k_B T$, exponential $E_i << k_B T$, lies above M.B.	For $E_i >> k_B T$; exponential where $E_i >> E_F$. If $E_F >> k_B T$ decreases abruptly near E_F.
6. The energy of M.B. system, such as in ideal monoatomic gas, may be zero at absolute zero.	The energy may be zero at absolute zero.	The energy at absolute zero, cannot be zero because all particles cannot come down to ground state due to Pauli's exclusion principle.

Q. 4. (c) Distinguish between Bosons and Fermions. Give one example each. **(4)**

Ans. Refer Q. 4. (a) End Term Examination May-June 2017.

Q. 5. (a) Derive Schrodinger's time independent wave equation. (5)

Ans. Consider a system of stationary waves associated with a moving particle. If the position coordinates of the particle are (x, y, z) and Ψ be the periodic displacement for the matter waves at any instant of time t, then we can represent the motion of the wave by a differential equation as follows.

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2} \quad ...(1)$$

where v is the velocity of wave associated with the particle. The solution of Eq. (1) given Ψ as a periodic diaplacement in terms of time *i.e.*

$$\Psi(x, y, z, t) = \Psi_0(x, y, z)e^{-i\omega t} \quad ...(2)$$

where Ψ_0 is the amplitude of the particle wave at the point (x, y, z) which is independent of time (t). It is a function of (x, y, z). *i.e.*, the position r and not of time t, Here

$$r = x\hat{i}+y\hat{j}+z\hat{k} \quad ...(3)$$

Eq. (2) may be expressed as

$$\Psi(r, t) = \Psi_0(r)e^{-i\omega t} \quad ...(4)$$

Differentiating Eq. (4) twice with respect to t, we get

$$\frac{\partial^2\Psi}{\partial t^2} = -\omega^2\Psi_0(r)e^{-i\omega t}$$

or

$$\frac{\partial^2\Psi}{\partial t^2} = -\omega^2\Psi \quad ...(5)$$

Substituting the value of $\frac{\partial^2\Psi}{\partial t^2}$ from this equation in Eq. (1), we get

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+\frac{\omega^2}{v^2}\psi = 0 \quad ....(6)$$

where $\omega = 2\pi\text{v} = 2\pi(v/\lambda)$ [as $u = \lambda\text{v}$]

so that

$$\frac{\omega}{u} = \frac{2\pi}{\lambda} \quad ...(7)$$

Also $$\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2} = \nabla^2\psi \quad ...(8)$$

where ∇^2 is known as Laplacian operator. Using Eqs. (6), (7) and (8), we have

$$\nabla^2\Psi+\frac{4\pi^2}{\lambda^2}\Psi = 0 \quad ...(9)$$

Also from the de-Broglie wave concept

$$\lambda = \frac{h}{mv}$$

Using this relation in Eq. (9) gives–

$$\nabla^2\Psi + \frac{4\pi^2 m^2 v^2}{h^2}\Psi = 0 \qquad ...(10)$$

If E and V are respectively the total energy and potential energy of the particle then its kinetic energy is given by

$$\frac{1}{2}mv^2 = E - V$$

$$m^2v^2 = 2m\,(E - V) \qquad ...(11)$$

The use of Eq. (11) in Eq. (10) gives

$$\nabla^2\Psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

or

$$\nabla^2\Psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \qquad ...(12)$$

This is the time independent Schroedinger equation, where the quantity Ψ is known as wave function.

For a freely moving or free particle V = 0. Therefore, Eq. (12) becomes

$$\nabla^2\Psi + \frac{2mE}{\hbar^2}\Psi = 0 \qquad ...(13)$$

This is called time independent Schroedinger equation for a free particle.

Q. 5. (b) What is a wave packet? Show that phase velocity of de-Broglie wave is greater than the velocity of light. (1 + 3)

Ans. When plane waves of slightly different wavelength travel simultaneously in the same direction along a straight line, through a dispersive medium then successive groups of waves are produced. These waves are called wave packets. Each wave group travels with a velocity called group velocity.

$$\text{Phase velocity} = \frac{w}{K} \qquad \text{....(1)}$$

Also, phase velocity $= V_p = \gamma\lambda$.....(2)

For an electromagnetic wave

$$E = h\gamma$$

or

$$\gamma = \frac{E}{h} \qquad \text{....(3)}$$

According to de-Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \text{....(4)}$$

From eqn. (2), (3) and (4), we get

$$V_p = \gamma\lambda$$

$$= \frac{E}{h} \times \frac{h}{mv} = \frac{E}{mv} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

$$\boxed{V_p = \frac{C^2}{V}} \qquad \text{...(5)}$$

Since C >> v, Eqn (5) implies that the phase velocity of de-Broglie wave is greater than the velocity of light.

Q. 5. (c) A particle is confined to an one-dimensional infinite potential well of width 0.2×10^{-9} m. It is found that when the energy of the particle is 230 eV, its eignefunction has 5 antiodes. Find the mass of the particle and show that it can never have energy equal to 1 Kev. (3.5)

Ans. Energy of particle of mass "m" in a box of length "l" is given by

$$E_n = \frac{n^2h^2}{8ml^2}$$

Given

$$l = 0.2 \times 10^{-9} \text{m} = 2 \times 10^{-10} \text{ m}$$
$$h = 6.63 \times 10^{-34} \text{ J} - \text{S}$$
$$n = 5$$
$$E = 230 \text{ eV} = 368 \times 10^{-19} \text{ J}$$
$$m = ?$$

$$368 \times 10^{-19} = \frac{25 \times (6.63 \times 10^{-34})^2}{8 \times m \times (2 \times 10^{-10})^2}$$

$$m = \frac{25 \times (6.63 \times 10^{-34})^2}{8 \times (2 \times 10^{-10})^2 \times 368 \times 10^{-19}}$$

$$= \frac{1098.93 \times 10^{-68}}{11776 \times 10^{-39}}$$

$$m = 0.0933 \times 10^{-29} \text{ kg} = 9.33 \times 10^{-31} \text{ kg}$$

$$\boxed{m = 9.33 \times 10^{-31} \text{kg}}$$

Solving for energy at $n = 1$

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.33 \times 10^{-31} \times (2 \times 10^{-10})^2} = 0.147 \times 10^{-17} \text{J}$$

$$E_1 = \frac{0.147 \times 10^{-17}}{1.6 \times 10^{-19}} = 9.187 \text{eV}$$

For $n = 2 \Rightarrow E_2 = 36.75$ eV

For $n = 3 \Rightarrow E_3 = 82.683$ eV

For $n = 4 \Rightarrow E_4 = 146.992$ eV

For $n = 5 \Rightarrow E_5 = 229.675$ eV

$\therefore$ $E_n < 1$ KeV Or $E_n < 1000$ eV

WITH ANSWER

Q.1. *Define microstate of a system of particles.*

Ans. The distinct arrangement of particles of a system is known as its microstate.

Q.2. *Define macrostate of a system of particles.*

Ans. The arrangements of the particles of a system without distinguishing them from one another is known as macrostate of the system.

Q.3. *What are static and dynamic systems?*

Ans. A static system is one whose constituent particles are at rest where dynamic system is one whose constituent particles are in constant state of motion and are not stationary.

Q.4. *Explain the terms position space, momentum space and phase space?*

Ans. The position of a particle in a three dimensional static system is completely defined by three position co-ordinates x, y, z. This three dimensional space is called position space.

In a dynamic system, the particles move with various velocities and possess mometa. Let p_x, p_y and p_z be the components of the momentum along three co-ordinates. Thus momentum 'p' of a particle is completely specified by the three momentum co-ordinates p_x, p_y and p_z in three dimensional space called momentum space.

Combination of position and momentum spaces is called phase space. Thus phase space has six dimensions. A particle in phase space is represented by a point having six co-ordinates x, y, z, p_x, p_y, p_z.

Q.5. *What is the concept of phase space? What is it dimensionality?*

Ans. The phase space is a six dimensional space in which x, y, z, p_x, p_y, p_z represent mutually perpendicular directions.

Q.6. *What are the main points of difference between classical and quantum statistics?*

Ans. (*i*) In classical statistics the particles of the system are *distinguishable* whereas in quantum statistics these are considered as *indistinguishable*.

(*ii*) In classical statistics size of the cell in phase space can be made as small as we like whereas in quantum statistics volume of the cell cannot be less than h^3 where h is Planck's constant.

Q.7. *What is the minimum size of phase space cell in classical and quantum statistics?*

Ans. Minimum size of phase space cell in classical statistics is zero whereas in quantum statistics it is h^3

Q.7. *What is the minimum size of phase space cell in classical and quantum statistics?*

Ans. Minimum size of phase space cell in classical statistics is zero whereas in quantum statistics it is h^3.

Q.8. *Why the elementary volume of a cell in phase space for quantum particle cannot be zero?*

Ans. For a quantum particle, we cannot choose h as small as possible. It is because quantum statistics is governed by the uncertainty principle, which is given as

$$dx\, dp_x \geq h$$

$$dy\, dp_y \geq h$$

$$dz\, dp_z \geq h$$

where h is Planck's constant

$$\therefore \qquad dx\, dy\, dz\, dp_x\, dp_y\, dp_z \geq h^3$$

$$d\tau \geq h^3$$

since $h = 6.63 \times 10^{-34}$ Js and $d\tau \geq h^3$, so elementary volume of a cell in phase space cannot be zero.

Q.9. *Write expression for Maxwell-Boltzman energy and velocity distribution functions for an ideal gas.*

Ans. Energy distribution function

$$n(u) = \frac{2\pi N}{(\pi kT)^{3/2}} \sqrt{u}\, e^{-u/kT}$$

velocity distribution function

$$n(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

LONG QUESTIONS

1. Define the terms, microstates, macrostate, static and dynamic system.
2. Explain by giving example macrostates and microstates.
3. Define basic postulates of statistical physics.
4. What is phase space? Derive an expression for the number of cells in phase space in terms of volume, in position and momentum space.
5. Discuss the division of phase space volume into elementary cells and show that the minimum size of an elementary cell in the phase space in quantum mechanical system is h^3.
6. What are different kinds of statistics? Explain them.
7. Derive Boltzmann distribution law.
8. Derive Maxwell-Boltzmann law of distribution of molecular energies and molecular speed for an ideal gas under equilibrium.

...ESTIONS

Q1. *Distinguish between classical and quantum statistics?*

Ans. *(i)* Classical statistics applies to particles which obey the laws of classical mechanics whereas quantum statistics is applied to particles obeying quantum mechanical laws.

(ii) The state of the particle in classical statistics is specified by giving its position and momentum whereas the state of a particle in quantum statistics is described by the wave function of particle.

Q2. *What was the need to introduce quantum statistics?*

Ans. Classical statistics successfully explained the energy and velocity distribution of molecules of an ideal gas but failed to explain several experimentally observed phenomenon such as energy distribution of electrons in metals, black body radiations and photo-electric effect etc. Since the behaviour of particles constituting these systems is governed by the laws of quantum mechanics, a need was felt to develop quantum statistics.

Q3. *Mention briefly the assumptions of B.E. and F.D. statistics.*

Ans. Assumptions of B.E. statistics

(i) The particles of the system are identical and indistinguishable.

(ii) Any no. of particles can occupy a single cell in phase space.

(iii) The size of the cell cannot be less than h^3 where h is Planck's constant.

(iv) B.E. statistics is applicable to particles known as bosons.

Assumptions of F.D. statistics

(i) the particles of the system are identical and indistinguishable.

(ii) There cannot be more than one particle in a single cell in phase space.

(iii) The size of the cell cannot be less than h^3 where h is Planck's constant.

(iv) F.D. statistics is applicable to particles known as fermions.

Q4. *What are the constraints which are obeyed by a photon gas at a given temperature?*

Ans. The photon gas obeys only one constraint *i.e.* the total energy of the photon gas remains constant. But the number of photons is not constant.

Q5. *Write a short note on Bose-Einstein statistics?*

Ans. In Bose-Einstein statistics, particles are treated as *indistinguishable* and they don't obey Pauli's exclusion principle and so any number of particles can occupy a singie cell in phace space. It applies to particles having integral spin angular momentum (in units of $\hbar$) *i.e.* photon (spin 1), mesons, α–particles (spin zero) etc. and such particles are called bosons.

Q6. *What is free electrons Fermi gas?*

Ans. The gas of free electrons and non-interacting electrons subject to Pauli principle is called free electron Fermi gas.

gas.

Q7. *What is a photon gas? Are the number of photons constant?*

Ans. A system of black body radiations contained in an encloser is called photon gas. No, the number of photons in the encloser does not remain constant.

Q8. *What is the value of α for a photon gas?*

Ans. Value of α is zero for a photon gas. It is because the number of photons in the system is not constant.

Q9. *How does free electron gas differ from ordinary gas.*

Ans. Free electrons gas differs from ordinary gas in the following ways:

(*i*) Free electron gas is constituted by electrons which are charged particles while the atoms or molecules which constitute the ordinary gas are neutral.

Q10. *What do you understand by photon gas and electron gas? Which statistics is obeyed by them?*

Ans. In metallic conductors there are free electrons called *conduction electrons* and these electrons are free to move about inside the volume of the conductor like a gas and is called electron gas. It obeys F.D. statistics. A hollow enclosure, at a given constant temperature would be filled with radiations (called photons) which are characteristics of that temperature. These photons are supposed to form a photon gas. It obeys B.E. statistics.

Q11. *What is meant by Fermi energy of a metal?*

Ans. The energy of the highest filled level in metal at 0 K is known as Fermi energy. All the energy states above it are completely empty and all the energy states below it are completely filled.

Q12. *What is Fermi gas? Give an example.*

Ans. An assembly of indistinguishable elementary particles having half integral spin, called fermions confined in a volume is known as Fermi gas. The conduction electrons in a metal is an example of Fermi gas.

Q13. *Do electrons have zero energy at 0 K? Why Or What do you mean by zero point energy for Fermi gas*

Ans. The electrons do not have zero energy at 0 K due to Pauli exclusion principle—rather electrons have a definite energy at 0 K. This energy is given by $\overline{\varepsilon} = \dfrac{3n\,\varepsilon_F(0)}{5}$.

Q14. *Explain the conditions under which B.E. and F.D. statistics yield to classical statistics.*

Ans. Both these statistics yield to classical statistics at high temperatures and low concentrations.

Q15. *Explain why at occupation index << 1 B.E. and F.D. statistics give the same result as is given by M.B. statistics?*

Q15. *Explain why at occupation index << 1 B.E. and F.D. statistics give the same result as is given by M.B. statistics?*

Ans. The expression for the occupation index n_i/g_i for these statistics is

$$\frac{n_i}{g_i} = \frac{1}{e^{\alpha} e^{u_i/kT}} \quad \text{M.B. statistics}$$

$$\frac{n_i}{g_i} = \frac{1}{e^{\alpha} e^{u_i/kT} - 1} \quad \text{B.E. statistics}$$

$$\frac{n_i}{g_i} = \frac{1}{e^{\alpha} e^{u_i/kT} + 1} \quad \text{F.D. statistics}$$

If $(n_i/g_i) << 1$, then denominators of R.H.S. of equations (*ii*) and (*iii*) are very large. That is $e^{\alpha} e^{u_i/kT} >> 1$.

Then 1 can be neglected in comparison to $e^{\alpha} e^{ui/kT}$. Therefore $(n_i/g_i) = \dfrac{1}{e^{\alpha} e^{u_i/kT}}$

Q16. *Distinguish between boson and fermion.*

Ans. The particles which obey Pauli's exclusion principle are called fermions Electrons, positrons, protons and neutrons are examples.

The particles which have integral spin and do not obey Pauli's exclusion principle are called boson. Photons, π-mesons are examples.

Q17. *Distinguish the following particles as bosons or fermions (i) Hydrogen atom (ii) $^3He^+$ (iii) α-particle (iv) $^6Li^+$ ion (v) $^7Li^+$ ion (vi) hydrogen molecule.*

Ans. *(i)* Hydrogen atom contains 1 proton and 1 electron *i.e.* 2 fermions. Then it is a boson.

(ii) $^3He^+$ nucleus contains 3 fermions so it is a fermion.

(iii) α-particle is a nucleus of atom. It contains $2n$ and $2p$ *i.e.* 4 fermions. Thus it is a boson.

(iv) $^6L_i^+$ nucleus has 6 fermions, so it is boson.

(v) $^7L_i^+$ nucleus has 7 fermions, so it is fermion.

(vi) Hydrogen molecule contains $2e$ and $2p$ *i.e.* 4 fermion so it is boson.

Q18. *What is F.D. distribution law? explain.*

Ans. F.D. distribution law is

(vi) Hydrogen molecule contains 2e and 2p i.e. 4 fermion so it is boson.

Q18. *What is F.D. distribution law? explain.*

Ans. F.D. distribution law is

$$n_i = \frac{g_i}{e^{\alpha} e^{u_i/kT} + 1}$$

where k is Boltzmann constant and T is absolute temperature and $\alpha = -\mu/kT$ where μ is chemical potential per mole, n_i is number of particles in ith energy state and g_i is the degenracy of the ith energy level and u_i the energy of a particle associated with this level.

Q19. *How do you define Fermi energy and Fermi level at absolute zero?*

Ans. At absolute zero all the levels below a certain level will be filled with electrons and all the levels above it will be empty. The level which divides the filled and vacant levels is called Fermi level at absolute zero and the energy of this level is called Fermi energy at 0 K.

LONG QUESTIONS

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LONG QUESTIONS

1. State and explain clearly the basic differences between classical and quantum statistics.
2. Differentiate between distinguishable and indistinguishable particles. Explain with examples.
3. Derive an expression for the most probable distribution of particles for a system obeying Bose-Einstein statistics.
4. Starting from basic assumption of B.E. Statistics, derive the relation

$$n_i = \frac{g_i}{e^{\alpha} e^{u_i/kT} - 1}$$

where the symbols have their usual meanings.
5. Write down the postulates of B-E statistics. Derive the distribution of particles governed by B-E statistics.
6. Discuss the salient features of black body radiation.
7. Starting from basic assumption of F.D. statistics derive the relation

$$n_i = \frac{g_i}{e^{\alpha} e^{u_i/kT} + 1}$$

where the symbols have their usual meanings.
8. Apply the Fermi-Dirac distribution law to derive the energy distribution of free electrons in a conductor.
9. Using F-D distribution law, discuss thé behaviour of electrons in conductor at 0 K and at higher temperature.
10. (a) Analyse the difference between classical M.B. statistics and quantum B.E. and F.D. statistics. Show that in limiting case both B.E. and F.D. statistics are reduced to M.B. statistics.
(b) How does free electron gas differ from ordinary gas.
11. Give the significance of Fermi energy of a metal at absolute zero temperature. Show that at any normal temperature, half of the total number of particles of an electron gas are below the fermi level.
12. Starting from basic postulates, obtain F.D. distribution law.
13. What is the difference between a boson and fermion. Find an expression for the energy distribution for electron gas in a metal.

...AL PROBLEMS