Graph Theory

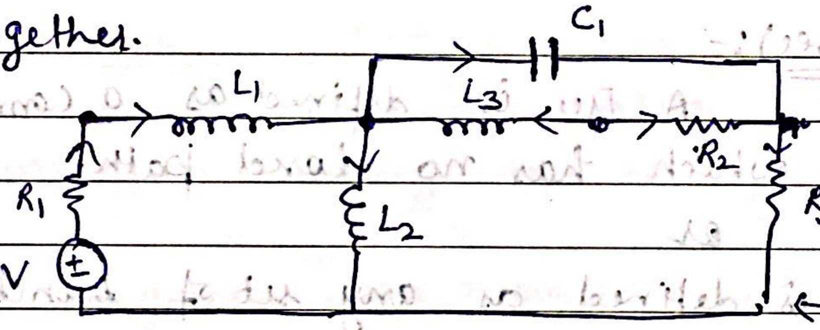
Graph :- Graph is defined as a collection of nodes and branches, which shows the geometrical interconnection of the elements of a network.

Branch :-

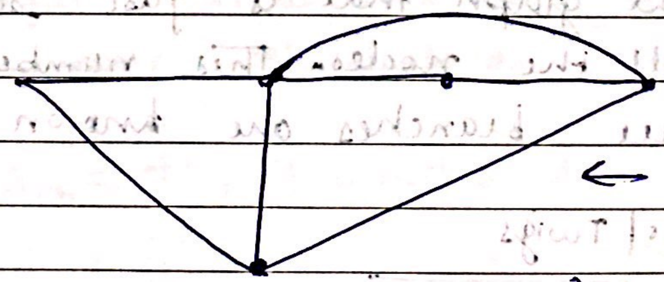
In graph, components are replaced by a line segments, these line segments are known as branches. 

Node :-

The end points of a branch or a common point at which two or more branches meet together.



← Network

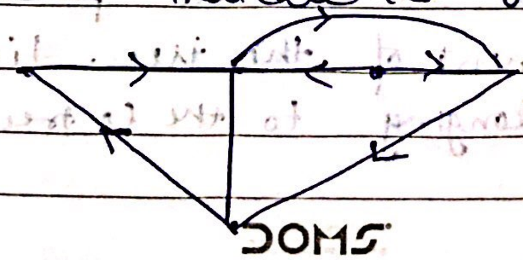


← Graph

(Undirected Graph.)

Directed Graph :-

A graph whose branches are oriented is known as directed or oriented graph or we can say each branch of graph carries an arrow to indicate its orientation.

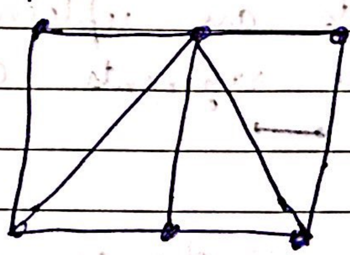


← Directed Graph

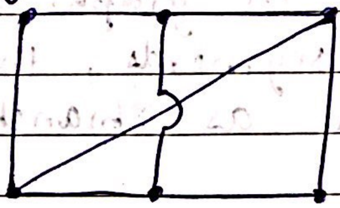
How to Draw a Graph:- (1) Replace All the elements by branches
(2) Short ckt the Voltage Source.
(3) open ckt the current Source.

Planar Graph:-

Graph is said to planar graph if that can be drawn on a sheet of paper without crossing lines.



Planar Graph.



Non-Planar Graph

~~Twigs~~ (Tree):-

A Tree is defined as a connected graph which has no closed path.

or

A tree is defined as any set of branches in the original graph that is just sufficient to connect all the nodes. This number is n-1.

The Tree branches are known as Twigs.

no. of Twigs

$$\boxed{n_t = n-1}$$

n → no. of Nodes.

Links or chords or Co-Tree:-

The branches of graph which are not in the tree form the Co-tree or Complement of the tree. Link is any branch belonging to the Co-tree.

* Graph = Tree ∪ Co-Tree
* Graph = Twigs ∪ Links.

No. of Links

$$nl = b - nt$$

b ↓ Total no. of Branches in a graph

nt ← No. of Twigs.

$$nl = b - (n-1)$$

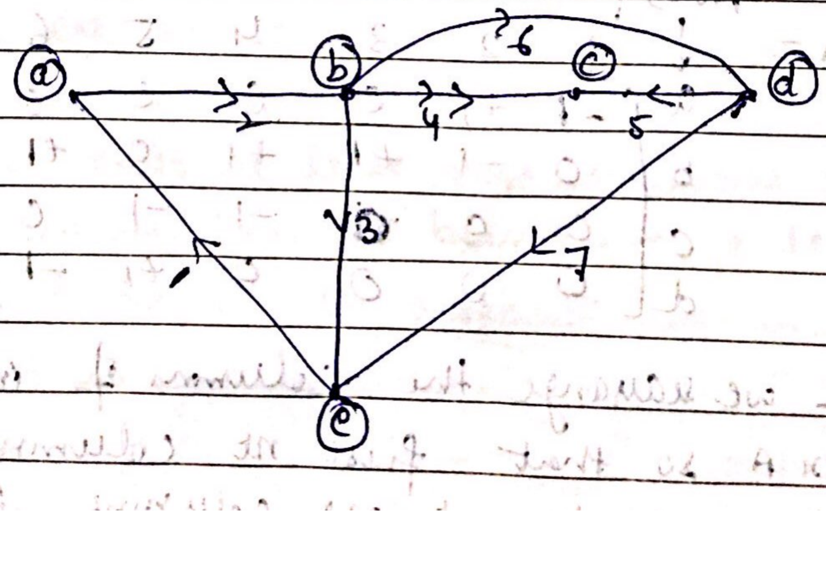
$$\boxed{nl = b - n + 1}$$

Incidence matrix:-

This matrix indicates which branch is entering and leaving which node.

for a graph with n nodes and b branches, the complete Incidence matrix A_c is an $n \times b$ matrix whose elements are defined as:-

a_{nb} = +1 ; if branch b leaves node n
-1 ; if branch b enters node n
0 ; if branch b is not incident with node n.



Complete ~~Incidence~~ Incidence matrix of this graph is :-

$$[A_a] = \begin{array}{c|ccccccc} \text{Nodes} \downarrow \,/\, \text{Branches} \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline a & -1 & +1 & 0 & 0 & 0 & 0 & 0 \\ b & 0 & -1 & +1 & +1 & 0 & +1 & 0 \\ c & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ d & 0 & 0 & 0 & 0 & +1 & -1 & +1 \\ e & +1 & 0 & -1 & 0 & 0 & 0 & -1 \end{array}$$

Since each branch b enters a single node and leaves a single other node, each column contains a single -1 and a single +1 with all elements equal to zero.

Reduced Incidence Matrix:-

By eliminating any one row, we get reduced incidence matrix A or incidence matrix A. The dimension of this matrix is $(n-1) \times b$. ie $n_t \times b$.

$$A = \begin{array}{c|ccccccc} \text{Nodes} \downarrow \,/\, \text{Branches} \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline a & -1 & +1 & 0 & 0 & 0 & 0 & 0 \\ b & 0 & -1 & +1 & +1 & 0 & +1 & 0 \\ c & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ d & 0 & 0 & 0 & 0 & +1 & -1 & +1 \end{array}$$

If we rearrange the columns of incident matrix A so that first n_t columns corresponds to twigs and next n_l columns to the links.

DOMS

$$
\begin{array}{c|cccc:ccc}
\text{Nodes} \downarrow \backslash \text{Branches} \rightarrow & & A_t & & & & A_l & \\
 & 1 & 3 & 4 & 7 & 2 & 5 & 6 \\
a & -1 & 0 & 0 & 0 & +1 & 0 & 0 \\
b & 0 & +1 & +1 & 0 & -1 & 0 & +1 \\
c & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\
d & 0 & 0 & 0 & +1 & 0 & +1 & -1
\end{array}
$$

$$[A] = \begin{bmatrix} -1 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & +1 & +1 & 0 & -1 & 0 & +1 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 & 0 & +1 & -1 \end{bmatrix}$$

$$[A] = [A_t : A_l]$$

Properties of Complete Incidence Matrix:-

1.) The sum of entries in any column is zero.
2.) The rank of complete incidence matrix of a oriented graph is $n-1$.
3.) The determinant of complete incidence matrix is always zero.

*Fundamental-Loop Matrix of Tiesets:-

If a link is added to a tree, the resulting graph contains one closed path, called a loop (or a circuit).

Properties of loop of a Graph:-

i.) There are exactly two paths between any pair of nodes in the circuit.
ii) There exist at least two branches in a Loop.
iii) Maximum possible branches in a loop are equal to no. of nodes in the graph.

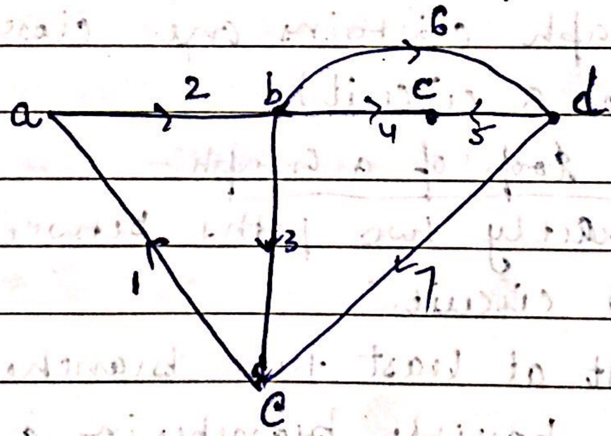
The loops which contains only one link are independent and are called as basic or fundamental loops (f-loops) or tiesets.

The no. of f-loops = no. of links
Orientation of f-loop is chosen to be same as that of its link.

Procedure for obtaining Tie-Set matrix:-

1.) Draw the Graph of given network.
2.) Select a Tree.
3.) No. of Tie-sets = No. of Links.
4.) Each Tie-set Contains one and only one link.
5.) The orientation of loop is defined by the orientation of the corresponding link.

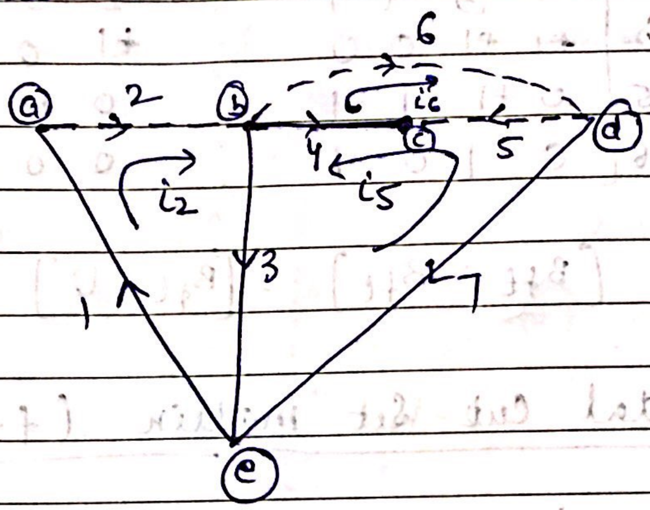
Step-1



Draw the Graph.

Step-2

Select a Tree



Step-3

No. of Tie-Sets = 3 (As there are 3 links)

i_2, i_5, i_6

Step-4

for a graph with l links and b branches the tie-set matrix is $nl \times b$ matrix, whose elements are defined as:—

b_{lb} = +1, if branch b is in the tie-set and orientation of tie-set & branch coincides

-1, if branch b is in the tie-set and orientation of tie-set & branch do not coincide

0, if branch b is not in the tie-set.

$$[B_f] = \begin{array}{c|ccccccc} \text{Tie-sets} \downarrow \; / \; \text{Branches} \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline i_2 & +1 & +1 & +1 & 0 & 0 & 0 & 0 \\ i_5 & 0 & 0 & +1 & -1 & +1 & 0 & -1 \\ i_6 & 0 & 0 & -1 & 0 & 0 & +1 & +1 \end{array}$$

__/__/____

$$\begin{array}{c c} & \text{Branches} \\ \text{Tiesets} & \begin{array}{cccc:ccc} 1 & 3 & 4 & 7 & 2 & 5 & 6 \end{array} \\ [B_f] = \begin{array}{c} i_2 \\ i_5 \\ i_6 \end{array} & \left[\begin{array}{cccc:ccc} +1 & +1 & 0 & 0 & +1 & 0 & 0 \\ 0 & +1 & -1 & -1 & 0 & +1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & +1 \end{array}\right] \end{array}$$

$$[B_f] = [B_{ft} : B_{fl}] = [B_{ft} : U]$$

✱ fundamental cut-set matrix (f-cutsets)

Cut-set is a minimal set of branches that if removed, divides a connected graph into two connected sub-graphs i.e. it separates the nodes of the graph into two groups.

Cut-sets containing only one twig are independent and are known as basic or fundamental cut-sets (f-cut-sets)

The no. of f-cutsets = No. of Twigs

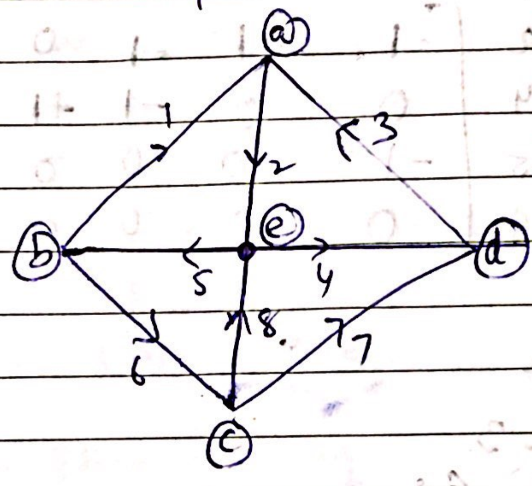
orientation of f-cut-set is chosen to be same as that of its twig.

Procedure for obtaining f-cut-set matrix:-

1. Draw the graph of given network.
2. Select a Tree.
3. No. of cut-sets = No. of Twigs.
4. Each cut-set contains one and only one twig.

(5) The orientation of cut-set is defined by the orientation of the corresponding twig.

Step 1:-



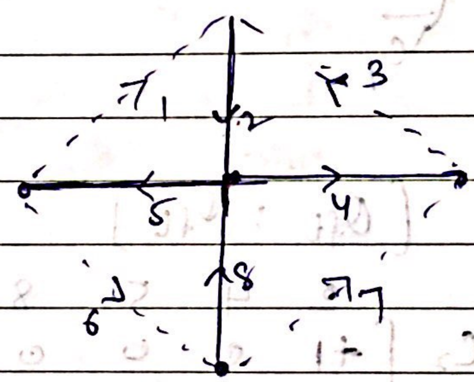
← Draw the Graph.

Step-2

$n = 5$, $nt = 4$

$b = 7$, No. of cut-set = 4

Select a Tree



Step 3:- No. of cut sets = No. of Twigs (4)

C_2, C_4, C_5, C_8

Step-4:-

$q_{Cb} = +1$ if branch b is in the cut-set and its orientation coincides with cut-set.

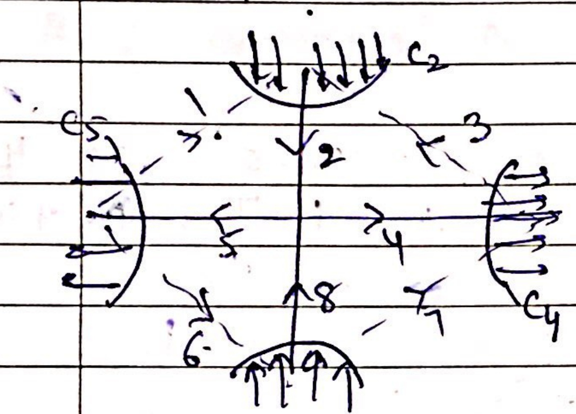
-1 if branch b is in the cut set and their direction do not coincides

0 if branch b is not in the cut set.

DOMS

Cut-Sets ↓ → Branches.

$$
[Q_f] = \begin{array}{c} \\ C_2 \\ C_4 \\ C_5 \\ C_8 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
-1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & +1 & 0 & 0 & +1 & 0 \\
-1 & 0 & 0 & 0 & +1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & +1 & +1
\end{bmatrix}
\end{array}
$$



$$[Q_f] = [Q_{ft} : Q_{fl}]$$

$$
= \begin{array}{c} \\ C_2 \\ C_4 \\ C_5 \\ C_8 \end{array}
\begin{array}{c}
\begin{array}{cccc:cccc} 2 & 4 & 5 & 8 & 1 & 3 & 6 & 7 \end{array} \\
\left[\begin{array}{cccc:cccc}
+1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\
0 & +1 & 0 & 0 & 0 & -1 & 0 & +1 \\
0 & 0 & +1 & 0 & -1 & 0 & -1 & 0 \\
0 & 0 & 0 & +1 & 0 & 0 & -1 & +1
\end{array}\right]
\end{array}
$$

$$[Q_f] = [U : Q_{fl}]$$

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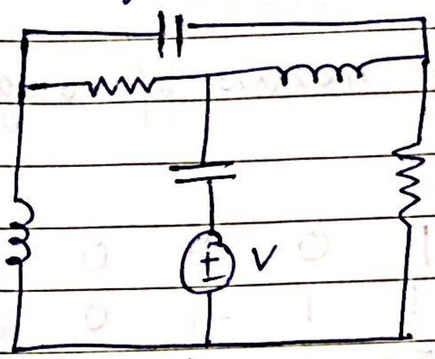
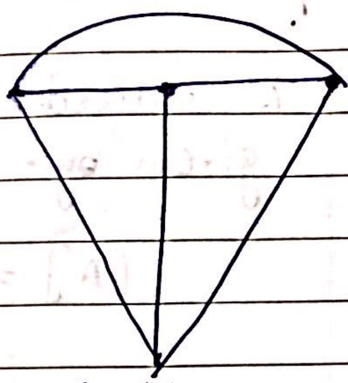
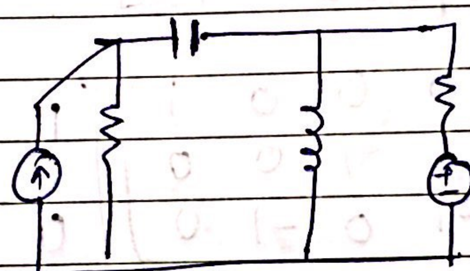
Q.1 Draw the graphs of given networks:-



(a) n/w

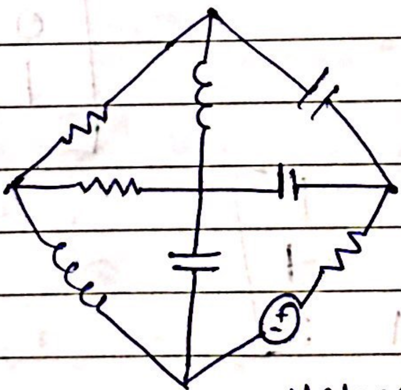


Graph

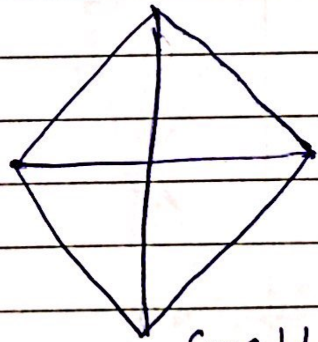


(b) n/w

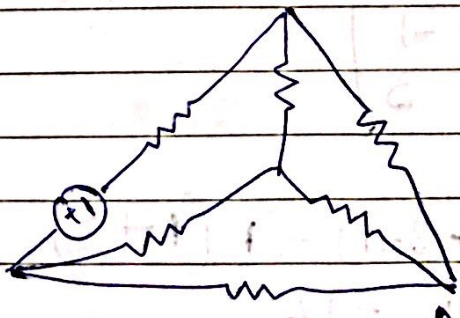
Graph.



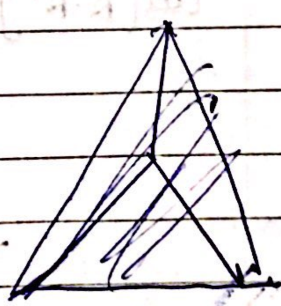
(c) Network.

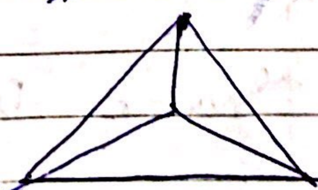


Graph.



(d) Network





Graph.

$$\boxed{\text{No. of Trees} = \det[A \cdot A^T]}$$

Q.1 A reduced incidence matrix of a graph is given by:-

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

Obtain the no. of possible Trees

Sol^n

$$A \cdot A^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$\det[A \cdot A^T] = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$$= 3(9-1) + 1(-3-1) - 1(1+3)$$

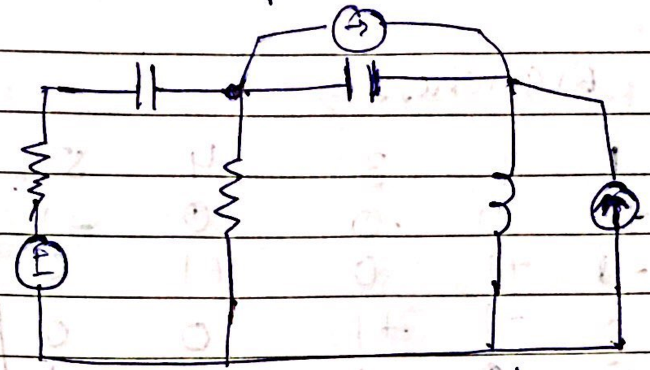
$$= 24 - 4 - 4 = 16$$

So $\boxed{\text{No of Trees} = 16}$ Ans.

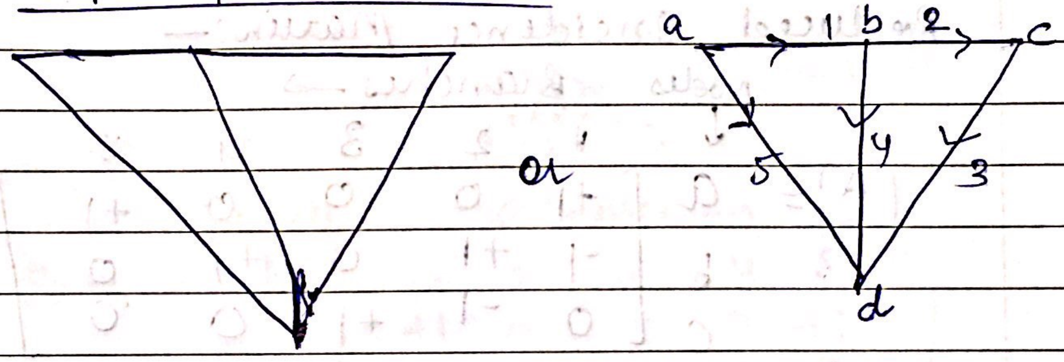
Q.2 Draw the possible Trees of given network and also obtain (1) Incidence matrix

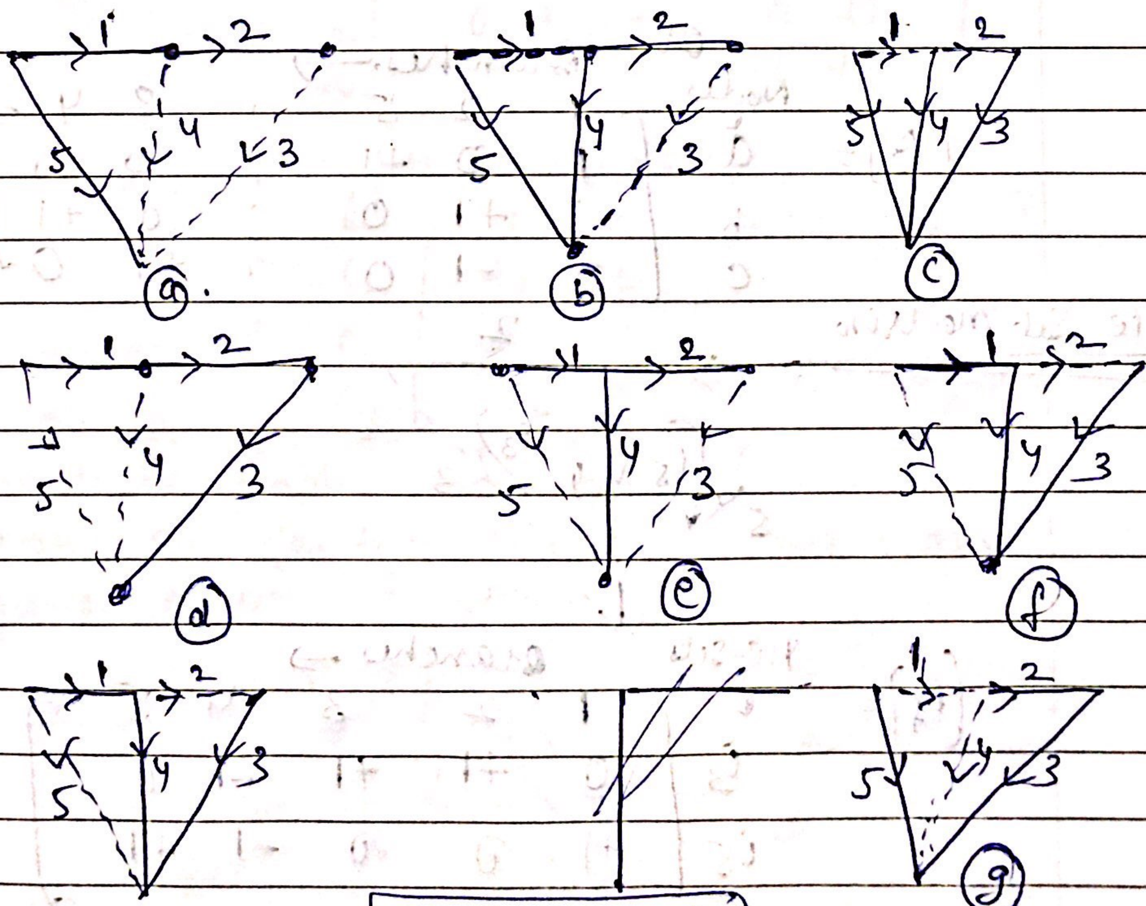
(2) Tie-Set matrix

(3) Cut-Set matrix



Sol^n Graph of the network





No. of trees = 8

The complete Incidence Matrix

Nodes ↓ Branches →

$$[A_c] = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} +1 & 0 & 0 & 0 & +1 \\ -1 & +1 & 0 & +1 & 0 \\ 0 & -1 & +1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

Reduced Incidence Matrix:—

Nodes ↓ Branches →

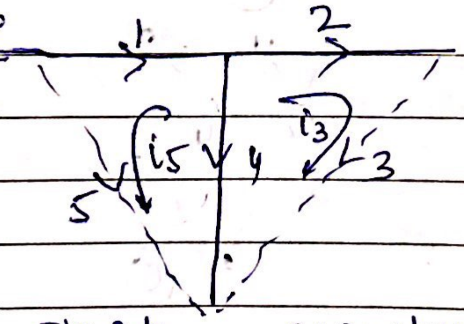
$$[A] = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} +1 & 0 & 0 & 0 & +1 \\ -1 & +1 & 0 & +1 & 0 \\ 0 & -1 & +1 & 0 & 0 \end{bmatrix} \end{array}$$

or

Nodes ↓ Branches →

$$[A] = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 5 & 3 & 4 \end{array} \\ \left[\begin{array}{ccc:cc} 1 & 0 & +1 & 0 & 0 \\ -1 & +1 & 0 & 0 & +1 \\ 0 & -1 & 0 & +1 & 0 \end{array}\right] \end{array}$$

Tie-Set Matrix



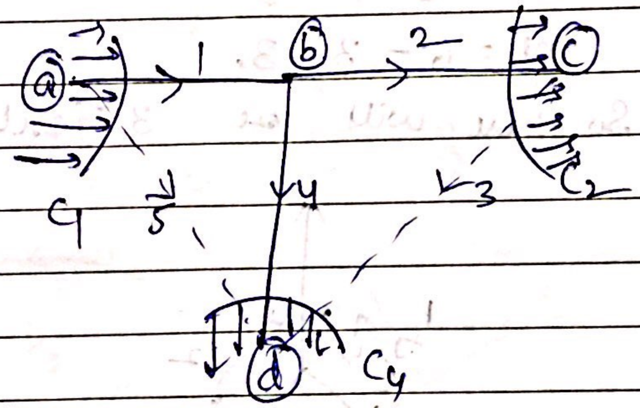
Tie-Sets ↓ Branches →

$$[B_f] = \begin{array}{c} \\ i_3 \\ i_5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & +1 & +1 & -1 & 0 \\ +1 & 0 & 0 & -1 & +1 \end{bmatrix} \end{array}$$

$$[B_f] = \begin{array}{c} \text{Tie-sets} \downarrow \\ \\ \\ \end{array} \quad \text{Branches} \rightarrow$$

$$\begin{array}{c|ccccc} & 1 & 2 & 4 & 3 & 5 \\ \hline i_3 & 0 & +1 & -1 & +1 & 0 \\ i_5 & +1 & 0 & -1 & 0 & +1 \end{array}$$

Cut-Set matrix



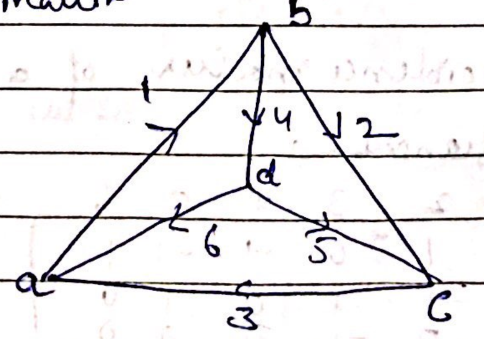
$$[Q_f] = \quad \text{Cut-Sets} \downarrow \quad \text{Branches} \rightarrow$$

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} +1 & 0 & 0 & 0 & +1 \\ 0 & +1 & -1 & 0 & 0 \\ 0 & 0 & +1 & +1 & +1 \end{bmatrix} \end{array}$$

$$[Q_f] = \quad \text{Cut-Sets} \downarrow \quad \text{Branches} \rightarrow$$

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccc:cc} 1 & 2 & 4 & 3 & 5 \end{array} \\ \left[\begin{array}{ccc:cc} +1 & 0 & 0 & 0 & +1 \\ 0 & +1 & 0 & -1 & 0 \\ 0 & 0 & +1 & +1 & +1 \end{array}\right] \end{array}$$

Q.3 Consider the graph shown in figure. Select a tree with twigs (4,5,6) for this tree and write down the tie-set matrix



Tieset

Sol^n

$n = 4$

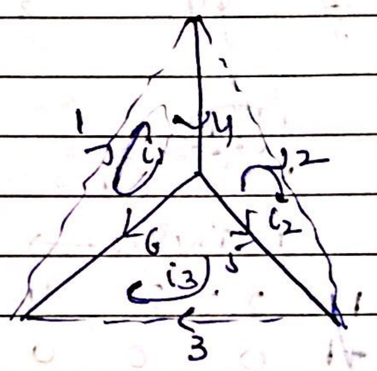
$b = 6$

$n_t = n-1 = 3$

$n_l = b-(n-1)$

$n_l = 6-3 = 3.$

So there will be 3 Tiesets :-



Tie-sets ↓ Branches →

$$[B_f] = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} +1 & 0 & 0 & +1 & 0 & +1 \\ 0 & +1 & 0 & -1 & -1 & 0 \\ 0 & 0 & +1 & 0 & +1 & -1 \end{bmatrix} \end{array}$$

or

Tie-Sets ↓ Branches →

$$[B_f] = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \end{array}\begin{array}{c} \begin{array}{cccccc} 4 & 5 & 6 & 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc|ccc} +1 & 0 & 1 & +1 & 0 & 0 \\ -1 & -1 & 0 & 0 & +1 & 0 \\ 0 & +1 & -1 & 0 & 0 & +1 \end{array}\right] \end{array}$$

Q.4 The reduced incidence matrix of a graph is given by

nodes ↓ Branches →

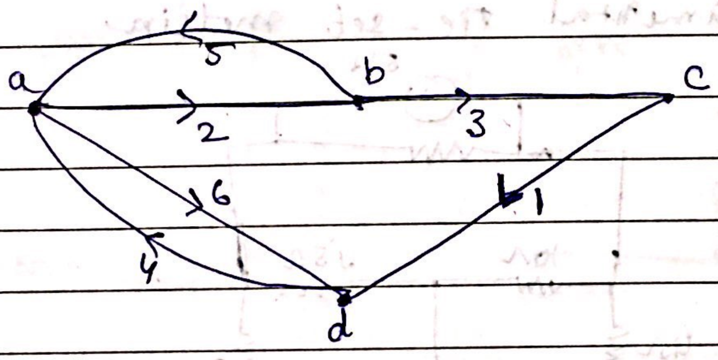
$$[A] = \begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Draw the oriented graph. Select a tree and find f- cutset matrix

Soln

Complete Incidence Matrix :-

$$A_C = \begin{array}{c|cccccc} \text{Nodes} \downarrow & \multicolumn{6}{c}{\text{Branches} \rightarrow} \\ & 1 & 2 & 3 & 4 & 5 & 6 \\ a & 0 & 1 & 0 & -1 & -1 & 1 \\ b & 0 & -1 & 1 & 0 & 1 & 0 \\ c & 1 & 0 & -1 & 0 & 0 & 0 \\ d & -1 & 0 & 0 & 1 & 0 & -1 \end{array}$$

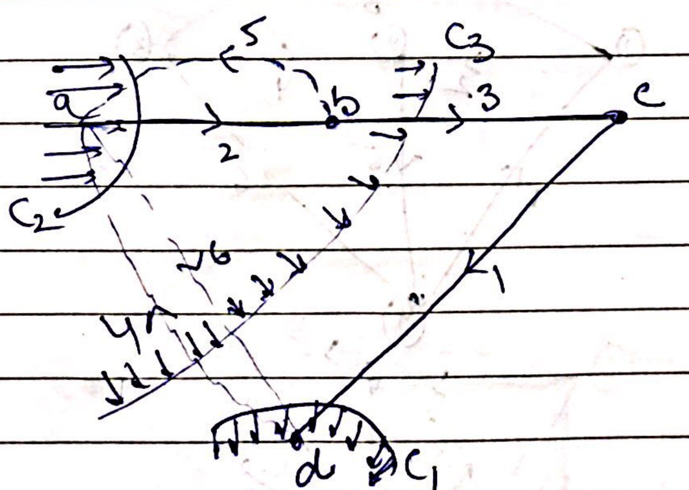


$n = 4$

$n_t = n-1 = 3.$

$n_l = b - n + 1 = 6 - 3 = 3$

Tree



$$Q_f = \begin{array}{c|cccccc} \text{Cut-Sets} \downarrow & \multicolumn{6}{c}{\text{Branches} \rightarrow} \\ & 1 & 2 & 3 & 4 & 5 & 6 \\ C_1 & +1 & 0 & 0 & -1 & 0 & +1 \\ C_2 & 0 & +1 & 0 & -1 & -1 & +1 \\ C_3 & 0 & 0 & +1 & -1 & 0 & +1 \end{array}$$

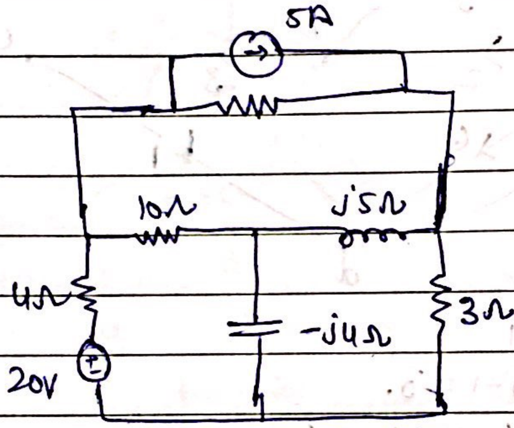
DOMS

$$\begin{array}{c|cccccc} \text{Cut-Sets} \downarrow & & & \text{Branches} \rightarrow & & & \\ & 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

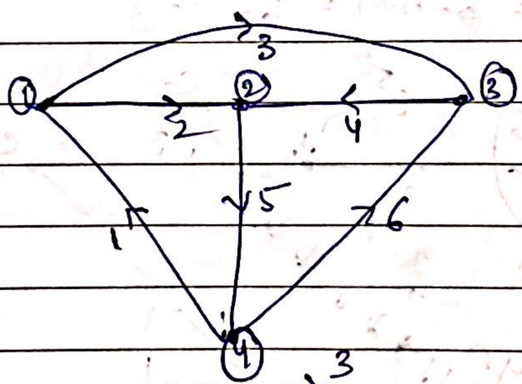
$$Q_f = \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$Q_f = [U : Q_{fl}]$$

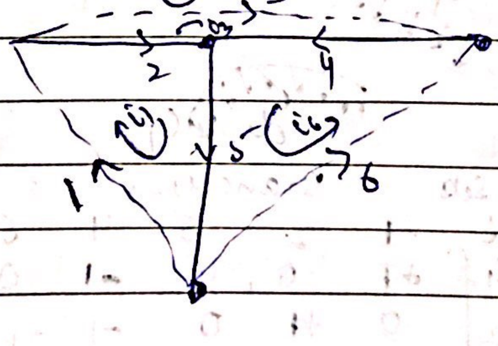
Q.5 For a given network draw the graph. Select 2,4,5 as tree branches and obtain fundamental tie-set matrix.



Sol^n



$n = 4$
$b = 6$
$n_t = 3$
$\therefore n_l = 3$

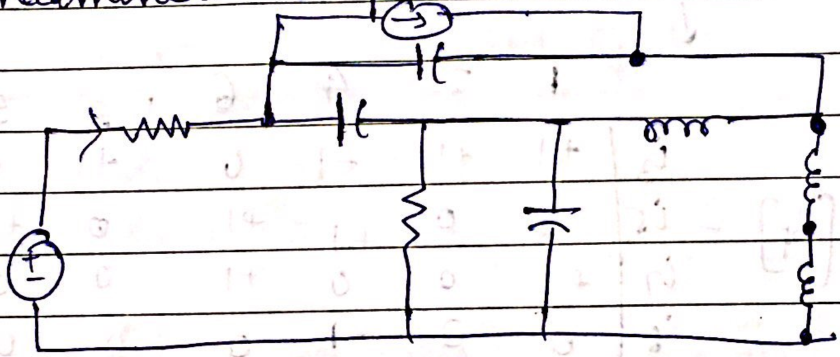


$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

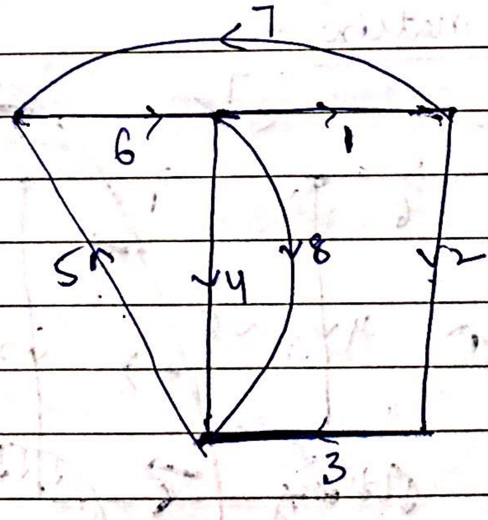
Tiesets ↓ Branches →

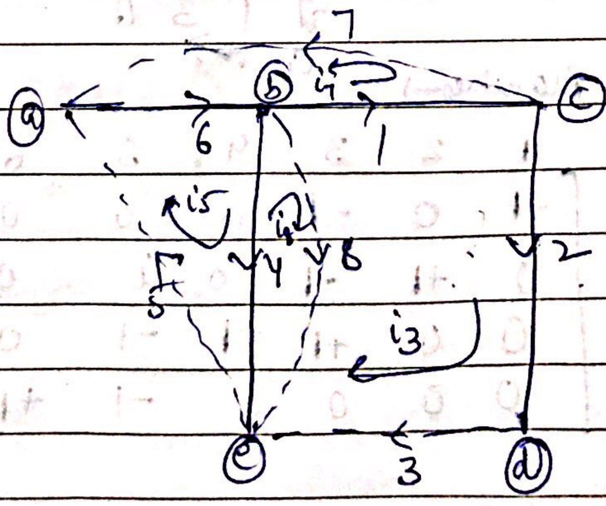
$$B_f = \begin{array}{c} i_1 \\ i_3 \\ i_6 \end{array}\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & +1 & +1 & 0 & 0 \\ 0 & 0 & 0 & +1 & +1 & +1 \end{bmatrix}$$

Q.6 Draw the graph of given circuit and obtain fundamental loop matrix and cut-set matrix.



Soln





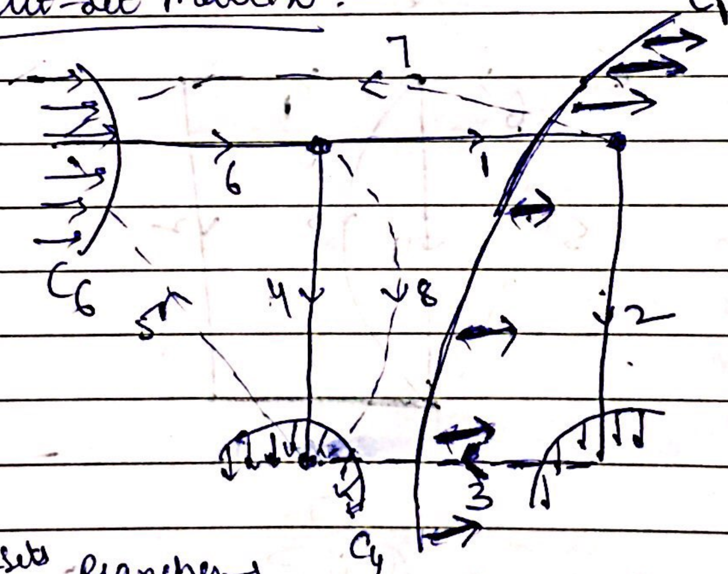
Tie-Sets ↓ Branches →

$$[B_f] = \begin{array}{c} \\ i_3 \\ i_5 \\ i_7 \\ i_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} +1 & +1 & +1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & +1 \end{bmatrix} \end{array}$$

Tie-Sets ↓ Branches →

$$[B_f] = \begin{array}{c} \\ i_3 \\ i_5 \\ i_7 \\ i_8 \end{array} \begin{array}{c} \begin{array}{cccc|cccc} 1 & 2 & 4 & 6 & 3 & 5 & 7 & 8 \end{array} \\ \left[\begin{array}{cccc|cccc} +1 & +1 & -1 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & +1 & +1 & 0 & +1 & 0 & 0 \\ +1 & 0 & 0 & +1 & 0 & 0 & +1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & +1 \end{array}\right] \end{array}$$

for Cut-set Matrix :-



Cut-Sets ↓ Branches →

$$[Q_f] = \begin{array}{c} \\ C_1 \\ C_2 \\ C_4 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} +1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & +1 & -1 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & -1 & +1 & -1 & 0 \end{bmatrix} \end{array}$$

$$
\begin{array}{c|cccc|cccc}
\text{Cut-Sets} \downarrow \;\; \text{Branches} \rightarrow & 1 & 2 & 4 & 6 & 3 & 5 & 7 & 8 \\
\hline
C_1 & +1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\
C_2 & 0 & +1 & 0 & 0 & -1 & 0 & 0 & 0 \\
C_4 & 0 & 0 & +1 & 0 & +1 & -1 & 0 & +1 \\
C_6 & 0 & 0 & 0 & +1 & 0 & -1 & -1 & 0
\end{array}
$$

$[Q_f] =$ the matrix above.

★ **Sub-matricies of A, B_f and Q_f :-**

$$A = [A_t : A_l]$$

$$B_f = [B_{ft} : B_{fl}] = [B_{ft} : U]$$

$$Q_f = [Q_{ft} : Q_{fl}] = [U : Q_{fl}]$$

Inter-Relation Among various Matricies:-

① **Relation Between A & B_f :-**

$$A \cdot B_f^T = 0$$

$$[A_t : A_l]\begin{bmatrix} B_{ft}^T \\ \vdots \\ U \end{bmatrix} = 0$$

$$A_t B_{ft}^T + A_l = 0$$

$$B_{ft}^T = -A_t^{-1} A_l$$

So $\boxed{B_{ft} = \left[-A_t^{-1} A_l\right]^T}$ — (A)

② Relation Between A & Q_f:—

KCL Equations

$$A I_b = 0$$

$$Q_f I_b = 0$$

where $I_b = \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix}$

So

$$\begin{bmatrix} A_t : A_l \end{bmatrix} \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix} = 0$$

$$A_t I_t + A_l I_l = 0$$

$$I_t = -A_t^{-1} A_l I_l \quad \text{—(1)}$$

and

$$\begin{bmatrix} Q_{ft} : Q_{fl} \end{bmatrix} \cdot \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix} = 0$$

$$\begin{bmatrix} U : Q_{fl} \end{bmatrix} \cdot \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix} = 0$$

$$I_t + Q_{fl} I_l = 0$$

$$I_t = -Q_{fl} I_l \quad \text{—(2)}$$

from eqn (1) & (2):—

$$\cancel{-} A_t^{-1} A_l I_l = \cancel{-} Q_{fl} I_l$$

$$\boxed{Q_{fl} = A_t^{-1} A_l} \quad \text{—(B)}$$

③ Relation Between B_f and Q_f:-

from Eqⁿ Ⓐ & Ⓑ :-

$$Q_{fl} = -B_{ft}^T$$

$$B_{ft} = -Q_{fl}^T$$

* Relationships Among Parameters :-

KVL : $B_f V_b = 0$

KCL : $A I_b = 0$

$Q_f I_b = 0$

$$V_b = \begin{bmatrix} V_t \\ \cdots \\ V_l \end{bmatrix} \quad \text{and} \quad I_b = \begin{bmatrix} I_t \\ \cdots \\ I_l \end{bmatrix}$$

① Relation Between Link Voltages and Twig Voltages:-

$$\begin{bmatrix} B_{ft} & \vdots & U \end{bmatrix} \begin{bmatrix} V_t \\ \cdots \\ V_l \end{bmatrix} = 0$$

$$B_{ft} V_t + V_l = 0$$

$$\boxed{\begin{gathered} V_l = -B_{ft} V_t \\ \text{or} \\ V_l = Q_{fl}^T V_t \end{gathered}}$$

② Relation Between Branch Voltages & Twig Voltages:-

$$V_b = \begin{bmatrix} V_t \\ \cdots \\ V_l \end{bmatrix} = \begin{bmatrix} U V_t \\ \cdots \\ Q_{fl}^T V_t \end{bmatrix}$$

$$\boxed{V_b = Q_f^T V_t}$$

DOMS

___/___/___

(4) Relation between Branch voltages and Node voltages:—

$$\boxed{V_b = A^T V_n}$$

V_n → Voltages of various Nodes

(5) Relation Between Twig Currents & Link Currents:—

$$I_t = -A_t^{-1} A_l I_l = -Q_{fl} I_l = B_{ft}^T I_l$$

(6) Relation Between Branch Currents & Link Currents:—

$$I_b = \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix} = \begin{bmatrix} B_{ft}^T I_l \\ \vdots \\ U I_l \end{bmatrix} = \begin{bmatrix} B_{ft}^T \\ \vdots \\ U \end{bmatrix} I_l$$

$$\boxed{I_b = B_f^T I_l}$$

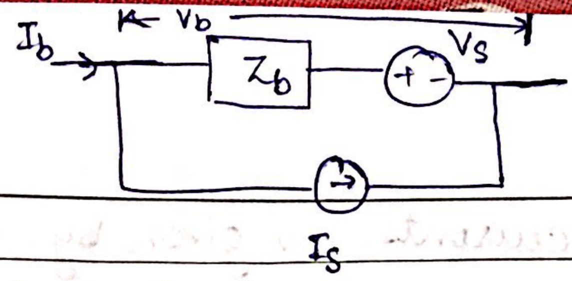
★ **Network Analysis:—**

For a general network with many branches, branch voltages and branch currents can be written as:—

$$V_b = V_s + Z_b(I_b - I_s) \quad —(1)$$

$$I_b = I_s + Y_b(V_b - V_s) \quad —(2)$$

where
V_b: Branch voltage Matrix of $b \times 1$
I_b: Branch current " of $b \times 1$
V_s: Voltage Source matrix of $b \times 1$
I_s: Current Source Matrix of $b \times 1$
Z_b: Branch Impedance matrix of $b \times b$
Y_b: Branch Admittance matrix of $b \times b$.



① Loop Analysis :-

The Branch Voltages are given by eqⁿ ① :-

$$V_b = V_s + Z_b\,(I_b - I_s)$$

Premultiplication by B_f Gives :-

$$B_f V_b = B_f V_s + B_f Z_b\,(I_b - I_s)$$

$$0 = B_f V_s + B_f Z_b\,(B_f^T I_l - I_s)$$

$$\left[\begin{array}{l}\text{Since } B_f V_b = 0 \\ \quad \text{and } B_f^T I_l = I_b\end{array}\right]$$

$$B_f Z_b I_s - B_f V_s = B_f Z_b B_f^T I_l$$

Let

$$B_f Z_b I_s - B_f V_s = V_l.$$

$$B_f Z_b B_f^T = Z_l$$

$$\boxed{V_l = Z_l\, I_l}$$

This is called as Matrix Loop Equation

where

$Z_l \rightarrow$ Loop Impedance Matrix

$I_l \rightarrow$ Loop current matrix

$$\boxed{I_l = Z_l^{-1} V_l}$$

DOMS

(5) Nodal Analysis:-

The branch currents are given by eqn (2):-

$$I_b = I_s + Y_b (V_b - V_s)$$

Premultiplication by A_1 gives:-

$$A I_b = A I_s + A Y_b (V_b - V_s)$$

$$0 = A I_s + A Y_b (V_b - V_s)$$

since $\begin{bmatrix} A I_b = 0 \\ V_b = A^T V_n \end{bmatrix}$

or

$$A Y_b V_s - A I_s = A Y_b A^T V_n$$

Let

$$A Y_b V_s - A I_s = I_n$$

$$A Y_b A^T = Y_n$$

$$\boxed{I_n = Y_n V_n}$$

This is called as Matrix Node equation

where Y_n : Node Admittance matrix

V_n : Node Voltage matrix

The node voltages are obtained by the relation:-

$$\boxed{V_n = Y_n^{-1} I_n}$$

$\_/\_/\_$

③ Cut-Set Analysis:-

The Branch currents are given by eq^n ②:-

$$I_b = I_s + Y_b (V_b - V_s)$$

Now

$$Q_f I_b = Q_f I_s + Q_f Y_b (V_b - V_s) = 0$$

Since $\boxed{Q_f I_b = 0}$

$$Q_f I_s + Q_f Y_b (V_b - V_s) = 0$$

Since $\boxed{V_b = Q_f^T V_t}$

$$Q_f [I_s + Y_b (Q_f^T V_t - V_s)] = 0$$

$$Q_f Y_b V_s - Q_f I_s = Q_f Y_b Q_f^T V_t$$

Let $Q_f Y_b V_s - Q_f I_s = I$

$$Q_f Y_b Q_f^T = Y_t$$

Then $\boxed{I = Y_t V_t}$

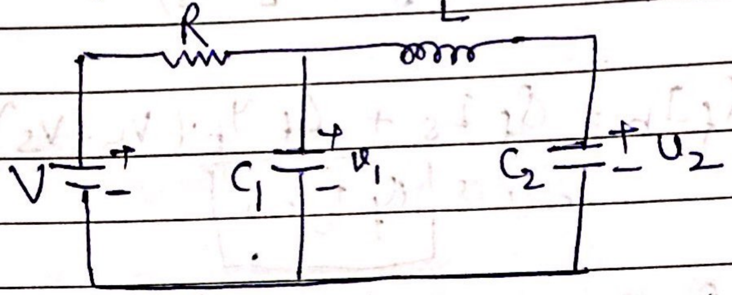
This is called as matrix cut-set equation or Node pair equation.

where

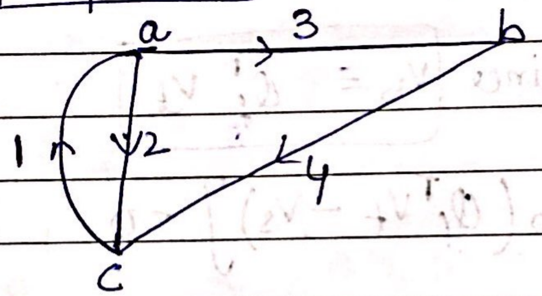
Y_t: Node pair Admittance matrix

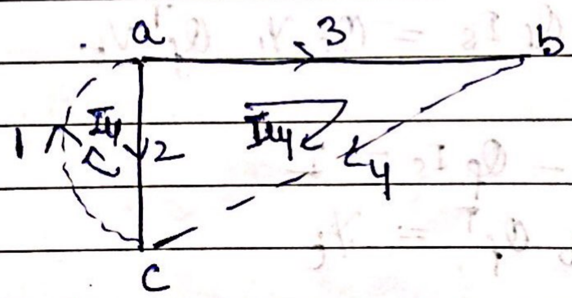
V_t: Tree Branch voltage matrix.

Q.7 Write the matrix loop equation for the n/w given, using the Loop Analysis.



Sol^n Graph of Network.—





Tie-Set matrix

Tiesets ↓ Branches →

$$\begin{array}{c} \\ I_{l_1} \\ I_{l_4} \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} +1 & +1 & 0 & 0 \\ 0 & -1 & +1 & +1 \end{bmatrix} \end{array}$$

Branch Admittance Matrix

$$Z_b = \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & \frac{1}{C_1 s} & 0 & 0 \\ 0 & 0 & Ls & 0 \\ 0 & 0 & 0 & \frac{1}{C_2 s} \end{bmatrix}$$

The loop Impedance matrix :-

$$Z_l = B_f Z_b B_f^T$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & \frac{1}{C_1 s} & 0 & 0 \\ 0 & 0 & Ls & 0 \\ 0 & 0 & 0 & \frac{1}{C_2 s} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} R & 0 \\ \frac{1}{C_1 s} & -\frac{1}{C_1 s} \\ 0 & Ls \\ 0 & \frac{1}{C_2 s} \end{bmatrix}$$

$$Z_l = \begin{bmatrix} R + \frac{1}{C_1 s} & -\frac{1}{C_1 s} \\ -\frac{1}{C_1 s} & Ls + \frac{1}{C_1 s} + \frac{1}{C_2 s} \end{bmatrix}$$

Voltage Source matrix:-

$$V_s = \begin{bmatrix} -V \\ v_1 \\ 0 \\ v_2 \end{bmatrix}$$

Current Source matrix:-

$$I_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence

$$V_l = B_f Z_b I_s - B_f V_s$$

$$= 0 - B_f V_s \quad [\text{As } I_s = 0]$$

$$= -\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -V \\ V_1 \\ 0 \\ V_2 \end{bmatrix}$$

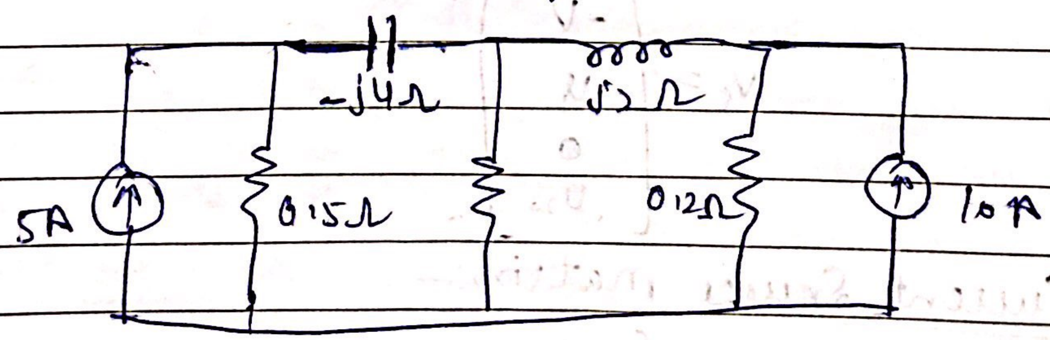
$$V_l = \begin{bmatrix} V - V_1 \\ V_1 - V_2 \end{bmatrix}.$$

So, the matrix loop equation can be written as :-

$$\begin{bmatrix} V - V_1 \\ V_1 - V_2 \end{bmatrix} = \begin{bmatrix} R + \frac{1}{C_1 s} & -\frac{1}{C_1 s} \\ -\frac{1}{C_1 s} & Ls + \frac{1}{C_1 s} + \frac{1}{C_2 s} \end{bmatrix} \begin{bmatrix} I_{l_1} \\ I_{l_2} \end{bmatrix}$$

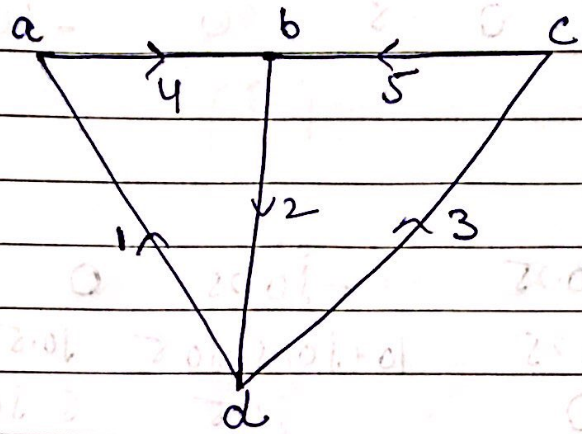
Ans

Q.8 For a given network, obtain the incidence matrix, The node admittance matrix and matrix node equation.



Sol^n

Graph of Network:—



If Node 'd' is taken as the reference.
Then Incidence Matrix is.—

$$[A] = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline a & -1 & 0 & 0 & 1 & 0 \\ b & 0 & 1 & 0 & -1 & -1 \\ c & 0 & 0 & -1 & 0 & 1 \end{array}$$

(Nodes ↓, Branches →)

The Branch Admittance matrix:—

$$Y_b = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & j0.25 & 0 \\ 0 & 0 & 0 & 0 & -j0.5 \end{bmatrix}$$

Node Admittance matrix:—

$$Y_n = A Y_b A^T$$

$$AY_b = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & j0.25 & 0 \\ 0 & 0 & 0 & 0 & -j0.5 \end{bmatrix}$$

DOMS

Then

$$AY_bA^T = \begin{bmatrix} -2 & 0 & 0 & j0.25 & 0 \\ 0 & 10 & 0 & -j0.25 & j0.5 \\ 0 & 0 & -5 & 0 & -j0.5 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

So

$$Y_n = \begin{bmatrix} 2+j0.25 & -j0.25 & 0 \\ -j0.25 & 10+j0.25-j0.5 & j0.5 \\ 0 & j0.5 & 5-j0.5 \end{bmatrix}$$

<u>Voltage Source Matrix:-</u>

$$[V_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

<u>Current Source Matrix</u>

$$[I_s] = \begin{bmatrix} 5 \\ 0 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

Then $I_n = AY_bV_s - AI_s$

$= 0 - AI_s$

$$= -\begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

DOMS

$$I_n = -\begin{bmatrix} -5 \\ 0 \\ -10 \end{bmatrix}$$

$$I_n = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix}$$

So the matrix Node equation :-

$$I_n = Y_n V_n$$

$$\begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix} = \begin{bmatrix} 2+j0.25 & -j0.25 & 0 \\ -j0.25 & 10-j0.25 & j0.5 \\ 0 & j0.5 & 5-j0.5 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Q.9 The reduced incidence matrix of a graph is given. Draw the graph and obtain the f-loop and f-cut set matrices. Also verify the result.

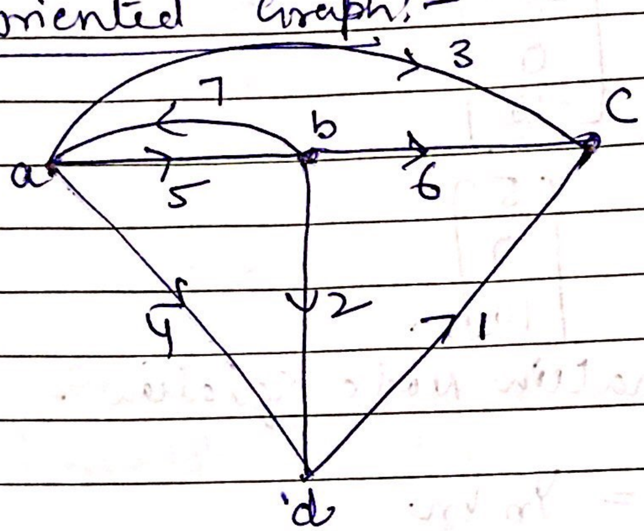
Sol^n

$$A = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & -1 & 0 & 0 & -1 & 0 \end{bmatrix} \end{array} \leftarrow \text{Given}$$

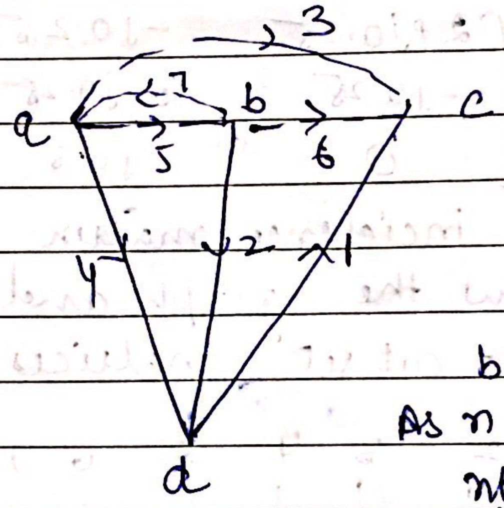
The Complete Incidence Matrix.

$$A_c = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & -1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The oriented Graph:-



Tree

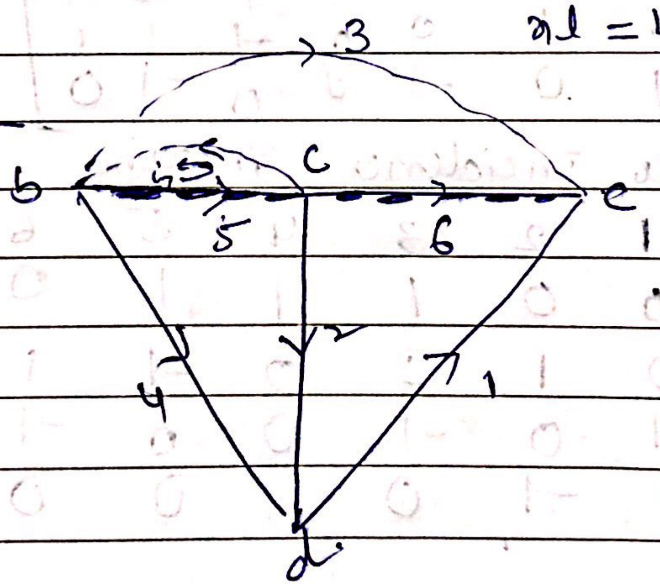


$b = 7$

As $n = 4$

$n_t = 3$

$n_l = 4$. = No. of f-loops



f-loop matrix:-

f-loops ↓ Branches →

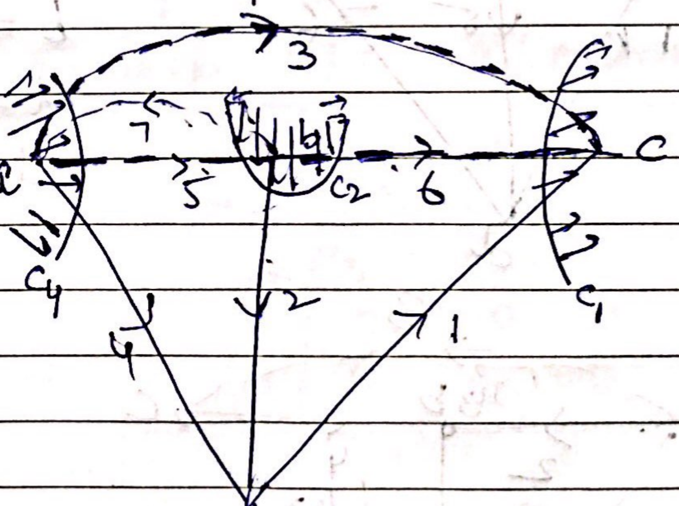
$$
\begin{array}{c c}
 & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
[B_f] = \begin{array}{c} i_3 \\ i_5 \\ i_6 \\ i_7 \end{array} & \begin{bmatrix} -1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

or

$$
\begin{array}{c c}
 & \begin{array}{ccc|cccc} \multicolumn{3}{c|}{\text{Twigs}} & \multicolumn{4}{c}{\text{Links}} \\ 1 & 2 & 4 & 3 & 5 & 6 & 7 \end{array} \\
[B_f] = \begin{array}{c} i_3 \\ i_5 \\ i_6 \\ i_7 \end{array} & \left[\begin{array}{ccc|cccc} -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]
\end{array}
$$

for f-Cut Set matrix

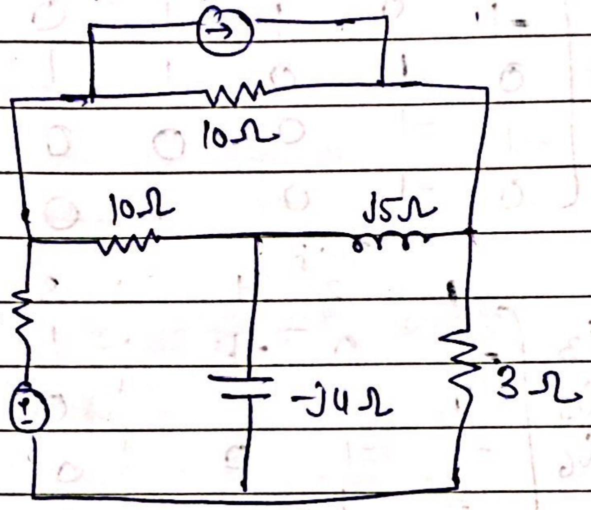
No of f-Cutsets = No. of Twigs = 3.



Cutsets ↓ Branches →

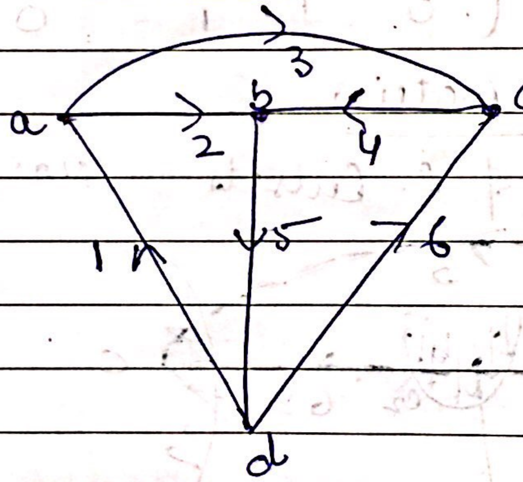
$$
\begin{array}{c c}
 & \begin{array}{ccc|cccc} 1 & 2 & 4 & 3 & 5 & 6 & 7 \end{array} \\
Q_f = \begin{array}{c} c_1 \\ c_2 \\ c_4 \end{array} & \left[\begin{array}{ccc|cccc} +1 & 0 & 0 & +1 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & +1 & +1 & +1 & 0 & -1 \end{array}\right]
\end{array}
$$

Q.10 For the network as shown in figure, draw the network graph, select 2,4,5 as tree branches, obtain the loop incidence matrix and loop equations



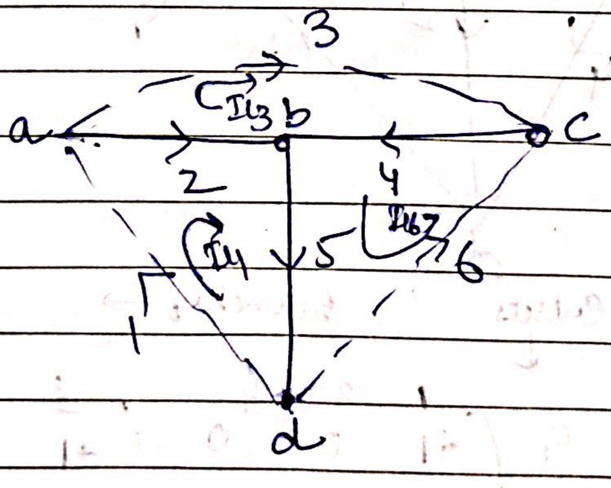
Soln.

Graph



$n = 4$
$b = 6$
$nt = 3$
$nl = 3$

Tree



__/__/____

f-loop matrix

Tiesets ↓ Branches →

$$[B_f] = \begin{array}{c} \\ i_{l_1} \\ i_{l_3} \\ i_{l_6} \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

The Branch Impedance Matrix:-

$$[Z_b] = \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & j5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now, The loop Impedance Matrix:-

$$Z_l = B_f Z_b B_f^T$$

$$= \begin{bmatrix} 4 & 10 & 0 & 0 & -j4 & 0 \\ 0 & -10 & 10 & j5 & 0 & 0 \\ 0 & 0 & 0 & j5 & -j4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Z_l = \begin{bmatrix} 14-j4 & -10 & -j4 \\ -10 & 20+j5 & j5 \\ -j4 & j5 & 3+j \end{bmatrix}$$

DOMS

The voltage and Current Source matricies are—

$$V_S = \begin{bmatrix} -20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad I_S = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So

$$V_l = B_f Z_b I_s - B_f V_S$$

$$= \begin{bmatrix} 4 & 10 & 0 & 0 & -j4 & 0 \\ 0 & -10 & 10 & j5 & 0 & 0 \\ 0 & 0 & 0 & j5 & -j4 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 50 \\ 0 \end{bmatrix} - \begin{bmatrix} -20 \\ 0 \\ 0 \end{bmatrix}$$

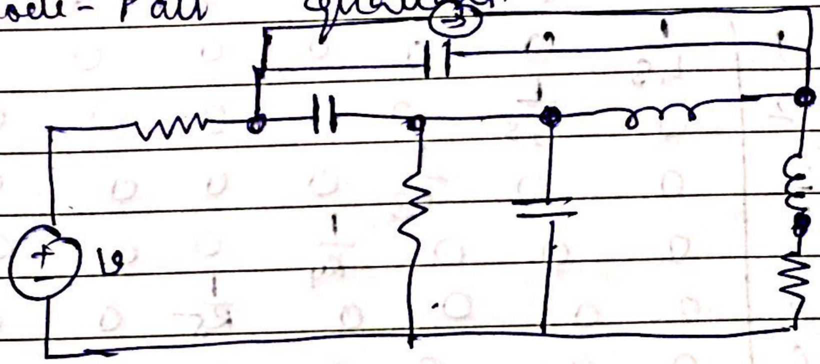
$$= \begin{bmatrix} 20 \\ 50 \\ 0 \end{bmatrix}$$

So, The matrix loop equation is :—

$$\begin{bmatrix} 20 \\ 50 \\ 0 \end{bmatrix} = \begin{bmatrix} 14-j4 & -10 & -j4 \\ -10 & 20+j5 & j5 \\ -j4 & j5 & 3+j \end{bmatrix} \begin{bmatrix} I_{l1} \\ I_{l3} \\ I_{l6} \end{bmatrix}$$

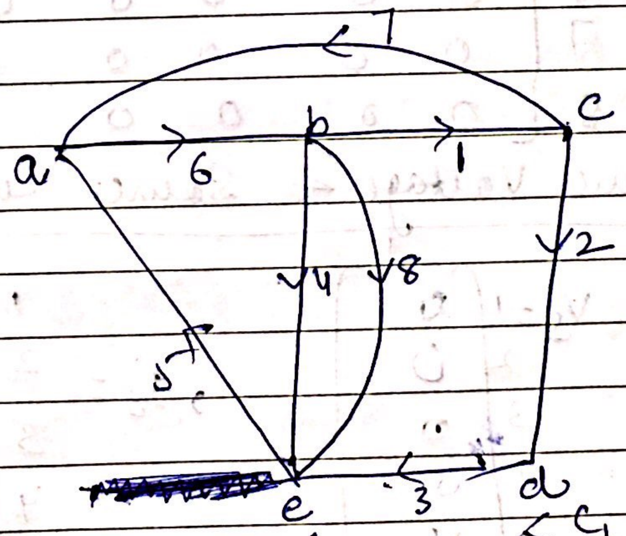
Ans.

Q.11 Draw the Directed Graph of the network as shown, choose a tree, write the cut-set matrix, Branch Admittance matrix and Node-Pair Equations.



Soln

Graph :—



$n = 5$

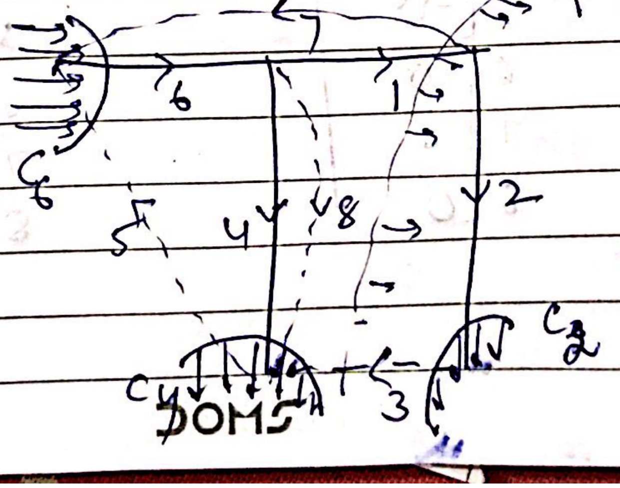
$b = 7$

$nt = 4$

$nl = 3$

4 No. of Cut-sets

Tree



Cut-Set matrix :—

Cut Sets ↓ Branches →

$$[Q_f] = \begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline C_1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ C_2 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ C_4 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 1 \\ C_6 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 \end{array}$$

Branch Admittance Matrix :—

$$[Y_b] = \begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & \frac{1}{L_1 s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & \frac{1}{L_2 s} & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & \frac{1}{R_3} & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & \frac{1}{R_4} & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & \frac{1}{R_5} & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & C_6 s & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & C_7 s & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_8 s \end{array}$$

The Source Voltage & Source Current matrices are :—

$$V_s = \begin{array}{c|c} 1 & 0 \\ 2 & 0 \\ 3 & 0 \\ 4 & 0 \\ 5 & -v \\ 6 & 0 \\ 7 & 0 \\ 8 & 0 \end{array} \quad , \quad I_s = \begin{array}{c|c} 1 & 0 \\ 2 & 0 \\ 3 & 0 \\ 4 & 0 \\ 5 & 0 \\ 6 & 0 \\ 7 & -i \\ 8 & 0 \end{array}$$

The Node-Pair Admittance Matrix

$$Y_t = Q_f Y_b Q_f^T$$

$$Y_t = \begin{bmatrix} \frac{1}{L_1 s} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 & -C_7 s & 0 \\ 0 & \frac{1}{L_2 s} & -\frac{1}{R_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_3} & \frac{1}{R_4} & -\frac{1}{R_5} & 0 & 0 & C_8 s \\ 0 & 0 & 0 & 0 & -\frac{1}{R_5} & C_6 s & -C_7 s & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$Y_t = \begin{bmatrix} \frac{1}{L_1 s} + \frac{1}{R_3 s} + C_7 s & \frac{1}{R_8} & -\frac{1}{R_3} & C_7 s \\ \frac{1}{R_3} & \frac{1}{L_2 s} + \frac{1}{R_3} & -\frac{1}{R_3} & 0 \\ -\frac{1}{R_3} & -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} + C_8 s & \frac{1}{R_5} \\ C_7 s & 0 & \frac{1}{R_5} & \frac{1}{R_5} + C_6 s + C_7 s \end{bmatrix}$$

The current matrix :-

$$I = Q_f Y_b V_s - Q_f I_s$$

$$= \begin{bmatrix} 0 \\ 0 \\ \frac{V}{R_5} \\ \frac{V}{R_5} \end{bmatrix} - \begin{bmatrix} i \\ 0 \\ 0 \\ i \end{bmatrix}$$

$$I = \begin{bmatrix} -i \\ 0 \\ \frac{v}{R_5} \\ \frac{v}{R_5} - i \end{bmatrix}$$

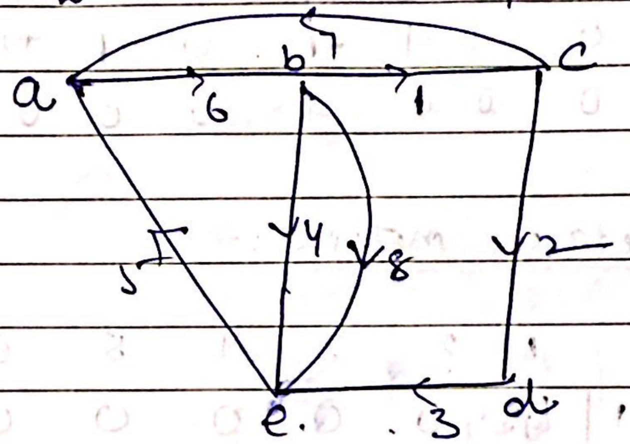
So, the Matrix Node Eq^n is :-

$$I = Y_t V_t$$

$$\begin{bmatrix} -v \\ 0 \\ \frac{v}{R_5} \\ \frac{v}{R_5} - i \end{bmatrix} = \begin{bmatrix} \frac{1}{L_1 s} + \frac{1}{R_3 s} + C_7 s & \frac{1}{R_3} & -\frac{1}{R_3} & C_7 s \\ \frac{1}{R_3} & \frac{1}{L_2 s} + \frac{1}{R_3} & -\frac{1}{R_3} & 0 \\ -\frac{1}{R_3} & -\frac{1}{R_3} & \frac{1}{R_8} + \frac{1}{R_4} + \frac{1}{R_5} + C_8 s & \frac{1}{R_5} \\ C_7 s & 0 & \frac{1}{R_5} & \frac{1}{R_5} + C_6 s + C_7 s \end{bmatrix}$$

Ans.

Q.12 for a given graph, write the incidence matrix A, loop matrix B and Branch Impedance Matrix. The determine the loop Impedance Matrix and write the loop equation in matrix form.

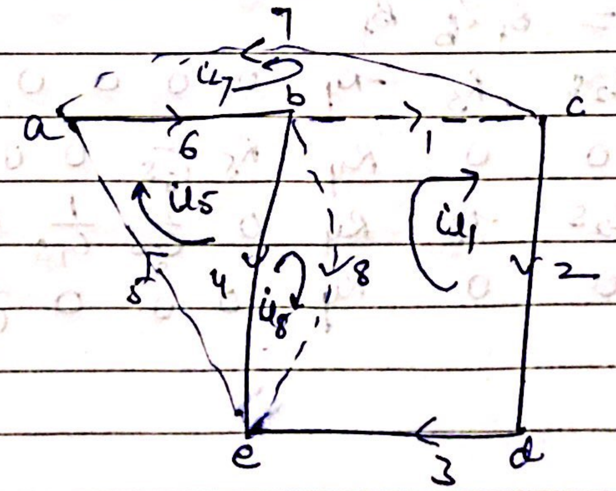


Sol^n let Node e is taken as reference node. So the incidence matrix is :—

Nodes $\rightarrow$ Branches
$\downarrow$

$$[A] = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & -1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Tree



Tie-Set Matrix.

Tiesets ↓ Branches →

$$
\begin{array}{cc}
 & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
[B_f] = \begin{array}{c} i_1 \\ i_5 \\ i_7 \\ i_8 \end{array} & \begin{bmatrix} +1 & +1 & +1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & +1 & +1 & 0 & 0 \\ 0 & -1 & -1 & +1 & 0 & +1 & +1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & +1 \end{bmatrix}
\end{array}
$$

Branch Impedance Matrix :-

$$
\begin{array}{cc}
 & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
[Z_b] = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} & \begin{bmatrix} L_1s & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_2s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_6s} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_7s} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_8s} \end{bmatrix}
\end{array}
$$

Now, The Loop Impedance Matrix is:-

$$Z_l = B_f Z_b B_f^T$$

$$
Z_l = \begin{bmatrix} L_1s & L_2s & R_3 & -R_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_4 & R_5 & \frac{1}{C_6s} & 0 & 0 \\ 0 & -L_2s & -R_3 & R_4 & 0 & \frac{1}{C_6s} & \frac{1}{C_7s} & 0 \\ 0 & 0 & 0 & R_4 & 0 & 0 & 0 & \frac{1}{C_8s} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ -1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$[Z_l] = \begin{bmatrix} L_1s + L_2s + R_3 + R_4 & -R_4 & -L_2s - R_3 - R_4 & R_4 \\ -R_4 & R_4 + R_5 + \frac{1}{C_6 s} & R_4 + \frac{1}{C_6 s} & -R_4 \\ -L_2s - R_3 - R_4 & R_4 + \frac{1}{C_6 s} & L_2s + R_3 + R_4 + \frac{1}{C_6 s} + \frac{1}{C_7 s} & -R_4 \\ R_4 & -R_4 & -R_4 & R_4 + \frac{1}{C_8 s} \end{bmatrix}$$

The Loop Voltage matrix

$$V_l = B_f Z_b I_s - B_f V_s$$

$$= \begin{bmatrix} 0 \\ 0 \\ -\frac{i}{C_7 s} \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -v \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ v \\ -\frac{i}{C_7 s} \\ 0 \end{bmatrix}$$

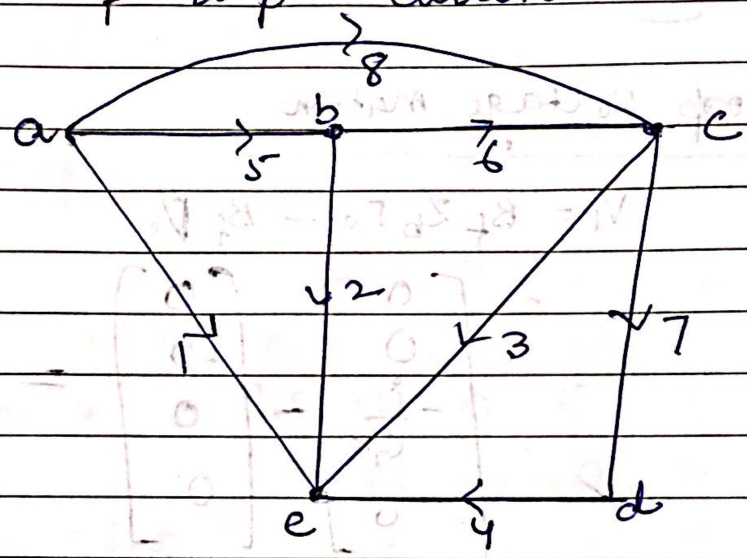
Hence, The matrix loop equation is:—

$$V_l = Z_l I_l$$

$$\begin{bmatrix} 0 \\ v \\ -\frac{i}{C_7 s} \\ 0 \end{bmatrix} = \begin{bmatrix} L_1s + L_2s + R_3 + R_4 & -R_4 & -L_2s - R_3 - R_4 & R_4 \\ -R_4 & R_4 + R_5 + \frac{1}{C_6 s} & R_4 + \frac{1}{C_6 s} & -R_4 \\ -L_2s - R_3 - R_4 & R_4 + \frac{1}{C_6 s} & L_2s + R_3 + R_4 + \frac{1}{C_6 s} + \frac{1}{C_7 s} & -R_4 \\ R_4 & -R_4 & -R_4 & R_4 + \frac{1}{C_8 s} \end{bmatrix} \begin{bmatrix} I_{l1} \\ I_{l5} \\ I_{l7} \\ I_{l8} \end{bmatrix}$$

Ans.

Q.13 Write the Incidence matrix of the graph given and then express branch voltages in terms of node voltages. Write the loop matrix B and express branch currents in terms of loop currents.



Sol^n Taking e as reference node:—

Nodes ↓ Branches →

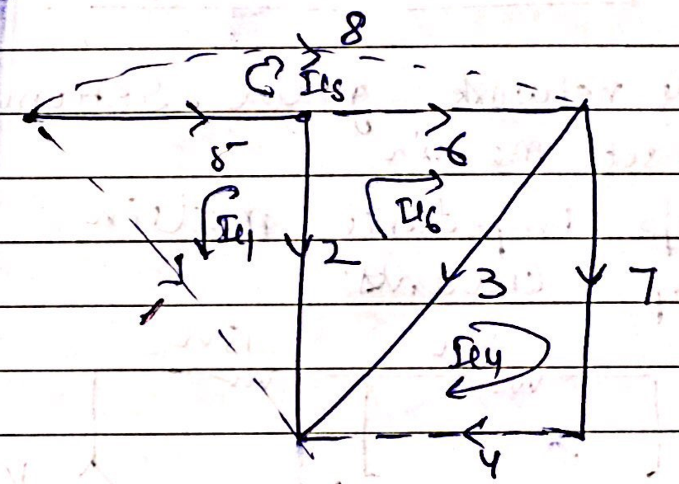
$$[A] = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

As, we know that:—

$$\boxed{V_b = A^T V_n}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix}$$

Tree



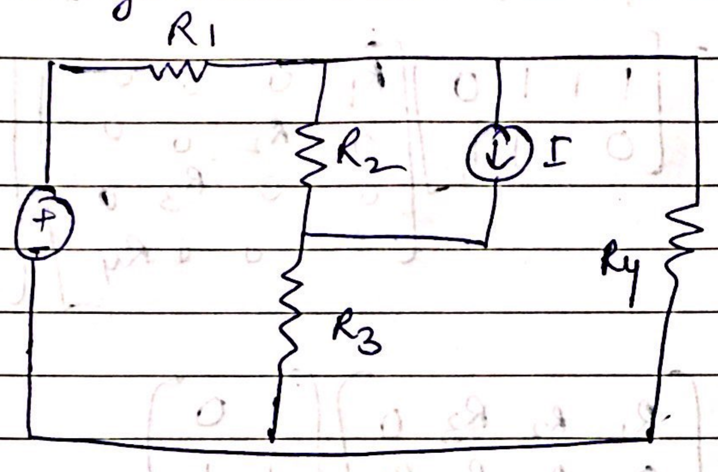
Now, As we know $\boxed{I_b = B_f^T I_l}$

So
$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{l1} \\ I_{l2} \\ I_{l3} \\ I_{l4} \end{bmatrix}$$

Ans.

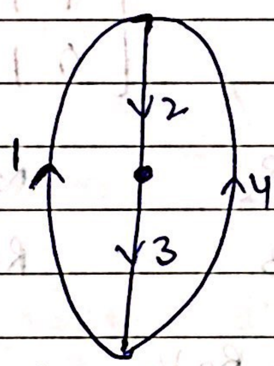
__/__/____

Q.14 Write the loop equation in matrix form for the given circuit.

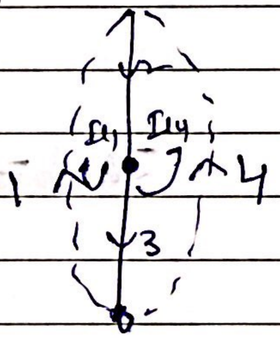


Soln

Graph



Tree



Tie-set matrix

$$[B_f] = \begin{array}{c} \\ I_1 \\ I_4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$[Z_b] = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & R_4 \end{bmatrix}$$

$$V_s = \begin{bmatrix} -V \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad I_s = \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix}$$

So, The loop impedance matrix :-

$$Z_l = B_f Z_b B_f^T$$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & R_4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_1 & R_2 & R_3 & 0 \\ 0 & R_2 & R_3 & R_4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$Z_l = \begin{bmatrix} R_1+R_2+R_3 & R_2+R_3 \\ R_2+R_3 & R_2+R_3+R_4 \end{bmatrix}$$

$$V_l = B_f Z_b I_s - B_f V_s$$

$$= \begin{bmatrix} R_1 & R_2 & R_3 & 0 \\ 0 & R_2 & R_3 & R_4 \end{bmatrix} \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -V \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} IR_2 \\ IR_2 \end{bmatrix} - \begin{bmatrix} -V \\ 0 \end{bmatrix} = \begin{bmatrix} IR_2+V \\ IR_2 \end{bmatrix}$$

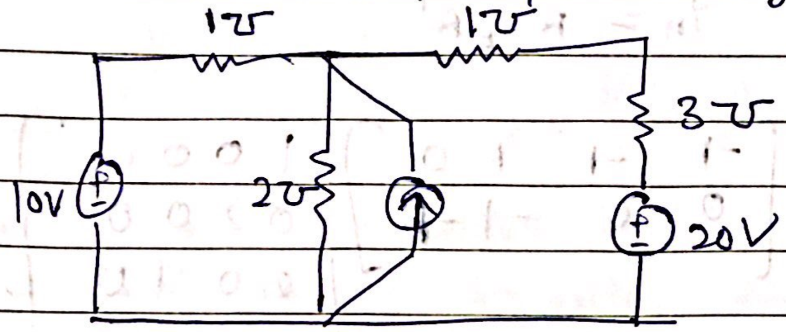
So, The loop eqn in matrix form:-

$$\begin{bmatrix} IR_2+V \\ IR_2 \end{bmatrix} = \begin{bmatrix} R_1+R_2+R_3 & R_2+R_3 \\ R_2+R_3 & R_2+R_3+R_4 \end{bmatrix} \begin{bmatrix} I_{l_1} \\ I_{l_2} \end{bmatrix}$$

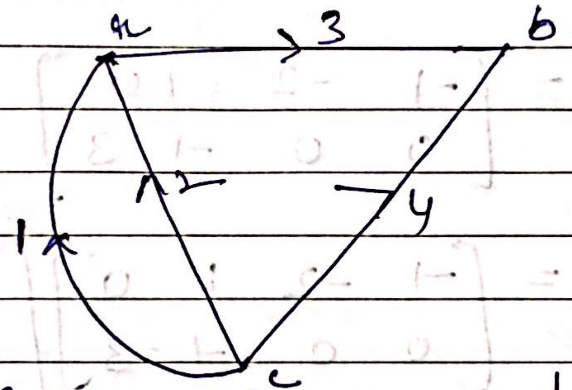
Ans

DOMS

Q.15 write the node eqn for the given network:—



Soln Graph



Taking c as reference node

$$[A] = \begin{array}{c c} & \text{Branches} \rightarrow \\ \text{Nodes} \downarrow & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \end{array} & \begin{bmatrix} -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

$$[Y_b] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$V_s = \begin{bmatrix} -10 \\ 0 \\ 0 \\ -20 \end{bmatrix} \quad \text{and } I_s = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

DOMS

So

$$Y_n = A Y_b A^T$$

$$AY_b = \begin{bmatrix} -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

So $$Y_n = \begin{bmatrix} -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -3 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -1 & 0 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$$

$$Y_n = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$$

And

$$I_n = A Y_b V_s - A I_s$$

$$= \begin{bmatrix} -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -3 \end{bmatrix} \begin{bmatrix} -10 \\ 0 \\ 0 \\ -20 \end{bmatrix} - \begin{bmatrix} -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

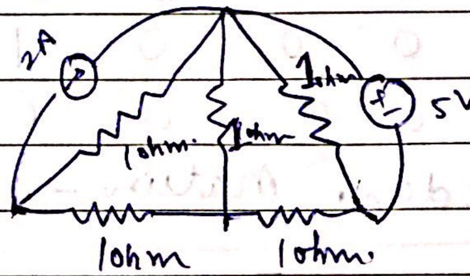
$$= \begin{bmatrix} 10 \\ 60 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 60 \end{bmatrix}$$

Hence, the node equations are:—

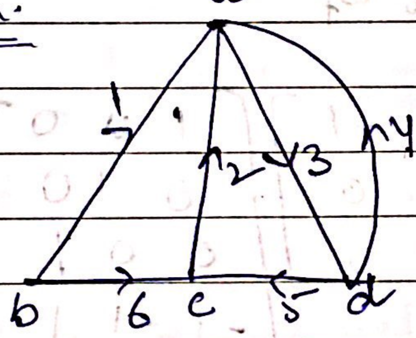
$I_n = Y_n V_n$

$$\begin{bmatrix} 11 \\ 60 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix}$$ Ans.

Q.16 for a given network, draw the graph, write the tie-set schedule. ~~and then obtain~~ ~~then~~



Sol^n Graph.



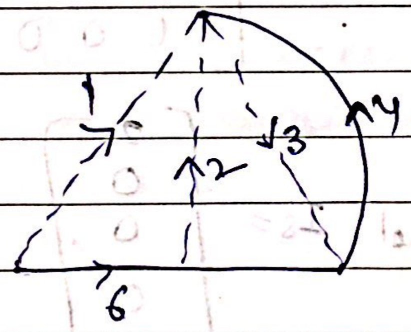
$n = 4$

$b = 6$

$nt = 4-1 = 3$

$nl = 3.$

Tree



Tie-Set matrix

$$[B_f] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

The Branch Impedance Matrix:—

$$[Z_b] = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

The loop Impedance Matrix:—

$$Z_l = B_f Z_b B_f^T$$

$$Z_l = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$V_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -5 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad I_s = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So

$$V_l = B_f Z_b I_s - B_f V_s$$

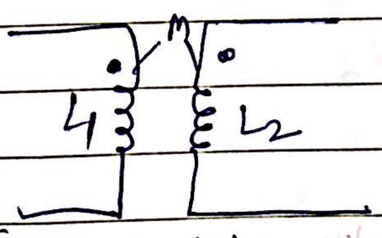
$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -5 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

$$V_l = \begin{bmatrix} -3 \\ -5 \\ +5 \end{bmatrix}$$ Ans

★ Network with Mutual Inductance:–

As we have seen earlier, if there is no mutual coupling b/w the branches, the branch impedance or admittance matrix is a diagonal matrix otherwise it may have non-zero elements in the off-diagonal locations also.

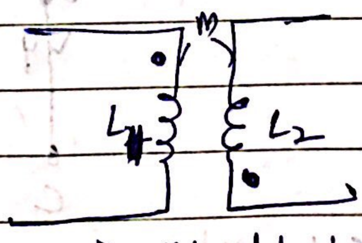


(with positive coupling)

↓

Branch Inductance

$$L_b = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}$$



(with opposing coupling)

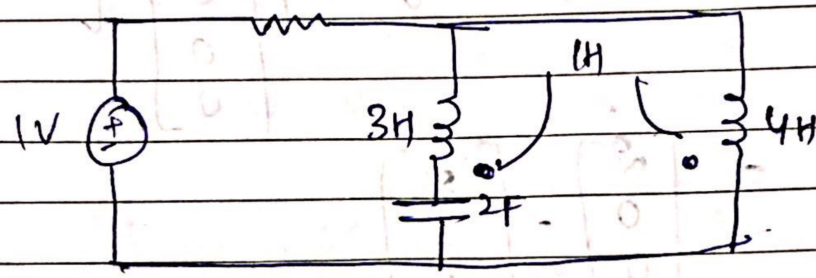
↓

Branch Inductance

$$L_b = \begin{bmatrix} L_1 & -M \\ -M & L_2 \end{bmatrix}$$

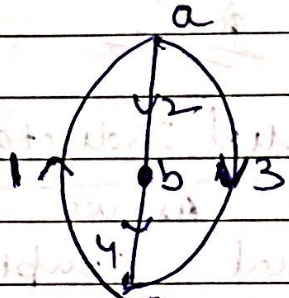
DOMS

Q.17 For the given network, write down the f-cut set matrix for the tree having 3H inductor and 2F capacitor as twigs and write the matrix cut-set equations.

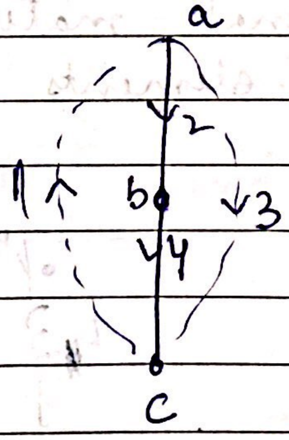


Soln Since the dots are on the same side, the mutual inductance would be positive.

Graph



Tree with 3H inductor & 2F capacitor as twigs.



f-cut matrix

Cut sets Branches →

$$[Q_f] = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline C_2 & -1 & 1 & 1 & 0 \\ C_4 & -1 & 0 & 1 & 1 \end{array}$$

$$Y_b = [Z_b]^{-1} = \begin{bmatrix} [1]^{-1} & 0 & 0 & 0 \\ 0 & \begin{bmatrix} 3s & s \\ s & 4s \end{bmatrix}^{-1} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & \left[\frac{1}{2s}\right]^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{4}{11}s & -\frac{1}{11}s & 0 \\ 0 & -\frac{1}{11}s & \frac{3}{11}s & 0 \\ 0 & 0 & 0 & 2s \end{bmatrix}$$

$$Q_f Y_b = \begin{bmatrix} -1 & \frac{3}{11s} & \frac{2}{11s} & 0 \\ -1 & -\frac{1}{11s} & \frac{3}{11s} & 2s \end{bmatrix}$$

$$Y_t = Q_f Y_b Q_f^T = \begin{bmatrix} -1 & \frac{3}{11s} & \frac{2}{11s} & 0 \\ -1 & -\frac{1}{11s} & \frac{3}{11s} & 2s \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{5}{11s} & 1 + \frac{2}{11s} \\ 1 + \frac{2}{11s} & 1 + \frac{3}{11s} + 2s \end{bmatrix}$$

$$I = Q_f Y_b V_s - Q_f E_s = Q_f Y_b V_s \quad [\text{since } I_s = 0]$$

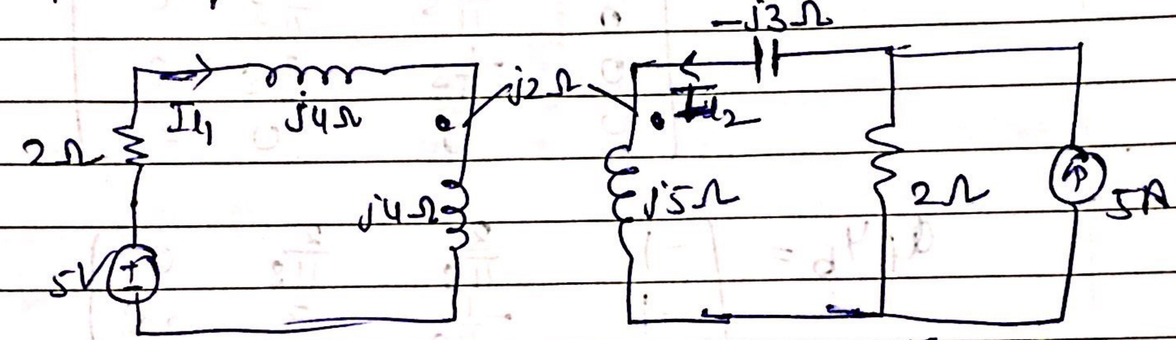
$$= \begin{bmatrix} -1 & \frac{3}{11s} & \frac{2}{11s} & 0 \\ -1 & -\frac{1}{11s} & \frac{3}{11s} & 2s \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Hence, The matrix cut-set eqn is:—

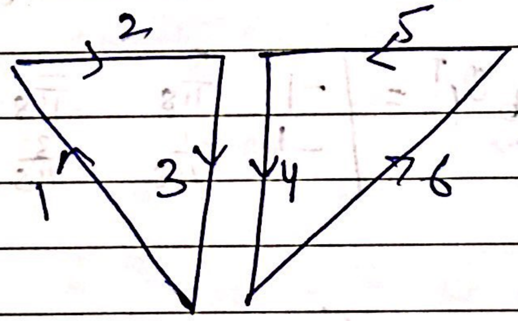
$$I = Y_t V_t$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+\frac{5}{11s} & 1+\frac{2}{11s} \\ 1+\frac{2}{11s} & 1+\frac{3}{11s}+2s \end{bmatrix} \begin{bmatrix} V_2 \\ V_4 \end{bmatrix}$$ Ans.

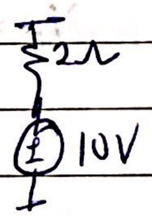
Q.18 for a given network (a) determine the f-loop matrix and f-loop equations



Soln Graph:—

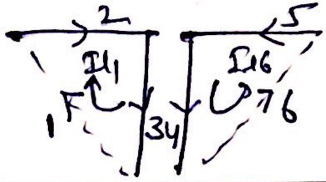


Source conversion



The Branch Impedance Matrix:→

$$Z_b = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & j4 & 0 & 0 & 0 & 0 \\ 0 & 0 & j4 & j2 & 0 & 0 \\ 0 & 0 & j2 & j5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \end{array}$$



_ / _ / _

The f-loop matrix:—

$$
B_f = \begin{array}{c|cccccc}
\text{f-loops} & \multicolumn{6}{c}{\text{Branches} \rightarrow} \\
 & 1 & 2 & 3 & 4 & 5 & 6 \\
I_{l1} & 1 & 1 & 1 & 0 & 0 & 0 \\
I_{l2} & 0 & 0 & 0 & 1 & 1 & 1
\end{array}
$$

$$B_f Z_b = \begin{bmatrix} 2 & j4 & j4 & j2 & 0 & 0 \\ 0 & 0 & j2 & j5 & -j3 & 2 \end{bmatrix}$$

So, the loop Impedance matrix:→

$$Z_l = B_f Z_b B_f^T$$

$$Z_l = \begin{bmatrix} 2 & j4 & j4 & j2 & 0 & 0 \\ 0 & 0 & j2 & j5 & -j3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$Z_l = \begin{bmatrix} 2+j8 & 2 \\ j2 & 2+j2 \end{bmatrix}$$

The loop voltage matrix:—

$$V_l = B_f Z_b I_s - B_f V_s$$

$$= 0 - \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 0 \\ 0 \\ 0 \\ 0 \\ -10 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

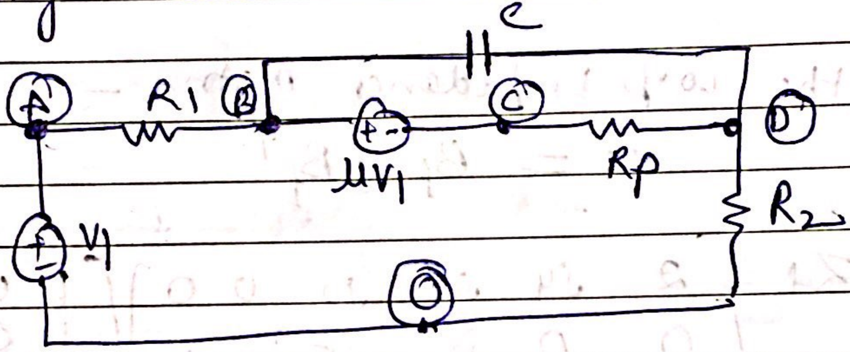
Hence, loop eq^{n}s are:—

$$V_l = Z_l I_l$$

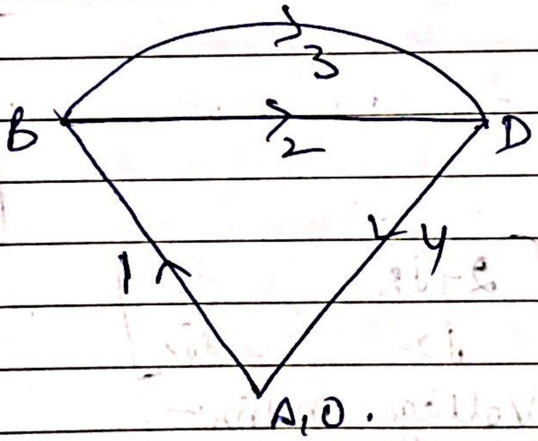
$$\begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 2+j8 & j2 \\ j2 & 2+j2 \end{bmatrix} \begin{bmatrix} I_{l1} \\ I_{l2} \end{bmatrix}$$ Ans.

DOMS

Q. 19 Draw the graph of given network and find a Tree of network. Considering 'O' as a datum node and assuming elements BD and BC as links, determine the tie-set schedule, branch impedance matrix and source voltage matrix. obtain loop equations using above said matricies.

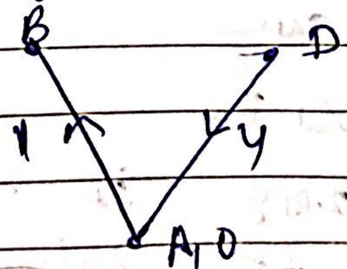


Sol^n

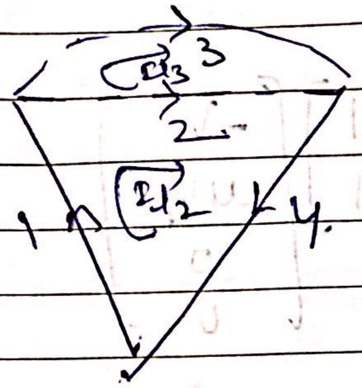


Branch 1 w.r. to R_1
" 2 " R_p
" 3 " C
" 4 " R_2

Tree with given condition i.e. BD & BC as links



As 2 Link $\longrightarrow$ 2 No. of Tie-sets.



Tie-set schedule:-

$$\begin{array}{c|c} & \text{Branches} \rightarrow \\ \text{Tiesets} \downarrow & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} I_2 \\ I_3 \end{array} & \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right] \end{array}$$

$[B_f] =$ (as above).

The Branch Impedance Matrix:-

$$Z_b = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_p & 0 & 0 \\ 0 & 0 & \frac{1}{Cs} & 0 \\ 0 & 0 & 0 & R_2 \end{bmatrix}$$

Source voltage matrix:-

$$V_s = \begin{bmatrix} -V_1 \\ \mu V_1 \\ 0 \\ 0 \end{bmatrix}$$

__/__/__

So, The Loop Voltage matrix:-

$$V_l = B_f Z_b I_s - B_f V_s$$

$$= 0 - \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -V_1 \\ \mu V_1 \\ 0 \\ 0 \end{bmatrix}$$

$$V_l = \begin{bmatrix} V_1 - \mu V_1 \\ V_1 \end{bmatrix}$$

Loop Impedance matrix:-

$$Z_l = B_f Z_b B_f^T$$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_p & 0 & 0 \\ 0 & 0 & \frac{1}{Cs} & 0 \\ 0 & 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} R_1 & R_1 \\ R_p & 0 \\ 0 & \frac{1}{Cs} \\ R_2 & R_2 \end{bmatrix}$$

$$Z_l = \begin{bmatrix} R_1 + R_p + R_2 & R_1 + R_2 \\ R_1 + R_2 & R_1 + \frac{1}{Cs} + R_2 \end{bmatrix}$$

Hence, matrix loop equations are:—

$$\begin{bmatrix} V_1 - \mu V_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} R_1 + R_p + R_2 & R_1 + R_2 \\ R_1 + R_2 & R_1 + \frac{1}{Cs} + R_2 \end{bmatrix} \begin{bmatrix} I_{l_2} \\ I_{l_3} \end{bmatrix}$$

Ans.

Duals and Duality:-

According to ohm's law

$$V = IR \quad \text{—(1)}$$

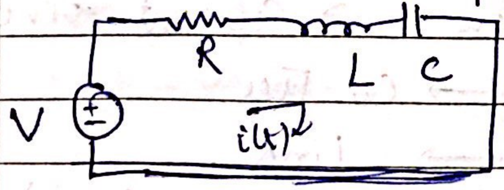
$$I = GV \quad \text{—(2)}$$

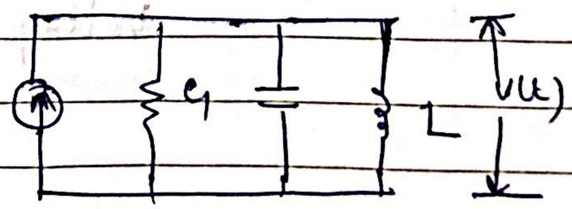
from eq^n (1) & (2), it is to be noted that either eq^n may be found from the other by interchanging each symbol by its pair opposite. i.e. I and V, G and R.

in open ckt $I = 0, V \neq 0$

in short ckt $V = 0, I \neq 0$

So, we can say that there is a systematic symmetry of opposites, or duality between resistance and conductance, voltage and current, open circuit and short-circuit i.e. R is the dual of G, G is dual of R, V is dual of I, I is dual of V, short circuit is dual of open-circuit, open circuit is dual of short circuit.





$$V = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt \qquad I = Gv(t) + C\frac{dv(t)}{dt} + \frac{1}{L}\int v(t)\,dt$$

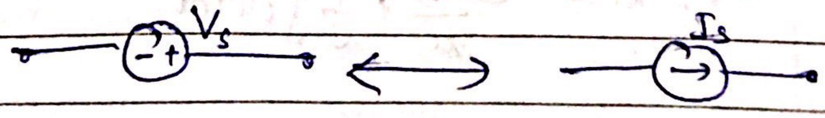
These two networks are called as dual of each other.

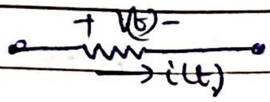
So, we can say, two networks are said to be dual of each other, if the mesh equations of one are the same as the node equations of the other.

→ In terms of graphs, we can say that two graphs G_1 and G_2 are said to be dual of each other, if the incidence matrix of any of them is a loop matrix of the other.

<u>Some of Dual Relations are as follows:-</u>

Voltage (V) ⟷ Current (I)
Resistance (R) ⟷ Conductance (G)
Inductance (L) ⟷ Capacitance (C)
Impedance (Z) ⟷ Admittance (Y)
Voltage Source (V_s) ⟷ Current Source (I_s)
KVL ⟷ KCL
Mesh ⟷ Node
Mesh Current ⟷ Node Voltage
Mesh Equations ⟷ Nodal Equations
Thevenin's Theorem ⟷ Norton's Theorem
Voltage Division ⟷ Current Division
Tree ⟷ Co-Tree
Twig ⟷ Link
Cut-Set ⟷ Tieset (Loop)





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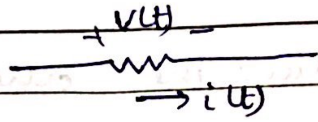
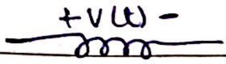


$v(t) = R\,i(t)$ $\quad$ $i(t) = G\,v(t)$



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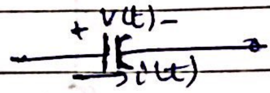
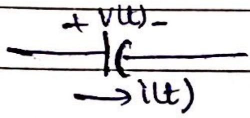


$v(t) = L\dfrac{di(t)}{dt}$ $\quad$ $i(t) = C\dfrac{dv(t)}{dt}$



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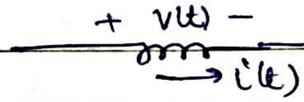
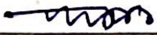


$v(t) = \dfrac{1}{C}\int^{t} i(t)\,dt + v_c(0^-)$ ⟷

$i(t) = \dfrac{1}{L}\int^{t} v(t)\,dt + i(0^-)$

Procedure for Constructing Dual of a Network:—

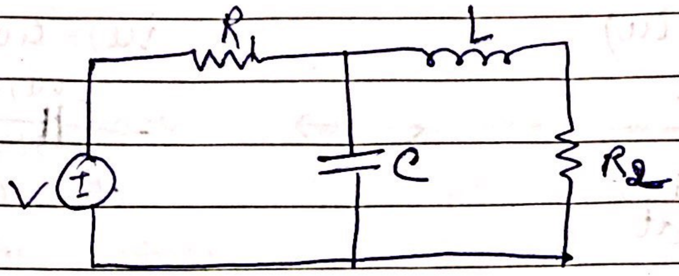
i) A dot is placed in the centre of each mesh. Label each node with a number or alphabet.

ii) Place an extra dot outside the network. This will be the reference or datum node.

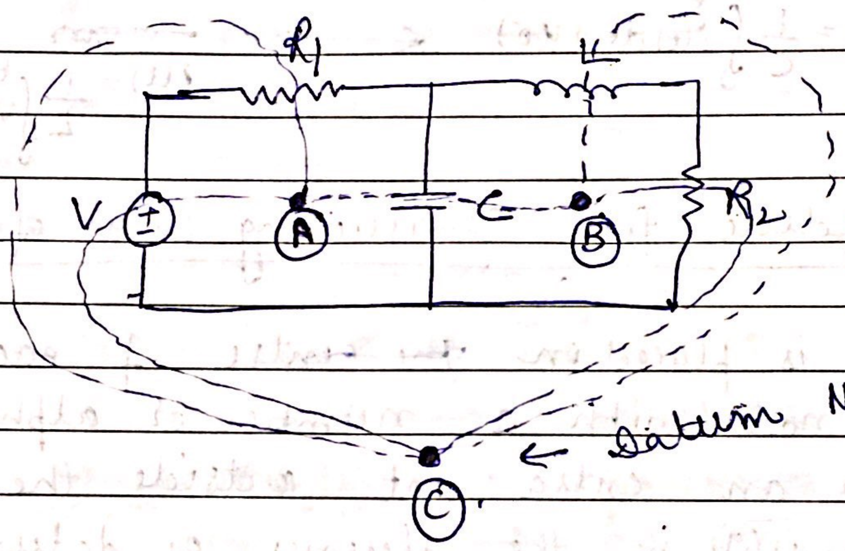
iii) Draw dashed line between the nodes in such a way that each line crosses only one network element and a dashed line has been drawn through every element in the original network.

iv) The dashed lines, along with the dual elements, constitute the dual of original network.

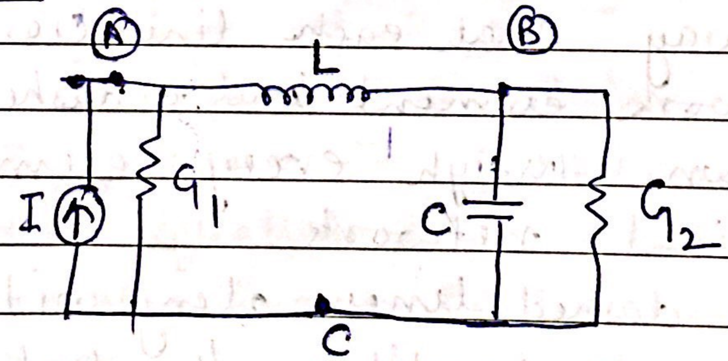
Q.20 Draw the dual of network given:—



Sol^n

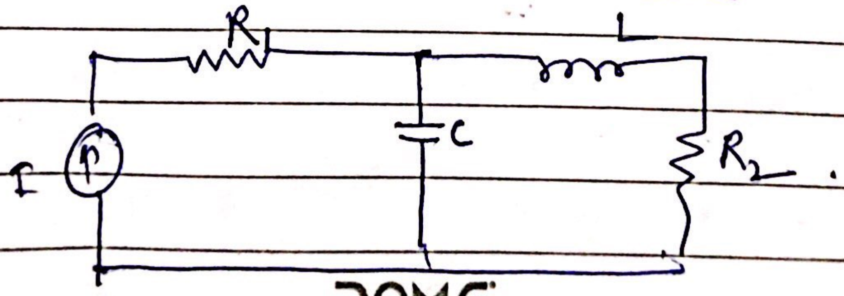


Dual Network:—

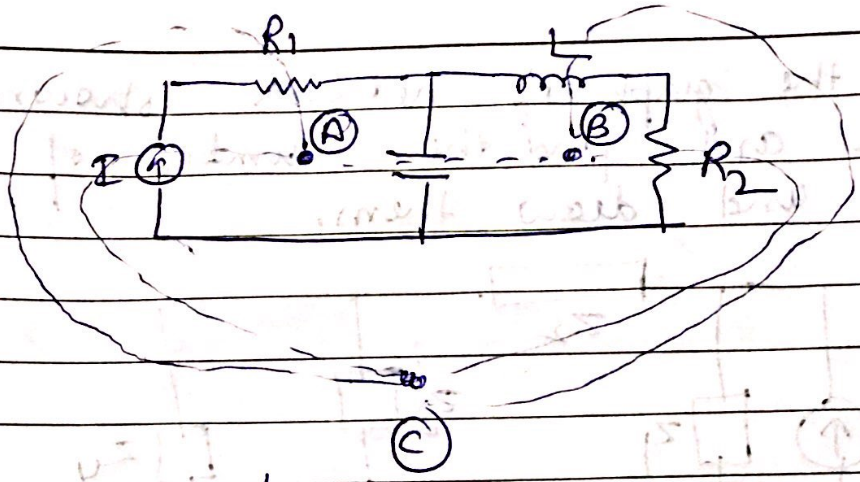


Ans

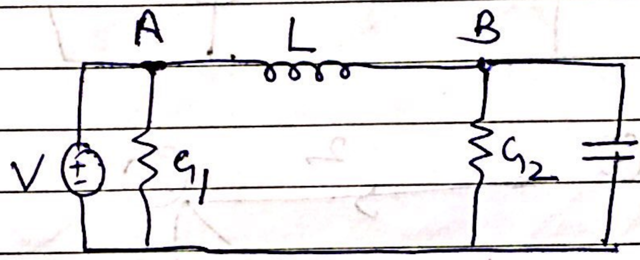
Q.21 Draw the Dual of Given Network:—



DOMS

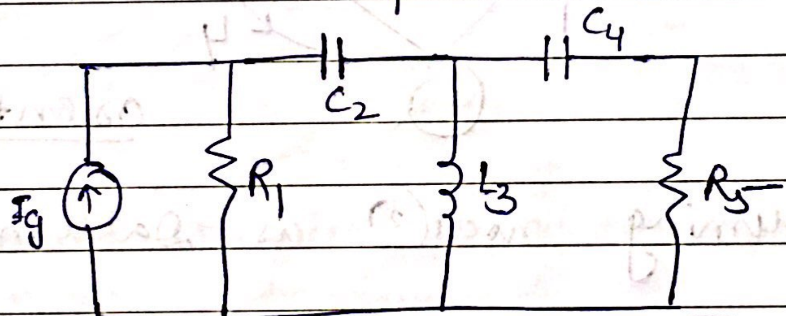


Dual Network

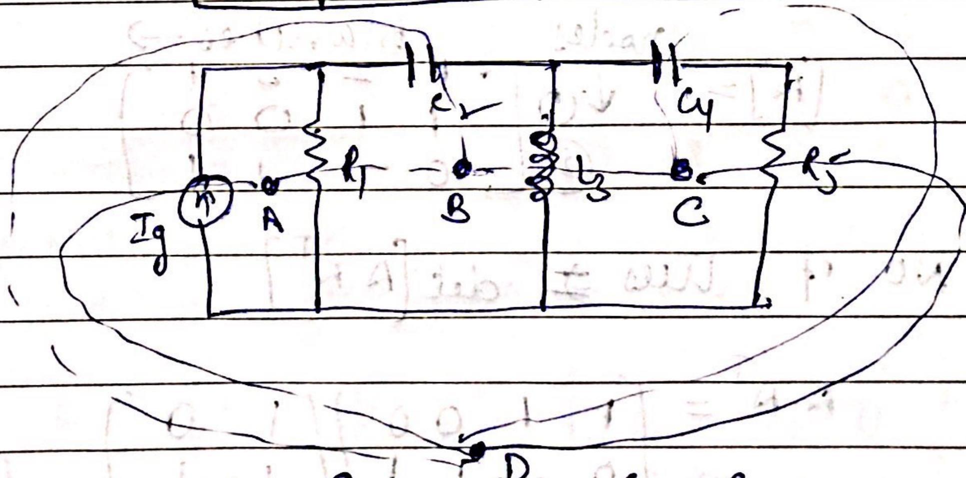


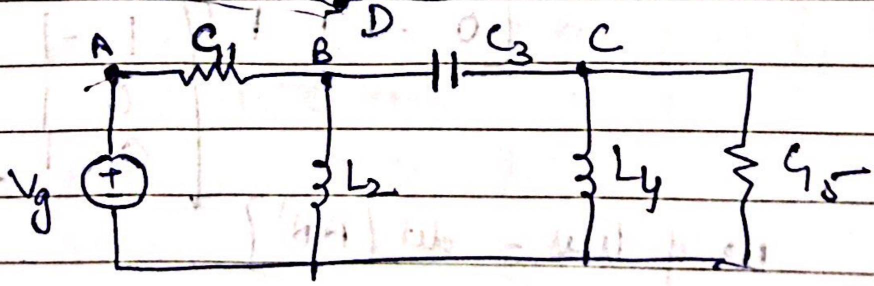
Ans

Q.22 Draw the Dual of the network shown in fig.



Sol^n

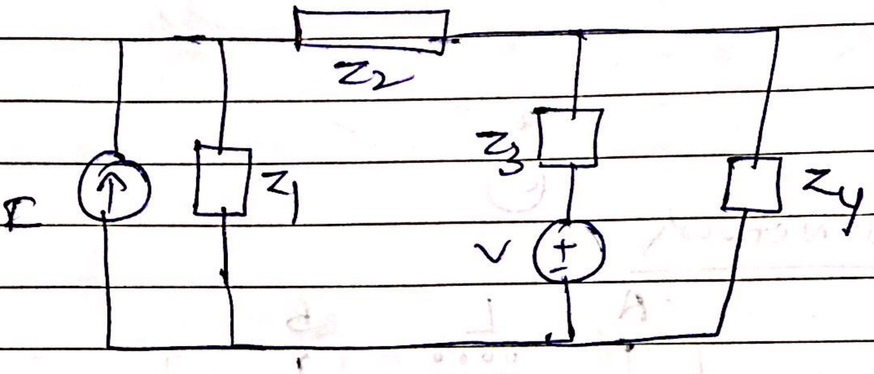




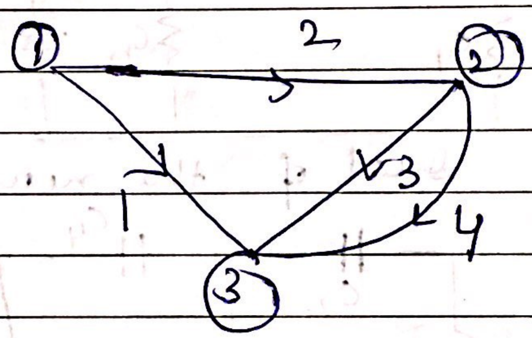
Ans

__/__/____

Q.23 Draw the graph of network shown in figure and find the number of possible trees and draw them.



Sol^n



Oriented Graph

Assuming node 3 as Datum Node:→

$$[A] = \begin{array}{c|c} & \text{Branches} \rightarrow \\ \text{nodes} \downarrow & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

No. of trees = $\det[A A^T]$

$$A \cdot A^T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

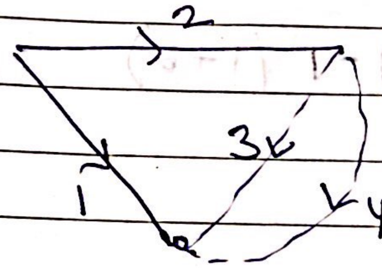
No of trees = $\det[A A^T]$

DOMS

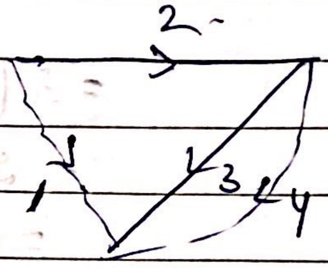
$$= \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6-1 = 5.$$

So, No of trees = 5

①



②



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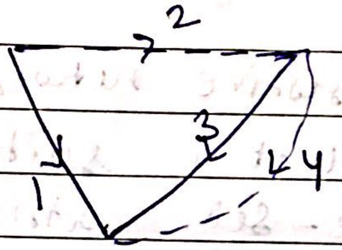
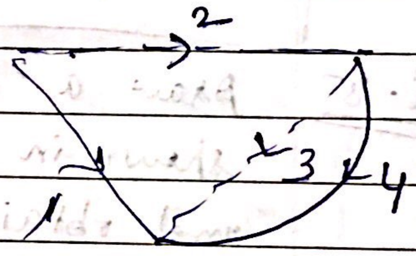
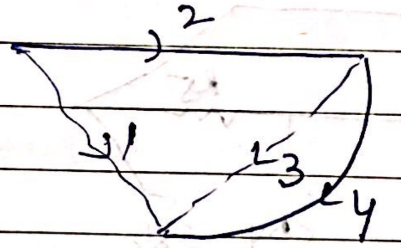


④



⑤.



Q.24 A reduced incidence matrix of a graph is given by:-

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}$$

obtain the no. of possible trees.

Solⁿ No. of Trees = $\det[A \cdot A^T]$

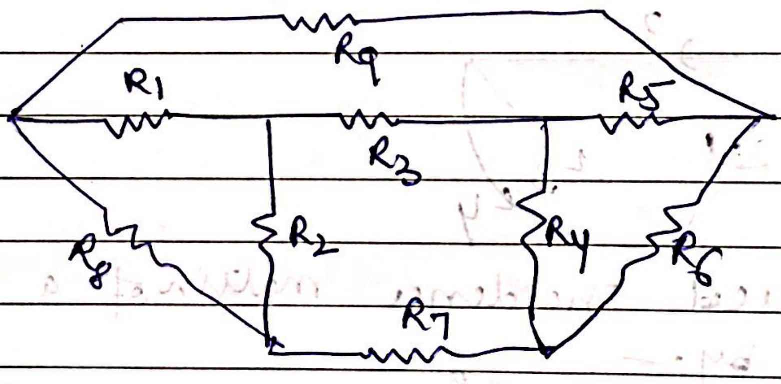
$$A \cdot A^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$\det[A \cdot A^T] = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

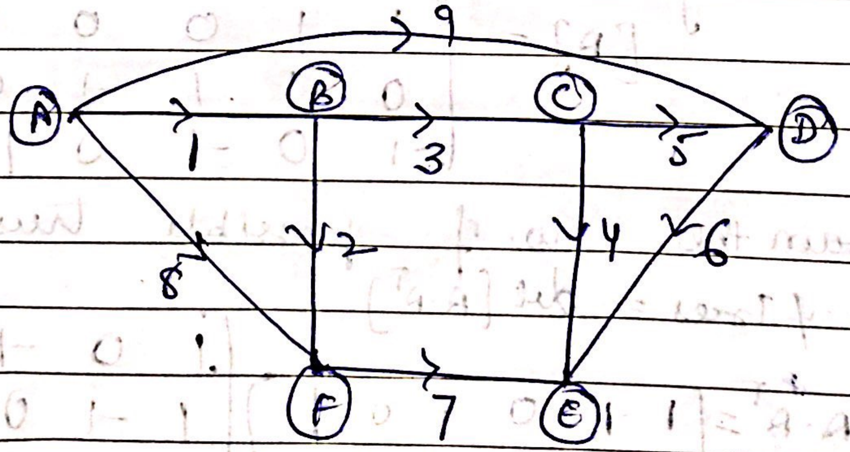
$$= 3(9-1) + 1(-3-1) - 1(1+3)$$
$$= 24 - 4 - 4$$
$$= 16.$$

$\boxed{\text{So, No of Trees} = 16}$.

Q. 25 Draw a graph of resistive network as shown in figure. Select a suitable tree and obtain the tie-set matrix.



Sol^n



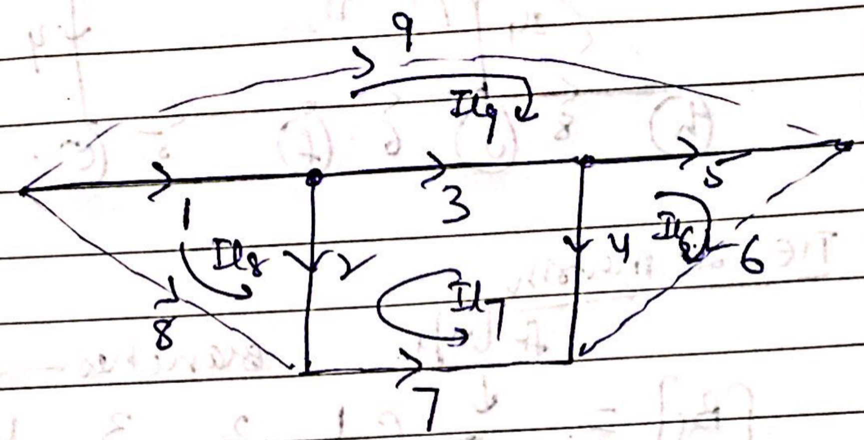
Graph

$n = 6$ $\quad$ $nt = 5$

$b = 9$ $\quad$ $nl = 4$

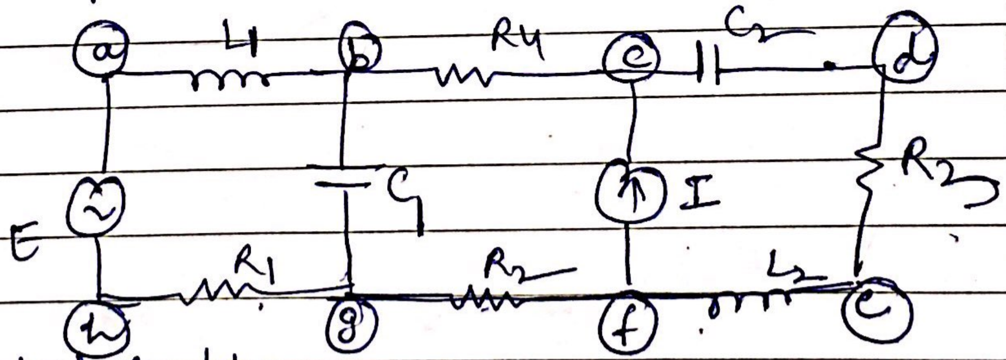
As $n_l = 4$, so No of Tie-sets = 4.

Tree

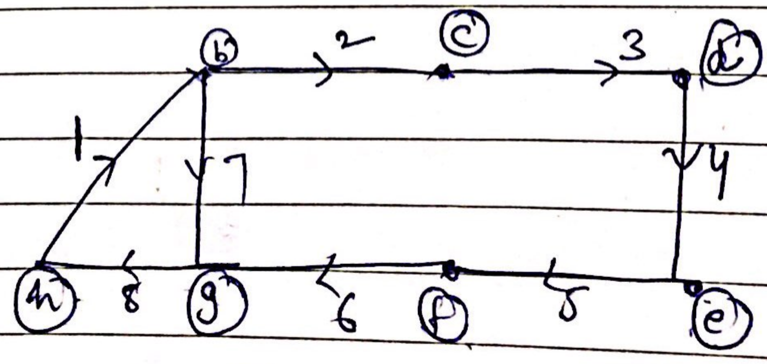


$$
\text{Tie-set matrix } [B_f] = \begin{array}{c} \text{f-loops} \downarrow \\ \\ I_{l_6} \\ I_{l_7} \\ I_{l_8} \\ I_{l_9} \end{array}
\begin{array}{c} \xrightarrow{\text{Branches}} \\ \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}
$$

Q.26 Develop the Tie-set matrix of given circuit.



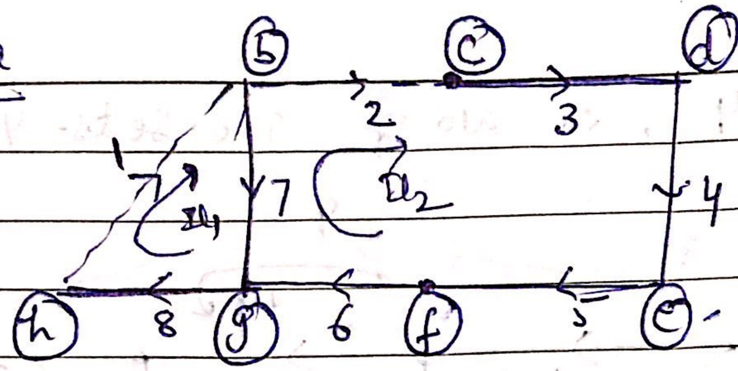
Soln Oriented Graph:-



$n = 7$ $\quad\quad$ $n_t = n - 1 = 6$

$b = 8$ $\quad\quad$ $n_l = 8 - 6 = 2.$

Tree



Tie-Set matrix

f loops ↓ Branches →

$$[B_f] = \begin{array}{c} \\ IL_1 \\ IL_2 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & -1 & 0 \end{bmatrix} \end{array}$$

Ans