Here $h = 0.1$. $\therefore$ $0.24 = 0.1 + (0.1)u$ or $u = 1.4$

Newton-Gregory forward difference formula is

$$y(.24) = y(.1) + u\,\Delta y(.1) + \frac{u(u-1)}{2!}\Delta^2 y(.1) + \frac{u(u-1)(u-2)}{3!}\Delta^3 y(.1) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y(.1)$$

$$\Rightarrow \quad 10^5 y(.24) = 10^5 y(.1) + u\,10^5 \Delta y(.1) + \frac{u(u-1)}{2!}10^5 \Delta^2 y(.1) + \frac{u(u-1)(u-2)}{3!}10^5 \Delta^3 y(.1) + \frac{u(u-1)(u-2)(u-3)}{4!}10^5 \Delta^4 y(.1)$$

$$\Rightarrow \quad 10^5 y(.24) = 110517 + (1.4)(11623) + \frac{(1.4)(1.4-1)}{2}(1223) + \frac{(1.4)(1.4-1)(1.4-2)}{3!}(127) + \frac{(1.4)(1.4-1)(1.4-2)(1.4-3)}{4!}(17)$$

$$= 127124.9088$$

$$\therefore \quad y(.24) = e^{.24} = 1.271249088$$

Example 4. *The following table gives the population of a town during the last six censuses. Estimate the population in 1913 by Newton's forward difference formula*

Year	*1911*	*1921*	*1931*	*1941*	*1951*	*1961*
Population (in thousands)	*12*	*15*	*20*	*27*	*39*	*52*

[U.P.T.U. (MCA) 2009]

Sol. Here, $a = 1911,\ h = 10,\ x = 1913$

$$\therefore \quad u = \frac{x-a}{h} = \frac{1913-1911}{10} = 0.2$$

Forward difference table is

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
1911	$\boxed{12}$					
		$\boxed{3}$				
1921	15		$\boxed{2}$			
		5		$\boxed{0}$		
1931	20		2		$\boxed{3}$	
		7		3		$\boxed{-10}$
1941	27		5		−7	
		12		−4		
1951	39		1			
		13				
1961	52					

Newton's forward difference formula is

$$f(1913) = f(1911) + u\Delta f(1911) + \frac{u(u-1)}{2!}\Delta^2 f(1911) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(1911)$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(1911) + \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\Delta^5 f(1911)$$

$$= 12 + (0.2)(3) + \frac{(.2)(-.8)}{2}(2) + \frac{(.2)(-.8)(-1.8)(-2.8)}{4!}(3)$$

$$+ \frac{(.2)(-.8)(-1.8)(-2.8)(-3.8)}{5!}(-10)$$

$f(1913) = 12.08384$ thousands

Hence the population of the town in the year 1913 $\simeq$ 12083.84 = 12084 (approximately).

Example 5. *From the table, estimate the number of students who obtained marks between 40 and 45.* (M.T.U. 2013)

Marks:	*30–40*	*40–50*	*50–60*	*60–70*	*70–80*
No. of students:	*31*	*42*	*51*	*35*	*31.*

Sol. Difference table is:

Marks less than (*x*)	*y*	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	$\boxed{31}$				
		$\boxed{42}$			
50	73		$\boxed{9}$		
		51		$\boxed{-25}$	
60	124		– 16		$\boxed{37}$
		35		12	
70	159		– 4		
		31			
80	190				

We shall find y_{45}, number of students with marks less than 45.

$$a = 40,\ h = 10,\ a + hu = 45.$$

$\therefore \quad 40 + 10u = 45 \quad \Rightarrow \quad u = .5$

By Newton's forward difference formula,

$$y(45) = y(40) + u\,\Delta\,y(40) + \frac{u(u-1)}{2!}\Delta^2 y(40)$$

$$+ \frac{u(u-1)(u-2)}{3!}\Delta^3 y(40) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y(40)$$

$$= 31 + (.5)(42) + \frac{(.5)(.5-1)}{2}(9) + \frac{(.5)(.5-1)(.5-2)}{6}(-25)$$

$$+ \frac{(.5)(.5-1)(.5-2)(.5-3)}{24}$$

$= 47.8672 \approx 48$

Hence number of students getting marks less than 45 = 48

By number of students getting marks less than 40 = 31

Hence number of students getting marks between 40 and 45 = 48 − 31 = 17.

Example 6. *Find the cubic polynomial which takes the following values:*

x:	*0*	*1*	*2*	*3*
f(x):	*1*	*2*	*1*	*10.*

Sol. Let us form the difference table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	$\boxed{1}$			
		$\boxed{1}$		
1	2		$\boxed{-2}$	
		−1		$\boxed{12}$
2	1		10	
		9		
3	10			

Here, $h = 1$. Hence using the formula, $x = a + hu$

and choosing $a = 0$, we get $x = u$

∴ By Newton's forward difference formula,

$$y = y_0 + x\,\Delta y_0 + \frac{x(x-1)}{2!}\Delta^2 y_0 + \frac{x(x-1)(x-2)}{3!}\Delta^3 y_0$$

$$= 1 + x(1) + \frac{x(x-1)}{2!}(-2) + \frac{x(x-1)(x-2)}{3!}(12)$$

$$= 2x^3 - 7x^2 + 6x + 1$$

Hence the required cubic polynomial is

$$y = f(x) = 2x^3 - 7x^2 + 6x + 1.$$

Example 7. *The following table gives the marks secured by 100 students in the Numer Analysis subject:*

Range of marks:	*30–40*	*40–50*	*50–60*	*60–70*	*70–80*
No. of students:	*25*	*35*	*22*	*11*	*7*

Use Newton's forward difference interpolation formula to find.

(i) the number of students who got more than 55 marks.

(ii) the number of students who secured marks in the range from 36 to 45.

Sol. The given table is re-arranged as follows:

Marks obtained	*No. of students*
Less than 40	25
Less than 50	60
Less than 60	82
Less than 70	93
Less than 80	100

(*i*) Here, $a = 40,\quad h = 10,\quad a + hu = 55$

$\therefore\quad 40 + 10u = 55 \quad\Rightarrow\quad u = 1.5$

First, we find the number of students who got less than 55 marks.

The difference table is as under:

Marks obtained less than	*No. of students* = *y*	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	$\boxed{25}$				
		$\boxed{35}$			
50	60		$\boxed{-13}$		
		22		$\boxed{2}$	
60	82		– 11		$\boxed{5}$
		11		7	
70	93		– 4		
		7			
80	100				

Applying Newton's forward difference formula,

$$y_{55} = y_{40} + u\,\Delta y_{40} + \frac{u(u-1)}{2!}\Delta^2 y_{40} + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_{40} + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_{40}$$

$$= 25 + (1.5)(35) + \frac{(1.5)(.5)}{2!}(-13) + \frac{(1.5)(.5)(-.5)}{3!}(2) + \frac{(1.5)(.5)(-.5)(-1.5)}{4!}(5)$$

$$= 71.6171875 \approx 72$$

There are 72 students who got less than 55 marks.

$\therefore$ No. of students who got more than 55 marks = 100 – 72 = 28

(*ii*) To calculate the number of students securing marks between 36 and 45, take the difference of y_{45} and y_{36}.

$$u = \frac{x-a}{h} = \frac{36-40}{10} = -.4$$

Also,

$$u = \frac{45-40}{10} = .5$$

By Newton's forward difference formula.

$$y_{36} = y_{40} + u\,\Delta y_{40} + \frac{u(u-1)}{2!}\Delta^2 y_{40} + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_{40}$$
$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_{40}$$

$$= 25 + (-.4)(35) + \frac{(-.4)(-1.4)}{2!}(-13) + \frac{(-.4)(-1.4)(-2.4)}{3!}(2)$$

$$+ \frac{(-.4)(-1.4)(-2.4)(-3.4)}{4!}(5) = 7.864$$

Also,
$$y_{45} = y_{40} + u\,\Delta y_{40} + \frac{u(u-1)}{2!}\Delta^2 y_{40} + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_{40}$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_{40}$$

$$= 25 + (.5)(35) + \frac{(.5)(-.5)}{2}(-13) + \frac{(.5)(-.5)(-1.5)}{6}(2)$$

$$+ \frac{(.5)(-.5)(-1.5)(-2.5)}{24}(5)$$

$$= 44.0546 \approx 44.$$

Hence the number of students who secured marks in the range from 36 to 45 $= y_{45} - y_{36} = 44 - 8 = 36$.

Example 8. *The following are the numbers of deaths in four successive ten year age groups. Find the number of deaths at 45–50 and 50–55.*

Age group:	*25–35*	*35–45*	*45–55*	*55–65*
Deaths:	*13229*	*18139*	*24225*	*31496.*

Sol. Difference table of cumulative frequencies:

Age upto x	*No. of deaths* $f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
35	$\boxed{13229}$			
		$\boxed{18139}$		
45	31368		$\boxed{6086}$	
		24225		$\boxed{1185}$
55	55593		7271	
		31496		
65	87089			

Here, $h = 10,\ a = 35,\ a + hu = 50$

$\therefore\quad 35 + 10u = 50 \quad\Rightarrow\quad u = 1.5$

By Newton's forward difference formula,

$$y_{50} = y_{35} + u\,\Delta y_{35} + \frac{u(u-1)}{2!}\Delta^2 y_{35} + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_{35}$$

$$= 13229 + (1.5)(18139) + \frac{(1.5)(.5)}{2}(6086) + \frac{(1.5)(.5)(-.5)}{6}(1185)$$

$$= 42645.6875 \approx 42646$$

$\therefore$ Deaths at the age beween 45 – 50 is 42646 – 31368 = 11278

and Deaths at the age between 50 – 55 is 55593 – 42646 = 12947.

ASSIGNMENT

1. Estimate the value of $f(22)$ from the following available data:

x:	20	25	30	35	40	45
$f(x)$:	354	332	291	260	231	204

2. Given that:

x:	1	2	3	4	5	6
$y(x)$:	0	1	8	27	64	125

Find the value of $f(2.5)$. [*U.P.T.U. (MCA) 2008*]

3. (*i*) The table below gives value of tan x for $.10 \le x \le .30$.

x:	.10	.15	.20	.25	.30
tan x:	.1003	.1511	.2027	.2553	.3093

Evaluate tan 0.12 using Newton's forward difference formula.

(*ii*) Find the value of sin 52° from the following table using Newton's forward difference formula:

θ:	45°	50°	55°	60°
sin θ:	0.7071	0.7660	0.8192	0.8660

4. (*i*) Fit a polynomial of degree 3 and hence determine $y(3.5)$ for the following data:

x:	3	4	5	6
y:	6	24	60	120

[*M.T.U. 2013, G.B.T.U. 2011, 2012*]

(*ii*) Find the interpolating polynomial to the following data and hence find the value of y for $x = 5$:

x:	4	6	8	10
$f(x)$:	1	3	8	16

(*G.B.T.U. 2013*)

(*iii*) Express the value of θ in terms of x using the following data:

x:	40	50	60	70	80	90
θ:	184	204	226	250	276	304

Also find θ at $x = 43$. (*M.T.U. 2012*)

5. (*i*) Obtain the value of $f(3.5)$ from the following data:

x:	3	4	5	6	7
$f(x)$:	3	6.6	15	22	35

(*G.B.T.U. 2010*)

(*ii*) Use Newton-Gregory formula to compute y at $x = 24$ from the following data:

x:	21	25	29	33	37
y:	18.4	17.8	17.1	16.3	15.5

[*G.B.T.U. (C.O.) 2011*]

6. (*i*) Find the cubic polynomial which takes the following values:

$$y(0) = 1, \quad y(1) = 0, \quad y(2) = 1 \text{ and } y(3) = 10$$

Hence or otherwise obtain $y(4)$.

(*ii*) Find the polynomial interpolating the data:

x:	0	1	2
y:	0	5	2

(*U.P.T.U. 2008*)

7. Ordinates $f(x)$ of a normal curve in terms of standard deviation x are given as

x:	1.00	1.02	1.04	1.06	1.08
$f(x)$:	0.2420	0.2371	0.2323	0.2275	0.2227

Find the ordinate for standard deviation $x = 1.025$.

8. Find the number of men getting wages between ₹ 10 and ₹ 15 from following table:

Wages (in ₹):	0–10	10–20	20–30	30–40
Frequency:	9	30	35	42

(G.B.T.U. 2011)

9. Following are the marks obtained by 492 candidates in a certain examination

Marks:	0–40	40–45	45–50	50–55	55–60	60–65
No. of candidates:	210	43	54	74	32	79

Find out the number of candidates who secured

(*a*) more than 48 but not more than 50 marks

(*b*) less than 48 but not less than 45 marks.

10. Find the number of students from the following data who secured marks not more than 45

Marks range:	30–40	40–50	50–60	60–70	70–80
No. of students :	35	48	70	40	22

11. Use Newton's forward difference formula to obtain the interpolating polynomial $f(x)$ satisfying the following data:

x:	1	2	3	4
$f(x)$:	26	18	4	1

If another point $x = 5$, $f(x) = 26$ is added to the above data, will the interpolating polynomial be the same as before or different. Explain why?

12. Find the polynomial of degree four which takes the following values:

x:	2	4	6	8	10
y:	0	0	1	0	0

(U.P.T.U. 2007)

13. Use Newton's method to find a polynomial $p(x)$ of lowest possible degree such that $p(n) = 2^n$ for $n = 0, 1, 2, 3, 4$.

14. Find the order of the polynomial which might be suitable for the following function:

x:	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7
$f(x)$:	0.577	0.568	0.556	0.540	0.520	0.497	0.471	0.442

Also find the value of $f(2.15)$ using difference formulae. [G.B.T.U. (MCA) 2010]

Answers

1. 352.22304

2. 3.375

3. (*i*) 0.1205 (*ii*) 0.7880

4. (*i*) $x^3 - 3x^2 + 2x$, 13.125 (*ii*) $\frac{3}{8}x^2 - \frac{11}{4}x + 6$, 1.625 (*iii*) $\frac{1}{100}x^2 + \frac{11}{10}x + 124$, 189.79

5. (*i*) 3.28125 (*ii*) 17.9571

6. (*i*) $x^3 - 2x^2 + 1$; 33 (*ii*) $9x - 4x^2$

7. 0.23589625

8. 15

9. (*a*) 27 (*b*) 27

10. 51

11. $\frac{17}{6}x^3 - 20x^2 + \frac{193}{6}x + 11$; no change since third differences are constant.

12. $\frac{1}{64}(x^4 - 24x^3 + 196x^2 - 624x + 640)$

13. $\frac{x^4}{24} - \frac{x^3}{12} + \frac{11}{12}x^2 + \frac{7x}{12} + 1$

14. 7th, 0.562425293.

4.35. NEWTON'S GREGORY BACKWARD INTERPOLATION FORMULA

| U.P.T.U. MCA (SUM) 2008 |

Let $y = f(x)$ be a function of x which assumes the values $f(a), f(a+h), f(a+2h), \ldots\ldots, f(a+nh)$ for $(n+1)$ equidistant values $a, a+h, a+2h, \ldots\ldots, a+nh$ of the independent variable x.

Let $f(x)$ be a polynomial of n^{th} degree.

Let, $$f(x) = A_0 + A_1(x-a-nh) + A_2(x-a-nh)(x-a-\overline{n-1}\,h) + \ldots\ldots + A_n(x-a-nh)(x-a-\overline{n-1}\,h)\ldots\ldots(x-a-h)$$

where $A_0, A_1, A_2, A_3, \ldots\ldots, A_n$ are to be determined. ...(1)

Put $x = a+nh, a+\overline{n-1}\,h, \ldots\ldots, a$ in (1) respectively.

Put $x = a+nh$, then $f(a+nh) = A_0$ ...(2)

Put $x = a+(n-1)h$, then

$$f(a+\overline{n-1}\,h) = A_0 - hA_1 = f(a+nh) - hA_1 \qquad | \text{ By (2)}$$

$$\Rightarrow \qquad A_1 = \frac{\nabla f(a+nh)}{h} \qquad \ldots(3)$$

Put $x = a+(n-2)h$, then

$$f(a+\overline{n-2}\,h) = A_0 - 2hA_1 + (-2h)(-h)A_2$$

$$\Rightarrow \qquad 2!\,h^2A_2 = f(a+\overline{n-2}\,h) - f(a+nh) + 2\nabla f(a+nh) = \nabla^2 f(a+nh)$$

$$A_2 = \frac{\nabla^2 f(a+nh)}{2!\,h^2} \qquad \ldots(4)$$

Proceeding, we get $$A_n = \frac{\nabla^n f(a+nh)}{n!\,h^n} \qquad \ldots(5)$$

Substituting the values in (1), we get

$$f(x) = f(a+nh) + (x-a-nh)\frac{\nabla f(a+nh)}{h} + \ldots\ldots + (x-a-nh)(x-a-\overline{n-1}\,h)\ldots\ldots(x-a-h)\frac{\nabla^n f(a+nh)}{n!\,h^n} \qquad \ldots(6)$$

Put $x = a+nh+uh$, then

$$x-a-nh = uh$$

and $$x-a-(n-1)h = (u+1)h$$

$$\vdots$$

$$x-a-h = (u+\overline{n-1})\,h$$

$\therefore$ (6) becomes,

$$f(x) = f(a+nh) + u\,\nabla f(a+nh) + \frac{u(u+1)}{2!}\nabla^2 f(a+nh) + \ldots\ldots + uh\,.\,(u+1)h\ldots\ldots(u+\overline{n-1})(h)\frac{\nabla^n f(a+nh)}{n!\,h^n}$$

or

$$\boxed{f(a+nh+uh) = f(a+nh) + u\,\nabla f(a+nh) + \frac{u(u+1)}{2!}\nabla^2 f(a+nh) + \ldots\ldots + \frac{u(u+1)\ldots\ldots(u+\overline{n-1})}{n!}\nabla^n f(a+nh)}$$

This formula is useful when the value of $f(x)$ is required near the end of the table.

EXAMPLES

Example 1. *The population of a town was as given. Estimate the population for the year 1925.*

Years (x):	*1891*	*1901*	*1911*	*1921*	*1931*
Population (y): *(in thousands)*	*46*	*66*	*81*	*93*	*101*

Sol. Here, $a + nh = 1931,\ h = 10,\quad a + nh + uh = 1925$

$$\therefore \quad u = \frac{1925 - 1931}{10} = -0.6$$

Difference table is:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
		20			
1901	66		– 5		
		15		2	
1911	81		– 3		$\boxed{-3}$
		12		$\boxed{-1}$	
1921	93		$\boxed{-4}$		
		$\boxed{8}$			
1931	$\boxed{101}$				

Applying Newton's Backward difference formula, we get

$$y_{1925} = y_{1931} + u\nabla y_{1931} + \frac{u(u+1)}{2!}\nabla^2 y_{1931} + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_{1931} + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_{1931}$$

$$= 101 + (-.6)(8) + \frac{(-.6)(.4)}{2!}(-4) + \frac{(-.6)(.4)(1.4)}{3!}(-1) + \frac{(-.6)(.4)(1.4)(2.4)}{4!}(-3)$$

$$= 96.8368 \text{ thousands.}$$

Hence the population for the year 1925 = 96836.8 ≈ 96837.

Example 2. *The population of a town is as follows:*

Year:	*1921*	*1931*	*1941*	*1951*	*1961*	*1971*
Population: *(in Lakhs)*	*20*	*24*	*29*	*36*	*46*	*51*

Estimate the increase in population during the period 1955 to 1961.

Sol. Here, $a + nh = 1971,\ h = 10,\ a + nh + uh = 1955$

$$\therefore \quad 1971 + 10u = 1955 \quad \Rightarrow \quad u = -1.6$$

Difference table is:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1921	20					
		4				
1931	24		1			
		5		1		
1941	29		2		0	
		7		1		$\boxed{-9}$
1951	36		3		$\boxed{-9}$	
		10		$\boxed{-8}$		
1961	46		$\boxed{-5}$			
		$\boxed{5}$				
1971	$\boxed{51}$					

Applying Newton's backward difference formula, we get

$$y_{1955} = y_{1971} + u\nabla y_{1971} + \frac{u(u+1)}{2!}\nabla^2 y_{1971} + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_{1971}$$

$$+ \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_{1971} + \frac{u(u+1)(u+2)(u+3)(u+4)}{5!}\nabla^5 y_{1971}$$

$$= 51 + (-1.6)(5) + \frac{(-1.6)(-0.6)}{2!}(-5) + \frac{(-1.6)(-0.6)(0.4)}{6}(-8)$$

$$+ \frac{(-1.6)(-0.6)(.4)(1.4)}{24}(-9) + \frac{(-1.6)(-0.6)(0.4)(1.4)(2.4)}{120}(-9)$$

$$= 39.789632$$

$\therefore$ Increase in population during period 1955 to 1961 is

$$= 46 - 39.789632 = 6.210368 \text{ Lakhs} = 621036.8 \approx 621037.$$

Example 3. *Evaluate from following table f(3.8) to three significant figures using Gregory-Newton backward interpolation formula.*

x:	*0*	*1*	*2*	*3*	*4*	
f(x):	*1*	*1.5*	*2.2*	*3.1*	*4.6*	(U.P.T.U. 2009)

Sol. Here, $a + nh = 4$, $h = 1$, $a + nh + uh = 3.8$

$$\therefore \qquad u = \frac{3.8 - 4}{1} = -0.2$$

Backward difference table is

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
0	1				
		0.5			
1	1.5		0.2		
		0.7		0	
2	2.2		0.2		$\boxed{0.4}$
		0.9		$\boxed{0.4}$	
3	3.1		$\boxed{0.6}$		
		$\boxed{1.5}$			
4	$\boxed{4.6}$				

By Newton's backward difference formula,

$$f(3.8) = f(4) + u\,\nabla f(4) + \frac{u\,(u+1)}{2}\,\nabla^2 f(4) + \frac{u\,(u+1)\,(u+2)}{3\,!}\,\nabla^3 f(4)$$

$$+ \frac{u\,(u+1)\,(u+2)\,(u+3)}{4\,!}\,\nabla^4 f(4)$$

$$= 4.6 + (-0.2)\,(1.5) + \frac{(-0.2)\,(0.8)}{2}\,(0.6) + \frac{(-0.2)\,(0.8)\,(1.8)}{3\,!}\,(.4)$$

$$+ \frac{(-.2)\,(.8)\,(1.8)\,(2.8)}{4\,!}\,(.4) = 4.219\ldots$$

Example 4. *Given log x for x = 40, 45, 50, 55, 60 and 65 according to the following table*

x:	40	45	50	55	60	65
log x:	1.60206	1.65321	1.69897	1.74036	1.77815	1.81291

Find the value of log 5875. [M.T.U. (MCA) 201…]

Sol. The difference table is:

x	$10^5 \log x = 10^5\, y_x$	$10^5\, \nabla y_x$	$10^5\, \nabla^2 y_x$	$10^5\, \nabla^3 y_x$	$10^5 \nabla^4 y_x$	$10^5\, \nabla^5 y_x$
40	160206					
		5115				
45	165321		− 539			
		4576		102		
50	169897		− 437		− 25	
		4139		77		$\boxed{5}$
55	174036		− 360		$\boxed{-20}$	
		3779		$\boxed{57}$		
60	177815		$\boxed{-303}$			
		$\boxed{3476}$				
65	$\boxed{181291}$					

Newton's Backward difference formula is

$$f(a+nh+uh) = f(a+nh) + u\nabla f(a+nh) + \frac{u(u+1)}{2\,!}\,\nabla^2 f(a+nh)$$

$$+ \frac{u(u+1)(u+2)}{3\,!}\,\nabla^3 f(a+nh) + \frac{u(u+1)(u+2)(u+3)}{4\,!}\,\nabla^4 f(a+nh)$$

$$+ \frac{u(u+1)(u+2)(u+3)(u+4)}{5\,!}\,\nabla^5 f(a+nh) \quad \ldots(1)$$

First we shall find the value of log(58.75).

Here, $\quad a + nh = 65,\ h = 5,\ a + nh + uh = 58.75$

$\therefore \quad 65 + 5u = 58.75 \quad \Rightarrow \quad u = -1.25$

From (1),

$$10^5 f(58.75) = 181291 + (-1.25)(3476) + \frac{(-1.25)(-.25)}{2!}(-303)$$

$$+ \frac{(-1.25)(-.25)(.75)}{3!}(57) + \frac{(-1.25)(-.25)(.75)(1.75)}{4!}(-20)$$

$$+ \frac{(-1.25)(-.25)(.75)(1.75)(2.75)}{5!}(5)$$

$\Rightarrow$ $10^5 f(58.75) = 176900.588$

$\therefore$ $f(58.75) = \log 58.75 = 176900.588 \times 10^{-5} = 1.76900588$

Hence,

$\log 5875 = 3.76900588$ | $\because$ Mantissa remain the same

Example 5. *From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policy maturing at the age of 63:*

Age:	45	50	55	60	65
Premium: (in rupees)	114.84	96.16	83.32	74.48	68.48

Sol. The difference table is:

Age (x)	Premium (in rupees) (y)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
45	114.84				
		– 18.68			
50	96.16		5.84		
		– 12.84		– 1.84	
55	83.32		4		$\boxed{.68}$
		– 8.84		$\boxed{-1.16}$	
60	74.48		$\boxed{2.84}$		
		$\boxed{-6}$			
65	$\boxed{68.48}$				

Here $a + nh = 65$, $h = 5$, $a + nh + uh = 63$

$\therefore$ $65 + 5u = 63 \Rightarrow u = -.4$

By Newton's backward difference formula,

$$y(63) = y(65) + u\nabla y(65) + \frac{u(u+1)}{2!}\nabla^2 y(65) + \frac{u(u+1)(u+2)}{3!}\nabla^3 y(65)$$

$$+ \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y(65)$$

$$= 68.48 + (-.4)(-6) + \frac{(-.4)(.6)}{2}(2.84) + \frac{(-.4)(.6)(1.6)}{6}(-1.16) + \frac{(-.4)(.6)(1.6)(2.6)}{24}(.68)$$

$$= 70.585152$$

ASSIGNMENT

1. Find a polynomial of degree three using Newton-Gregory backward difference formula wh takes the following values. Hence find $y(7)$:

x:	3	4	5	6
y:	6	24	60	120

(G.B.T.U. 20

2. The population of a twon in decennial census is as under. Estimate the population for the ye 1955:

Year:	1921	1931	1941	1951	1961
Population (in lacs):	46	66	81	93	101

(M.T.U. 20

3. Estimate the value of $f(42)$ from the following available data:

x:	20	25	30	35	40	45
$f(x)$:	354	332	291	260	231	204

4. The table below gives the value of $\tan x$ for $0.10 \le x \le 0.30$:

x:	0.10	0.15	0.20	0.25	0.30
$y = \tan x$:	0.1003	0.1511	0.2027	0.2553	0.3093

Find: (*i*) tan 0.50 (*ii*) tan 0.26 (*iii*) tan 0.40.

5. (*i*) Given:

x:	1	2	3	4	5	6	7	8
$f(x)$:	1	8	27	64	125	216	343	512

Find $f(7.5)$ using Newton's Backward difference formula.

(*ii*) Compute $f(8)$ from the following data:

x:	1	3	5	7	9
$f(x)$:	9	21	81	237	537

(G.B.T.U. 20

6. Using Newton's backward difference formula, find the value of $e^{-1.9}$ from the following table values of e^{-x}:

x:	1	1.25	1.50	1.75	2.00
e^{-x}:	0.3679	0.2865	0.2231	0.1738	0.1353

7. If $y(10) = 35.3$, $y(15) = 32.4$, $y(20) = 29.2$, $y(25) = 26.1$, $y(30) = 23.2$ and $y(35) = 20.5$, find $y($ using Newton's forward as well as backward interpolation formula. Also explain why the diff ence (if any) in the result occur. (U.P.T.U. 20

8. From the following table of values of x and $f(x)$, determine (*i*) $f(0.23)$ (*ii*) $f(0.29)$:

x:	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$:	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

9. From the following table, find the value of tan 17°

$\theta°$:	0	4	8	12	16	20	24
$\tan \theta°$:	0	0.0699	0.1405	0.2126	0.2867	0.3640	0.4402

10. From the following table:

x:	10°	20°	30°	40°	50°	60°	70°	80°
$\cos x$:	0.9848	0.9397	0.8660	0.7630	0.6428	0.5000	0.3420	0.1737

Calculate cos 25° and cos 73° using Gregory Newton formula. (U.P.T.U. 20

Answers

1. $x^3 - 3x^2 + 2x$, 210 2. 9683680 3. 219
4. (*i*) .5543 (*ii*) .2662 (*iii*) .4241
5. (*i*) 421.875 (*ii*) 366 6. 0.1496
7. 34.22007, 34.30866 8. (*i*) 1.6751 (*ii*) 1.7081
9. 0.3057 10. 0.9063, 0.2923

4.36. CENTRAL DIFFERENCE INTERPOLATION FORMULAE

We shall study now the central difference formulae most suited for interpolation near the middle of a tabulated set.

4.37. GAUSS' FORWARD DIFFERENCE FORMULA

Newton's Gregory forward difference formula is

$$f(a+hu) = f(a) + u\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a) + \ldots\ldots \quad \ldots(1)$$

Put $a = 0$, $h = 1$, we get

$$f(u) = f(0) + u\Delta f(0) + \frac{u(u-1)}{2!}\Delta^2 f(0) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(0) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(0) + \ldots\ldots \quad \ldots(2)$$

Now, $\Delta^3 f(-1) = \Delta^2 f(0) - \Delta^2 f(-1) \Rightarrow \Delta^2 f(0) = \Delta^3 f(-1) + \Delta^2 f(-1)$

Also, $\Delta^4 f(-1) = \Delta^3 f(0) - \Delta^3 f(-1) \Rightarrow \Delta^3 f(0) = \Delta^4 f(-1) + \Delta^3 f(-1)$

and $\Delta^5 f(-1) = \Delta^4 f(0) - \Delta^4 f(-1) \Rightarrow \Delta^4 f(0) = \Delta^5 f(-1) + \Delta^4 f(-1)$ and so on.

$\therefore$ From (2),

$$f(u) = f(0) + u\Delta f(0) + \frac{u(u-1)}{2!}\{\Delta^2 f(-1) + \Delta^3 f(-1)\} + \frac{u(u-1)(u-2)}{3!}\{\Delta^3 f(-1) + \Delta^4 f(-1)\} + \frac{u(u-1)(u-2)(u-3)}{4!}\{\Delta^4 f(-1) + \Delta^5 f(-1)\} + \ldots\ldots$$

$$= f(0) + u\Delta f(0) + \frac{u(u-1)}{2!}\Delta^2 f(-1) + \frac{u(u-1)}{2}\left\{1 + \frac{u-2}{3}\right\}\Delta^3 f(-1) + \frac{u(u-1)(u-2)}{6}\left\{1 + \frac{u-3}{4}\right\}\Delta^4 f(-1) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^5 f(-1) + \ldots\ldots$$

$$= f(0) + u\Delta f(0) + \frac{u(u-1)}{2!}\Delta^2 f(-1) + \frac{(u+1)u(u-1)}{3!}\Delta^3 f(-1) + \frac{(u+1)u(u-1)(u-2)}{4!}\Delta^4 f(-1) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^5 f(-1) + \ldots\ldots \quad \ldots(3)$$