Euler's Method :-

$$\boxed{y_{n+1} = y_n + h f(x_n, y_n)}$$

$n = 0, 1, 2 \ldots$

Ex Using Euler's method, find an approximate value of y corresponding to $x = 1$, given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$

Sol We take $n = 10$ and $h = 0.1$ which is sufficiently small. The various calculations are as follows :-

$y_0 = 1$, $x_0 = h = 0.1$, $f(x, y) = x + y$

$\therefore$ we have

$y_0 = 1$, $x_0 = h = 0.1$, $f(x, y) = x + y$

$y_1 = y_0 + h f(x_0, y_0) = 1.00 + 0.1 (1.00) = 1.10$

$y_2 = y_1 + h f(x_1, y_1)$

$= 1.10 + 0.1 (1.20) = 1.22$

$y_3 = y_2 + h f(x_2, y_2)$

$= 1.22 + 0.1 (1.42) = 1.36$

$y_4 = y_3 + h f(x_3, y_3)$

$= 1.36 + 0.1 (1.66) = 1.53$

$$y_5 = y_4 + h f(x_4, y_4)$$
$$= 1.53 + 0.1\,(1.93) = 1.72$$

$$y_6 = y_5 + h f(x_5, y_5)$$
$$= 1.72 + 0.1\,(2.22) = 1.94$$

$$y_7 = y_6 + h f(x_6, y_6) = 1.94 + 0.1(2.54) = 2.19$$

$$y_8 = y_7 + h f(x_7, y_7) = 2.19 + 0.1(2.89) = 2.48$$

$$y_9 = y_8 + h f(x_8, y_8) = 2.48 + 0.1\,(3.89) = 2.81$$

$$y_{10} = y_9 + h f(x_9, y_9) = 2.81 + 0.1\,(3.71) = 3.18$$

Thus the required approximate value of $y = 3.18$

Given $\frac{dy}{dx} = \frac{y-x}{y+x}$, with $y=1$ for $x=0$. find y approximately for $x=0.1$ By Euler's Method in five steps.

Since we want to reach $x=0.1$ from $x=0$ in five steps, say x_1, x_2, x_3, x_4, x_5. we shall determine the values of y at $x = 0.02, 0.04, 0.06, 0.08$ and 0.1 respectively

Therefore we have.

$y_0 = 1$, $x_0 = 0$, $h = 0.02$, $f(x,y) = \frac{y-x}{y+x}$

$$y_1 = y_0 + h f(x_0, y_0)$$

$$= 1 + (0.02)\left[\frac{1-0}{1+0}\right] = 1.02$$

$$y_2 = y_1 + h f(x_1, y_1)$$

$$= 1.02 + (0.02)\frac{1.02 - 0.02}{1.02 + 0.02}$$

$$= 1.0392$$

$$y_3 = y_2 + h f(x_2, y_2)$$

$$= 1.0392 + (0.02) \cdot \frac{1.0392 - 0.04}{1.0392 + 0.04}$$

$$= 1.0577$$

$$y_4 = y_3 + h f(x_3 y_3) = 1.0577 + (0.02) \times \frac{1.0577 - 0.06}{1.0577 + 0.06}$$

$$= 1.0756$$

$$y_5 = 1.0756 + (0.02) \cdot \frac{1.0756 - 0.08}{1.0756 + 0.08}$$

$$= 1.0928$$

Hence for $x = 0.1$, y is 1.0928

in the given equation, it is asked to solve the equation by Euler's method in four steps than the value of h will be

$h = \frac{0.1}{4} = 0.025 = 0.025$ and we will continue this process till y_4.

Using Euler's method, compute $y(0.5)$ for differential equation $\frac{dy}{dx} = y^2 - x^2$ with $y = 1$ when $x = 0$

Given $\frac{dy}{dx} = xy$ with $y^{(1)} = 5$. Find the solution in the interval $[1, 1.5]$ using $h = 0.1$

Modified Euler's Method (Predictor - Corrector Method)

$$y_1 = y_0 + h f(x_0 y_0)$$

$$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_0 + h, y_1)]$$

$$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0 y_0) + f(x_0 + h, y_1^{(1)})]$$

$$y_2 = y_1 + h f(x_0 + h, y_1)$$

$$y_2^{(1)} = y_1 + \frac{h}{2}[f(x_0 + h, y_1) + f(x_0 + 2h, y_2)]$$

We repeat this process until y_2 becomes stationary. Then we proceed to calculate y_3 as above and so on.

Solve $\frac{dy}{dx} = 1 - y$, $y(0) = 0$ in the range $0 \le x \le 0.2$ by taking $h = 0.1$

Sol: By Euler's Method

$$y^{(1)} = y_0 + h f(x_0, y_0)$$

$$= 0 + (0.1)(1 - 0) = 0.1$$

Using Euler's method, compute $y(0.5)$ for differential equation $\frac{dy}{dx} = y^2 - x^2$ with $y=1$ when $x=0$

Given $\frac{dy}{dx} = xy$ with $y(1) = 5$. Find the Solution in the interval $[1, 1.5]$ using $h=0.1$

odified Euler's Method (Predictor - Corrector Method)

$$y_1 = y_0 + h f(x_0, y_0)$$

$$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_0 + h, y_1)]$$

$$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_0 + h, y_1^{(1)})]$$

$$y_2 = y_1 + h f(x_0 + h, y_1)$$

$$y_2^{(1)} = y_1 + \frac{h}{2}[f(x_0 + h, y_1) + f(x_0 + 2h, y_2)]$$

We repeat this process until y_2 becomes stationary. Then we proceed to calculate y_3 as done and so on.

Solve $\frac{dy}{dx} = 1 - y$, $y(0) = 0$ in the range $0 \le x \le 0.2$ taking $h = 0.1$

By Euler's Method

$$y_1^{(1)} = y_0 + h f(x_0, y_0)$$
$$= 0 + (0.1)(1 - 0) = 0.1$$

Now by formula

first approximation :-

$y_1 = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1,y_1).\ y_1^{(1)}]$

$= 0 + \frac{0.1}{2}[1 + (1-0.1)]$

$= 0.095$

$y_1^{(2)} = y_0 + \left(\frac{h}{2}\right)[f(x_0,y_0) + f(x_1, 0.095)]$

$= 0 + \left(\frac{1}{2}\right)[1+1-0.095]$

$= 0.0952$

$y_1^{(3)} = 0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, 0.0952)]$

$= \frac{1}{2}[1+1-0.0952]$

$= 0.09524$

$y_2^{(1)} = y_1 + h(f(x_1,y_1)$

$= 0.09524 + (0.1)[1 - 0.09524]$

the first approximation to y_2 is given by

$= y_1 + \left(\frac{h}{2}\right)[f(x,y_1) + f(x_2,y_2).\ y_2^{(1)}]$

$= 0.09524 + \left(\frac{0.1}{2}\right)[(1-0.09524) + (1-0.1857163)]$

$= 0.09524 + 0.0859522 = 0.1811922$

the 2nd approximation to y_2

$= y_1 + \left(\frac{h}{2}\right)[f(x_1,y_1) + f(x_1, 0.1811922)]$

$= 0.09524 + \left(\frac{0.1}{2}\right)[(1-0.09524) + (1-0.1811922)]$

$= 0.18141839$

Using Euler's modified, obtain a solution to the equation $\frac{dy}{dx} = x + |\sqrt{y}| = f(x, y)$ with initial condition $y = 1$ at $x = 0$ for the range $0 \le x \le 0.6$ in steps of 0.2

find $y(2.2)$ using Euler's modified method for $\frac{dy}{dx} = -xy^2$, where $y(2) = 1$ (Take $h = 0.1$)

Find $y(0.2)$ and $y(0.5)$. Given

$\frac{dy}{dx} = \log_{10}(x+y)$

with initial condition $y = 1$ for $x = 0$

Ex. Using Euler's modified, obtain a Solution to the equation $\frac{dy}{dx} = x + |\sqrt{y}| = f(x, y)$ with initial Condition $y = 1$ at $x = 0$ for the range $0 \le x \le 0.6$ in steps of 0.2

Ex find $y(2.2)$ using Euler's modified Method for $\frac{dy}{dx} = -xy^2$, where $y(2) = 1$ (Take $h = 0.1$)

Ex Find $y(0.2)$ and $y(0.5)$. Given

$$\frac{dy}{dx} = \log_{10}(x+y)$$

with initial Condition $y = 1$ for $x = 0$