Graph Theory (1)

→ When all the element (eg. resistors, inductors, capacitor etc) in a network are replaced by lines with circles, or dots at both ends, the configuration is then called graph of n/w

The graph of any network is drawn by keeping all the points of intersection of two or more branches and representing the network element of lines, voltages and current sources by their internal impedences. The point of intersection of two or more branches is called a node.

Terminology used in Network graph:-

→ Branch:- A branch is a line segment representing one network element or combination of element connected between two points. Though the line segment with its two distinct end points represents a branch. It does not indicate anything about the type of element.

→ Node:- A node point is defined an end point of a line segment and exist at the junction between two branches or at ends of an isolated branch.

→ Tree:- It is an interconnected open set of branches which include all the nodes of given graph. In a tree of graph there can not be any closed loop.

→ Twig (Tree branch):- It is any branch of a tree.

→ Tree link or chord:- It is that branch of the graph that does not belong to the particular tree. It is simply called link.

EX'-

→ Loop:-
This is the closed contour selected in graph.

→ Cutset:- It is that set of element or branches of a graph that seperates two main part of a network.

→ Tie set:- It is a unique set with respect to a given tree of a connected graph containing one link.

→ Directed or oriented graph:-
A graph is said to be directed or oriented when all nodes and branches are numbered and direction are assigned to the branches by arrow.

→ Relation between twig and links:-

No. of twigs in a particular tree = N-1

No. of link in " " " = B-N+1

where B = Branches of graph

N = Nodes " "

→ Properties of a Tree in a graph:-

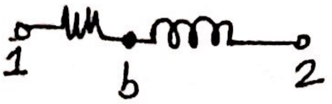
① It consist of all the nodes of graph

② If the graph has 'N' no. of nodes the tree will have (N-1) branches

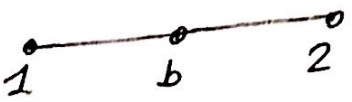
③ There will be no closed path in the tree.

④ There can be many possible different tree for a given graph depending on the no. of nodes and branches.

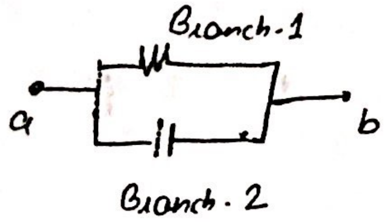
EX:-



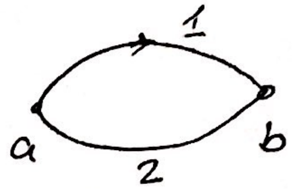
(2)



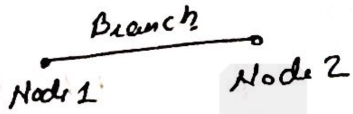
Node - 3
Branch - 2

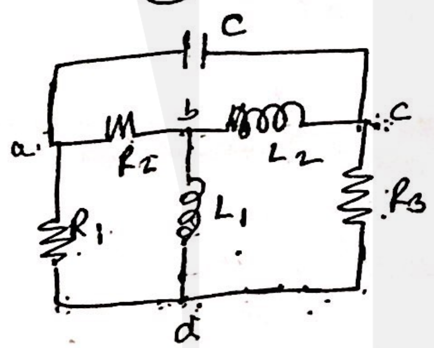


Node - 2
Branch - 2

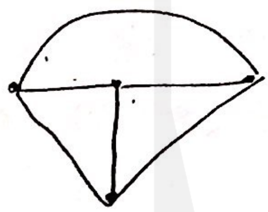


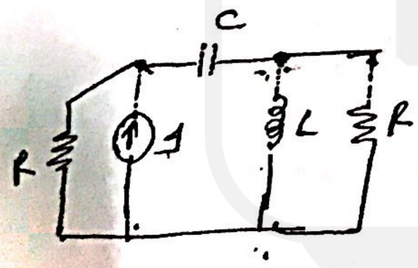
~~Node~~

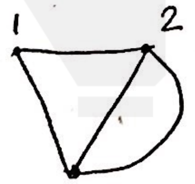




Node 4
Branch - 6



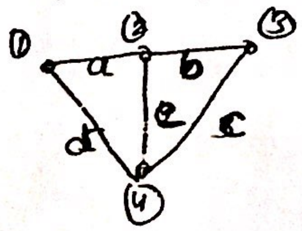




Reference node.

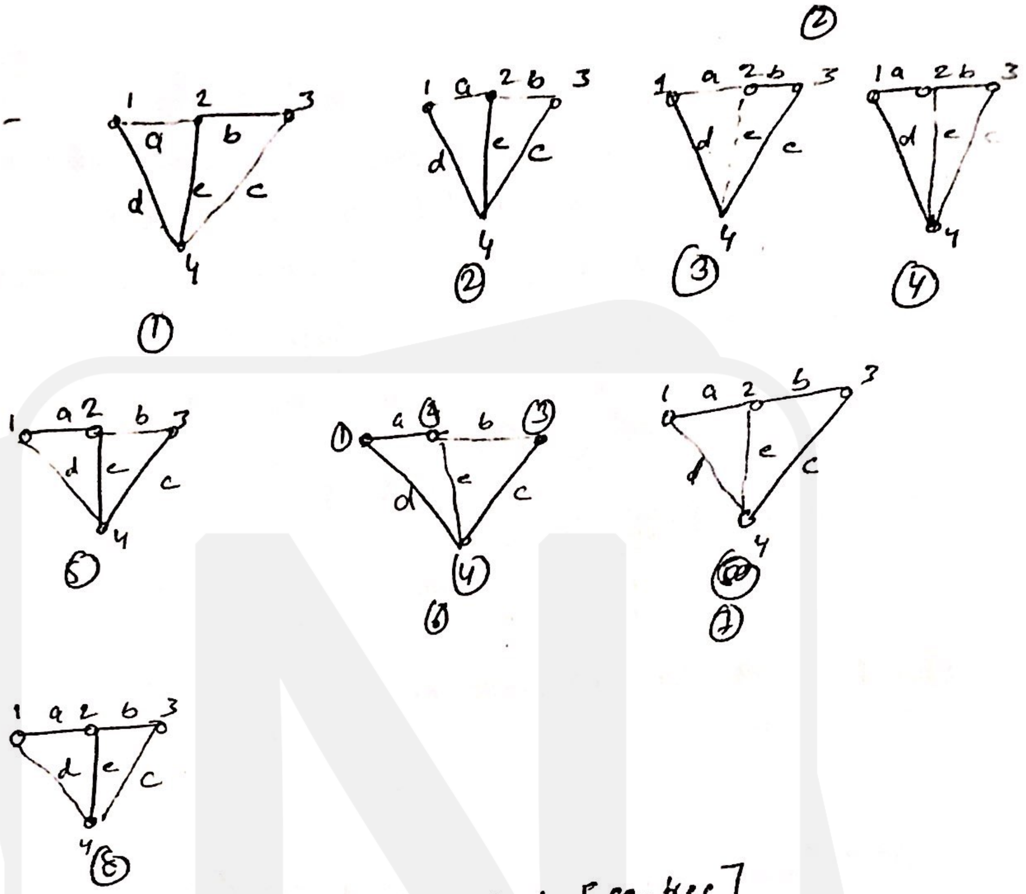
Here current source is present so it is open ckt so no branch is drawn for current source

Q) Fig showing the graph of a network. Show the tree, twigs and links.



Incidence Matrix :-

Tree :-



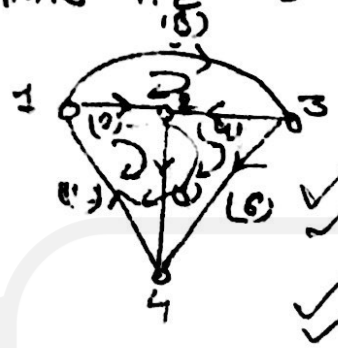
Tree (Twigs)	Link [co-tree]
1. [b e d]	[a, c]
2. [c e d]	[a, b]
3. [d c b]	[a e]
4. [d a b]	[e c]
5. [a e c]	[b d]
6. [a d c]	[b e]
7. [a b c]	[d e]
8. [a b e]	[c d]

Tie set or Fundamental loop (circuit) — B_f

→ loop which contain only one link are independent are called basic fundamental loop or tie set.

→ No. of fundamental loop or tie set is equal to no. of links i.e. $b-n+1$ Imp

EX:-

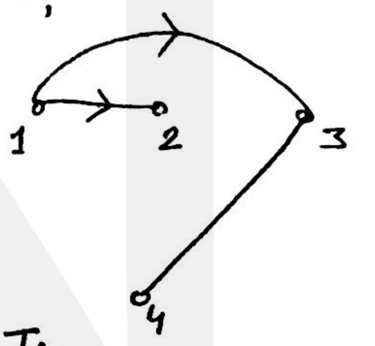


Procedure for obtaining Tie set matrix [fundamental loop matrix]:-

Step1:- Arbitrarily a tree is selected in the graph

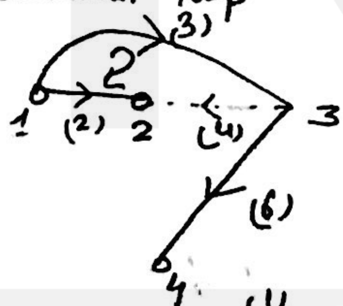
Step2:- Form the fundamental loop with each [single] link in the graph for entire tree

Step 1:- A tree is selected;

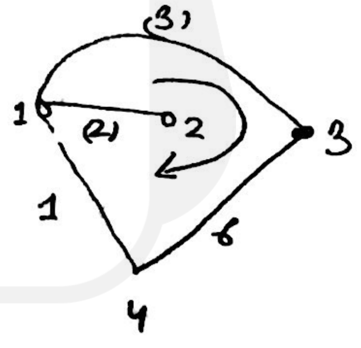


Tree

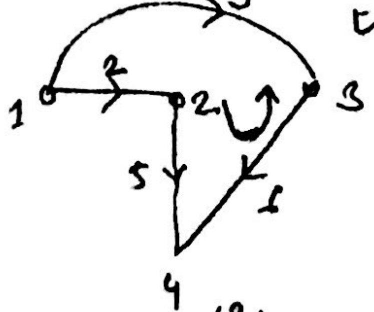
Step 2:- Fundamental loop



(1)



(2)



(3)

Step 3 Assume direction of loop current oriented in the same direction as that of link.

- It will be '+1' if branch direction will be same as loop direction else it will be '-1'

Graph Theory

x

- When all elements [e.g. resistors, inductors, capacitor etc] in a network are replaced by lines with circle or dots at both ends, the configuration is then called the graph of network.

Terminology used in Graph:-

Branch → A branch is a line segment representing one network element or a combination of element connected between two points

Node → A node point is defined as an end point of a line segment and exist at the junc^n between two branches or at the ends of an isolated branch.

Path – Path in a network constitute a set of element traversed such that no node is passed through again

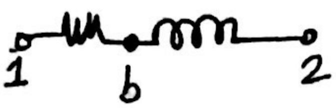
Tree – It is an interconnected open set of branches which include all nodes of given graph. In a tree of graph there cannot be any closed loop.
Each tree has (n-1) branch where n = no. of nodes

Twigs :- The tree branches are called twigs.
No. of twigs = n-1

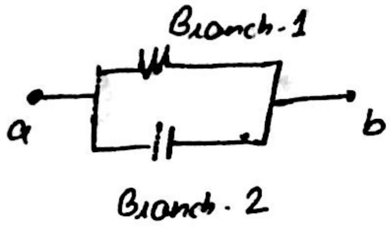
Link – It is that branch of graph that does not belong to a particular tree
No. of links = b-n+1

Loop – closed contour selected in a graph.

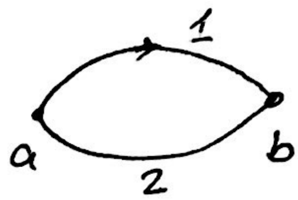
Ex:-



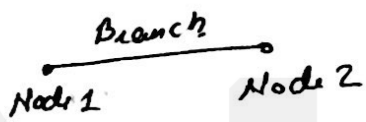
Node - 3
Branch - 2

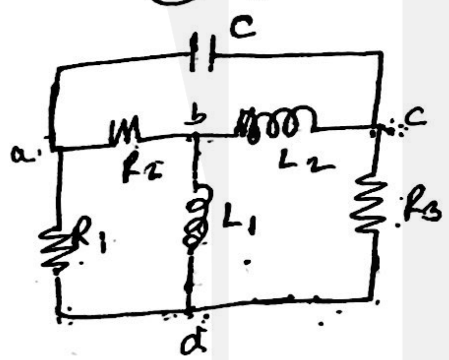


Node - 2
Branch - 2

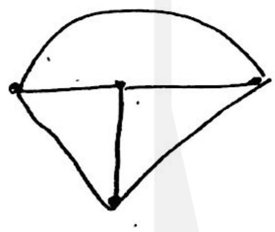


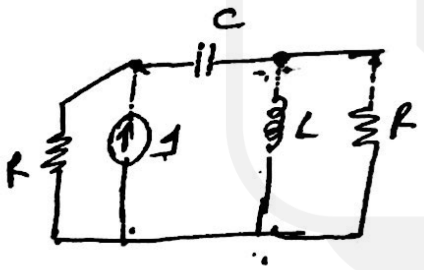
~~Node~~





Node 4
Branch - 6

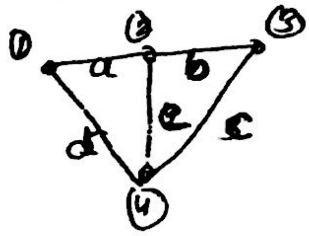




Reference node.

Here current source is present so it is open ckt so no branch is drawn for current source

Q) Fig showing the graph of a network. Show the tree, twigs and links.



Incidence Matrix:-

Any oriented graph can be described completely in a compact matrix form. Here we specify the orientation of each branch in graph and nodes at which this branch is incident. This matrix is called incident matrix

→ When one row is deleted from the complete incidence matrix, the remaining matrix are termed as reduced incidence matrix.

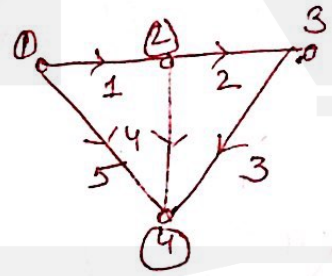
Formation of Incidence matrix:- [Ai]

→ This matrix shows which branch is incident to which node.

Each row of matrix being representing the corresponding node and each column of matrix represent the branch.

→ If branch 'j' is incident at node 'i' but oriented away from node then matrix element will be '+1' and if oriented toward nodes then it will be '-1'

Q) obtain the incidence matrix of given graph.



Branch

$$A_i = \begin{array}{c} \text{Node} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

Properties of Incidence matrix:-

1. Algebraic sum of the column entries of an incidence matrix is zero.

2. Determinant of incidence matrix of a closed loop is zero.

Reduced Incidence Matrix [A]:-

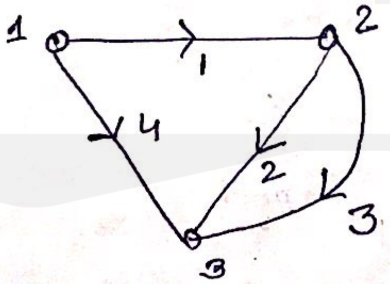
When one row is deleted from complete incidence matrix [Ai] the remaining matrix is called a reduced incidence matrix [A]

Q.) An incidence matrix is given by

$$[A_i] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 \end{bmatrix}$$

Draw the graph corresponding to this matrix.

$$[A_i] = \begin{array}{c|cccc} \text{Node} \backslash \text{Branch} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 1 \\ 2 & -1 & 1 & 1 & 0 \\ 3 & 0 & -1 & -1 & -1 \end{array}$$



Develop the reduced matrix from given incidence matrix.

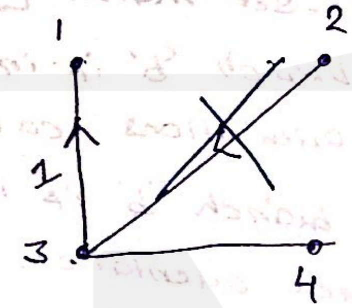
$$[A] = \begin{array}{c|cccc} \text{Node} \backslash \text{Branch} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 1 \\ 2 & -1 & 1 & 1 & 0 \end{array}$$

For reduce matrix delete one row from given incidence matrix

Q) Incidence matrix of a graph is given below. Draw the directed graph.

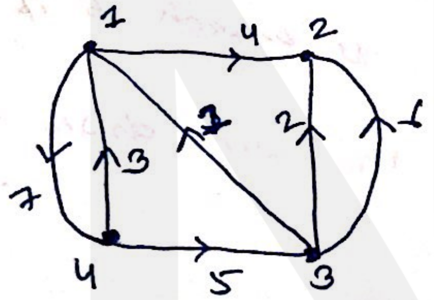
(4)

$$
\begin{array}{c|ccccccc}
 & & & & \text{Branch} & & & \\
\text{Node} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
1 & -1 & 0 & -1 & 1 & 0 & 0 & 1 \\
2 & 0 & -1 & 0 & -1 & 0 & -1 & 0 \\
3 & 1 & 1 & 0 & 0 & -1 & 1 & 0 \\
4 & 0 & 0 & 1 & 0 & 1 & 0 & -1
\end{array}
$$



At node 1 branch '1' and branch '3' is incident

At node '2' Branch 2 and 3 is incident



similarly draw for another nodes

* Fundamental Tie-set Matrix or Fundamental loop matrix [B]:-

→ A fundamental loop or a fundamental tie-set of a graph with respect to a tree is a loop formed by only one link associated with other twigs.

→ No. of fundamental loop or Tie-set

$= B-(N-1)$

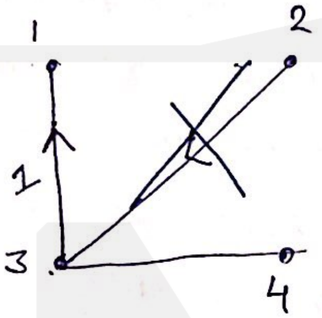
where B → Branch

N → Nodes

Scanned by CamScanner

Q) Incidence matrix of a graph is given below. Draw the directed graph. (4)

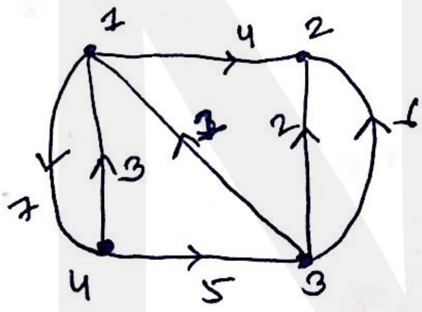
$$
\begin{array}{c|c}
 & \text{Branch} \\
\text{Node} & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} &
\begin{bmatrix}
-1 & 0 & -1 & 1 & 0 & 0 & 1 \\
0 & -1 & 0 & -1 & 0 & -1 & 0 \\
1 & 1 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & -1
\end{bmatrix}
\end{array}
$$



At node 1 branch '1' and branch '3' is incident

At node '2' Branch 2 and 3 is incident

similarly draw for another nodes



\# Fundamental Tie-set Matrix or Fundamental loop matrix [B]:-

→ A fundamental loop or a fundamental tie-set of a graph with respect to a tree is a loop formed by only one link associated with other twigs.

→ No. of fundamental loop or Tie-set

$= B - (N-1)$

where B → Branch

N → Nodes

* Procedure of obtaining Fundamental Tie-Set matrix or loop matrix:-

→ 1] Arbitrarily a tree is selected in the graph.

→ 2] Form fundamental loop with each links in the graph for entire tree.

→ 3] Assume direction of loop current oriented in the same direction as that of link.

→ 4] Form fundamental tie-set matrix $[b_{lb}]$ where;

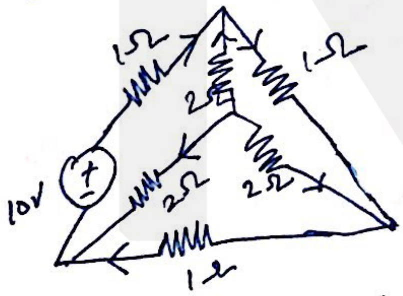
$b_{lb} = 1$; if branch 'b' is in loop 'l' and their orientations coincide.

$= -1$; if branch 'b' is in loop 'l' and their orientation donot coincide.

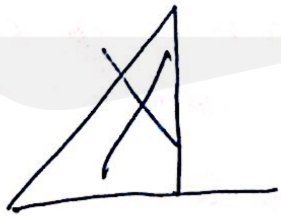
$= 0$; if branch 'b' is not in loop 'l'.

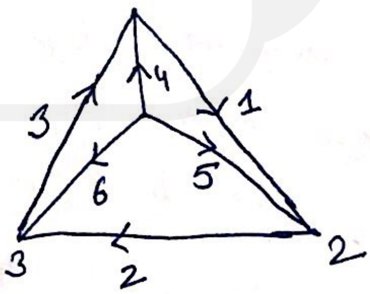
5] Write down the tie-set matrix

Q] Draw the graph and write down the tie-set matrix



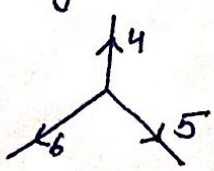
Sol^n:-

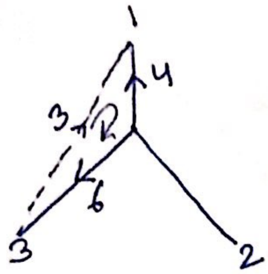




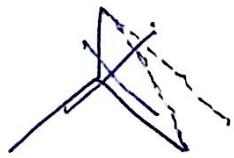
Oriented Graph

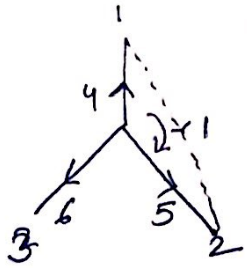
Step 1:- Randomly select a tree



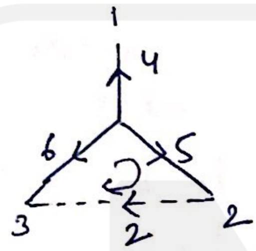


Tie set-1





Tie-Set-2



Tie-Set-3

Tie-set 1 [4 6 3]   Tieset 2 [1 4 5]   Tieset 3 [6 5 2]

$$
\begin{array}{c|c}
 & \text{Branch} \\
\text{Tie set} & \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline \end{bmatrix}
\end{array}
$$

$$
\begin{array}{c}
\text{Tie set} \\ 1 \\ 2 \\ 3
\end{array}
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
0 & 0 & 1 & -1 & 0 & 1 \\
1 & 0 & 0 & 1 & -1 & 0 \\
0 & 1 & 0 & 0 & 1 & -1
\end{bmatrix}
$$

* Cutset Matrix:- $[Q_f]$

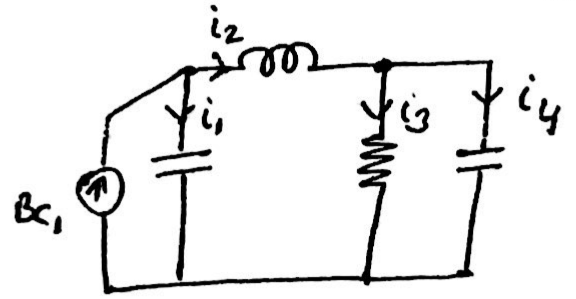
→ A fundamental cutset of a graph with respect to a tree is a cutset formed by one and only one twig and a set of links.

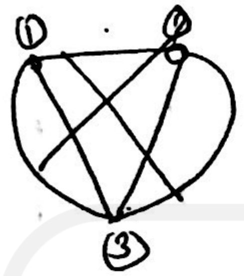
→ For a graph having 'N' nodes there will be (N-1) fundamental cutset.

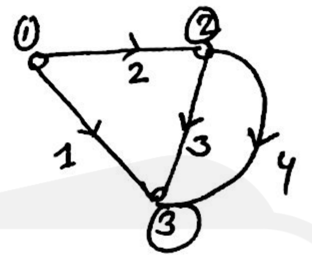
⇒ Orientation of cutset is so chosen that it coincides with orientation of its twig.

→ cutset is a minimal set of branches of graph, removal of which cuts the graph into two part. It seperate the node of graph into two graph each being in one of two parts.

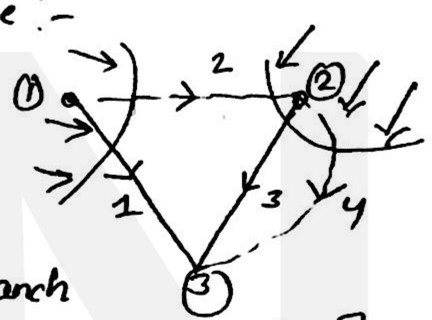
Q1' Develop the fundamental cutset matrix (5)







Select a tree :-



$$
\begin{array}{c|cccc}
 & \multicolumn{4}{c}{\text{Branch}} \\
\text{cutset} & 1 & 2 & 3 & 4 \\
\hline
1 & 1 & 1 & 0 & 0 \\
2 & 0 & -1 & 1 & 1
\end{array}
$$

Procedure of forming Fundamental cutset matrix :-

Step1 :- Arbitrarily a tree is selected in the [single] graph.

Step 2 :- Form fundamental cutset with each twig in the graph for entire tree.

Step3 :- Assume direction of cutset oriented in the same direction of concerned twig.

→ If direction of links is same as that of concerned twig then it will be '+1' else opposite than '-1'.

Fundamental loop or tie set B_f

Tie set 1 = [2 3 4]

Tie set 2 : [1 3 6]

Tie set 3 [2 3 5 6].

Tie set Matrix

Branches →

$$B_f = \begin{array}{c} \text{Tie set} \downarrow \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & -1 \end{bmatrix} \end{array}$$

Cut set matrix :- $[Q_f]$

→ A fundamental cut set of a graph with respect to a tree is a cutset formed by one and only one twig and a set of links.

→ For a graph having 'N' nodes there will be (N-1) fundamental cutset.

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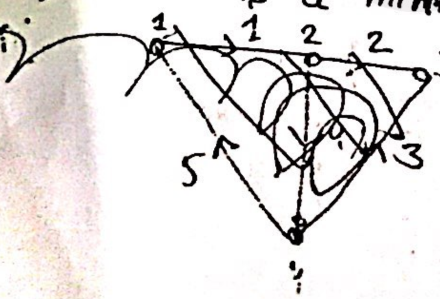
Ex:



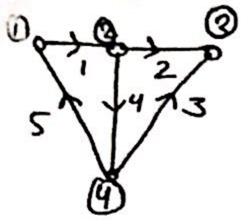
Q) For a given graph, find out the cutset matrix

(6)

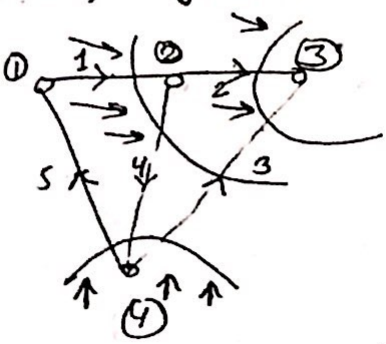
Sol:- Select a tree :- for given tree [1, 2, 5]

twig → 3

link → 2



Graph



cutset 1 [2, 3]

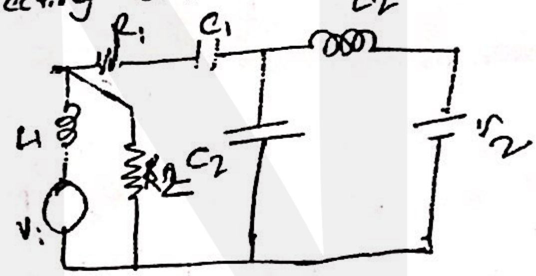
cutset 2 [1 4 3]

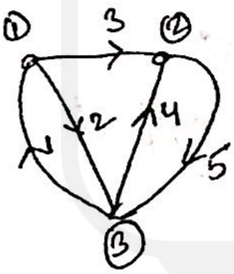
cutset 3 [5 3 4]

Branches

$$\begin{array}{c|c} \text{cutset} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix} \end{array}$$

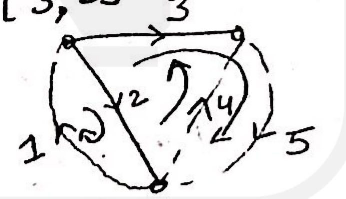
Develop the graph of network shown and obtain the tie set matrix selecting a tree.





Select a tree first

Tree [3, 2] is selected



Tie set 1 [1, 2]

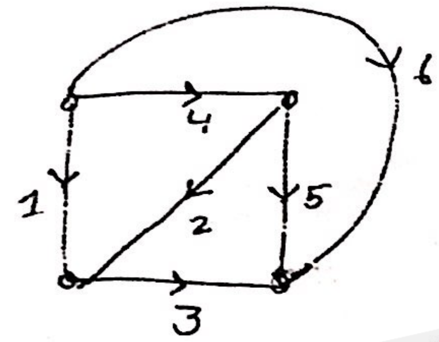
Tie set 2 [2 3 4]

Tie set 3 [2 3 5]

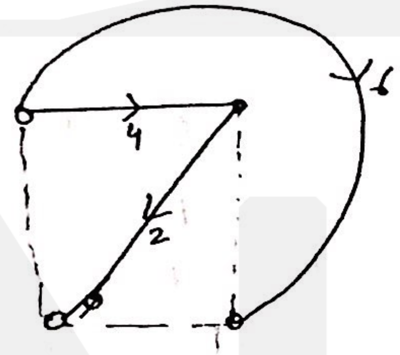
Branch

$$B_f : \begin{array}{c|c} \text{Tieset} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

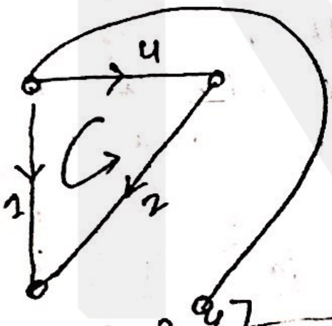
An oriented graph of a certain circuit shown in fig. For tree [2, 4, 6], Find $[Q_f]$ and B_f and hence show that $[Q_f][B_f]^T = 0$

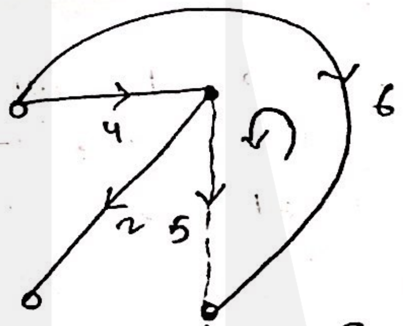


For tree



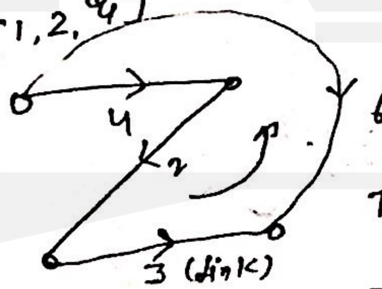
Fundamental Tie set matrix:-





Tieset 1 [1, 2, 4]

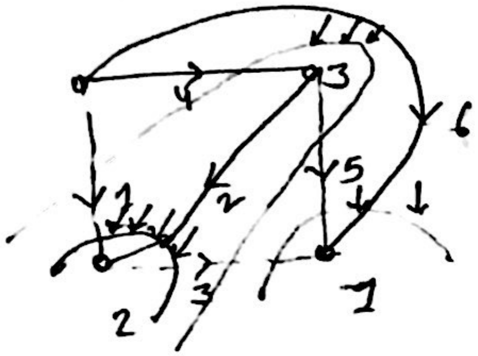
Tieset 2 [4 5 6]



Tieset 3 [2 3 4 6]

$$B_f : \begin{array}{c|c} \text{Tieset} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

Cutset matrix :-



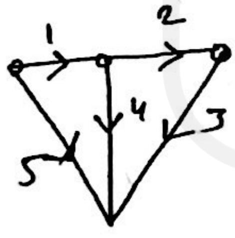
Cutset 1 [3 5 6]

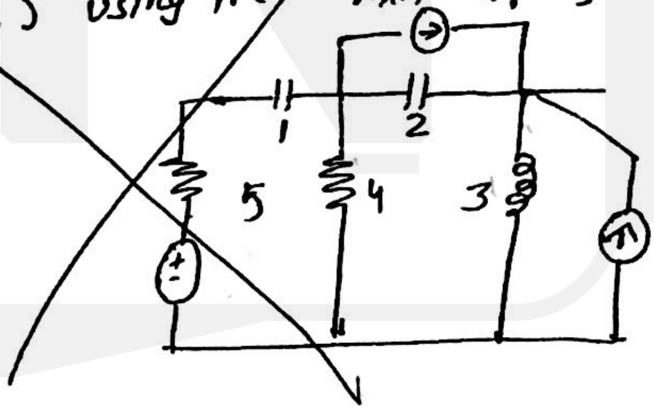
Cutset 2 [1 2 3]

Cutset 3 [1 4 5 3]

$$Q_f = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

Q) For the n/w determine all tree and corresponding co-tree for network and consider the tree formed by branch (1, 2, 5) using tree write A, B_f and Q_f





B-tree form

Q) The reduced incidence matrix of a graph is given by (9)

Nodes / Branches →

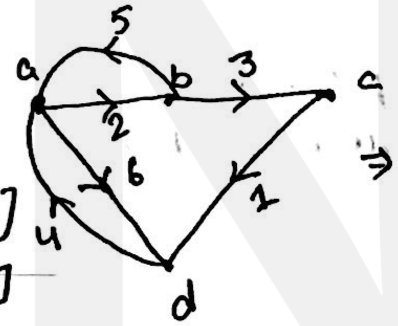
$$\begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Draw the oriented graph. Select a tree and find f-cutset matrix.

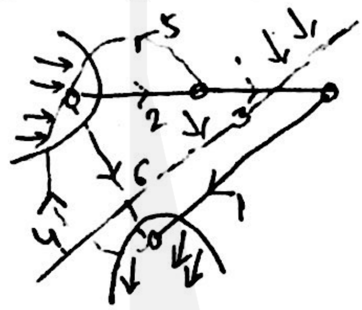
$$\begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array} \Rightarrow$$

Incidence Matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



$\Rightarrow$ Select a tree



Cutset 1 [2 4 5 6]

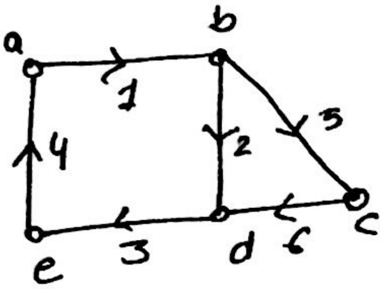
cutset 2 [1 4 6]

Cutset 3 [3 4 6]

Branches

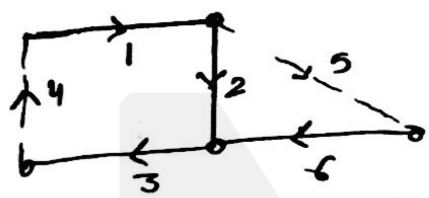
$$\begin{array}{c} \text{cutset} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & -1 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

Show the cutset for the graph and develop the fundamental cutset matrix



Step 1:- First select an arbitrary tree.

Tree [1 2 3 6] is selected



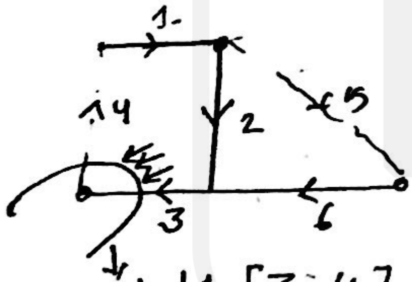
Twigs [1 2 3 6]

Link [1 4 5]

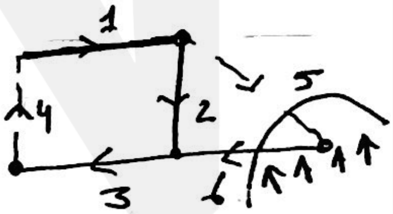
Step 2:- Form fundamental cutset with single ~~twig~~ branch and no. of links.

Total no. of cutset = N-1

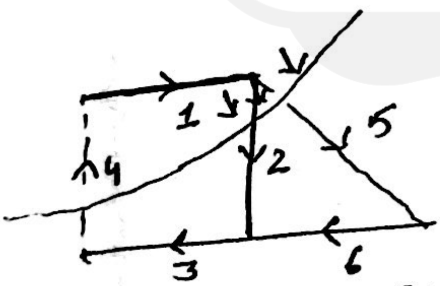
= 5-1 = 4



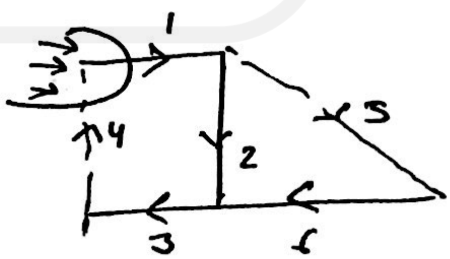
Cutset 1 [3 4]



Cutset 2 [6 5]



Cutset 3 [4 2 5]

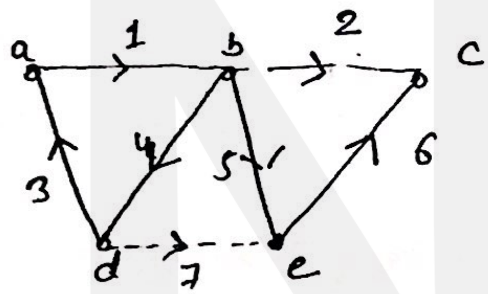


Cutset 4 [1 4]

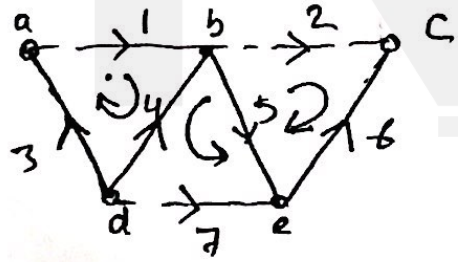
Cutset Matrix :-

$$\begin{array}{c|cccccc} & \multicolumn{6}{c}{\text{Branch}} \\ \text{Cutset} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & -1 & 1 \\ 3 & 0 & 1 & 0 & -1 & 1 & 0 \\ 4 & 1 & 0 & 0 & -1 & 0 & 0 \end{array}$$

Q) Find the cutset and tieset matrix of given graph



For ties set :- $B - N + 1 = 7 - 5 + 1 = 3$



Tieset 1 [1 3 4]

Tieset 2 [4 5 7]

Ties et 3 [2 5 6]

Tieset matrix :-

$$\begin{array}{c|ccccccc} & \multicolumn{7}{c}{\text{Branch}} \\ \text{Tieset} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 & -1 & -1 & 0 \end{array}$$

9

Q1 The reduced incidence matrix of a graph is given by

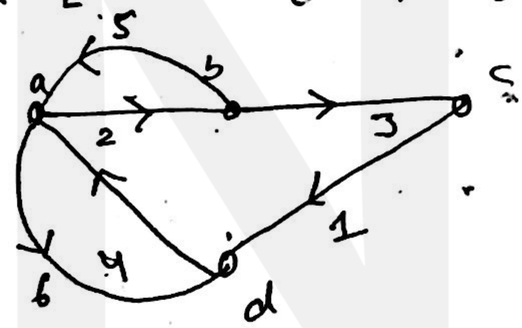
Nodes | Branches

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

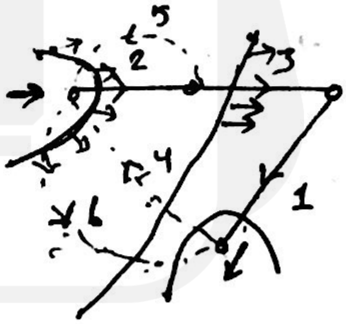
Draw the oriented graph. select a tree and find f-cut set matrix.

Sol^n:- Incidence Matrix:-

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$



cutset matrix:- select a tree:-



cutset 1:- cutset 1 [4 5 2 6]

cutset 2 [1 4 6]

cutset 3 [3 4 6]

Branch

$$\begin{array}{c|c} \text{cut set} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} & 1 & & -1 & -1 & 1 \\ 1 & & & -1 & & 1 \\ & & 1 & -1 & & 1 \end{bmatrix} \end{array}$$

Two Port Network

→ Port → If the current entering one terminal of a pair is equal and opposite to the current leaving the other terminal of the pair, then this type of terminal pair is called as a "port".

→ Two port network :-

→ Each port consists of two terminals, one for entering the current and other for leaving.

So two port network represented by a box assumed to consist of a passive network of linear resistor, inductors, capacitor and dependent source.

→ In order to describe the relationship between the port voltage and port current of a linear port n/w, two linear eq^n are required among the four variables.

Relationship of two port variables :-

	Function Express	In terms of	Matrix eq^n
a) Open ckt impedence [Z]	V_1, V_2	I_1, I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
b) Short ckt admittance [Y]	I_1, I_2	V_1, V_2	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Transmission or chain [T] $\quad V_1, I_1 \quad V_2, -I_2 \quad \begin{bmatrix}V_1\\I_1\end{bmatrix} = \begin{bmatrix}A & B\\C & D\end{bmatrix}$

Inverse Transmission [T'] $\quad V_2, I_2 \quad V_1, -I_1 \quad \begin{bmatrix}V_2\\I_2\end{bmatrix} = \begin{bmatrix}A' & B'\\C' & D'\end{bmatrix}$

Hybrid (h) $\quad V_1, I_2 \quad I_1, V_2 \quad \begin{bmatrix}V_1\\I_2\end{bmatrix} = \begin{bmatrix}h_{11} & h_{12}\\h_{21} & h_{22}\end{bmatrix}\begin{bmatrix}I_1\\V_2\end{bmatrix}$

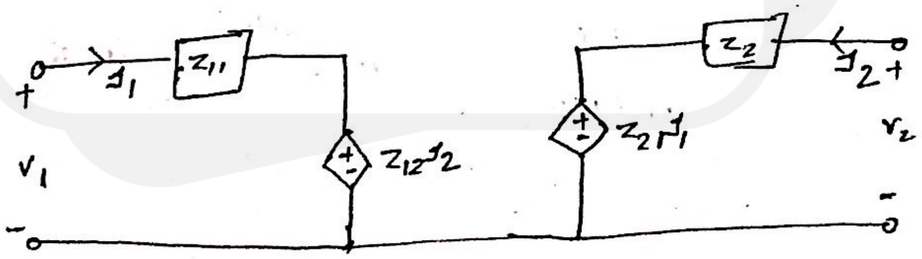
Inverse hybrid (g) $\quad I_1, V_2 \quad V_1, I_2 \quad \begin{bmatrix}I_1\\V_2\end{bmatrix} = \begin{bmatrix}g_{11} & g_{12}\\g_{21} & g_{22}\end{bmatrix}\begin{bmatrix}V_1\\I_2\end{bmatrix}$

Open ckt Impedence [Z] parameters :-

$$(V_1, V_2) = f(I_1, I_2)$$

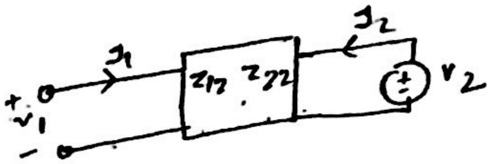
$$\begin{bmatrix}V_1\\V_2\end{bmatrix} = \begin{bmatrix}Z_{11} & Z_{12}\\Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}I_1\\I_2\end{bmatrix}$$

$$\left.\begin{aligned}V_1 &= Z_{11}I_1 + Z_{12}I_2\\V_2 &= Z_{21}I_1 + Z_{22}I_2\end{aligned}\right\} \longrightarrow (a)$$



Equivalent ckt of a two port network in terms of z-parameters.

Determination of Z-parameters:-



$$V_1 = I_1 z_{11} + I_2 z_{12}$$

$$V_2 = I_1 z_{21} + I_2 z_{22}$$

Case 1:- when $I_2 = 0$ [o/p port open ckt'd]

$$V_1 = I_1 z_{11} + 0$$

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = z_{11} \rightarrow \text{I/p driving point impedence with o/p port open ckt'd.}$$

$$V_2 = I_1 z_{22} + I_2 z_{21}.$$

$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = z_{21} \rightarrow \text{Forward transfer impedence with the o/p port open ckt'd.}$$

Case 2:- when $I_1 = 0$ [I/p port open ckt'd]

$$V_1 = I_2 z_{12}$$

$$\left.\frac{V_1}{I_2}\right|_{I_1=0} = z_{12} \rightarrow \text{Reverse transfer impedence with I/p port open ckt'd}$$

$$V_2 = I_2 z_{22}$$

$$\left.\frac{V_2}{I_2}\right|_{I_1=0} = z_{22} \rightarrow \text{o/p driving point impedence with I/p port open ckt'd.}$$

b) Short ckt Admittance [Y] parameter :-

$$(I_1, I_2) = f(V_1, V_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

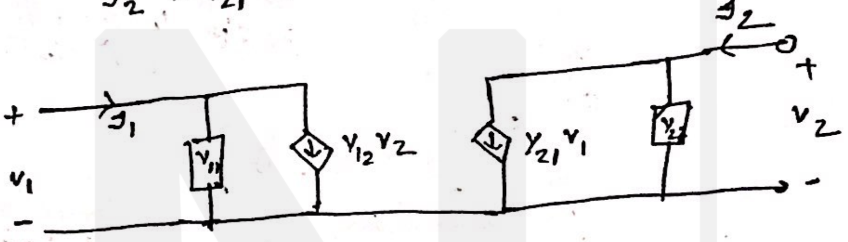
$$[I] = [Y][V]$$

Where $[Y]$ = short ckt admittance parameter.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

where $Y_{12} V_2$ and $Y_{21} V_1$ are voltage controlled current source



Case 1 :- $V_2 = 0$ [o/p port short circuited]

$$I_1 = Y_{11} V_1$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0}$$ → [I/p driving point admittance with the o/p port short ck'd]

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$$ [forward transfer admittance with o/p port short ck'd]

Case 2 :- $V_1 = 0$ [I/p port short ck'd]

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$ → [reverse transfer admittance with I/p port short ck'd]

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$$ [o/p driving point admittance with the I/p port short ck'd]

Short circuit admittance matrix = [open circuit impedance matrix]$^{-1}$

$$[Y] = [Z]^{-1}$$

And $Y_{ij} \neq \frac{1}{Z_{ij}}$ i.e $Y_{11} \neq \frac{1}{Z_{11}}$ and

$Y_{12} \neq \frac{1}{Z_{12}}$ etc.

Transmission (T) or chain or ABCD parameter;

Expressing one port variables in terms of other port variable i.e

$(V_1, I_1) = f(V_2, -I_2)$

→ T- parameter are used in the analysis of power transmission line

→ The I/p and o/p ports are called sending and recieving end respectively.

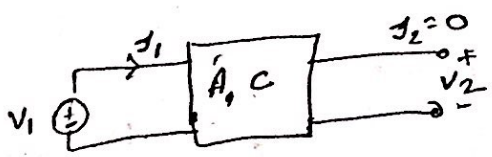
→ o/p port current is considered outward, therefore negative sign arises with I_2

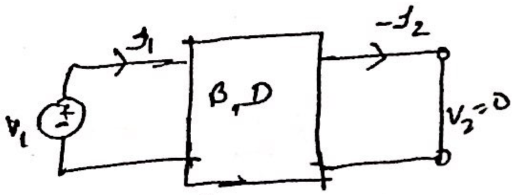
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The matrix equatn defines A, B, C, D parameter, where matrix is known as Transmission (T) or ABCD matrix.

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Determination of T- parameter :-





→ In order to determine the T- parameter, open and short the o/p port (recieving end) and applied some voltage 'V_1' to I/p port to obtain A, C and B, D

Case I:- $I_2 = 0$ [o/p port open ckt]

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$ [reverse voltage ratio with receiving end open ckt]

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$ [reverse transfer admittance with receiving end open ckt]

Case II :- $V_2 = 0$ [o/p short ckt]

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$ [reverse transfer impedance with receiving end short ckt]

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$ [reverse current ratio with receiving end short ckt]

Inverse Transmission (T') parameter:-

Expressing o/p port variables in terms of I/p port variables;

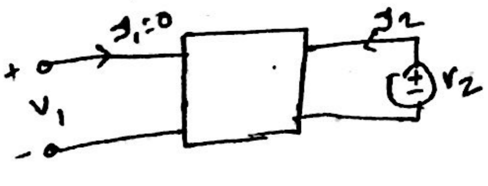
$$(V_2, I_2) = f(V_1, -I_1)$$

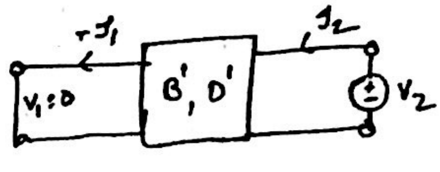
$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

where matrix consisting of A', B', C', D' is called as inverse transmission matrix;

$$V_2 = A'V_1 + B'(-I_1)$$
$$I_2 = C'V_1 + D'(-I_1)$$

Determination of T' parameter:-





1 :- $I_1 = 0$ input port is open ckt'd

$$V_2 = A'V_1 + B'(-I_1)$$

$$I_2 = C'V_1 + D'(-I_1)$$

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0}$$ ⇒ forward voltage ratio with sending end open ckt'd

$$C' = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$ ⇒ Transfer admittance

Case II :- $V_1 = 0$ ⇒ I/P is short ckt'd

$$B' = \left.\frac{V_2}{-I_1}\right|_{V_1=0}$$ ⇒ Transfer impedance with sending end short ckt'd

$$D' = \left.\frac{I_2}{-I_1}\right|_{V_1=0}$$ ⇒ forward current ratio.

Hybrid (h) Parameter :-

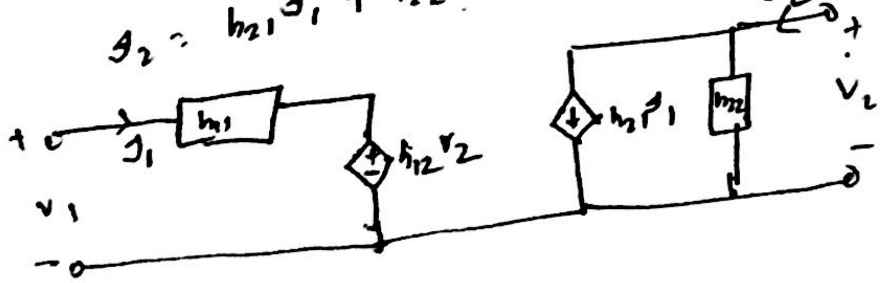
In this case, voltage of I/P port and ~~voltage of~~ current of o/p port are expressed in terms of current of I/P port and voltage of o/p ports.

$$(V_1, I_2) = f(I_1, V_2)$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

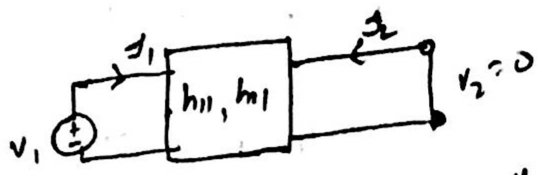


Determination of h parameter :-

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

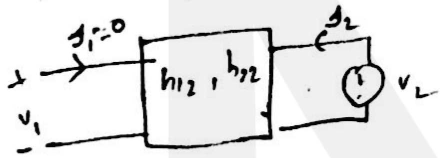
Case I :- $V_2 = 0 \Rightarrow$ o/p port short ckt'd



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \Rightarrow \text{I/p impedance}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \Rightarrow \text{forward current gain}$$

Case II $I_1 = 0 \Rightarrow$ I/p port open ckt'd



$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \Rightarrow \text{Reverse voltage gain}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \Rightarrow \text{O/p admittance}$$

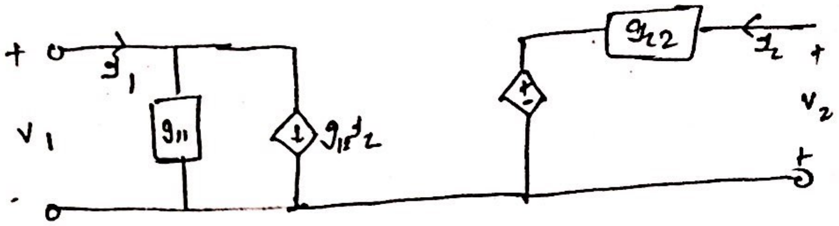
Inverse Hybrid parameter [g] :-

$$[g] = [h]^{-1}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$



Determination of g. parameter :-

Case I :, $I_2 = 0 \Rightarrow$ o/p port open ckt'd.

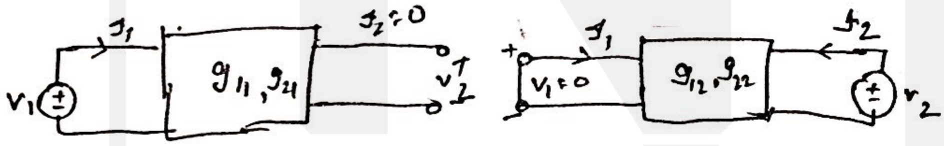
$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \Rightarrow \text{i/p admittance}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \Rightarrow \text{forward voltage gain}$$

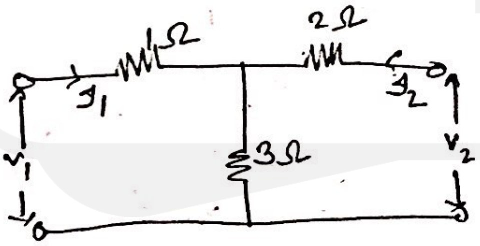
Case II :— $V_1 = 0 \Rightarrow$ i/p port short ckt

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \Rightarrow \text{reverse current gain}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \Rightarrow \text{o/p impedance.}$$



EX:-



Determine
i) Z, ii) Y
iii) T, iv) T'
v) h and vi) g

$V_1 = I_1 \times 1 + 3(I_1 + I_2)$

$V_1 = 4I_1 + 3I_2$ —— (I)

$V_2 = 2I_2 + 3(I_1 + I_2)$

$V_2 = 3I_1 + 5I_2$ —— (II)

a) z- parameter :- $(V_1, V_2) = f(I_1, I_2)$

Case I :- $I_2 = 0$ and case II :- $I_1 = 0$

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{V_1}{V_1/4} = 4\Omega$$

$$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \Rightarrow 3\Omega.$$

Case II :- $I_1 = 0$

$$V_1 = 3I_2$$

$$Z_{12} = \frac{V_1}{I_2} = 3\Omega$$

$$z_{22} = \frac{V_2}{I_2} = 5\Omega$$

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

Y- parameters :- $(I_1, I_2) = f(V_1, V_2)$

case I :- $V_2 = 0$ Case II $V_1 = 0$

$$V_1 = 4I_1 + 3I_2$$

$$V_2 = 3I_1 + 5I_2 \Rightarrow 3I_1 = -5I_2$$

$$V_1 = \frac{11}{5} I_1$$

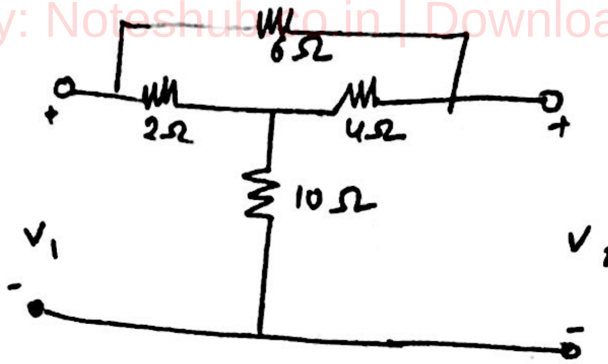
$$V_1 = -\frac{11}{3} I_2$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \Rightarrow \frac{5}{11}\ \mho$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = -\frac{3}{11}\ \mho$$

Case II

Q1



Obtain the open ckt Z-parameters of the n/w;

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

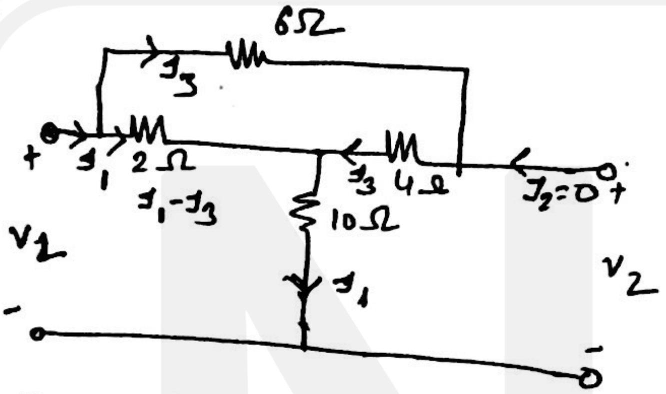
Case:- $I_2 = 0$

$-6I_3 - 4I_3 - 2(I_3 - I_1) = 0$

$-12 I_3 + 2 I_1 = 0$

$-12 I_3 = 2 I_1$

$I_3 = I_1/6$



$-6I_3 - 4I_3 - 2(I_1 - I_3) = 0$

$-12 I_3 - 2I_1 = 0$

-12

$$V_1 = 2(I_1 - I_3) + 10 I_1 \Rightarrow V_1 = 12 I_1 - 2I_3 \quad \text{—(i)}$$

$$V_2 = 4 I_3 + 10 I_1 \quad \text{—(ii)}$$

$$-6 I_3 - 4 I_3 - 2(I_3 - I_1) = 0$$

$$-12 I_3 + 2 I_1 = 0$$

$$-12 I_3 = -2 I_1$$

$$\boxed{I_3 = I_1/6}$$

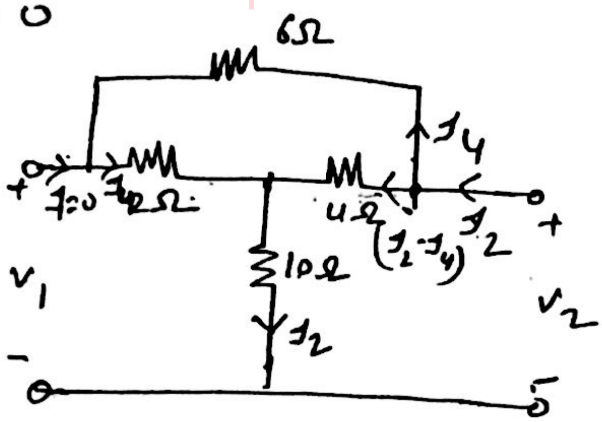
$$V_1 = 12 I_1 - \frac{I_1}{3}$$

$$V_2 = \frac{2I_1}{3} + 10 I_1 = \frac{32}{3} I_1$$

$$Z_{11} = 35/3\ \Omega$$

$$Z_{21} = 32/3\ \Omega$$

Case II :- $I_1 = 0$



$I_1 \quad I_2$
$I_1 + I_2$
$I_2 + I_3 = I_2 - I_4$

$V_1 = 2I_4 + 10 I_2$ —— (I)

$V_2 = 4(I_2 - I_4) + 10 I_2$

$V_2 = 14 I_2 - 4 I_2$ —— (II)

$-6I_4 - 2I_4 - 4(I_2 - I_4) = 0$

$+2I_4$

$-6I_2 - 2I_4 - 4(I_4 - I_2) = 0$

$-2I_2 - 6I_4 = 0$

$-2I_2 = 6I_4$

$I_2/3 = I_4$

$-6I_4 - 2I_4 - 4(I_2 - I_4) = 0$

$-4I_4 + 4I_2 = 0$

$\therefore I_2$

$6I_4 + 4(I_2 - I_4) = 0$

$4I_4 + 4I_2 = 0$

$I_4 = -I_2$

$-6I_4 - 2I_4 - 4(I_4 - I_2) = 0$

$-12I_4 + 4I_2 = 0$

$12I_4 = 4I_2$

$3I_4 = I_2$

$-6I_4 - 2I_4 - 4(I_4 - I_2) = 0$

$-12I_4 + 4I_2 = 0$

$12I_4 = 4I_2$

$3I_4 = I_2$

$I_4 = I_2/3$

$$V_1 = \frac{2I_2}{3} + 10I_2 = \frac{32}{3} I_2$$

$$V_2 = 14I_2 - \frac{4I_2}{3} = \frac{38}{3} I_2$$

$$z_{12} = \frac{32}{3}\ \Omega$$

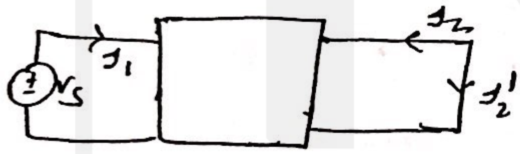
$$z_{22} = \frac{38}{3}\ \Omega$$

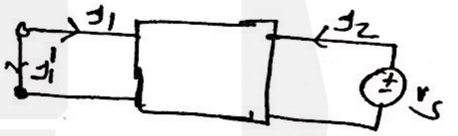
Condition for Reciprocity:-

Reciprocal:- A two port network is said to be reciprocal if ratio of excitation to response is invariant to an interchange of position of excitation and response in the n/w.

→ Network containing resistors, inductors and capacitors are generally reciprocal.

→ Network additionally have dependent source are generally non reciprocal.





$[V_1 = V_S,\ I_1 = I_1,\ V_2 = 0,\ I_2 = -I_2']$ $\quad$ $[V_2 = V_S,\ I_2 = I_2,\ V_1 = 0,\ I_1 = -I_1']$

$$\frac{V_S}{I_2'} = \frac{V_S}{I_1'}$$

In terms of z-parameter:-

$$\boxed{z_{12} = z_{21}}$$

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

From fig a) $V_S = z_{11} I_1 - z_{12} I_2'$

$0 = z_{21} I_1 - z_{22} I_2'$

$\Rightarrow z_{21} I_1 = z_{22} I_2'$

on solving ; $I_2' = \dfrac{V_S Z_{21}}{Z_{11}Z_{22} - Z_{21}Z_{12}}$

From fig b) ; $V_2 = V_S$, $I_2 = I_2$, $V_1 = 0$, $I_1 = -I_1'$

$$\left.\begin{aligned} 0 &= -Z_{11}I_1' + Z_{12}I_2 \\ V_S &= -Z_{21}I_1' + Z_{22}I_2 \end{aligned}\right] \rightarrow b)$$

On solving b)

$$I_1' = \frac{V_S Z_{12}}{Z_{11}Z_{22} - Z_{21}Z_{12}}$$

comparing I_1' and I_2',

$\boxed{Z_{12} = Z_{21}}$ → condition of reciprocity

→ In terms of Y-parameter :-

$$\boxed{Y_{12} = Y_{21}}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

In same manner as in Z-parameters.

→ In terms of T-parameter :-

$$V_1 = AV_2 + B(-I_2)$$
$$I_1 = CV_2 + D(-I_2)$$

From fig. a) $V_1 = V_S$, $I_1 = I_1$, $V_2 = 0$, $I_2 = -I_2'$

$$I_2' = V_S/B$$

From fig b) ; $V_2 = V_S$, $I_2 = I_2$, $V_1 = 0$, $I_1 = -I_1'$

$$-I_1' = CV_S - DI_2$$
$$= CV_S + DI_2'$$
$$-I_1' = CV_S + D\times\frac{V_S}{B}$$
$$-BI_1 = \frac{BC + D}{B}$$

6

From fig by $V_2 = V_S$, $I_2 = I_2$, $V_1 = 0$, $I_1 = -I_1'$

$$0 = AV_S - BI_2$$

$$\frac{AV_S}{B} = I_2$$

$$-I_1' = CV_S - DI_2$$

$$= CV_S - \frac{DAV_S}{B}$$

$$-I_1' = \frac{(BC - AD)V_S}{B}$$

$$I_1' = \frac{(AD - BC)V_S}{B}$$

on comparing I_1' and I_2' ;

$$AD - BC = 1 \quad \text{or} \quad \Delta T = 1$$

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$$

In terms of T-parameter :-

$$A'D' - B'C' = 1 \quad \text{or} \quad \Delta T' = 1$$

$$\begin{vmatrix} A' & B' \\ C' & D' \end{vmatrix} = 1$$

- In terms of h-parameter :-

$$\boxed{h_{12} = -h_{21}}$$

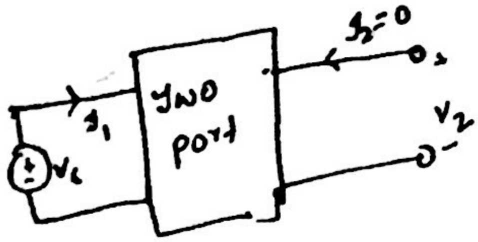
similarly find I_1' and I_2' and compare

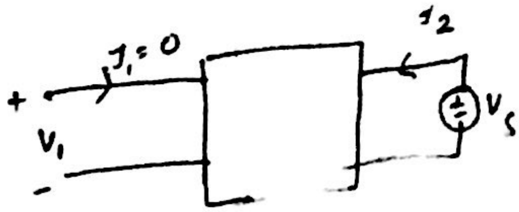
- In terms of g-parameter :-

$$\boxed{g_{12} = -g_{21}}$$

Condition for Symmetry:

A two port network is said to be symmetrical if the ports can be interchanged without changing the port voltage and current.





$V_1 = V_s$, $I_1 = I_1$, $I_2 = 0$, $V_2 = V_2$.

$V_1 = V_1$, $I_1 = 0$, $V_s = V_2$, $I_2 = I_2$

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$$

In terms of Z-parameter,

$$\boxed{Z_{11} = Z_{22}}$$

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

From fig a) $V_s = Z_{11} I_1$

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = Z_{11}$$

From fig b) $V_s = Z_{22} I_2$.

$$\left.\frac{V_s}{I_2}\right|_{I_1=0} = Z_{22}$$

$\therefore$ $Z_{11} = Z_{22}$

In terms of Y-parameter:

$$\boxed{Y_{11} = Y_{22}}$$

In terms of T

$A = D$

Inverse T

$A' = D'$

$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$

$$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$$

In terms of T-parameter:-

$$\boxed{A = D}$$

In terms of T' parameter:-

$$\boxed{A' = D'}$$

In terms of h-parameter:-

$$h_{11}h_{22} - h_{12}h_{21} = 1 \quad ; \quad \Delta h = 1$$

$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$

In terms of g-parameter:-

$$g_{11}g_{22} - g_{12}g_{21} = 1; \quad \Delta g = 1$$

$$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$$

Relation b/w Parameter sets:-

Table no — 8.3, Page no - 282 From KM Soni.

Interconnections of Two Port Networks:-

Two port network may be interconnected in various config. such as series, parallel, cascade, series-parallel and parallel-series connection.

Series Connection:-

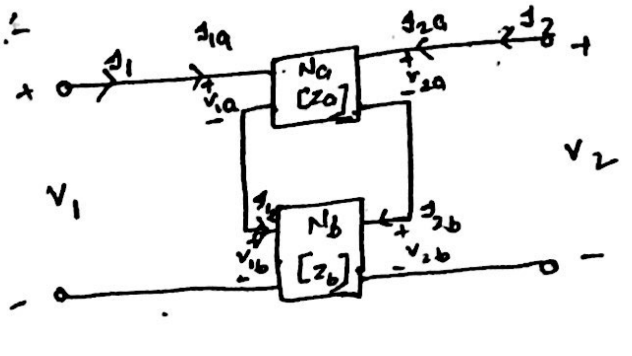


Fig showing a series connection of two two port network N_a and N_b with open ckt z-parameter z_a' and z_b' for network N_a;

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

For Network N_b;

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

Then their series connection requires that;

$$I_1 = I_{1a} = I_{1b} \,,\quad I_2 = I_{2a} = I_{2b}$$

$$V_1 = V_{1a} + V_{1b}$$

$$V_2 = V_{2a} + V_{2b}$$

$$V_1 = V_{1a} + V_{1b} = (Z_{11} I_{1a} + Z_{12} I_{2a}) + (Z_{11b} I_{1b} + Z_{12b} I_{2b})$$

$$= (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b}) I_2$$

$$\left[\text{Since } I_1 = I_{1a} = I_{1b} \text{ and } I_2 = I_{2a} = I_{2b}\right]$$

and similarly; $V_2 = V_{2a} + V_{2b}$

$$= \cancel{(Z_{21a} + Z_{22a})}$$

$$= (Z_{21a} I_{1a} + Z_{22a} I_{2a}) + (Z_{21b} I_{1b} + Z_{22b} I_{2b})$$

$$= (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$$

In matrix form;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{11} = Z_{11a} + Z_{11b}$$

$$Z_{12} = Z_{12a} + Z_{12b}$$

$$Z_{21} = Z_{21a} + Z_{21b}$$

$$Z_{22} = Z_{22a} + Z_{22b}$$

or in matrix form;

$$[Z] = [Z_a] + [Z_b]$$

Thus, the overall z-parameter matrix for series connected two port n/w is simply the sum of z-parameter matrices of each individual two port n/w connected in series.

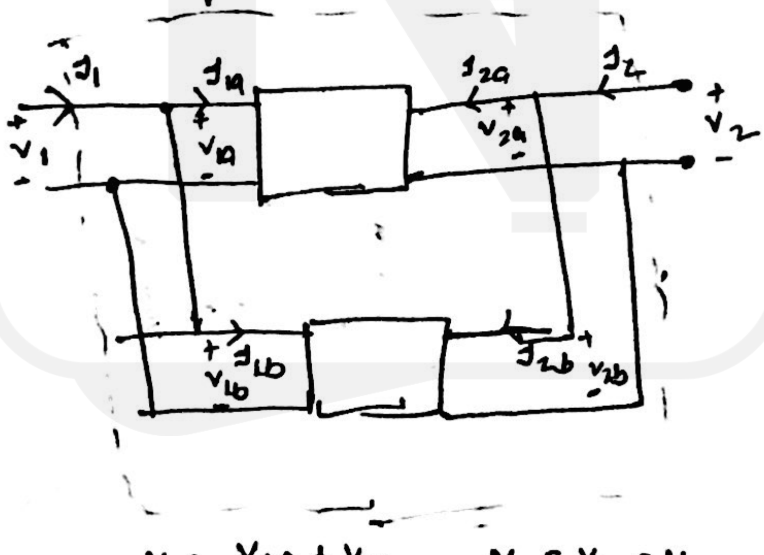
Parallel Connection :-

Fig. shows a parallel connection of two port n/w Na and N_b with short ckt y-parameter Y_a and Y_b respectively i.e for network Na;

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

Similarly for network. N_b;

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$



$V_1 = V_{1a} + V_{1b}$

$V_2 = V_{2a} + V_{2b}$

$V_1 = V_{1a} = V_{1b}$

$V_2 = V_{2a} = V_{2b}$

$I_1 = I_{1a} + I_{1b}$

$I_2 = I_{2a} + I_{2b}$

$$I_1 = I_{1a} + I_{1b}$$

$$= (Y_{11a}V_{1a} + Y_{12a}V_{2a}) + (Y_{11b}V_{1b} + Y_{12b}V_{2b})$$

and $V_{1a} = V_{1b} = V_1$ and $V_{2a} = V_{2b} = V_2$

$$\Rightarrow (Y_{11a} + Y_{11b})V_1 + (Y_{12a} + Y_{12b})V_2$$

$$\therefore \boxed{\begin{aligned} Y_{11} &= Y_{11a} + Y_{11b} \\ Y_{12} &= Y_{12a} + Y_{12b} \end{aligned}}$$

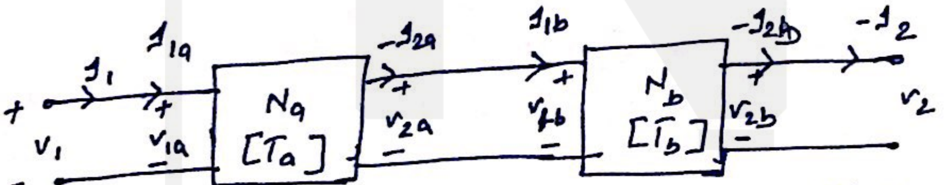
and $I_2 = I_{2a} + I_{2b}$

$$= (Y_{21a}V_{1a} + Y_{22a}V_{2a}) + (Y_{21b}V_{1b} + Y_{22b}V_{2b})$$

$$= (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$$

$$\therefore \boxed{\begin{aligned} Y_{21} &= Y_{21a} + Y_{21b} \\ Y_{22} &= Y_{22a} + Y_{22b} \end{aligned}}$$

Cascade connection:- (Tandem connection)
Two port network are said to be connected in cascade if output port of first becomes the input port of second



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \underbrace{\begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}}_{(b)}$$

and $I_{1a} = I_1 \qquad -I_{2a} = I_{1b} \qquad I_2 = I_{2b}$

$V_1 = V_{1a} \qquad V_{2a} = V_{1b} \qquad V_{2b} = V_2$

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

and value of $\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$ from eqn (b)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$$\therefore A = A_a A_b + B_a C_b$$

$$B = A_b B_b + B_a D_b$$

$$C = C_a A_b + D_a C_b$$

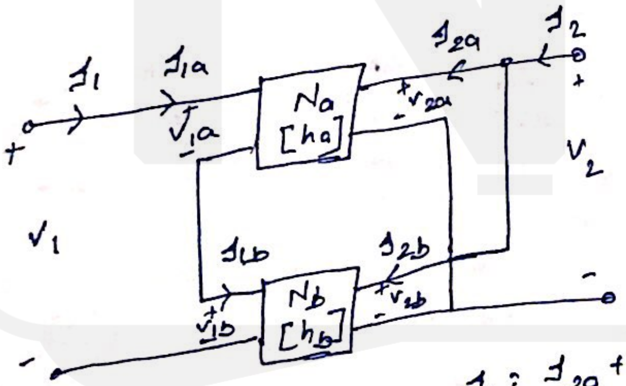
$$D = C_a B_b + D_a D_b$$

$$\therefore [T] = [T_a] \cdot [T_b]$$

$\therefore$ Therefore the overall T-parameter matrix for cascade connected two port network is simply the matrix product of the T-parameter matrices of each individual two port network in cascade.

Series - Parallel connection:-

Two two port networks are said to be connected in series-parallel if the input ports are connected in series while o/p port are connected in parallel.



$$I_1 = I_{1a} = I_{1b}$$

$$V_1 = V_{1a} + V_{1b}$$

$$I_2 = I_{2a} + I_{2b}$$

$$V_2 = V_{2a} = V_{2b}$$

For network N_a;

$$\begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} h_{11a} & h_{12a} \\ h_{21a} & h_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix}$$

For network N_b;

$$\begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} h_{11b} & h_{12b} \\ h_{21b} & h_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix}$$

$$= \begin{bmatrix} h_{11a}+h_{11b} & h_{12a}+h_{12b} \\ h_{21a}+h_{21b} & h_{22a}+h_{22b} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$\therefore$ $h_{11} = h_{11a}+h_{11b}$

$h_{12} = h_{12a}+h_{12b}$

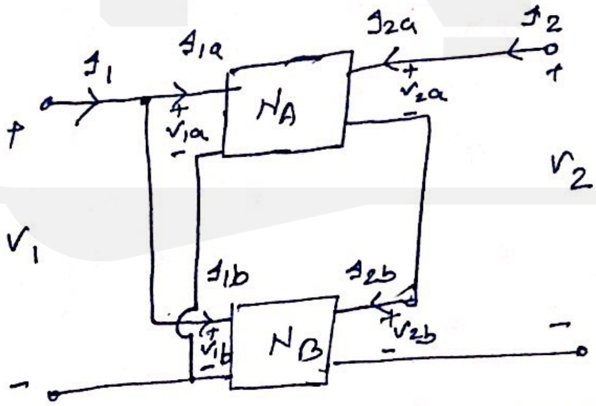
$h_{21} = h_{21a}+h_{21b}$

$h_{22} = h_{22a}+h_{22b}$

$\therefore$ $[h] = [h_a]+[h_b]$

Therefore the overall h-parameter matrix for series-parallel connected two port network is simply the sum of h-parameter matric of each individual two port network connected in series parallel.

* Parallel - Series Connection:-



As similar to previous case;

$g_{11} = g_{11a}+g_{11b}$ $\qquad$ $g_{21} = g_{21a}+g_{21b}$

$g_{12} = g_{12a}+g_{12b}$ $\qquad$ $g_{22} = g_{22a}+g_{22b}$

$[g] = [g_a]+[g_b]$

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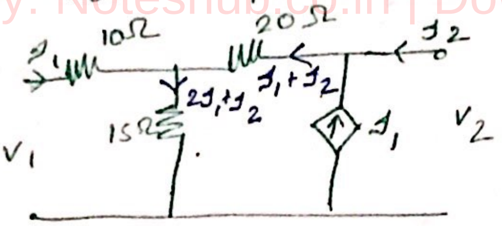
Q) Determine the Z-parameter of the network.



$V_1 = 10I_1 + 15(2I_1 + I_2)$

$V_1 = 10I_1 + 30I_1 + 15I_2$

$V_1 = 40I_1 + 15I_2$ — (I)

$V_2 = 20(I_1 + I_2) + 15(2I_1 + I_2)$

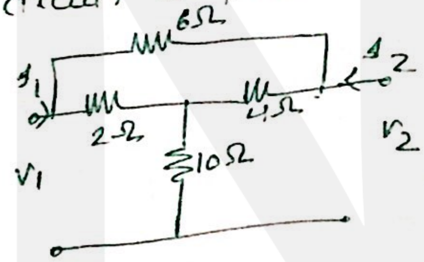
$V_2 = 50I_1 + 35I_2$ — (II)

Comparing eqn (I) and (II) with $V_1 = z_{11}I_1 + z_{12}I_2$
$V_2 = z_{21}I_1 + z_{22}I_2$

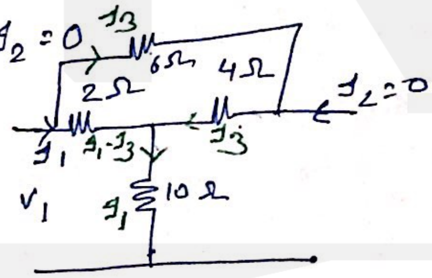
$\therefore\ z_{11} = 40\ \Omega \qquad z_{21} = 50\ \Omega$

$z_{12} = 50\ \Omega \qquad z_{22} = 35\ \Omega$

Q) Obtain the open circuit Z-parameter of network.



Case 1 :-



$V_1 = 2(I_1 - I_3) + 10I_1 = 12I_1 - 2I_3$

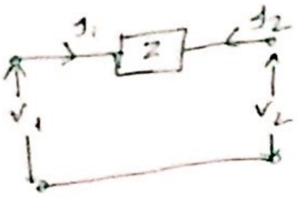
$V_2 = 4I_3 + 10I_1$

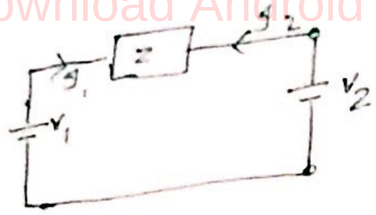
$I_3 = I_1 \times \frac{2}{2+6+4} = \frac{I_1}{6}$ [using current division rule]

$\therefore\ V_1 = 12I_1 - 2\times\frac{I_1}{6} \Rightarrow V_1 = \frac{35}{3}I_1$

$V_2 = 4\frac{I_1}{6} + 10I_1 = \frac{32}{3}I_1$

Q) Determine Z and Y-parameter;





Sol^n:- $V_1 = I_1 Z + V_2$

$I_1 = -I_2$

and For Z-parameter;

$V_1 = z_{11} I_1 + z_{12} I_2$

$V_2 = z_{21} I_1 + z_{22} I_2$

For finding the z_{11} and z_{21} → $I_2 = 0$

but $I_1 = -I_2 = 0$

and similarly z_{12} and z_{22} → $I_1 = 0$

but $I_1 = -I_2 = 0$

So Z-parameter don't exist, as I_1 and I_2 are not independent.

→ For Y-parameter;

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

For finding Y_{11} and Y_{21} → $V_2 = 0$

$V_1 = I_1 Z + V_2$ and $I_1 = -I_2$

$V_2 = 0 \Rightarrow V_1 = I_1 Z$

and $Y_{11} = \frac{I_1}{V_1} = \frac{1}{Z}$

$Y_{21} = \frac{I_2}{V_1}$ and $I_2 = -I_1$

$= \frac{-I_1}{V_1} = -\frac{1}{Z}$

For finding Y_{12} and Y_{22} → $V_1 = 0$

$V_1 = I_1 Z + V_2 \Rightarrow 0 = I_1 Z + V_2$

$Y_{12} = \frac{I_1}{V_2} =$

$\Rightarrow I_1 Z = -V_2$

$\Rightarrow \frac{I_1}{V} = -\frac{1}{Z} = Y_{12}$

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

$V_1 = V_2 + I_1 Z \Rightarrow 0 = V_2 + I_1 Z$

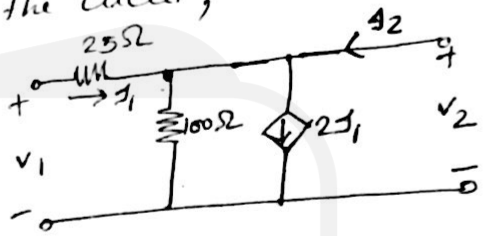
$I_1 = -I_2 \Rightarrow 0 = V_2 - I_2 Z$

$\Rightarrow V_2 = +I_2 Z$

$= +\frac{1}{2} = \frac{I_2}{V_2} = Y_{22} = +\frac{1}{2}$

$$Y = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Q) Find short circuit parameter of the circuit;



~~I_{10}~~

Applying KCL;

$I_1 + I_2 = \frac{V_2}{100} + 2I_1$

$\Rightarrow -I_1 + I_2 = \frac{V_2}{100}$ —(i)

$I_1 = \frac{V_1 - V_2}{25}$

So ; $I_1 = \frac{V_1}{25} - \frac{V_2}{25}$ —(ii)

~~$I_{20} = \frac{V_1}{100}$~~

From eq^n (i) $-I_1 + I_2 = \frac{V_2}{100}$

$-\left[\frac{V_1 - V_2}{25}\right] + I_2 = \frac{V_2}{100}$

$I_2 = \frac{V_2}{100} + \frac{V_1 - V_2}{25}$

$= \frac{V_2 + 4V_1 - 4V_2}{100}$

$I_2 = \frac{1}{25}V_1 - \frac{3V_2}{100}$ —(iii)

Comparing eq^n (ii) and eq^n (iii) with

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

$$Y_H = \begin{bmatrix} \frac{1}{25} & -\frac{1}{25} \\ \frac{1}{25} & -\frac{3}{100} \end{bmatrix}$$

$$I_1 = I_{1a} + I_{1b}$$
$$= (Y_{11a}V_{1a} + Y_{12}V_{2a}) + (Y_{11b}V_{1b} + Y_{12b}V_{2b})$$
$$= (Y_{11a} + Y_{11b})V_1 + (Y_{12a} + Y_{12b})V_2$$

Since $V_1 = V_{1a} = V_{1b}$ and $V_2 = V_{2a} = V_{2b}$

and similarly;

$$I_2 = I_{2a} + I_{2b}$$
$$= (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$$

In matrix form;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{11} = Y_{11a} + Y_{11b}$$
$$Y_{12} = Y_{12a} + Y_{12b}$$
$$Y_{21} = Y_{21a} + Y_{21b}$$
$$Y_{22} = Y_{22a} + Y_{22b}$$

or in matrix form;

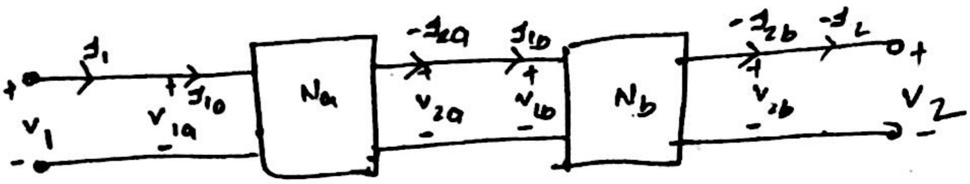
$$[Y] = [Y_a] + [Y_b]$$

The overall Y-parameter matrix for parallel connected two port network is simply the sum of Y-parameter matrices of each individual two port n/w connected in parallel.

→ Parallel connection is called as parallel-parallel connection since both i/p and o/p ports are parallel connected.

Cascade Connection :-

Two two port n/w are said to be connected in cascade if the o/p port of the first becomes the i/p port of second.



If the T_a and T_b are T. parameters of n/w N_a and N_b;

For network N_a;

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

For network N_b;

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

Then their cascade connection requires that.

$I_1 = I_{1a}$ $\quad -I_{2a} = I_{1b}$ $\quad I_{2b} = I_2'$

$V_1 = V_{1a}$ $\quad V_{2a} = V_{1b}$ $\quad V_{2b} = V_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

Putting the value of V_{1b} and I_{1b}

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

and $V_{2b} = V_2$ and $I_{2b} = I_2$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

In matrix form;

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

In eq^n form;

$$A = A_a A_b + B_a C_b$$

$$B = A_a B_b + B_a D_b$$

$$C = C_a A_b + D_a C_b$$

$$D = C_a B_b + D_a D_b$$

or in matrix form;

$$[T] = [T_a] \cdot [T_b]$$

The overall T-parameter matrix for cascade connected two port n/w is simply the matrix the product of T-parameter matrices of each individual two port network in cascade.

Note:- For T' parameter overall T' parameter matrix for cascaded two port n/w is simply the matrix product of T' parameter matrices for each individual two port n/w in reverse order.

~~Neha Alihasika~~

$$[T] = [T_b'][T_a']$$

Series - Parallel Connection :-

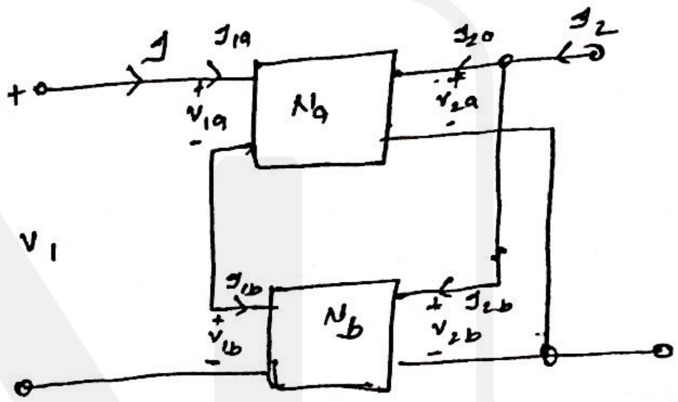
Two port networks are said to connected in series - parallel if input ports are connected in series while o/p port are connected in parallel ;

$$\left.\begin{aligned} V_1 &= V_{1a} + V_{1b} \\ I_1 &= I_{1a} = I_{1b} \end{aligned}\right] \rightarrow I/P$$

$$\left.\begin{aligned} V_2 &= V_{2a} = V_{2b} \\ I_2 &= I_{2a} + I_{2b} \end{aligned}\right] \rightarrow O/P$$

N_a

$$\begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} h_{11a} & h_{12a} \\ h_{21a} & h_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix}$$



For network N_b ;

$$\begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} h_{11b} & h_{12b} \\ h_{21b} & h_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix}$$

Relation ;

$$\begin{bmatrix} h_{11} = h_{11a} + h_{11b} \\ h_{12} = h_{12a} + h_{12b} \\ h_{21} = h_{21a} + h_{21b} \\ h_{22} = h_{22a} + h_{22b} \end{bmatrix}$$

in matrix form ;

$$[h] = [h_a] + [h_b]$$

4) Parallel-series Connection:-

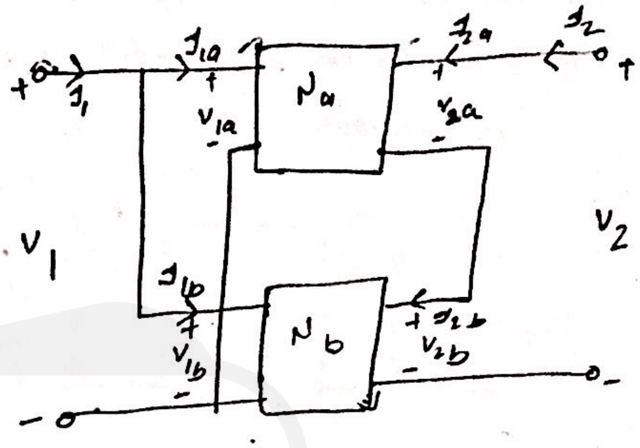
I/p ports are connected in parallel, while o/p ports are connected in series:

$\therefore \quad V_1 = V_{1a} = V_{1b}$

$I_1 = I_{1a} + I_{1b}$

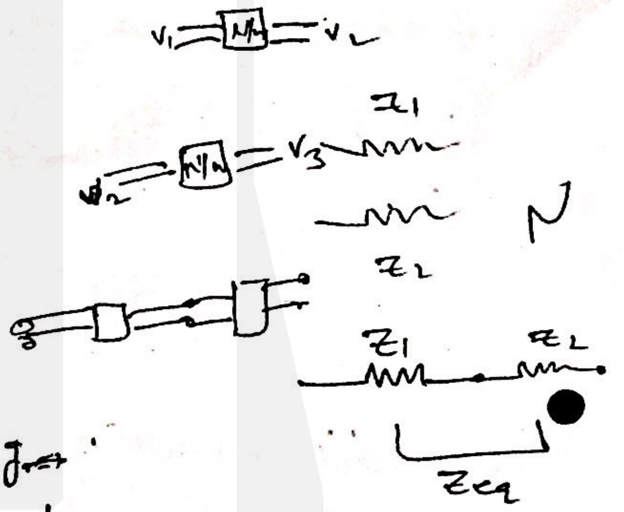
$V_2 = V_{2a} + V_{2b}$

$I_2 = I_{2a} = I_{2b}$



$$\therefore \begin{bmatrix} g_{11} = g_{11a} + g_{11b} \\ g_{21} = g_{21a} + g_{21b} \\ g_{12} = g_{12a} + g_{12b} \\ g_{22} = g_{22a} + g_{22b} \end{bmatrix}$$

or $[g] = [g_a] + [g_b]$



$= Z_1 + Z_2$

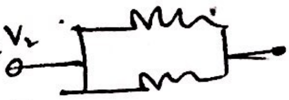
h.

V_1, V_2

$I_1 \; I_2$

$V_1 = h_{11} I_1 + h_{12} V_2$

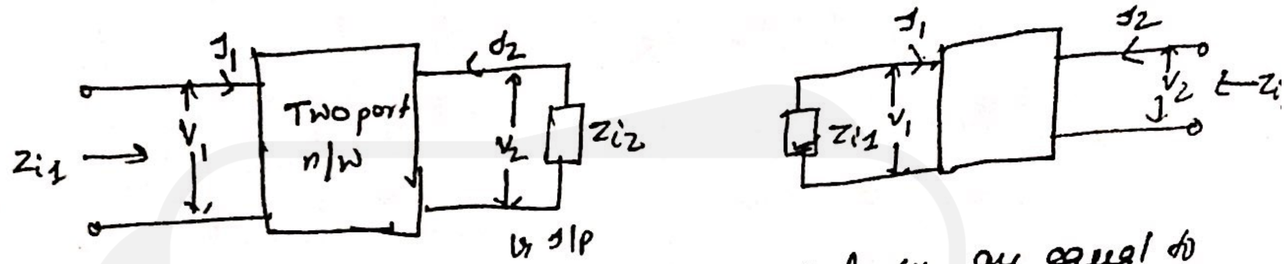
$I_2 = h_{21} I_1 + h_{22} V_2$



$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$Y_{eq} = Y_1 + Y_2$

- **Image Impedance:-**
In a two port n/w, if the impedance at I/p port with impedance Z_{i2} connected across o/p port be Z_{i1} and the impedance at o/p port with impedance Z_{i1} connected across I/p port be Z_{i2} then Z_{i1} and Z_{i2} are termed as image impedance of the network.



- For symmetrical n/w, image impedances are equal to each other i.e. $Z_{i1} = Z_{i2}$ and is called characteristic or iterative impedance Z_c;

$$Z_{i1} = V_1/I_1 \quad \text{(driving point impedance at I/p port)}$$

$$Z_{i2} = V_2/I_2 \quad \text{(driving point impedance at o/p port)}$$

- → Image impedances in terms of I/p and o/p impedance:-
$Z_{i1} = Z_{ip}$ and $Z_{i2} = Z_{op}$

→ Image impedance in term of T-parameters:-

$$Z_{i1} = Z_{ip}$$

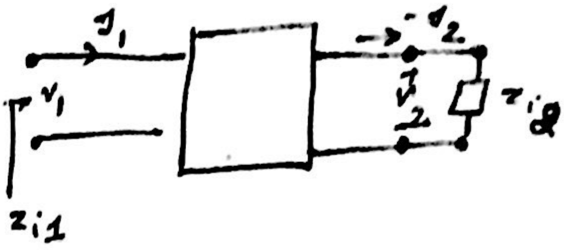
$$Z_{i1} = \frac{A Z_{i2} + B}{C Z_{i2} + D}$$

$$Z_{i2} = Z_{op}$$

$$Z_{i2} = \frac{D Z_{i1} + B}{C Z_{i1} + A}$$

$V_1 = AV_2 + B(-I_2)$

$I_1 = CV_2 + D(-I_2)$



$V_2 = z_{i2} \times -I_2$

$$V_1 = A(z_{i2} \times -I_2) + B(-I_2)$$
$$= -(A z_{i2} + B) I_2$$

$$I_1 = C(z_{i2} \times -I_2) - D I_2$$
$$= -(C z_{i2} + D) I_2$$

$$\frac{V_1}{I_1} = \frac{-(A z_{i2} + B) I_2}{-(z_{i2} C + D) I_2}$$

$$z_{i1} = \frac{V_1}{I_1} = \frac{A z_{i2} + B}{C z_{i2} + D} \quad \text{—(A)}$$

$$z_{i2} = \frac{V_2}{I_2} = \frac{D z_{i1} + B}{C z_{i1} + A} \quad \text{—(B)}$$

Solving eq^n (A) and (B) ;

$$z_{i1} = \sqrt{\frac{AB}{CD}} \; ; \quad \text{and } z_{i2} = \sqrt{\frac{BD}{AC}}$$

→ Derivation of voltage and current ratios as follows:-

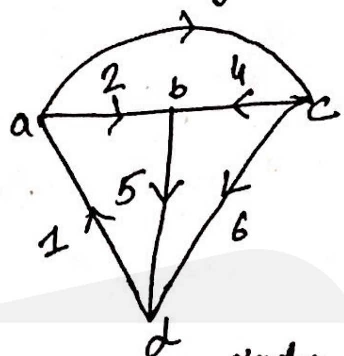
T-parameter eq^n :-

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

and $z_{i2} = -V_2/I_2 \Rightarrow I_2 = \dfrac{-V_2}{z_{i2}}$

Reduced ~~Incid~~ Incidence matrix :-

If in the all incidence matrix, the informat^n^ about a particular node is completely omitted i.e one of the rows is eleminated, it is called Reduced incidence matrix.



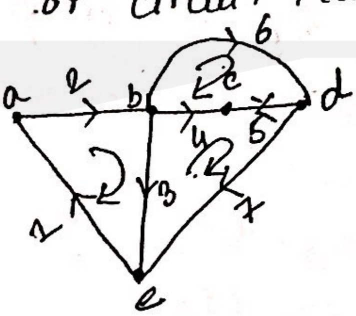
Incidence matrix ; A_c =

$$\begin{array}{c|cccccc} & & & \text{Branch} & & & \\ \text{Nodes} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline a & -1 & 1 & 1 & 0 & 0 & 0 \\ b & 0 & -1 & 0 & -1 & 1 & 0 \\ c & 0 & 0 & -1 & 1 & 0 & [illegible] \\ d & 1 & 0 & 0 & 0 & -1 & -1 \end{array}$$

If informat^n^ about node 'd' is omitted, fourth row is eleminated, it results in reduced ~~row~~ incidence matrix

$$A_c = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Loop matrix or Circuit Matrix [B] :-



Loop 1 :- [1, 2, 3]

Loop 2 :- (4, 6, 5)

Loop 3 :- (3, 4, 5, 7)

Loop 4 :- (1, 2, 6, 7)

Loop 5 :- (3, 6, 7)

Loop 6 :- (1, 2, 4, 5, 7)

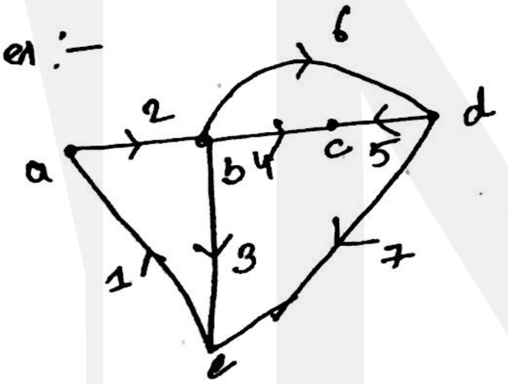
Incidence Matrix :- [A] Incidence matrix of a particular graph, is a matrix which gives complete informat^n regarding the connection of various branches to the nodes and orientation of these branches towards the node.

The procedure for drawing incidence matrix is ;

For a graph with 'n' nodes and 'b' branch, the complete incidence matrix A_c is an $n \times b$ matrix whose elements are defined by ;

$$a_{nb} = +1 ; \text{ if branch 'b' leaves node 'n'}$$
$$= -1 \text{ if " 'b' enters " 'n'}$$
$$= 0 ; \text{ " " " is not incident with 'n'}$$

For ex :-



Nodes / Branch

$$A_c = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline a & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ b & 0 & -1 & 1 & 1 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & -1 & 1 & 0 & 1 \\ d & 0 & 0 & 0 & 0 & 1 & -1 & 1 \\ e & 1 & 0 & -1 & 0 & 0 & 0 & -1 \end{array}$$

Properties of Complete Incidence Matrix :-

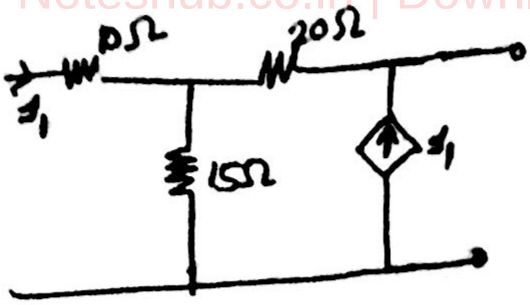
→ The sum of the entries in any column is zero

→ The determinant of complete incidence matrix is always zero

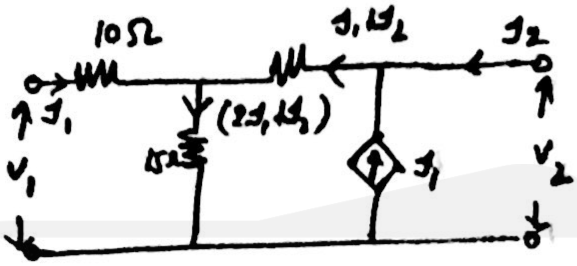
→ Rank of a complete incidence matrix of a connected graph is "n-1"

→ Another feature of all incidence matrix is that the determinant of the incidence matrix of a particular loop which is called sub matrix is always zero

9]



Determine the z-param of network



$$V_1 = 10I_1 + (2I_1+I_2)\times 15$$

$$V_1 = 40I_1 + 15I_2 \quad \text{——(I)}$$

$$V_2 = (2I_1+I_2)\times 15 + 20\,(I_1+I_2)$$

$$V_2 = 50I_1 + 35I_2 \quad \text{——(II)}$$

Now comparing eq^n (I) and (II) w.r.t. to

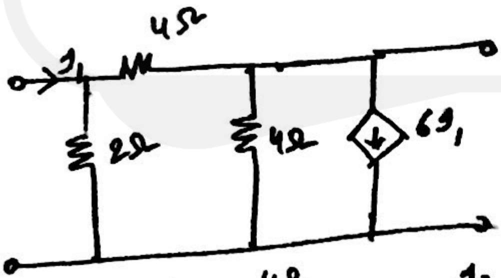
$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_1$$

$$\therefore \quad z_{11} = 40\Omega \qquad z_{21} = 50\Omega$$

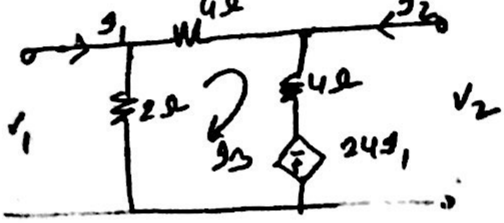
$$z_{12} = 15\Omega \qquad z_{22} = 35\Omega$$

2]



Calculate z-parameter of the n/w.

Sol^n :-



$V_1 = 2(I_1 - I_3)$ —(I)

$24I_1 = 2(I_3 - I_1) + 4I_3 + 4(I_3 + I_2)$

$\Rightarrow 26I_1 - 4I_2 - 10I_3 = 0$ —(II)

$V_2 = 4(I_2 + I_3) - 24I_1$

$V_2 = -24I_1 + 4I_2 + 4I_3$ —(III)

Put the value of I_3 from eq^n (II) in eq^n (I) and (III)

$$I_3 = \frac{26I_1 - 4I_2}{10}$$

$$\therefore V_1 = 2I_1 - 2\left(\frac{26I_1 - 4I_2}{10}\right)$$

$$V_1 = 2I_1 - 5.2I_1 + .8I_2$$

$$V_1 = -3.2I_1 + 0.8I_2 \quad \text{—(IV)}$$

$$V_2 = -24I_1 + 4I_2 + 4I_3$$

$$= -24I_1 + 4I_2 + 4\left(\frac{26I_1 - 4I_2}{10}\right)$$

$$V_2 = -13.6I_1 + 2.4I_2 \quad \text{—(V)}$$

from eq^n (IV) and (V) comparing with the eq^n;

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

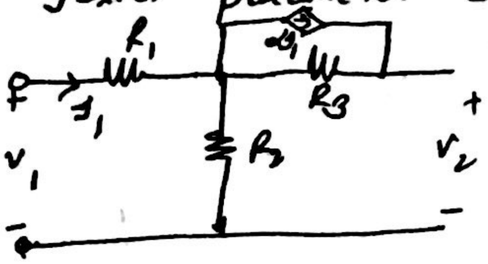
$z_{11} = -3.2\,\Omega$

$z_{21} = -13.6\,\Omega$

$z_{12} = 0.8\,\Omega$

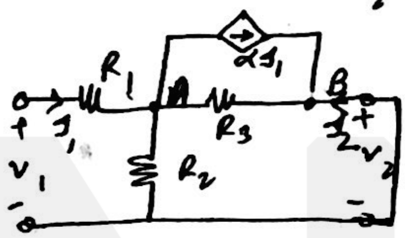
$z_{22} = 2.4\,\Omega$

2] Find the hybrid parameter for the n/w.



Sol^n:- Case 1:- $V_2 = 0$; $h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$



Applying ~~KCL~~ at Node A;

At node A; 'V' potential is developed

$\therefore$ $I_1 = \frac{V}{R_2} + \frac{V}{R_3} + \alpha I_1$

$\Rightarrow V = \frac{(1-\alpha) I_1 R_2 R_3}{R_2 + R_3}$

~~KCL add~~ Applying KVL;

$V_1 = I_1 R_1 + V$

$= I_1 R_1 + \frac{(1-\alpha) I_1 R_2 R_3}{R_2 + R_3}$

$\therefore$ $\frac{V_1}{I_1} = \frac{R_1(R_2+R_3) + (1-\alpha) R_2 R_3}{R_2 + R_3}$

$\therefore$ $\boxed{h_{11} = \frac{V_1}{I_1} = \frac{R_1(R_2+R_3) + (1-\alpha) R_2 R_3}{R_2 + R_3}}$

KCL at node B;

$$\frac{V}{R_3} + \alpha I_1 + I_2 = 0$$

$$I_2 = -\frac{V}{R_3} - \alpha I_1$$

Putting the value of V;

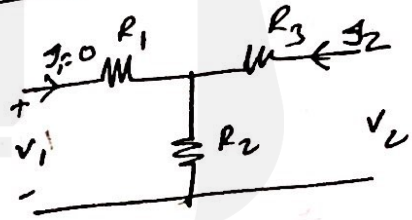
$$I_2 = -\frac{1}{R_3}\left[\frac{(1-\alpha) I_1 R_2 R_3}{R_2+R_3}\right] - \alpha I_1$$

$$I_2 = \frac{I_1}{R_3}\left[\frac{-(1-\alpha) R_2 R_3}{R_2+R_3} - \alpha\right]$$

$$\frac{I_2}{I_1} = \frac{-R_2 + \alpha R_2 - \alpha R_2 - \alpha R_3}{R_2+R_3}$$

$$\boxed{h_{21} = \frac{I_2}{I_1} = -\frac{(R_2 + \alpha R_3)}{R_2+R_3}}$$

Case 2:- $I_1 = 0$;



$$V_2 = I_2(R_2+R_3)$$

$$V_2 = I_2 R_2 + I_2 R_3 \quad \text{—(i)}$$

$$V_1 = I_2 R_2 \quad \text{—(ii)}$$

$$\therefore \; h_{12} = \frac{V_1}{V_2} = \frac{I_2 R_2}{I_2(R_2+R_3)} = \frac{R_2}{R_2+R_3}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{I_2}{I_2 R_2 + I_2 R_3}$$

$$h_{22} = \frac{1}{R_2+R_3}$$

Note:— Do the Open circuit & Short circuit Impedance [Derivation], Input and output impedance and Image Impedance by Yourself

[Page no – 301
KH. Soni.]

Interconversion of parameters :-

→ It is necessary to convert from one set of parameter to another ; for ex - all transistor problem cannot be solved by using h-parameter such as if we want to express α-parameter in terms of β-parameter we have to write β-parameter equation and then by algebraic manipulation, rewrite the eq^n as needed for α-parameters.

Z-parameter in term of other parameter :-

a) 'Z' in terms of 'Y'.

$$[I] = [Y][V]$$

$$[V] = [Z][I]$$

Therefore $[Z] = [Y]^{-1}$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$= \frac{1}{\Delta Y}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\therefore \quad Z_{11} = \frac{Y_{22}}{\Delta Y} \;;\quad Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} \;;\quad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

b) 'Z' in terms of T-parameter :-

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—(a)}$$

$$V_2 = Z_{21} I_1 + Z_{22} \quad \text{—(b)}$$

and $\quad V_1 = A V_2 + B(-I_2) \quad$ —(c)

$$I_1 = C V_2 + D(-I_2) \Rightarrow V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \quad \text{—(d)}$$

on comparing (d) and (b) we will get ;

$$Z_{21} = \frac{1}{C} \quad \text{and} \quad Z_{22} = \frac{D}{C}$$

and on putting the value of 'V_2' in eq^n (C)

$$V_1 = A\left(\frac{1}{C} I_1 + \frac{D}{C} I_2\right) + B(-I_2)$$

$$V_1 = \frac{A}{C} I_1 + \frac{AD - CB}{C} I_2 \quad \text{—(e)}$$

on comparing (b) and (e)

$$z_{11} = \frac{A}{C} \quad , \quad z_{12} = \frac{AD-BC}{C}$$

$$z_{12} = \frac{\Delta T}{C}$$

z-parameter in terms of T'-parameter:-

similarly to previous conversion,

$$z_{11} = \frac{D'}{C'} \qquad z_{12} = \frac{1}{C'}$$

$$z_{21} = \frac{\Delta T'}{C'} \qquad z_{22} = \frac{A'}{C'}$$

z-parameter in terms of h-parameter:-

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—(a)} \quad \& \quad V_1 = z_{11} I_1 + z_{12} I_2 \quad \text{—(c)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—(b)} \qquad V_2 = z_{21} I_1 + z_{22} I_2 \quad \text{—(d)}$$

from (b); $V_2 = \frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$ —(e)

on comparing (d) and (e)

$$z_{21} = \frac{-h_{21}}{h_{22}} \quad \text{and} \quad z_{22} = \frac{1}{h_{22}}$$

and putting the value of V_2 in eq^n (a)

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$$

$$= \left(h_{11} - \frac{h_{12}h_{21}}{h_{22}}\right) I_1 + \frac{h_{12}}{h_{22}} I_2 \quad \text{—(f)}$$

on comparing eq^n (f) and (c)

$$z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

$$z_{12} = \frac{h_{12}}{h_{22}}$$

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parameter in terms of g-parameter:-

As similar to above case;

$z_{11} = 1/g_{11}$ $\quad$ $z_{12} = \frac{-g_{12}}{g_{11}}$

$z_{21} = \frac{g_{21}}{g_{11}}$ $\quad$ $z_{22} = \frac{\Delta g}{g_{11}}$

Y-parameter in terms of other parameter:-

* In terms of z-parameter;

$Y_{11} = \frac{z_{22}}{\Delta z}$ $\quad$ $Y_{12} = \frac{-z_{12}}{\Delta z}$

$Y_{21} = \frac{-z_{21}}{\Delta z}$ $\quad$ $Y_{22} = \frac{z_{11}}{\Delta z}$

* In terms of T-parameter;

$Y_{11} = D/B$ $\quad$ $Y_{12} = -\Delta T/B$

$Y_{21} = -1/B$ $\quad$ $Y_{22} = A/B$

* In terms of T' parameter

$Y_{11} = A'/B'$ $\quad$ $Y_{12} = -1/B$

$Y_{21} = \frac{-\Delta T'}{B'}$ $\quad$ $Y_{22} = D'/B'$

* In terms of h-parameter

$Y_{11} = 1/h_{11}$ $\quad$ $Y_{12} = \frac{-h_{12}}{h_{11}}$

$Y_{21} = \frac{h_{21}}{h_{11}}$ $\quad$ $Y_{22} = \Delta h/h_{11}$

* in term of g-parameter

$Y_{11} = \frac{\Delta g}{g_{22}}$ $\quad$ $Y_{12} = \frac{g_{12}}{g_{22}}$

$Y_{21} = \frac{-g_{21}}{g_{22}}$ $\quad$ $Y_{22} = 1/g_{22}$

T-parameter in term of other parameter:-

* in terms of z-parameter;

$V_1 = AV_2 - BI_2$ —— (I)

$I_1 = CV_2 - DI_2$ —— (II)

$V_1 = z_{11}I_1 + z_{12}I_2$ —— (III)

$V_2 = z_{21}I_1 + z_{22}I_2$ —— (IV)

from eq^n (IV)

$\Rightarrow I_1 = \frac{1}{z_{21}}V_2 - \frac{z_{22}}{z_{21}}I_2$ —— (V)

on comparing (II) and (V)

$C = \frac{1}{z_{21}}$ $\quad$ $D = \frac{z_{22}}{z_{21}}$

putting the value of 'I_1' from eq^n (V) to eq^n (III)

$V_1 = z_{11}\left[\frac{1}{z_{21}}V_2 - \frac{z_{22}}{z_{21}}I_2\right] + z_{12}I_2$

$V_1 = \frac{z_{11}}{z_{21}}V_2 + \left(z_{12} - \frac{z_{22}}{z_{21}}\right)I_2$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left(\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right) I_2$$

$$A = \frac{Z_{11}}{Z_{21}} \quad \text{and} \quad B = \frac{\Delta Z}{Z_{21}}$$

* T-parameter in terms of Y-parameter;

As similar to above case;

$$A = \frac{-Y_{22}}{Y_{21}} \; ; \; B = -\frac{1}{Y_{21}}$$

$$C = \frac{-\Delta Y}{Y_{21}} \; , \; D = \frac{-Y_{11}}{Y_{21}}$$

* T parameter in terms of T'-parameter:-

$$V_1 = AV_2 + B(-I_2)$$
$$I_1 = CV_2 + D(-I_2)$$

$$V_2 = A'V_1 + B'(-I_1) \quad \text{---(i)}$$
$$I_2 = C'V_1 + D'(-I_1) \quad \text{---(ii)}$$
$$I_1 = \frac{C'}{D'} V_1 - \frac{1}{D'} I_2$$

Putting the value of I_1 in eq^n (i)

$$V_2 = A'V_1 - B'\left(\frac{C'}{D'} V_1 - \frac{1}{D'} I_2\right)$$
$$V_2 = \left(A' - \frac{B'C'}{D'}\right)V_1 + \frac{B'}{D'} I_2$$
$$V_2 = \left(\frac{A'D' - B'C'}{D'}\right)V_1 + \frac{B'}{D'} I_2$$

-------,

$$V_1 = \frac{D'}{A'D' - B'C'} V_2 - \frac{B'}{A'D' - B'C'} I_2$$

$$\therefore A = \frac{D'}{\Delta T'} \; , \; B = \frac{B'}{\Delta T'}$$

Similarly $C = \frac{C'}{\Delta T'}$, $D = \frac{A'}{\Delta T'}$

* T-parameter in terms of h-parameter:-

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{---(a)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{---(b)}$$

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$I_1 = \frac{-h_{22}}{h_{21}} V_2 + \left(\frac{-1}{h_{21}}\right)(-I_2)$$

$$C = \frac{-h_{22}}{h_{21}} \; , \; D = -\frac{1}{h_{21}}$$

Putting the value of I_1 in eq^n (a)

$$V_1 = h_{11} \times \left\{\left(\frac{-h_{22}}{h_{21}}\right)V_2 + \left(-\frac{1}{h_{21}}\right)(-I_2)\right\} + h_{12} V_2$$

$$V_1 = \left(\frac{-h_{11}h_{22} + h_{12}h_{21}}{h_{21}}\right)V_2 + \left(\frac{-h_{11}}{h_{21}}\right)(-I_2)$$

$$\therefore A = -\frac{\Delta h}{h_{21}} \quad \text{and} \quad B = \frac{-h_{11}}{h_{21}}$$

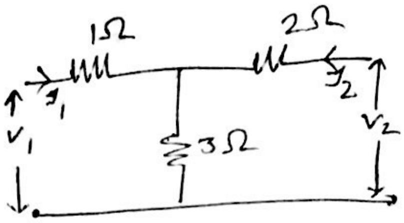
T parameter in terms of g-parameter;

$$A = \frac{1}{g_{21}} \; , \; B = \frac{g_{22}}{g_{21}}$$

$$C = \frac{g_{11}}{g_{21}} \quad D = \frac{\Delta g}{g_{21}}$$

For the network calculate Z and Y- parameter

Applying KVL eq^n;



$V_1 = 1 \times I_1 + 3(I_1 + I_2)$

$V_1 = 4I_1 + 3I_2$ ——(I)

$V_2 = 2I_2 + 3(I_1 + I_2)$ ——

$V_2 = 3I_1 + 5I_2$ ——(II)

For Z- parameter;

$$\left.\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}\right\} \text{——(a)}$$

on comparing eq^n (I) and (II) with eq^n (a)

$Z_{11} = 4\Omega$, $Z_{12} = 3\Omega$

$Z_{21} = 3\Omega$, $Z_{22} = 5\Omega$

For Y- parameter;

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

When $V_2 = 0$; $V_1 = 4I_1 + 3I_2$ ——(b)

$V_2 = 3I_1 + 5I_2$ ——(c)

then $3I_1 = -5I_2 \Rightarrow I_2 = -\frac{3}{5}$

putting the value of I_2 in eq^n (b)

$V_1 = 4I_1 - 3 \times \frac{3}{5} I_1$

$V_1 = \left(\frac{20-9}{5}\right) I_1 \Rightarrow V_1 = \frac{11}{5} I_1$

and $Y_{11} = \frac{I_1}{V_1} = \frac{5}{11}$ ℧

and for $Y_{21} = \frac{I_2}{V_1}$

$V_1 = 4I_1 + 3I_2$

and $I_1 = -\frac{5}{3} I_2 \Rightarrow V_1 = 4 \times -\frac{5}{3} I_2 + 3I_2$

$V_1 = -\frac{11}{3} I_2$

So; $Y_{21} = \frac{I_2}{V_1} = -\frac{3}{11}\,\mho$

When $V_1 = 0$; $V_1 = 4I_1 + 3I_2$

$V_2 = 3I_1 + 5I_2$

$V_1 = 0 \Rightarrow 4I_1 = -3I_2$

$V_2 = 3I_1 + 5I_2$

$Y_{12} = \frac{I_1}{V_2}$ So putting the value of $I_2 = -\frac{4}{3}I_1$

$V_2 = 3I_1 - 5\times\frac{4}{3}I_1$

$V_2 = -\frac{11}{3}I_1$

$Y_{12} = \frac{I_1}{V_2} = -\frac{3}{11}\,\mho$

and $Y_{22} = \frac{I_2}{V_2}$ $V_2 = 3I_1 + 5I_2$

and $I_1 = -\frac{3}{4}I_2$

$V_2 = 3\times -\frac{3}{4}I_2 + 5I_2$

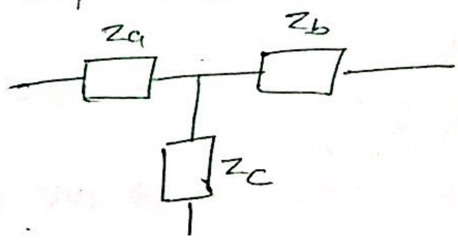
$V_2 = +\frac{11}{4}I_2$

So. $Y_{22} = \frac{I_2}{V_2} = \frac{4}{11}\,\mho$

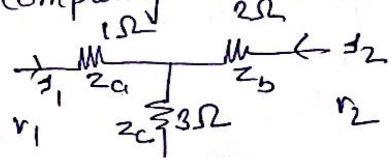
So $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}$

$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & -\frac{3}{11} \\ -\frac{3}{11} & \frac{4}{11} \end{bmatrix}$

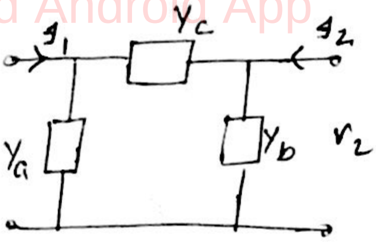
Q) For the T-network, obtain the Z-parameter.



Sol^n:– on comparing the circuit with



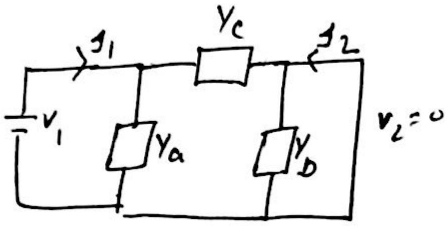
Obtain Y-parameter of the network;



$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

Case I :- $V_2 = 0$



$V_1 = (I_1 + I_2) \times \frac{1}{Y_a}$

and also;

$V_1 = \frac{-I_2}{Y_c}$

$I_2 = -V_1 Y_c$

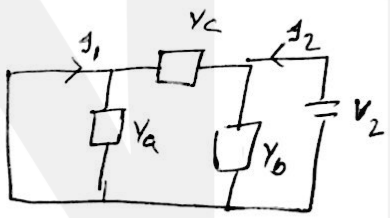
$\therefore V_1 = (I_1 - V_1 Y_c) \times \frac{1}{Y_a}$

$V_1 + \frac{V_1 Y_c}{Y_a} = \frac{I_1}{Y_a} \Rightarrow \frac{I_1}{V_1} = Y_{11} = \left(1 + \frac{Y_c}{Y_a}\right) \times Y_a$

$\boxed{Y_{11} = (Y_a + Y_c)}$

$Y_{21} = \frac{I_2}{V_1} \Rightarrow -Y_c$

Case II $V_1 = 0$



$V_2 = \frac{1}{Y_b}(I_1 + I_2)$

and $V_2 = \frac{-I_1}{Y_c}$

$Y_{12} = \frac{I_1}{V_2} = -Y_c$

$Y_{22} = \frac{I_2}{V_2}$

$V_2 = \frac{1}{Y_b}(I_1 + I_2)$

and $I_1 = -V_2 Y_c$

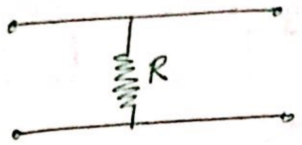
$V_2 = \frac{1}{Y_b}(-V_2 Y_c + I_2)$

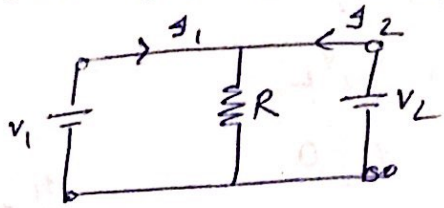
$V_2\left(1 + \frac{Y_c}{Y_b}\right) = \frac{I_2}{Y_b}$

$\Rightarrow Y_b + Y_c = \frac{I_2}{V_2} = Y_{22}$

$Y = \begin{bmatrix} Y_a + Y_c & -Y_c \\ -Y_c & Y_b + Y_c \end{bmatrix}$

Q) Determine 'Z' and 'Y' parameter for the network.





$$V_1 = R(I_1 + I_2)$$
$$V_2 = R(I_1 + I_2)$$
$$\therefore \ V_1 = V_2$$

For Z-parameter; $\quad V_1 = Z_{11} I_1 + Z_{12} I_2$
$\quad V_2 = Z_{21} I_1 + Z_{22} I_2$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \quad \text{and} \quad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$$Z_{11} = \frac{V_1}{I_1} = R \qquad Z_{21} = R$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} \qquad Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$$= R \qquad\qquad = R$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$

For Y-parameter; $\quad I_1 = Y_{11} \cancel{I_1} + Y_{12} V_2$
$\quad I_2 = Y_{21} V_1 + Y_{22} V_2$

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}$$ but when $V_2 = 0$
then $V_1 = V_2 = 0$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$ similarly for Y_{12} and Y_{22}
$V_1 = 0$ and $V_1 = V_2 = 0$

So in this case Y-parameter don't exist.

Determine the transmission parameter of circuit

$V_1 = 10I_1 + 5(2I_1+I_2)$

$V_1 = 20I_1 + 5I_2$ — (i)

$V_2 = 10(I_1+I_2) + 5(2I_1+I_2)$

$V_2 = 20I_1 + 15I_2$ — (ii)

Case 1:— $I_2 = 0$, $A = \frac{V_1}{V_2}$

$C = \frac{I_1}{V_2}$

$$\left.\begin{array}{l} V_1 = 20I_1 \\ V_2 = 20I_1 \end{array}\right\} \Rightarrow A = \frac{V_1}{V_2} = 1$$

$$\Rightarrow C = \frac{I_1}{V_2} = \frac{1}{20} \mho$$

Case 2:— $V_2 = 0$

$B = \frac{V_1}{-I_2}$

$D = \frac{-I_1}{I_2}$

$V_1 = 20I_1 + 5I_2$

$0 = 20I_1 + 15I_2 \Rightarrow -20I_1 = 15I_2$

$\Rightarrow \frac{-I_1}{I_2} = \frac{\cancel{15}\,3}{\cancel{20}\,4} = D$

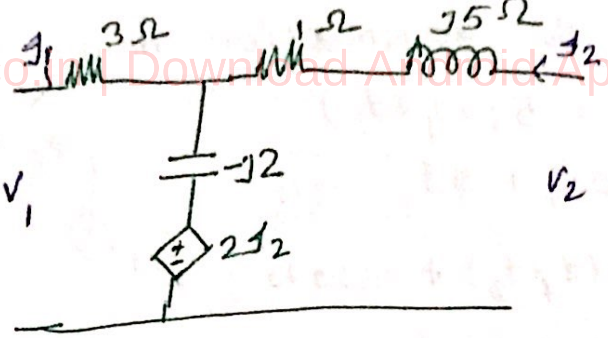
$\Rightarrow D = 3/4$

and $I_1 = -\frac{3}{4} I_2$

$V_1 = \cancel{20}^{5} \times -\frac{3}{\cancel{4}} I_2 + 5I_2$

$V_1 = -10I_2 \Rightarrow \frac{V_1}{I_2} = -10 = B$

Q) Calculate the z-parameters of network and show that network is neither reciprocal nor symmetrical.



$V_1 = 3I_2 - j2 \times (I_1 + I_2) + 2I_2$

$V_1 = (3 - j2) I_1 + (2 - j2) I_2$ — (I)

$V_2 = j5 \times I_2 + 1 \times I_2 - j2(I_1 + I_2) + 2I_2$

$V_2 = -j2 \times I_1 + (3 + j3) I_2$ — (II)

from eq^n (I) and (II)

$Z_{11} = (3 - j2)\,\Omega$

$Z_{12} = (2 - j2)\,\Omega$

$Z_{21} = -j2\,\Omega$

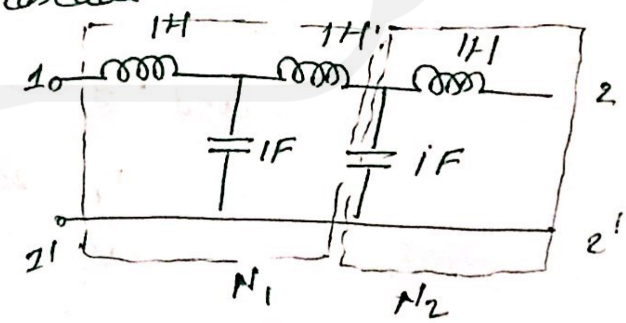
$Z_{22} = (3 + j3)\,\Omega$

For network to be reciprocal, $Z_{12} = Z_{21}$

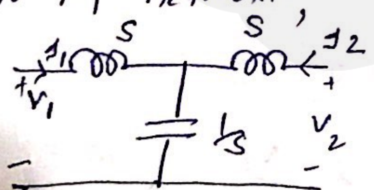
and $Z_{12} \neq Z_{21}$ so network is not reciprocal.

For network to be symmetry, $Z_{11} \neq Z_{22}$.

Q) Determine the transmission parameter of network shown in fig using concept of interconnection of two port networks 'N₁' and 'N₂' in cascade.



For N_1 network:



$V_1 = S I_1 + \frac{1}{S}(I_1 + I_2)$

$V_1 = \left(S + \frac{1}{S}\right) I_1 + \frac{1}{S} I_2$ — (I)

$V_2 = S I_2 + \frac{1}{S}(I_1 + I_2)$

$V_2 = \frac{1}{S} I_1 + \left(S + \frac{1}{S}\right) I_2$ — (II)

$V_1 = \left(\frac{s^2+1}{s}\right) I_1 + \frac{1}{s} I_2$

$V_1 = -\left(\frac{s^2+1}{s}\right)(s^2+1) I_2 + \frac{1}{s} I_2$

$V_1 = \left(\frac{-(s^2+1)^2+1}{s}\right) I_2$

$= \left(\frac{-(s^4+1+2s^2)+1}{s}\right) I_2$

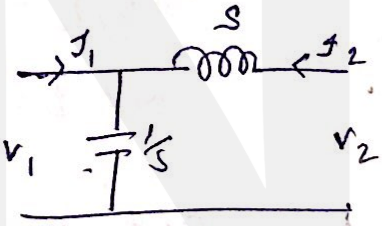
$V_1 = \left(\frac{-s^4-\cancel{1}-2s^2+\cancel{1}}{s}\right) I_2$

$\Rightarrow V_1 = -\frac{\cancel{s}(s^3+2s)}{\cancel{s}} I_2$

$\Rightarrow \frac{-V_1}{I_2} = B = (s^3+2s)$

For network 1; $\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} = \begin{bmatrix} 1+s^2 & 2s+s^3 \\ s & 1+s^2 \end{bmatrix}$

For network 2;



$V_1 = \frac{1}{s}(I_1+I_2)$

$V_2 = sI_2 + (I_1+I_2)\frac{1}{s}$

$\begin{bmatrix} 1 & s \\ s & 1+s^2 \end{bmatrix}$

$V_1 = \frac{1}{s} I_1 + \frac{1}{s} I_2$ —(I)

$V_2 = \cancel{sI_2 + (}\ \frac{1}{s} I_1 + (s+\frac{1}{s}) I_2$ —(II)

Case 1:- $I_2 = 0$;

$V_1 = \frac{1}{s} I_1$ —(a) (from eq^n (I))

$V_2 = \frac{1}{s} I_1$ —(b) from eq^n (II)

$\therefore C = \frac{I_1}{V_2} = s$ [from (b)]

and $A = \frac{V_1}{V_2} = 1$ [from a]

For T- network; Case 1; $I_2 = 0$

$$V_1 = (s + 1/s) I_1 + \frac{1}{s} I_2$$

$$V_1 = (s + 1/s) I_1 \quad \text{—— (a)}$$

$$V_2 = \frac{1}{s} I_1 \quad \text{—— (b)}$$

$$\Rightarrow \boxed{C = \frac{I_1}{V_2} = s}$$

and (a) ÷ (b)

$$A = \frac{V_1}{V_2} = \frac{s + 1/s}{1/s} = s^2 + 1$$

Case 2:- $V_2 = 0$

$$V_1 = \frac{1}{s} I_2 + (s + 1/s) I_1$$

$$V_2 = (s + 1/s) I_2 + \frac{1}{s} I_1$$

$$\Rightarrow 0 = (s + 1/s) I_2 + \frac{1}{s} I_1$$

$$(s + 1/s) I_2 = -\frac{1}{s} I_1$$

and $D = \frac{-I_1}{I_2} = \cancel{\frac{1/s}{s+1/s}}$ s^2+1

and $B = \frac{-V_1}{I_2}$

$$\cancel{V_1 = \frac{1}{s} I_2 + \left(\frac{s^2+1}{s}\right) I_1}$$

$$\cancel{\text{and } I_1 = -\frac{s^2+1}{\cancel{s}} I_2}$$

$$\cancel{V_1 = \left(\frac{s^2+1}{s}\right) I_2 + \frac{1}{s} I_1}$$

$$\cancel{\text{and } I_1 = (s^2+1) I_2}$$

$$\cancel{V_1 = \left(\frac{s^2+1}{s}\right) I_2 + \frac{1}{s} (s^2+1) I_2}$$

$$\cancel{= \frac{1}{s} I_2 - \frac{(s^2+1)(s^2+1)}{s} I_2 = \left(\frac{2s^2+2}{s}\right) I_2}$$

$$\cancel{= s^4 + 2s^4 + 1}$$

$$V_1 = (s + 1/s) I_1 + \frac{1}{s} I_2$$

and $(s + 1/s) I_2 = -\frac{1}{s} I_1$

$$\Rightarrow -(s^2+1) I_2 = I_1$$

Putting the value of I_1; $\cancel{V_1 = \frac{(s^2+1)}{s} \frac{1}{s}(s^2+1)}$

2. $V_2 = 0$

$V_1 = \frac{1}{s} I_1 + \frac{1}{s} I_2$ —— (1)

$V_2 = \frac{1}{s} I_1 + \left(s + \frac{1}{s}\right) I_2$ —— (11)

$0 = \frac{1}{s} I_1 + \left(s + \frac{1}{s}\right) I_2$ —— (111)

and $D = -I_1 / I_2$

$\Rightarrow -\frac{1}{\cancel{s}} I_1 = \left(\frac{s^2 + 1}{\cancel{s}}\right) I_2$

$= \boxed{\frac{-I_1}{I_2} = s^2 + 1 = D}$

and $B = \frac{-V_1}{I_2}$

$\Rightarrow V_1 = \frac{1}{s} I_1 + \frac{1}{s} I_2$

and from eq^n(111)

$I_1 = -(s^2+1) I_2$

$\therefore V_1 = \frac{-(s^2+1)}{s} I_2 + \frac{1}{s} I_2$

$= \frac{\{-(s^2+1)+1\}}{s} I_2$

$V_1 = -\frac{s^{\cancel{2}}}{\cancel{s}} I_2$

and $B = \frac{-V_1}{I_2}$

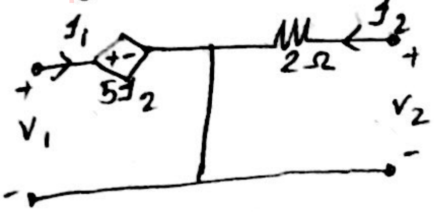
$= +s$

$\therefore$ For 2 N/O $\begin{bmatrix} 1 & s \\ s & 1+s^2 \end{bmatrix}$

and overall T-parameter:-

$[T] = [T_1][T_2] = \begin{bmatrix} 1+s^2 & 2s+s^3 \\ s & 1+s^2 \end{bmatrix} \begin{bmatrix} 1 & s \\ s & 1+s^2 \end{bmatrix} = \begin{bmatrix} 1+3s^2+s^4 & 3s+4s^3+s^5 \\ 2s+s^3 & 1+3s^2+s^4 \end{bmatrix}$

Q) Find the Z-parameters of following circuit



Sol^n:- Applying KVL in left hand side;

$$V_1 = 5I_2 \quad \text{—(i)}$$

Applying KVL in right hand side;

$$V_2 = 2I_2 \quad \text{—(ii)}$$

and we know;

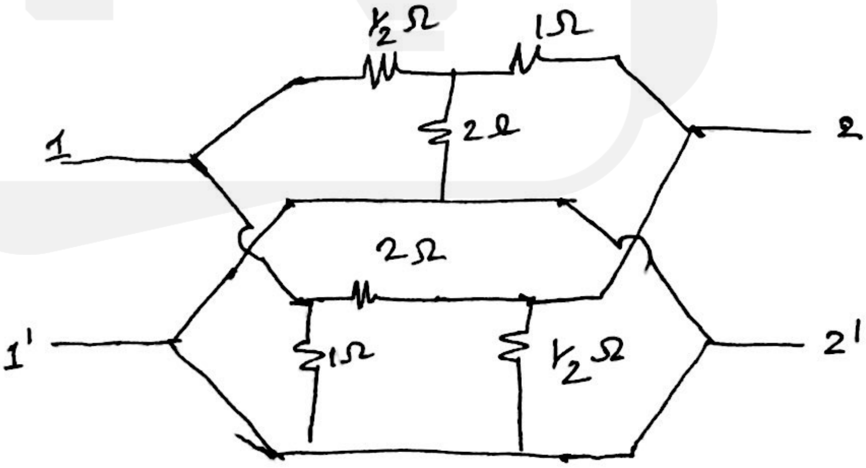
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$\therefore \quad Z_{11} = 0, \quad Z_{12} = 5\Omega$$

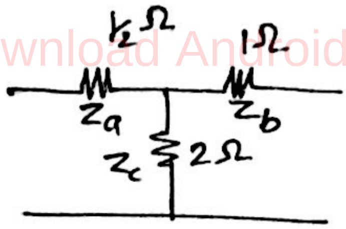
$$Z_{21} = 0 \quad Z_{22} = 2\Omega$$

Q) The n/w shown in fig. consist of resistive 'T' and a resistive π-n/w connected in parallel. For the element values given, determine the Y-parameters.



For n/w A ;

previous ques; (page no. 1)



$$[Z] = \begin{bmatrix} Z_a+Z_c & Z_c \\ Z_c & Z_b+Z_c \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}+2 & 2 \\ 2 & 2+1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & 2 \\ 2 & 3 \end{bmatrix}$$

and for Y- n/w ;

$$[Y] = [Z]^{-1} = \begin{bmatrix} \frac{5}{2} & 2 \\ 2 & 3 \end{bmatrix}^{-1}$$

For inverse of Z ; first transpose the given matrix

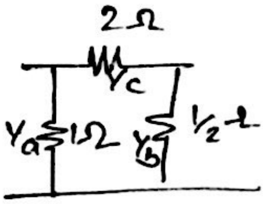
$$[Z^{-1}] = \frac{[Z]^T}{|Z|} = \frac{\begin{bmatrix} 3 & -2 \\ -2 & \frac{5}{2} \end{bmatrix}}{\frac{15}{2}-4}$$

$$= \begin{bmatrix} 3 & -2 \\ -2 & \frac{5}{2} \end{bmatrix} \times \frac{2}{7}$$

$$[Y_A] = [Z^{-1}] = \begin{bmatrix} \frac{6}{7} & -\frac{4}{7} \\ -\frac{4}{7} & \frac{5}{7} \end{bmatrix}$$

For B -n/w ;

In previous ques it has been found (page no. 2)



$$[Y_B] = \begin{bmatrix} Y_a+Y_c & -Y_c \\ -Y_c & Y_b+Y_c \end{bmatrix}$$

$$= \begin{bmatrix} 1+\frac{1}{2} & -2 \\ -2 & 2+\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

Network function:-

A network function exhibit the relationship between the transform of source or excitation to the transform of the response for an electrical network.

Driving point Impedance and Admittance:-

Driving point impedance of a one port network is defined as;

$$Z(s) = \frac{V(s)}{I(s)}$$

While driving point admittance is given as;

$$Y(s) = \frac{I(s)}{V(s)}$$

⇒ For two port network, the driving point impedance and admittance at port 1 is defined as;

$$\boxed{Z_{11}(s) = \frac{V_1(s)}{I_1(s)}}$$

$$\boxed{Y_{11}(s) = \frac{I_1(s)}{V_1(s)}}$$

While driving point impedance and admittance at port 2 is defined as;

$$\boxed{Z_{22}(s) = \frac{V_2(s)}{I_2(s)}}$$

$$\boxed{Y_{22}(s) = \frac{I_2(s)}{V_2(s)}}$$

Transfer impedence and admittance:-

→ Transfer impedence is defined as ratio of transfo voltage at the output port to the transformed cu. at the input port of two port network.

Ratio of one voltage to another current or one current to other voltage

Thus;

$$\left[Z_{12}(s) = \frac{V_2(s)}{I_1(s)} \right]$$

$Z_{12} = V_1/I_2$

→ Transfer admittance is defined as ratio of current transform at output to the voltage transform at input port.

Thus;

$$\left[Y_{12}(s) = \frac{I_2(s)}{V_1(s)} \right]$$

$Y_{12} = I_1/V_2$

Voltage and Current Transfer Ratio:-

Voltage transfer ratio is the ratio of voltage transform at output port to the voltage transform at the input port. It is usually denoted by G(s)

$$\left[G_{12}(s) = \frac{V_2(s)}{V_1(s)} \right]$$

→ Current transfer ratio is the ratio of transform of current at output port to that at the input port of two port network gives the current transfer ratio and denoted by $\alpha(s)$

$$\left[\alpha_{12}(s) = \frac{I_2(s)}{I_1(s)} \right]$$

Immitance :-

Because of similarity of impedence and admittance (i) here two quantities are assigned as immitance. An immitance is thus an impedence or an admittance.

Immitance function for circuit element

Elements	Impedence funcⁿ $Z(S)$	Admittance funcⁿ $Y(S)$
(1) R	R	$1/R = G$
(2) L	SL	$\frac{1}{LS}$
(3) C	$1/SC$	SC

Concept of poles and zeroes in a Network funcⁿ :-

A network funcⁿ $H(S)$ may be written as ;

$$H(S) = \frac{A(S)}{B(S)} = \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} S + a_n}{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} S + b_m}$$

where $a_0, a_1, \dots a_n$ and $b_0, b_1, \dots b_n$ are the coefficient of polynomial $A(S)$ and $B(S)$.

Factorising the numerator and denominator, the network funcⁿ can be ;

$$H(S) = \frac{A(S)}{B(S)} = \frac{a_0 (S-Z_1)(S-Z_2) \dots (S-Z_n)}{b_0 (S-P_1)(S-P_2) \dots (S-P_m)}$$

where $Z_1, Z_2, \dots Z_n$ are n-roots for $A(S) = 0$

$P_1, P_2, \dots P_m$ are m-roots for $B(S) = 0$

a_0/b_0 is a constant called scale factor

⇒ $z_1, z_2, \ldots z_n$ are called zeros and denoted by a "small circle" in the complex freq. diagram while $P_1, P_2, \ldots P_m$ are called poles and denoted by "cross" on the plot

from following location

⇒ The network func^n $H(s)$ becomes zero if "s" is equal to any of the zeros and become infinity when 's' is equal to any of the poles.

Note:- ① when "$n > m$" we have poles at infinity of multiplicity or degree of $(n-m)$.

② when "$n < m$" we have zeroes at infinity of multiplicity or degree of $(m-n)$

EX:-
$$T(s) = \frac{(2s+4)(s+4)}{(s+2)(s^2+2s+2)}$$

~~Poles~~

Poles will be

$(s+2) = 0 \Rightarrow s = -2$

$(s^2+2s+2) = 0 \Rightarrow s = -1 \pm J$

zeroes will be

$(2s+4) = 0 \Rightarrow s = -2$

$(s+4) = 0 \Rightarrow s = -4$

Poles; $s_1 = -2$, $s_2 = -1+J$, $s_3 = -1-J$

zeroes $s_1 = -2$ $s_2 = -4$, $s_3 = \infty$

Restriction on location of poles and zeroes in driving point immitance func^n:-

The following list provides the listing of restrictions of location of poles and zeros in driving point immitance func^n:

$$H(s) = \frac{A(s)}{B(s)}$$

① Coefficient of $A(s)$ and $B(s)$ = Real and positive

② poles and zeros if complex must occur in conjugate pairs.

③ Real parts of poles and zeros must be zero or negative.

④. Polynomial $A(s)$ or $B(s)$ cannot have any missing term between those of highest and lowest degree value unless all even order or all odd order terms are missing

⑤ Degree of $A(s)$ and $B(s)$ may differ by zero or one only.

⑥ lowest degree in $A(s)$ and $B(s)$ may differ in degree by at most one.

Q) check whether driving point impedence $Z(s)$ is suitable for representing a passive network.

$$Z(s) = \frac{s^4 - s^3 + 2s^2}{(s+5)} = \frac{A(s)}{B(s)} = \frac{s^2(s^2 - s + 2)}{(s+5)}$$

Sol^n:- ① One coeff. in $A(s)$ is -ve.

② $s = -5$,
$s = 0, -0.5 \pm \sqrt{7/4}$

Numerator gives double zero at S=0. This is not,

③ degree of numerator = 4 and degree of denominator = 1
difference of 3 exist between degree of numerator and denominator and is not permitted.

④ Lowest degree in numerator is 2 and in denominator is zero and difference is 2 ⇒ not permitted.

So given funcⁿ is not suitable for representing the driving point impedence.

⇒ Necessary Condition for Transfer function :-

The limitation on poles and zero location in transfer function are as follows:-

① Coefficient of B(s) must be +ve and coeff. of A(s) may or may not be +ve.

② poles and zeros if complex must be conjugate.

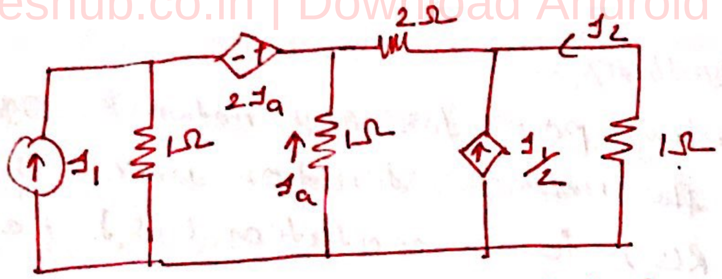
③ Real part of poles must be negative or zero

④ polynomial A(s) may have missing term between lowest and highest degree but B(s) should not have any missing term b/w highest and lowest degree unless all even or odd terms are missing

⑤ Degree of A(s) may be zero, independent of degree of B(s)

⑥ For voltage or current transfer ratio, maxᵐ degree of A(s) must equal the degree of B(s)

⑦ For transfer impedence and admittance, the maxᵐ degree of A(s) must equal the degree of B(s) pluse one



Q) Find current ratio transfer func^n (α).

sol^n:- current ratio transfer func^n

$$\alpha = \frac{I_2}{I_1}$$

$I_1 = I - 2I_a - I_a$

$I_1 = I - 3I_a$

and $I + I_a + I_1/2 + I_2 = 0$

$I = -(I_a + I_1/2 + I_2)$

$I_1 = -(I_a + I_1/2 + I_2) - 3I_a$

$I_a = -1/4(3I_1/2 + I_2)$

KVL in loop 2;

$-I_a \times 1 + 2(I_1/2 + I_2) + I_2 \times 1 = 0$

$\Rightarrow 3I_2 + I_1 - I_a = 0$

$\Rightarrow 3I_2 + I_1 + 1/4(3I_1/2 + I_2) = 0$

$= 3I_2 + \frac{1}{4}I_2 + \frac{3I_1}{8} + I_1 = 0$

$= \frac{13}{4}I_2 + \frac{11}{8}I_1 = 0$

$= \alpha = \frac{I_2}{I_1} = -\frac{11}{26}$

Network Synthesis:-

The starting point for any network synthesis problem is the network function which is the ratio of response R(s) to the excitation E(s) i.e;

$$T(s) = \frac{R(s)}{E(s)}$$

→ The first step in a synthesis procedure is to determine whether T(s) can be realized as a physical passive network.

a) Causality:- The response of network must be zero for $t<0$. i.e voltage cannot appear between any pair of terminal in the network before a current is impressed.

b) Stability:-
For the network to be stable, the following conditions must be satisfied:-

a) T(s) cannot have poles in the right half of s-plane.

b) T(s) should not have any multiple poles in the $j\omega$-axis.

c) The degree of numerator of T(s) cannot exceed the degree of denominator by more than unity.

Hurwitz Polynomial:-

The denominator polynomial P(s) of the system function is termed as Hurwitz polynomial.

the polynomial P(s) is said to be Hurwitz if

(i) P(s) is real when s is real

(ii) the roots of P(s) have real parts which are to be zero or negative.

Properties of Hurwitz Polynomial:-

$$P(s) = \alpha_n s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_1 s + \alpha_0$$

(1) All coeff. 'α_i' must be positive.

(2) Between the highest and lowest order term, none of coefficient may be zero. [unless the polynomial is even or odd). [should not be missing terms in 's']

(3) $P(s) = M(s) + N(s)$

where $M(s) =$ even part of $P(s)$

$N(s) =$ odd part of $P(s)$

Both $M(s)$ and $N(s)$ should have roots on $j\omega$-axis only.

(4) If P(s) is either even or odd, all its roots are on the $j\omega$-axis including origin.

(5) The continued fraction expansion of ratio of odd to even part or even to odd part of Hurwitz polynomial yield all positive quotient terms.

$$\psi(s) = \frac{N(s)}{M(s)} \text{ or } \frac{M(s)}{N(s)}$$

$$= q_1 s + \cfrac{1}{q_2 s + \cfrac{1}{q_3 s + \cfrac{1}{\cdots + \cfrac{1}{q_n s}}}}$$

When quotient $q_1, q_2, \ldots q_n$ must be positive.

(6) In case if polynomial is either even or only odd it is not possible to obtain continued fraction expansion. in such cases, the polynomial $P(s)$ is Hurwitz if ratio of $P(s)$ and its derivative $P'(s)$ gives a continued fraction expansion.

Conditions to check polynomial is Hurwitz or not :-

(i) All coeff. of polynomial should be +ve, no missing term except all even or odd.

(ii) $P(s) = M(s) + N(s)$

All continued fraction of $\frac{M(s)}{N(s)}$ or $\frac{N(s)}{M(s)}$ should be +ve.

Q) $P(s) = s^4 + s^3 + 2s^2 + 3s + 2$

check whether Hurwitz or not.

(I) $P(s) = s^4 + s^3 + 2s^2 + 3s + 2$

All coeff are +ve, no missing term.

(II) Even part $M(s) : s^4 + 2s^2 + 2$

Odd part $N(s) : s^3 + 3s$

So; $\frac{M(s)}{N(s)} = \frac{s^4 + 2s^2 + 2}{s^3 + 3s}$

Now continued fraction of $\frac{M(s)}{N(s)}$

$$\frac{M(s)}{N(s)} = \frac{s^4+2s^2+2}{s^3+3s}$$

$s^3+3s \,)\, s^4+2s^2+2 \,(\, s$
$\quad s^4+3s^2$
$\quad -$
$\quad \overline{-s^2+2 \,)\, s^3+3s \,(\, -s}$
$\qquad s^3-2s$
$\qquad -\;\; +$
$\qquad \overline{5s \,)\, -s^2+2 \,(\, -s/5}$
$\qquad\quad -s^2$
$\qquad\quad +$
$\qquad\quad \overline{2 \,)\, 5s \,(\, 5s/2}$
$\qquad\qquad 5s$
$\qquad\qquad \overline{\times}$

Quotient of continued fraction is negative. so it is not Hurwitz polynomial.

Note:— ① For pure even Hurwitz polynomial means all roots are on $j\omega$ axis

② For pure odd Hurwitz polynomial means all roots are on $j\omega$-axis including origin.

Q) $P(s) = s^6+6s^4+11s^2+6$ check Hurwitz or not.

$P(s) = s^6+6s^4+11s^2+6$

① All coeff. are +ve and pure even.

② so continued fraction; $\frac{P(s)}{P'(s)}$

$P'(s) = 6s^5 + 24s^3 + 22s$

$$\frac{P(s)}{P'(s)} = \frac{s^6 + 6s^4 + 11s^2 + 6}{6s^5 + 24s^3 + 22s}$$

$$6s^5 + 24s^3 + 22s \,\big)\, s^6 + 6s^4 + 11s^2 + 6 \,\big(\, s/6$$
$$\underline{s^6 + 4s^4 + 11/3\, s^2}$$
$$2s^4 + \tfrac{22}{3}s^2 + 6 \,\big)\, 6s^5 + 24s^3 + 22s \,\big(\, 3s$$
$$\underline{6s^5 + 22s^3 + 18s}$$
$$2s^3 + 4s \,\big)\, 2s^4 + \tfrac{22}{3}s^2 + 6 \,\big(\, s$$
$$\underline{2s^4 + 4s^2}$$
$$\tfrac{10}{3}s^2 + 6 \,\big)\, 2s^3 + 4s \,\big(\, \tfrac{3s}{5}$$
$$\underline{2s^3 + \tfrac{18}{5}s}$$
$$\tfrac{2}{5}s \,\big)\, \tfrac{10}{3}s^2 + 6 \,\big(\, \tfrac{25s}{3}$$
$$\underline{\tfrac{10}{3}s^2 + 6}$$
$$\times$$

All quotient of $\frac{P(s)}{P'(s)}$ is +ve. So it is hurwitz.

$F(s) = s^4 + s^3 + 3s^2 + 2s + 2$. check whether Hurwitz or not.

① All coeff. are +ve and no missing term.

② $M(s) = s^4 + 3s^2 + 2$

$N(s) = s^3 + 2s$

$$\frac{M(s)}{N(s)} = \frac{s^4 + 3s^2 + 2}{s^3 + 2s}$$

$$\begin{array}{r|l}
s^3+2s \;) & s^4 + 3s^2 + 2 \;(s \\
 & s^4 + 2s^2 \\
 & - \quad - \\
\hline
s^2+2 \;) & s^3 + 2s \;(s \\
 & s^3 + 2s \\
\hline
 & \times
\end{array}$$

It is said premature termination:

Note :- In Premature termination there are roots on $j\omega$ - axis given by last divisor.

If $W(s) = s^2 + 2$

$W'(s) = 2s$

$\frac{W(s)}{W'(s)}$ = quoteint should be positive then polynomial will be hurwitz.

$$\begin{array}{r|l}
2s \;) & s^2 + 2 \;(\frac{s}{2} \\
 & s^2 \\
\hline
\times \;\; 2 \;) & 2s \;(s \\
 & 2s \\
\hline
 & \times
\end{array}$$

quotient of $\frac{W(s)}{W'(s)}$ is positive so it is HURWITZ

Positive Real Function [PRF]

Driving point impedance func^n $Z(s)$ as well as driving admittance $Y(s)$ of one port network can be represented the form of:

$$F(s) = \frac{A(s)}{B(s)}$$

PRF represent physically realizable and stable passive driving point immitance func^n

$$= \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}$$

The function $F(s)$ is called a positive Real function if

a) $F(s)$ is real for 's' real

b) $B(s)$ is hurwitz polynomial

c) If $F(s)$ has poles on $j\omega$ axis, the poles are simple and residues therefore are real and +ve.

d) Real $F(j\omega) \geq 0$ for all values of ω.

Procedure for testing PRF:-

① $F(s) = \frac{A(s)}{B(s)}$ is PRF if

① $F(s)$ is real for 's' real

② $B(s)$ is hurwitz polynomial and $T(s)$ have no poles on right half of s-plane.

③ If $T(s)$ have poles on $j\omega$-axis then poles are simple and residues there are of real and positive.

④ $F(s) = \frac{A(s)}{B(s)} = \frac{M_1 + N_1}{M_2 + N_2}$

Wh...

determine

Where M_1, M_2 = even part of $F(s)$

N_1, N_2 = odd part of $F(s)$

determine $M_1 M_2 - N_1 N_2$ and put $s = j\omega$ and if $A(\omega^2) \geq 0$ for all ω $0 \leq \omega < \infty$. so it is PRF.

Q) $H(s) = \dfrac{s^2+10s+4}{s+2}$ check PRF or not.

$$H(s) = \frac{s^2+10s+4}{s+2}$$

$$\begin{array}{r} s+2 \,)\, s^2+10s+4 \,(\, s \\ s^2+2s \\ \hline 8s+4 \,)\, s+2 \,(\, \tfrac{1}{8} \\ s+\tfrac{1}{2} \\ \hline \tfrac{3}{2} \,)\, 8s+4 \end{array}$$

$N(s) = s^2+10s+4$

$D(s) = s+2$

$= s = -2$

(1) $D(s)$ is hurwitz polynomial

(2) No poles on $j\omega$ axis.

(3) $H(s) = \dfrac{N(s)}{D(s)} = \dfrac{s^2+10s+4}{s+2} = \dfrac{M_1+N_1}{M_2+N_2}$

even part $\left[\begin{array}{l} M_1(s) = s^2+4 \\ M_2(s) = 2 \end{array}\right.$ $\quad$ $\left.\begin{array}{l} N_1(s) = 10s \\ N_2(s) = s \end{array}\right]$ odd part

$M_1 M_2 - N_1 N_2 = (s^2+4)\times 2 - 10s \times s$

$= -8s^2+8$

Put $s = j\omega$

$= -8(j\omega)^2+8$

$A(\omega^2) = 8\omega^2+8 > 0$ for every value of 'ω'.

So it is PRF

Q) $H(s) = \frac{2s^2+5}{s(s^2+1)}$

(i) $D(s) = s(s^2+1)$

$s = 0, \pm j \Rightarrow$ Hurwitz

(ii) Poles on $j\omega$-axis $s = \pm j$

so residue on these poles;

$$H(s) = \frac{2s^2+5}{s(s^2+1)} = \frac{2s^2+5}{s^3+s}$$

$$= \frac{2s^2+5}{s(s-j)(s+j)}$$

$$= \frac{A}{s} + \frac{B}{s-j} + \frac{C}{s+j}$$

Residue of $H(s)$ at a pole $s = j$

$$\cancel{\frac{2s^2+5}{s(s-j)(s+j)} = \frac{A}{s} + \frac{B}{s-j} + \frac{C}{s+j}}$$

$$\text{Res}[J(s), j] = \lim_{s \to j} (s-j) \times F(s)$$

$$= \lim_{s \to j} \cancel{(s-j)} \times \frac{2s^2+5}{s(\cancel{(s-j)}(s+j))}$$

$$= \frac{2(j)^2+5}{j(j+j)} = \frac{-2+5}{j \times 2j}$$

$$= \frac{3}{2j^2} = \frac{-3}{2}$$

Residue of $H(s)$ at a pole $s = -j$

$$\text{Res}[J(s), -j] = \lim_{s \to j} \cancel{(s+j)} \times \frac{2s^2+5}{s(s-j)\cancel{(s+j)}}$$

$$= \frac{2(-j)^2+5}{-j \times -2j} = \frac{-3}{2}$$

idue at $s = \pm j$ is real but negative.
So it is not PRF.

$$H(s) = \frac{2s^2+2s+1}{s^3+2s^2+s+2}$$

(i) For D(s) to Hurwitz;

$$s^3+2s^2+s+2 = D(s)$$

$$M(s) = 2s^2+2$$

$$N(s) = s^3+s$$

$$\frac{N(s)}{M(s)} = \frac{s^3+s}{2s^2+2}$$

$$2s^2+2 \overline{) \; s^3+s \;} \left(\frac{s}{2} \right.$$
$$\underline{s^3+s}$$
$$\times$$

Premature termination

$$\therefore \; W(s) = 2s^2+2$$

$$W'(s) = 4s$$

$$\frac{W(s)}{W'(s)} = \frac{2s^2+2}{4s}$$

$$4s \overline{) \; 2s^2+2 \;} \left(\frac{s}{2} \right.$$
$$\underline{2s^2}$$
$$2 \overline{) \; 4s \;} \left(2s \right.$$
$$\underline{4s}$$
$$\times$$

quotient is +ve so it is Hurwitz

(ii) For poles; $s^3+2s^2+s+2 = (s^2+1)(s+2)$

$$s = \pm j, \; -2$$

Poles are on $j\omega$ axis;

Therefore residue at $s = j$;

$$F(s) \text{ at } s = j = (s-j) \times \frac{2s^2+2s+1}{s^3+2s^2+s+2}$$

$$= (s-j) \times \frac{2s^2+2s+1}{(s^2+1)(s+2)}$$

$$= \cancel{(s-j)} \frac{2s^2+2s+1}{(s+j)\cancel{(s-j)}(s+2)}$$

$$= \frac{2(j)^2+2j+1}{(j+j)(j+2)}$$

$$= \frac{-2+2j+1}{2j \times (j+2)}$$

$$= \frac{2j-1}{2j^2+4j} = \frac{\cancel{2j-1}}{-1\cancel{j}+2}$$

$$= \frac{2j-1}{-2+4j} = \frac{\cancel{2j-1}}{2(\cancel{2j-1})}$$

$$= \frac{1}{2}$$

at $s = -j$ Residue will be;

$$F(s) \text{ at } s = -j = \cancel{(s+j)} \times \frac{2s^2+2s+1}{\cancel{(s+j)}\cancel{(s-j)}(s+2)}$$

$$= \frac{2(-j)^2+2(-j)+1}{(-j-j)(-j+2)}$$

$$= \frac{2 \times -1 - 2j + 1}{-2j(-j+2)}$$

$$= \frac{-1-2j}{2j^2-4j} = \frac{-(1+2j)}{2(1-2j)}$$

$$\frac{-(1+2j)}{-2-2j} = \frac{-j \times -j}{j^2} \quad 2(1-j)$$

$2-2j$

$-1-2j$

$-2 \quad 2j$

$$= \frac{-1-2J}{-2-4J}$$

$$= \frac{\cancel{-}(1+\cancel{2J})}{\cancel{-}2(1+\cancel{2J})} = \frac{1}{2}$$

So residue are real and +ve.

(iii) $F(s) = \dfrac{2s^2+2s+1}{s^3+2s^2+s+2}$

$M_1(s) = 2s^2+1 \qquad N_1(s) = 2s$

$M_2(s) = 2s^2+2 \qquad N_2(s) = s^3+s$

$$A(\omega^2) = M_1M_2 - N_1N_2 \geq 0$$

$$= (2s^2+1)(2s^2+2) - 2s(s^3+s)$$

$$= 4s^4+6s^2+2-2s^4-2s^2$$

$$= 2s^4+4s^2+2$$

Put $s = j\omega$

$$= 2(j\omega)^4 + 4(j\omega)^2 + 2$$

$$= 2j^4\omega^4 + 4j^2\omega^2 + 2$$

$$= 2\omega^4 - 4\omega^2 + 2$$

$$= 2(\omega^4 - 2\omega^2 + 1)$$

$$= 2(\omega^2-1)^2 \geq 0 \text{ for all } \omega.$$

$x^2 - x$
$x^2 - 2x + 1$
$(x-1)^2$

Therefore $F(s)$ is always a PRF.

Properties of Positive Real func^n:-

(1) $T(s)$ is PRF then $\frac{1}{T(s)}$ is also PRF. If $Z(s)$ is PRF then $1/Z(s) = Y(s)$ is also PRF

~~(2) If two impedances~~

(2) Sum of PRF function is also PRF i.e sum of two

impedence connected in series or two admittances in parallel, resultant admittance or impedence is PRF.

Synthesis of Network:-

For the synthesis of one port network with two kind of element either LC, RL or RC network has to choose.

→ There are a number of methods of synthesizing (or realizing) a one port network :-

(I) Foster -1 form ⎤ ⇒ by partial fraction
(II) Foster -II Form ⎦

(III) Cauer - 1 form ⎤ ⇒ by continued fraction.
(IV) Cauer - II form. ⎦

① LC Immitance Function:-

Properties :-

① $Z(S) = \frac{even}{odd}$ or $\frac{odd}{even}$

② All the poles and zeroes of Z(S) are on $j\omega$-axis including origin

③ Poles and zeroes are interlaced so residues are +ve.
$\underbrace{\text{interlaced}}_{\text{alternate}}$

② RC impedence (Z_{RC}) or RL admittance (Y_{RL}) function:

Properties:-

① All poles and zeros are on -ve real axis including origin and interlaced with first singularity is pole

L impedence Properties ①

ii. Residue of $Z_{RC}(s)$ or $Y_{RL}(s)$ are +ve.

RL impedance (Z_{RL}) or RC admittance (Y_{RC}) function:

Properties:-

① All the poles and zeros are on -ve real axis including origin and interlaced with first singularity is zero.

② Residue of $\frac{Y_{RC}}{s}$ or $\frac{Z_{RL}}{s}$ are positive.

Foster-1 form:-

$$Z(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$$

① First check whether LC or RL or RC immitance using properties.

② This one is LC immitance function.

③ Second step is to do partial fraction of $Z(s)$.

$$Z(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$$

or;

$$\frac{Z(s)}{s} = \frac{s^2+4}{2(s^2+1)(s^2+9)}$$

Let $s^2 = x$

$$\frac{Z(s)}{s} = \frac{(x+4)}{2(x+1)(x+9)}$$

$$\frac{Z(s)}{s} = \frac{1}{2}\left[\frac{(x+4)}{(x+1)(x+9)}\right]$$

so, $\frac{x+4}{(x+1)(x+9)} = \frac{A}{x+1} + \frac{B}{x+9}$

$$x+4 = A(x+9) + B(x+1)$$

$$A + B = 1$$
$$9A + B = 4$$
$$- \quad - \quad -$$
$$-8A = -3$$
$$A = 3/8 \; ; \quad B = 1 - 3/8 = 5/8$$

$$\therefore \frac{x+4}{(x+9)(x+1)} = \frac{3}{8(x+1)} + \frac{5}{8(x+9)}$$

Therefore final eqⁿ will be;

$$\frac{Z(s)}{s} = \frac{1}{2}\left[\frac{3}{8(x+1)} + \frac{5}{8(x+9)}\right]$$

and $x = s^2$

$$Z(s) = \frac{1}{2} \times s \left[\frac{3}{8(s^2+1)} + \frac{5}{8(s^2+9)}\right]$$

or

$$Z(s) = \frac{3s}{16(s^2+1)} + \frac{5s}{16(s^2+9)}$$

$$Z(s) = \frac{3s}{16s^2+16} + \frac{5s}{16s^2+144}$$

$$\Rightarrow Z(s) = \frac{1}{\left(\frac{16s^2}{3s} + \frac{16}{3s}\right)} + \frac{1}{\left(\frac{16s^2}{5s} + \frac{144}{5s}\right)}$$

$$\Rightarrow Z(s) = \frac{1}{\left(\frac{16s}{3} + \frac{16}{3s}\right)} + \frac{1}{\left(\frac{16s}{5} + \frac{144}{5s}\right)}$$

$$Z(S) = \underbrace{\frac{1}{\left(\frac{16}{3}\right)s + \frac{1}{\left(\frac{3}{16}\right)s}}}_{Z_1} + \underbrace{\frac{1}{\left(\frac{16}{5}\right)s + \frac{1}{\left(\frac{5}{144}\right)s}}}_{Z_2}$$

$$Z_1 = \frac{1}{Y_1} \qquad Z_2 = 1/Y_2$$

$$Y_1 = \left(\frac{16}{3}\right)s + \frac{1}{\left(\frac{3}{16}\right)s}$$

$$Y_2 = \left(\frac{16}{5}\right)s + \frac{1}{\left(\frac{5}{144}\right)s}$$

$Z_L = sL$ & $Z_C = 1/sC$

or

$Y_L = \frac{1}{sL}$ or $Y_C = sC$

$$\Rightarrow Y_1 = \frac{16}{3}s + \frac{1}{\left(\frac{3}{16}\right)s}$$

$$= Y_{11} + Y_{12}$$

$$Y_{11} = \frac{16}{3}s \Rightarrow \boxed{C = \frac{16}{3}\ F}$$

$$Y_{12} = \frac{1}{\left(\frac{3}{16}\right)s} \Rightarrow \boxed{L = \frac{3}{16}\ H}$$

$$Y_2 = \frac{16}{5}s + \frac{1}{\left(\frac{5}{144}\right)s}$$

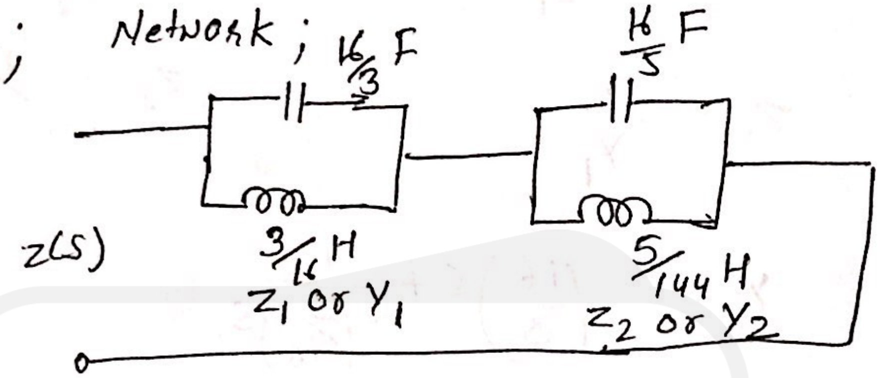
$$= Y_{21} + Y_{22}$$

so, $Y_{21} = \frac{16}{5}s \Rightarrow \boxed{C = \frac{16}{5}\ F}$

$$Y_{22} = \frac{1}{\left(\frac{5}{144}\right)s}$$

So; $\boxed{L = \frac{5}{144}\ H}$

Therefore; Network;



Note; Z_1 and Z_2 will be in series

Y_1 and Y_2 will be in parallel.

Foster-II form:-

$$Z(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$$

$$Y(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

$$\Rightarrow \frac{Y(s)}{s} = 2\left[\frac{(s^2+1)(s^2+9)}{s^2(s^2+4)}\right]$$

Let $s^2 = x$

$$\Rightarrow \frac{Y(s)}{s} = 2\left[\frac{(x+1)(x+9)}{x(x+4)}\right]$$

$$\frac{Y(s)}{s} = 2\left[\frac{(x+1)(x+9)}{x(x+4)}\right]$$

$$\frac{(x+1)(x+9)}{x(x+4)} = \frac{x^2+10x+9}{x^2+4x}$$

$$x^2+4x \,\big)\, x^2+10x+9 \,\big(\, 1$$
$$\underline{\begin{matrix} x^2+4x \\ - \quad - \end{matrix}}$$
$$6x+9$$

$$\Rightarrow \frac{(x+1)(x+9)}{x(x+4)} = 1 + \frac{6x+9}{x^2+4x}$$

$$\frac{6x+9}{x(x+4)} = \frac{A}{x} + \frac{B}{x+4}$$

$$A = 9/4 \quad , \quad B = 15/4$$

$$\therefore \frac{(x+1)(x+9)}{x(x+4)} = 1 + \frac{9}{4x} + \frac{15}{4(x+4)}$$

$$\frac{Y(s)}{s} = 2\left[1 + \frac{9}{4x} + \frac{15}{4(x+4)}\right]$$

$$Y(s) = 2s + \frac{9s}{4x} + \frac{15s}{4(x+4)}$$

$x = s^2$

$$Y(s) = 2s + \frac{2 \times 9 \cancel{s}}{{}_2\cancel{4} s^{\cancel{2}}} + \frac{2 \times 15 s}{{}_2\cancel{4}(s^2+4)}$$

$$Y(s) = 2s + \frac{9}{2s} + \frac{15s}{2(s^2+4)}$$

$$Y(s) = 2s + \frac{9}{2s} + \frac{1}{\left(\frac{2}{15}\right)s + \frac{8}{15s}}$$

$$Y(s) = \underbrace{[2s]}_{Y_1} + \underbrace{\left[\frac{1}{\frac{2}{9}s}\right]}_{Y_2} + \underbrace{\left[\frac{1}{\frac{2}{15}s + \frac{1}{\left(\frac{15}{8}\right)s}}\right]}_{Y_3}$$

$Y = Cs$

$Y = \frac{1}{Ls}$

$Y_1 = Cs = 2s$

$\boxed{C = 2F}$

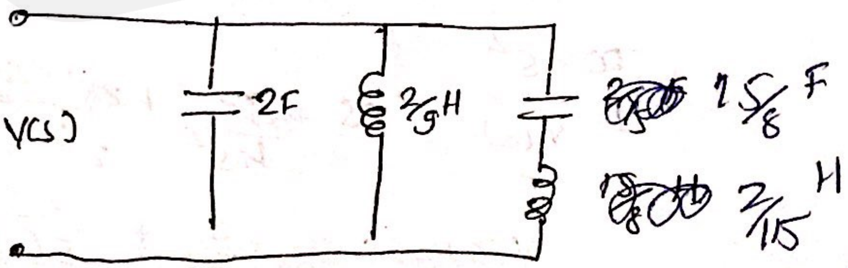
$Y_2 = \frac{1}{\left(\frac{2}{9}\right)s} \Rightarrow \boxed{L = \frac{2}{9} H}$

$$Y_3 = \frac{1}{\frac{2}{15}s + \frac{1}{\left(\frac{15}{8}\right)s}}$$

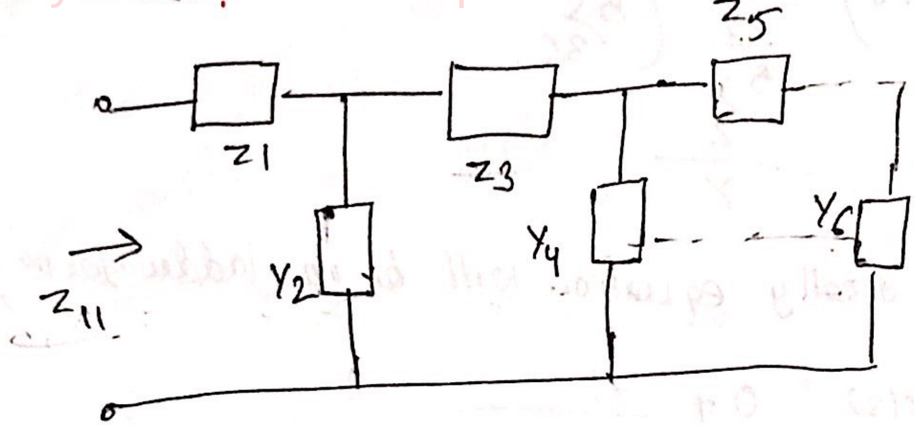
$$Y_3 = \frac{1}{Z_{31} + Z_{32}}$$

$Z_{31} = \frac{2}{15}s \Rightarrow \boxed{L = \frac{2}{15} H}$

$Z_{32} = \frac{1}{\left(\frac{15}{8}\right)s} \Rightarrow \boxed{C = \frac{15}{8} F}$



Ladder Network:-



$Z_{56} = Z_5 + \frac{1}{Y_6}$

$\frac{1}{Z_{56}} + \ldots$

$$Z = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6 + \ldots}}}}}$$

- This equation is known as continued fraction. It may be simplified to determine 'Z' for a given ladder network.

Cauer 1 - form:- Continued fraction of Z(S)

1st type Question:-

$$Z(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$$

$$= \frac{s^3+4s}{2s^4+20s^2+18}$$

So, $$Z(s) = \cfrac{1}{\cfrac{2s^4+20s^2+18}{s^3+4s}}$$

Note:- This eq^n is in the form of $\cfrac{1}{\cfrac{2s^4+20s^2+18}{s^3+4s}}$

therefore $Z_1 = 0$ as

$Z(s) = Z_1 + \frac{1}{Y_2}$

$= 0 + \cfrac{1}{2s + \cfrac{s^3+4s}{12s^2+18}}$

$s^3+4s \,)\, 2s^4+20s^2+18 \,(\, 2s$

$2s^4+8s^2$

$- \quad -$

$12s^2+18 \,)\, s^3+4s \,(\, s/12$

$s^3 + \frac{3}{2}s$

$-$

$\frac{5s}{2} \,)\, 12s^2+18 \,(\, \frac{24}{5}s$

$12s^2 -$

18) $\frac{5s}{2}\left(\frac{5s}{36}\right.$
$\frac{5s}{2}$
$\overline{\times}$

So finally equation will be in ladder form;

$$z(s) = 0 + \cfrac{1}{2s + \cfrac{1}{\frac{s}{12} + \cfrac{1}{\frac{24}{5}s + \cfrac{1}{\left(\frac{5s}{36}\right)}}}}$$

z_1, Y_2, z_3, Y_4, z_5

$z_1 = 0$

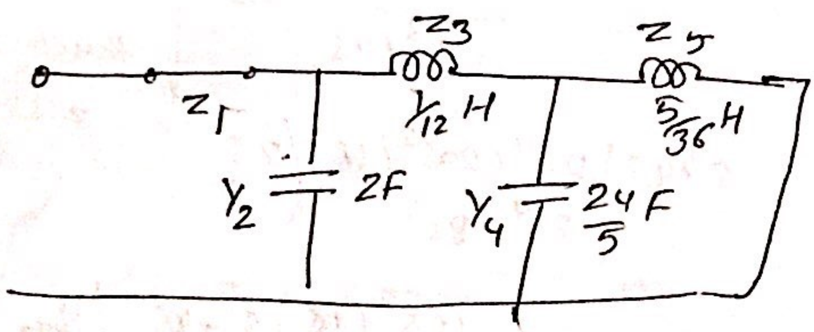
$Y_2 = 2s \Rightarrow \boxed{C = 2F}$

$z_3 = \frac{s}{12} \Rightarrow \boxed{L = \frac{1}{12} H}$

$Y_4 = \frac{24}{5}s \Rightarrow \boxed{C = \frac{24}{5} F}$

$z_5 = \frac{5s}{36} \Rightarrow \boxed{L = \frac{5}{36} H}$

So ckt is



C-1 form:— 2nd type Question

Q) $$Z(s) = \frac{s^5 + 5s^3 + 4s}{s^4 + 3s^2 + 1}$$

$s^4+3s^2+1 \,)\, s^5 + 5s^3 + 4s \,(\, s \rightarrow Z_1$
$\quad s^5 + 3s^3 + s$
$\quad \overline{\quad 2s^3 + 3s \,)\, s^4 + 3s^2 + 1 \,(\, \frac{s}{2} \rightarrow Y_2}$
$\qquad s^4 + \frac{3}{2}s^2$
$\qquad \overline{\quad \frac{3}{2}s^2 + 1 \,)\, 2s^3 + 3s \,(\, \frac{4}{3}s \rightarrow Z_3}$
$\qquad\quad 2s^3 + \frac{4}{3}s$
$\qquad\quad \overline{\quad \frac{5}{3}s \,)\, \frac{3}{2}s^2 + 1 \,(\, \frac{9}{10}s \rightarrow Y_4}$
$\qquad\qquad \frac{3}{2}s^2$
$\qquad\qquad \overline{\quad 1 \,)\, \frac{5}{3}s \,(\, \frac{5}{3}s \rightarrow Z_5}$
$\qquad\qquad\quad \frac{5}{3}s$
$\qquad\qquad\quad \overline{\quad \times \quad}$

$$Z(s) = Z_1 + \cfrac{1}{Y_1 + \cfrac{1}{Z_2 + \cfrac{1}{Y_3 + \cdots}}}$$

$$= \underset{Z_1}{s} + \cfrac{1}{\underset{Y_2}{\frac{s}{2}} + \cfrac{1}{\underset{Z_3}{\frac{4}{3}s} + \cfrac{1}{\underset{Y_4}{\frac{9}{10}s} + \cfrac{1}{\underset{Z_5}{\frac{5}{3}s}}}}}$$

$Z_1 = s = Ls \Rightarrow \boxed{L = 1H}$

$Y_2 = \frac{s}{2} = Cs \Rightarrow \boxed{C = \frac{1}{2}F}$

~~$Z_3 = \frac{4s}{3s} \rightarrow \frac{1}{Cs} \Rightarrow$~~ $\boxed{\cancel{C = \frac{3}{4}}F}$

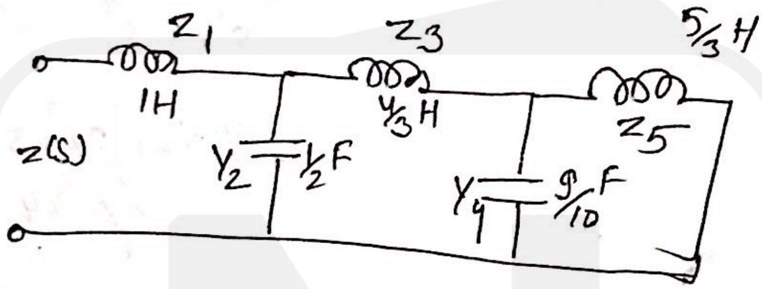
$Z_3 = \frac{4}{3}s \Rightarrow Ls \Rightarrow \boxed{L = \frac{4}{3} H}$

$Y_4 = \frac{9}{10} s \Rightarrow$

$= Cs \Rightarrow \boxed{C = \frac{9}{10} F}$

$Z_5 = \frac{5}{3} s \Rightarrow \boxed{L = \frac{5}{3} H}$

ckt network:-



* Cauer II - form:-

Continued fraction of Z(s) where $s = \frac{1}{p}$

Type 1 question:-

$$Z(s) = \frac{s^4 + 9s^2 + 8}{s^3 + 4s}$$

Put $s = \frac{1}{p}$

$$Z(s) = \frac{\frac{1}{p^4} + \frac{8}{p^2} + 8}{\frac{1}{p^3} + \frac{4}{p}}$$

$$Z(s) = \frac{\frac{8p^4 + 8p^2 + 1}{p^4}}{\frac{4p^2 + 1}{p^3}}$$

$$\Rightarrow \frac{8p^4 + 8p^2 + 1}{p(4p^2 + 1)}$$

$$= \frac{8p^4 + 8p^2 + 1}{4p^3 + p}.$$

Cauer II form :-

Q) $Z(s) = \dfrac{s^4 + 9s^2 + 8}{s^3 + 4s}$

Type 1 :-

(1) Arrange in ascending order first.

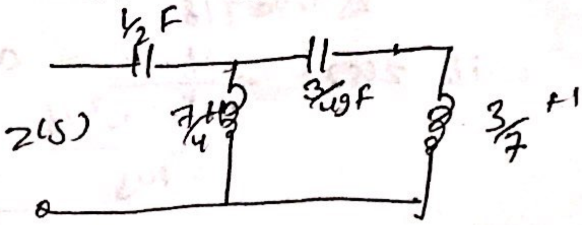
$$Z(s) = \frac{8 + 9s^2 + s^4}{4s + s^3}$$

(2) Continued fraction expansion of

$$Z(s) = \frac{8 + 9s^2 + s^4}{4s + s^3}$$

$$
\begin{array}{l}
4s + s^3 \;)\; 8 + 9s^2 + s^4 \;\left(\; \dfrac{2}{s} \rightarrow Z_1 \right. \\
\qquad\qquad\; 8 + 2s^2 \\
\qquad\qquad\; - \quad - \\
\hline
\qquad\qquad\qquad 7s^2 + s^4 \;)\; 4s + s^3 \;\left(\; \dfrac{4}{7s} \rightarrow Y_2 \right. \\
\qquad\qquad\qquad\qquad\qquad\quad 4s + \dfrac{4}{7}s^3 \\
\qquad\qquad\qquad\qquad\qquad\quad - \quad - \\
\hline
\qquad\qquad\qquad\qquad\qquad \dfrac{3}{7}s^3 \;)\; 7s^2 + s^4 \;\left(\; \dfrac{49}{3s} \rightarrow Z_3 \right. \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad 7s^2 - \\
\hline
\qquad\qquad\qquad\qquad\qquad\qquad s^4 \;)\; \dfrac{3}{7}s^3 \;\left(\; \dfrac{3}{7s} \rightarrow Y_4 \right. \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad \dfrac{3}{7}s^3 \\
\hline
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \times
\end{array}
$$

Therefore network would be



for. Continued

Note:—

→ For synthesise of network, first check whether its LC, RC or RL immitance using the properties for each immitance defined

→ Then synthesize the network using the method specify as for foster-1 form use partial fraction of $Z(s)$ and for for foster-II also used partial fraction of $Y(s)$ if $Z(s)$ is provided in question then find out the equation for $Y(s) = \frac{1}{Z(s)}$ then use foster II form.

→ For Cauer 1 and cauer II form use continuous fraction and draw the ladder network. And for cauer II form if numerator power is greater than denominator power then arrange in ascending form and then use continuous fraction and if numerator power is less than denominator power then put $s = \frac{1}{p}$ and then use continuous fraction. or if numerator power is less than denominator power arrange the $Z(s)$ or $Y(s)$ in terms of $\frac{1}{Z(s)}$

For ex $Z(s) = \frac{s^3+4s}{2s^4+20s^2+18}$ is given then for cauer-II form write $Z(s) = \cfrac{1}{\cfrac{2s^4+20s^2+18}{s^3+4s}}$ and then do the continued frac^n.

form:-

continued fraction of $Z(s)$ when $s = 1/p$

Type II Question:-

Q) $Z(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$

[Note:- Numerator power is < denominator power so put $s = 1/p$]

$Z(s) = \frac{s^3+4s}{2s^4+20s^2+18}$

$\Rightarrow$ Put $s = 1/p$ we get;

$Z(s) = \frac{\frac{1}{p^3}+\frac{4}{p}}{\frac{2}{p^4}+\frac{20}{p^2}+18}$

$\Rightarrow Z(s) = \frac{4p^3+p}{18p^4+20p^2+2}$

$\Rightarrow Z(s) = \frac{1}{\frac{18p^4+20p^2+2}{4p^3+p}}$

Another method for this ques:-

$Z(s) = \frac{s^3+4s}{2s^4+20s^2+18}$

$\equiv \frac{1}{\frac{2s^4+20s^2+18}{s^3+4s}}$

and now do the continued fraction of $\frac{2s^4+20s^2+18}{s^3+4s}$

$$
\begin{array}{l}
4p^3+p \;)\; 18p^4+20p^2+2 \;(\; \frac{9}{2}p \\
\qquad\quad 18p^4+\frac{9}{2}p^2 \\
\qquad\quad - \quad - \\
\qquad\quad \frac{31}{2}p^2+2 \;)\; 4p^3+p \;(\; \frac{8}{31}p \\
\qquad\qquad\qquad 4p^3+\frac{16}{31}p \\
\qquad\qquad\qquad - \\
\qquad\qquad\qquad \frac{15}{31}p \;)\; \frac{31}{2}p^2+2 \;(\; \frac{(31)^2}{30}p \\
\qquad\qquad\qquad\qquad \frac{31}{2}p^2 \\
\qquad\qquad\qquad\qquad - \\
\qquad\qquad\qquad\qquad 2 \;)\; \frac{15}{31}p \;(\; \frac{15}{62}p \\
\qquad\qquad\qquad\qquad\qquad \frac{15p}{31}
\end{array}
$$

$$z(s) = 0 + \cfrac{1}{\cfrac{9p}{2} + \cfrac{1}{\left(\cfrac{8p}{31}\right) + \cfrac{1}{\cfrac{961p}{30} + \cfrac{1}{\cfrac{15p}{62}}}}}$$

z_1, Y_2, z_3, y_4, z_5

soln As $s = 1/p$

$\Rightarrow p = 1/s$

$z_1 = 0$

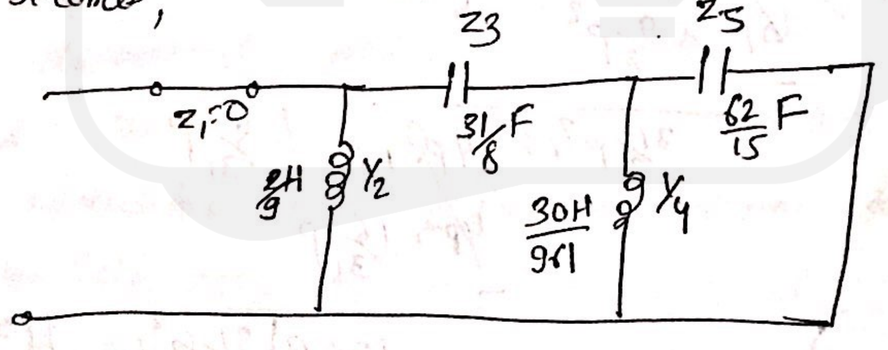
$Y_2 = \frac{9p}{2} = \frac{9}{2s} = \frac{1}{(2/9)s} \Rightarrow \boxed{L = \frac{2}{9} H}$

$z_3 = \frac{8p}{31} \Rightarrow \frac{8}{31s} \Rightarrow \frac{1}{\left(\frac{31}{8}\right)s} \Rightarrow \boxed{C = \frac{31}{8} F}$

$Y_4 = \frac{961p}{30} \Rightarrow \frac{961}{30s} \Rightarrow \frac{1}{\frac{30}{961}s} \Rightarrow \boxed{L = \frac{30}{961} H}$

$z_5 = \frac{15p}{62} \Rightarrow \frac{15}{62s} \Rightarrow \frac{1}{\frac{62}{15}s} \Rightarrow \boxed{C = \frac{62}{15} F}$

so ckt becomes;



* Note:- if numerator power < denominator power the $z_1 = 0$ for both cauer 1 and cauer II form

* if numerator power > denominator power then there will be value of z_1 for both cauer 1 and cauer II form

Q1 An impedence funcⁿ is given by;

$$Z(S) = \frac{(S+1)(S+4)}{S(S+2)(S+5)}$$

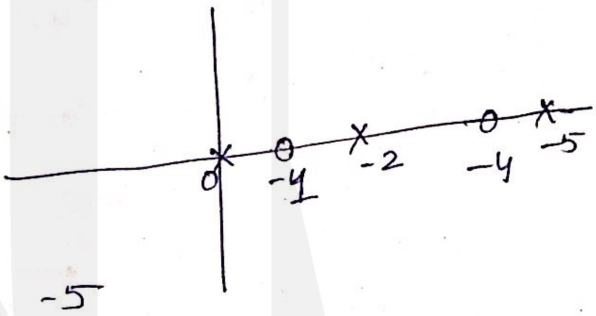
Synthesize the network using
i) Foster - I form
ii) Foster - II form
iii) Cauer - I form
iv) Cauer - II form.

$$Z(S) = \frac{(S+1)(S+4)}{S(S+2)(S+5)}$$

Step 1:- Check whether the network is LC immitance, RC or RL immitance.

→ Z(S) is RC impedence.

i) $P_1, P_2, P_3 = 0, -2, -5$
$Z_1, Z_2 = -1, -4$



ii) Residue at $S = 0, -2, -5$

at $S = 0$; Residue $= \cancel{S} \times \frac{(S+1)(S+4)}{\cancel{S}(S+2)(S+5)} = \frac{4}{10} = \frac{2}{5}$

Residue at $S = -2$; $\cancel{(S+2)} \frac{(S+1)(S+4)}{S\cancel{(S+2)}(S+5)} = \frac{\cancel{+6}}{} \frac{1}{3}$

Residue at $S = -5$ $\cancel{(S+5)} \frac{(S+1)(S+4)}{S(S+2)\cancel{(S+5)}} = \frac{4}{15}$

→ So poles and zeroes lying in negative real axis with first singularity is pole

→ Residue are real and positive
Therefore Network is RC impedence.

Step 2:- Partial fraction of Z(S)

$$Z(S) = \frac{A}{S} + \frac{B}{S+2} + \frac{C}{S+5}$$

on solving further we get the value of

$$A = \frac{2}{5}, \quad B = \frac{1}{3}, \quad C = \frac{4}{15}$$

$$\therefore \; Z(S) = \frac{2}{5S} + \frac{1}{3(S+2)} + \frac{4}{15(S+5)}$$

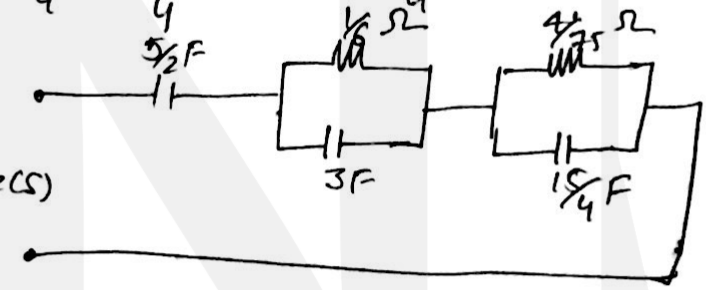
$$= \frac{1}{\frac{5}{2}S} + \frac{1}{3S+6} + \frac{1}{\frac{15}{4}S + \frac{75}{4}}$$

$$Y_1(S) = \frac{5}{2}S \Rightarrow C_1 = \frac{5}{2} F$$

$$Y_2(S) = 3S+6 \Rightarrow C_2 = 3F, \; R_1 = \frac{1}{6}\Omega$$

$$Y_3(S) = \frac{15}{4}S + \frac{75}{4} \Rightarrow C_3 = \frac{15}{4} F, \; R_2 = \frac{4}{75}\Omega$$

$\therefore$



Fosle - II form:-

$$Z(S) = \frac{(S+1)(S+4)}{S(S+2)(S+5)}$$

$$Y(S) = \frac{S(S+2)(S+5)}{(S+1)(S+4)}$$

$$\frac{Y(S)}{S} = \frac{(S+2)(S+5)}{(S+1)(S+4)}$$

$\therefore$ this is RL admittance as all conditions are satisfied.

$$\frac{Y(S)}{S} = \frac{S^2+7S+10}{S^2+5S+4}$$

step 2:- Partial fraction of $\frac{Y(S)}{S}$.

$$s^2+5s+4 \overline{\big) s^2+7s+10 } \big(1$$
$$\underline{s^2+5s+4}$$
$$2s+6$$

$$\frac{Y(s)}{s} = 1 + \frac{2s+6}{(s+1)(s+4)}$$

Now partial fraction of $\frac{2s+6}{(s+1)(s+4)}$

$$\frac{2s+6}{(s+1)(s+4)} = \frac{A}{(s+1)} + \frac{B}{(s+4)}$$

on solving we got the value of $A = 4/3$, $B = 2/3$

$$\frac{Y(s)}{s} = 1 + \frac{4}{3(s+1)} + \frac{2}{3(s+4)}$$

$$Y(s) = s + \frac{s}{\frac{3}{4}s + \frac{3}{4}} + \frac{s}{\frac{3}{2}s + 6}$$

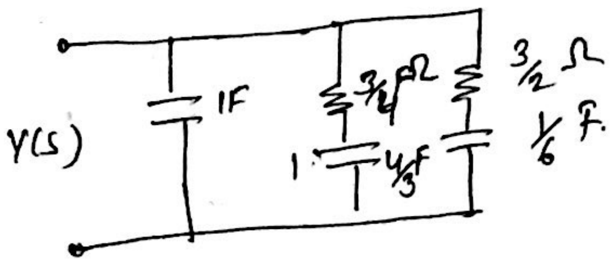
$$Y(s) = s + \frac{1}{\frac{3}{4} + \frac{3}{4s}} + \frac{1}{\frac{3}{2} + \frac{6}{s}}$$

$Y_1(s) = s$ $\qquad C_1 = 1F$

$Z_1(s) = \frac{3}{4} + \frac{3}{4s}$ $\qquad R_1 = \frac{3}{4}\Omega$, $C_2 = \frac{4}{3}F$

$Z_2(s) = \frac{3}{2} + \frac{6}{s}$ $\qquad R_2 = \frac{3}{2}\Omega$, $C_3 = \frac{1}{6}F$

$$\therefore Y(s) = Y_1 + \frac{1}{Z_1} + \frac{1}{Z_2}$$



Cauer 1 form :-

$$Z(s) = \frac{(s+1)(s+4)}{s(s+2)(s+5)} = \frac{s^2+5s+4}{s^3+7s^2+10s}$$

$$Z(s) = \frac{1}{\dfrac{s^3+7s^2+10s}{s^2+5s+4}}$$

Now continued fraction of $\dfrac{s^3+7s^2+10s}{s^2+5s+4}$

$s^2+5s+4 \,)\, s^3+7s^2+10s \,(\, s \rightarrow Y_1$

$\quad s^3+5s^2+4s$

$\quad 2s^2+6s \,)\, s^2+5s+4 \,(\, \frac{1}{2} \rightarrow Z_2$

$\quad\quad s^2+3s$

$\quad\quad 2s+4 \,)\, 2s^2+6s \,(\, s \rightarrow Y_3$

$\quad\quad\quad 2s^2+4s$

$\quad\quad\quad 2s \,)\, 2s+4 \,(\, 1 \rightarrow$

$\quad\quad\quad\quad 2s$

$\quad\quad\quad\quad 4 \,)\, 2s \,(\, \frac{s}{2}$

$\quad\quad\quad\quad\quad 2s$

$\quad\quad\quad\quad\quad \times\times$

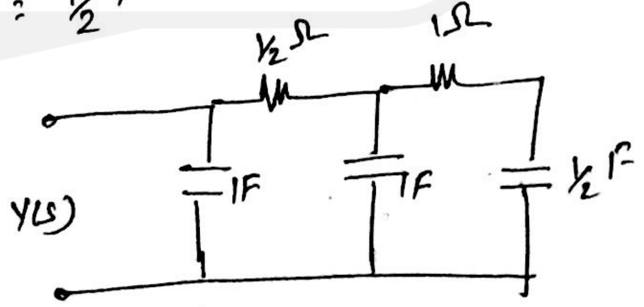
$Y_1 = s \Rightarrow C_1 = 1F$

$Z_2 = \frac{1}{2} \Rightarrow R_2 = \frac{1}{2}\Omega$

$Y_3 = s \Rightarrow C_3 = 1F$

$Z_4 = 1 \Rightarrow R_4 = 1\Omega$

$Y_5 = \frac{s}{2} \Rightarrow C_5 = \frac{1}{2}F$



Cauer - II form :-

(3)

$$z(s) = \frac{s^2+5s+4}{s^3+7s^2+10s}$$

Numerator power < Denominator power.

$$s = 1/p$$

$\therefore$

$$z(s) = \frac{\frac{1}{p^2} + \frac{5}{p} + 4}{\frac{1}{p^3} + \frac{7}{p^2} + \frac{10}{p}}$$

$$z(s) = \frac{\frac{1+5p+4p^2}{\cancel{p^2}}}{\frac{1+7p+10p^2}{\cancel{p^3}\ p}}$$

$$= \frac{p+5p^2+4p^3}{1+7p+10p^2}$$

$$= \frac{4p^3+5p^2+p}{10p^2+7p+1}$$

$$10p^2+7p+1 \,)\, 4p^3+5p^2+p \,\left(\frac{4}{10}p\right.$$

$$4p^3+\frac{28}{10}p^2+\frac{4}{10}p$$

$$- \quad -$$

$$\frac{22}{10}p^2+\frac{6}{10}p \,)\, 10p^2+7p+1 \,\left(\frac{100}{22}\right.$$

$$10p^2+\frac{30}{11}p$$

$$\frac{47}{11}p+1 \,)\, \frac{22}{10}p^2+\frac{6}{10}p \,\left(\frac{11}{47}\times\frac{22}{10}p\right.$$

$$\frac{22}{10}p^2+\frac{121}{235}p$$

$$\times \quad \frac{20}{235}p \,)\, \frac{47}{11}p+1 \,\left(\frac{220}{44}\right.$$

$$\frac{47}{11}p+\cancel{20}$$

$$1 \,)\, \frac{20}{}p \,(\frac{20}{}$$

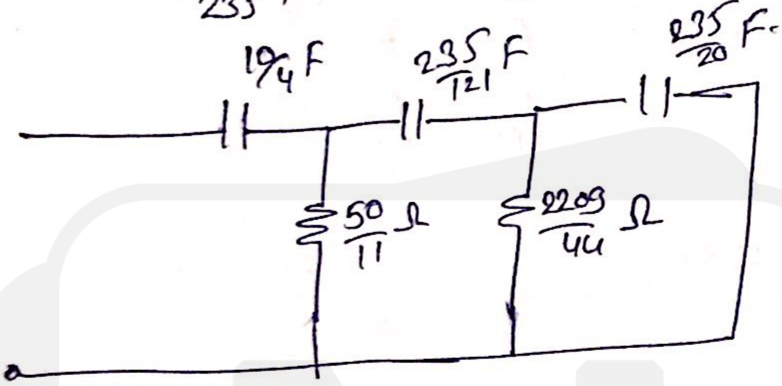
$Z_1 = \frac{4}{10}P = \frac{4}{10s} \Rightarrow C_1 = \frac{10}{4}F$

$Y_2 = \frac{100}{22} = \frac{50}{11} \Rightarrow R_2 = \frac{50}{11}$

$Z_3 = \frac{121}{235}P = \frac{121}{235s} \Rightarrow C_3 = \frac{235}{121}F$

$Y_4 = \frac{2209}{44} \Rightarrow R_4 = \frac{2209}{44}\Omega$

$Z_5 = \frac{20}{235}P = \frac{20}{235s} \Rightarrow C_5 = \frac{235}{20}F$



Q) $Z(s) = \frac{s(s+2)(s+5)}{(s+1)(s+4)}$

Step 1:- Checking of RC, RL and LC

- Poles and zeroes

An impedance funct at the input of network is represented by;

$$Z(s) = \frac{s^2+5s+4}{s^2+2s}$$

Express it in both the Foster forms.

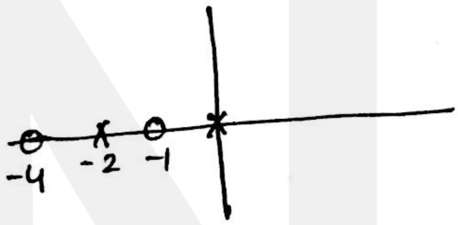
$$Z(s) = \frac{s^2+5s+4}{s^2+2s} = \frac{(s+1)(s+4)}{s(s+2)}$$

Step1:- check whether its RC, LC or RL immitance.

$P_1, P_2 = 0, -2$

$Z_1, Z_2 = -1, -4$

Poles are on negative real axis.



Poles are alternate and first singularity is pole

Residue of $Z(s)$ at $P_1 = 0$, $P_2 = -2$

at $P_1 = 0 \Rightarrow \quad \cancel{(s-0)} \times \frac{(s+4)(s+1)}{\cancel{s}(s+2)}$

$= \frac{4}{2} = 2$

at $P_2 = -2 \quad \cancel{(s+2)} \frac{(s+1)(s+4)}{s\cancel{(s+2)}}$

$= \frac{-1 \times 2}{-2} = 1$

Residue are Real and +ve.

Therefore it is RC imittance.

Step2:- $Z(s) = \frac{(s+1)(s+4)}{s(s+2)}$

Partial fraction of $Z(s)$

Foster

$$Z(s) = \frac{s^2+5s+4}{s^2+2s}$$

$$s^2+2s \overline{\big)\, s^2+5s+4 \,}\big(1$$
$$\underline{s^2+2s}$$
$$3s+4$$

$$\therefore Z(s) = 1 + \frac{3s+4}{s(s+2)}$$

Partial fraction of $Z(s) = \dfrac{3s+4}{s(s+2)}$

$$= \frac{A}{s} + \frac{B}{s+2}$$

on solving $A = 2,\ B = 1$

$$\therefore Z(s) = 1 + \frac{2}{s} + \frac{1}{s+2}$$

$$Z(s) = 1 + \frac{1}{s/2} + \frac{1}{s+2}$$

$$= Z_1 + Z_2 + Z_3$$

$Z_1 = 1 \Rightarrow R = 1\Omega$

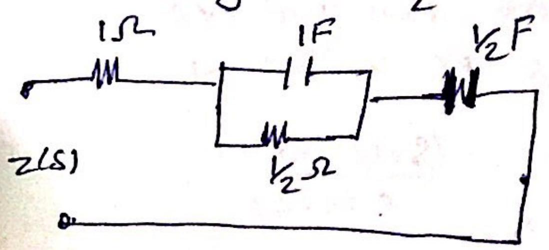
$Z_2 = \frac{1}{s/2} \Rightarrow C = \frac{1}{2} F$

$$Z_3 = \frac{1}{s+2} = \frac{1}{Y_3}$$

$Y_3 = s+2$

$C_3 = 1F$

$R_3 = \frac{1}{2}\Omega$



Foster-II Form:

$$Z(s) = \frac{s^2+5s+4}{s^2+2s}$$ [convert the Z(s) into Y(s)]

$$\Rightarrow Y(s) = \frac{s^2+2s}{s^2+5s+4}$$

$$Y(s) = \frac{s(s+2)}{(s+1)(s+4)}$$

$$\Rightarrow \frac{Y(s)}{s} = \frac{s+2}{(s+1)(s+4)}$$

$$\frac{Y(s)}{s} = \frac{A}{s+1} + \frac{B}{s+4}$$

$$\Rightarrow A(s+4) + B(s+1) = s+2$$

$$A+B = 1$$
$$4A+B = 2$$
$$-3A = -1 \Rightarrow A = \frac{1}{3}$$ and $B = \frac{2}{3}$

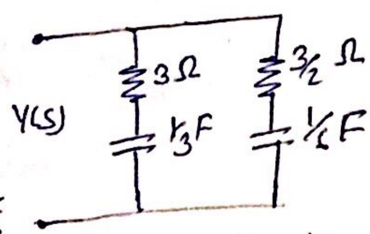
$$\Rightarrow \frac{Y(s)}{s} = \frac{1/3}{s+1} + \frac{2/3}{s+4}$$

$$Y(s) = \frac{\frac{1}{3}s}{s+1} + \frac{\frac{2}{3}s}{s+4}$$

$$Y(s) = \frac{s}{3s+3} + \frac{2s}{3s+12}$$

$$Y(s) = \frac{1}{3+\frac{3}{s}} + \frac{1}{\frac{3}{2}+\frac{6}{s}}$$

$$Y(s) = Y_1 + Y_2$$
$$= \frac{1}{Z_1} + \frac{1}{Z_2}$$



Foster-II N/W

$Z_1 = 3 + \frac{3}{s}$ $\quad$ $Z_2 = \frac{3}{2} + \frac{6}{s}$

$R_1 = 3\Omega$ $\quad$ $R_2 = \frac{3}{2}\Omega$

$C_1 = \frac{1}{3}F$ $\quad$ $C_2 = \frac{1}{6}F$

Cauer - 1 form:-

$$Z(s) = \frac{s^2+5s+4}{s^2+2s}$$

$$\begin{array}{l} s^2+2s \,)\, s^2+5s+4 \,(\, 1 \to R_1 \\ \qquad\qquad s^2+2s \\ \hline \qquad\qquad 3s+4 \,)\, s^2+2s \,(\, \frac{s}{3} \to Y_2 \\ \qquad\qquad\qquad s^2+\frac{4}{3}s \\ \hline \qquad\qquad\qquad \frac{2}{3}s \,)\, 3s+4 \,(\, \frac{9}{2} \to R_3 \\ \qquad\qquad\qquad\qquad 3s \\ \hline \qquad\qquad\qquad\qquad 4 \,)\, \frac{2s}{3} \,(\, \frac{s}{6} \to Y_4 \\ \qquad\qquad\qquad\qquad\quad \frac{2s}{3} \\ \hline \qquad\qquad\qquad\qquad\quad \times \end{array}$$

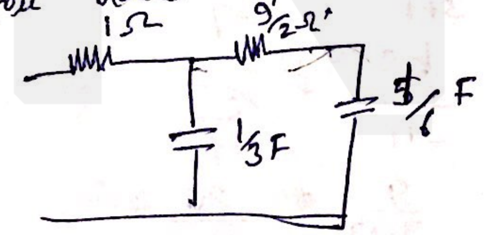
$R_1 = 1\Omega$

$Y_2 = Cs = \frac{1}{3}F$

$R_3 = \frac{9}{2}\Omega$

$Y_4 = \frac{s}{6} = \frac{1}{6}F$

Therefore ladder n/w.



Cauer - II form:-

$$Z(s) = \frac{s^2+5s+4}{s^2+2s}$$

Step 1:- Arrange in ascending order.

$$Z(s) = \frac{4+5s+s^2}{2s+s^2}$$

Step 2:- continued fract^n of $Z(s)$

$$
\begin{array}{l}
2s+s^2 \overline{)\, 4+5s+s^2 \,}\left(\frac{2}{s} \rightarrow z_1 \right. \\
\quad\ \ 4+2s \\
\quad\ \ \overline{\qquad\quad} \\
\qquad 3s+s^2 \overline{)\, 2s+s^2 \,}\left(\frac{2}{3} \rightarrow Y_2 \right. \\
\qquad\qquad\quad 2s+\frac{2}{3}s^2 \\
\qquad\qquad\quad \overline{\qquad\quad} \\
\qquad\qquad\quad \frac{1}{3}s^2 \overline{)\, 3s+s^2 \,}\left(\frac{9}{s} \rightarrow z_3 \right. \\
\qquad\qquad\qquad\quad 3s \\
\qquad\qquad\qquad\quad \overline{\qquad} \\
\qquad\qquad\qquad\quad s^2 \overline{)\, \frac{1}{3}s^2 \,}\left(\frac{1}{3} \rightarrow Y_4 \right. \\
\qquad\qquad\qquad\qquad \frac{1}{3}s^2 \\
\qquad\qquad\qquad\qquad \overline{\quad\times\quad}
\end{array}
$$

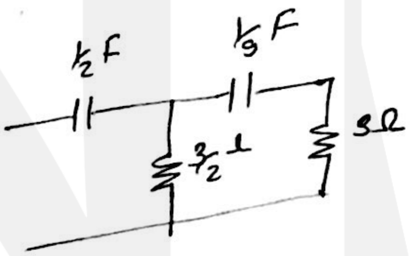
$z_1 = \frac{2}{s} \Rightarrow C_1 = \frac{1}{2}F$

$Y_2 = \frac{2}{3} \Rightarrow R_2 = \frac{3}{2}\Omega$ $\left[\begin{array}{l} Z_1 = R \\ Y_1 = \frac{1}{R} \end{array} \right]$

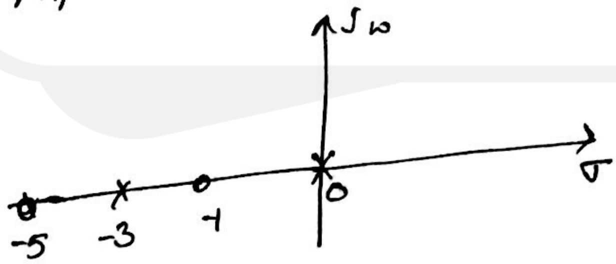
$z_3 = \frac{9}{s} \Rightarrow C_3 = \frac{1}{9}F$

$Y_4 = \frac{1}{3} \Rightarrow R_4 = 3\Omega$

Ladder structure:-



Q) An impedence func^n has pole-zero pattern. If $z(-2) = 3$. Synthesize the impedence in foster-second form.



From pole zero pattern;

$$Z(s) = \frac{k(s-z_1)(s-z_2)}{(s-P_1)(s-P_2)}$$

$z_1 = -1 \qquad P_1 = 0$

$z_2 = -5 \qquad P_2 = -3$

$$z(s) = \frac{k(s+1)(s+5)}{s(s+3)}$$

For value of k; $z(-2) = 3$

$$z(-2) = \frac{k(-2+1)(-2+5)}{-2(-2+3)}$$

$$3 = \frac{k \times -1 \times 3}{-2 \times 1}$$

$$3 = \frac{k \times \cancel{-}3}{\cancel{-}2}$$

$$\boxed{k = 2}$$

$$z(s) = \frac{2(s+1)(s+5)}{s(s+3)}$$

Step 1:- check LC, RC, and RL immittance

By looking at pole-zero diag; first singularity is pole and interlaced with zero.

$\therefore$ It is RC immitance func^n as all conditions are satisfied [check all three condition for RC immitance]

Step 2:- convert it into $Y(s)$

$$\therefore Y(s) = \frac{s(s+3)}{2(s+1)(s+5)}$$

$$\frac{Y(s)}{s} = \frac{(s+3)}{2(s+1)(s+5)}$$

Step 3:- partial fract^n of $\frac{Y(s)}{s} = \frac{(s+3)}{2(s+1)(s+5)}$

$$\therefore \frac{Y(s)}{s} = \frac{A}{s+1} + \frac{B}{s+5}$$

on solving $A = 1/4$, $B = 1/4$

$$\frac{Y(s)}{s} = \frac{1}{4(s+1)} + \frac{1}{4(s+5)}$$

$$Y(s) = \frac{s}{4(s+1)} + \frac{s}{4(s+5)}$$

$$\frac{Y(s)}{s} = \frac{1}{4}\left[\frac{A}{s+1} + \frac{B}{s+5}\right]$$

$$\frac{3(s+3)}{(s+1)(s+5)} = \frac{A(s+5) + B(s+1)}{(s+1)(s+5)}$$

$$3(s+3) = (A+B)s + (5A+B)$$

$A + B = 3$ — (I)

$5A + B = 3$ — (II)

$\cancel{-4A = 3}$

$\cancel{A = -\frac{3}{4}}$

$\cancel{A + B = 3}$

$\cancel{-\frac{3}{4} + B = 3 \Rightarrow B = \frac{3+3}{4} = \frac{15}{4}}$

on solving eq^n (I) & (II)

$A = \frac{1}{2}$

$B = \frac{1}{2}$

$$\cancel{\frac{Y(s)}{s} = \frac{1}{2}\left[\frac{-3}{4(s+1)} + \frac{15}{4(s+5)}\right]}$$

$$\frac{Y(s)}{s} = \frac{1}{2}\left[\frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{2}}{s+5}\right]$$

$$\Rightarrow Y(s) = \frac{\frac{1}{4}s}{s+1} + \frac{\frac{1}{4}s}{s+5}$$

$$\Rightarrow Y(s) = \frac{1}{\left(4 + \frac{4}{s}\right)} + \frac{1}{\left(4 + \frac{20}{s}\right)}$$

$$Y(s) = Y_1 + Y_2$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{4 + \frac{4}{s}}$$

$$Z_1 = 4 + \frac{4}{s} \Rightarrow R_1 = 4\Omega$$

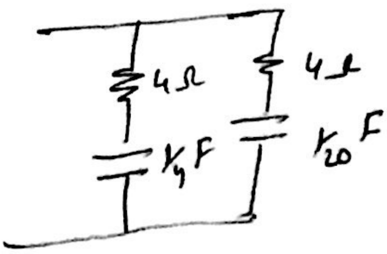
$$C_1 = \frac{1}{4}F$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{4 + \frac{20}{s}}$$

$$Z_2 = 4 + \frac{20}{s}$$

$$R_2 = 4\Omega$$

$$C_2 = \frac{1}{20}F$$



$$Y(s) = \frac{1}{\left(4+\frac{4}{s}\right)} + \frac{1}{\left(4+\frac{20}{s}\right)}$$

$$Y(s) = Y_1 + Y_2$$

$$Y_1 = \frac{1}{4+\frac{4}{s}} = \frac{1}{Z_1}$$

$$Z_1 = 4+\frac{4}{s}$$

$$Y_2 = \frac{1}{4+\frac{20}{s}} = \frac{1}{Z_2}$$

$$Z_2 = 4+\frac{20}{s}$$

$$\frac{\frac{1}{4}s}{s+1}$$

$$\frac{\frac{1}{4}s \times 1}{(s+1)}$$

$$\frac{Y(s)}{s} = \frac{s(s+3)}{2(s+1)(s+5)}$$

$$\frac{A}{s+1} + \frac{B}{s+5}$$

$$s^2+3s = A(s+5)+B(s+1)$$

$$s^2+3s = (A+B)s + 5A+B$$

$$A+B = 3$$

$$5A+B = 0$$

$$-4A = 3 \Rightarrow A = -\frac{3}{4}$$

$$\frac{\frac{1}{4}s}{s+1} = \frac{\frac{4}{s}\cdot\frac{1}{4}s}{\frac{4}{s}(s+1)} = \frac{1}{4+\frac{4}{s}} = \frac{1}{Z_1}$$

$$Y = \frac{1}{Z_1} =$$

$$Z_1 = 4+\frac{4}{s}$$

$$\frac{1}{Cs} = \frac{1}{4}F$$

$$R = 4\Omega$$

$$\frac{\frac{1}{4}s}{s+1}$$

$$\frac{\frac{4}{s}\times\frac{1}{4}s}{\frac{4}{s}(s+1)}$$

$$\left(4+\frac{4}{s}\right)$$

Filter:-

→ A filter is electronic device that can remove specific range of freq. from signal. Resonant circuit in AC select narrow band of freq. and reject other. Reactive n/w are called filters

→ Filter n/w widely used in communication system to seperate voice channel in carrier freq in telephone ckt.

classification of filter:-

① Active Filter

② Passive Filter

→ Passive filter:- Made up of entirely passive component such as resistors, capacitor, inductors.

→ Active filter:- it is made up of active component such as op-amp in order to filter the signal.

→ Active filter may contain active as well as passive component

→ passive filter can be used at high freq. by using inductors For Active filter freq. range is dependent on bandwidth of amplifiers. compared to active filter properties of passive filter may changes when load is changed.

Parameter of Filters:-

① characteristic Impedence (Z_c or Z_0):-

The characteristic impedence of a filter must be chosen such that the filter may fit into a given line or between two type of equipment.

② Pass Band:-

Band in which filter have to pass all frequencies without reduction in magnitude are refered as pass band

③ Stop Band:-

Band in which filter have to attenuate frequencies are referred as stop band.

④ cut off freq (f_c):-

The frequency which seperates the pass band and stop band is defined as cut off freq.

⑤ Attenuation:- It is a loss of signal.

→ It can be expressed in decibels (dB) or Nepers or Bels

$$\text{Attenuation in dB} = 10\log_{10}\frac{P_i}{P_o}$$

$$= 20\log_{10}\frac{V_i}{V_o}$$

$$= 20\log_{10}\frac{I_i}{I_o}$$

$$\text{Attenuation in Nepers} = \frac{1}{2}\log_e \frac{P_i}{P_o}$$

$$= \log_e \frac{V_i}{V_o}$$

$$= \log_e \frac{I_i}{I_o}$$

$$\text{Attenuation in Bels} = \log_{10}\frac{P_i}{P_o}$$

$$= 2\log_{10}\frac{V_i}{V_o}$$

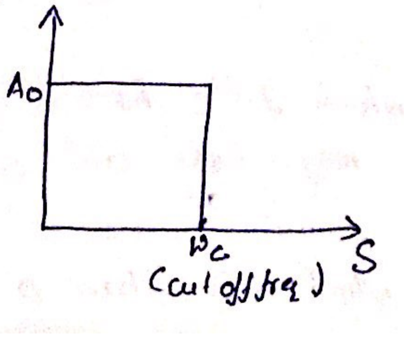
$$= 2\log_{10}\frac{I_i}{I_o}$$

Attenuation in dB = 8.686 × Attenuation in Nepers

= 10 × Attenuation in Bels.

Classification of Filters:-

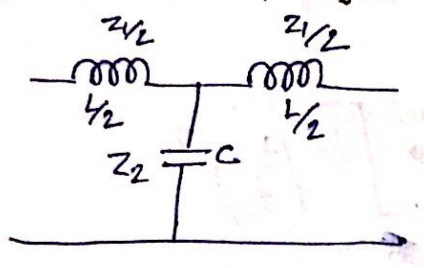
① Low Pass filter:-



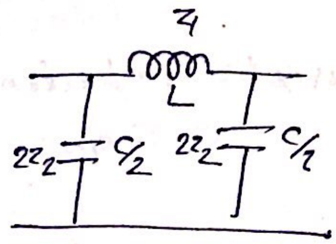
characteristic of LPF

LPF reject all freq. above cut off freq 'f_c' or 'ω_c'. thus the pass band for LPF is 0 to f_c and stop band is the freq. range above 'f_c'.

T-n/w and π-n/w for LPF:-



T-n/w



π-n/w.

characteristic parameter for LPF:-

① Design impedence or load resistance;

$$\cancel{Z_1 Z_2 = R_0 = \sqrt{\tfrac{L}{C}}}$$

$$Z_1 Z_2 = k^2 = \frac{L}{C} = R_0^2$$

$$\boxed{R_0 = \sqrt{L/C}}$$

where Z_1 = Series impedence
Z_2 = shunt "

② cut off freq:-

$$\omega_c = \frac{2}{\sqrt{LC}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

③ Attenuation constant:-

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right) \quad [\text{stop band}]$$

$$\alpha = 0 \quad [\text{pass band}]$$

④ Phase constant:-

$$\beta = \pi \quad [\text{stop band}]$$

$$\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right) \quad [\text{pass band}]$$

⑤ Filter component value:-

$$Z_1 Z_2 = k^2 \Rightarrow R_0 = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

$$\boxed{L = \frac{K}{\pi f_c} \qquad C = \frac{1}{\pi f_c K}}$$

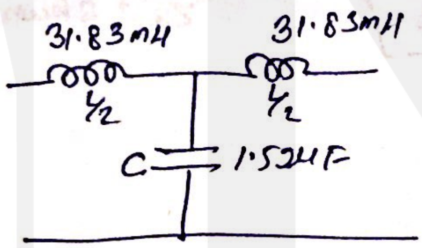
(b) characteristic impedance:

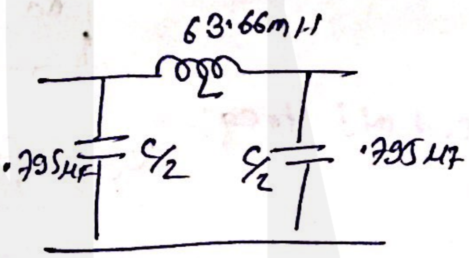
For T- n/w; $Z_{0T} = R_0 \sqrt{1-\left(\frac{f}{f_c}\right)^2}$

For π- N/w $Z_{0\pi} = \dfrac{R_0}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$

Q) Design a LPF for both 'T' and π - n/w having cut off freq. of 1 kHz to operate with terminated load resistance of 2000 Ω and find the freq. at which filter offers attenuat^{n} of 19.1 dB.

Sol^{n}:-





$f_c = 1 \times 10^3$ Hz

$R_0 = 2000\,\Omega = K$

$L = \dfrac{K}{\pi f_c} = \dfrac{2000}{\pi \times 10^3} = 63.66\,mH$

$C = \dfrac{1}{\pi f_c K} = \dfrac{1}{\pi \times 10^3 \times 2000} = 1.59\,\mu F$

Attenuat^{n} in dB = 19.1

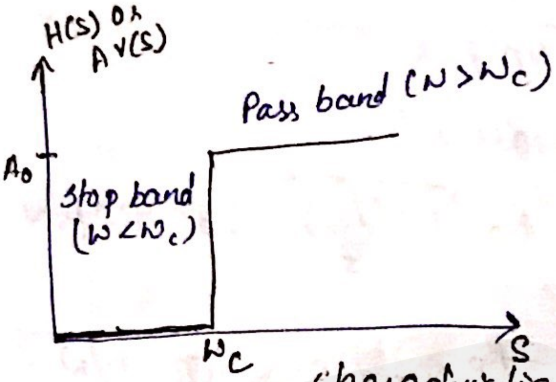
" " nepers = $\dfrac{19.1}{8.686} = 2.2$ nepers

$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$

$2.2 = 2\cosh^{-1}\left(\frac{f}{1\,kHz}\right) \Rightarrow f = 1.67\,kHz$

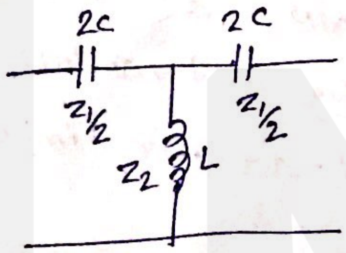
High Pass filter:-

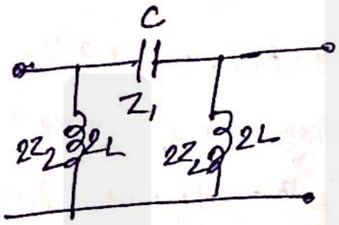
HPF reject all frequencies below cut off freq. thus pass band and stop band of HP filter are the freq. range above f_c and below f_c.



characteristic of HPF

characteristic Parameter :-





① Design impedance or load resistance:-

$$K^2 = R_0^2 = L/C$$

$$R_0 = \sqrt{L/C}$$

② Cut off freq :-

$$\omega_c = \frac{1}{2\pi\sqrt{LC}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

③ Attenuation constant:-

$$\alpha = 0 \rightarrow \text{[Pass band]}$$

$$\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right) \quad \text{[stop band]}$$

④ Phase constant :-

$$\beta = \pi \quad \text{[Pass band]}$$

$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right) \quad \text{[stop band]}$$

(5) Filter component value:-

$$L = \frac{k}{4\pi f_c}$$

$$C = \frac{1}{4\pi f_c k}$$

(6) characteristic impedence:-

$$Z_{OT} = R_0\sqrt{1-\left(\frac{f_c}{f}\right)^2} \rightarrow T\text{-}n/w$$

$$Z_{O\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \rightarrow \pi\text{-}n/w.$$

Q) Find the component value of π-n/w constant k ~~pass~~ high pass filter having cut off freq. of 8 KHz and nominal characteristic impedence of 600 Ω. Hence find its characteristic impedence and phase constant at f = 12KHz.

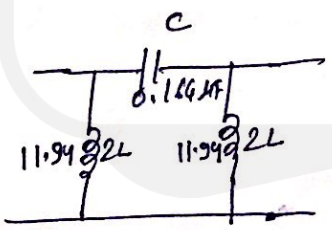
Sol^n:-

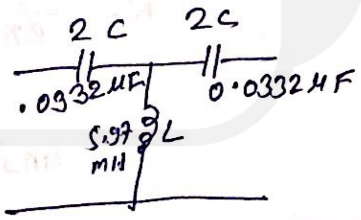
$$R_0 = k = \sqrt{\frac{L}{C}} = 600$$

$$f_c = 8000Hz$$

$$L = \frac{k}{4\pi f_c} = \frac{600}{4\pi \times 8000} = 5.97\ mH$$

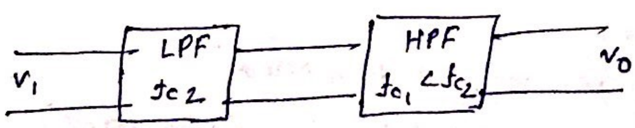
$$C = \frac{1}{4\pi k f_c} = \frac{1}{4\pi \times 600 \times 8000} = 0.0166\ \mu F$$





Band Pass Filter:-

Band Pass filter is generally obtained by series or cascade connection of LPF and HPF



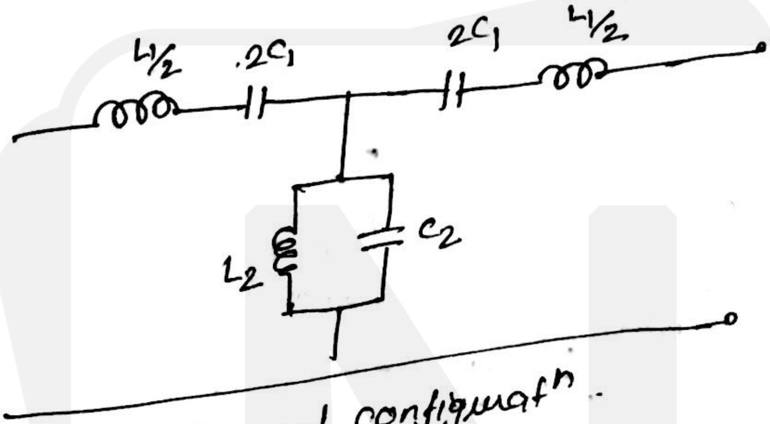
Characteristic of BPF

Pass band for BPF → $\omega_1 \to \omega_2$

$f_{c_1} \to f_{c_2}$

Stop band for BPF = $0 \to \omega_1$, $0 \to f_{c_1}$

$\omega_2 \to \infty$ $f_{c_2} \to \infty$



General configuratn.

characteristic Parameter:-

$$R_0 = K_0 = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}$$

→ cut off freq; $f_0 = \sqrt{f_{c_1} f_{c_2}}$

cut off freq for BPF is equal to geometric mean of two cut off freq.

→ Filter component values:-

$$C_1 = \frac{f_{c_2} - f_{c_1}}{4\pi R_0 f_{c_1} f_{c_2}}$$

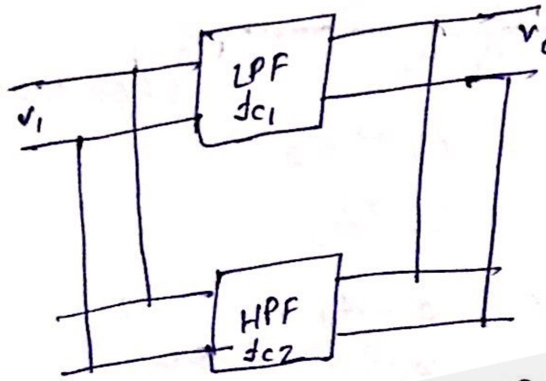
$$C_2 = \frac{1}{\pi R_0 (f_{c_2} - f_{c_1})}$$

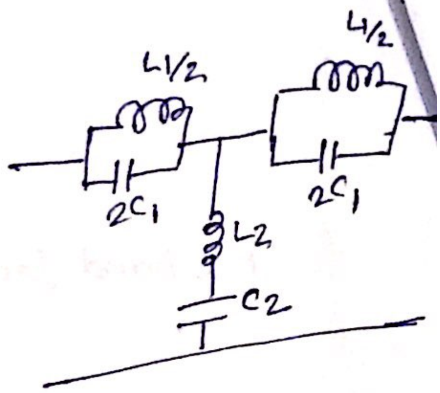
$$L_1 = \frac{R_0}{\pi (f_{c_2} - f_{c_1})}$$

$$L_2 = \frac{(f_{c_2} - f_{c_1}) R_0}{4\pi f_{c_1} f_{c_2}}$$

Band Stop filter:-

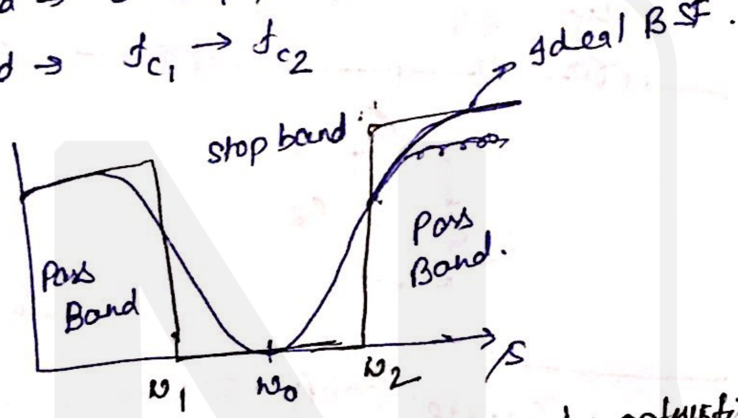
Band stop filter is obtained by parallel connection of LPF and HPF.





Pass band → $0 \to f_{c_1}$, $f_{c_2} \to \infty$

Stop band → $f_{c_1} \to f_{c_2}$



$\Delta\omega = \omega_2 - \omega_1$

characteristic of BSF

Characteristic Parameter:-

(1) Design impedance or load impedance;

$$K = R_0 = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}}$$

(2) cutt off freq:-

$$f_0 = \sqrt{f_{c_1} f_{c_2}}$$

(3) Filter component values;

$$L_1 = \frac{R_0 (f_{c_2} - f_{c_1})}{\pi f_{c_1} f_{c_2}} \qquad C_1 = \frac{1}{4\pi R_0 (f_{c_2} - f_{c_1})}$$

$$L_2 = \frac{R_0}{4\pi (f_{c_2} - f_{c_1})} \qquad C_2 = \frac{f_{c_2} - f_{c_1}}{\pi R_0 f_{c_1} f_{c_2}}$$

Q: Design a Constant K - Band Pass filter with cut off freq. (5)
of 3KHz and 7.5KHz and nominal characteristic impedance of 900 Ω

$R_0 = 900\ \Omega$

$f_{c_1} = 3KHz$

$f_{c_2} = 7.5\ KHz$

$$L_1 = \frac{R_0}{\pi(f_{c_2} - f_{c_1})} = \frac{900}{\pi(7.5-3)\times 10^3} = 63.66\,mH$$

$$C_1 = \frac{f_{c_2} - f_{c_1}}{4\pi R_0 f_{c_1} f_{c_2}} = \frac{(7.5-3)\times 10^3}{4\pi \times 900 \times 7.5\times 3\times 10^6} = 0.017\,\mu F$$

$$L_2 = \frac{(f_{c_2} - f_{c_1}) R_0}{4\pi f_{c_1} f_{c_2}} = \frac{(7.5-3)\times 10^3 \times 900}{4\pi \times 7.5\times 3\times 10^6} = 14.32\,mH$$

$$C_2 = \frac{1}{\pi R_0 (f_{c_2} - f_{c_1})}$$

$$= \frac{1}{\pi \times 900(7.5-3)\times 10^3} = 0.078\,\mu F$$