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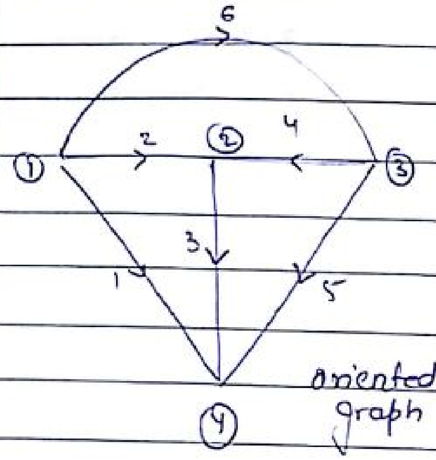
UNIT – 3

Que. With the help of oriented graph. Find twig and link matrix.

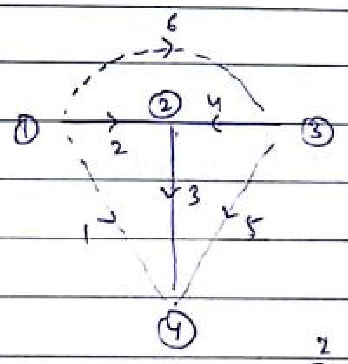


oriented graph

$$[A] = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

If the arrow from node is
- in = −1
- out = 1

$$\text{Reduced } [A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$



Tree

Links: 1, 5, 6

Twigs: 2, 3, 4

$$\text{Reduced } [A] = \begin{array}{c} \begin{array}{cccccc} 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

Twigs (columns 2, 3, 4) Links (columns 1, 5, 6)

$$\text{Twig Matrix } [T] = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \qquad \text{Link Matrix } [L] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

The determinant of Link Matrix is 1 or −1

$$\text{Det}[T] = 1(-1) - 0 + 0$$
$$= -1$$

* <u>No. of Trees from a Graph</u>

$$\text{Determinant}\left[\,\left|[B]\cdot[B]^T\right|\,\right]$$

$$= \left|\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}\right|$$

$$= \left|\begin{bmatrix} 1+1+1 & -1 & -1 \\ -1 & 1+1+1 & -1 \\ -1 & -1 & 1+1+1 \end{bmatrix}\right|$$

$$= \left|\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}\right| = 3(9-1)+1(-3-1)-1(1+3)$$
$$= 24-4-4$$
$$= 16$$

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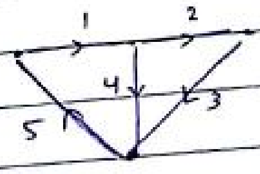
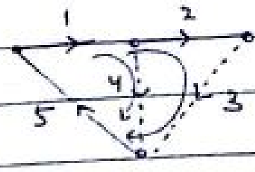
* Fundamental Tie-Set Matrix

A fundamental loop or tie-set matrix of a graph w.r.t. a tree is a loop for by only one link associated with other twigs





(L_1) loop 1 : (1, 4, 5)

(L_2) loop 2 : (1, 2, 3, 5)

If the arrows
- in the direction of loop = 1
- opp. to the direction of loop = -1

Fundamental loop matrix

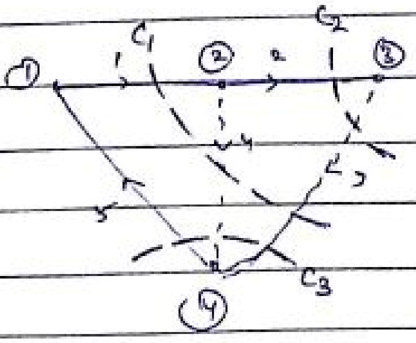
$$[D] = \begin{array}{c} \\ L_1 \\ L_2 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

* Cut-Set Matrix

It consist of one and only one branch of the network tree together with any links which must be cut to divide the network into two parts.

Assumptions:

1. Cut is always $(n-1)$ in a tree [n: no. of nodes]
2. Cut should be like this only, one node is included
3. Cut should be like this only, one solid line (twig) is included.

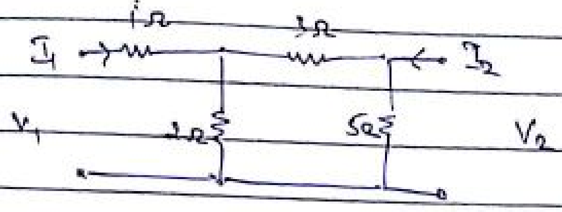


$$[C] = \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} \begin{bmatrix} 1 & -1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

If arrow from the arc is
- outward = -1
- inward = 1

Two Port Network Analysis

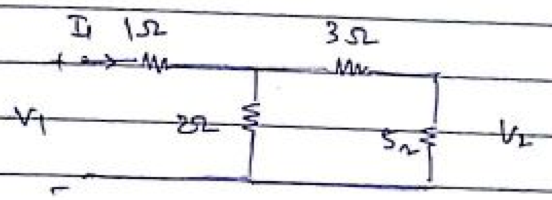
- Z → Parameter
- Y → Parameter
- T (ABCD) - Parameter
- h - parameter



Z-PARAMETER

$I_2 = 0$	$I_1 = 0$
$Z_{11} = \frac{V_1}{I_1}$	$Z_{12} = \frac{V_1}{I_2}$
$Z_{21} = \frac{V_2}{I_1}$	$Z_{22} = \frac{V_2}{I_2}$

Case I: $I_2 = 0$



$$R_{ev} = \left(\frac{16}{10} + 1\right) = 2.6\ \Omega$$

$$V_1 = R_{ev} \cdot I_1$$

$$\Rightarrow V_1 = 2.6 \cdot I_1$$

$$\frac{V_1}{I_1} = Z_{11} = 2.6\ \Omega$$

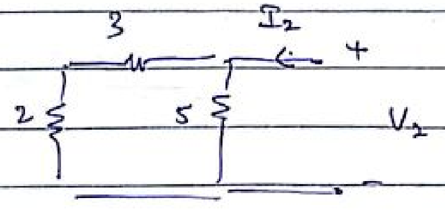
$V_2 = 5 \cdot I_{5\Omega}$

$\Rightarrow 5\, I_{5\Omega} = I_1 \cdot \frac{2}{10}$

$\Rightarrow V_2 = I_1$

$\Rightarrow \frac{V_2}{I_1} = Z_{21} = 1\Omega$

Case II : $I_1 = 0$



$R_{eq} = \frac{5 \cdot 5}{10} = 2.5\,\Omega$

$V_2 = R_{eq} \cdot I_2$

$\frac{V_2}{I_2} = Z_{22} = 2.5\,\Omega$

$V_1 = 2 \cdot I_{2\Omega}$

$I_{2\Omega} = I_2 \cdot \frac{5}{10}$

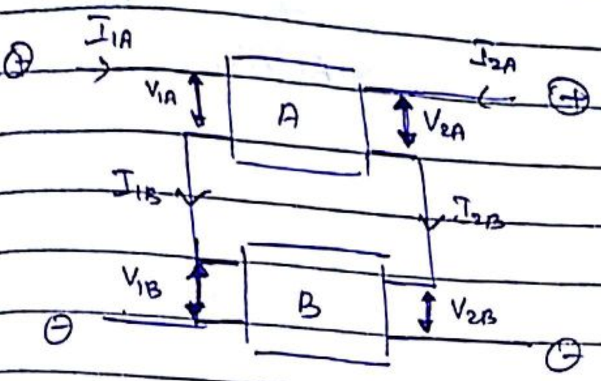
$V_1 = 2 \cdot \frac{I_2}{2}$

$\frac{V_1}{I_2} = Z_{12} = 1\,\Omega$

Inter Connection of 2-Port Network

series – Z Parallel – Y Cascade – T

① Series Connection



For Network A

$$V_{1A} = Z_{11A} I_{1A} + Z_{12A} \cdot I_{2A}$$
$$V_{2A} = Z_{21A} \cdot I_{1A} + Z_{22A} \cdot I_{2A}$$

For Network B

$$V_{1B} = Z_{11B} \cdot I_{1B} + Z_{12B} \cdot I_{2B}$$
$$V_{2B} = Z_{21B} \cdot I_{1B} + Z_{22B} \cdot I_{2B}$$

Refer to given configuration, i.e series connection

If
$$I_1 = I_{1A} = I_{1B}$$
$$I_2 = I_{2A} = I_{2B}$$

$$V_1 = V_{1A} + V_{1B}$$
$$V_2 = V_{2A} + V_{2B}$$

$$\therefore V_1 = (Z_{11A} \cdot I_{1A} + Z_{12A} \cdot I_{2A}) + (Z_{11B} \cdot I_{1B} + Z_{12B} \cdot I_{2B})$$
$$= (Z_{11A} + Z_{11B}) I_1 + (Z_{12A} + Z_{12B}) I_2$$

Similarly

$$V_2 = (Z_{21A} + Z_{21B}) I_1 + (Z_{22A} + Z_{22B}) I_2$$

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Addition $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11A} + Z_{11B} & Z_{12A} + Z_{12B} \\ Z_{21A} + Z_{21B} & Z_{22A} + Z_{22B} \end{bmatrix}$

Definitions

- Linear element
- Active Elements
- Unilateral Ckt.
- Linear ckt.
- Non Linear Element
- Passive Elements
- Bilateral

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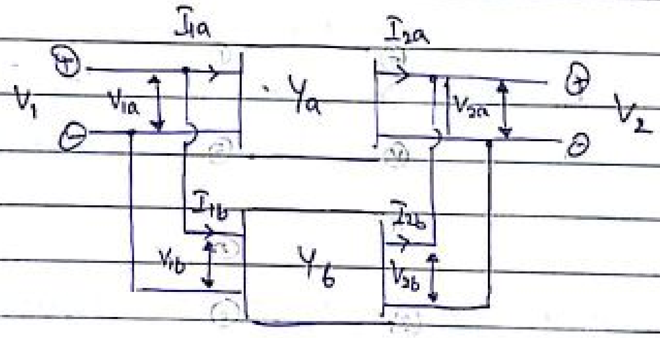
② Parallel connection



$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$V_1 = V_{1a} = V_{2b}$

$V_2 = V_{2a} = V_{2b}$

$I_1 = I_{1a} + I_{1b}$

$I_2 = I_{2a} + I_{2b}$

$I_{1a} = Y_{11a} \cdot V_{1a} + Y_{12a} \cdot V_{2a}$ —— ①

$I_{2a} = Y_{21a} \cdot V_{1a} + Y_{22a} V_{2a}$ —— ②

$$I_{1b} = y_{11b}V_{1b} + y_{12b}V_{2b} \quad \text{——— (3)}$$
$$I_{2b} = y_{21b}V_{1b} + y_{22b}V_{2b} \quad \text{——— (4)}$$

Adding (1) & (3)

$$\Rightarrow I_1 = (y_{11a} + y_{11b})V_1 + (y_{12a} + y_{12b})V_2 \quad \text{——— (5)}$$

Adding (2) & (4)

$$\Rightarrow I_2 = (y_{21a} + y_{21b})V_1 + (y_{22a} + y_{22b})V_2 \quad \text{——— (6)}$$

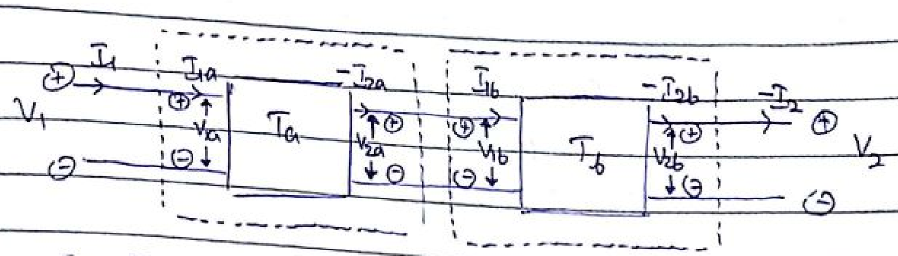
On comparing

additn

$$\begin{bmatrix} (y_{11a} + y_{11b}) & (y_{12a} + y_{12b}) \\ (y_{21a} + y_{21b}) & (y_{22a} + y_{22b}) \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = [y]$$

Imp.

(3) Cascade Connection



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \quad \text{——— (1)}$$

cases : $I_2 = 0$
$V_2 = 0$

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix} \quad \text{——— (2)}$$

$I_1 = I_{1a}$ $I_{1b} = -I_{2b}$ $I_2 = I_{2b}$

$V_1 = V_{1a}$ $V_{1b} = V_{2a}$ $V_2 = V_{2b}$

Using (1)

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} = \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

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Substituting ②

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Multiplication

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$$[T] = [T_a][T_b]$$

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Reciprocity

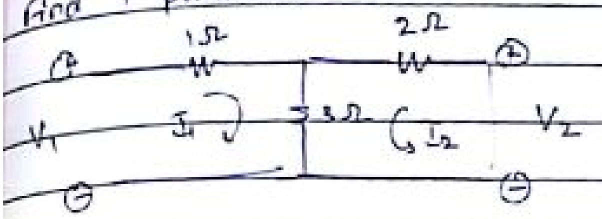
A two port network is said to be reciprocal if the ratio of the excitation to response is invariant to an interchange of the pos.ⁿ of the excitation and response in the ckt.

Symmetry

A two port network is said to be symmetrical if the ports can be interchanged without changing the port, ~~vol~~ voltages & current

Parameter	Relation b/w V and I	Condⁿ for symmetry	Condⁿ for reciprocity
Z	$(V_1, V_2) = F(I_1, I_2)$	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
Y	$(I_1, I_2) = F(V_1, V_2)$	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
T A, B, C, D	$(V_1, I_1) = F(V_2, I_2)$	$A = D$	$AD - BC = 1$
h	$(V_1, I_2) = F(V_2, I_1)$	$h_{11}h_{22} - h_{12}h_{21} = 1$	$h_{12} = -h_{21}$

Find Y parameter



$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$

$Y_{11} = \dfrac{I_1}{V_1}$ $\quad Y_{21} = \dfrac{I_2}{V_1}$

$Y_{12} = \dfrac{I_1}{V_2}$ $\quad Y_{22} = \dfrac{I_2}{V_2}$

Case I: $V_2 = 0$

$V_1 = I_1 + 3(I_1 + I_2)$

$0 = 2 \cdot I_2 + 3(I_1 + I_2)$

$\Rightarrow -3I_1 = 5I_2$

$\therefore V_1 = 4I_1 - \dfrac{9}{5} I_1$

$= \dfrac{11}{5} I_1$

$Y_{11} = \dfrac{I_1}{V_1} = \dfrac{5}{11}\ \Omega^{-1}$

$V_1 = 4 \cdot \left(-\dfrac{5}{3} I_2\right) + 3 I_2$

$= \dfrac{-11}{3} I_2$

$Y_{21} = \dfrac{I_2}{V_1} = \dfrac{-3}{11}\ \Omega^{-1}$

Case II: $V_1 = 0$

$0 = I_1 + 3(I_1 + I_2)$

$\Rightarrow -4I_1 = 3I_2$

$V_2 = 2I_2 + 3(I_1 + I_2)$

$= 5I_2 + 3I_1$

$$\therefore V_2 = 5I_2 + 3\left(\frac{-3}{4}I_2\right)$$

$$= \left(\frac{20-9}{4}\right)I_2$$

$$= \frac{11}{4}I_2$$

$$\Rightarrow \frac{I_2}{V_2} = \frac{4}{11}\Omega^{-1} = Y_{22}$$

$$V_2 = 5\left(\frac{-4}{3}I_1\right) + 3I_1$$

$$= \left(\frac{-20+9}{3}\right)I_1$$

$$= \frac{-11}{3}I_1$$

$$\Rightarrow \frac{I_1}{V_2} = Y_{12} = \frac{-3}{11}\Omega^{-1}$$

$Y_{11} \neq Y_{22}$ $\therefore$ The given port network is not symmetric

$Y_{21} = Y_{12}$ $\therefore$ It's reciprocal

d) ABCD Parameter

Case I $I_2 = 0$

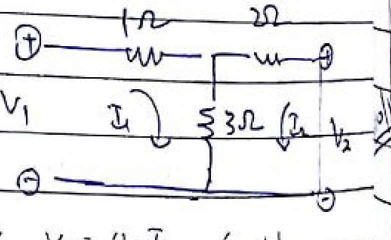
$$A = \frac{V_1}{V_2}$$

$$C = \frac{I_1}{V_2}$$

Case 2 $V_2 = 0$

$$B = \frac{V_1}{-I_2}$$

$$D = \frac{I_1}{-I_2}$$



$V_1 = \frac{11}{5} \cdot I_1$ $\frac{1}{5} + 1$

Case I : $I_1 = 0$

$V_2 = I_2 4$

$V_2 = 3I_2$ +

$$\Rightarrow C = \frac{I_1}{V_2} = \frac{1}{3}\ \Omega^{-1}$$

$$V_1 = \frac{V_2}{3} \cdot 4$$

$$\Rightarrow A = \frac{V_1}{V_2} = \frac{4}{3}$$

Case II : $V_2 = 0$

$$\cancel{V_1 = 1 I_1 + 1\, 3(I_1 + I_2)}$$

$$V_1 = I_1 + 3(I_1 + I_2)$$

$$= 4I_1 + 3I_2$$

$$0 = 2I_2 + 3(I_1 + I_2)$$

$$5I_2 = -3I_1$$

$$\frac{I_1}{-I_2} = D = \frac{5}{3}$$

$$V_1 = 4\cdot\left(\frac{-5}{3} I_2\right) + 3I_2$$

$$= \frac{-11}{3} I_2$$

$$\frac{V_1}{-I_2} = \frac{11}{3}\ \Omega$$

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Network Functions

	$Z(s)$	$Y(s)$
R	R	$1/R = G$
L	sL	$1/sL$
C	$1/sC$	sC

Immitance → Impedance
→ Admittance

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$$Z_{21}(S) = \frac{V_2(S)}{I_1(S)}$$ Forward Impedance Transfer funct^n

$$Z_{12}(S) = \frac{V_1(S)}{I_2(S)}$$ Reverse Impedance T.F

$$Y_{21}(S) = \frac{I_2(S)}{V_1(S)}$$ Fwd Admittance T.F.

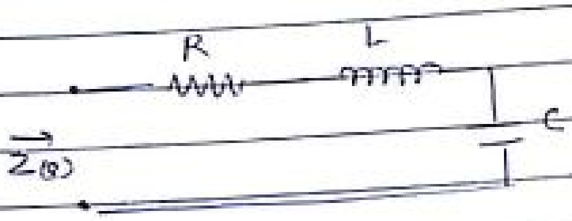
$$Y_{12}(S) = \frac{I_1(S)}{V_2(S)}$$ Rev. Admittance T.F.

$$\beta_{21}(S) = \frac{V_2(S)}{V_1(S)}$$ Fwd. Voltage TF

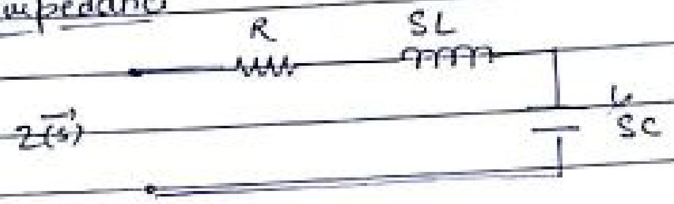
$$\beta_{12}(S) = \frac{V_1(S)}{V_2(S)}$$ Rev. Voltage T.F.

$$\alpha_{21}(S) = \frac{I_2(S)}{I_1(S)}$$ Fwd current T.F

$$\alpha_{12}(S) = \frac{I_1(S)}{I_2(S)}$$ Rev. Current T.F



Impedance



Admittance

$$Z(s) = Z_1(s) + Z_2(s) + Z_3(s)$$

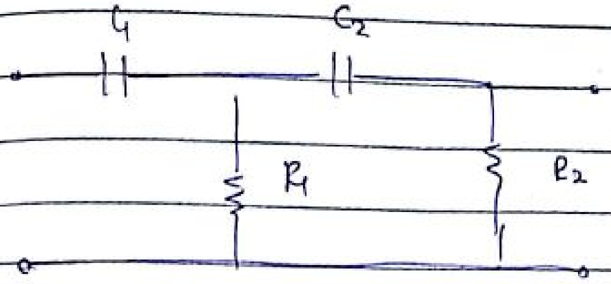
$$= R + SL + \frac{1}{SC}$$

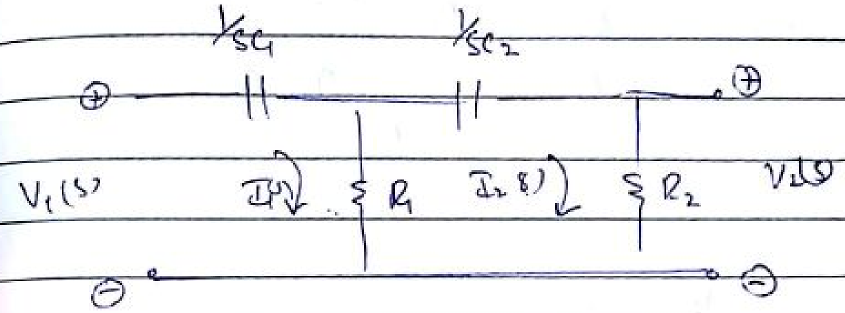
$$= \frac{RSC + S^2LC + 1}{SC}$$

$$= \frac{LC\left(S^2 + \left(\frac{R}{L}\right)S + \frac{1}{LC}\right)}{SC}$$

$$= \frac{L}{S}\left(S^2 + \left(\frac{R}{L}\right)S + \frac{1}{LC}\right)$$

1) Find the exp. for voltage transfer function of given network





loop 1

$$V_1(s) = \frac{I_1}{SC_1} + R_1\left[I_1(s) - I_2(s)\right]$$

$$V_1(s) = I_1(s)\left(R_1 + \frac{1}{SC_1}\right) - I_2(s)R_1 \quad \text{——} (1)$$

loop 2

$$\frac{I_2(s)}{SC_2} + R_2 \cdot I_2(s) + R_1\left[I_2(s) - I_1(s)\right] = 0$$

$$\Rightarrow I_2(s)\left(R_1 + R_2 + \frac{1}{SC_2}\right) - I_1(s) \cdot R_1 = 0 \quad \text{——} (2)$$

$$V_2(s) = R_2 \cdot I_2(s) \quad \text{——} (3)$$

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$$\beta_{21}(s) = \frac{V_2(s)}{V_1(s)} - \frac{R_2 I_2(s)}{V_1(s)}$$

CRAMMER RULE

- $a_1 x + b_1 y = c_1$
- $a_2 x + b_2 y = c_2$

$$\Delta = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

$$\Delta x = \begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix} \qquad \Delta y = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}$$

$$x = \frac{\Delta x}{\Delta} \qquad y = \frac{\Delta y}{\Delta}$$

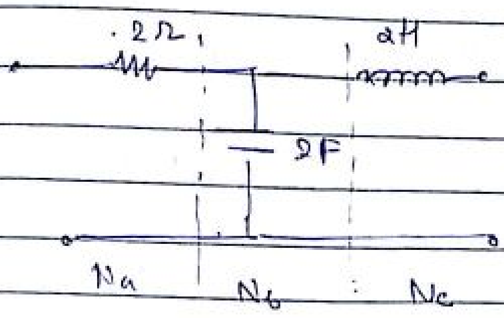
$$I_2(s) = \frac{\begin{vmatrix} R_1 + 1/sC_1 & V_1(s) \\ -R_1 & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + 1/sC_1 & -R_1 \\ -R_1 & R_1 + R_2 + \frac{1}{sC_2} \end{vmatrix}}$$

$$= \frac{V_1(s) \cdot R_1}{R_1 \cdot R_2 + \frac{R_1}{sC_2} + \frac{R_1 + R_2}{sC_1} + \frac{1}{s^2 C_1 C_2}}$$

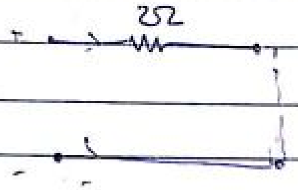
$$\therefore \beta_{21}(s) = \frac{\left(\frac{R_1 R_2 V_1(s)}{R_1 R_2 + \frac{R_1}{sC_2} + \frac{R_1 + R_2}{sC_1} + \frac{1}{s^2 C_1 C_2}} \right)}{V_1(s)}$$

$$= \left(\frac{R_1 R_2}{R_1 R_2 + \frac{R_1}{sC_2} + \frac{R_1 + R_2}{sC_1} + \frac{1}{s^2 C_1 C_2}} \right)$$

Q. Determine the transmission parameter of a T-network considering 3 sections. Assuming connected in cascade manner.



$N_a \rightarrow$



Case I : $I_2 = 0$

$A = \frac{V_1}{V_2}$ $\quad C = \frac{I_1}{V_2}$

- $V_1 = V_2$

$\Rightarrow \boxed{A = 1}$

- $I_1 = I_2 = 0$ no close loop

$\Rightarrow \boxed{C = 0\,\Omega}$

Case II : $V_2 = 0$

$B = \frac{V_1}{-I_2}$ $\quad D = \frac{I_1}{-I_2}$

$V_1 = I_1 \cdot 2$

but $I_1 = -I_2$

$\Rightarrow \boxed{B = \frac{V_1}{-I_1} = 2\,\Omega}$

$\boxed{D = 1}$

$$\therefore [T_a] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

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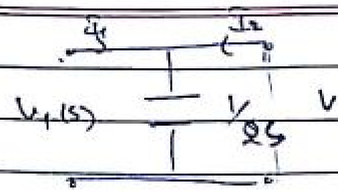
$N_b \longrightarrow$



Case I : $\underset{\sim\sim}{I_2 = 0}$

$V_1 = \frac{I_1}{2s}$ & $V_2 = \frac{I_1}{2s}$

$\boxed{C = \frac{I_1}{V_2} = 2s}\ \Omega^{-1}$

$\boxed{A = \frac{V_1}{V_2} = 1}$

case II : $\underset{\sim\sim}{V_2 = 0}$

$V_1(s) = \frac{1}{2s}(I_1 + I_2)$

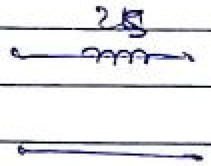
$0 = \frac{1}{2s}(I_1 + I_2)$

$\Rightarrow I_1 = -I_2$

$\boxed{D = \frac{I_1}{-I_2} = 1}$ $\boxed{B = 0}$

$$[T_B] = \begin{bmatrix} 1 & 0 \\ 2s & 1 \end{bmatrix}$$

$N_c \longrightarrow$



$$[T_C] = \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix}$$

By cascade property

$$[T] = [T_a][T_b][T_c]$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2s & 1 \end{bmatrix} \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4s & 2 \\ 2s & 1 \end{bmatrix} \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+4s & 2s(1+4s)+2 \\ 2s & 4s^2+1 \end{bmatrix}$$

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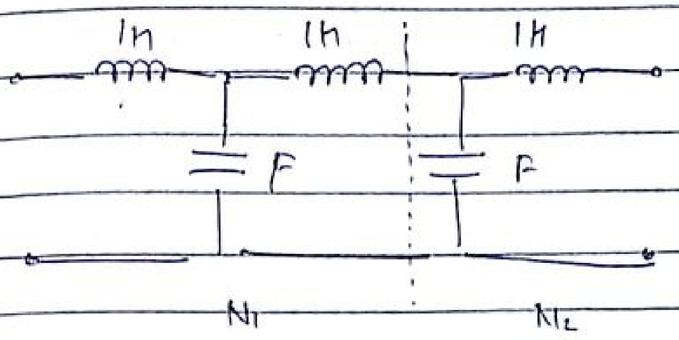
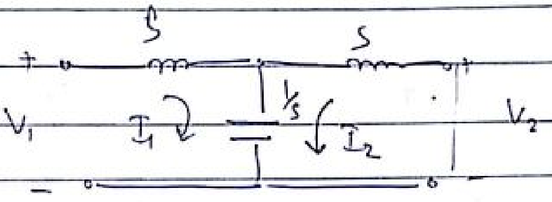
Q. Find the T parameters of the network, using the concept of interconnection of two port network in cascade.



$[T] = [T_1][T_2]$

For N1



Case I : $I_2 = 0$

$V_1 = I_1 S + \frac{1}{S} I_1$

$V_1 = I_1\left(S + \frac{1}{S}\right)$

$V_2 = I_1 \cdot \frac{1}{S}$

$\boxed{C = \frac{I_1}{V_2} = S\ \Omega^{-1}}$

$\boxed{A = \frac{V_1}{V_2} = S^2+1}$

Case II : $V_2 = 0$

$V_1 = \frac{(S^2+1)}{S}$

$V_1 = I_1 S + \frac{1}{S}(I_1 + I_2)$

$I_2 \cdot S + \frac{1}{S}(I_2 + I_1) = 0$

$\left(\frac{S^2+1}{S}\right) I_2 = -\frac{I_1}{S}$

$V_1 = -S(S^2+1) I_2 + \frac{1}{S} \cdot I_2$

$+ (S^2+1) I_2$

$\frac{V_1}{-I_2} = S(S^2+1) - \frac{1}{S} + (S^2+1)$

$= S^4 + S - 1 + S^3 + S$

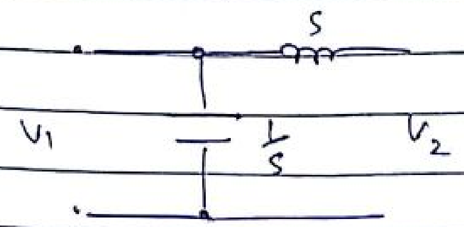
$V_1 = -\frac{(S^2+1)}{S}(S^2+1) I_2 + \frac{I_2}{S}$

$$\frac{V_1}{-I_2} = \frac{s^4+2s^2+1-1}{s}$$

$$\boxed{B = s^3 + 2s \ \Omega}$$

$$\boxed{D = \frac{I_1}{-I_2} = s^2+1}$$

For N_2



Case I $I_2 = 0$

$$V_1 = I_1 \cdot \frac{1}{s}$$

$$V_2 = \frac{I_1}{s}$$

$$\boxed{C = \frac{I_1}{V_2} = s = \Omega^{-1}}$$

$$\boxed{A = 1}$$

$$I_1 = -I_2(s^2+1)$$

$$\boxed{D = \frac{I_1}{-I_2} = s^2+1}$$

$$\cancel{V_1 = \left(\frac{-I_2}{s^2+1}\right) + I_1} \qquad V_1 = \left(\frac{-I_2(s^2+1)+I_2}{s}\right)$$

$$\boxed{B = \frac{V_1}{-I_2} = s \ \Omega}$$

Case II: $V_2 = 0$

$$V_1 = (I_1 + I_2)\frac{1}{s}$$

$$0 = \frac{sI_2}{s} + \frac{(I_1+I_2)}{s}$$

$$\Rightarrow 0 = \frac{s^2 I_2}{s} + \frac{I_1+I_2}{s}\left(\frac{s^2+1}{s}\right)$$

$$\Rightarrow I_1 = -2I_2$$

$$\cancel{\boxed{D = \frac{I_1}{-I_2} = 2}}$$

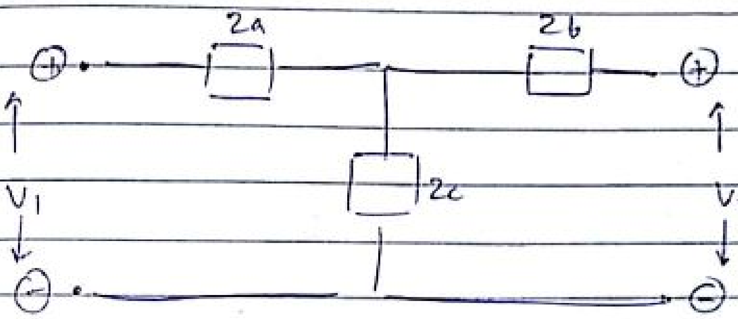
$$\cancel{B = \frac{V_1}{-I_2} = \frac{1}{2s}}$$

$$[T] = [T_1][T_2]$$

$$= \begin{bmatrix} s^2+1 & s^3+2s \\ s & s^2+1 \end{bmatrix} \begin{bmatrix} 1 & s \\ s & s^2+1 \end{bmatrix}$$

$$= \begin{bmatrix} (s^2+1+s^4+2s^2) & (s^3+s+s^5+2s^3+s^3+2s) \\ (s+s^3+s) & (s^2+s^4+2s^2+1) \end{bmatrix}$$

T-configuration 2-Port Network



Z-Parameter

$\boxed{I_2 = 0}$	$\boxed{I_1 = 0}$
$Z_{11} = \dfrac{V_1}{I_1}$	$Z_{12} = \dfrac{V_1}{I_1}$
$Z_{21} = \dfrac{V_2}{I_1}$	$Z_{22} = \dfrac{V_2}{I_2}$

when $I_2 = 0$

$$V_1 = I_1(Z_a + Z_c)$$

$$Z_{11} = \frac{V_1}{I_1} = Z_a + Z_c$$

$$V_2 = I_1 \cdot Z_c$$

$$Z_{21} = \frac{V_2}{I_1} = Z_c$$

when $I_1 = 0$

$$V_2 = I_2(Z_b + Z_c)$$

$$Z_{22} = \frac{V_2}{I_2} = Z_b + Z_c$$

$$V_1 = Z_c \cdot I_2$$

$$Z_{12} = \frac{V_1}{I_2} = Z_c$$

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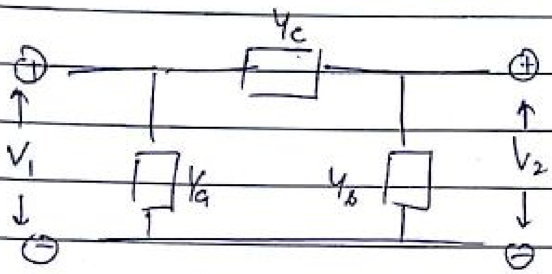
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix}$$

- $Z_{11} = Z_a + Z_c$
- $Z_{12} = Z_{21} = Z_c$
- $Z_{22} = Z_b + Z_c$
- $Z_a = Z_{11} - Z_{12}$
- $Z_b = Z_{22} - Z_{12}$
- $Z_c = Z_{21} = Z_{12}$

Π configuration 2 Port Network

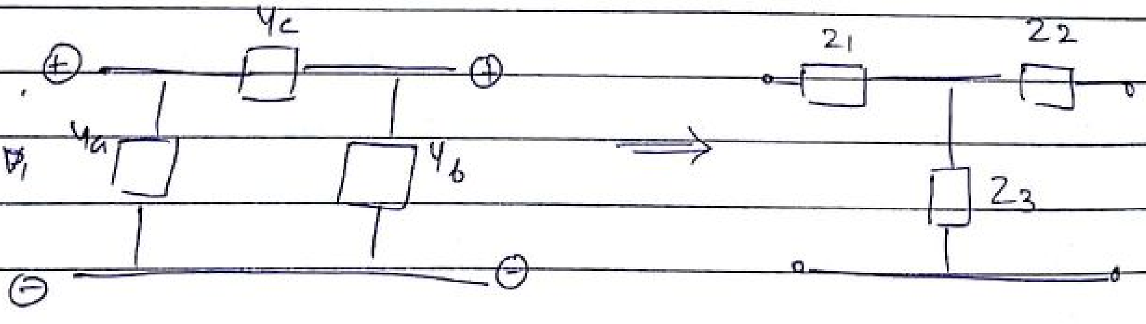


Check

$$\begin{bmatrix} Y_{11} & Y_{11} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_a + Y_c & -Y_c \\ -Y_c & Y_b + Y_c \end{bmatrix}$$

- $Y_a = Y_{11} + Y_{12}$
- $Y_b = -Y_{12} = -Y_{21}$
- $Y_c = Y_{22} + Y_{12}$

Conversion



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$$\Delta Y = Y_a Y_b + Y_b Y_c + Y_c Y_a$$

$$Z_1 = \frac{Y_b}{\Delta Y}$$

$$Z_2 = \frac{Y_a}{\Delta Y}$$

$$Z_3 = \frac{Y_c}{\Delta Y}$$

$\boxed{Z \text{ to } Y \Rightarrow T \text{ to } \pi}$

$$Y_a = \frac{Z_2}{\Delta Z} \qquad \Delta Z = Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3$$

$$Y_b = \frac{Z_1}{\Delta Z}$$

$$Y_c = \frac{Z_3}{\Delta Z}$$

Image Impedance

If we see a two port network from input end and short the O/P end, with load then the virtual load appears at I/P end.

i.e. Z_{i1}

Similarly if load is connected at I/P end then img. of that load appears at O/P end

i.e. Z_{i2}

So Z_{i1} and Z_{i2} is called image / iterative impedance

If $Z_{i1} = Z_{i2}$ then network is symmetrical

$$\boxed{Z_{img} = \sqrt{Z_{sc}\, Z_{oc}}}$$

Image Impedance

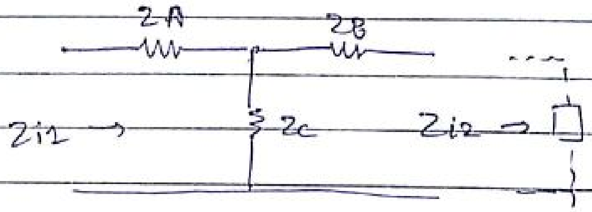

Fig.(a)

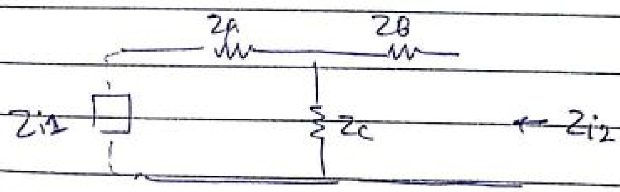

Fig (b)

Fig (a) $Z_{i1} = Z_A + \left[Z_C \,||\, (Z_B + Z_{i2}) \right]$

$$= Z_A + \frac{Z_C (Z_B + Z_{i2})}{Z_B + Z_C + Z_{i2}}$$

$$\Rightarrow Z_{i1} (Z_B + Z_C + Z_{i2}) = Z_A (Z_B + Z_C + Z_{i2}) + Z_C (Z_B + Z_{i2})$$

$$\Rightarrow Z_{i1} \cdot Z_{i2} + Z_{i1} (Z_B + Z_C) - Z_{i2} (Z_A + Z_C) = \underline{Z_A Z_B + Z_B Z_C + Z_A Z_C} = \sum Z_A \cdot Z_B \quad (1)$$

Dt. 11/10/17 B+

Pg.

CG

Q $P(s) = 4s^6 + 2s^5 + 17s^4 + 8s^3 + 16s^2 + 6s + 3$

$e(s) = 4s^6 + 17s^4 + 16s^2 + 3$

$o(s) = 2s^5 + 8s^3 + 6s$

$\frac{e(s)}{o(s)} = 2s^5 + 8s^3 + 6s \,\big)\, 4s^6 + 17s^4 + 16s^2 + 3 \,\big(\, 2s$

$- \; 4s^6 + 16s^4 + 12s^2$

$* \; s^4 + 4s^2 + 3 \,\big)\, 2s^5 + 8s^3 + 6s \,\big(\, 2s$

$2s^5 + 8s^3 + 6s$

$\times$

~~All the coeff should be +ve~~

~~thee eqn end terminates abruptly~~

The continued fract^n expansion terminates abruptly. The last divisor $\{s^4 + 4s^2 + 3\}$ is therefore one of the factor of given polynomial. [P(s)]

So the find another factor for P(s), divide P(s) with last divisor obtained in continued fract^n expansion.

$4s^2 + 2s + 1$

$* \; s^4 + 4s^2 + 3 \,\big)\, 4s^6 + 2s^5 + 17s^4 + 8s^3 + 16s^2 + 6s + 3$

$- \; 4s^6 + 16s^4 + 12s^2$

$2s^5 + s^4 + 8s^3 + 4s^2 + 6s + 3$

$- \; 2s^5 + 8s^3 + 6s$

$s^4 + 4s^2 + 3$

$- \; s^4 + 4s^2 + 3$

$\times$

Premium

Dt.
Pg.

B+

$\therefore P(s) = (s^4 + 4s^2 + 3)(4s^2 + 2s + 1)$

$= P_1(s) \cdot P_2(s)$

$\rightarrow P_1(s) = s^4 + 4s^2 + 3$

Condition:
1st all coeff. ~~sho~~ should be +ve
2nd either all odd / even ~~term~~ should be missing
3rd.

$e_1(s) = s^4 + 4s^2 + 3$

$O_1(s) = \frac{d\, e(s)}{ds}$

$= 4s^3 + 8s$

$\frac{e_1(s)}{O_1(s)} = 4s^3 + 8s \,\big|\, s^4 + 4s^2 + 3 \,\big|\, s/4$

$s^4 + 2s^2$

$2s^2 + 3 \,\big|\, 4s^3 + 8s \,\big|\, 2s$

$4s^3 + 6s$

$2s^1 \,\big|\, 2s^2 + 3 \,\big|\, s$

$2s^2$

$3^0 \,\big|\, 2s \,\big|\, 2s/3$

$2s$

0

order

*

$\therefore$ Hurwitz conditions

$\rightarrow P_2(s) = 4s^2 + 2s + 1$

$e_2(s) = 4s^2 + 1$

$O_2(s) = 2s$

$\frac{e_2(s)}{O_2(s)} = 2s^1 \,\big|\, 4s^2 + 1 \,\big|\, 2s$

$4s^2$

$1 \,\big|\, 2s \,\big|\, 2s$

$2s$

$\times$

$\therefore$ Hurwitz

Premium

Dt.
Pg.

B+

Z - Parameters in terms of Y - Parameter

We know $Z = \frac{1}{Y}$

$$\Rightarrow [Z] = [Y]^{-1}$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix} \cdot \frac{1}{\Delta Y}$$

$\Delta Y \longrightarrow$ determinant $= Y_{11} \cdot Y_{22} - Y_{12} \cdot Y_{21}$

$$\therefore Z_{11} = \frac{Y_{22}}{\Delta Y} \qquad Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} \qquad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

Z - Parameter in terms of T - Parameter

$$V_1 = Z_{11} I_1 + Z_{12} \cdot I_2 \quad \text{--- (1)} \qquad V_1 = A V_2 + B(-I_2) \quad \text{--- (3)}$$

$$V_2 = Z_{21} I_1 + Z_{22} \cdot I_2 \quad \text{--- (2)} \qquad I_1 = C V_2 + D(-I_2) \quad \text{--- (4)}$$

Using eq^n (4)

$$C V_2 = I_1 + D I_2$$

$$V_2 = \frac{I_1}{C} + \frac{D}{C} I_2 \quad \text{--- (5)}$$

comparing (2) & (5)

$$\Rightarrow \boxed{Z_{21} = \frac{1}{C}} \qquad \boxed{Z_{22} = \frac{D}{C}}$$

- PRF, filter design, numerical based on filter passive filter only

Dt. Pg.

B+

Using (5) & Eq (3)

$$V_1 = \frac{A}{C} \cdot I_1 + \frac{AD}{C} I_2 + B(-I_2)$$

$$V_1 = \frac{A}{C} I_1 + I_2 \left(\frac{AD - BC}{D}\right) \quad \text{—— (6)}$$

Comparing (1) and (6)

$$\Rightarrow \boxed{z_{11} = \frac{A}{C}} \qquad \boxed{z_{12} = \frac{AD-BC}{C}}$$

h parameter in terms of T parameter

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{— (1)} \qquad V_1 = AV_2 + B(-I_2) \quad \text{— (3)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad (4) \qquad I_1 = C V_2 + D(-I_2) \quad \text{— (4)}$$

$$D I_2 = C V_2 - I_1$$

$$I_2 = \frac{-I_1}{D} + \frac{C}{D} V_2 \quad \text{—— (5)}$$

$$\boxed{h_{21} = \frac{-1}{D}} \qquad \boxed{h_{22} = \frac{C}{D}}$$

$$V_1 = AV_2 - B\left(\frac{-I_1}{D} + \frac{C}{D} V_2\right)$$

$$= \left(\frac{AD - BC}{D}\right) V_2 + \frac{B}{D} I_1$$

$$\boxed{h_{11} = \frac{B}{D}} \qquad \boxed{h_{12} = \frac{AD - BC}{D}}$$

Premium

Properties of Positive Real Function (PRF)

$$F(s) = \frac{P(s)}{Q(s)}$$

$$= \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m \cdot s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}$$

The necessary & sufficient condⁿ for a rational network F(s) to be real & positive are stated as

1. F(s) must not have any pole in the right side of the s-plane, i.e. both P(s) and Q(s) of F(s) are Hurwitz.
   This condition can be checked by continued fractⁿ expansion of the odd to even part and even to odd part of P(s) and Q(s)

2. Real part of F(s) must be greater or equal to 0 for all ω

$$Re[F(j\omega)] \geq 0 \quad \text{for all } \omega$$

where $A(\omega^2) = M_1(j\omega) M_2(j\omega) - N_1(j\omega) \cdot N_2(j\omega)$

where M_1 & M_2 are the even part of P(s) & Q(s)
N_1 & N_2 are the odd parts of P(s) & Q(s)

3. This condⁿ is tested for the poles of F(s) lying on the jω axis. This is done by using the partial fraction expansion of F(s) to check whether the residues of the poles on the jω axis are +ve and real

Dt.

Pg. B+

Q. $F(s) = \dfrac{s^2+6s+5}{s^2+9s+14}$

$F(s) = \dfrac{P(s)}{Q(s)}$

$\therefore P(s) = s^2+6s+5$ | $Q(s) = s^2+9s+14$

$e_p(s) = s^2+5$

$O_p(s) = 6s$

$e_p(s) = s^2+14$

$O_p(s) = 9s$

$6s \,|\, s^2+5 \,|\, s/6$

s^2+

$5 \,|\, 6s \,|\, 6s/5$

$6s$

$\times$

$9s \,|\, s^2+14 \,|\, s/9$

$-s^2$

$14 \,|\, 9s \,|\, 9s/14$

$9s$

$\times$

$\therefore$ Both $P(s)$ & $Q(s)$ are Hurwitz polynomial.

$\therefore$ $F(s)$ is Hurwitz polynomial

$Re[F(j\omega)] \geq 0$

$A(\omega^2) = (s^2+5)(s^2+14) - (6s)(9s) \geq 0$

$s^4 + 19s^2 + 70 - 54s^2 \geq 0$

$s^4 - 35s^2 + 70 \geq 0$

replace 's' by '$j\omega$'

$\omega^4 + 35\omega^2 + 70 \geq 0$

$A(\omega^2) \geq 0 \Rightarrow F(s)$ is PRF

The funct^n is positive degree

$\therefore$

Premium

Q. $F(s) = \dfrac{s^2 + \frac{3s}{4} + \frac{1}{4}}{s^2 + s + 4}$

Ans: $P(s) = s^2 + \frac{3s}{4} + \frac{3}{4}$

$\frac{3s}{4} \,\Big|\, s^2 + \frac{3}{4} \,\Big|\, \frac{s4}{3}$

s^2

$\frac{3}{4} \,\Big|\, \frac{3s}{4} \,\Big|\, s$

X

$Q(s) = s^2 + s + 4$

$s \,\Big|\, s^2 + 4 \,\Big|\, s$

s^2

$4 \,|\, s \,|\, s/4$

X

$A(\omega^2) = \left(s^2 + \frac{3}{4}\right)(s^2 + 4) - \frac{3s}{4} \cdot s \geq 0$

$4 - \frac{1}{4}$

$s^4 + s^2 \frac{19}{4} + 3 - \frac{3s^2}{4} \geq 0$

$s^4 + \frac{16 s^2}{4} + 3 \geq 0$

$\cancel{\omega^4 - \frac{14\omega^2}{4} + 1} \geq 0$

$\omega^4 - 4\omega^2 + 3 \geq 0$

Sturm Test

$\omega^2 = x$

$P_0(x) = x^2 - 4x + 3$

$P'_0(x) = 2x - 4$

$\dfrac{P_0(x)}{P'_0(x)}$

$$\begin{array}{r|l} & x/2 - 1 \\ 2x-4 & x^2 - 4x + 3 \\ & x^2 - 2x \\ \hline & -2x + 3 \\ & -2x + 4 \\ \hline & -1 \end{array} = -P_1(x)$$

	Sign of $P_0(x)$	Singn of $P_0''(x)$	Sign of $P_1(x)$	Total ~~Sign of~~ sign ~~P_1 to~~ change
$x=0$	+	−	+	2
$x=\infty$	+	+	+	0

No. of total sign change $= 2 - 0 = 2$

∴ The given funct^n is not physically reliable

Q. $A(\omega^2) = 3\omega^4 - 12\omega^2 + 4$

$P_e(\omega) = 3\omega^4 - 12\omega^2 + 4$
$P_o(\omega) = 12\omega^3 - 24\omega$

$$\begin{array}{r|l} & \omega/4 - 1/\omega 2 \\ 12\omega^3 - 24\omega & 3\omega^4 - 12\omega^2 + 4 \\ & \omega^4 - 6\omega^2 \\ \hline & -6\omega^2 + 4 \\ & -6\omega^2 + 12 \\ \hline & -8 \end{array} = -P_1(x)$$

Dt.
Pg. B+

	Sign of $P(x)$	Sign of $P'(x)$	Sign of $P_1(x)$	Total sign changes
$x=0$	+	+		
$x=\infty$	+	+		

Q $F(s) = \frac{2s^2+2s+1}{s^3+2s^2+s+2}$

$P(s) = 2s^2+2s+1$

$$2s \,\overline{)\,2s^2+1\,}\, s$$
$$2s^2$$
$$1 \,\overline{)\,2s\,}\, 2s$$
$$\times$$

$Q(s) = s^3+2s^2+s+2$

$$s^3+s \,\overline{)\,2s^2+2\,}\, 2/s$$
$$2s^2+2$$
$$\times$$

$$s^3+s \,\overline{)\, s^3+2s^2+s+2 \,}\, 1+2/s$$
$$s^3 \quad +s$$
$$2s^2 \quad +2$$
$$2s^2 \quad +2$$
$$\times$$

$\therefore Q(s) = (s^3+s)\left(1+\frac{2}{s}\right)$

$Q_1(s) = s^3+s$ $\qquad$ $Q_2(s) = 1+\frac{2}{s}$

$Q_{o1}(s) = s^3+s$ $\qquad$ $Q_{o2}(s) = 2/s$

$Q_{e1}(s) = 3s^2+1$ $\qquad$ $Q_{e2}(s) = 1$

Premium

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Dt.
Pg.

B+

$s^3+s \overline{) 3s^2+1 } \; 3/s$

$3s^2+3$

$-2 \overline{) s^3+s }$

Q5 check whether the given polymial

$F(s) = s^5 + 2s^3 + s$

$F_o(s) = s^5 + 2s^3 + s$

$F_e(s) = 3s^4 + 6s^2 + 1$

8/10/17

Filter Designing

1. Foster I
2. Foster II } Partial fraction
3. Couier I
4. Couier II } continued fraction expansion

Coues

$$Z(s) = \frac{s(s+2)(s+5)}{(s+1)(s+4)}$$

	$Z(s)$	$Y(s)$
R (resistor)	R	$1/R$
L (inductor)	sL	$1/sL$
C (capacitor)	$1/sC$	sC

$$\Rightarrow \frac{Z(s)}{s} = \frac{s^2+7s+10}{s^2+5s+4}$$

$s^2+5s+4 \overline{) s^2+7s+10 } \; 1$

s^2+5s+4

$2s+6$

Premium

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$$\Rightarrow \frac{Z(s)}{s} = 1 + \underbrace{\frac{2s+6}{s^2+7s+10}}_{Z^*(s)}$$

$$Z^*(s) = \frac{2s+6}{(s+1)(s+4)}$$

$$= \frac{A}{s+1} + \frac{B}{s+4}$$

$$A = \frac{4}{3} \qquad B = \frac{2}{3}$$

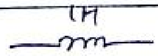
$$Z^*(s) = \frac{4}{3}\frac{1}{(s+1)} + \frac{2}{3}\frac{1}{(s+4)}$$

$$\therefore \frac{Z(s)}{s} = 1 + \frac{4}{3(s+1)} + \frac{2}{3(s+4)}$$

$$Z(s) = s + \frac{4s/3}{(s+1)} + \frac{2s/3}{(s+4)}$$

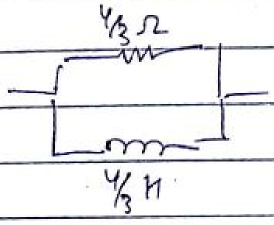
$$= Z_1(s) + Z_2(s) + Z_3(s)$$

- $Z_1(s) = 1/s \longrightarrow$



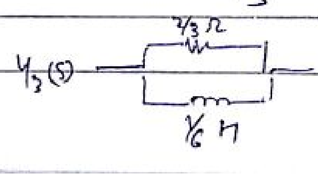
- $Z_2(s) = 4s/3 \,/\, (s+1)$

$$Y_2(s) = \frac{s+1}{4s/3} = \frac{1}{4/3} + \frac{1}{4s/3}$$



- $$Z_3(s) = \frac{2s/3}{s+4}$$

$$Y_3(s) = \frac{s+4}{\frac{2s}{3}} = \frac{1}{2/3} + \frac{4}{s/6}$$



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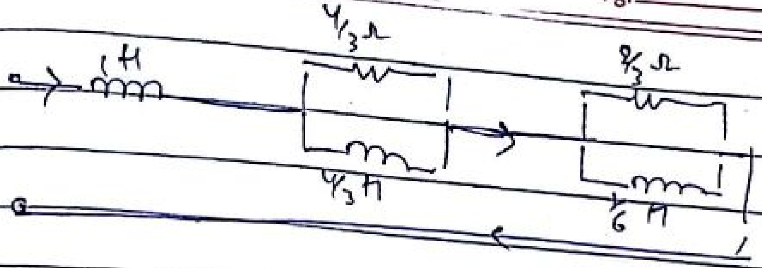


Foster Form II

* For foster II form we always solve our funct" in admittance form using partial fraction

$$Z(s) = \frac{s(s+2)(s+5)}{(s+1)(s+4)}$$

$$\therefore Y(s) = \frac{(s+1)(s+4)}{s(s+2)(s+5)} \qquad \frac{(-1)(2)}{(-2)(3)}$$

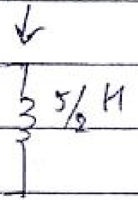
$$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$A = \frac{4}{10} = \frac{2}{5}$$

$$B = \frac{1}{3}$$

$$C = \frac{(-4)(-1)}{(-5)(-3)} = \frac{4}{15}$$

$$= \frac{2}{5s} + \frac{1}{3(s+2)} + \frac{4}{15(s+5)}$$

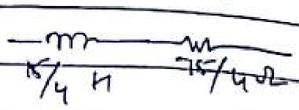


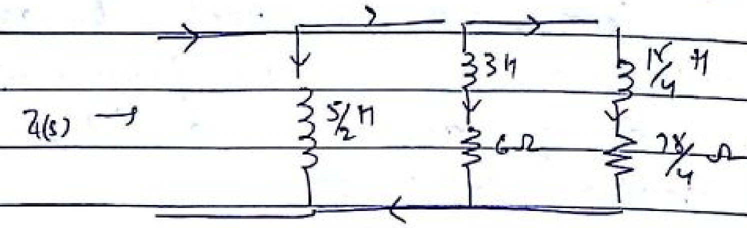
$$Y_{(2)}(s) = \frac{1}{3(s+2)}$$

$$\rightarrow Z_2(s) = 3s + 6$$

$$Y_3(s) = \frac{4}{15(s+5)}$$

$$Z_3(s) = \frac{15s}{4} + \frac{75}{4}$$





~~Foster~~'s Cauer Form I

* The Highest power of s should be at numerator side

$$Z(s) = \frac{s(s+2)(s+5)}{(s+1)(s+4)}$$

$$= \frac{s^3 + 7s^2 + 10s}{s^2 + 5s + 4}$$

$$s^2 + 5s + 4 \,\big)\, s^3 + 7s^2 + 10s \,\big(\, s \quad Z_1(s)$$

$$-\; s^3 + 5s^2 + 4s$$

$$2s^2 + 6s \,\big)\, s^2 + 5s + 4 \,\big(\, \tfrac{1}{2} \quad Y_2(s)$$

$$-\; s^2 + 3s$$

$$2s + 4 \,\big)\, 2s^2 + 6s \,\big(\, s \quad Z_3(s)$$

$$-\; 2s^2 + 4s$$

$$2s \,\big)\, 2s + 4 \,\big(\, 1 \quad Y_4(s)$$

$$-\; 2s \qquad Z_5(s)$$

$$4 \,\big)\, 2s \,\big(\, s/2$$

$$\times$$

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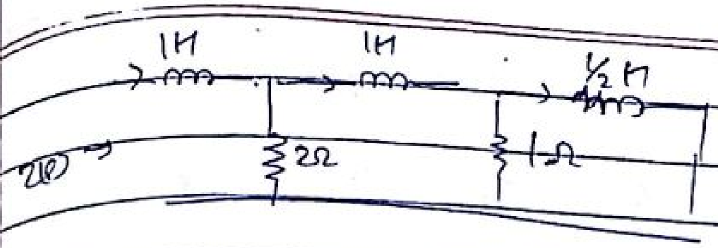


* In Cauer II form realise the funct^n by rearranging the numerator and denominator polynomial in ascending order of degree s and take continued fraction expansion

$$Z(s) = \frac{s(s+2)(s+5)}{(s+1)(s+4)}$$

$$= \frac{10s + 7s^2 + s^3}{4 + 5s + s^2}$$

$$4 + 5s + s^2 \;\Big|\; 10s + 7s^2 + s^3 \;\Big|\; \frac{10s}{4} = \frac{5}{2}s$$

$$- \quad 10s + \frac{25}{2}s^2 + \frac{5}{2}s^3$$

$$-\frac{11}{2}s^2 - \frac{3}{2}s^3 \;\Big|\; 4 + 5s + s^2 \;\Big|\; \frac{-2}{11s^2}$$

Since we get -ve quotient term in continued fraction expansion. To avoid this we take reciprocal of our given funct^n

Dt.
Pg.

B+

$$\therefore Y(s) = \frac{4+5s+s^2}{10s+7s^2+s^3}$$

$$10s+7s^2+s^3 \,\big)\, 4+5s+s^2 \,\big(\, \frac{4}{10s} = \frac{2}{5s} \quad Y_1(s)$$

$$\frac{2}{5} \qquad 4 + \frac{14s}{5} + \frac{2s^2}{5}$$

$$\frac{11s}{5}+\frac{3s^2}{5} \,\big)\, 10s+7s^2+s^3 \,\big(\, \frac{5\times10}{11}$$

$$\frac{5\times10}{11}\times 3 \qquad 10s + \frac{30s^2}{11} + \cancel{6s^3}$$

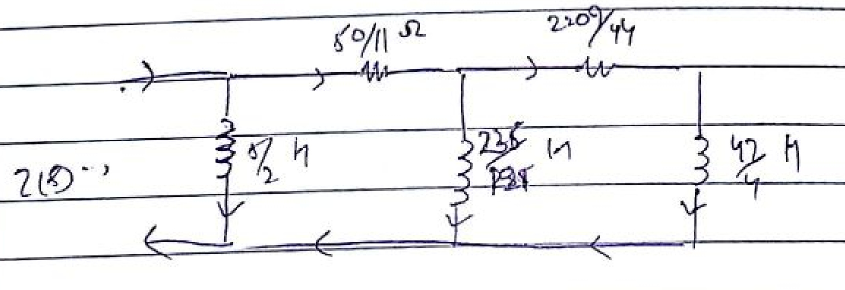
$$\frac{47}{11}s^2+s^3 \,\big)\, \frac{11s}{5}+\frac{3s^2}{5} \,\big(\, 47$$

$77 - 30 = 47$

$$Y_1(s) = \frac{2}{5s} \qquad Z_4(s) = \frac{2209}{44}$$

$$Z_2(s) = \frac{50}{11} \qquad Y_5(s) = \frac{4}{47s}$$

$$Y_3(s) = \frac{121}{235s}$$



Premium