* Complex no. and their geometrical representations-
Quadratic equations of the type $x^2+1=0$ gave rise to a new type of nos.

ie $x^2+1=0 \Rightarrow x^2=-1 \Rightarrow x=\sqrt{-1}$

Euler first defined the no. i, called iota or imaginary unit, as the no. whose square is -1.
With the help of i, Square root of a -ve no. can interpreted as

$$\sqrt{-25}=5i, \quad \sqrt{-9}=3i, \quad \sqrt{-7}=\sqrt{7}\,i$$

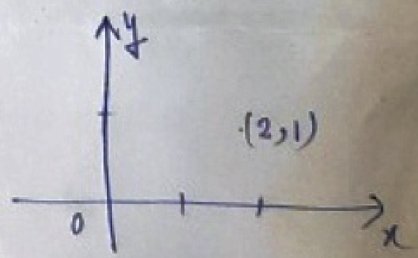
These are the imaginary nos.

A complex no. is of the form $a+ib$, where a & b are real nos. and $i=\sqrt{-1}$.

It is denoted by $z=a+ib$, then a & b are real & imaginary parts of z, denoted by $Re(z)$ & $Im(z)$, respectively.

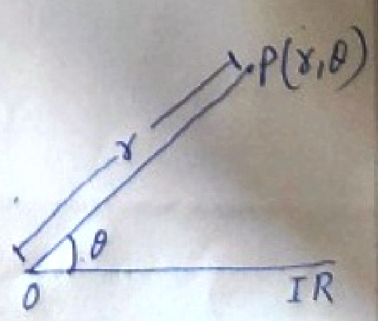
- A real nos. R are uniquely represented by the points of a real line.
- A complex nos. C are uniquely represented by the points of a co-ordinate plane

$$a+ib \rightarrow (a,b)$$



- A co-ordinate plane used for representing complex no. is called an Argand plane or Gaussian plane.

*Polar form of complex number:-



$$z=x+iy \quad (1)$$

Put $x=r\cos\theta$, $y=r\sin\theta$ in (1)

$$z=r(\cos\theta+i\sin\theta), \text{ where } r\in[0,\infty)$$

and $\theta=[0,2\pi)$. Here, r is the distance of z from a fixed point O called pole, and θ is the angle made by OP in the positive direction.

~~apr~~ $\arg(z)$ or $\operatorname{amp}(z)=\tan^{-1}(y/x)$; amplitude of z is $|z|=\sqrt{x^2+y^2}$

Graphing in polar coordinates:-

(I) Symmetry in cartesian coordinates

(a) A curve is symmetrical about the x-axis, if all the powers of y in the equation of the given curve are even
Eg $y^2 = 4x$

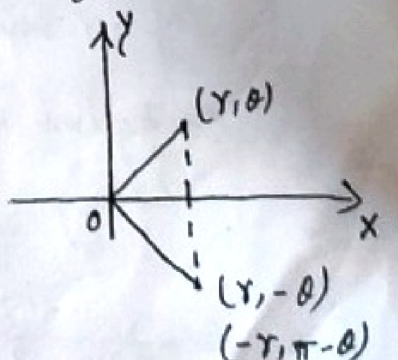
(b) A curve is symmetrical about the y-axis, if all the powers of x in the equation of the given curve are even.
Eg $x^2 = 4y$.

(c) A curve is symmetrical about the line $y=x$, if the equation of the curve remain unchange on interchanging x and y.

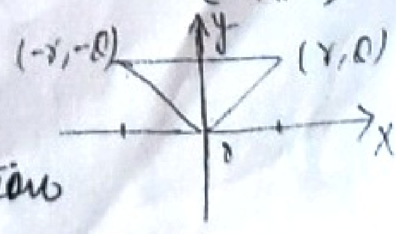
II Symmetry in polar coordinates:-
The graph of a polar ~~coordinates~~ equation can be evaluated for three types of symmetry.

(a) A graph is symmetric with respect to the polar axis (x-axis), if replacing (r,θ) with $(r,-\theta)$ or $(-r, \pi-\theta)$ yield an equivalent equation

Eg $r = 1-\cos\theta \rightarrow (r,\theta)$
$\Rightarrow r = 1-\cos(-\theta) \rightarrow (r,-\theta)$
$\Rightarrow r = 1-\cos\theta \rightarrow (r,\theta)$

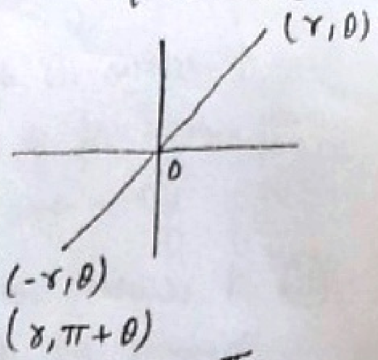


(b) A graph is symmetric with respect to the line $\theta = \pi/2$ (y-axis), if replacing (r,θ) with $(-r,-\theta)$ or $(r, \pi-\theta)$ yield an equivalent equation



Eg $r^2 = 1-\cos\theta \Rightarrow r^2 = 1-\cos\theta$ $\boxed{\text{For } -r, -\theta}$

(c) The graph is symmetric with respect to the polar (origin) if replacing (r, θ) with $\begin{cases}(-r, \theta) \\ (r, \pi+\theta)\end{cases}$ yield an equivalent equation.



Eg $\quad r = 1 - \cos\theta \quad (r,\theta)$

$\quad -r = 1 - \cos\theta \quad (-r,\theta)$

Q. Find the cartesian coordinates

$$r = 3, \ \theta = 0^\circ$$

We know that the transformations

$$x = r\cos\theta$$

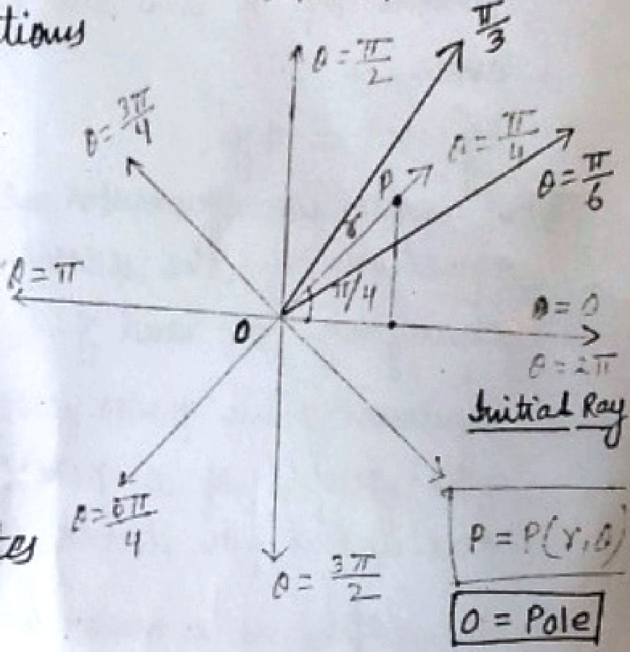
$$y = r\sin\theta$$

$\therefore \quad x = 3\cos 0^\circ = 3$

$\quad\ \ y = 3\sin 0^\circ = 0$

$\therefore \quad (r,\theta)$ ie $(3, 0^\circ)$

$\Rightarrow \quad (x,y)$ ie $(3,0)$



Q. Find the cartesian coordinates

$$r = \sqrt{2}, \ \theta = \pi/4$$

we know that

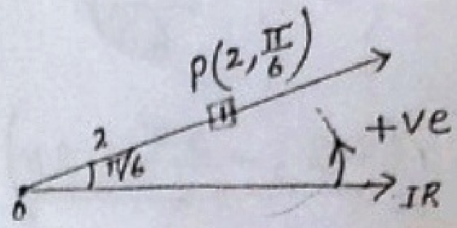
$$x = r\cos\theta \quad \Big| \quad x = \sqrt{2}\cos\frac{\pi}{4} = \sqrt{2} \times \frac{1}{\sqrt{2}} = 1$$

$$y = r\sin\theta \quad \Big| \quad y = \sqrt{2}\sin\pi/4 = \sqrt{2} \times \frac{1}{\sqrt{2}} = 1$$

$\therefore$ Cartesian coordinates $(1,1)$.

Q Find all Polar coordinates of the point $(2, \pi/6)$

(I) $\left(2, \frac{\pi}{6}\right)$



After one rotation

$\frac{\pi}{6} + 2\pi$

After second rotation

$\left(\frac{\pi}{6} + 2\pi\right) + 2\pi = \frac{\pi}{6} + 4\pi$

After third rotation

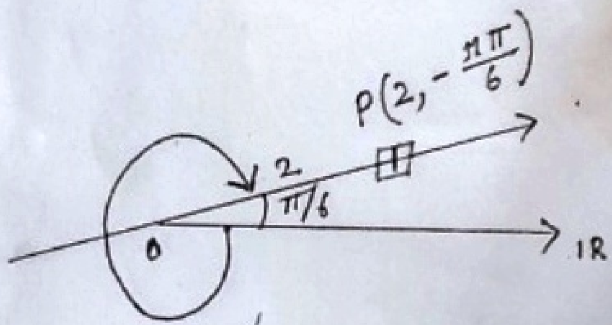
$\left(\frac{\pi}{6} + 4\pi\right) + 2\pi = \frac{\pi}{6} + 6\pi$

And so on

$\frac{\pi}{6} + 2n\pi, \; n \in N.$

$\therefore \left(2, \frac{\pi}{6} + 2n\pi\right).$

— x — x — x —



(II) $\left(2, -\frac{11\pi}{6}\right)$

$-2\pi + \frac{\pi}{6}$

$\frac{-12\pi + \pi}{6} = \frac{-11\pi}{6}$

$\left(2, -2n\pi - \frac{11\pi}{6}\right)$

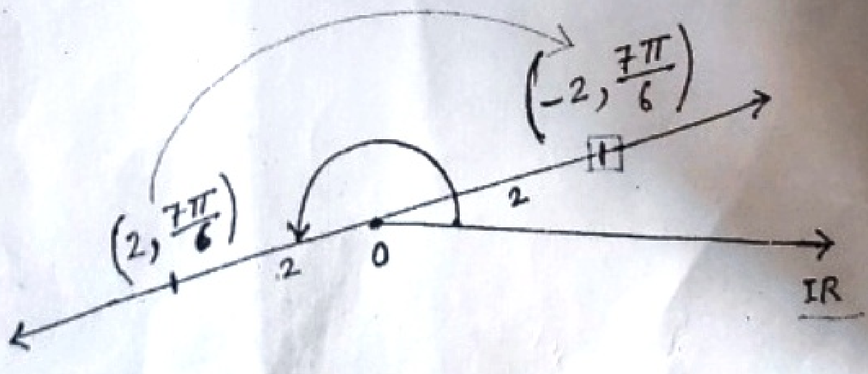
or $\left(2, -\frac{11\pi}{6} - 2n\pi\right), \; n \in N$

— x — x — x — x — x —

(III) $\left(-2, \frac{7\pi}{6}\right)$

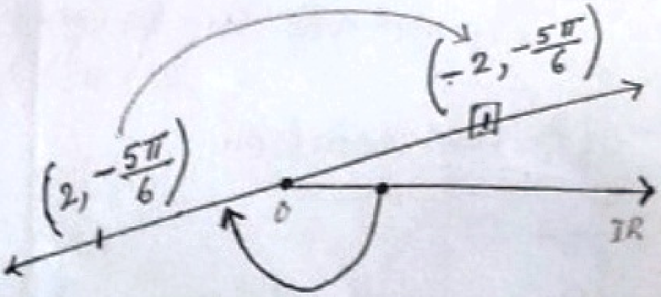
$\left(-2, \frac{7\pi}{6} + 2\pi\right)$

$\left(-2, \frac{7\pi}{6} + 4\pi\right)$



$\left(-2, \frac{7\pi}{6} + 2n\pi\right), \; n \in N$

(IV) $\left(-2, -\frac{5\pi}{6}\right)$



$\left(-2, -\frac{5\pi}{6} - 2\pi\right)$

$\left(-2, -\frac{5\pi}{6} - 4\pi\right)$

$\left(-2, -\frac{5\pi}{6} - 2n\pi\right)$, $n \in \mathbb{N}$.

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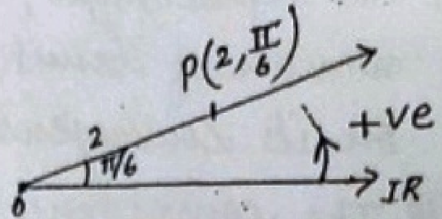
Q Find all Polar coordinates of the point $(2, \pi/6)$



$\left(2, \frac{\pi}{6}\right)$

After one rotation

$\frac{\pi}{6} + 2\pi$

After second rotation

$\left(\frac{\pi}{6} + 2\pi\right) + 2\pi = \frac{\pi}{6} + 4\pi$

After third rotation

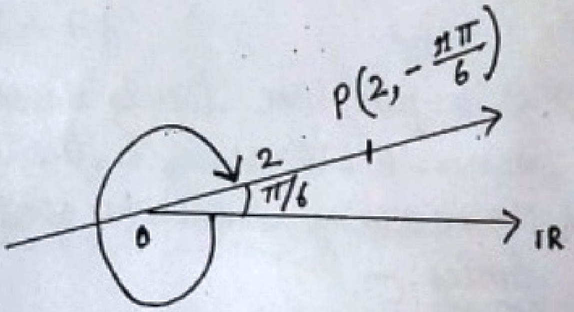
$\left(\frac{\pi}{6} + 4\pi\right) + 2\pi = \frac{\pi}{6} + 6\pi$

And so on

$\frac{\pi}{6} + 2n\pi, \; n \in N.$

$\therefore \left(2, \frac{\pi}{6} + 2n\pi\right).$

—x—x—x—



$\left(2, -\frac{11\pi}{6}\right)$

$-2\pi + \frac{\pi}{6}$

$\frac{-12\pi + \pi}{6} = \frac{-11\pi}{6}$

$\left(2, -2n\pi - \frac{11\pi}{6}\right)$

or $\left(2, -\frac{11\pi}{6} - 2n\pi\right).$