Circuits & Systems

Signal – A physical quantity which can be a function of one or more variables like frequency, temp., pressure.
It represents a physical phenomenon.

System – A physical entity which gives set of outputs after processing their input.
System can be hardware or software.

→ Classification of signals –

I) Discrete time and Continuous time signal –
A signal is said to be continuous time signal if it is defined for all the continuous values of time variable 't'

A discrete time signal is defined only for some specific instants of time variable 'n'.
It can have values only for integer instants of time variable 'n'.

II) Continuous value and Discrete value signals –
Continuous valued functions can have any value b/w $-\infty$ to $+\infty$; whereas discrete valued can have value within a particular specified range.

III) Deterministic and Random signals –
Deterministic signals are ones whose nature and amplitude can be predicted whereas this probable prediction is not possible for random signals. Only the probability function can be predicted.

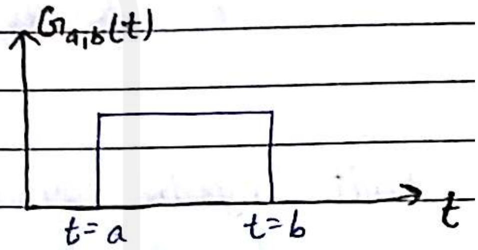
Date

Unit RAM signal - represented by 'r'

$$r(t) = 0 \ , \text{ for } t<0$$
$$= t \ , \text{ for } t \geq 0$$

GATE Signal - Rectangular pulse of unit height starting at $t=a$ and ending at $t=b$

$G_{a,b}(t) =$



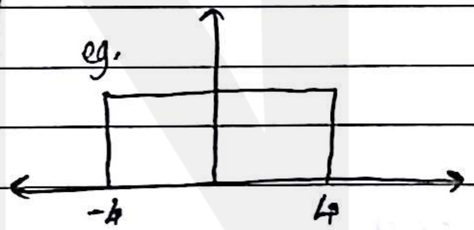
$$\boxed{G_{a,b}(t) = u(t-a) - u(t-b)}$$

starting point ; end point

eg.

$u(t-0) - u(t-1)$

eg.



$G(t) = u(t+4) - u(t-4)$

→ Transformation of signals -

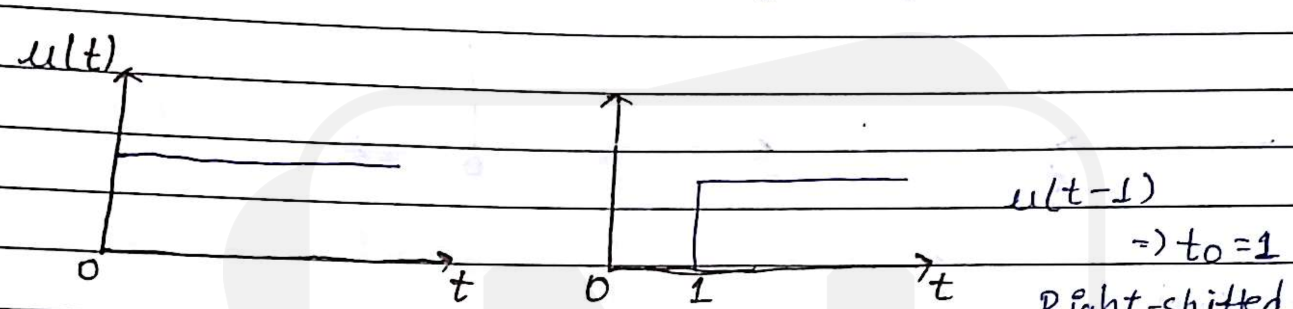
Signals can be transformed by 3 processes -

i) Time shifting
ii) Time scaling
iii) Time reversal

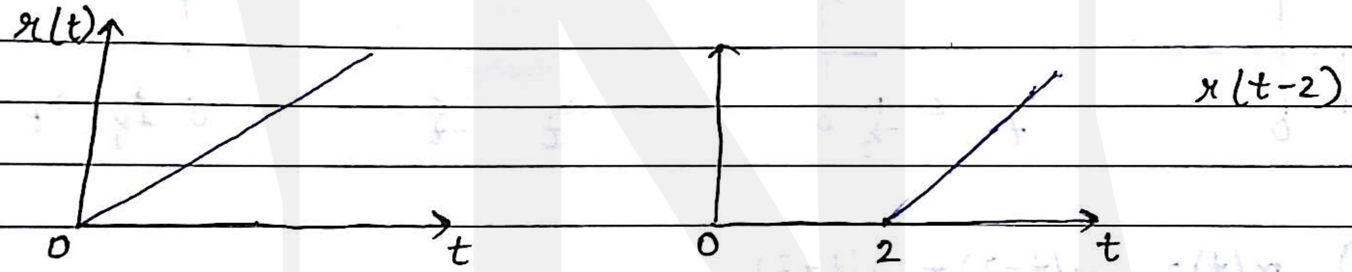
Time shifting - Signal can be time shifted by replacing time variab[le] t with $(t-t_0)$, where t_0 is known as shifting fac[tor]

If $t_0 > 0$, signal is right-shifted and this operation results in delay of the signal by t_0.

If $t_0 < 0$, signal is left-shifted and this operation results in advancement of the signal by t_0.

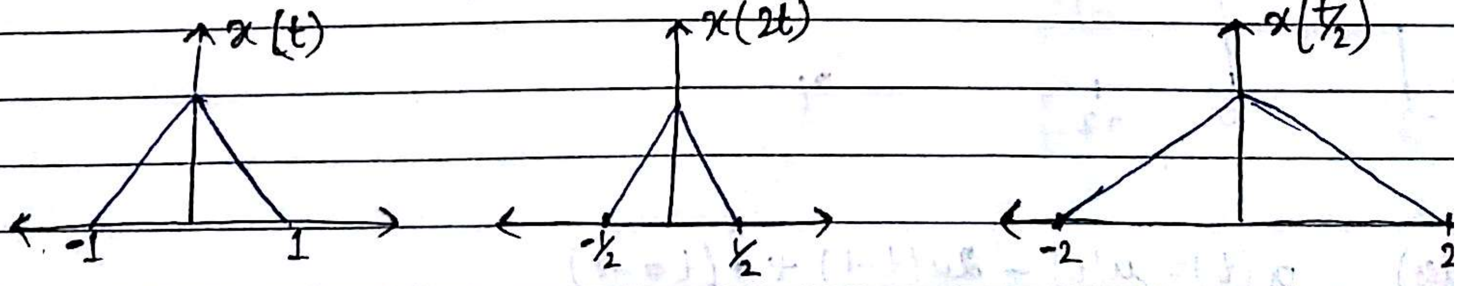


Right-shifted / Delayed



2) Time Scaling - Replace t by at, where a is scaling factor
a cannot be zero

If $a > 1$, scaling results in compression of the signal
If $0 < a < 1$, scaling results in expansion of the signal

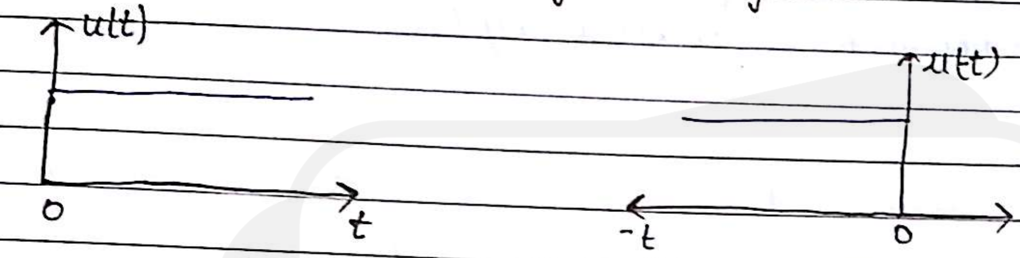


Compression

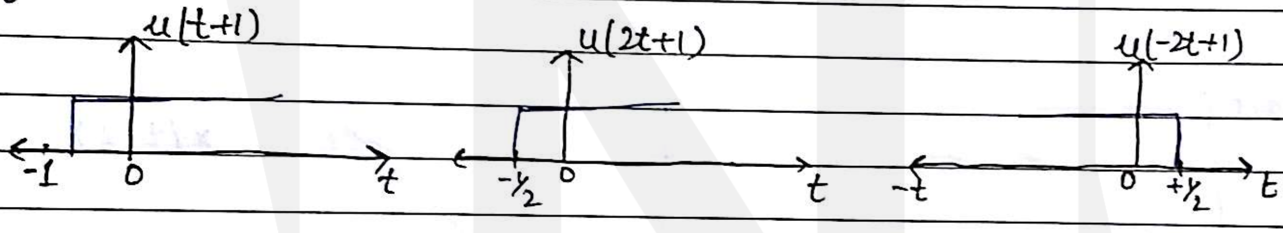
Expansion

SSK – ~~Scat~~ Shifting, Scaling, Reversal

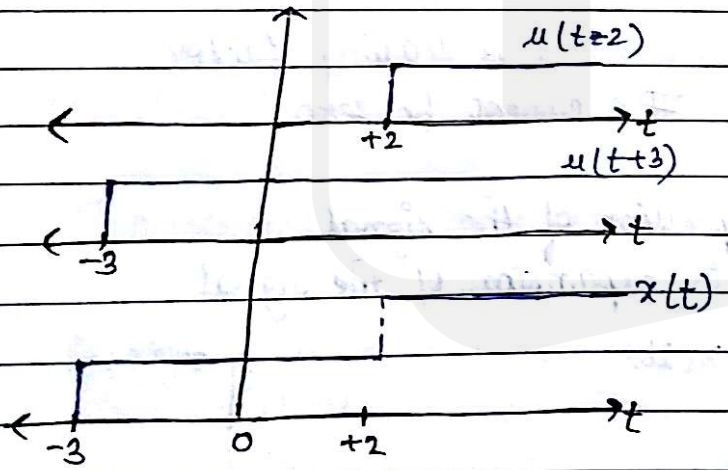
3) Time Reversal – Replace t by −t
Result of this operation is known as Folding / Reflection / Time reversal about the time origin t=0.
Mirror image about y-axis



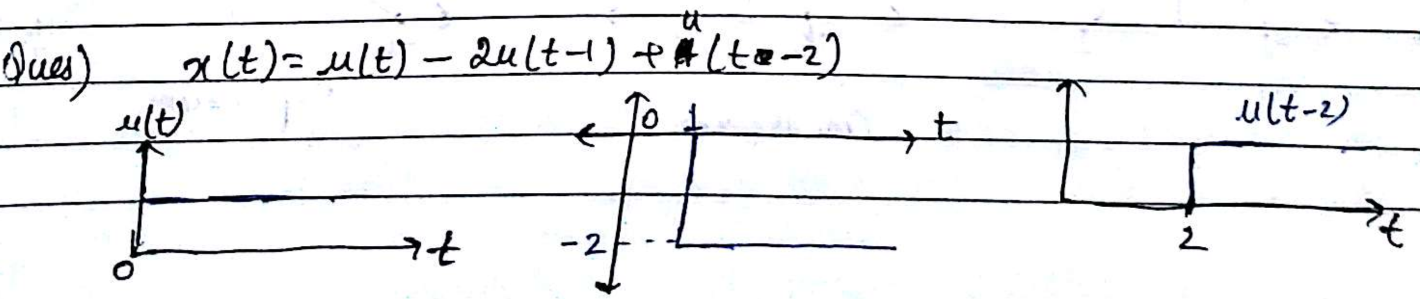
eg. $u(-2t+1)$

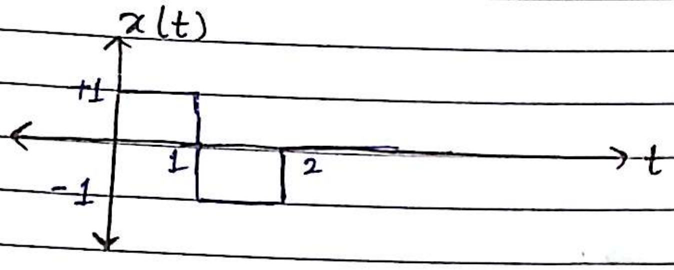


Ques) $x(t) = u(t-2) + u(t+3)$



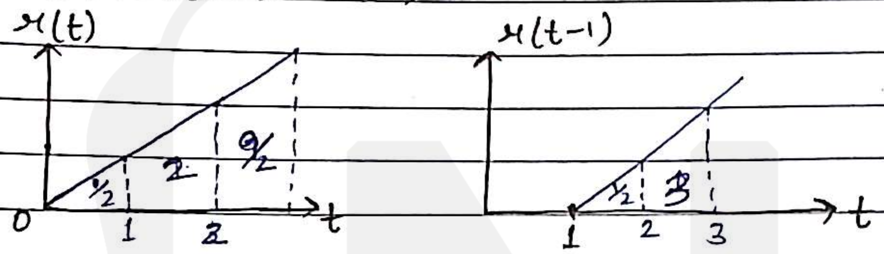
Ques) $x(t) = u(t) - 2u(t-1) + u(t-2)$

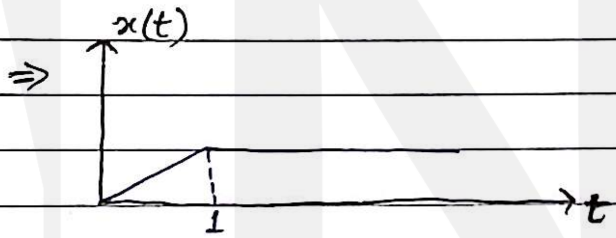




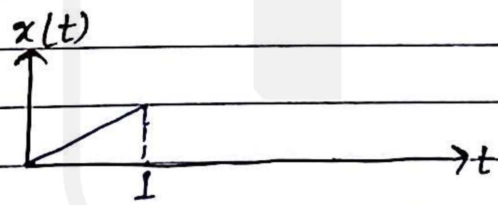
Ques) Draw the given signal –

$r(t) - r(t-1) = x(t)$

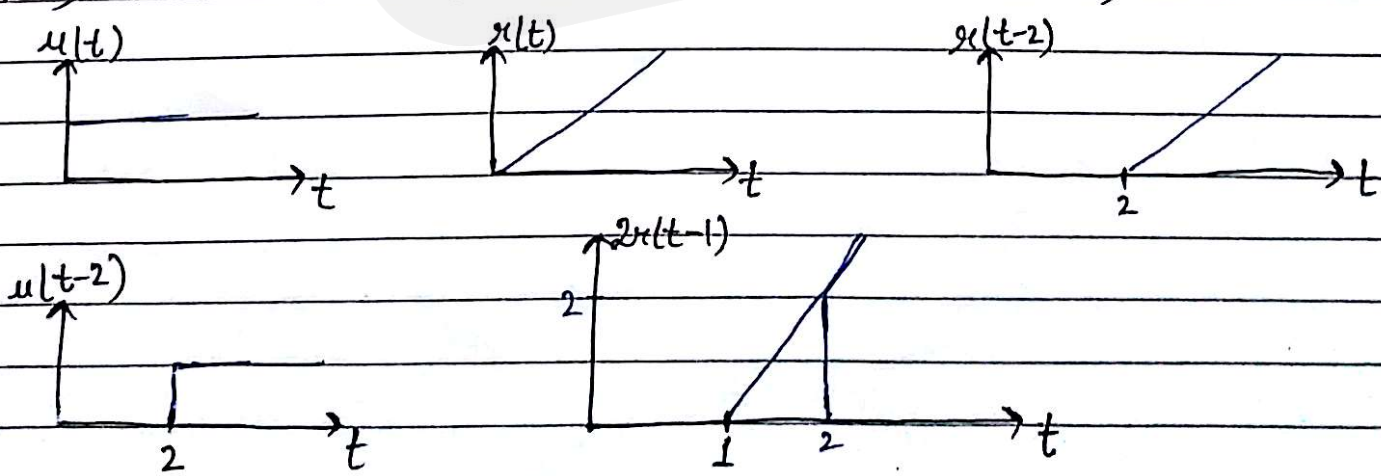


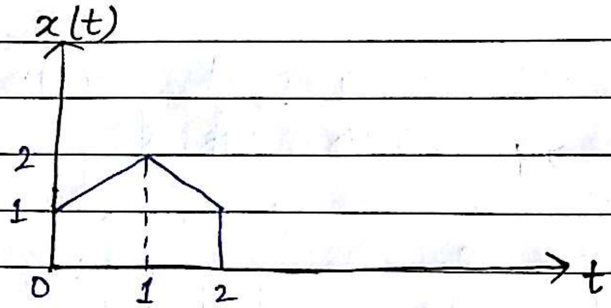


Ques) $x(t) = r(t) - r(t-1) - u(t-1)$



Ques) $x(t) = u(t) + r(t) - 2r(t-1) + r(t-2) - u(t-2)$





→ Causal, Anti-causal & Non-causal signals-

A signal is said to be a "Causal signal" if it has values for $t \geq 0$

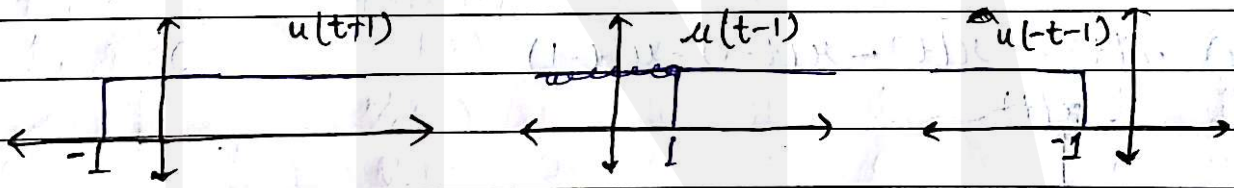
Anti-causal- signals that have values only for negative values of time variable 't' or 'n'.

OR

For discrete signals, anti-causal signals are those signals that have values ranging from $-\infty$ to -1.

eg. $u(-t-1)$

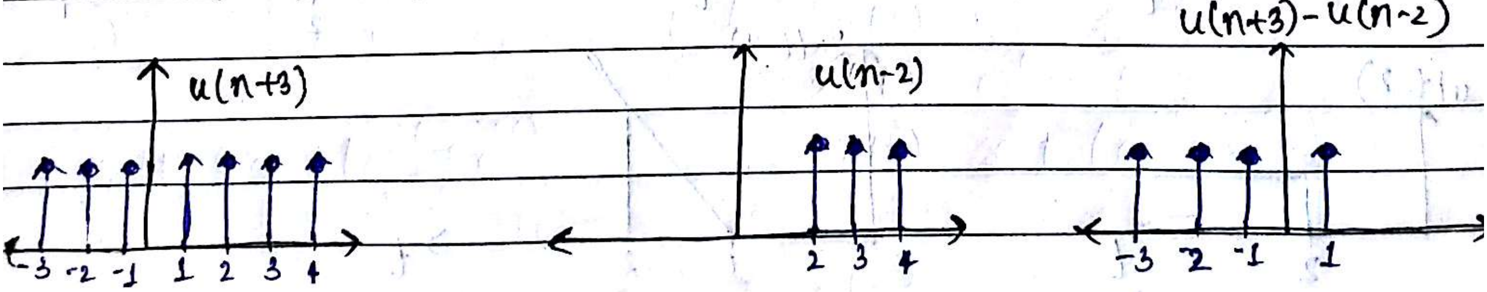
eg.



Non-causal signals- Can have values for all possible values for time variable

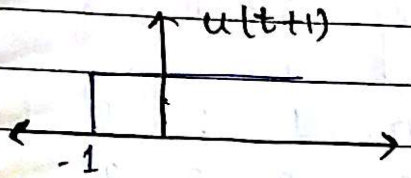
Ques) Check for the causality of given signals-

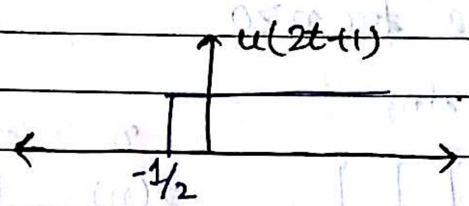
(i) $u(n+3) - u(n-2)$



Non-causal

Ques) $u(2t+1)$





→ Even & Odd Signals-

Even signals- symmetrical about vertical axis

i.e. $\boxed{x(t) = x(-t)}$ eg. cost

Odd signals- not symmetrical about vertical axis

$\boxed{\begin{array}{c} x(t) \neq x(-t) \\ x(-t) = -x(t) \end{array}}$ eg. sint

$e^n \begin{cases} \text{even fn}, & n \text{ is even} \\ \text{odd fn}, & n \text{ is odd} \end{cases}$

1) Even + Even = Even
2) Odd + Odd = Odd
3) Even * Even = Even
4) Odd * Odd = Even
5) Even + Odd = Neither even nor odd

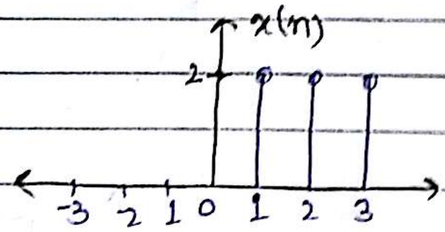
→ Calculation of even & odd part of a signal-

$$\boxed{x_e(t) = \frac{x(t) + x(-t)}{2}} \qquad \boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$$

For discrete signals, replace 't' with 'n'.

Date

eg. $x(n) = 0$ for $n<0$
$= 2$ for $n \geq 0$



n	-3	-2	-1	0	1	2	3
$x(n)$	0	0	0	2	2	2	2
~~$x(-n)$~~	~~2~~	~~2~~	~~2~~	~~2~~	~~0~~	~~0~~	~~0~~
~~$x(-n)$~~	~~0~~	~~0~~	~~0~~	~~2~~	~~2~~	~~2~~	~~2~~
$x_e(n)$	1	1	1	2	1	1	1
$x_o(n)$	-1	-1	-1	0	1	1	1

$$x_e(n) = \begin{cases} 1, & n<0 \ \& \ n>0 \\ 2, & \cancel{n \geq 0} \ n=0 \end{cases}$$

$x_o(n) = 0$, for $n=0$
$= -1$, $n<0$
$= 1$, $n>0$

Ques) Obtain the odd & even part of signal $u(n)$.

$$u(n) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$$

n	-3	-2	-1	0	1	2	3
$u(n)$	0	0	0	1	1	1	1
~~$u(-n)$~~	~~0~~	~~0~~	~~0~~	~~1~~	~~1~~	~~1~~	~~1~~
$u(-n)$	0	0	0	1	1	1	1
$u_e(n)$	0	0	0	1	1	1	1
$u_o(n)$	0	0	0	0	0	0	0

$$u_n(e) = \begin{cases} 1/2, & n<0 \\ 1/2, & n \geq 0 \\ 1 & n=0 \end{cases}$$

$u_o(n) = \cancel{0 \ \forall n}$
$-1/2$, $n<0$
0, $n=0$
$1/2$, $n>0$

Ques) Calculate even & odd part for $x(t) = e^{-2t} \cos t$

$x(-t) = e^{2t} \cos t$

$$x_e(t) = \frac{\cos t \left(e^{-2t} + e^{2t}\right)}{2}$$

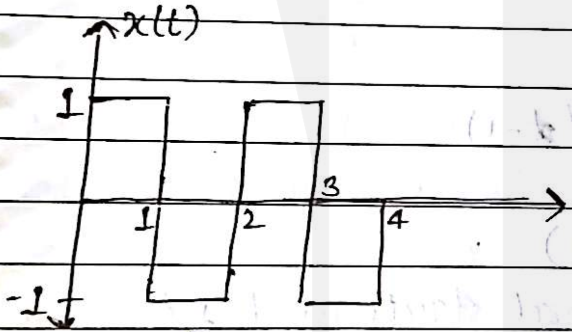
$$x_o(t) = \frac{\cos t \left(e^{-2t} - e^{+2t}\right)}{2}$$

Ques) Obtain even & odd part for $y(n)$

$$y(n) = \begin{cases} -1, & n<0 \\ 1, & n=0 \\ 2, & n>0 \end{cases}$$

n	-3	-2	-1	0	1	2	3
$y(n)$	-1	-1	-1	1	2	2	2
$y(-n)$	2	2	2	1	-1	-1	-1
$y_e(n)$	$1/2$	$1/2$	$1/2$	1	$1/2$	$1/2$	$1/2$
$y_o(n)$	$-3/2$	$-3/2$	$-3/2$	0	$3/2$	$3/2$	$3/2$

→ Waveform Synthesis

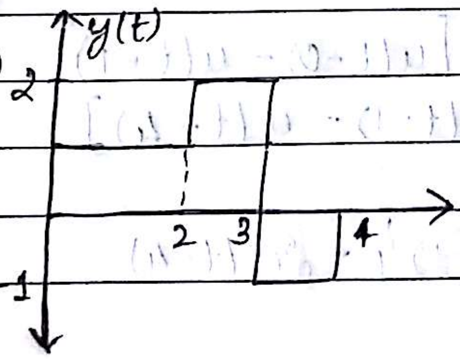


$$x(t) = 1[u(t-0) - u(t-1)] + (-1)[u(t-1) - u(t-2)] + 1[u(t-2) + u(t-3)] + (-1)[u(t-3) - u(t-4)]$$

$$= u(t) - u(t-1) - u(t-1) + u(t-2) + u(t-2) + \cancel{u(t-3)} - \cancel{u(t-3)} + u(t-4)$$

$$\underline{x(t)} = u(t) - 2u(t-1) + 2u(t-2) + u(t-4)$$

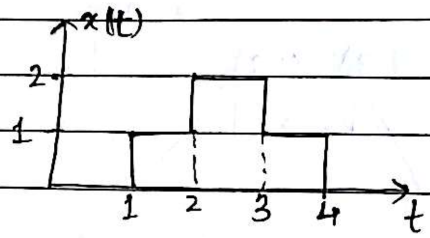
es)



$$y(t) = 1[u(t-0) - u(t-2)] + 2[u(t-2) - u(t-3)] + (-1)[u(t-3) - u(t-4)]$$

$$\underline{y(t) = u(t) + u(t-2) - 3u(t-3) + u(t-4)}$$

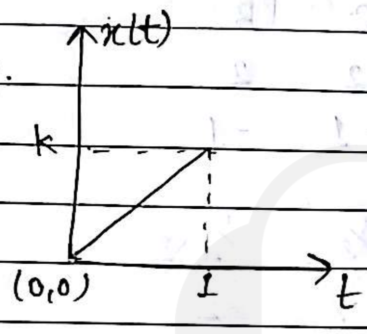
eg.



$$x(t) = 1[u(t-\cancel{0}) - u(t-1)] + 2[u(t-2) - u(t-3)] + 1[u(t-3) - u(t-4)]$$

$$\cancel{= u(t)} = u(t-1) + u(t-2) - u(t-3) - u(t-4)$$

eg.



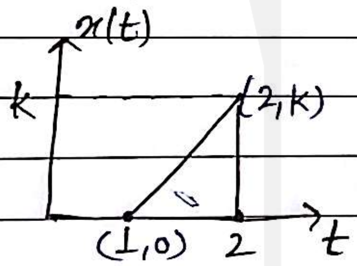
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{k-0}{1-0}(x-0)$$

or $$x(t) - 0 = \frac{k-0}{1-0}(t-0)$$

$$\boxed{x(t) = kt}$$

Ques) Synthesise the given waveform



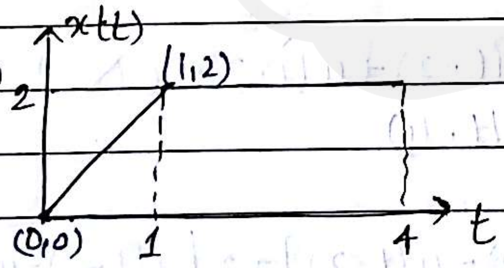
$$x(t)\cancel{y} - 0 = \frac{k-0}{2-1}(t-1)$$

$$x(t) = k(t-1)$$

And the gate signal starts at $t=1$ & ends at t

$$\Rightarrow \quad x(t) = k(t-1)[u(t-1) - u(t-2)]$$

Ques)



$$x(t)\cancel{y} - 0 = \frac{2-0}{1-0}(t-0)$$

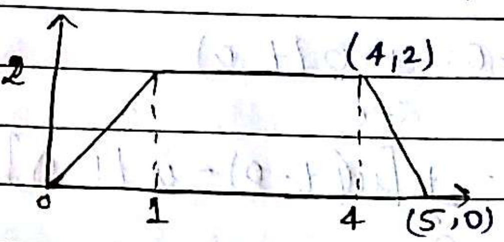
$$x(t) = 2t[u(t-0) - u(t-1)] + 2[u(t-1) - u(t-4)]$$

or $$x(t) = 2tu(t) + \cancel{2u\{(t-1) - t(t-1)\}} - 2u(t-4)$$

$$2u(t-1)\{1-t\}$$

Date

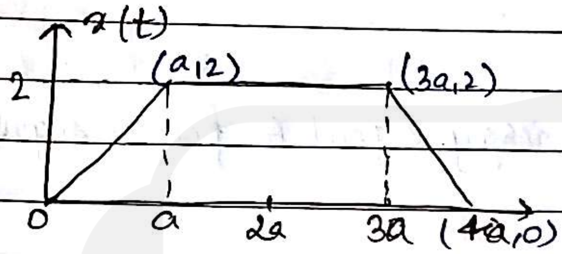
Ques)



$x(t) - 0 = \frac{2-0}{4-5}(t-5)$

$x(t) = -2(t-5)[u(t-4) - u(t-5)]$
$+ 2tu(t) + 2u(t-1)\{1-t\}$
$- 2u(t-4)$

Ques)



$x(t) - 0 = \frac{2-0}{3a-4a}(x-4a)$

$x(t) = \frac{2}{a}(4a-x)$

$$x(t) = \frac{2}{a}(t)[u(t) - u(t-a)] + 2[u(t-a) - u(t-3a)]$$
$$- \frac{2}{a}(t-4a)[u(t-3a) - u(t-4a)]$$

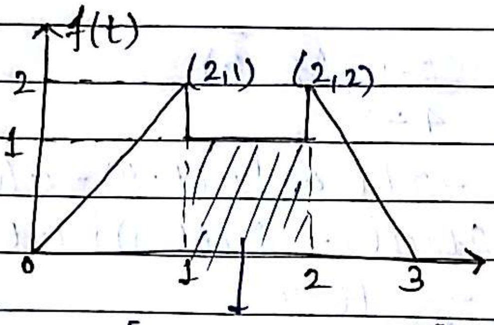
We know that, $r(t) = \begin{cases} tu(t) & , t > 0 \\ 0 & , t < 0 \end{cases}$

$$x(t) = \frac{1}{a}tu(t) - \frac{1}{a}tu(t-a) + u(t-a) - u(t-3a) - \frac{1}{a}tu(t-3a)$$
$$+ 4u(t-3a) + \frac{1}{a}tu(t-4a) - 4u(t-4a)$$

#

$$tu(t-a) = (t-a+a)u(t-a)$$
$$= \underbrace{(t-a)u(t-a)} + au(t-a)$$
$$= r(t-a) +$$

Ques)



$2[u(t-1)-u(t-2)]$

$$f(t)-0 = \frac{1-0}{2-0}(t-0)$$

$$f(t) = \frac{t}{2}[u(t-0)-u(t-1)]$$

Ques) Check whether the given signal is energy signal or power signal.

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

Energy, $\boxed{E_x = \sum_{n=-\infty}^{\infty} |x(n)|^2}$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \cdot 1$$

$$= 1 + \left(\frac{1}{4}\right)^1 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \cdots \quad \text{(Infinite G.P.)}$$

$$= \frac{a}{1-r} = \frac{1}{1-(1/4)} = \frac{4}{3}$$

Power, $\boxed{P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n}$

Sum of a finite G.P. $= \frac{a(1-r^n)}{(1-r)}$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\frac{1\left\{1-\left(\frac{1}{4}\right)^n\right\}}{1-1/4} \right]$$

$$= \frac{1}{\infty} = \underline{\underline{0}}$$

$$\lim_{t\to\infty} \frac{\sin 2t}{2t} = 0 \qquad \lim_{t\to 0} \frac{\sin 2t}{2t} = 1$$

Page Subhash

Date

Ques) Check whether the given signal is energy or power signal:-

$x(t) = A\cos t$

$$E_x = \int_{-\infty}^{\infty} A^2\cos^2 t \, dt = 2\int_0^{\infty} A^2 \cos^2 t \, dt$$

$$= 2A^2 \int_0^{\infty} \left(\frac{1+\cos 2t}{2}\right)^2 dt$$

$$= \frac{A^2}{2}\int_0^{\infty} (1 + \cancel{\cos^2 2t}\cos^2 2t)\, dt$$

$$= \frac{A^2}{2}\left[t\Big|_0^{\infty} + x\right.$$

$$= \infty$$

$\Rightarrow$ Energy signal

Ques) $x(t) = e^{j(t+\pi/4)} = e^{i\theta} = \cos\theta + i\sin\theta$

$\Rightarrow |e^{i\theta}| = |\cos\theta + i\sin\theta| = 1$

$$E_x = \int_{-\infty}^{\infty} 1^2 \, dt = \left[t\Big|_{-\infty}^{\infty}\right. = \infty$$

Energy signal

Now, $$P_x = \lim_{t\to\infty} \frac{1}{2T}\int_{-T}^{T} 1\, dt = \lim_{t\to\infty}\frac{1}{2T}\left[t\Big|_{-T}^{T}\right.$$

$$= \lim_{t\to\infty} \frac{1}{2T}\cdot 2T$$

$$= \lim_{t\to\infty} 1$$

$$= 1$$

Power signal

Date

→ Periodic and Aperiodic Signals -

A signal is said to be periodic if it repeats itself after a regular period of time

i.e. $x(t) = x(t+T)$, $x(n) = x(n+N)$

Fundamental time period (t_0) – The smallest value of time period t after which the signal starts repeating itself

eg. for $x(t) = \sin t$

$\boxed{t_0 = 2\pi}$

eg. $x(n) = e^{j\omega_0 n}$

$x(n+N) = e^{j\omega_0(n+N)} = e^{j\omega_0 n} + e^{j\omega_0 N}$

also, $\omega_0 = 2\pi f$

$\Rightarrow N = \frac{2\pi}{\omega_0}$

$\Rightarrow x(n+N) = e^{j\omega_0 n} + e^{j\frac{2\pi}{\cancel{N}}\cancel{N}}$ $[e^{i2\pi} = 1]$

$\boxed{N = \frac{2\pi}{\omega_0}(m)}$ → to make time integer as discrete values can only have integer values

fundamental time period

eg. $\sin\sqrt{2}\,n$

t_0 cannot be irrational, in terms of π and also not in terms of j (complex no.).

Ques) Check for the periodicity of $x(n) = \sin \omega n \cdot u(n)$

$u(n)$ exist only for $n \geq 0$.

whereas periodic fns exist from $(-\infty, \infty)$.

$\Rightarrow$ $\sin \omega n \cdot u(n)$ is aperiodic

Ques) $y(n) = y_1(n) + y_2(n) + y_3(n) + y_4(N)$

$N_1 = 40$, $N_2 = 64$, $N_3 = 50$, $N_4 = 100$

$\frac{N_1}{N_2} = \frac{40}{64} = \cancel{\frac{20}{27}} \ \frac{5}{8}$ | $\frac{N_1}{N_3} = \frac{40}{50} = \frac{4}{5}$ | $\frac{N_1}{N_4} = \frac{40}{100} = \frac{2}{5}$

ratios should be RATIONAL

A no. is said to be a rational no. if it can be written in the form p/q where p and q are integers, $p \neq 0$, $q \neq 0$.

To calculate t_0 of composite fn = LCM of denominator $\times$ t_0 of first sig

$$= \text{LCM}(8, 5, 5) \times 40$$
$$= 1600$$

Ques) $x(n) = x_1(n) + x_2(n) + x_3(n)$

$N_1 = 4$, $N_2 = 1.25$, $N_3 = \sqrt{2}s$

$\frac{N_1}{N_2} = \frac{4}{1.25}$

$\underbrace{\frac{N_1}{N_3} = \frac{4}{\sqrt{2}}}_{\text{irrational}}$

Date

Ques) Check for the periodicity $e^{j(t+\pi/4)\,u(n)}$

$u(n)$ → cannot be periodic

⇒ Signal is not periodic

Ques) $e^{(-1+j)n}$

$$= e^{j\left(\frac{-1}{j}+j\right)n} = e^{j\left(\frac{-1}{j}+1\right)n}$$

$$= e^{j\left(\frac{-j}{j^2}+1\right)n}$$

$$= e^{j(j+1)n}$$

$$\Rightarrow \boxed{\omega_0 = j+1}$$

→ Linear & Non-Linear Systems -

1)
2) Stable & Unstable
3) Static & Dynamic

A system is said to be linear if it follows the 'Superposition Theorem'.

Superposition Theorem implies that response resulting from several input signals can be computed as sum of responses resulting from each input signal alone.

If $x_n(t)$ is input, output should be $y_n(t)$.

i.e.

$$\text{i/p}\begin{cases} x_1(t) \rightarrow y_1(t) \\ x_2(t) \rightarrow y_2(t) \\ x_3(t) \rightarrow y_3(t) \end{cases}\text{o/p}$$

A system is linear if it follows the following steps-

1) $x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$ [Additive]

2) $\alpha x_1(t) \longrightarrow \alpha y_1(t)$ [Scaling property]

\# Superposition Principle states that response to weighted (scaling) sum of input signals must be equal to weighted sum of outputs corresponding to each of these individual input signals.

$ax_1(t) \rightarrow$, $bx_2(t) \rightarrow$ [sum] $\rightarrow ay_1(t) + by_2(t) \leftarrow$ [sum] $\leftarrow ax_1(t) + bx_2(t)$

eg. Check if $y(t) = t \cdot x(t)$ is a linear signal

Step 1: $ay_1(t) = at\, x_1(t)$ — (i)

$by_2(t) = bt\, x_2(t)$ — (ii)

Adding (ii) & (iii),

$ay_1(t) + by_2(t) = t\,[ax_1(t) + bx_2(t)]$ — (iii)

Step 2: $y_3(t) = t\,x_3(t)$

where $x_3(t) = ax_1(t) + bx_2(t)$

$\Rightarrow ay_1(t) + by_2(t) = t\,[ax_1(t) + bx_2(t)]$ — (iv)

eqⁿs (iii) & (iv) are same, hence $y(t) = t \cdot x(t)$ is a linear signal

Ques) check for linearity : $y(n) = 2x(n) - 3$

Step 1: $ay_1(n) = a2x_1(n) - 3$

Add $by_2(n) = b2x_2(n) - 3$

$ay_1(n) + by_2(n) = 2\,[ax_1(n) + bx_2(n) - 3]$ — (i)

Step 2: $y_3(n) = 2x_3(n) - 3$

$ay_1(n) + by_2(n) = 2\,[ax_1(n) + bx_2(n)] - 3$ — (ii)

eqⁿs (ii) & (iii) are not same,

$\Rightarrow y_1(t) = t \cdot x(t)$ is a non-linear system.

Ques) $y(n) = x^2(n)$

Step 1: $ay_1(n) = ax_1^2(n)$

$by_2(n) = \cancel{bg}\, bx_2^2(n)$

$ay_1(n) + by_2(n) = ax_1^2(n) + bx_2^2(n)$ —— (I)

Step 2: $y_3(n) = x_3^2(n)$

$ay_1(n) + by_2(n) = [ax_1(n) + bx_2(n)]^2$ —— (II)

(I) $\neq$ (II) $\Rightarrow$ non-linear

Ques) $y(n) = x(n+1) - x(n-1)$

Step 1: $ay_1(n) = a[x_1(n+1) - x_1(n-1)]$

$by_2(n) = b[x_2(n+1) - x_2(n-1)]$

$ay_1(n) + by_2(n) = a[x_1(n+1) - x_1(n-1)] + b[x_2(n+1) - \cancel{b}x_2(n-1)]$ —— (I)

Step 2: $y_3(n) = x_3(n+1) - x_3(n-1)$

$ay_1(n) + by_2(n) = \{ax_1(n+1) + bx_2(n+1)\}$
$+ \{ax_2(n-1) + bx_1(n-1)\}$ —— (II)

(I) = (II) $\Rightarrow$ Linear

Ques) $\frac{dy}{dt} + 3t\,y(t) = t^2 x(t)$

$a\frac{dy_1}{dt} + 3at\,y_1(t) = at^2 x_1(t)$

$b\frac{dy_2}{dt} + 3bt\,y_2(t) = bt^2 x_2(t)$

$a\frac{dy_1}{dt} + b\frac{dy_2}{dt} + 3at\,y_1(t) + 3bt\,y_2(t) = at^2 x_1(t) + bt^2 x_2(t)$

→ Time variant & Time in-variant System-

A system is said to be time invariant if the behaviour and characteristics of system are fixed overtime.

OR

A system is said to be time invariant if time-shifting input signal results in an identical time shift in the output.

How to check for time variancy-

1) Delay (only in input)
2) Replace t by $(t-t_0)$ in both input and output

eg. $y(t) = t.x(t)$

i) Delay: $y(t,t_0) = t\, x(t-t_0)$ — ①

ii) Replace: $y(t-t_0) = (t-t_0)\, x(t-t_0)$ — ⑪

① ≠ ⑪ ⇒ Time variant

Ques) $y(t) = x(t)\sin t$

$y(t,t_0) = x(t-t_0)\sin t$ — ①

$y(t-t_0) = x(t-t_0)\sin(t-t_0)$ — ⑪

① ≠ ⑪ ⇒ Time variant

Ques) $y(t) = x(t^2)$

$y(t,t_0) = x(t^2-t_0)$ — ①

$y(t,t_0) = x(t-t_0)^2$ — ⑪

① ≠ ⑪ ⇒ Time variant

→ Invertible & Non-Invertible System-

In case multiple inputs give a signal output, then it is not possible to predict the responding input by looking at output. Such systems are non-invertible systems.

→ Laplace Transform-

It is a powerful tool in solving circuit problems.
It is generally used in solving differential of circuit.
It transforms time signal into s domain where s is a laplace operator having operator $(s = \pm\sigma \mp j\omega)$.
OR we can say that it transforms time domain signal to frequency domain.

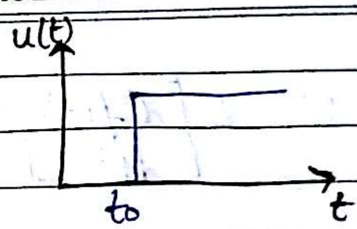
Formulae-

$$\boxed{L[f(t)] = F(s) = \int_0^\infty f(t) \cdot e^{-st}\, dt}$$

1) $L[1] = 1/s$
2) $L[K] = K/s$
3) $L[u(t)] = 1/s$
4) $L[\delta(t)] = 1/s$
5) $L[t] = 1/s^2$
6) $L[r(t)] = 1/s^2$
7) $L[e^{\pm at}] = \dfrac{1}{s \mp a}$
8) $L[\sin\omega t] = \dfrac{\omega}{s^2+\omega^2}$
9) $L[\cos\omega t] = \dfrac{s}{s^2+\omega^2}$
10) $L[e^{at}\sin\omega t] = \dfrac{\omega}{(s-a)^2+\omega^2}$
11) $L[e^{at}\cos\omega t] = \dfrac{s-a}{(s-a)^2+\omega^2}$
12) $L[e^{\pm at} f(t)] = F(s \mp a)$

Date

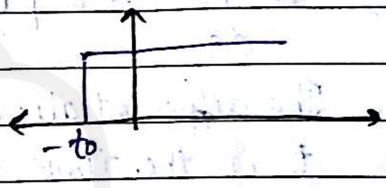
Ques) $L[u(t-t_0)] = ? = \int_0^\infty u(t-t_0)\, e^{-st} dt$



$$= \int_{t_0}^\infty u(t-t_0) e^{-st} dt$$

$$= \int_{t_0}^\infty 1 \cdot e^{-st} dt = \frac{e^{-t_0 s}}{s}$$

Ques) $L[u(t+t_0)] = ? = \int_0^\infty u(t+t_0) e^{-st} dt$



$$= \int_{-t_0}^\infty u(t-t_0) e^{-st} dt$$

$$= \frac{e^{t_0 s}}{s}$$

\# Time-shifting property – In case any func^n is delayed/advanced by a time factor say t_0, then the LT of this fn will be $e^{\pm t_0 s}$ times LT of normal function.

i.e. $\boxed{L[f(t \pm t_0)] = e^{\pm t_0 s} F(s)}$

Properties of Laplace Transform –

1) Multiplication by a constant : $L[K f(t)] = K \cdot F(s)$

2) Sum and difference : $L[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$

3) Frequency shifting :

4) $L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^+)$

5) $L\left[\frac{d^2 f(t)}{dt^2}\right] = s^2F(s) - s\,f(0^+) - f'(0)$

6) $L\left[\int_0^t f(t)\,dt\right] = \frac{f(0^+)}{s} + \frac{1}{s}F(s)$

7) $\frac{dF(s)}{ds} = -L\left[t \cdot f(t)\right]$ [Diff. in s domain]

The differentiation in s domain corresponds to multiplication by t in the time domain.

8) $\int_s^{\infty} F(s)\,ds = L\left[\frac{f(t)}{t}\right]$ [~~Diff.~~ Integration in s domain]

The Integration of F(s) in s domain corresponds to division by t in time domain

Initial Value Theorem-

$$\boxed{f(0^+) = \lim_{s\to\infty} sF(s)}$$

Final Value Theorem-

$$\boxed{f(\infty) = \lim_{s\to 0} sF(s)}$$

1. $f(t) = 9 - 2e^{-5t}$

$$F(s) = \frac{9}{s} - \frac{2}{s+5} = \frac{9s+45-2s}{s^2+5s} = \frac{7s+45}{s(s+5)}$$

Now, $f(0^+) = \lim_{s\to\infty} \cancel{s}\cdot\frac{7s+45}{\cancel{s}(s+5)} = \lim_{s\to\infty} \frac{\cancel{s}(7+45/s)}{\cancel{s}(1+5/s)}$

$$= \boxed{7}$$

Date

Ques) $sF(s) = \frac{(2s+1)s}{s^2+2s+5}$

$$\lim_{s\to\infty} \frac{s^2(2+2/s)}{s^2(1+2/s+5/s^2)} = \boxed{2}$$

Ques) Obtain initial and final value and $f(t)$.

$$F(s) = (5s+3)/s(s+1)$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s+3}{(s+1)} = 5$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{5s+3}{s+1} = 3$$

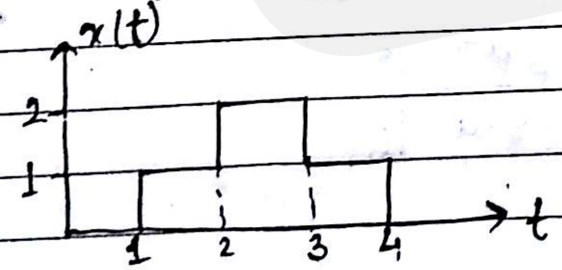
Now, $F(s) = \frac{5s+3}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$

$\Rightarrow F(s) = \frac{3}{s} + \frac{2}{s+1}$

Applying Inverse Laplace, we get

$f(t) = 3 + 2e^{-t}$

Ques) Obtain Laplace Transform of given waveform –



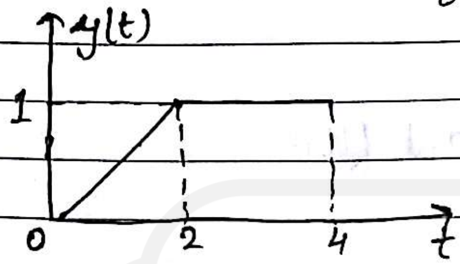
$x(t) = 1[u(t-1) - u(t-2)] + 2[u(t-2) - u(t-3)] + 1[u(t-3) - u(t-4)]$

$x(t) = u(t-1) + u(t-2) - u(t-3) - u(t-4)$

Date

$$X(s) = \frac{1}{s}e^{-1s} + \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s} - \frac{e^{-4s}}{s}$$

Ques) Obtain the Laplace Transform



$$y(t) = \frac{1}{2}\left[u(t) - u(t-2)\right] + \left[u(t-2) - u(t-4)\right]$$

$$= \frac{1}{2}\left[r(t) - (t-2+2)u(t-2)\right]$$

$$\Rightarrow \quad y(s) = \frac{1}{2}\left[\frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s}\right]$$

ues) Find out the LT of $x(t) = u(t-3a) + \delta(t+3a) + r(t-4a)$

$$X(s) = \frac{1}{s}e^{-3as} + e^{3as} + \frac{1}{s^2}e^{-4as}$$

ues) $$F(s) = \int_s^{\infty} e^{-at} u(t-b) e^{-st}\, dt$$

$$= \int_b^{\infty} e^{-at} e^{-st}\, dt = \int_s^{\infty} e^{-t(s+a)}\, dt$$

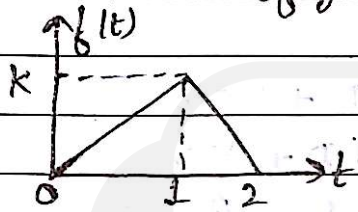
$$= \left[\frac{e^{-(s+a)}}{(-s+a)}\right]_b^{\infty}$$

Date

Ques) $i(t) = 1.5\,(1-e^{-4t})\,u(t) - 1.5\,[1-e^{-4(t-0.1)}]\,u(t-0.1)$

$$I(s) = \frac{1.5}{s} - \frac{1.5}{s+4} - \frac{1.5\,e^{-0.1s}}{s} + \frac{1.5}{s+4}e^{-0.1s}$$

Ques) Obtain the LT of given waveform



$f(t) = K\,tu(t) - 2K\,(t-1)u(t-1)$
$+\; K(t-2)u(t-2)$

since $tu(t) = r(t)$

$\Rightarrow f(t) = Kr(t) - 2Kr(t-1) + Kr(t-2)$

$$F(s) = \frac{K}{s^2} - \frac{2K\,e^{-s}}{s^2} + \frac{Ke^{-2s}}{s^2}$$

$$F(s) = K\left\{\frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}\right]$$

Unit-II

Page: Subbash
Date:

25/Aug/17

→ Solⁿ of Differential Eqⁿs –

I) First order & second order homogeneous eqⁿ –

$\frac{dy(x)}{dt} + y(t) = 0$

Replace D $+ 1 = 0$

$\boxed{P_1 = -1}$

$C.F. = ke^{P_1 t}$

$C.F = ke^{-t}$

$\frac{d^2y(x)}{dt} + 3\frac{dy(x)}{dt} + 2y(t) = 0$

$D^2 + 3D + 2 = 0$

$(D+1)(D+2) = 0$

$P_1 = -1, \quad P_2 = -2$

$C.F. = k_1 e^{P_1 t} + k_2 e^{P_2 t}$

$= k_1 e^{-t} + k_2 e^{-2t}$

II) First/second order Non-homogeneous eqⁿ containing constant or exponential fn on right hand side –

Case I: ~~Constant~~ Exponential fn on right side

~~Make~~ Put RHS=0

Find PI and C.F.

Solⁿ = PI + C.F.

eg. $\frac{dy(t)}{dt} + y = 5e^{2t} \,/\, e^{at}$

A.E. $D + 1 = 0$

$D = -1$

1) $C.F = ke^{-t}$

2) $P.I. = \frac{5e^{2t}}{D+1}$

Replace D by 2 (i.e. replace D by a for e^{at})

Date

$\Rightarrow$ $P.I. = \dfrac{5e^{2t}}{2+1} = \dfrac{5e^{2t}}{3}$

Now, $y = CF + PI = ke^{-t} + \dfrac{5e^{2t}}{3}$

eg. $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = e^{2t}$

A.E $\Rightarrow$ $D^2 + 3D + 2 = 0$

$(D+1)(D+2) = 0$

$D = -2, -1$

Now, $P.I = \dfrac{e^{2t}}{D^2+3D+2}$

Put $D =$

II] Constant on RHS –

eg. $\dfrac{dy}{dt} + y = 5$

A.E] $D + 1 = 0$

$D = -1$

$C.F = ke^{-t}$

$P.I. = \dfrac{5e^{0t}}{D+1}$

eg.) $\dfrac{d^2y}{dt^2} + \dfrac{3dy}{dt} + 2y = 5$

A.E. $D^2 + 3D + 2 = 0$

$D = -1, -2$

$P.I. = \dfrac{5e^{0t}}{D^2+3D+2}$

Put $D = 0$

$\Rightarrow P.I. =$

Type 3: First & Second order Non-homogeneous eq^n with RHS as a sine or cosine fn.

eg. $\frac{dy}{dt} + yt = \sin 5t$

A.E $\Rightarrow$ $D+1=0$

$D=-1$

$C.F = Ke^{-t}$

$P.I. = \frac{\sin 5t}{D+1}$

Replace D^2 by $-a^2$,

To do so, we need to make $D \to D^2$,

$\therefore$ Rationalizing the fraction,

$$= \frac{\sin 5t}{(D+1)} \frac{(D-1)}{(D-1)}$$

$$= \frac{(D-1)\sin 5t}{D^2-1}$$

Put $D^2 = -a^2 = -(5)^2$

$$= \frac{(D-1)\sin 5t}{-25-1}$$

$$= \frac{(D-1)\sin 5t}{-26}$$

Equating $D = \frac{d}{dt}$,

$$P.I. = \frac{d}{dt}\left(\frac{\sin 5t}{-26}\right) - \frac{\sin 5t}{(-26)}$$

$$= \frac{-1}{26} 5\cos 5t + \frac{\sin 5t}{26}$$

$$P.I. = \frac{\sin 5t}{26} - \frac{5\cos 5t}{26}$$

and, $\{ y = C.F. + P.I. \}$

II^nd Order:-

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \sin 5t$$

$$D^2 + 3D + 2 = 0$$

$$P.I. = \frac{\sin 5t}{D^2 + 3D + 2}$$

Put $D^2 = -(5)^2$

$$= \frac{\sin 5t}{-25 + 3D + 2}$$

$$= \frac{\sin 5t}{3D - 23}$$

$$= \frac{\sin 5t}{(3D - 23)} \cdot \frac{(3D + 23)}{(3D + 23)}$$

$$= \frac{(3D + 23)\sin 5t}{9D^2 - 529}$$

$$= \frac{(3D + 23)\sin 5t}{-225 - 529}$$

Again equate $D = \frac{d}{dt}$ & solve.

→ Circuit Analysis by Classical Method-

Transient Response- Whenever a circuit is switched from one condition to another, then there is a transition period during which the current & voltage change from former values to new one. This period is known as "Transition time" & the value of current & voltage are known as "Transient Response".

Steady State Response - When the transient period is over OR the transition has died out, the circuit is said to be in steady state.
~~Zero input response~~

Zero Input Response- The value of current & voltage which results from initial condition when applied input is zero.

Zero State Response- The values of current & voltage for an input which is applied when all the initial conditions are zero. Such networks/circuits are said to be at rest.

→ **Order of Differential Eqⁿ**-
If a circuit contains only one storage element (L or C) ⇒ 1st order
" two " ⇒ 2nd order

- The circuit changes are assumed ~~a~~to occur at time $t=0$ & represented by a switch.

- Switching ON/OFF at $t=0$ does not disturb the storage elements so that $I(0^-) = I(0^+)$.

In case of capacitor $V_c(0^-) = V_c(0^+)$

Date

→ Voltage - current relationship for storage elements -

1) $V = IR$; $I = V/R$

2) $V_L(t) = L\dfrac{dI(t)}{dt}$; $I(t) = \dfrac{1}{L}\displaystyle\int_{-\infty}^{t} V_L(t)\,dt$

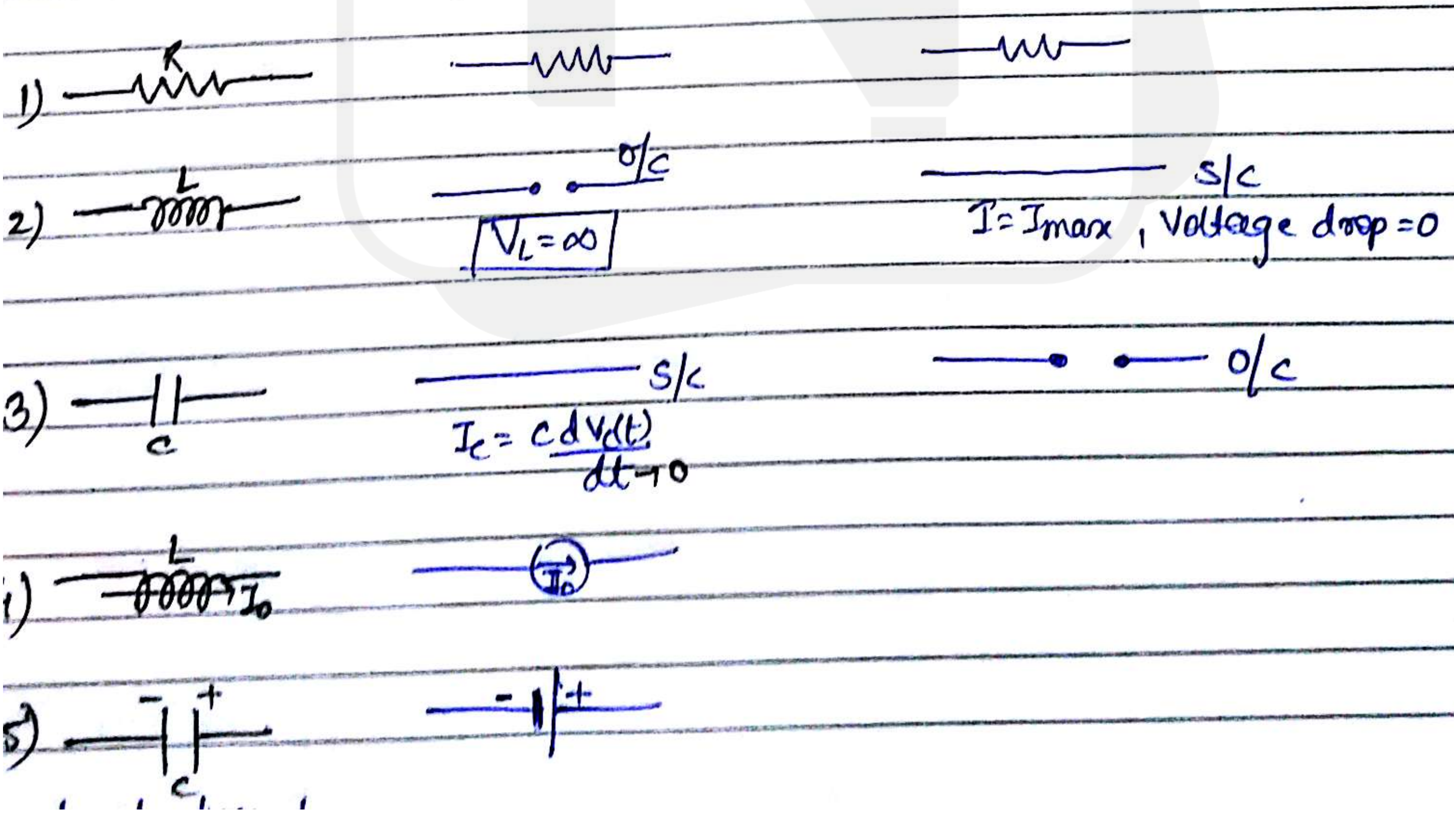
or $I(t) = \dfrac{1}{L}\displaystyle\int_{0}^{t} V_L(t)\,dt + I_L(0^-)$

3) $V_C(t) = \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} I_C(t)\,dt$; $I_C(t) = C\cdot\dfrac{dV_C(t)}{dt}$

$= \dfrac{1}{C}\displaystyle\int_{0}^{t} I_C(t)\,dt + V_C(t)$

→ Initial / Boundary conditions in a circuit -

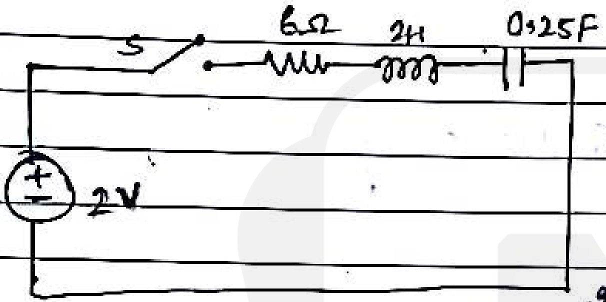
Circuit Element | Eq. ckt at $t = 0^+$ | Eq. ckt. at $t = \infty$



→ ~~Circuit Analysis~~

Ques Calculate $i(0^+)$, $\frac{di}{dt}(0^+)$, $\frac{d^2i}{dt}(0^+)$, $i(t)$ → transient response

of a given circuit where source is of 2V and resistance is of 6Ω, inductor = 2H, capacitor = 0.25F when the switch S is closed at time t=0.



Applying KVL,

$$V_s = 6i(t) + 2\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)dt + V_c(0^-)$$

$$2 = 6i(t) + 2\frac{di}{dt} + \frac{4}{8}\int_0^t i(t)dt + \underbrace{V_c(0^-)}_{0}$$ —— (1) (no initial charge)

$$2 = v(t)16 + 16\int_0^t v(t)dt + 4\frac{dv(t)}{dt}$$

$$2 = 4\frac{dv(0^+)}{dt} \Rightarrow \boxed{\frac{dv(0^+)}{dt} = \frac{1}{2}}$$

Double diff.,

$$0 = 16\frac{dv(t)}{dt} + \frac{1}{2}v(t)dt + 4\frac{d^2v(t)}{dt^2}$$

$$0 = 16\left(\frac{1}{2}\right) + 0 + \cancel{4\frac{d}{dt}}\ 4\frac{d^2v(t)}{dt^2}$$

$$\boxed{\frac{d^2v(t)}{dt^2} = -2}$$

~~Applying KVL,~~

~~$V = Ri(t) + L\frac{di(t)}{dt}$~~

~~$V = Ri + LDi$~~

~~$LD + R = 0$~~

~~$D = -\frac{R}{L}$~~

~~$\Rightarrow i(t) = Ke^{-\frac{R}{L}t} + \frac{V}{R}$~~

~~C.F. = $Ke^{D_1 t}$~~

~~$= Ke^{-\frac{R}{L}t}$~~

~~P.I. = $\frac{V}{LD+R}$~~

~~Put $D=0$~~

~~$= \frac{V}{0+R} = \frac{V}{R}$~~

——— x ———

Applying $i(0^+) = 0$

Because a $t=0^+$, L behaves as open ckt. (Initial condⁿ)

Putting $t = 0^+$,

i.e. at $i(0^+)$

eqⁿ (1) becomes, $2 = 6\underbrace{i(0^+)}_{\to 0} + 4\int_0^t \underbrace{i(0^+)}_{\to 0}dt + 2\frac{di(0^+)}{dt}$

$\cancel{2} = \cancel{2}\frac{di(0^+)}{dt}$

or $\boxed{\frac{di(0^+)}{dt} = 1}$ — (II)

Double diff. of eqⁿ (1),

$0 = 6\frac{di(t)}{dt} + 2\frac{d^2 i(t)}{dt^2} + 4i(t)$ — (III)

Put $\frac{d}{dt} = D$,

$6Di + 2D^2(i) + 4(i) = 0$

or $3D + D^2 + 2 = 0$

$$D^2 + 3D + 2 = 0$$
$$(D+1)(D+2) = 0$$
$$D = -1, \quad D = -2$$

Now, $C.F = k_1 e^{-D_1 t} + k_2 e^{-D_2 t}$

$$i(t) = k_1 e^{-t} + k_2 e^{-2t} \quad \text{—— (IV)}$$

Apply initial condition in eq^n (IV), [To determine values of k_1 & k_2]

i.e. $t = 0$

$$\underbrace{i(0^+)}_{0} = k_1 + k_2 \quad \text{—— (V)}$$

Diff. eq^n (IV) w.r.t t,

$$\frac{di(t)}{dt} = -k_1 e^{-t} - 2k_2 e^{-2t}$$

Put $t=0$,

$$\underbrace{\frac{di(0^+)}{dt}}_{1 \text{ (from (II))}} = -k_1 - 2k_2$$

$$1 = -k_1 - 2k_2$$

and, $0 = k_1 + k_2$

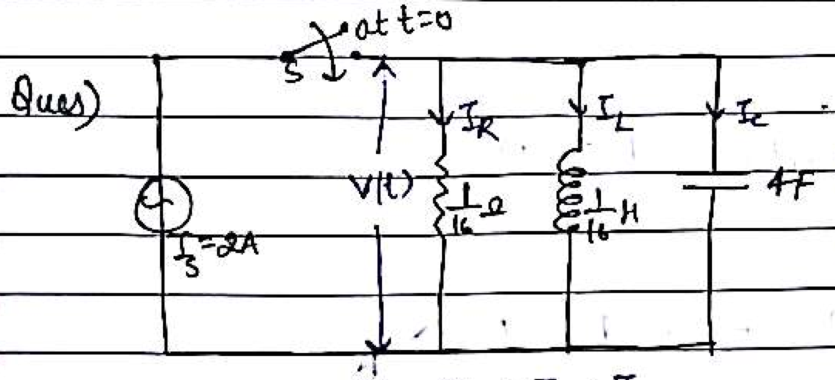
$$1 = -k_2$$

$\Rightarrow \quad k_1 = 1, \quad k_2 = -1$

Putting these values in eq^n (IV),

$$\boxed{i(t) = e^{-t} - e^{-2t}}$$

Ques)



$i(0^+)=?$

$v(t)=?$

$\frac{dv(0^+)}{dt}=?$

$\frac{d^2v(0^+)}{dt^2}=?$

$I_S = I_R + I_L + I_C$

Applying KCL,

$$\underset{I_S}{\underset{\parallel}{2}} = \underset{I_R}{\underset{\parallel}{\frac{v(t)}{R}}} + \underset{I_L}{\underset{\parallel}{\left\{\frac{1}{L}\int_0^t v(t) + \underbrace{I_L(0^-)}_{\to 0}\right\}}} + \underset{I_C}{\underset{\parallel}{C\cdot\frac{dv(t)}{dt}}}$$

Since all three elements are connected in parallel, voltage across each is same.

$$\Rightarrow \quad 2 = 16v(t) + 16\int_0^t v(t)dt + 4\frac{dv(t)}{dt} \quad \text{—(I)}$$

At initial cond^n, C acts as short circuited element.

$\Rightarrow R_C = 0$

$\Rightarrow$ Entire current flows through this branch

$\Rightarrow$ Voltage across s/c = 0.

$\Rightarrow \quad v(0^+)=0$

Putting $v(0^+)=0$ in eq^n (I),

$$2 = 0 + 0 + 4\frac{dv(0^+)}{dt}$$

or $$\boxed{\frac{dv(0^+)}{dt} = \frac{1}{2}} \quad \text{—(II)}$$

Date

Double diff. of eq^n ①,

$$0 = 16\frac{dv(t)}{dt} + 16v(t) + 4\frac{d^2v(t)}{dt^2}$$

Put $v(0^+) = 0$

$$0 = 16 \cdot \underbrace{\frac{dv(0^+)}{dt}}_{= \frac{1}{2}} + 0 + 4\frac{d^2v(0^+)}{dt^2}$$

$$0 = 8 + 4\frac{d^2v(0^+)}{dt^2}$$

$$\Rightarrow \boxed{\frac{d^2v(0^+)}{dt^2} = -2}$$

~~0 = 16~~ Diff. eq^n ①,

$$0 = 16\frac{dv(t)}{dt} + 16v(t) + 4\frac{d^2v(t)}{dt^2}$$

$$0 = 16D + 4D^2 + 16$$

or $D^2 + 4D + 4 = 0$

$(D+2)^2 = 0$

or $D = -2$

$CF = K_1e^{-Dt} + K_2e^{-Dt}$

$v(t) = K_1e^{-2t} + tK_2e^{-2t}$ — (iii)

Applying initial cond^n, ~~te~~ $v(0^+) = 0$

$t = 0^+$

$$\underbrace{v(0^+)}_{0} = K_1 + 0K_2$$

$$\boxed{K_1 = 0}$$

Diff. eq^n (iii) w.r.t. t,

$$\frac{dv(t)}{dt} = -2K_1e^{-2t} + \left(K_2e^{-2t} - 2K_2te^{-2t}\right)$$

Date

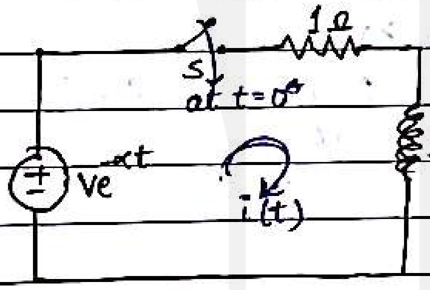
Again put $t = 0^+$,

$$\frac{dv(0^+)}{dt} = -\alpha \underbrace{k_1}_{0} + K_2$$

or $$\boxed{\frac{1}{\alpha} = k_2}$$ {from (II)}

$$\Rightarrow \boxed{v(t) = \frac{1}{2} t e^{-\alpha t}}$$

Ques) For a given ckt., voltage source is an exponential fn $v(t) = Ve^{-\alpha t}$ where α is constant, the switch S is closed at $t=0$, $R = 1\Omega$, $L = 1H$. The ckt. is shown below. Calculate $i(t)$.



Applying KVL,

$$Ve^{-\alpha t} = i(t)R + L\frac{di(t)}{dt}$$

$$\Rightarrow Ve^{-\alpha t} = i(t) + \frac{di(t)}{dt} \quad \text{—(I)}$$

or $0 = D + 1$

$D = -1$

$\therefore$ $CF = ke^{-t}$

$$P.I. = \frac{Ve^{-\alpha t}}{-\alpha + 1}$$

or $$i(t) = ke^{-t} + \frac{Ve^{-\alpha t}}{1-\alpha} \quad \text{—(II)}$$

Put $t = 0$,

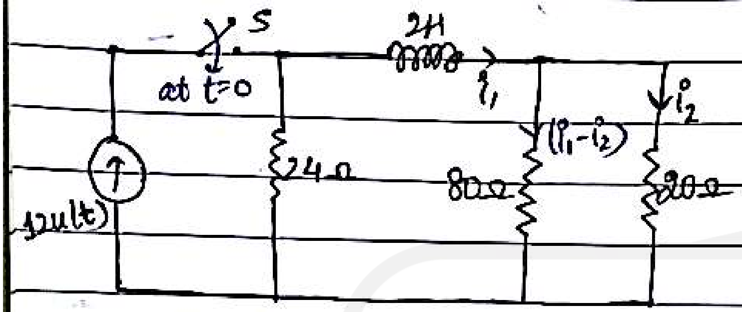
$$0 = K + \frac{V}{1-\alpha} \quad \text{or} \quad K = \frac{V}{\alpha - 1}$$

$$\Rightarrow i(t) = \frac{Ve^{-t}}{\alpha - 1} + \frac{Ve^{-t}}{1-\alpha}$$

Ques) Calculate the current i_1, i_2 and voltage V_1 of the given network

i) at $t=0^+$ iii) at $t=\infty$

ii) at $t=0^-$ iv) at $t=50ms$



Since input ~~volt~~ signal is an unit signal (which exists only for $t \geq 0$);

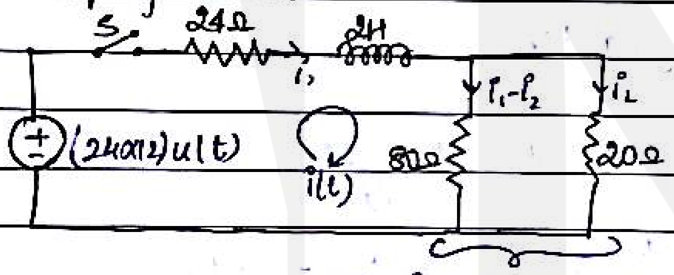
$\Rightarrow i_1(0^-)=0, \; i_2(0^-)=0$

$V(0^-)=0$

~~Applying KVL,~~ Also at $t=0^+$, L behaves as open ckt.

$i_1(0^+)=0; \; i_2(0^+)=0$

Simplified ckt:



Applying KVL,

$$288 = 24i_1 + 2\frac{di_1}{dt} + 80(i_1-i_2) \quad \text{---} (1)$$

$$i_2 = i_1 \times \frac{80}{80+20} = 0.8i_1$$

$\Rightarrow$ eqn (1) becomes, $288 = 24i_1 + 2\frac{di_1}{dt} + 80(i_1 - 0.8i_1)$

$$288 = 24i_1 + 2\frac{di_1}{dt} + 16i_1$$

$$144 = 20i_1 + \frac{di_1}{dt}$$

Now, $D+20=0$

$D=-20$

Now, ~~$i_1(t)$~~ CF $= ke^{-20t}$

$P.I. = \frac{144}{20} = 7.7$

~~$i(t) =$~~

Now, $i_1(t) = ke^{-20t} + 7.7$

at $t=0^+$, $i(0^+)=0$

$0 = k + 7.7$

$\Rightarrow k = -7.7$

$$\boxed{i_1(t) = -7.7e^{-20t} + 7.7}$$

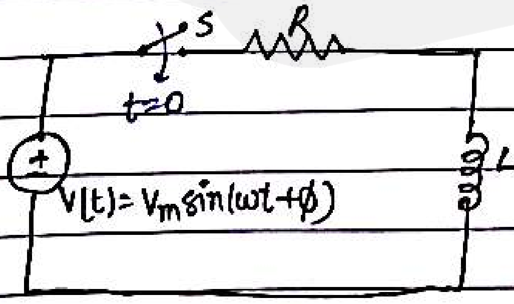
Now, at $t=\infty$, $i_1(\infty) = -7.7\underbrace{e^{-\infty}}_{\to 0} + 7.7$

$\therefore i_1(\infty) = 7.7mA$.

and, $i_2(t) = \frac{4}{5} i_1$

$\therefore i_2(\infty) = \frac{4}{5} \times \frac{144}{20} = \frac{144}{25}$

Ques) Consider a series RL ckt. excited by sinusoidal voltage $v(t) = V_m \sin(\omega t + \phi)$ as shown in the ckt.



Applying KVL,

$V(t) = iR + L\frac{di}{dt}$

$\frac{V(t)}{L} = \frac{R}{L}i + \frac{di}{dt}$

At $D + \frac{R}{L} = 0$

$\Rightarrow D = -R/L$

$$C.F = ke^{-\frac{R}{L}t}$$

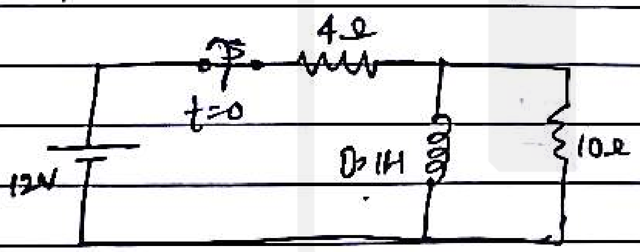
$$P.I. = \frac{v(t)}{(D+R/L)} \times \frac{(D-R/L)}{(D-R/L)}$$

$$= \frac{(D-R/L)\,v(t)}{D^2-(R/L)^2}$$

Replace D^2 with $-\omega^2$,

$$i(t) = ke^{-\frac{R}{L}t} - \frac{V_m}{R^2+\omega^2L^2}\left[\omega L\cos(\omega t+\phi) - R\sin(\omega t+\phi)\right]$$

Ques) A 12V battery is suddenly disconnected at t=0. Find the inductor current ~~at t=0~~ and $V_L(t)$.



at $t=\infty$, L behaves as short circuit
⇒ no current flows through 10Ω

⇒ $i = \frac{12}{4} = 3A$

Applying KVL at $t=0^-$,

→ Solutions of Circuits Using Laplace Transform:-

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = 5u(t) \quad \text{——} \ (1)$$

Taking Laplace transform on both sides,

$$L\left(\frac{d^2y}{dt^2}\right) = s^2Y(s) - sy(0^+) - y'(0^+)$$

$$L\left(\frac{dy}{dt}\right) = sY(s) - y(0^+)$$

$$L\,y(t) = Y(s)$$

$$L\{5u(t)\} = \frac{5}{s}$$

$\therefore$ Eq^n (1) becomes,

$$s^2Y(s) - sy(0^+) - y'(0^+) + 3sY(s) - 3y(0^+) + 2Y(s) = \frac{5}{s} \quad \text{——} \ (11)$$

Given that: Value of $y(0^+) = -1$

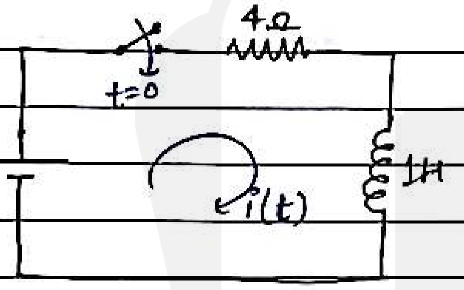
$y'(0^+) = 2$

$\therefore$ Eq^n (11) becomes,

$$s^2Y(s) - s(-1) - (2) + 3sY(s) + 2Y(s) = 5/s$$

or $\quad Y(s)\left[s^2 + 3s + 2\right]$

Ques)



Assume initial current through inductor $= 3A$

Applying KVL,

$$48 = 4i(t) + \frac{di(t)}{dt} \quad \text{—— (1)}$$

Taking Laplace on both sides,

$$\frac{48}{s} = 4\,I(s) + sI(s) - i(0^+)$$

$$\frac{48}{s} = (s+4)\,I(s) - 3$$

$$I(s) = \frac{3s+48}{s(s+4)}$$

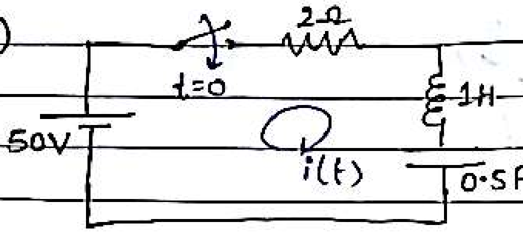
$$= \frac{A}{s} + \frac{B}{s-4}$$

$$= \frac{12}{s} - \frac{9}{(s+4)}$$

→ Applying Inverse LT,

$$I(t) = 12 - 9e^{-4t}$$

Ques)



$$50 = 2i(t) + \frac{di(t)}{dt} + 2\int_0^t i\,dt$$

Applying Laplace,

$$2I(s) + [sI(s) - \underbrace{i(0^+)}_{0}] + 2\frac{I(s)}{s} = \frac{50}{s}$$

$$I(s)\left[2 + s + \frac{2}{s}\right] = \frac{50}{s}$$

$$I(s)\left[s^2 + 2s + 2\right] = 50$$

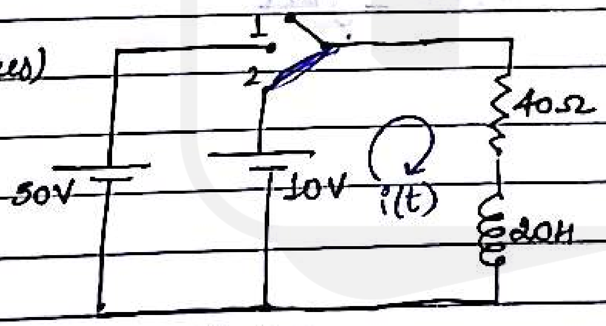
or $$I(s) = \frac{50}{s^2+2s+2}$$

$$= \frac{50}{\{(s+1)^2 + 1\}}$$

Applying Inverse LT,

$$= 50(\sin t)e^{-t}$$

Ques)



Switch is held in pos. 1 for a long time, then it is moved to pos. 2.

$$i(0^+) = i(0^-) = \frac{50}{40} = 5/4$$

Applying KVL,

$$10 = 40i(t) + 20\frac{di(t)}{dt}$$

Applying LT,

$$\frac{10}{s} = 4I(s) + 20[sI(s) - i(0^+)]$$

Date

$$\frac{10}{s} = 40\,I(s) + 20s\,I(s) - \cancel{20}^{\,5}\left(\frac{5}{\cancel{4}}\right)$$

$$\frac{10}{s} + 25 = I(s)\left[40 + 20s\right]$$

$$\frac{25s + 10}{20s^2 + 40s} = I(s)$$

$$I(s) = \frac{5s+2}{4s(s+2)}$$

Applying partial fraction,

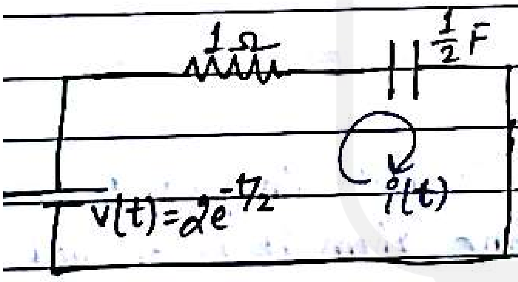
$$I(s) = \frac{1}{4s} + \frac{1}{s+2}$$

Applying ILT,

$$\boxed{i(t) = \frac{1}{4} + e^{-2}}$$

Ques) Find the current $i(t)$ if the voltage source is $v(t) = 2e^{-t/2}$

$v_c(0^-) = 0$, $R = 1$, $C = \frac{1}{2}$ F.



$$v(t) = 1\,i(t) + 2\int_0^t i\,dt$$

Graph Theory

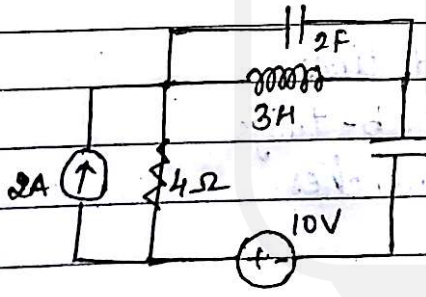
A graph is a geometrical representation of any network / circuit which consists of branchs and nodes.

Branch: Circuit elements like R, C, L are replaced by straight lines known as branch which does not give any idea about the type of element. Representation - 'b'.

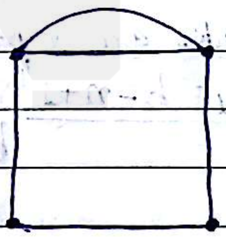
Node: The end points of a branch or common point of intersection of two or more branches.
Nodes are represented by 'n'.

→ How to draw a graph?

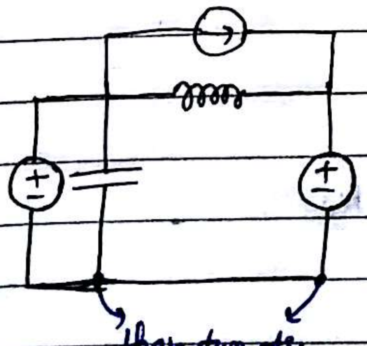
Step-1) Replace all the ckt. elements by st. lines.
2) Replace ideal voltage source by short ckt.
3) Replace ideal current source by open ckt.
4) If two elements are in series, then these elements can be replaced by either one branch or by two branches.



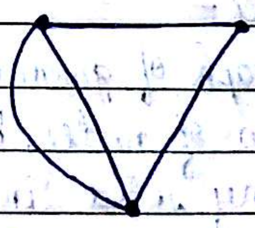
→



$n = 4$
$b = 5$

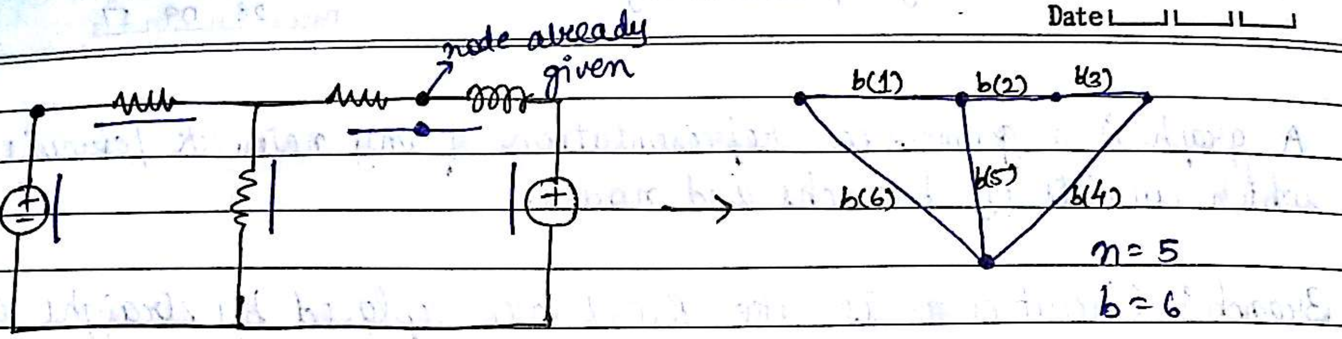


→



Always try to minimize the no. of nodes

Date



→ Planar Graph – A graph is said to be planar if there are no cross lines when drawn on paper.

→ Non-planar Graph – A graph which cannot be drawn on a paper without cross lines

→ Tree –

- A tree is open set of interconnected branches without any closed loop
- In tree, there exists only one path b/w any two nodes.
- The tree branch is represented by solid lines & is known as Twig.
- No. of twigs, $\boxed{n_t = n-1}$

  n = no. of nodes

- Link or Co-tree – Links are represented by dotted lines
- No. of links, $\boxed{n_l = b-n+1} = b-(n-1) = b - \text{twigs}$

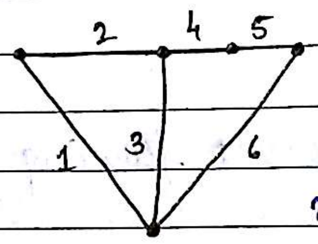
  b = total no. of branches

Graph is union of Tree & Co-tree.

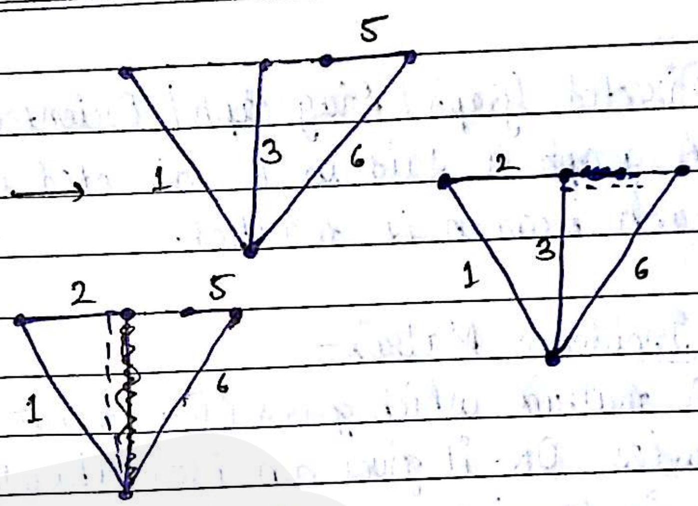
eg. To draw a tree –

i) Draw the graph of given circuit
ii) Count the no. of nodes, branches, twigs, links
iii) Draw as many solid lines as n_t.
iv) " dotted lines as n_l.

eg.

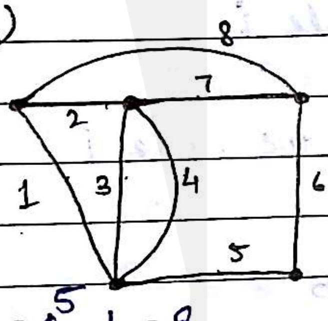


$n = 5$
$b = 6$
$n_t = 4$
$n_l = 2$

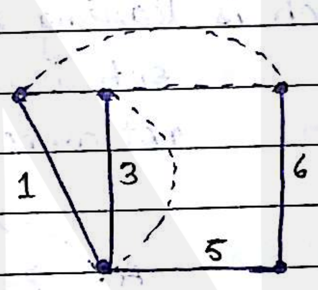


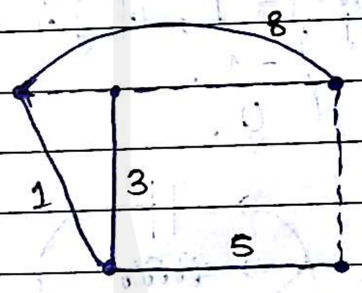
A single graph can have multiple no. of trees

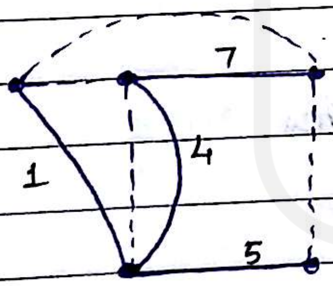
Ques)

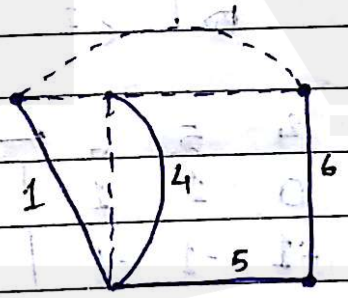


$n = 5, b = 8$
$n_t = 4, n_l = 4$









Date

→ **Directed Graph / Diagraph / Oriented Graph** –

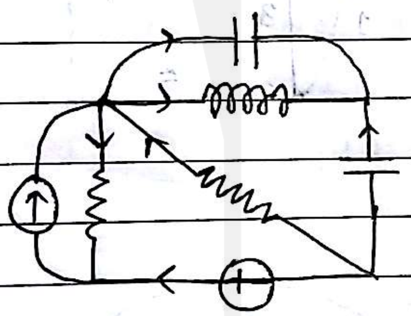
A graph is said to be directed if dirn of current flowing in each branch is marked.

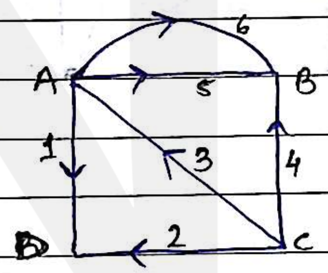
→ **Incidence Matrix** –

- A matrix which gives idea about orientation of branches to the nodes OR it gives an idea about a particular branch whether it is leaving or entering a particular node.
- Represented by '$[A_c]$'

$$a_{ij} = \begin{cases} +1, & \text{if } j\text{th branch is leaving the node } i \\ -1, & \text{,, entering ,,} \\ 0, & \text{,, not connected to the node } i \end{cases}$$

eg.





$$[A_c] = \begin{array}{c} \text{nodes} \downarrow \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \leftarrow \text{branches} \\ \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

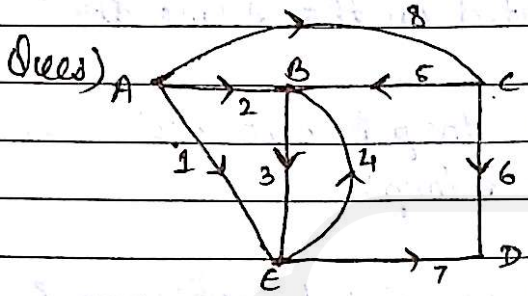
If direction of branches is not specified in the graph, we can assume any direction.

Date

→ Properties of complete incidence matrix [Ac] -

1) Sum of each column of [Ac] is zero.
2) Determinant of [Ac], ie. $|A_c| = 0$ always.

Ques)



	1	2	3	4	5	6	7	8
A	1	1	0	0	0	0	0	1
B	0	-1	1	-1	-1	0	0	0
C	0	0	0	0	1	1	0	-1
D	0	0	0	0	0	-1	-1	0
E	-1	0	-1	1	0	0	1	0

→ Reduced Incidence Matrix [A] -

- If a node in a graph is taken as reference node or datum node, then the incidence matrix obtained will be reduced IM.
- Any node of given graph can be taken as reference node if this does not change the behaviour of the circuit.

No. of Trees of a graph = Det $[A\,A^T]$

- Reduced IM can also be written as $[A] = [\underset{\text{twigs}}{A_t} \cdot \underset{\text{links}}{A_l}]$

→ Fundamental Loop Matrix / ~~Friseet~~ Tie-set Matrix -

- It is a matrix used to find branch current in terms of link current for a given tree of a graph.
- It is formed by only one link associated with other twigs.
- No. of tie-sets or fundamental loops = no. of links
- KVL is applicable in each loop

Procedure for obtaining tie-sets—

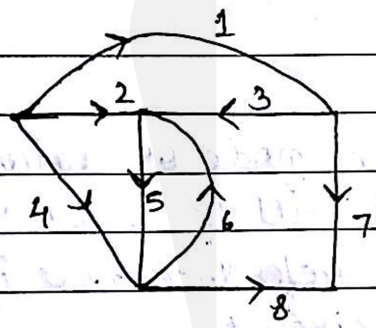
i) Draw the graph

ii) Form the tie-sets with each link in the graph for the entire tree.

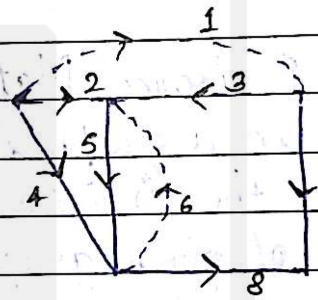
iii) Assume the direction of tie-sets oriented in the same dir^n as that of link.

iv) $b_{ij} = \begin{cases} -1, & \text{if the jth branch is in the loop i, but dir}^n \text{ are different (for branch \& loop)} \\ +1, & \text{dir}^n \text{ is same} \\ 0, & \text{if the jth branch is not associated with loop i.} \end{cases}$

eg.



$n = 5$
$b = 8$
$n_t = 4$
$n_l = 4$



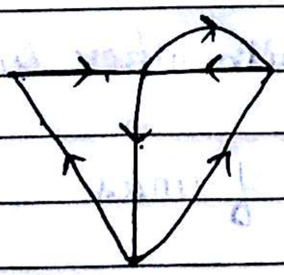
Tie-set matrix is represented by $[B_f]$

tie-sets ↓

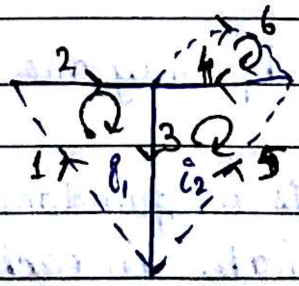
		1	2	3	4	5	6	7	8
(2,4,5)	i_1	0	+1	0	-1	+1	0	0	0
(5,6)	i_2	0	0	0	0	1	1	0	0
(5,7,3,8)	i_3	0	0	+1	0	+1	0	-1	+1
(1,4,7,8)	i_4	1	0	0	-1	0	0	1	-1

Each tie-set loop chosen should have only one dotted line.

Ques)



$n = 4$
$b = 6$
$n_l = 3$
$n_t = 3$



Date

$$[B_f] = \begin{array}{c|cccccc} & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \\ & J_1 & J_2 & J_3 & J_4 & J_5 & J_6 \\ & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline i_1 & 1 & 1 & 1 & 0 & 0 & 0 \\ i_2 & 0 & 0 & +1 & +1 & +1 & 0 \\ i_3 & 0 & 0 & 0 & 1 & 0 & 1 \end{array}$$

<u>NOTE:</u> 1) Dir^n of link determines the dir^n of loop.

2) No. of loops = no. of links

3) $\boxed{B_f = [\underbrace{B_{ft}}_{\text{twigs}} : \underbrace{B_{fl}}_{\text{links}}] = [B_{ft} : U]}$

4) Tie-set gives branch current in terms of links current.

5) If $j_1, j_2 \dots j_8$ are the corresponding branch currents, then (for B_f matrix shown above),

$J_1 = i_1$ $\quad$ $J_3 = i_1 + i_2$ $\quad$ $J_5 = i_2$

$J_2 = i_1$ $\quad$ $J_4 = i_2 + i_3$ $\quad$ $J_6 = i_3$

6) Tie-set matrix gives loop eq^n by adding voltage of each branch in a particular tie set.

Let $V_1, V_2 \dots V_6$ be the branch voltages.

Then

$V_1 + V_2 + V_3 = 0$, for i_1

$V_3 + V_4 + V_5 = 0$, for i_2

$V_4 + V_6 = 0$, for i_3

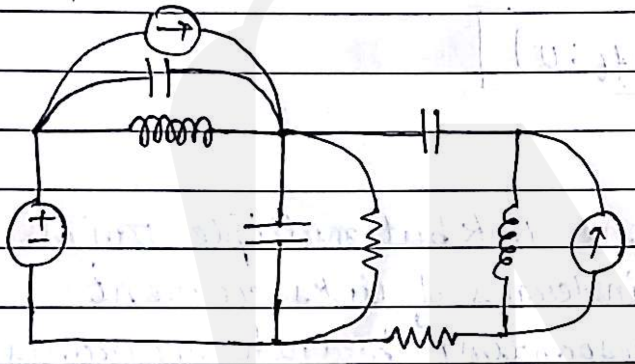
\# row-wise sum $\rightarrow$ voltage

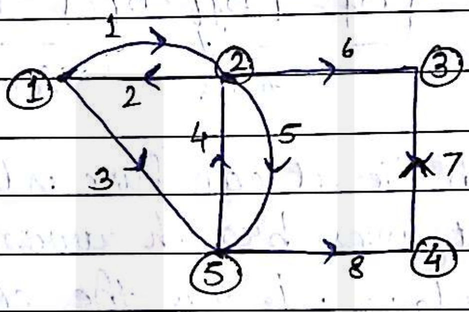
column wise sum $\downarrow$ current

→ Procedure for obtaining a Cut-set matrix :-

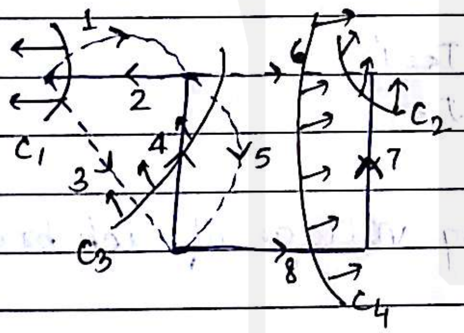
1) Draw the graph of the given circuit.
2) Select a tree.
3) Count the no. of twigs; ~~and~~ the no. of cut-sets are equal to the no. of ~~trees~~ twigs.
4) Dir^n of cut-set depends upon the dir^n of twig.
5) One cut-set must have a twig but not more than one.

eg.





$n = 5$
$b = 8$
$n_t = 4$
$n_l = 4$



$$q_{ij} = \begin{cases} +1, & \text{if } j\text{th branch is included in the cut-set } i \text{ \& the dir}^n \text{ is same} \\ -1, & \text{dir}^n \text{ opposite} \\ 0, & j\text{th branch not associated with cut-set } i \end{cases}$$

Cut-set matrix,

$$Q_f = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} -1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & +1 & 0 \\ 0 & 0 & -1 & +1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 1 \end{bmatrix} \end{array}$$

Cut-set matrix can also be written as:

$$[Q_f] = [Q_{ft} : Q_{fl}] = [U : Q_{fl}]$$

$$[Q_f] = \begin{array}{c} \begin{array}{cccccccc} 2 & 7 & 4 & 8 & 1 & 3 & 5 & 6 \end{array} \\ \left[\begin{array}{cccc:cccc} -1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & +1 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & +1 \end{array}\right] \end{array}$$

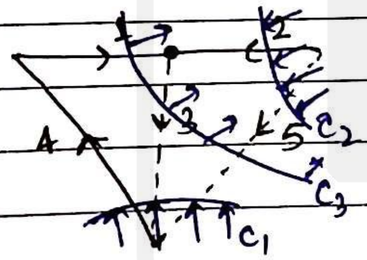
To check whether our $[Q_f]$ is correct, verify the presence of atleast one diagonal matrix in the matrix $[Q_{ft}]$.

NOTES:

1) KCL eq^n can be obtained from $[Q_f]$.
If $I_1, I_2 \ldots I_8$ be branch currents, then for each row

$$\left.\begin{array}{l} -I_1 + I_2 - I_3 = 0 \\ I_6 + I_7 = 0 \\ -I_3 + I_4 - I_5 - I_6 = 0 \\ I_6 + I_8 = 0 \end{array}\right\} \quad [Q_f I_b] = 0$$

Ques)



$$Q_f = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & -1 \end{array}\right] \end{array}$$

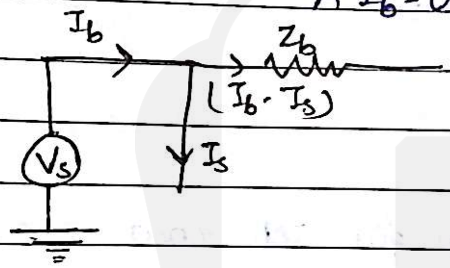
$$Q_f = \begin{array}{c} \begin{array}{ccccc} 4 & 2 & 1 & 3 & 5 \end{array} \\ \left[\begin{array}{ccc:cc} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right] \end{array}$$

Date

→ Interrelation b/w various matrices & various parameters —

1) $[A] = [A_t : A_l]$
2) $[B_f] = [B_{ft} : B_{fl}] = [B_{ft} : U]$
3) $[Q_f] = [Q_{ft} : Q_{fl}] = [U : Q_{fl}]$
4) KVL eq^n : $B_f V_b = 0$
5) KCL eq^n :- $Q_f I_b = 0$

$A I_b = 0$

6) 

$$\boxed{\begin{aligned} V_b &= V_s + Z_b(I_b - I_s) \\ I_b &= I_s + Y_b(V_b - V_s) \end{aligned}}$$

7) A and B_f are orthogonal i.e. $\boxed{A(B_f)^T = 0}$
8) B_f and Q_f are orthogonal i.e. $\boxed{B_f(Q_f)^T = 0}$

→ Relation b/w matrices :

1) $A(B_f)^T = 0$

$[A_t : A_l]\begin{bmatrix} B_{ft}^T \\ U \end{bmatrix} = 0$ $[U^T = U]$

$A_t B_f^T + A_l = 0$

or $\boxed{B_{ft}^T = -A_l(A_t^{-1})}$ — ①

(2) $B_f.(Q_f)^T = 0$

$[B_{ft} : U]\begin{bmatrix} U \\ Q_{fl}^T \end{bmatrix} = 0$

$B_{ft} + Q_{fl}^T = 0$

$\boxed{Q_{fl}^T = -B_{ft}}$ — ⑪

3) A and Q_f

from ① & ⑪, $\boxed{Q_{fl} = A_l(A_t^{-1})}$

Relation among parameters

① Relation b/w link voltages & twig voltages.

V_l & V_t

$$B_f V_b = 0$$

$$[B_{ft} : U] \begin{bmatrix} V_t \\ \vdots \\ V_l \end{bmatrix} = 0$$

$$B_{ft} V_t + V_l = 0$$

$$\Rightarrow \boxed{V_l = -B_{ft} V_t}$$

Also,

$$\Rightarrow \boxed{V_l = Q_{fl}^T V_t} \qquad \left\{ \because \boxed{B_{ft} = -Q_{fl}^T} \right\}$$

② Relation b/w V_b & V_t (branch, twig)

$$[V_b] = \begin{bmatrix} V_t \\ \vdots \\ V_l \end{bmatrix}$$

$$[V_b] = \begin{bmatrix} V_t \cdot U \\ \vdots \\ Q_{fl}^T V_t \end{bmatrix} = V_t \begin{bmatrix} U \\ \vdots \\ Q_{fl}^T \end{bmatrix}$$

$$\boxed{V_b = V_t Q_f^T}$$

③ V_b & V_l

$$[V_b] = \begin{bmatrix} V_t \\ \vdots \\ V_l \end{bmatrix} = \begin{bmatrix} -B_{ft}^{-1} V_l \\ \vdots \\ U \cdot V_l \end{bmatrix} = V_l \begin{bmatrix} -B_{ft}^{-1} \\ U \end{bmatrix}$$

$$\boxed{V_b = -B_f^{-1} V_l}$$

(4) I_t & I_l

$$Q_f I_b = 0$$

$$[U \vdots Q_{fl}] \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix} = 0$$

$$I_t + Q_{fl} I_l = 0$$

$$\boxed{I_t = -Q_{fl} I_l}$$

(5) ~~$A I_b = 0$~~

$$A I_b = 0$$

$$[A_t \vdots A_l] \begin{bmatrix} I_t \\ I_l \end{bmatrix} = 0$$

$$A_t I_t + A_l I_l = 0$$

~~$A_t = -A_l I_l$~~

$$\boxed{I_t = A_l A_t^{-1} I_l}$$

$$\therefore \boxed{I_t = -Q_{fl} I_l = A_l A_t^{-1} I_l = B_{ft}^T I_l}$$

(6) I_b & I_l

$$[I_b] = \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix} = \begin{bmatrix} B_{ft}^T I_l \\ \vdots \\ I_l U \end{bmatrix} = B_f^T I_l$$

$$\boxed{I_b = B_f^T I_l}$$

(7) V_b & V_n

$$\Rightarrow \boxed{V_b = A^T V_n}$$

Network Analysis using Graph theory:

$$V_b = V_s + Z_b (I_b - I_s) \quad \text{--- (1)}$$

$$I_b = I_s + Y_b (V_b - V_s) \quad \text{--- (2)}$$

where, V_b is the branch voltage matrix, of size 6×1

I_b : Branch current matrix, of size 6×1

V_s : voltage source matrix, of size 6×1
I_s : current source matrix, — 6×1
$Z_b \rightarrow$ branch impedence matrix, — of 6×6

$Y_b \rightarrow$ branch Admittance matrix, of 6×6

* Loop Analysis

$$V_b = V_s + Z_b (I_b - I_s)$$

Premultiply by fundamental tie set matrix :—

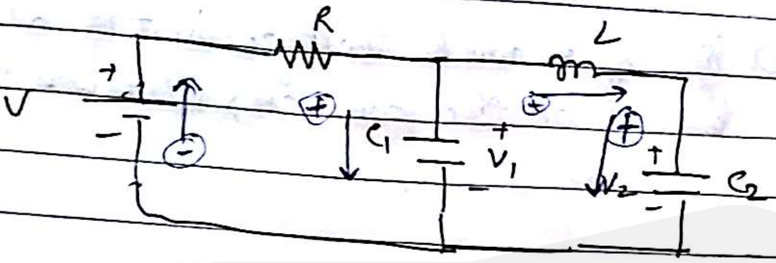
$$B_f V_b = B_f V_s + B_f Z_b (I_b - I_s)$$

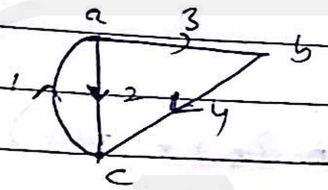
$$B_f V_b = B_f V_s + B_f Z_b (B_f^T I_\ell - I_s)$$

$$0 = B_f V_s + B_f Z_b B_f^T I_\ell - B_f Z_b I_s$$

$$\underbrace{B_f Z_b I_s - B_f V_s}_{V_\ell} = \underbrace{B_f Z_b B_f^T}_{Z_\ell} I_\ell$$

Q Obtain the matrix loop eqn of the given network :-

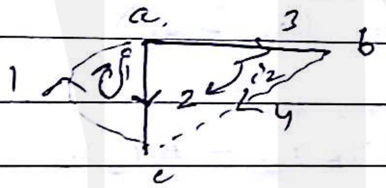




Impedance of $C_1 \Rightarrow \frac{1}{C_1 s}$

$n = 3$
$b = 4$
$t = 2$
$l = 2$

Impedance of Inductor $\Rightarrow Ls$



$$V_s = \begin{bmatrix} -V \\ V_1 \\ 0 \\ V_2 \end{bmatrix}$$

$$B_f = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

$$Z_b = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & \frac{1}{C_1 s} & 0 & 0 \\ 0 & 0 & Ls & 0 \\ 0 & 0 & 0 & \frac{1}{C_2 s} \end{bmatrix} \end{array}$$

$$I_s = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$I_l = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$V_l = -B_f V_s$ $\qquad$ $Z_l = B_f Z_b B_f^T$

$$V_l = \begin{bmatrix} V - V_1 \\ V_1 - V_2 \end{bmatrix}$$

$$Z_l = \begin{bmatrix} R + \frac{1}{C_1 s} & -\frac{1}{C_1 s} \\ -\frac{1}{C_1 s} & Ls + \frac{1}{C_1 s} + \frac{1}{C_2 s} \end{bmatrix}$$

Sign Convention for Voltage source matrix

The polarity of branch voltage is +ve at the tail end of the arrow on the Branch, otherwise -ve.

* Nodal Analysis

$$I_b = I_s + Y_b(V_b - V_s) \quad \text{——} (1)$$

Premultiply (1) by the reduced incidence matrix.

$$A I_b = A I_s + A Y_b (V_b - V_s)$$

$$0 = A I_s + A Y_b V_b - A Y_b V_s$$

$$\{V_b = A^T V_n\}$$

$$0 = A I_s + A Y_b A^T V_n - A Y_b V_s$$

$$\underbrace{A Y_b V_s - A I_s}_{I_n} = \underbrace{A Y_b A^T}_{Y_n} V_n$$

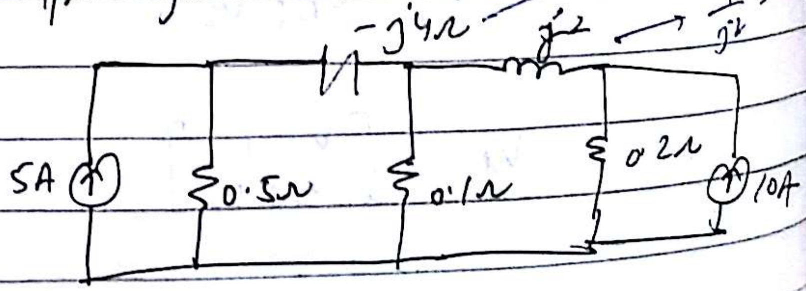
$$\boxed{I_n = Y_n V_n}$$

Matrix Node eqn

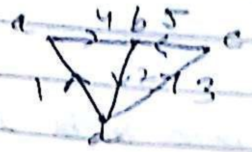
Q Write matrix node eqn for a given network:-

$\frac{1}{-j4} = \frac{j}{4}$

$\frac{1}{j2} = -0.5j$



Sol^n



$n = 4$
$b = 5$

Taking 'd' node as datum / reference node :-

$$[A] = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline a & -1 & 0 & 0 & 1 & 0 \\ b & 0 & 1 & 0 & -1 & -1 \\ c & 0 & 0 & -1 & 0 & 1 \end{array}$$

$$[Y_b] = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 10 & 0 & 0 & 0 \\ 3 & 0 & 0 & 5 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0.25j & 0 \\ 5 & 0 & 0 & 0 & 0 & -0.5j \end{array}$$

$$V_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad I_s = \begin{bmatrix} 5 \\ 0 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

$I_n = [\ \ ]$
$Y_n = [\ \ ]$

✩ Cut Set Analysis

$$I_b = I_s + Y_b(V_b - V_s)$$

Premultiplying ~~bot~~ by Q_f :-

$$Q_f I_b = Q_f I_s + Q_f Y_b V_b - Q_f Y_b V_s$$

$$\{V_b = Q_f^T V_t\}$$

$$\cancel{0 =}$$

$$0 = Q_f I_s + Q_f Y_b Q_f^T V_t - Q_f V_b V_s$$

$$Q_f V_b V_s - Q_f I_s = \underbrace{Q_f Y_b Q_f^T}_{} V_t$$

matrix cut set matrix

$$\boxed{I_t = Y_t V_t}$$

Duality

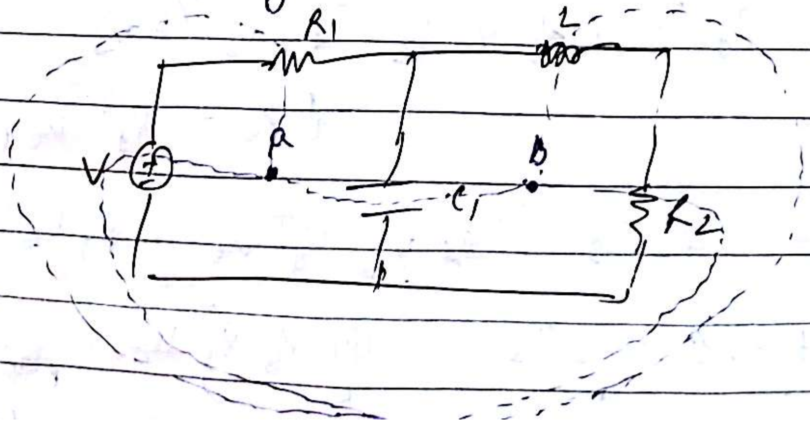
Two networks are said to be dual of each other if incidence matrix of one network is same as that of other.

Procedure to obtain the dual of a network :

Steps
1. Mark (•) in the centre of each loop.
2. Mark an extra dot (•) outside the ckt, which is known as reference / datum node.
3. Draw the dotted line such that each ckt element is cut by only one line
4. Draw the dual network by connecting all the nodes to the dual of ckt element.

series ⟷ parallel
V ⟷ I
R ⟷ G
L ⟷ C
node ⟷ loop
KCL eqn ⟷ KVL eqn

tree ⟷ co-tree
twig ⟷ link



$P_2(s) = s^3 + 2s^2 + 2s + 1$

$s^2 = -4$

$= \pm 2j$

$\pm\sqrt{2}\,\sqrt{j}$

$P(s)$

$s^4 + 4$

$= (s^2 - 2s + 2)(s^2 + 2s + 2)$

$\downarrow$ Non-Hurwitz $\downarrow$ Hurwitz

$\therefore P(s)$ is non Hurwitz

$\therefore P(s)$ is also non-Hurwitz.

Positive Real function

A function is said to be +ve real if ~~satisfied~~ satisfy the following condition:-

* If $T(s)$ is real for 's' is real.
* $T(s)$ polynomial is always Hurwitz polynomial $D(s)$ (check only denominator)
* $T(s)$ may have poles on $j\omega$ axis.
* These poles are simple poles & their residues are real & +ve.
* The real part of $T(s)$ is ≥ 0

Necessary & sufficient conditions for a function to be positive real are as follows:-

Condition 1 $T(s)$ must not have any poles in the right half of the 's' plane, i.e. denominator $D(s)$ must be Hurwitz polynomial

Page No.
Date / /

4. Only simple poles can exist on the imaginary axis.

5. The poles and zeros of a PRF are real or occur in conjugate pairs.

6. The difference in highest powers of N(s) & D(s) may differ at most by unity.

7. The difference in lowest powers of N(s) & D(s) may differ at most by unity.

Ques Check whether the given fn is a PRF :-

(i) $$F(s) = \frac{s^2+6s+8}{s^2+4s+3}$$

C1 $D(s) = s^2+4s+3$

Yes, it is Hurwitz

C2 ✓

C3 $M_1(s) = s^2+8$

$N_1(s) = 6s$

$M_2(s) = s^2+3$

$N_2(s) = 4s$

$$M_1 M_2 - N_1 N_2 = (s^2+8)(s^2+3) - 24s^2 \geq 0$$

$$= s^4+11s^2+24-24s^2 \geq 0$$

$$= s^4-13s^2+24 \geq 0$$

put $s=j\omega$

$\Rightarrow \omega^4+13\omega^2+24 \geq 0$ for all ω

$\therefore$ it is a PRF

4. Only simple poles can exist on the imaginary axis.

5. The poles and zeros of a PRF are real or occur in conjugate pairs.

6. The difference in highest powers of $N(s)$ & $D(s)$ may differ at most by unity.

7. The difference in lowest powers of $N(s)$ & $D(s)$ may differ at most by unity.

Ques Check whether the given fn is a PRF :-

(i) $F(s) = \dfrac{s^2+6s+8}{s^2+4s+3}$

C1 $D(s) = s^2+4s+3$

Yes, it is Hurwitz

C2 ✗

C3 $M_1(s) = s^2+8$

$N_1(s) = 6s$

$M_2(s) = s^2+3$

$N_2(s) = 4s$

$M_1M_2 - N_1N_2 = (s^2+8)(s^2+3) - 24s^2 \geq 0$

$= s^4+11s^2+24-24s^2 \geq 0$

$= s^4-13s^2+24 \geq 0$

put $s=j\omega$

$\Rightarrow \omega^4+13\omega^2+24 \geq 0$ for all ω

$\therefore$ it is a PRF

(ii) $Z(s) = \dfrac{s^3+5s^2+9s+3}{s^3+4s^2+7s+9}$

Sol:

C1 s^3+4s^2+7s+9

$E(s) = 4s^2+9$

$O(s) = s^3+7s$

$$4s^2+9 \overline{\big) s^3+7s \big(} \frac{s}{4}$$

$$s^3+\frac{9}{4}s$$

$$26s \overline{\big) 4s^2+9}$$

Yes it is Hurwitz

C3: $m_1(s) = 5s^2+3$

$n_1(s) = s^3+9s$

$m_2(s) = 4s^2+9$

$n_2(s) = s^3+7s$

$\Rightarrow (5s^2+3)(4s^2+9) - (s^3+9s)(s^3+7s)$

$\Rightarrow 20s^4+57s^2+27 - s^6 - 16s^4 - 63s^2$

$\Rightarrow -s^6+20s^4-6s^2+27$

$s = j\omega$

$-j^6\omega^6+20\omega^4+6\omega^2+27$

$\omega^6+20\omega^4+6\omega^2+27 \geq 0$ for all ω

Yes it is PRF

(ii) $F(s) = \frac{2s^2+5}{s(s^2+1)}$

C1 $D(s) = s^3+s$

yes Hurwitz

C2 roots $= \pm j$

Solving by partial

$= \frac{A}{s} + \frac{Bs+C}{s^2+1}$

$= \frac{5}{s} + \frac{-3}{s^2+1}$

Note :- check only ~~poles~~ residues at imaginary poles only i.e. for B & C here 'A' doesn't matter.

As the residue of complex poles are -ve, ∴ NOT a PRF

Check for +ve Realness :-

Q $F(s) = \frac{s+2}{s+1}$

C1 $s=-1$ ∴ lying on left side ∴ Hurwitz

C2 ✓

C3 ? put $s=j\omega$

$Re[F(j\omega)] \geq 0$

$\Rightarrow \frac{j\omega+2}{j\omega+1} \frac{(j\omega-1)}{(j\omega-1)}$

$= \frac{-\omega^2+j\omega-2}{(-2)} \Rightarrow \frac{-\omega^2-2}{-2} \Rightarrow \frac{\omega^2+2}{2} \geq 0$

∴ yes PRF.

Q $y(s) = \frac{s^2+2s+20}{s+10}$

S) C1 $s=-10$ Hurwitz

C3 $M_1(s) = s^2+20$
$N_1(s) = 2s$
$M_2(s) = 10$
$N_2(s) = s$

$(s^2+20)(10) - (2s)(s)$

$10s^2 + 200 - 2s^2$

$8s^2 + 200$

put $s = j\omega$

$-8\omega^2 + 200 \not> 0$ not always

Q $F(s) = \frac{s^2+s+6}{s^2+s+1}$

C1 ~~Hurwitz~~

C2 ✗

C3 $M_1(s) = s^2+6$

$N_1(s) = s$

$M_2(s) = s^2+1$

$N_2(s) = s$

$(s^2+6)(s^2+1) - s^2$ $\qquad$ $\omega^4 - 6\omega^2 + 6 \not> 0$

$s^4+7s^2+6-s^2$

s^4+6s^2+6

As we can't conclude whether the real part of $Re[F(j\omega)] \geq 0$ or not.
In that case, Sturm's test is done to check the true realness.

$$\omega^4 - 6\omega^2 + 6$$

put $\omega^2 = x$

$$A_0(x) = x^2 - 6x + 6$$

$$\frac{dA_0}{dx} = A_1(x) = 2x - 6$$

$$\boxed{\frac{A_0(x)}{A_1(x)} = (K_1 x - K_0) - \frac{A_3(x)}{A_0(x)}}$$

$$\frac{A_0(x)}{A_1(x)}$$

$$\begin{array}{r|l} & \frac{x}{2} - \frac{3}{2} \\ 2x-6 & x^2 - 6x + 6 \\ & x^2 - 3x \\ \hline & -3x + 6 \\ & -3x + 9 \\ \hline & -3 \end{array}$$

$$\frac{A_0(x)}{A_1(x)} = (2x-6) + \frac{(-3)}{x^2 - 6x + 6}$$

$$A_3(x) = 3$$

	A_0	A_1	A_3	sign changes
$x = 0$	+	−	+	2
$x = \infty$	+	+	+	0

If same; then PRF
otherwise it can't be PRF.

Unit-IV

Page No.
Date / /

7/Oct/17

Network Functions–

$H(s) = \frac{N(s)}{D(s)}$

Laplace operator

$$= \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \dots + b_m}$$

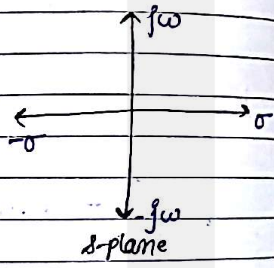
zeroes

$$= \frac{a_0 (s-z_1)(s-z_2)\dots\dots(s-z_n)}{b_0 (s-p_1)(s-p_2)\dots\dots(s-p_m)}$$

Poles

$= \frac{a_0}{b_0} = K$ (scale factor)

$\boxed{s = \pm\sigma \pm j\omega}$



zeroes represented by o
poles " x

eg. $N(s) = \frac{2s(s+1)}{(s+2)(s^2+2s+2)}$

Roots of numerator gives zeroes
Roots of denominator give poles

$\therefore z_1$ at $s=0$; z_2 at $s=-1$

Page No.
Date / /

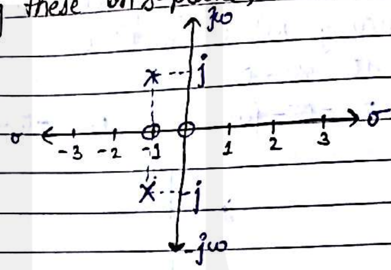
for poles,

$P_1 = -2$

$P_2 = (s^2+2s+2) = 0$

$P_2 = -1+j, -1-j$

Plotting these on s-plane,



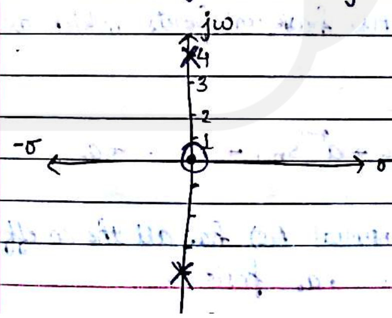
Ques) $N(s) = \frac{2s}{(s^2+16)} = \frac{2s}{(s+j4)(s-j4)}$

scale factor

Scale factor: Ratio of co-efficient of highest power terms of N^r & D^r.

Zero is at $s=0$.

Poles: $P_1 = -j4$, $P_2 = +j4$



Page No.
Date / /

Ques) $f(t) = e^{-\sigma t} \cos\omega t$

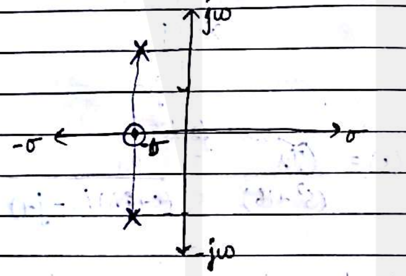
$$L\{f(t)\} = \frac{s+\sigma}{(s+\sigma)^2+\omega^2}$$

$$= \frac{s+\sigma}{(s+\sigma+j\omega)(s+\sigma-j\omega)}$$

Putting $N^r = 0$ & $D^r = 0$

Z_1 at $s = -\sigma$

$P_1 = -\sigma - j\omega$ $\quad P_2 = -\sigma + j\omega$



Properties of Hurwitz Polynomial–

It is a polynomial satisfying the following conditions– P(s)

1) P(s) is real when s is real
2) Roots of P(s) have real parts which are 0 or -ve.

Properties–

$P(s) = a^n s_n + a^{n-1} s_{n-1} + \dots + a_0$

1) the polynomial P(s) has all the co-efficients $a^n, a^{n-1}, \dots, a_0$ positive.

Page No.
Date / /

2) There should not be any missing terms unless the polynomial is even or odd.

3) Both even & odd parts of Hurwitz polynomial have roots only on $j\omega$ axis.

4) The continued fraction expansion of the ratio of odd to even parts or even to odd parts of a Hurwitz polynomial must give all the positive quotient terms.

5) If P(s) is Hurwitz poly and W(s) is a multiplicative factor, then
P(s) W(s) is also a Hurwitz poly. if
W(s) is also a Hurwitz poly.

6) In case of either only even or only odd poly., it is not possible to obtain continued fraction expansion.
In such cases, a poly. P(s) is Hurwitz if the ratio of P(s) and its derivative gives a continued fraction expansion with all positive quotient terms.

eg. Check whether the given poly. is Hurwitz or not
$P(s) = s^4 + s^3 + 5s^2 + 3s + 4$

Step 1) All the co-efficients are +ve.
Step 2) Not applicable

3) even part : $s^4 + 5s^2 + 4$ — $M(s)$
odd part : $s^3 + 3s$ — $N(s)$

Now find $\frac{M(s)}{N(s)}$ → because $M(s)$ has higher degree

Otherwise find $\frac{N(s)}{M(s)}$

$$
\begin{array}{l}
s^3+3s \,)\; s^4 + 5s^2 + 4 \;(\; s \\
\qquad\quad -s^4 + 3s^2 \\
\qquad\qquad 2s^2 + 4 \,)\; s^3 + 3s \;(\; \tfrac{1}{2}s \\
\qquad\qquad\qquad\quad s^3 + 2s \\
\qquad\qquad\qquad\qquad s \,)\; 2s^2 + 4 \;(\; 2s \\
\qquad\qquad\qquad\qquad\quad 2s^2 + 4s \\
\qquad\qquad\qquad\qquad\qquad 4 - 4s \,)\; s \;(
\end{array}
$$

till remainder becomes equal to zero

If no -ve co-efficient occurs in the successive quotients, then it's a Hewitz poly.

Ques) Find the value of a so that the given polynomial is Herwitz.

$P(s) = s^4 + s^3 + as^2 + 2s + 3$

$M(n): s^4 + as^2 + 3$

$N(n): s^3 + 2s$

$s^3+2s \,)\, s^4+as^2+3 \,(\, s$

$s^4 + 2s^2$

$(a-2)s^2 + 3 \,)\, s^3 + 2s \,(\, s/(a-2) \Rightarrow a-2>0$

$\cancel{(a-2)}s^3 + \frac{3s}{(a-2)}$ $\quad a>2$

$\underbrace{\left\{2 - \frac{3}{(a-2)}\right\}}_{>0} s \,)\, (a-2)s^2+3 \,($

$\left\{2 - \frac{3}{a-2}\right\} s > 0$

$2(a-2) - 3 > 0$

$2a - 4 - 3 > 0$

$\boxed{a > 7/2}$

Ques) Find the range of values of m so that the given poly. is Herwitz.

$P(s) = 2s^4 + s^3 + ms^2 + s + 2$

$M(s): 2s^4 + ms^2 + 2$

$N(s): s^3 + s$

$s^3+s \,)\, 2s^4 + ms^2 + 2 \,(\, 2s$

$2s^4 + 2s^2$

$(m-2)s^2 + 2 \,)\, s^3 + s \,(\, \frac{s}{(m-2)} \Rightarrow m>2$

$s^3 + \frac{2s}{m-2}$

$\left(1 - \frac{2}{m-2}\right) s \,)\, \,($

$1 - \frac{2}{m-2} > 0$

$m - 2 - 2 > 0$

$m > 4 \Rightarrow \boxed{m \in (4, \infty)}$

Ques) $P(s) = 4s^6 + 2s^5 + 17s^4 + 8s^3 + 16s^2 + 6s + 3$

$M(s): 4s^6 + 17s^4 + 16s^2 + 3$

$N(s): 2s^5 + 8s^3 + 6s$

$2s^5+8s^3+6s \,)\, 4s^6 + 17s^4 + 16s^2 + 3 \,(\, 2s$

$4s^6 + 16s^4 + 12s^2$

$s^4 + 4s^2 + 3 \,)\, 2s^5 + 8s^3 + 6s \,(\, 2s$

$2s^5 + 8s^3 + 6s$

$\times$

NOTE: If the continued fraction expansion suddenly terminates, then the last divisor is the common factor. We have to then determine the other factor by dividing $P(s)$ by this divisor.

These factors are then checked individually whether they are Hurwitz or not.

If both factors are Hurwitz, then given polynomial is Hurwitz; otherwise not.

$$
\begin{array}{r|l}
 & 4s^2+2s+1 \\
s^4+4s^2+3 & 4s^6+2s^5+17s^4+8s^3+16s^2+6s+3 \\
 & 4s^6+16s^4+12s^2 \\
\hline
 & 2s^5+s^4+8s^3+4s^2+6s+3 \\
 & 2s^5+8s^3+6s \\
\hline
 & s^4+4s^2+3 \\
 & s^4+4s^2+3 \\
\hline
 & \times
\end{array}
$$

$\therefore$ Factors are: $(s^4+4s^2+3)\,\underbrace{(4s^2+2s+1)}_{\text{Check Individually}}$

Ques) $P(s) = s^3+2s^2+3s+6$

$M(s)$: $2s^2+6$

$N(s)$: s^3+3s

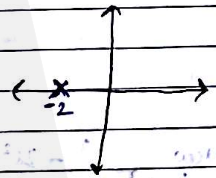
$$
\begin{array}{r|l}
 & \frac{s}{2} \\
2s^2+6 & s^3+3s \\
 & s^3+3s \\
\hline
 & \times
\end{array}
$$

$$
\begin{array}{r|l}
 & \frac{s}{2}+1 \\
2s^2+6 & s^3+2s^2+3s+6 \\
 & s^3+3s \\
\hline
 & 2s^2+6 \\
 & 2s^2+6 \\
\hline
 & \times
\end{array}
$$

Factors are: $(2s^2+6)\left(\frac{s}{2}+1\right)$

$$
\begin{array}{r|l}
 & \frac{s}{2} \\
4s & 2s^2+6 \\
 & 2s^2 \\
\hline
 & 6
\end{array}
\qquad
\begin{array}{r|l}
 & \frac{4s}{6} \\
6 & 4s \\
 & 4s \\
\hline
 & \times
\end{array}
$$

$\frac{s+2}{2}$



Lies in $-$ve x-axis

$\therefore$ It is a Hurwitz poly.

Ques) $P(s) = s^5+9s^4+7s^3+s^2+4s$

Constant term is missing.

$\therefore$ It is not a Hurwitz poly.

NOTE:

1) Quadratic eqⁿ is always a Hurwitz poly.

Ques) $P(s) = s^7 + 2s^6 + 2s^5 + s^4 + 4s^3 + 8s^2 + 8s + 4$

$M(s): 2s^6 + s^4 + 8s^2 + 4$

$N(s): s^7 + 2s^5 + 4s^3 + 8s$

$$2s^6+s^4+8s^2+4 \overline{\big) s^7 + 2s^5 + 4s^3 + 8s} \Big(\frac{s}{2}$$
$$s^7 + \frac{s^5}{2} + 4s^3 + 2s$$
$$\frac{3}{2}s^5 + 6s \overline{\big) 2s^6 + s^4 + 8s^2 + 4} \Big(\frac{4s}{3}$$
$$2s^6 + \frac{24s^2}{3}$$
$$s^4 + 4 \overline{\big) \frac{3}{2}s^5 + 6s} \Big(\frac{3s}{2}$$
$$\frac{3}{2}s^5 + 6s$$
$$\times$$

Now,
$$s^3 + 2s^2 + 2s + 1$$
$$s^4+4 \overline{\big) s^7 + 2s^6 + 2s^5 + s^4 + 4s^3 + 8s^2 + 8s + 4} \Big($$
$$s^7 + 4s^3$$
$$2s^6 + 2s^5 + s^4 + 8s^2 + 8s + 4$$
$$2s^6 + 8s^2$$
$$2s^5 + s^4 + 8s + 4$$
$$2s^5 + 8s$$
$$s^4 + 4$$
$$s^4 + 4$$
$$0$$

Network Synthesis –

Can be synthesised in 4 basic funcⁿ –

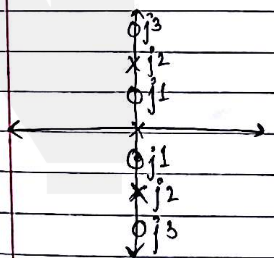
1) Foster-I / Foster series $\rightarrow Z(s)$
2) Foster-II / ,, parallel $\rightarrow Y(s)$
3) Cauer-I / } Immittance
4) Cauer-II /

In case of Foster-I, starting fn must be impedance or $Z(s)$

Forster-II, starting fn must be ampedance fn $Y(s)$

Ques) An impedance fn has a pole-zero diagram as shown and $Z(-2) = \frac{-130}{16}$

Synthesise foster-I diagram.



scale factor

$$\Rightarrow Z(s) = K\frac{(s^2+1)(s^2+9)}{s(s^2+4)}$$

or $$\frac{-130}{16} = K\frac{[(-2)^2+1][(-2)^2+9]}{(-2)[(-2)^2+4]}$$

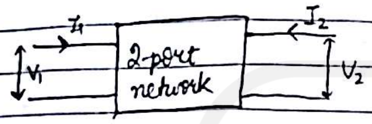
$$\Rightarrow \boxed{K = 2}$$

$$\therefore Z(s) = 2\frac{(s^2+1)(s^2+9)}{s(s^2+4)}$$

Two-port Networks

12/10/17

Page No.
Date / /



Z Parameter–

$$(V_1, V_2) = f(I_1, I_2)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \quad V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—— (I)}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—— (II)}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

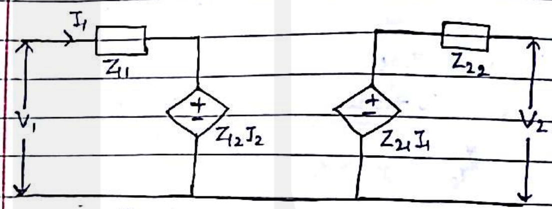
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

There are two types of fn–
1) Driving point fn – ratio of quantity of same port
eg. Z_{11}, Z_{22}
2) Transfer fn – ratio of quantities of diff. ports
eg. Z_{12}, Z_{21}

Transfer fns are further classified into –
i) Forward – ratio of o/p by i/p
ii) Reverse – ratio of i/p by o/p

Page No.
Date / /

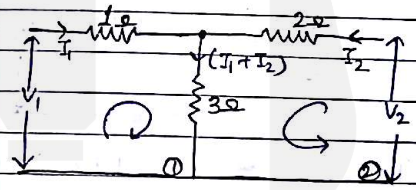
$Z_{11} \rightarrow$ Driving pt. I/p impedance when o/p is O/C
$Z_{22} \rightarrow$ Driving pt. o/p impedance when i/p is O/C
$Z_{12} \rightarrow$ Reverse Transfer Impedance, I/p is o/c
$Z_{21} \rightarrow$ Forward Transfer Impedance, o/p is o/c



Eq. ckt. of Z-parameter

$\left.\begin{matrix} Z_{12} I_2 \\ Z_{21} I_1 \end{matrix}\right\}$ current dependent voltage sources

Ques)



Determine the values of Z-parameters.

Applying KVL in

loop ①: $V_1 = I_1(1) + 3(I_1 + I_2)$

$$V_1 = \underbrace{4}_{Z_{11}} I_1 + \underbrace{3}_{Z_{12}} I_2 \quad \text{—— (I)}$$

loop ②: $V_2 = \underbrace{3}_{Z_{21}} I_1 + \underbrace{5}_{Z_{22}} I_2 \quad \text{—— (II)}$

$\therefore Z = \begin{bmatrix} 4 & 3 \end{bmatrix}$

OR

Case I: when $I_2 = 0$

$V_1 = 4I_1$ $\quad$ $V_2 = 3I_1$

$\frac{V_1}{I_1} = 4$ $\quad$ $\frac{V_2}{I_1} = 3$

$\boxed{Z_{11} = 4}$ $\quad$ $\boxed{Z_{21} = 3}$

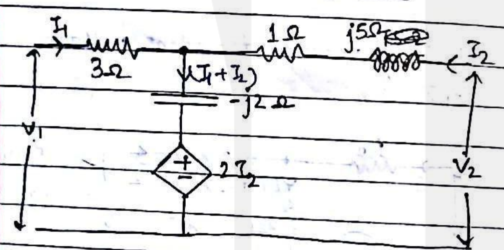
Case II: when $I_1 = 0$

$V_2 = 2I_2 + 3I_2$ $\quad$ $V_1 = 3I_2$

$\frac{V_2}{I_2} = 5$ $\quad$ $\frac{V_1}{I_2} = 3$

$\boxed{Z_{22} = 5}$ $\quad$ $\boxed{Z_{12} = 3}$

Ques)



Case I: When $I_2 = 0$

$V_1 = 3I_1 - j2(I_1)$

$\frac{V_1}{I_1} = (3 - j2)$

$\boxed{Z_{11} = (3-j2)\Omega}$

$V_2 = (-j2)I_1$

$\frac{V_2}{I_1} = -j2$

$\boxed{Z_{21} = (-j2)\Omega}$

when $I_1 = 0$

$V_1 = 2I_2 - j2I_2$

$\frac{V_1}{I_2} = (2 - j2)$

$\cancel{Z_{22}}$ $\boxed{Z_{12} = (2-j2)\Omega}$

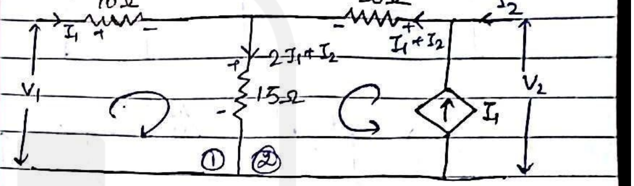
$V_2 = (j5 + 1 - j2 + 2)\, I_2$

$\frac{V_2}{I_2} = 3 + 3j$

$\boxed{Z_{22} = (3+3j)\Omega}$

Page No.
Date / /

Ques) Obtain the Z-parameters



\# In case there is a current source in the ckt., look for a resistance connected in parallel to the current source and then convert them into voltage source.

Applying KVL in loop 1,

$V_1 - 10I_1 - 15(2I_1 + I_2) = 0$

$V_1 = 40I_1 + 15I_2$

$Z_{11} = 40\Omega$, $Z_{12} = 15\Omega$

Applying KVL in loop 2,

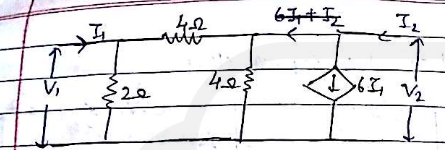
$V_2 - 20(I_1 + I_2) - 15(2I_1 + I_2) = 0$

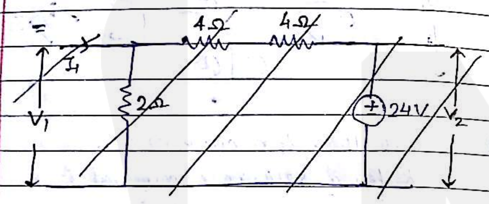
$V_2 - 20I_1 - 20I_2 - 30I_1 - 15I_2 = 0$

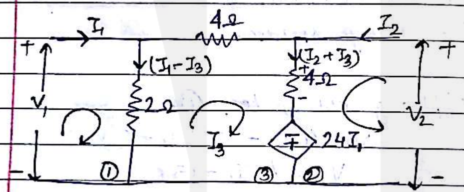
$V_2 = 50I_1 + 35I_2$

$Z_{21} = 50\Omega$, $Z_{22} = 35\Omega$

Ques) Obtain the Z parameters of the given network.







Applying KVL in loop

① : $V_1 = 2(I_1 - I_3)$ —— ①

② : $V_2 = 4(I_2 + I_3) - 24I_1$ —— ⑪

③ : $-2(I_3 - I_1) - 4I_3 - 4(I_2 + I_3) + 24I_1 = 0$

$-2I_3 + 2I_1 - 4I_3 - 4I_2 - 4I_3 + 24I_1 = 0$

$10I_3 = 26I_1 - 4I_2$

$I_3 = 2.6I_1 - 0.4I_2$

Putting this value of I_3 in eqⁿ ①,

$V_1 = 2I_1 - 2(2.6I_1 - 0.4I_2)$

$V_1 = -3.2I_1 + 0.8I_2$

$Z_{11} = -3.2\,\Omega$, $Z_{12} = 0.8\,\Omega$

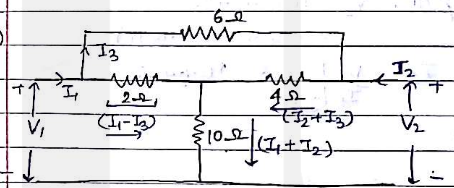
eqⁿ ⑪, $V_2 = 4I_2 + 4I_3 - 24I_1$

$V_2 = 4I_2 + 4(2.6I_1 - 0.4I_2) - 24I_1$

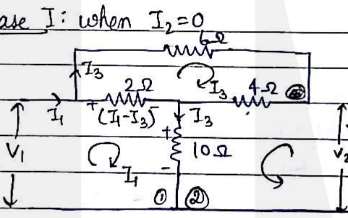
$V_2 = -14.4I_1 + 2.4I_2$

$Z_{21} = -14.4\,\Omega$, $Z_{22} = 2.4\,\Omega$

Ques)



Case I: when $I_2 = 0$



⇒ 6Ω & 4Ω are in series

Applying KVL in loop ①,

$V_1 - 2(I_1 - I_3) - 10I_1 = 0$ —— ①

loop ②, $V_2 - 4I_3 - 10I_1 = 0$ —— ⑪

Applying current distribution (for parallel resistances)

$I_3 = I_1 \times \frac{2}{(2+10)}$ ⇒ $I_3 = 0.16\,I_1$

Putting this value in eq^n ①,

$$V_1 - 12 I_1 + 2 I_3 = 0$$

$$V_1 - 12 I_1 + 2(0.16 I_1) = 0$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 11.68\ \Omega$$

eq^n ⑪, $V_2 = 10 I_1 + 4 I_3$

$$= 10 I_1 + 4(0.16 I_1)$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 10.64\ \Omega$$

→ Y-Parameters / ~~Impedance~~ Admittance Parameters - short circuit parameters

$$(I_1, I_2) = f(V_1, V_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

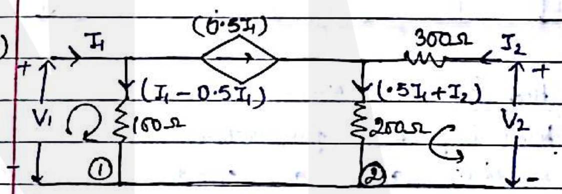
$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—①}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—⑪}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Ques)



KVL in loop ①, $V_1 - 100(I_1 - 0.5 I_1) = 0$

$$V_1 = 50 I_1$$

$$Y_{11} = 50\Omega,\ Y_{12} = 0\ \Omega$$

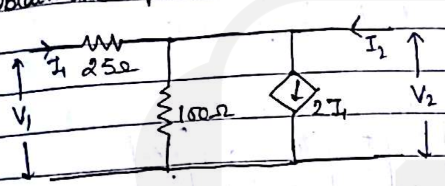
loop ②, $V_2 - 300 I_2 - 200(0.5 I_1 + I_2) = 0$

$$V_2 = 100 I_1 + 500 I_2$$

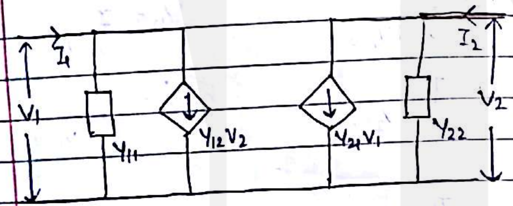
$$Y_{21} = 100\ \mho,\ Y_{22} = 500\ \mho$$

Page No.
Date / /

Ques) Obtain the Y-parameters.



Equivalent ckt. of Y parameters-



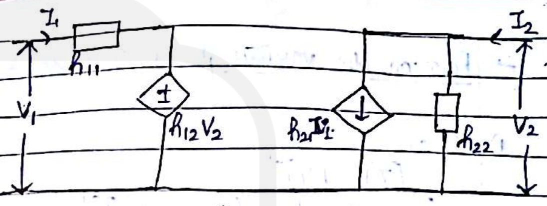
→ Hybrid Parameters-
Most important parameters used in transistor modelling as they are combination of Y and Z parameters.

$$(V_1, I_2) = f(I_1, V_2)$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \quad V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—— (I)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—— (II)}$$

Page No.
Date / /



$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} (\Omega) \qquad h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} (\mho)$$

→ Transmission / Chain / ABCD / T Parameters-
Are current used in power transmission lines.

$$(V_1, I_1) = f(V_2, -I_2)$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2 \quad \text{—— (I)}$$
$$I_1 = CV_2 - DI_2 \quad \text{—— (II)}$$

→ Inverse hybrid parameters-

$$(I_1, V_2) = f(V_1, I_2)$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$

Page No.
Date / /

→ Relation b/w various parameters set –

1) Z and Y

$$[Z] = [Y]^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

2) Z in terms of ABCD –

$$V_1 = AV_2 - BI_2 \quad \text{—(i)}$$

$$I_1 = CV_2 - DI_2 \quad \text{—(ii)}$$

or $CV_2 = I_1 + DI_2$

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \quad \text{—(iii)}$$

$$\boxed{Z_{21} = \frac{1}{C}} \quad , \quad \boxed{Z_{22} = \frac{D}{C}}$$

Put value of V_2 in eq^n (i),

$$V_1 = \frac{A}{C} I_1 + \frac{AD}{C} I_2 - BI_2$$

$$= \frac{A}{C} I_1 + \left(\frac{\overbrace{AD - BC}^{=\Delta T}}{C}\right) I_2$$

Page No.
Date / /

3) Y in terms of H –