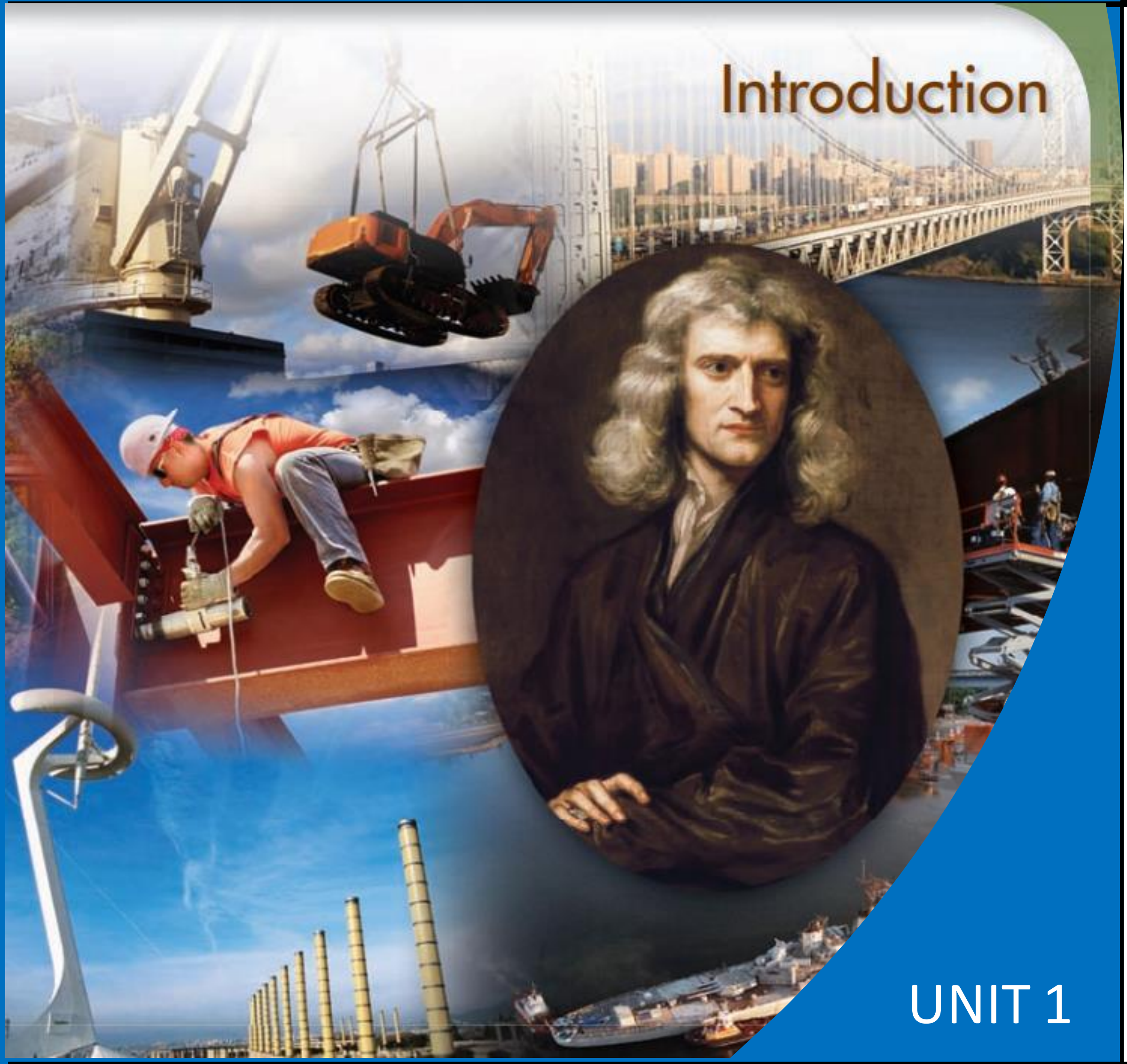
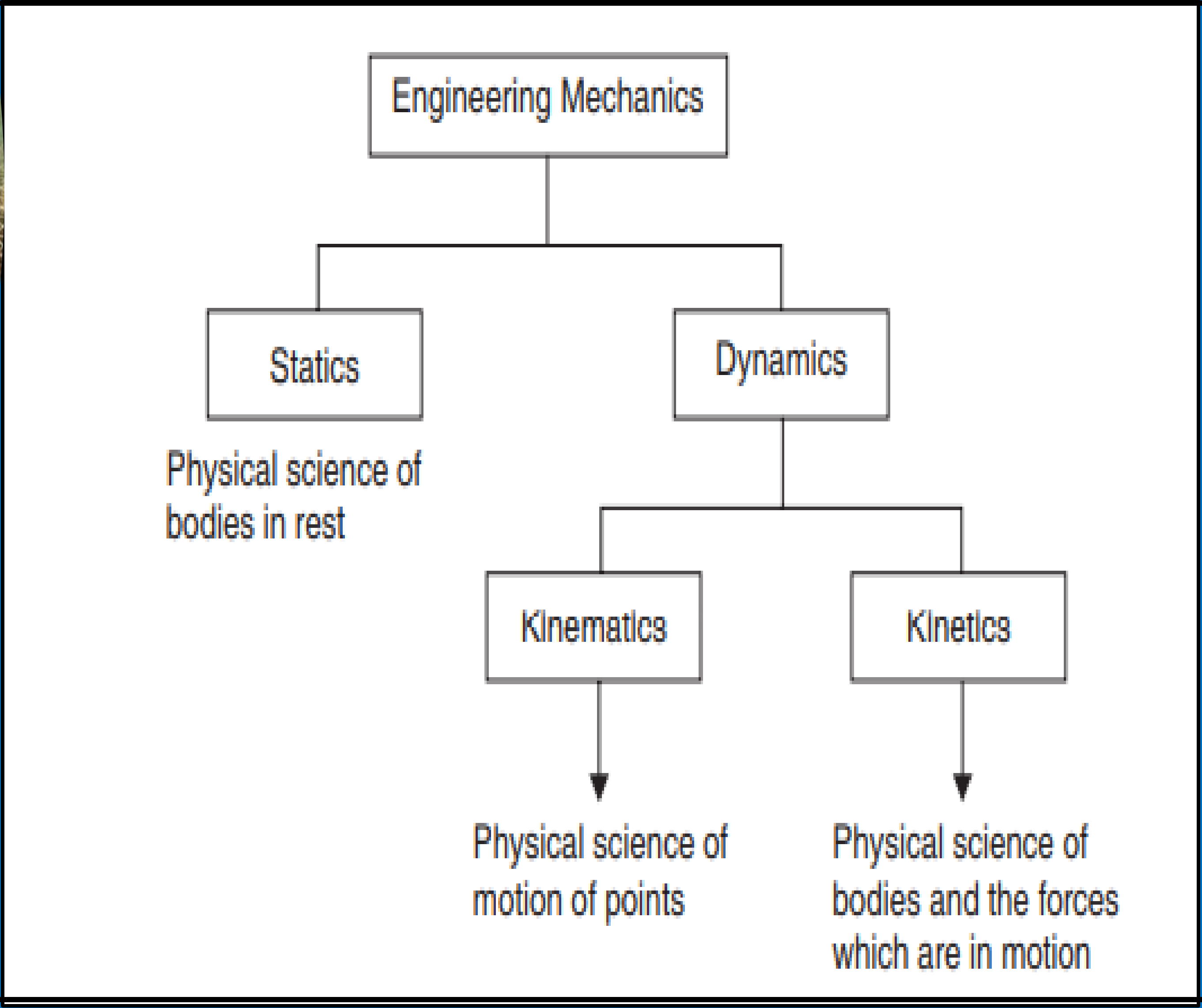
ENGINEERING MECHANICS



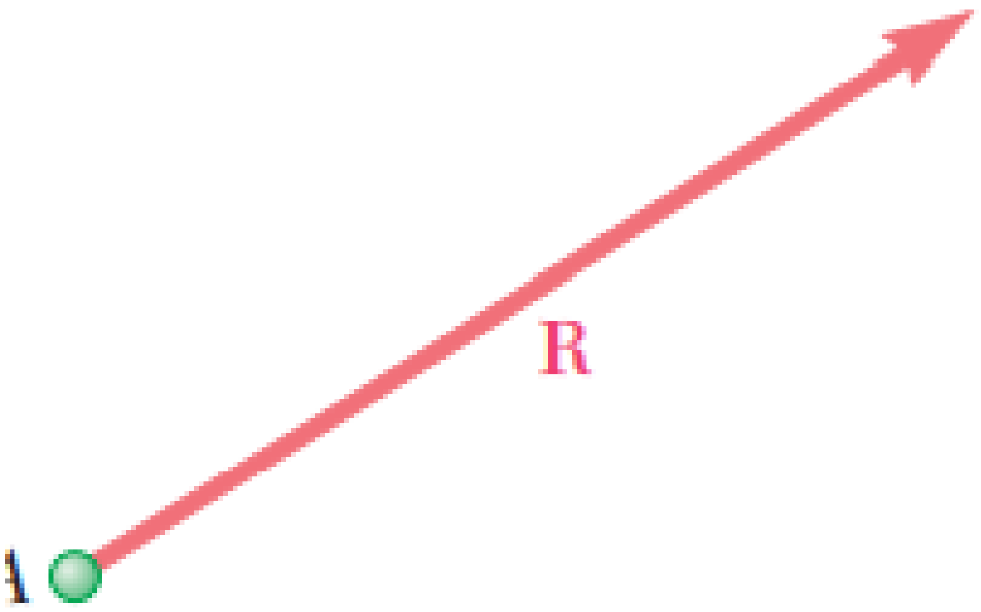


WHAT IS MECHANICS?

- Mechanics can be defined as that science which describes and predicts the conditions of rest or motion of bodies under the action of forces. It is divided into three parts: mechanics of rigid bodies, mechanics of deformable bodies, and mechanics of fluids.
- The mechanics of rigid bodies is subdivided into statics and dynamics, the former dealing with bodies at rest, the latter with bodies in motion. In this part of the study of mechanics, bodies are assumed to be perfectly rigid.
- You will study the conditions of rest or motion of particles and rigid bodies in terms of the four basic concepts(space, time, mass, and force).
- By particle we mean a very small amount of matter which may be assumed to occupy a single point in space.
- A rigid body is a combination of many particles occupying fixed positions with respect to each other.

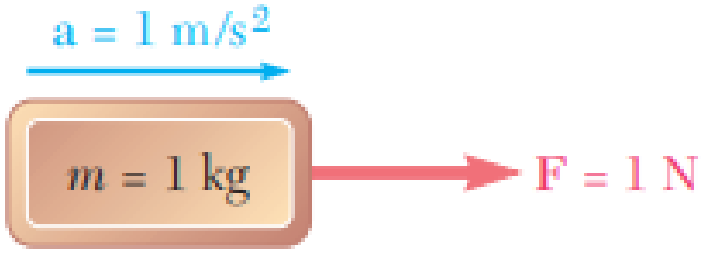
FORCE: One of the important term needed to be understand for study of mechanics

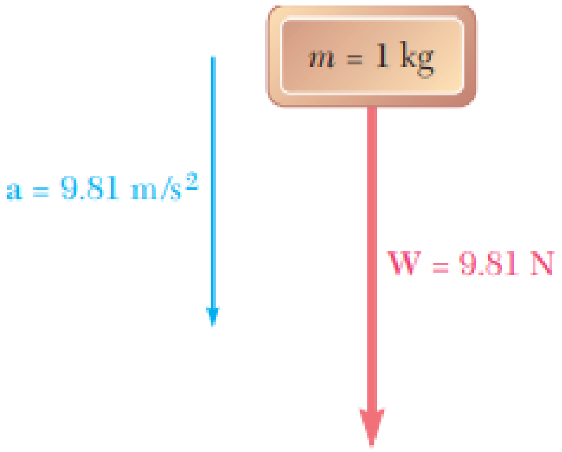
- ❖A force represents the action of one body on another. It can be exerted by actual contact or at a distance, as in the case of gravitational forces and magnetic forces.
- ❖A force is characterized by its point of application, its magnitude, and its direction; a force is represented by a vector.



SYSTEMS OF UNITS

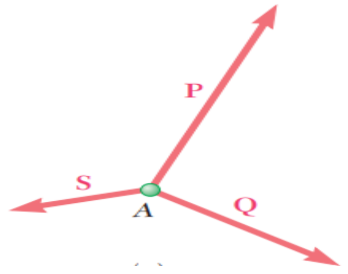
- International System of Units (SI Units). Basic units are the units of length, mass, and time, and they are called, respectively, the meter (m), the kilogram (kg), and the second (s).
- The unit of force is a derived unit. It is called the newton (N) and is defined as the force which gives an acceleration of 1 m/s ^{2} to a mass of 1 kg .

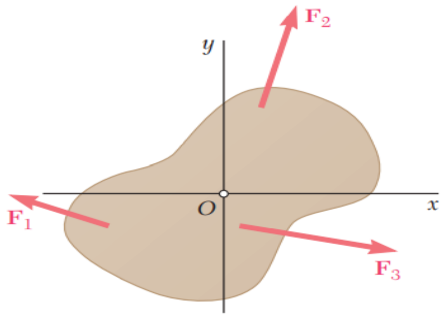


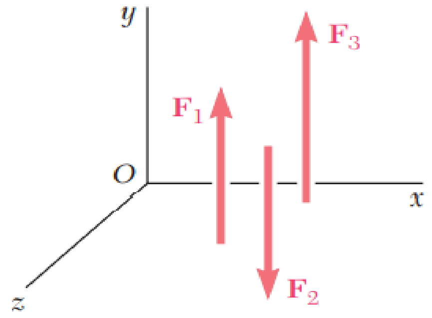


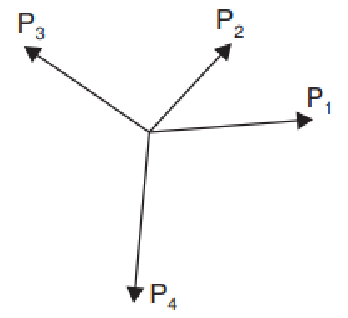
SYSTEM OF FORCES

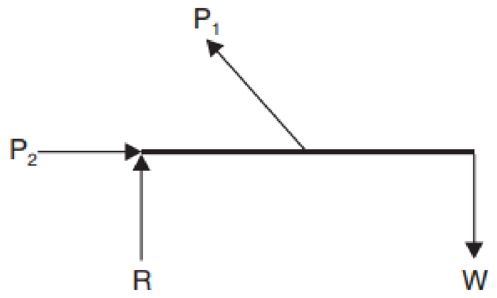
- ❖Concurrent forces: are applied at the same point and can therefore be added directly to obtain their resultant R . Thus, they always reduce to a single force.
- ❖Coplanar forces: act in the same plane, which may be assumed to be the plane of the figure
- ❖Parallel forces: Parallel forces have parallel lines of action and may or may not have the same sense.

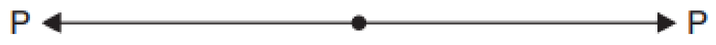


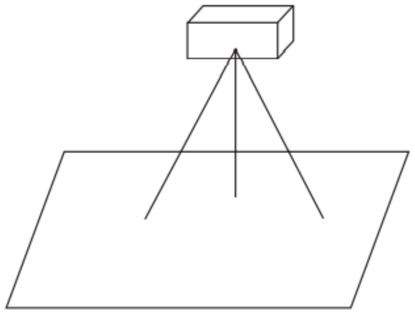


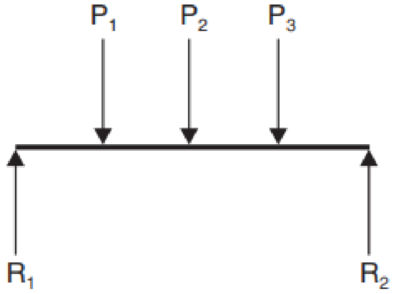






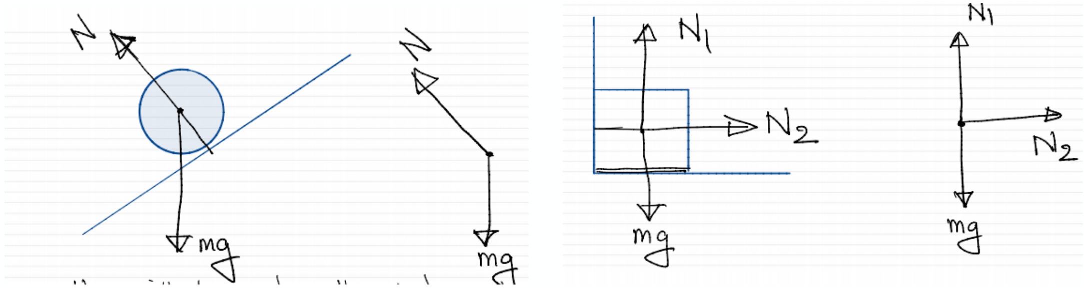


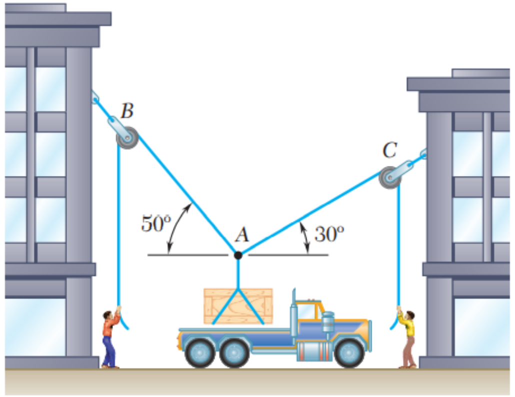


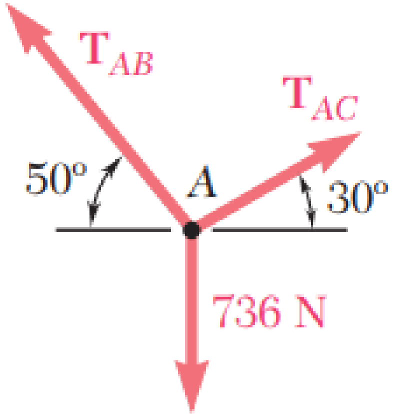


SYSTEM OF FORCES

- ❖Co-planar concurrent collinear system
- ❖Co-planar concurrent non-parallel system
- ❖Co-planar non concurrent parallel system
- ❖Co-planar non concurrent nonparallel system
- ❖Non-Coplanar concurrent system







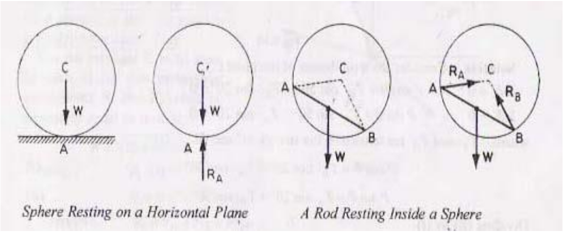
Force Diagram

- ❖ Space diagram: A sketch showing the physical conditions of the problem is known as a space diagram.
- ❖ Free body diagram: diagram showing all the forces acting on the particle or body . To draw the free-body diagram of a body we remove all the supports (like wall, floor, hinge or any other body) and replace them by the reactions which these supports exert on the body.

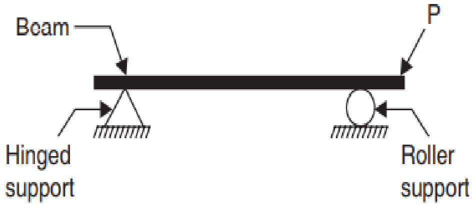
- **CONSTRAINT, ACTION AND REACTION**
- A body is not always free to move in all directions. The restriction to the free motion of a body in any direction is called a constraint.

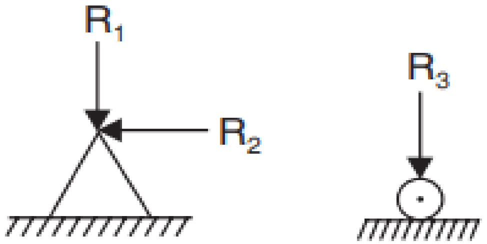
TYPES OF SUPPORTS AND SUPPORT REACTIONS

- A support is an element which bears the weight of a beam and keep it in equilibrium. Eg.
- **1. Frictionless Support.** The reaction acts normal to the surface at the point of contact .
- **2. Roller and Knife Edge Supports.** The roller and the knife edge restrict the motion normal to the surface of the beam. So, reaction $\mathbf{R}_A$ and $\mathbf{R}_B$ shall act normal to the surface at the points of contact A and B



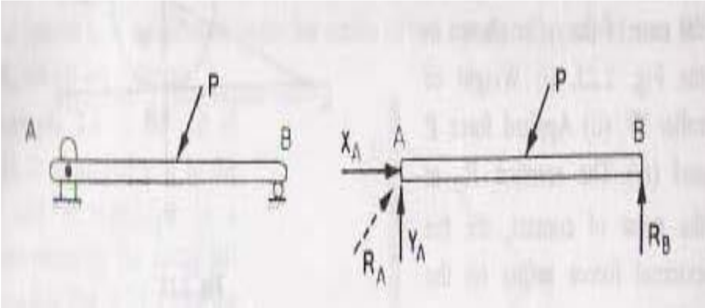
Sphere Resting on a Horizontal Plane *A Rod Resting Inside a Sphere*



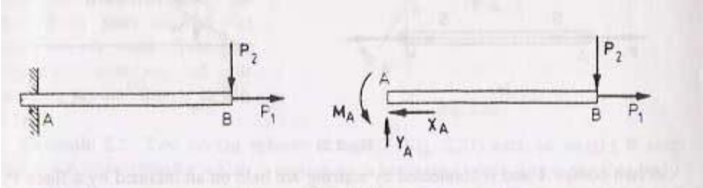


TYPES OF SUPPORTS AND SUPPORT REACTIONS

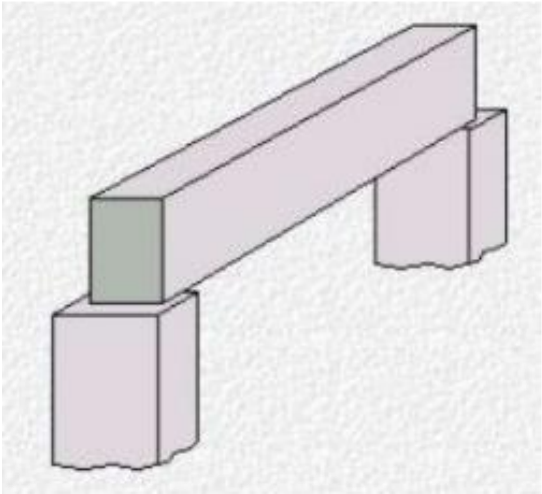
❖ **3. Hinged Support.** The hinge restricts the motion of the end A of the beam AB both in the horizontal as well as vertical directions.



❖ **4. Built-in-Support (Fixed).** If the end A of a beam AB is embedded in the concrete, it restricts the motion of the end A in the horizontal and the vertical directions. It also restricts the rotation of the beam AB about the point A.

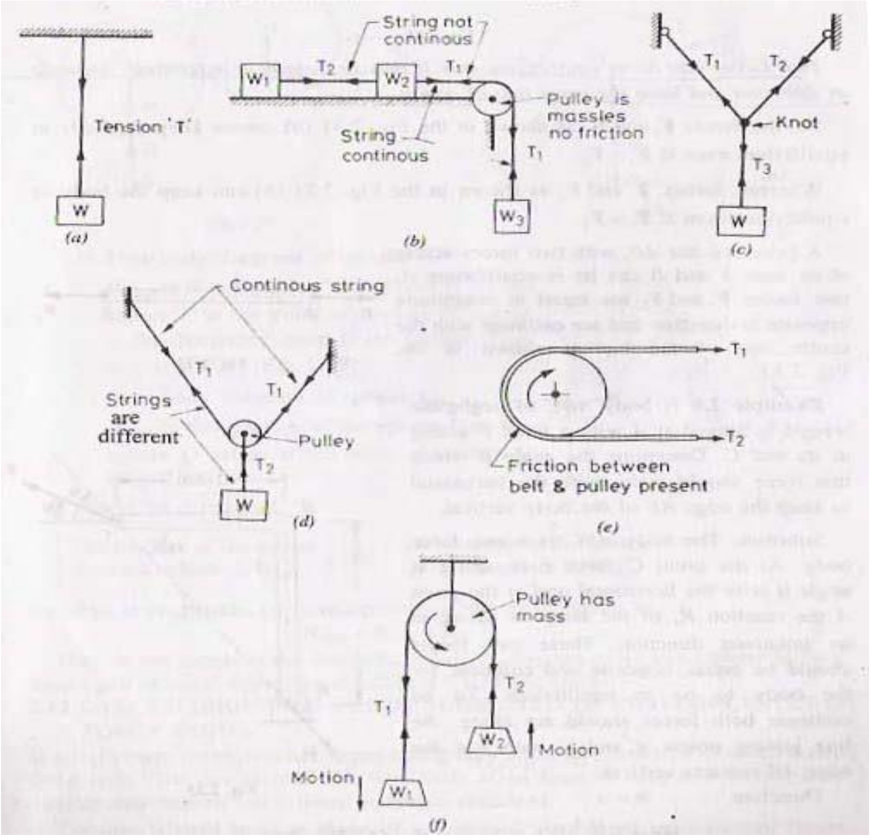


Example of Various types support

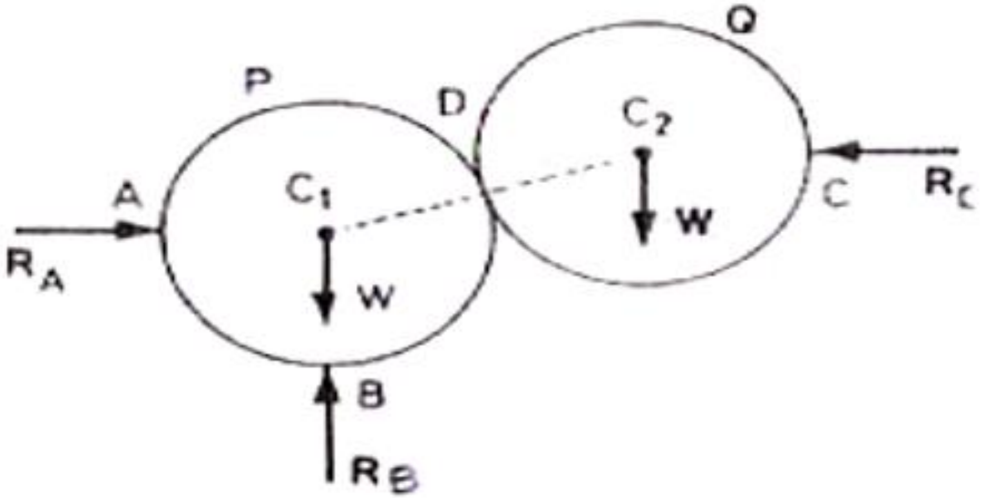


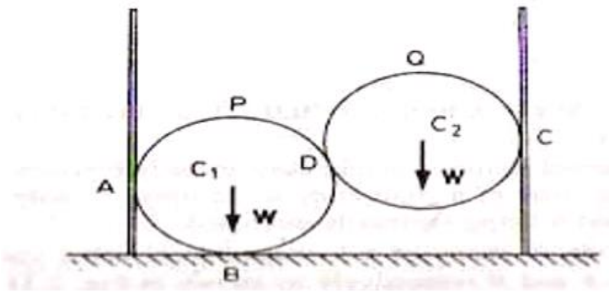
While drawing FBD for Tension in the String, Rope, Belt, Cable and Chain following must be consider:

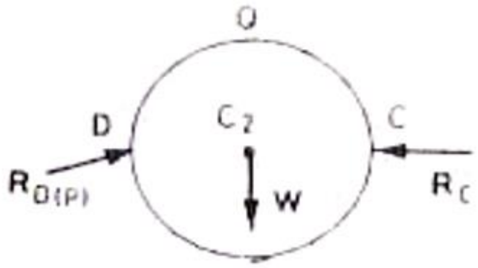
- ❖ The weight W supported at the end of a string attached to a fixed support. Tension in the string is an internal force(T).
- ❖ For a continuous rope, belt, cable and chain passing over a pulley, the tension remains same throughout provided,
- ❖ These string, rope, cable, pully etc., are assumed to be mass less and friction less.

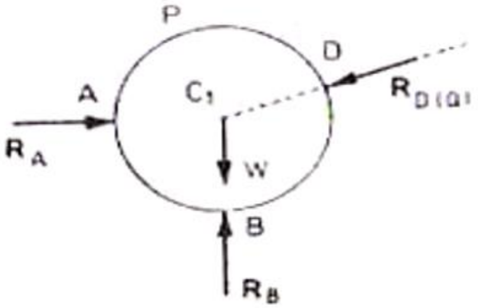


Free-body diagram of spheres P and Q.



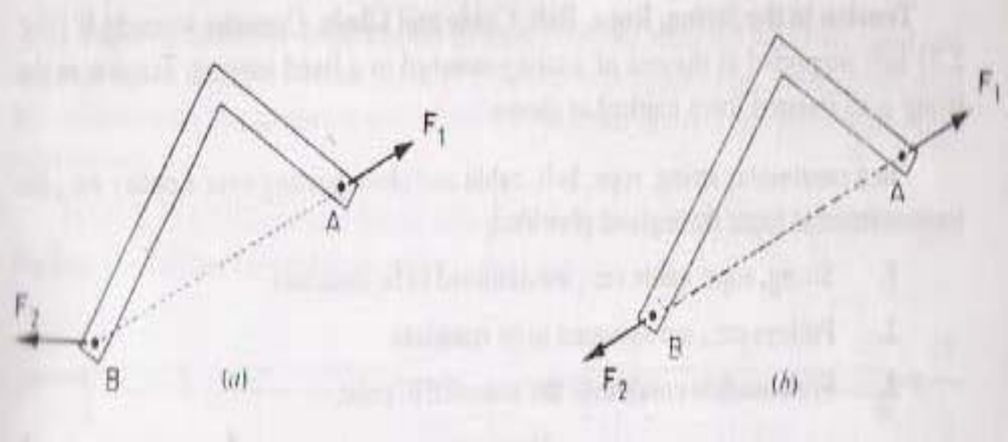






EQUILIBRIUM OF A BODY SUBJECTED TO TWO FORCES (TWO FORCE BODY)

- If a rigid body is subjected to forces acting only at the two points, it is called a two-force body. These forces can be in equilibrium only if they are equal in magnitude, opposite in direction and have the same lines of action.

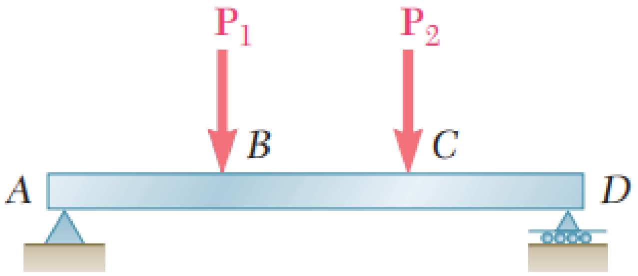


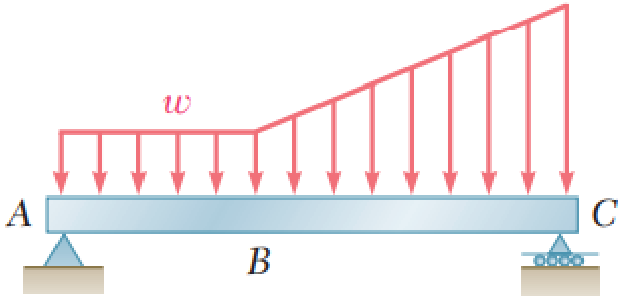
VARIOUS TYPES OF LOADING

❖A structural member designed to support loads applied at various points along the member is known as a beam. In most cases, the loads are perpendicular to the axis of the beam and will cause only shear and bending in the beam.

(a). Concentrated loads (N= Newtons)

(b). Distributed load : UDL(uniformly distributed load) and UVL(uniformly varying load) N/m, kN/m.

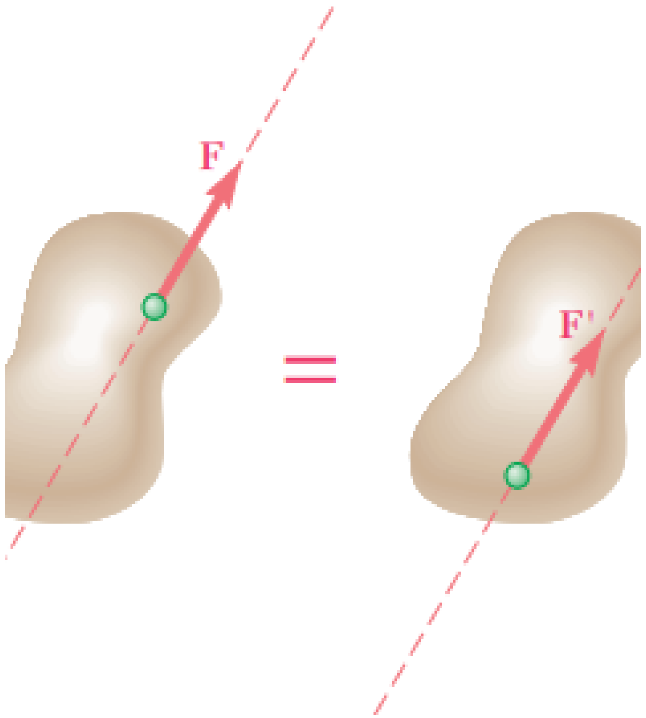




The study of elementary mechanics rests on following fundamental principles based on experimental evidence.

- ❑Principle of transmissibility
- ❑ Law of polygon
- ❑Triangle Law
- ❑Law of parallelogram of forces
- ❑ Lami’s Theorem
- ❑Moment of Force
- ❑Varignon’s theorem
- ❑Static equilibrium condition
- ❑Equilibrant forces
- ❑ Resolution of force
- ❑Couple.

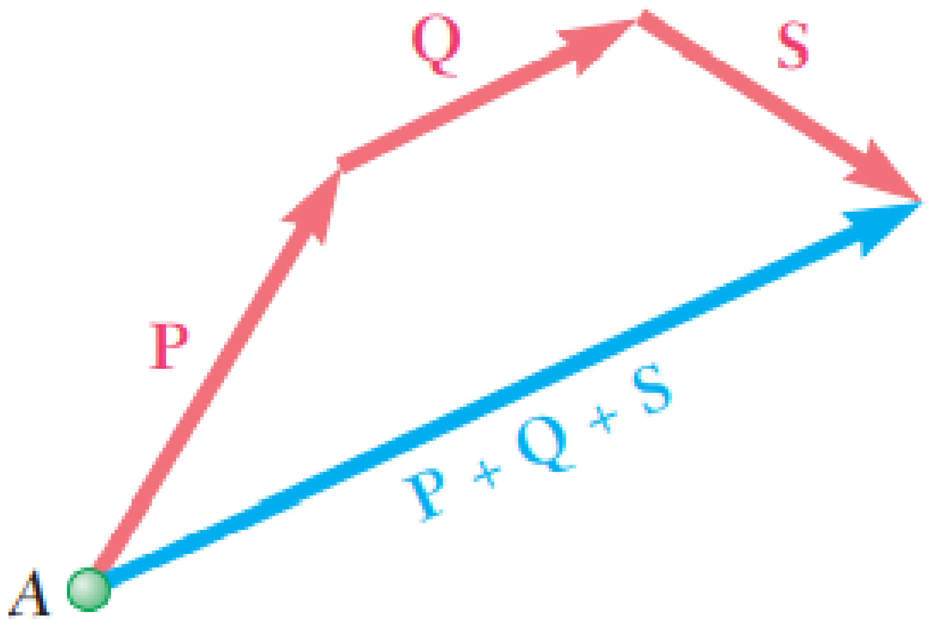
❖Principle of Transmissibility



This states that the conditions of equilibrium or of motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action

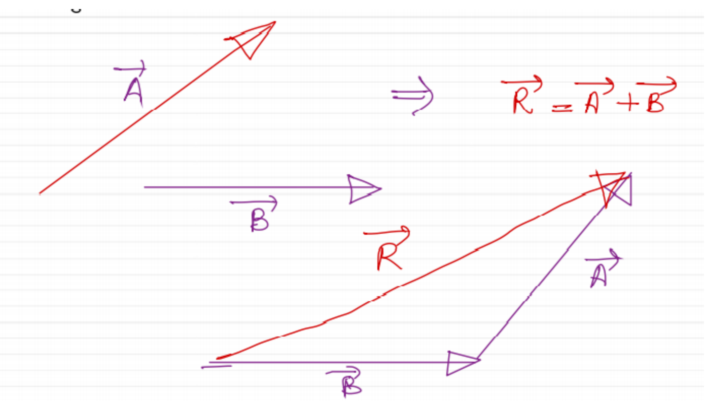
Law of Polygon: Resultant of several concurrent forces

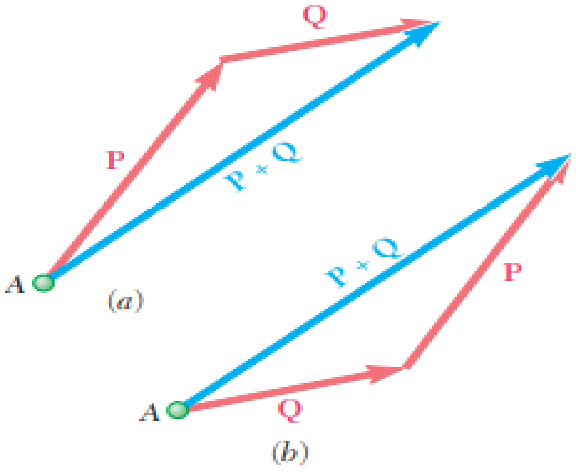
- ❖If the given vectors are coplanar, i.e., if they are contained in the same plane, their sum can be easily obtained graphically.
- ❖by arranging the given vectors in tip-to-tail fashion and connecting the tail of the first vector with the tip of the last one. This is known as the polygon rule for the addition of vectors.

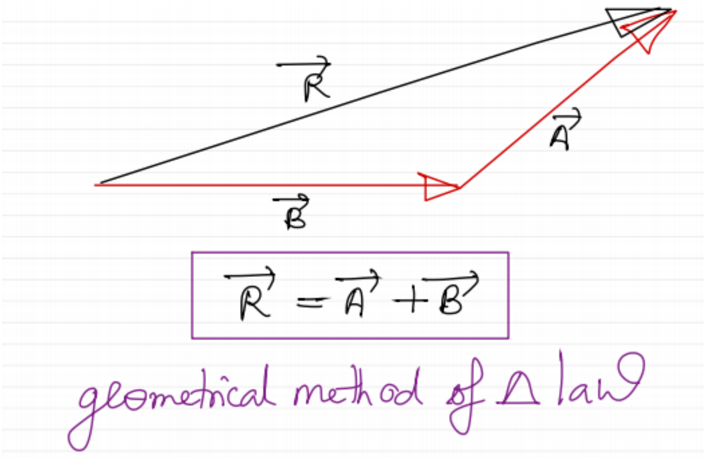


Triangle Law

- The sum of the two vectors can thus be found by arranging P and Q in tip-to-tail fashion and then connecting the tail of P with the tip of Q .

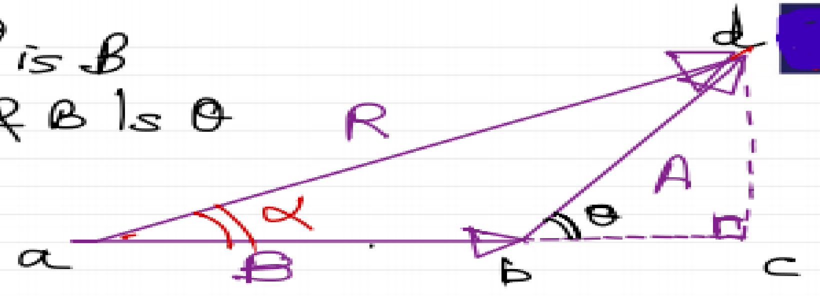






Triangle law: Analytical Approach

let mag of $\vec{A}$ is A, mag of $\vec{B}$ is B
mag of $\vec{R}$ is R angle b/w A & B is θ



$\triangle acd$

$$ac^2 + cd^2 = ad^2 \quad \text{—} (1)$$

$$ac = ab + bc$$

$$(ab + bc)^2 + cd^2 = ad^2$$

$$(B + bc)^2 + cd^2 = R^2$$

$$(B + A\cos\theta)^2 + (A\sin\theta)^2 = R^2$$

$$B^2 + A^2\cos^2\theta + 2AB\cos\theta + A^2\sin^2\theta = R^2$$

$$A^2(\sin^2\theta + \cos^2\theta) + B^2 + 2AB\cos\theta = R^2$$

$$\boxed{A^2 + B^2 + 2AB\cos\theta = R^2}$$

$\triangle bcd$

$$\sin\theta = \frac{cd}{bd} = \frac{cd}{A}$$

$$\therefore cd = A\sin\theta \quad \text{—} (2$$

$$\cos\theta = \frac{bc}{bd} = \frac{bc}{A}$$

$$bc = A\cos\theta \quad \text{—} (3$$

Continue...

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

If $\cos\theta = 1 \quad \theta = 0$

$$R^2 = A^2 + B^2 + 2AB$$

$$R^2 = (A+B)^2$$

$$R_{MAX} = A + B$$

If $\cos\theta = -1 \quad \theta = 180^\circ$

$$R^2 = A^2 + B^2 - 2AB$$

$$R^2 = (A-B)^2$$

$$R_{min} = A - B$$

If $\theta = 90^\circ$

$\cos\theta = 0$

$$R^2 = A^2 + B^2$$

$$R = \sqrt{A^2 + B^2}$$

It has to be noted that the value of the resultant will always lie between R_{min} & R_{MAX}.

$$R_{min} \leq R \leq R_{MAX}$$

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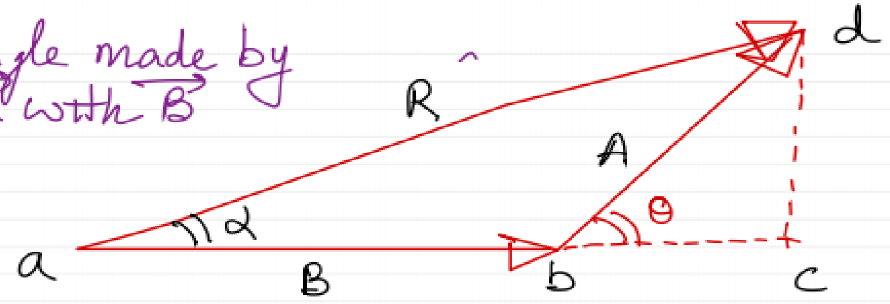
α is the angle made by resultant $\vec{R}$ with $\vec{B}$



$$\tan\alpha = \frac{cd}{ac} = \frac{cd}{ab + bc}$$

$$\boxed{\tan\alpha = \frac{A\sin\theta}{B + A\cos\theta}}$$

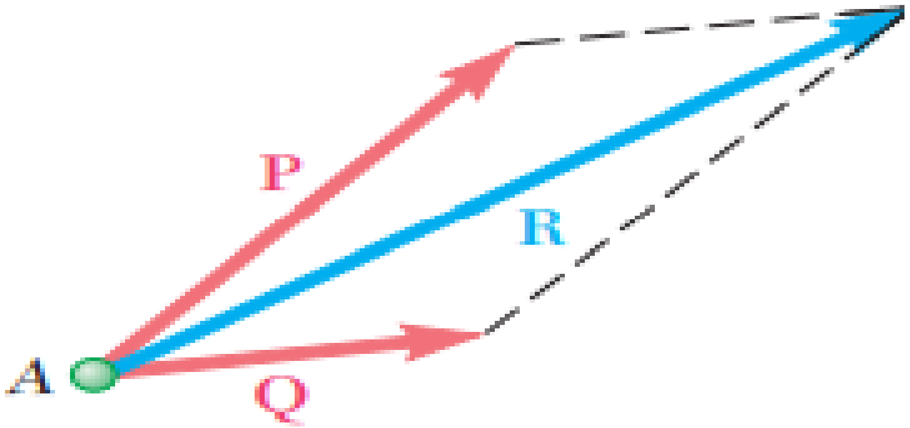
$$\cos\alpha = \frac{ac}{ad}$$

$$\cos\alpha = \frac{B + A\cos\theta}{R}$$

$$\sin\alpha = \frac{cd}{ad} = \frac{A\sin\theta}{R}$$

The Parallelogram Law for the Addition of Forces:

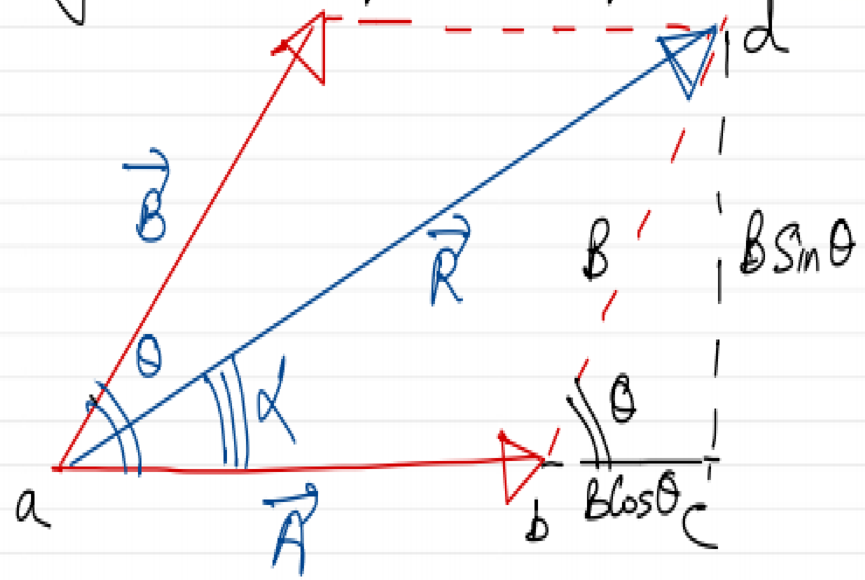
❖This states that two forces acting on a particle may be replaced by a single force, called their resultant, obtained by drawing the diagonal of the parallelogram which has sides equal to the given forces



Law of Parallelogram: Analytical Approach

parallogm law of addition | $\parallel^m$ law of add^n

$$\vec{R} = \vec{A} + \vec{B}$$



$$ac^2 + cd^2 = ad^2$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

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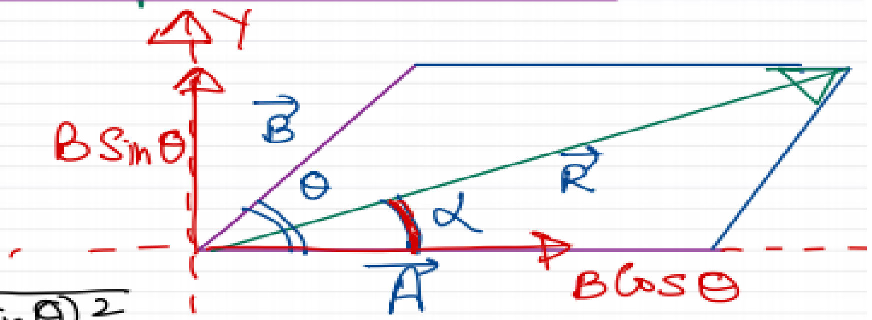
Proving llgm law by component method

$$X = A + B\cos\theta$$

$$Y = B\sin\theta$$



$$R = \sqrt{X^2 + Y^2}$$

$$R = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$$

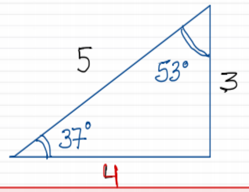
$$R = \sqrt{A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta}$$

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

$$\tan\alpha = \frac{Y}{X}$$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

where α is the angle made the resultant with $\vec{A}$



$\sin 37 = \frac{3}{5}$ $\quad$ $\sin 53 = \frac{4}{5}$

$\cos 37 = \frac{4}{5}$ $\quad$ $\cos 53 = \frac{3}{5}$

$\tan 37 = \frac{3}{4}$ $\quad$ $\tan 53 = \frac{4}{3}$

Static equilibrium condition

- When several forces act on a particle or rigid body, is said to be in equilibrium if there is no unbalanced force acting on it i.e., resultant of all the forces acting on it is zero.
- Analytically sum of F_x and F_y component must be zero.
- Graphically the force polygon must be close.

Forces in equilibrium

Equilibrium means net force acting is zero

$$\sum \vec{F} = 0$$

$$\sum F_x = 0$$

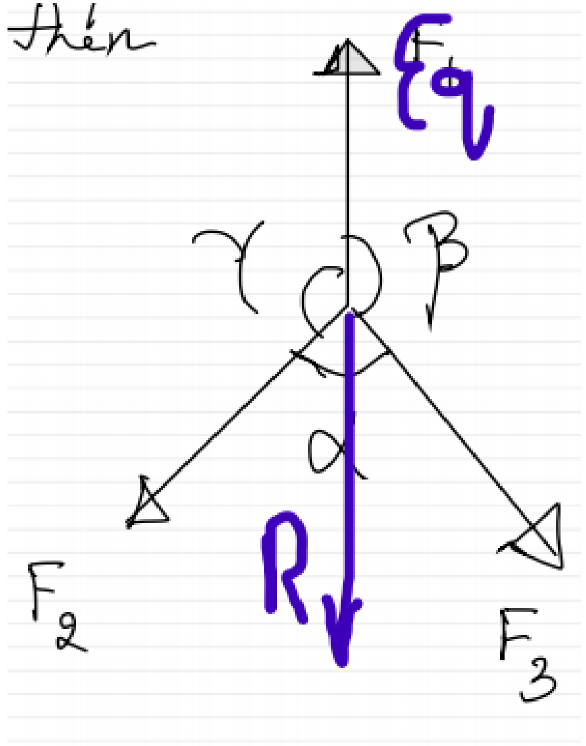
$$\sum F_y = 0$$

will be helpful in solving the problem.

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

Equilibrant Forces

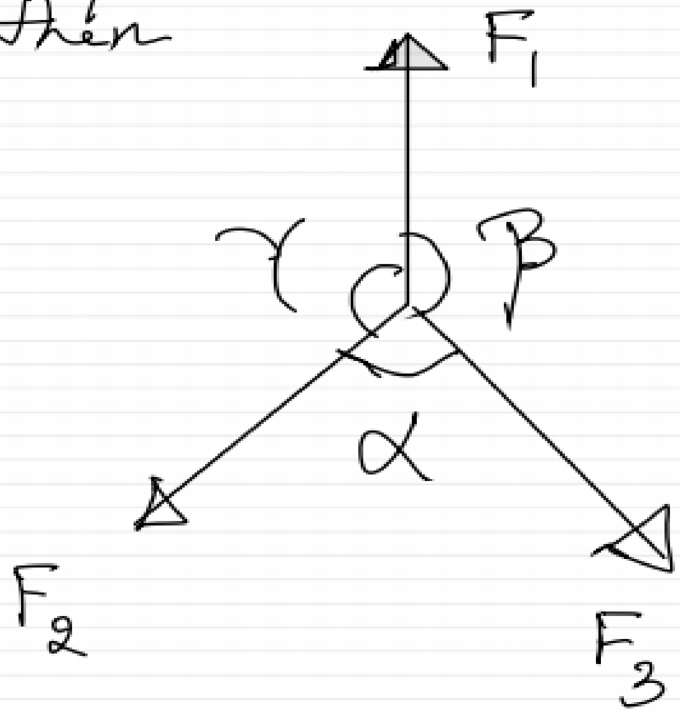
❖Under static equilibrium conditions the force which is equal in magnitude and opposite in direction to the resultant force acting on a particle is called equilibrant force.



Lami's Theorem

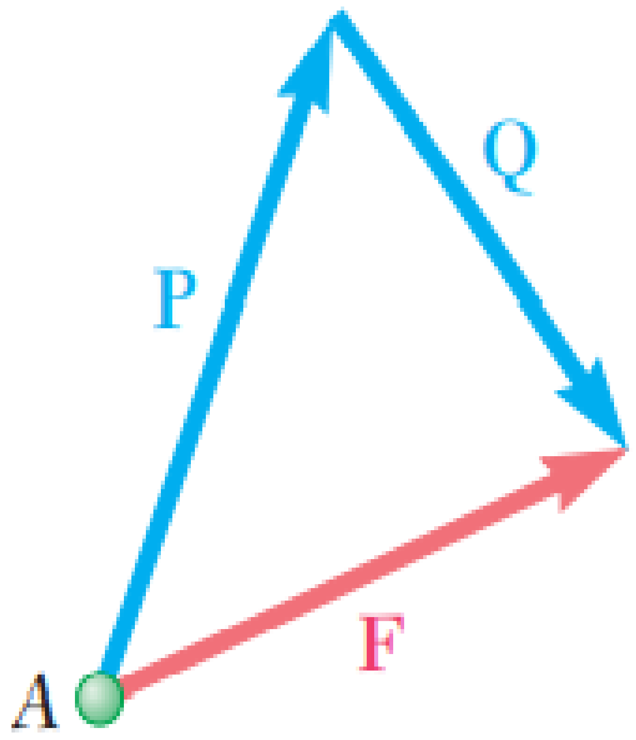
If we have 3 forces acting on a particle such that the particle is in equilibrium then

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$



RESOLUTION OF A FORCE INTO COMPONENTS

- ❖ We have seen that two or more forces acting on a particle may be replaced by a single force which has the same effect on the particle.
- ❖ Conversely, a single force F acting on a particle may be replaced by two or more forces which, together, have the same effect on the particle. These forces are called the components of the original force F , and the process of substituting them for F is called resolving the force F into components.
- ❖ The number of ways in which a given force F may be resolved into two components is unlimited. Two cases are of particular interest:
- ❖ One of the Two Components, P , Is Known: The second component, Q , is obtained by applying the triangle rule and joining the tip of P to the tip of F



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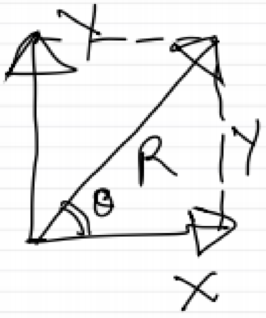
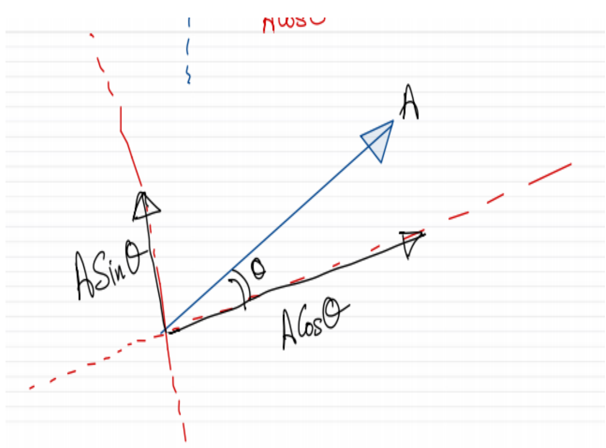
Addition of Vectors using Components [redacted]

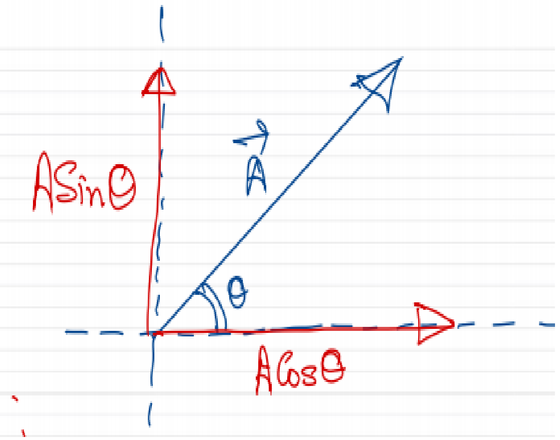
1. Choose a suitable X & Y axes if it is not given in the problem situation.
2. Now resolve all the Vectors along these X & Y axis.
3. Add all Components along X axis together & let us call as X
4. Add all Components along Y axis together & let us call it as Y
5. Hence their resultant R will be

$$\boxed{R = \sqrt{X^2 + Y^2}}$$

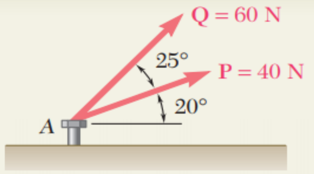


6. $\boxed{\tan\theta = \frac{Y}{X}}$ θ is the angle b/w R & X.



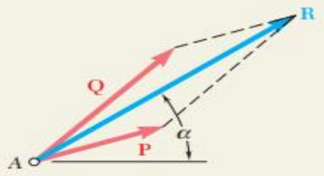


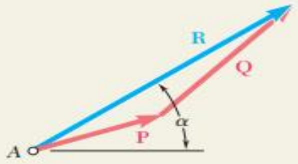
Example:

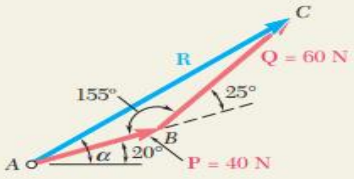


SAMPLE PROBLEM 2.1

The two forces **P** and **Q** act on a bolt A. Determine their resultant.







SOLUTION

Graphical Solution. A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant are measured and found to be

$$R = 98\text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98\text{ N} \measuredangle 35° \blacktriangleleft$$

The triangle rule may also be used. Forces **P** and **Q** are drawn in tip-to-tail fashion. Again the magnitude and direction of the resultant are measured.

$$R = 98\text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98\text{ N} \measuredangle 35° \blacktriangleleft$$

Trigonometric Solution. The triangle rule is again used; two sides and the included angle are known. We apply the law of cosines.

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$R^2 = (40\text{ N})^2 + (60\text{ N})^2 - 2(40\text{ N})(60\text{ N})\cos 155°$$
$$R = 97.73\text{ N}$$

Now, applying the law of sines, we write

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60\text{ N}} = \frac{\sin 155°}{97.73\text{ N}} \tag{1}$$

Solving Eq. (1) for sin A, we have

$$\sin A = \frac{(60\text{ N})\sin 155°}{97.73\text{ N}}$$

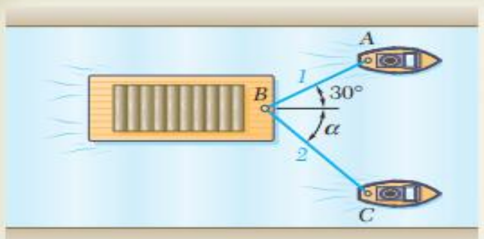
Using a calculator, we first compute the quotient, then its arc sine, and obtain

$$A = 15.04° \qquad \alpha = 20° + A = 35.04°$$

We use 3 significant figures to record the answer (cf. Sec. 1.6):

$$\mathbf{R} = 97.7\text{ N} \measuredangle 35.0° \blacktriangleleft$$

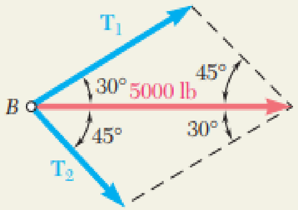
Example



SAMPLE PROBLEM 2.2

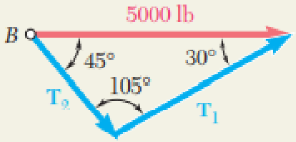
A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (*a*) the tension in each of the ropes knowing that $\alpha = 45°$, (*b*) the value of α for which the tension in rope 2 is minimum.

SOLUTION



a. Tension for $\alpha = 45°$. ***Graphical Solution.*** The parallelogram law is used; the diagonal (resultant) is known to be equal to 5000 lb and to be directed to the right. The sides are drawn parallel to the ropes. If the drawing is done to scale, we measure

$$T_1 = 3700 \text{ lb} \qquad T_2 = 2600 \text{ lb} \quad \blacktriangleleft$$

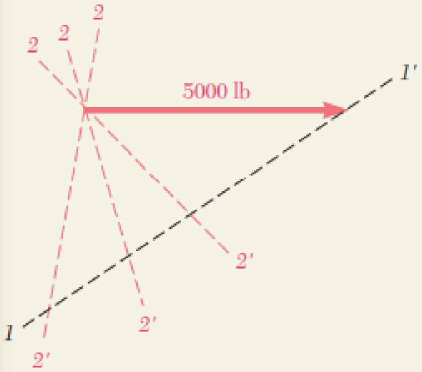


Trigonometric Solution. The triangle rule can be used. We note that the triangle shown represents half of the parallelogram shown above. Using the law of sines, we write

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000 \text{ lb}}{\sin 105°}$$

With a calculator, we first compute and store the value of the last quotient. Multiplying this value successively by sin 45° and sin 30°, we obtain

$$T_1 = 3660 \text{ lb} \qquad T_2 = 2590 \text{ lb} \quad \blacktriangleleft$$



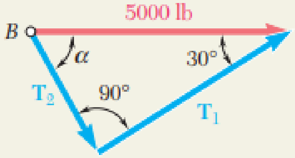
b. Value of α for Minimum T_2. To determine the value of α for which the tension in rope 2 is minimum, the triangle rule is again used. In the sketch shown, line *1-1′* is the known direction of $\mathbf{T}_1$. Several possible directions of $\mathbf{T}_2$ are shown by the lines *2-2′*. We note that the minimum value of T_2 occurs when $\mathbf{T}_1$ and $\mathbf{T}_2$ are perpendicular. The minimum value of T_2 is

$$T_2 = (5000 \text{ lb}) \sin 30° = 2500 \text{ lb}$$

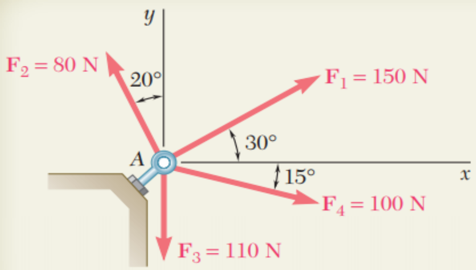
Corresponding values of T_1 and α are

$$T_1 = (5000 \text{ lb}) \cos 30° = 4330 \text{ lb}$$

$$\alpha = 90° - 30° \qquad \alpha = 60° \quad \blacktriangleleft$$



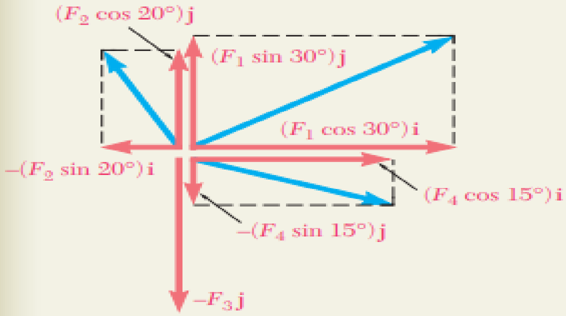
Example



SAMPLE PROBLEM 2.3

Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.

SOLUTION



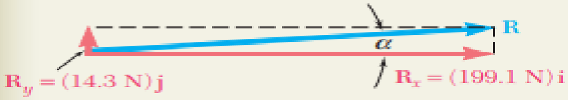
The x and y components of each force are determined by trigonometry as shown and are entered in the table below. According to the convention adopted in Sec. 2.7, the scalar number representing a force component is positive if the force component has the same sense as the corresponding coordinate axis. Thus, x components acting to the right and y components acting upward are represented by positive numbers.

Force	Magnitude, N	x Component, N	y Component, N
$\mathbf{F}_1$	150	+129.9	+75.0
$\mathbf{F}_2$	80	−27.4	+75.2
$\mathbf{F}_3$	110	0	−110.0
$\mathbf{F}_4$	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$

Thus, the resultant $\mathbf{R}$ of the four forces is

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} \qquad \mathbf{R} = (199.1\text{ N})\mathbf{i} + (14.3\text{ N})\mathbf{j} \quad ◀$$

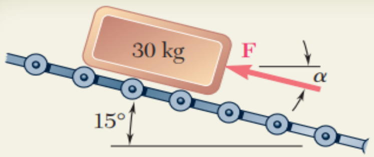
The magnitude and direction of the resultant may now be determined. From the triangle shown, we have



$$\tan\alpha = \frac{R_y}{R_x} = \frac{14.3\text{ N}}{199.1\text{ N}} \qquad \alpha = 4.1°$$

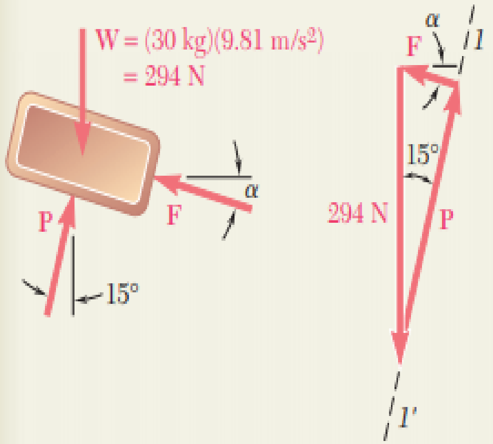
$$R = \frac{14.3\text{ N}}{\sin\alpha} = 199.6\text{ N} \qquad \mathbf{R} = 199.6\text{ N} \measuredangle 4.1° \quad ◀$$

Example



SAMPLE PROBLEM 2.5

Determine the magnitude and direction of the smallest force **F** which will maintain the package shown in equilibrium. Note that the force exerted by the rollers on the package is perpendicular to the incline.



SOLUTION

Free-Body Diagram. We choose the package as a free body, assuming that it can be treated as a particle. We draw the corresponding free-body diagram.

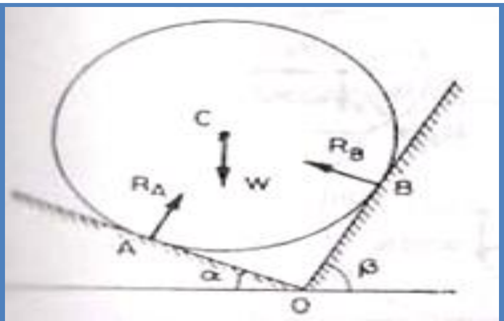
Equilibrium Condition. Since only three forces act on the free body, we draw a force triangle to express that it is in equilibrium. Line *1-1′* represents the known direction of **P**. In order to obtain the minimum value of the force **F**, we choose the direction of **F** perpendicular to that of **P**. From the geometry of the triangle obtained, we find

$$F = (294 \text{ N}) \sin 15° = 76.1 \text{ N} \qquad \alpha = 15°$$

$$\mathbf{F} = 76.1 \text{ N} \measuredangle 15°$$

Example

A smooth circular cylinder of weight W and radius r rests in a V-shaped groove whose sides are inclined at angles α and β to the horizontal as shown. Find the reaction R_A and R_B at the points of contact. Given $\alpha = 25^\circ$, $\beta = 65^\circ$, W = 500 N.



Writing the equation of equilibrium

$\sum F_X = 0$: $R_A \sin\alpha - R_B \sin\beta = 0$

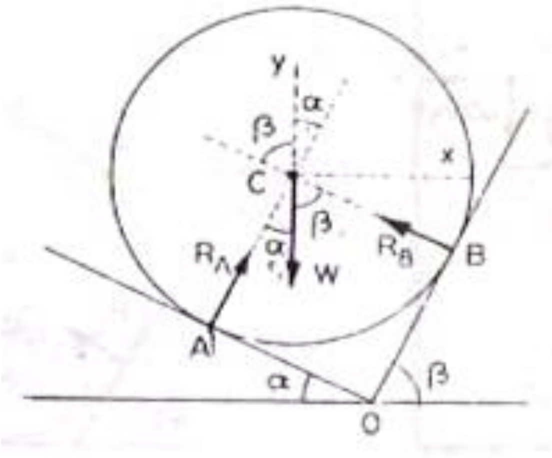
$\sum F_Y = 0$: $R_A \cos\alpha - R_B \cos\beta + W = 0$

After solving for R_A and R_B we get the same results as obtained earlier

Substituting, $\alpha = 25^\circ$, $\beta = 65^\circ$, W = 500 N

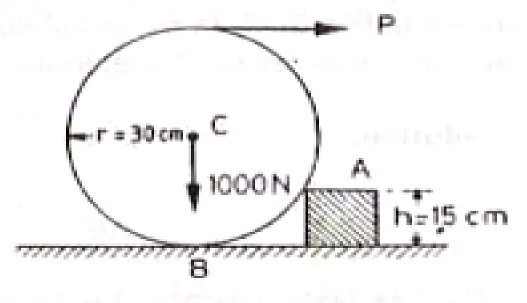
$R_A = \dfrac{500 \sin 65^\circ}{\sin 90^\circ}$ $R_B = \dfrac{500 \sin 25^\circ}{\sin 90^\circ}$

R_A = 453.2 N R_B = 211.3 N   Ans.



Example

A uniform wheel of 60.0 cm diameter and weighing 1000 N rests against a rectangular block 15 cm high lying on a horizontal plane as shown in the Fig. It is to be pulled over this block by a horizontal force P applied to the end of a string would round the circumference of the wheel. Find the force P when is just about to roll over the block.



$\sum F_X = 0$: $P - R_A \sin \theta = 0$ or $R_A \sin \theta = P$

$\sum F_Y = 0$: $R_A \cos \theta - W = 0$ or $R_A \cos \theta = W$

Dividing (i) by (ii) $\tan \theta = P/W$

In triangle ADE, $\tan \theta = DA / DE$, $DE = 2r - h$, $DA = \sqrt{CA^2 - CD^2}$, $DA = \sqrt{r^2 - (r-h)^2}$

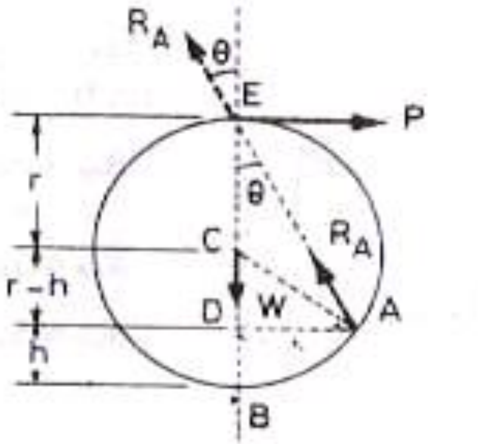
$$\tan \theta = \frac{\sqrt{r^2 - (r-h)^2}}{(2r - h)} = \frac{\sqrt{(2rh - h^2)}}{(2r - h)}$$

$$\tan \theta = \frac{\sqrt{h}\sqrt{(2r-h)}}{(2r - h)} = \frac{\sqrt{h}}{\sqrt{(2r - h)}} = \frac{\sqrt{0.15}}{\sqrt{(0.6-0.15)}}$$

$\tan \theta = 0.577$

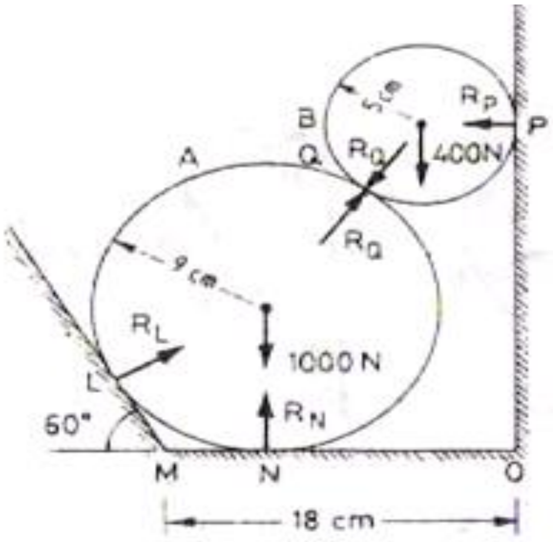
But $P/W = \tan \theta$

Therefore, $P = W \tan \theta = 1000 \times 0.577 = 577$ N Ans.



Example

Two cylinders A and B rest in a horizontal channel as shown in Fig. The cylinder A has a weight of 1000 N and radius of 9.0 cm. The cylinder B has a weight of 400 N and a radius of 5.0 cm. The channel is 18.0 cm wide at the bottom with one side vertical. The other side is inclined at an angle 60˚ with the horizontal. Find the reactions.



Writing the equations of equilibrium for cylinder B.

$\sum F_X = 0$: $R_Q \sin 33.86^\circ - R_P = 0$ (i)

$\sum F_Y = 0$: $R_Q \cos 33.86^\circ - 400 = 0$ (ii)

From (ii) $R_Q = \dfrac{400}{\text{Cos}33.86}$, $R_Q = 481.9$ N

From (i) $R_P = R_Q \sin 33.86^\circ$, $R_P = 268.5$ N Ans.

Writing the equations of equilibrium for cylinder A.

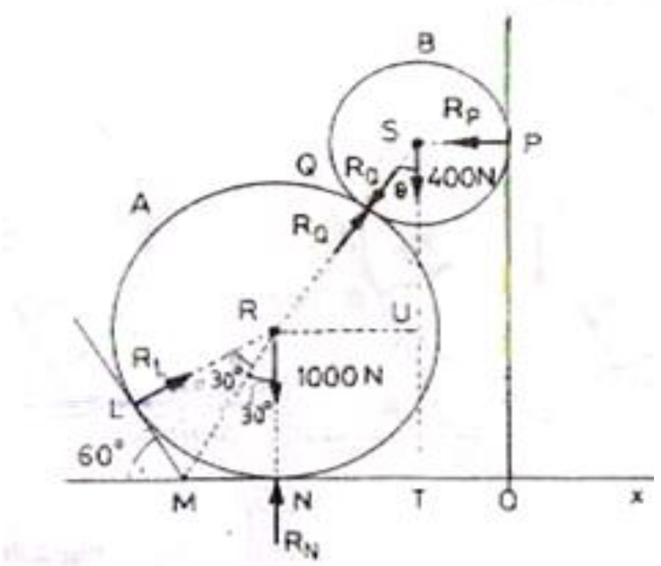
$\sum F_X = 0$: $R_L \sin 60^\circ - R_Q \sin 33.86^\circ = 0$

$R_L = \dfrac{R_Q \sin 33.86}{\text{Sin } 60^\circ} = \dfrac{481.9 \times 0.5571}{0.866} =$ 310 N Ans.

$\sum F_y = 0$: $R_N - 1000 - R_Q \cos 33.86^\circ + R_L \cos 60^\circ = 0$

$R_N = 1000 + 481.9 \times 0.830 - 310 \times 0.5$

$R_N = 1248.2$ N Ans.



Example:

The cylinder shown in fig. have same diameter but the cylinder 1 weighs 200N and cylinder 2 weighs 150 N .Find the reaction at the supports.

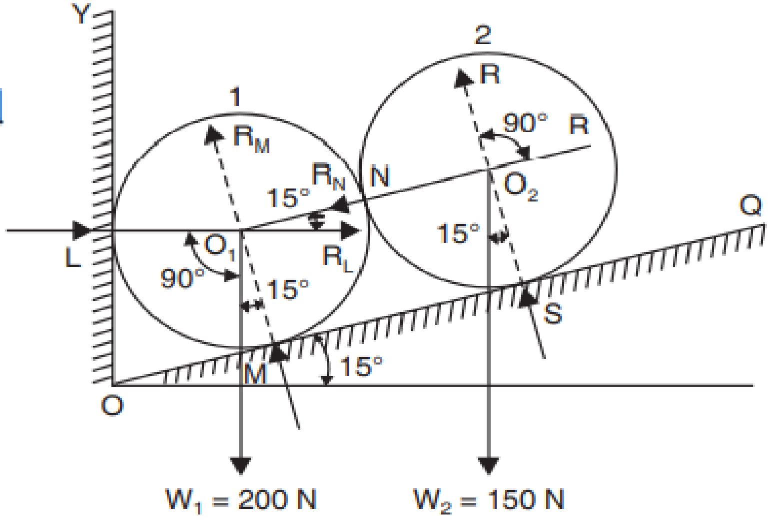
Sol. Refer to Fig. 2.44.

The following forces keep the cylinder '2' *in equilibrium*

(*i*) Weight of cylinder, W_2 (= 150 N) acting vertically downward ;

(*ii*) Reaction (R_s) acting at right angle to OQ ; and

(*iii*) Reaction of cylinder '1' (R_N) in the direction O_1O_2 (action and reaction are equal and opposite).



Applying Lami's theorem :

$$\frac{R_N}{\sin(180° - 15°)} = \frac{R_S}{\sin(90° + 15°)} = \frac{W_2}{\sin 90°}$$

or $$\frac{R_N}{\sin 15°} = \frac{R_S}{\sin 105°} = \frac{150}{1}$$

$\therefore$ $$R_N = 150 \sin 15° = 38.82 \text{ N}$$

and $$R_S = 150 \sin 105° = 144.89 \text{ N}$$

Continue…

The forces which keep the cylinder '1' in equilibrium are :

(*i*) Weight of the cylinder W_1 (= 200 N) acting vertically down ;

(*ii*) Reaction R_L acting at right angle to OY ;

(*iii*) Reaction R_M acting at right angle to OQ ;

(*iv*) R_N, the pressure of the cylinder '2' acting in the direction O_2O_1.

Since in this case, number of forces acting are four, so we *cannot apply Lami's theorem here which is applicable in a case where there are only three forces*. The unknown in this case can be determined by resolving the forces along O_1O_2 and in a direction *perpendicular to* O_1O_2.

$\therefore \quad R_L \cos 15° - W_1 \sin 15° - R_N = 0 \quad$ ...(*i*)

and $\quad R_M - R_L \sin 15° - W_1 \cos 15° = 0 \quad$ ...(*ii*)

From eqn. (*i*),

$$R_L \times 0.9659 - 200 \times 0.2588 - 38.82 = 0$$

or $\quad R_L \times 0.9659 - 51.76 - 38.82$

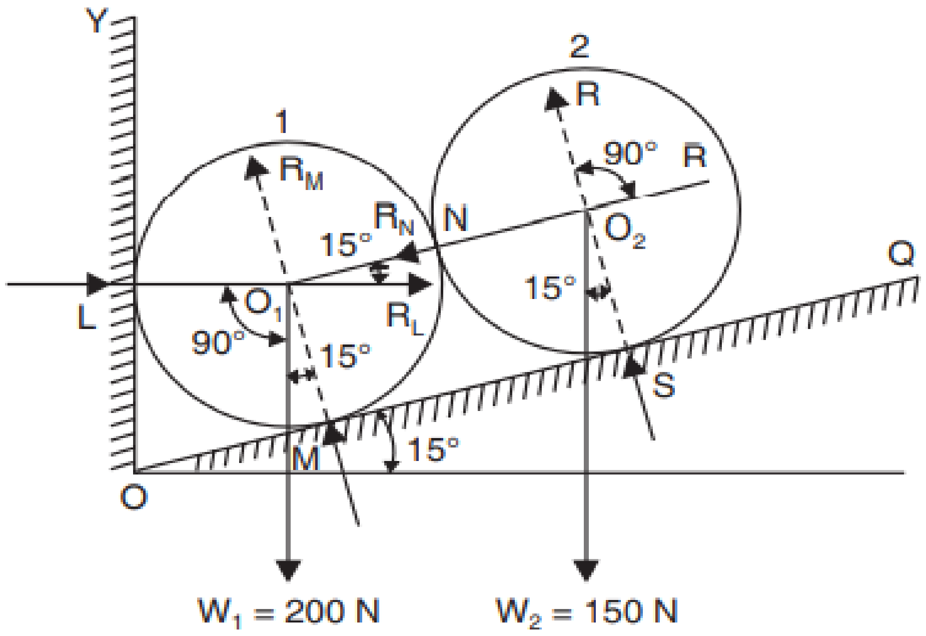
or $\quad \mathbf{R_L = 93.78\ N.\ (Ans.)}$

Putting the value of R_L in eqn. (*ii*), we get

$$R_M - 93.78 \times 0.2588 - 200 \times 0.9659 = 0$$

$$R_M - 24.27 - 193.18 = 0$$

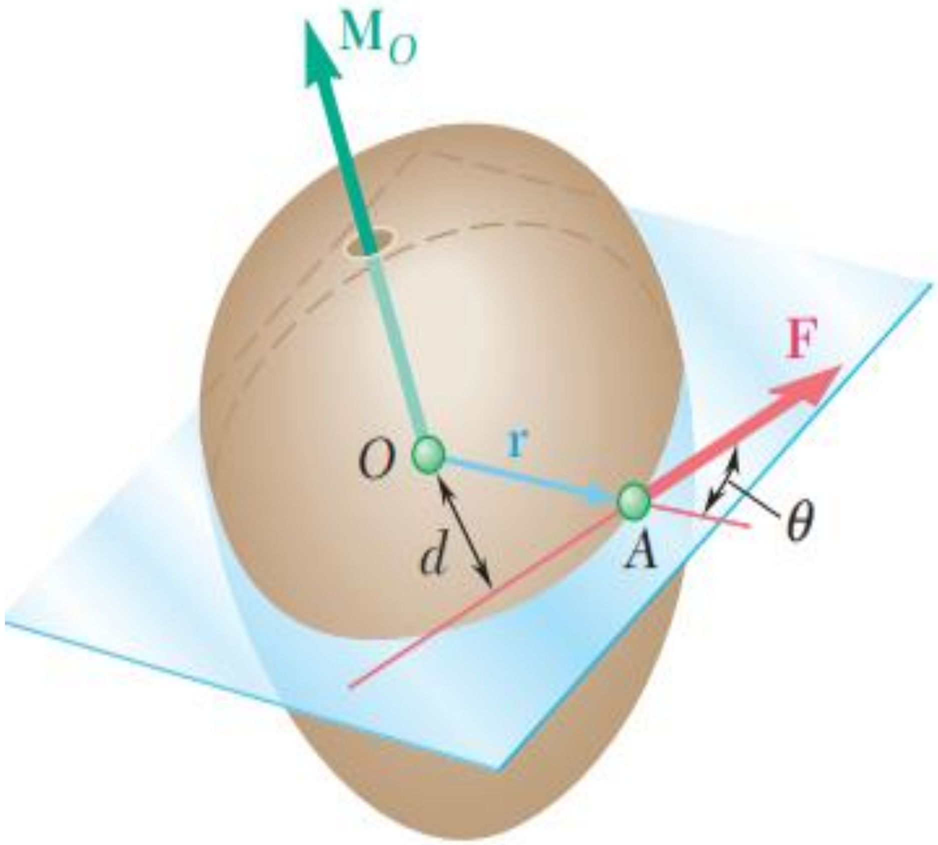
or $\quad \mathbf{R_M = 217.45\ N.\ (Ans.)}$



MOMENT OF A FORCE ABOUT A POINT

- Let us now consider a force F acting on a rigid body. As we know, the force F is represented by a vector which defines its magnitude and direction. However, the effect of the force on the rigid body depends also upon its point of application “A” . The position of “A” can be conveniently defined by the vector “r” which joins the fixed reference point O with A ; this vector is known as the position vector of A.
- We will define the moment of F about O as the vector product of r and F :

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$



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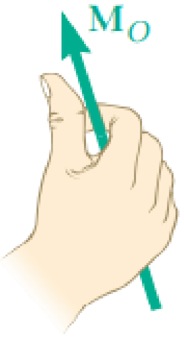
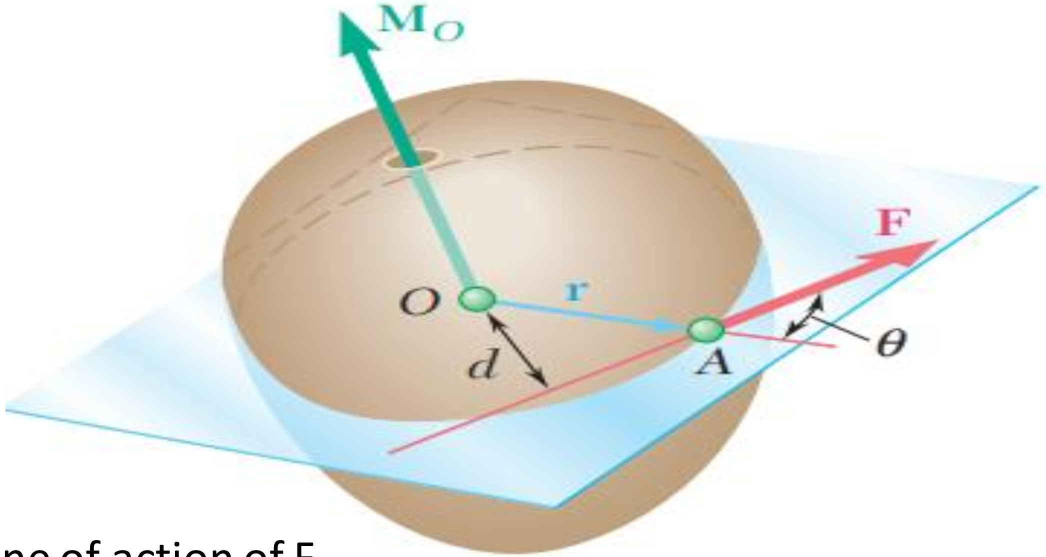
$$M_O = rF \sin\theta = Fd$$



- The sense of M_O is furnished by a variation of the right-hand rule: Close your right hand and hold it so that your fingers are curled in the sense of the rotation that F would impart to the rigid body about a fixed axis directed along the line of action of M_O ; your thumb will indicate the sense of the moment M_O.
- Finally, denoting by u the angle between the lines of action of the position vector r and the force F , we find that the magnitude of the moment of F about O is

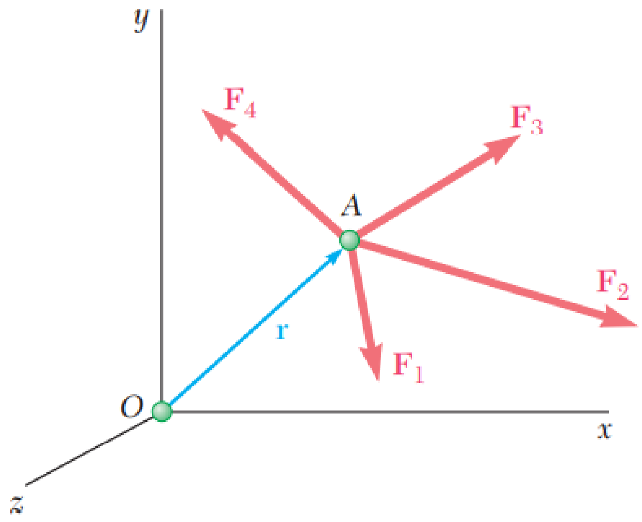


- where d represents the perpendicular distance from O to the line of action of F .
- In the SI system of units, where a force is expressed in newtons (N) and a distance in meters (m), the moment of a force is expressed in newton-meters (Nm).

VARIGNON’S THEOREM

❖The moment about a given point O of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about the same point O.

❖The moment of a force about an axis is equal to the sum of the moments of its components about the same axis



$$\mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \cdots) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \cdots$$

VARIGNON'S THEOREM

Moment of the force **F** about O,

F d=F (OA cos ϴ) = OA(F cos ϴ)

$F\ d = OA\ F_x$ . . . (a)

Moment of the force F1 about O,

$F_1 d_1 = F_1 (OA \cos \Theta_1) = OA(F_1 \cos \Theta_1)$

$F_1 d_1 = OA\ F_{x1}$ . . . (b)

Moment of the force $\mathbf{F_2}$ $F_2 d_2$ about o,

$F_2 d_2 = F_2 (OA \cos \Theta_2) = OA(F_2 \cos \Theta_2)$

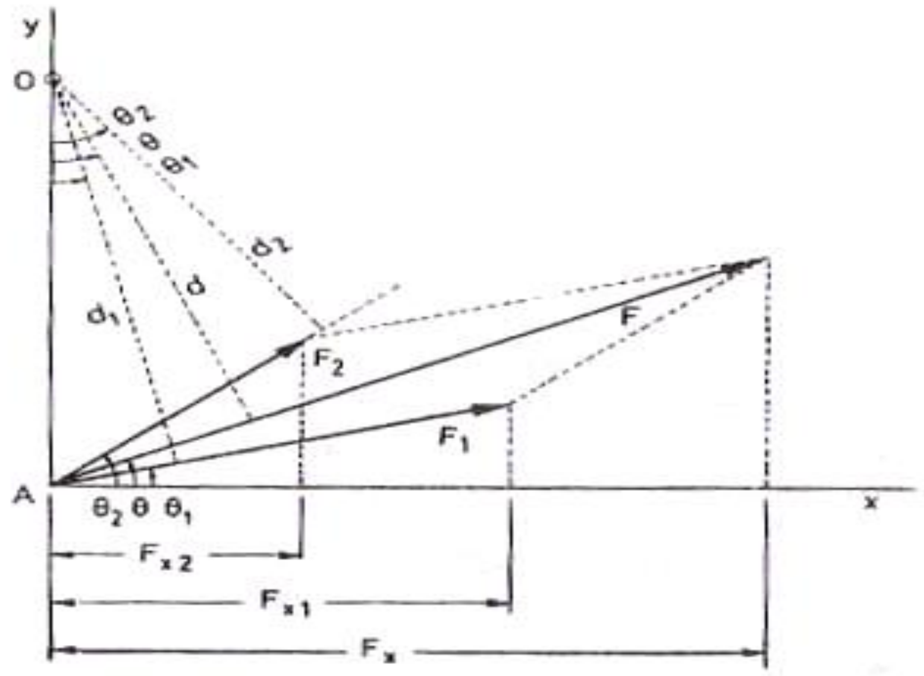
$F_2 d_2 = OA\ F_{x2}$ . . . (c)

Adding (b) and (c)

$F_1 d_1 + F_2 d_2 = OA\ (F_{x1} + F_{x2})$ . . . (d)

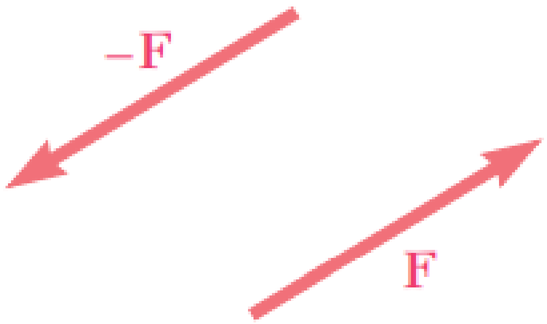
But, $F_x = F_{x1} + F_{x2}$

So from (a) and (d) , $Fd = F_1 d_1 + F_2 d_2$



MOMENT OF A COUPLE

- Two forces F and -F having the same magnitude, parallel lines of action, and opposite sense are said to form a couple.
- Clearly, the sum of the components of the two forces in any direction is zero. The sum of the moments of the two forces about a given point, however, is not zero. While the two forces will not translate the body on which they act, they will tend to make it rotate.



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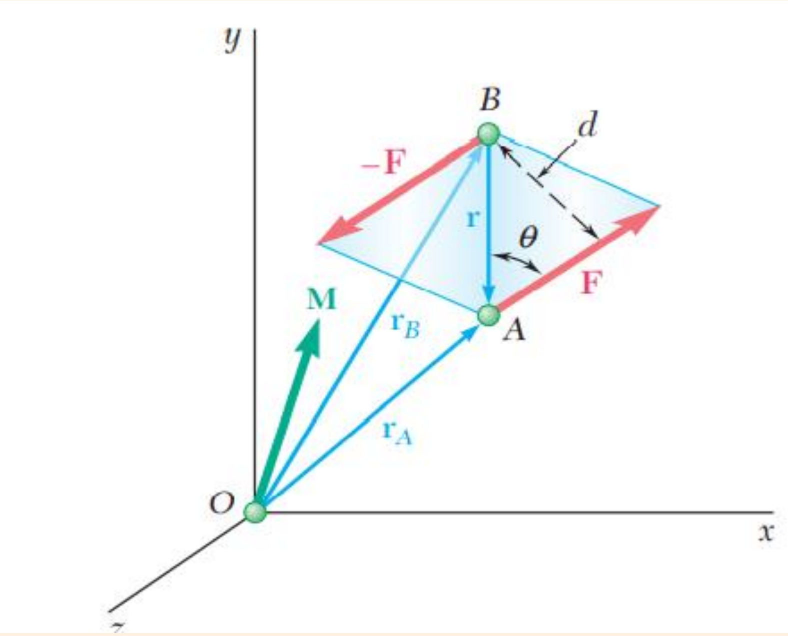


- Denoting by r_A and r_B , respectively, the position vectors of the points of application of F and –F. we find that the sum of the moments of the two forces about O is

$$r_A \times F + r_B \times (-F) = (r_A - r_B) \times F$$

- Setting r_A - r_B = r , where r is the vector joining the points of application of the two forces, we conclude that the sum of the moments of F and -F about O is represented by the vector.

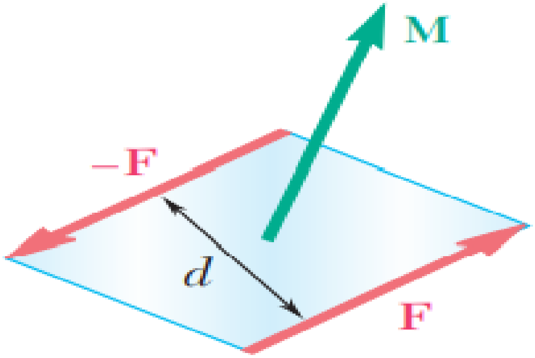
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Continue…

- The vector M is called the moment of the couple; it is a vector perpendicular to the plane containing the two forces, and its magnitude is

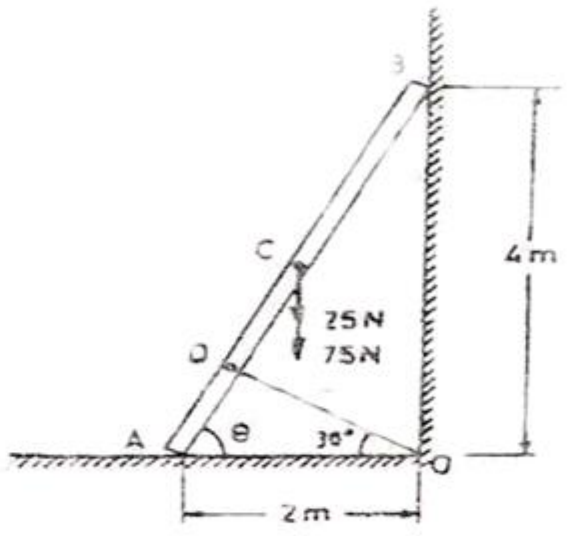
$$M = rF \sin \theta = Fd$$

- where d is the perpendicular distance between the lines of action of F and -F. The sense of M is defined by the right-hand rule



Example

A man weighing 75 N stands on the middle rung of a 25 N ladder sting on a smooth floor and against a wall. The ladder is prevented from slipping by a string OD. Find the tension in the string and reactions at A and B as shown in Fig.



Solution. Angle Θ which ladder makes with the horizontal

$\tan \Theta = 4/2 = 2, \quad \Theta = 63.43^\circ$

Consider the free-body diagram of the ladder.

Equation of equilibrium are,

$\sum F_x=0 \quad T \cos 30^\circ - R_B=0$ ... (i)

$\sum F_y=0 \quad R_A - (75+25) – T \sin 30^\circ=0, \quad R_A – T \sin 30^\circ = 100$ ... (ii)

Taking moments about O.

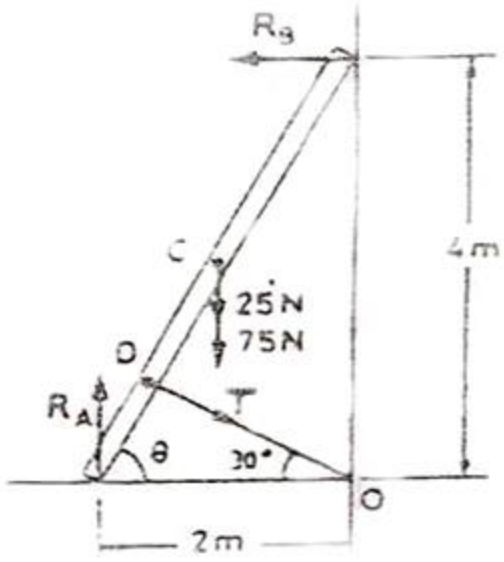
$\sum M_0 = 0 : \quad R_B \times 4 + 75 \times 1 + 25 \times 1 - R_A \times 2 = 0, \quad 2R_A=4R_B = 100$ ... (iii)

From (i) $\quad T = \dfrac{R_B}{\cos 30^\circ}$

Substituting in (ii), $\quad R_A – 0.577\, R_B = 100$ ... (iv)

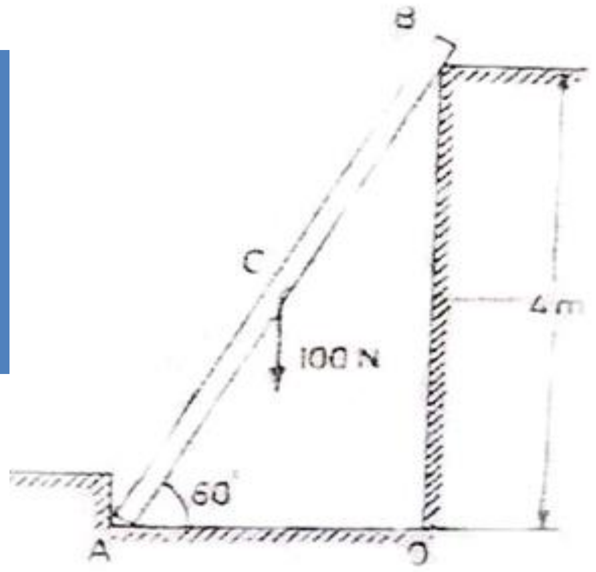
Solving equations (iv) and (iii) simultaneously, $R_B = 35.13$ N , $R_A = 120.26$ N Ans.

From (i), $T = 40.56$ N Ans.



Example

A man weighing 100 N stands on the middle rung of the ladder whose weight can be neglected. The end A rests on the ground against a and the end B rests on the corner of a wall as shown. Find the reaction at A and B. Neglected friction between the ladder and the ground and the ladder and the wall.



$\sum F_x = 0$: $\quad X_A - R_B \sin 60^\circ = 0 \quad$ ... (i)

$\sum F_y = 0$: $\quad Y_A - W + R_B \cos 60^\circ = 0 \quad$ ... (ii)

Taking moments about A

$\sum M_A = 0$: $\quad R_B(AB) - 100\,(AD) = 0$

where, $AB = OB \sec 30^\circ$

$AB = 4 \times 1.54 = 4.616$ m

$$RB = \frac{100 \times 1.154}{4.616}$$

where,

$AD = CD \times \tan 30^\circ$

$AD = 2 \times 0.577$

$AD = 1.154$ M

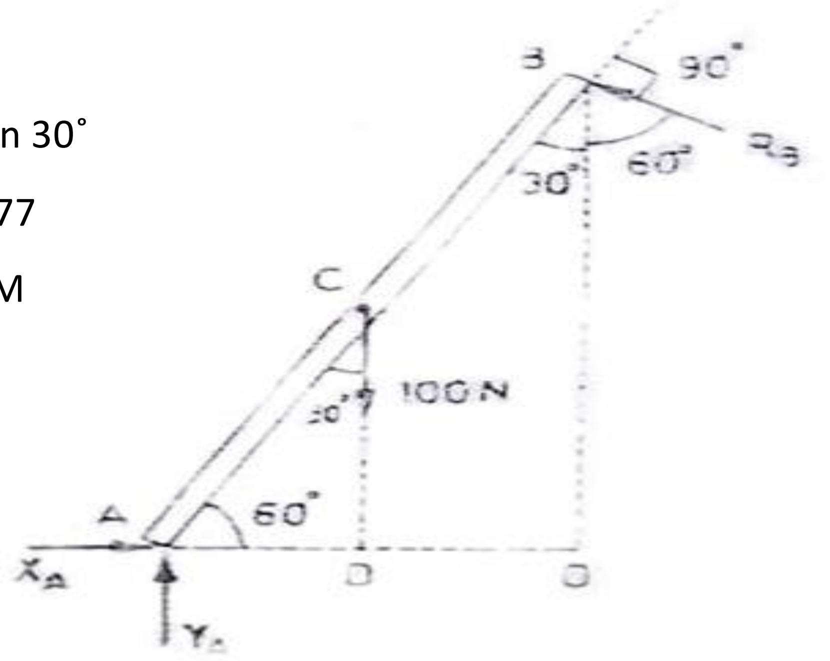
RB = 25 N Ans.

From (i) $\quad X_A = R_B \sin 60^\circ = 25 \times 0.866$

$X_A = 21.65$ N $\quad$ Ans.

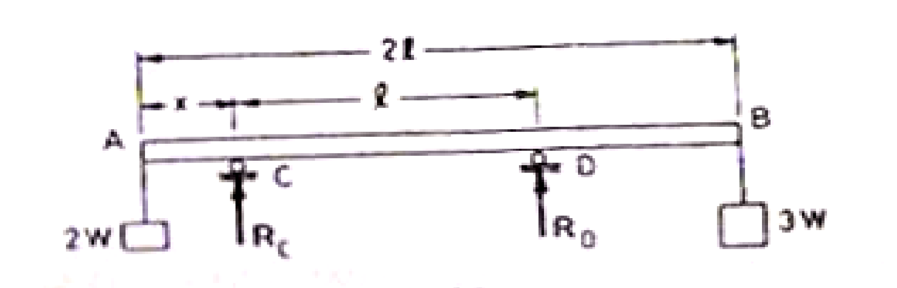
From (ii) $\quad Y_A = W - R_B \cos 60^\circ = 100 - 25 \times 0.5$

$Y_A = 87.5$ N $\quad$ Ans.



Example

A bar AB of length 2l and negligible weight rests on two roller supports C and D placed at a distance l apart. The bar supports two vertical loads as shown. For the reactions at the supports to be equal find the distance x, of the end A of the bar, from the support C



Forces acting on the bar AB are :

Reactions R_C, R_1, ; 2W and 3W

Writing the equations of equilibrium of the bar

$\sum F_y = 0$: $R_C + R_D - 2W - 3W = 0$

But, $R_C = R_D$ (Given)

Therefore, $R_C = R_D =$ <u>5W/2</u>

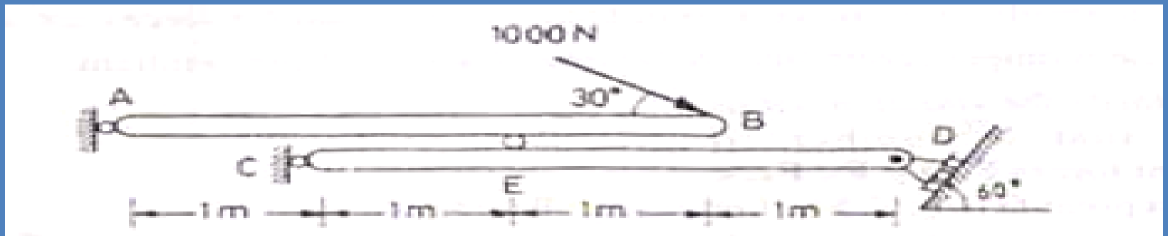
<u>Now Take moment about A:</u>

$R_C * X + R_D *l - 3W*2l = 0$

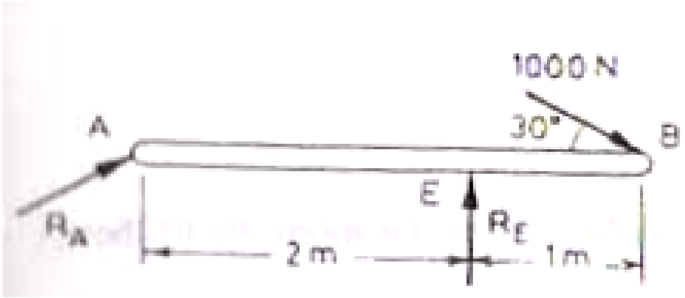
Substituting value $R_C = R_D =$ <u>5W/2 in above equation find the value of X</u>

Example

Two beams AB and CD are arranged and supported as shown. Find the reaction at D due to a force of 1000 N acting at B as shown in Fig.



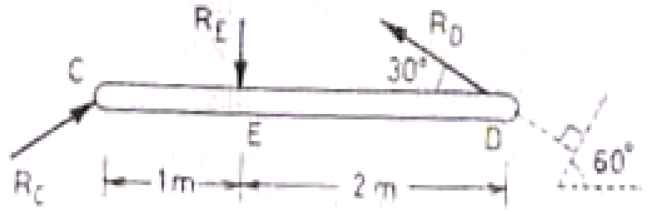
Consider the free-body diagram of the beam AB, where RA is the reaction at A and RE is the reaction at E. Taking moment about A,



$\sum M_A = 0 : \quad R_E \times 2 - 1000(3 \sin 30^\circ) = 0$

$R_E = \dfrac{3000 \times 0.5}{2} = 750$ N

Consider the free-body diagram on the beam CD, where R_C is the reaction at C and R_D is the reaction at D. Taking moment about C,



$\sum M_C = 0 : \quad R_D(3 \sin 30^\circ) - R_E\,(1) = 0 \quad , \qquad R_D\,(3 \sin 30^\circ) - 750(1) = 0$

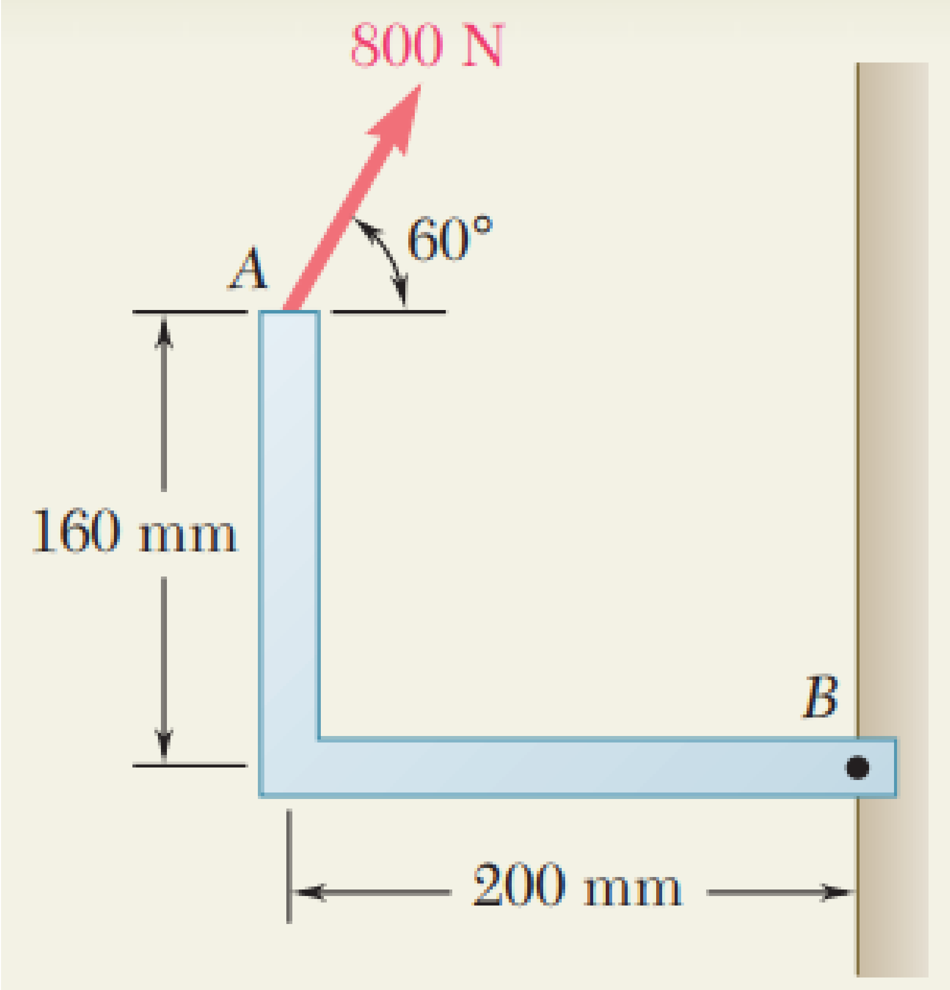
$R_D = \dfrac{750}{3 \sin 30^\circ} = \dfrac{750}{3 \times ½}$

$R_D = 500$ N Ans.

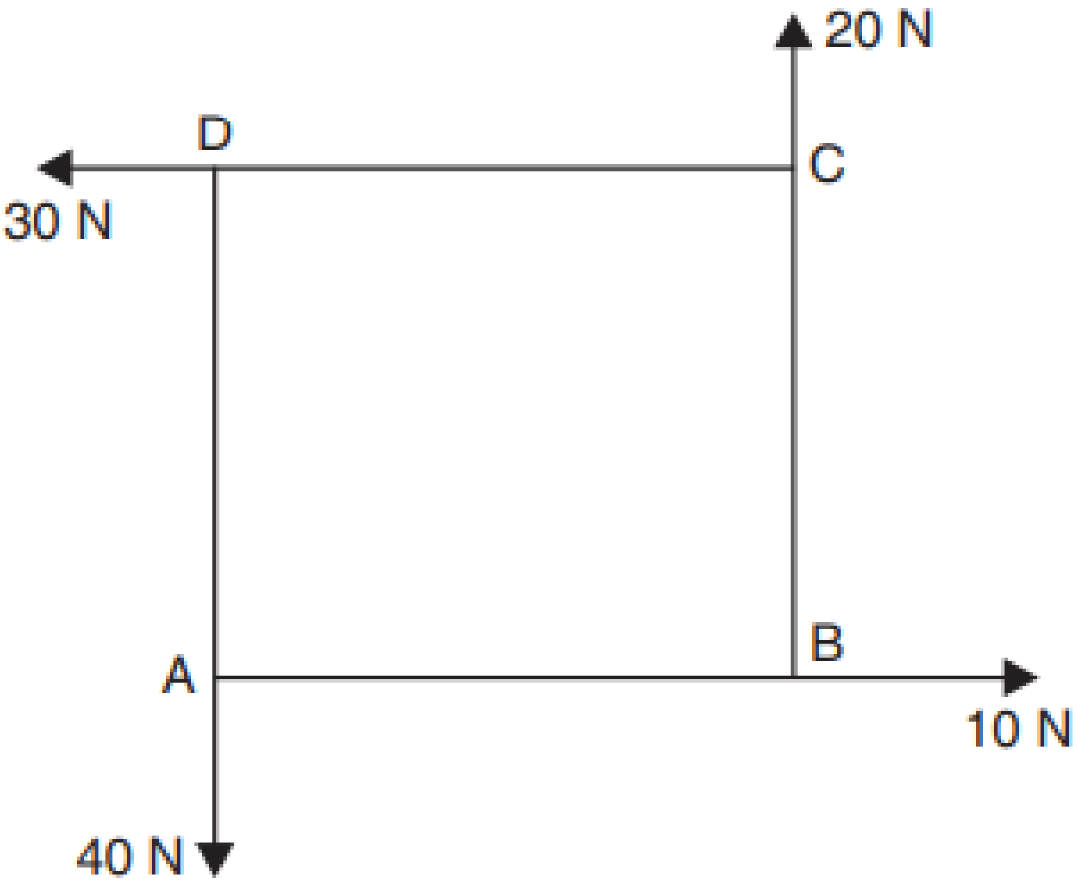
Questions:

SAMPLE PROBLEM 3.2

A force of 800 N acts on a bracket as shown. Determine the moment of the force about *B*.

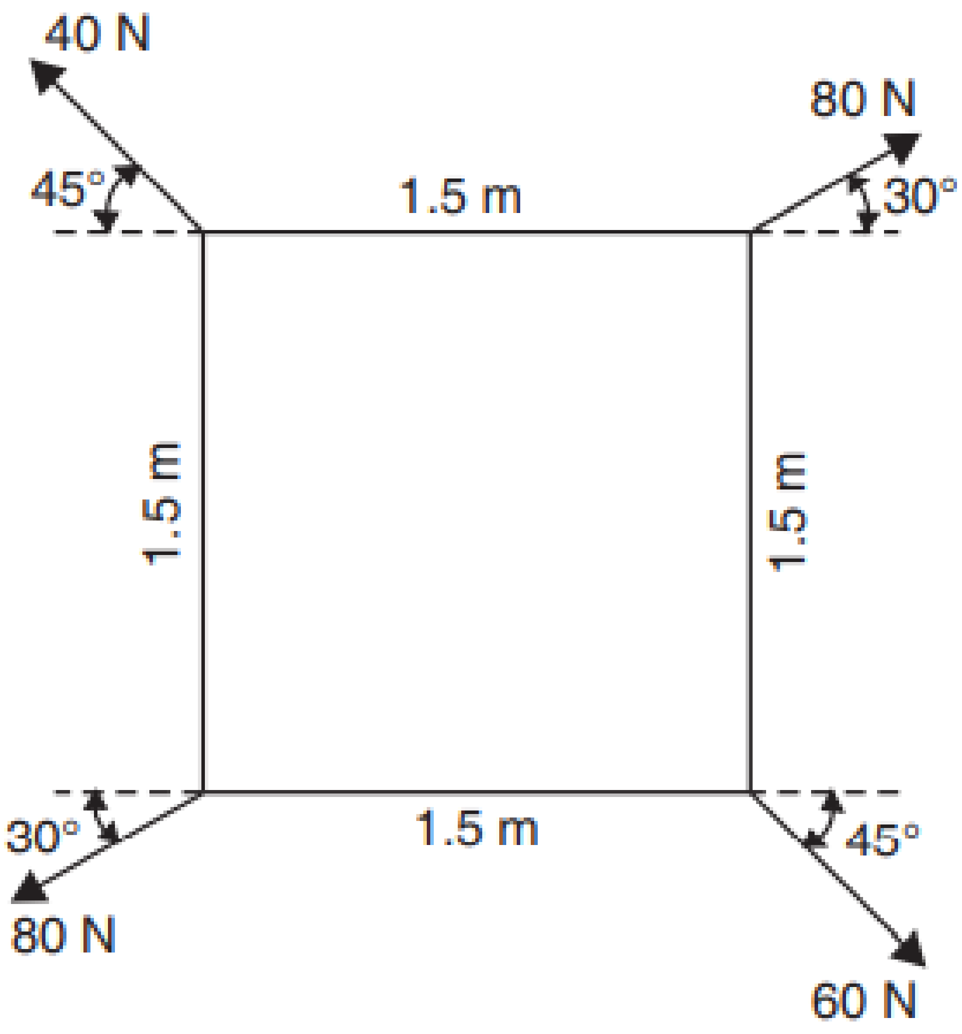


Example 3.8. *Four forces equal to 10 N, 20 N, 30 N and 40 N are respectively acting along the four sides (1 m each) of a square ABCD, taken in order. Find the magnitude, direction and position of the resultant force.*

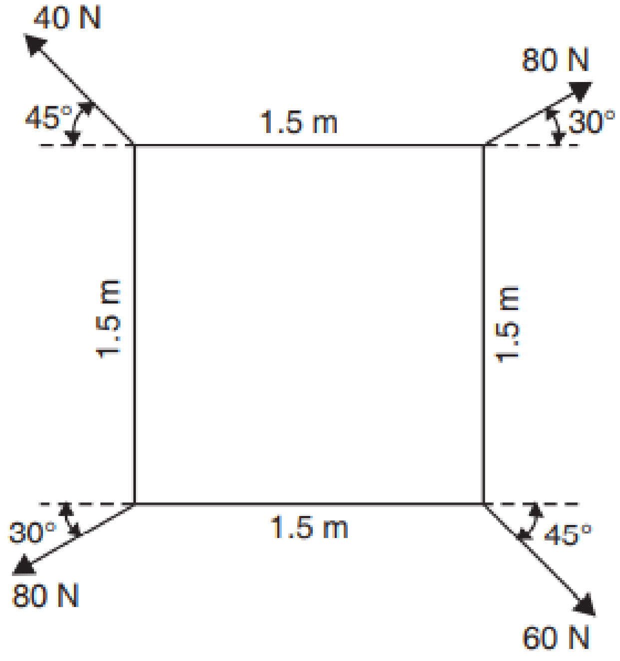


Questions

Questions



Example 3.9. *A body is under the action of four coplanar forces as shown in Fig. 3.36. Find the magnitude, direction and position of resultant of the given force system.*



Magnitude of resultant, R = ?

Resolving the forces *horizontally*,

$$\Sigma H = 80 \cos 30° + 60 \cos 45° - 80 \cos 30° - 40 \cos 45°$$
$$= 80 \times 0.866 + 60 \times 0.707 - 80 \times 0.866 - 40 \times 0.707 = 14.14 \text{ N.}$$

Resolving the forces *vertically* :

$$\Sigma V = 80 \sin 30° - 60 \sin 45° - 80 \sin 30° + 40 \sin 45° = -14.14 \text{ N}$$

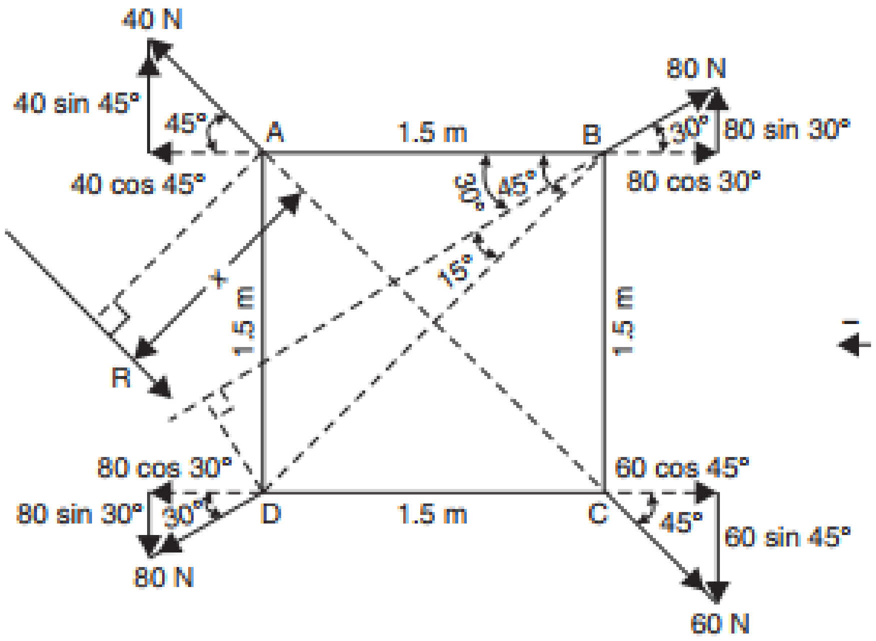
Resultant,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(14.14)^2 + (-14.14)^2}$$
$$= 20 \text{ N}$$
$$\boldsymbol{R = 20 \text{ N. (Ans.)}}$$

Direction of resultant, α = ?

The resultant will act at an angle with the horizontal so that,

$$\tan \alpha = \frac{\Sigma V}{\Sigma H} = \frac{14.14}{14.14} = 1$$
$$\alpha = 45°. \text{ (Fig. 3.38).}$$



Solution

Position of the resultant = ?

The position of the resultant 'R' can be determined by using the relation :

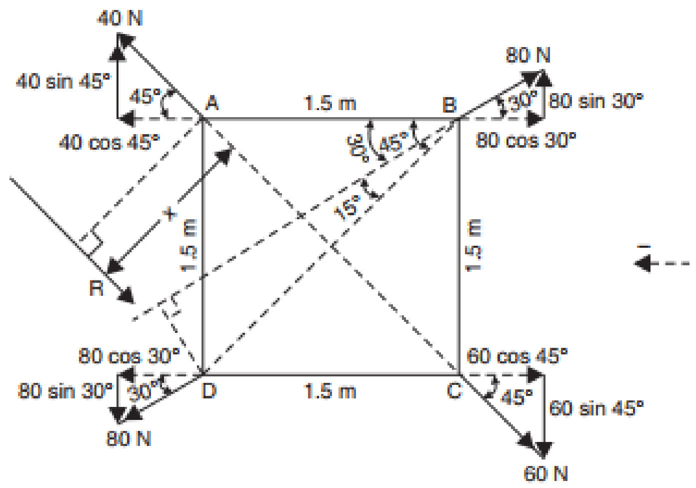
Moment of resultant about A = algebraic sum of moments of the rectangular components of all forces about A.

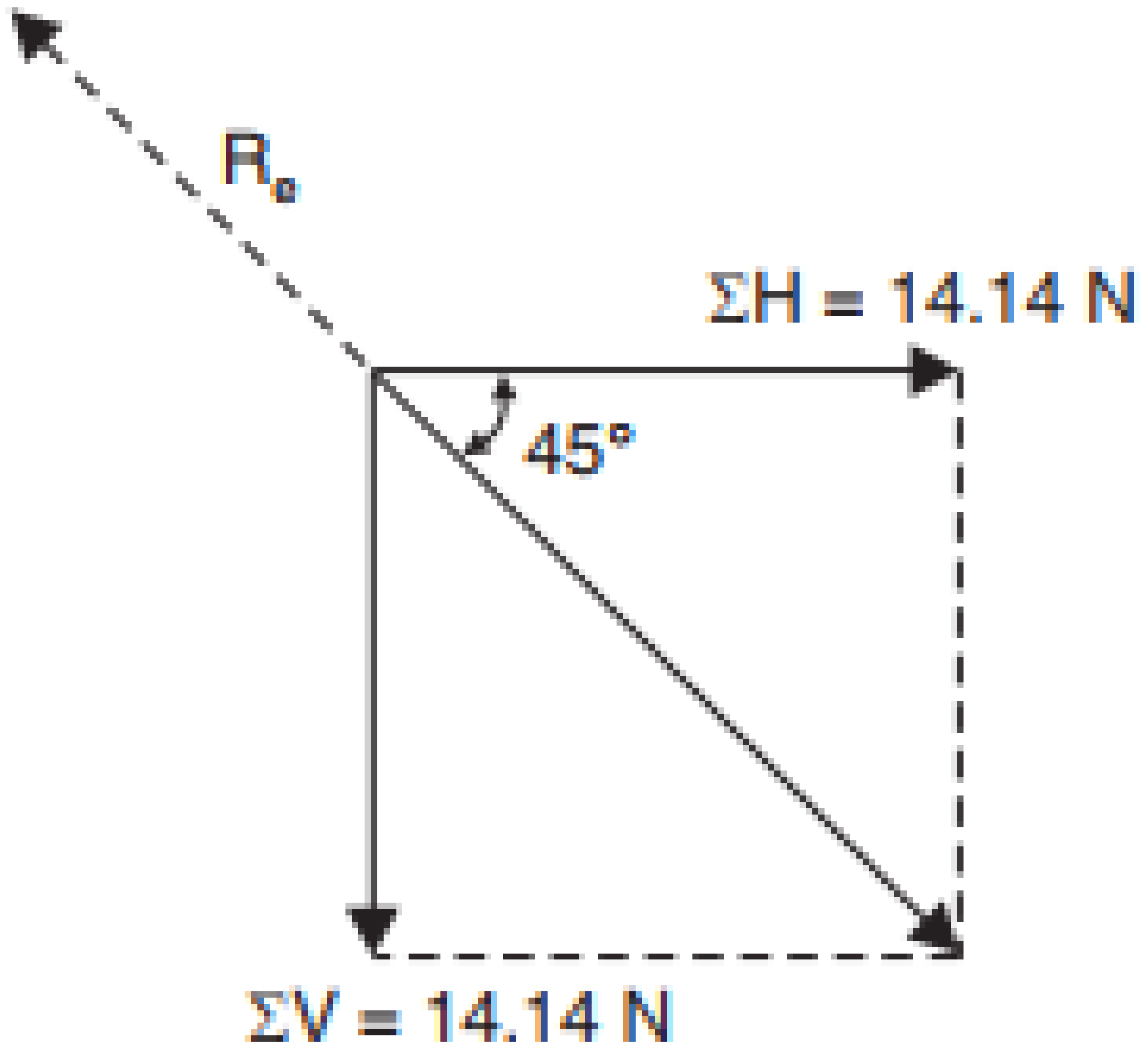
$$\begin{aligned} -R \times x &= 40 \sin 45° \times 0 \\ &+ 40 \cos 45° \times 0 \\ &+ 80 \sin 30° \times 1.5 \\ &+ 80 \cos 30° \times 0 \\ &+ 60 \cos 45° \times 1.5 - 60 \sin 45° \times 1.5 - 80 \cos 30° \times 1.5 \\ &+ 80 \sin 30° \times 0 \end{aligned}$$

$$-20 \times x = 0 + 0 + 60 + 0 + 63.63 - 63.63 - 103.92 + 0$$

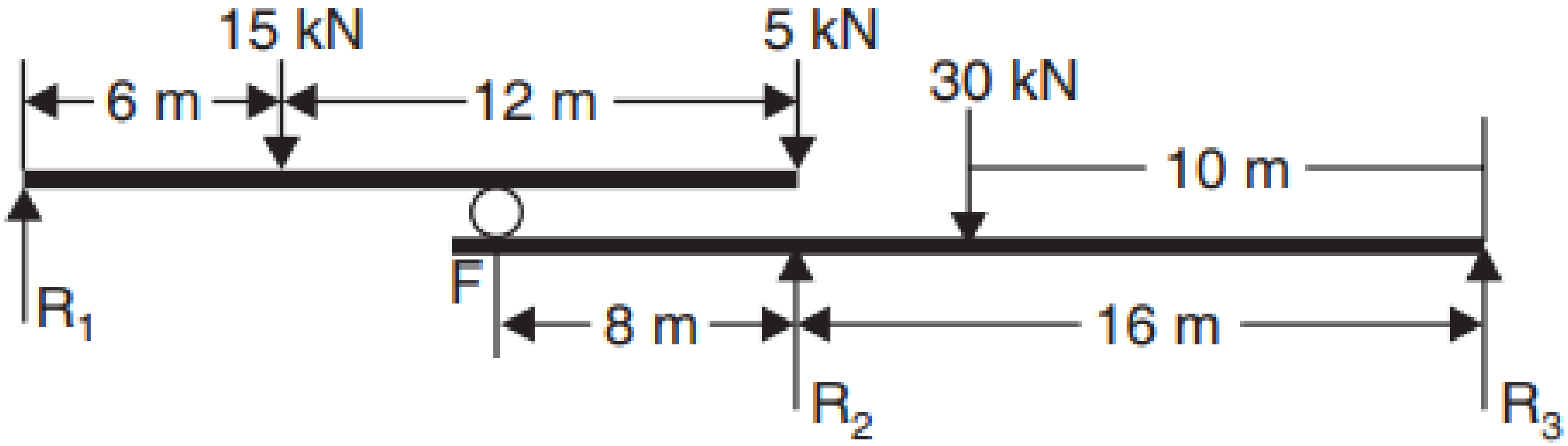
or $\quad -20\,x = -43.92$

or $\quad$ **x = 2.196 m from A (Fig. 3.37). (Ans.)**





Question



Example 3.20. *Find the reaction R_1, R_2 and R_3 in the case of two beams placed one over the other and loaded as shown in Fig. 3.50.*

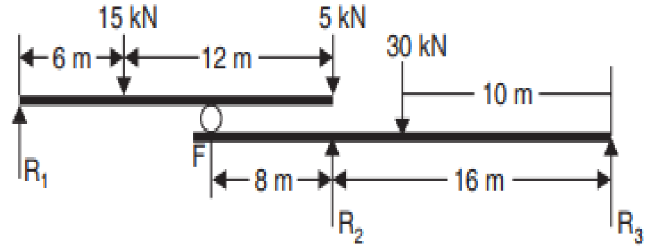


Fig. 3.50

Taking moments about F for the **top beam**

$$R_1 \times 10 + 5 \times 8 = 15 \times 4$$

$\therefore$ $\qquad \boldsymbol{R_1 = 2}$ **kN. (Ans.)**

Out of total load of (15 + 5 = 20 kN) 20 kN, rection R_1 takes up 2 kN and remaining (20 – 2 = 18 kN) 18 kN acts at the edge of the bottom beam as shown in Fig. 3.51.

Taking moments about R_3,

$$R_2 \times 16 = 18 \times 24 + 30 \times 10$$

$$= 432 + 300$$

$\therefore$ $\qquad \boldsymbol{R_3 = 45.765}$ **kN. (Ans.)**

Now, $\qquad \Sigma V = 0$

$\therefore$ $\qquad R_2 + R_3 = 18 + 30$

or $\qquad 45.75 + R_3 = 48$

or $\qquad \boldsymbol{R_3 = 2.25}$ **kN. (Ans.)**

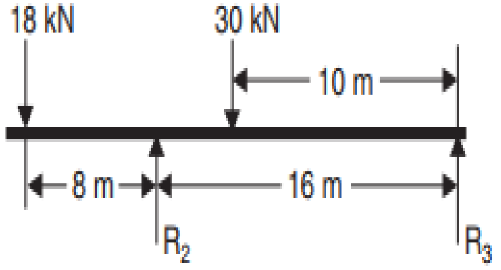
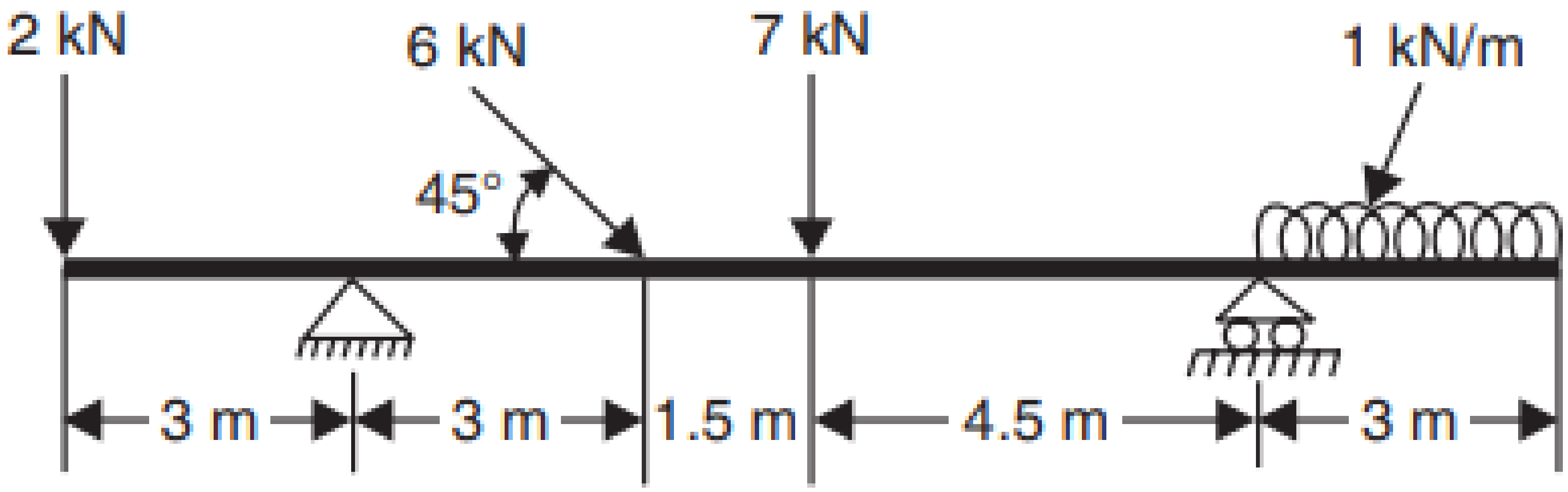


Fig. 3.51

Questions



A simply supported overhanging beam 15 metres long carries a system of loads as shown in Fig. 3.58. Determine the reactions at the supports. [**Ans.** 7.75 kN ; 9.27 kN acting at 26° 36′ with the vertical]