STA 3032 Exam 1 Topics

$A \cup B$ or

$A \cap B$ and

Set Theory

$\mathbb{R} \times \mathbb{R}$

$\mathbb{R}^2$

- Set Operations: Union, complementation, intersection.
- Set Containment and disjoint sets — A, B disjoint if $A \cap B = \emptyset$
- Cartesian Product of two sets — ordered pairs (a,b), $a \in \mathbb{R}$, $b \in \mathbb{R}$
- Set builder notation
- Represent the sample space of a particular experiment as a set
  - Represent events as sets
- Know what these sets are: $\mathbb{Z}, \mathbb{N}, \mathbb{R}, \emptyset, \mathbb{R}^n$.
- Laws: Distributive, Demorgan's.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$P(B|A)$ $\quad P\left(\left(\bigcup_{n=1}^{\infty} A_n\right) \cap B\right) = P\left(\bigcup_{n=1}^{\infty} (A_n \cap B)\right)$

Probability Theory

- Probability functions: map from S to $[0, 1]$.
  - Memorize the axioms of probability.
- Be able to use the axioms of probability to determine if a function P is a probability function
- The set theory that appears in the axioms of probability: Disjoint sets, infinite union of sets.
- Probabilities of events in a finite sample space where each outcome has equal probability.
  - Counting rules: multiplication rule
  - Know the definition of the choose function: $\binom{n}{x}$ → $\binom{n}{x} p^x (1-p)^{n-x}$
  - Use these to calculate probabilities → $X = \#$ of heads in n flips
- Proofs of general probability properties: see the proofs for $P(A^c) = 1 - P(A)$, and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, etc. from class.

  H _ H _
- Conditional probability.
- Independent events (do the DIY suggested in class: indep. events A, B imply $P(A|B) = P(A)$)
- Mutually exclusive and complementary events
- Law of total probability: understand the proof.
- Bayes rule: understand the proof.

Random Variables

$P(X \in A) = P(\{\omega \in S : X(\omega) \in A\})$

see HW2 Q3

- Definition of a random variable. Definition of $P(X \in A)$.
- Continuous vs. discrete random variables (pdf, pmf)
- Probability mass and probability density functions
  - The two properties which make a function a valid probability density/mass function.

    ↳ nonnegative

    ↳ $\sum_x f(x) = 1$

    $\int_{-\infty}^{\infty} f(x)\,dx = 1$
- Cumulative distribution functions
  - Know the definition → $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$ if cont.
  - Taking the derivative yields the pdf
- Calculating probabilities
  - With the cdf
  - With the pdf/pmf — 1,17 in notes
  - In the discrete OR continuous case
  - Over an infinite OR finite set
- Expected values
  - Know how to calculate given a pmf/pdf

    $E(X^2) = \int x^2 f(x)\,dx$
- Variance → $V(X) = E(X^2) - E(X)^2$
- Covariance → $\mathrm{cov}(X,Y) = E(XY) - E(X)E(Y)$

$$\frac{d}{dt}\int_{-\infty}^{\infty} e^{tx} f(x)\,dx = \int_{-\infty}^{\infty} x e^{tx} f(x)\,dx = \int x f(x)\,dx = E(x)$$

$t=0$, $e^0 = 1$

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$$

- Moment generating function: know the definition!
  - Using the moment generating to calculate moments. Using the moments to calculate the variance.

$$E(X) = M_X'(t)\big|_{t=0}$$
$$E(X^2) = M_X''(t)\big|_{t=0} \quad \cdots \quad E(X^k) = \frac{d}{dt^k} M_X(t)\big|_{t=0}$$

- Joint distributions
  - Calculating probabilities
  - Calculating expectations (e.g. $E(XY)$)
  - Finding marginal distributions
  - Finding conditional distributions know defn
- Independent random variables: know the definition. Know how to determine if two random variables are independent.
- Distribution families: some parameters may be left as variables when calculating expectations and probabilities.

$$f(x,y) = f_X(x) f_Y(y)$$

$a(e^y x)$

$e^y + x$

have $f(x,y)$

$$f_X(x) = \int_0^1 f(x,y)\,dy \qquad 0 \le y \le 1$$

$$f_Y(y) = \int_0^1 f(x,y)\,dx$$

$B = [0,1] \times [0,1]$

$$P(X \in A) = \int_A f_X(x)\,dx$$

$\subset \mathbb{R}$

$$P((X,Y) \in B) = \iint_B f(x,y)\,dx\,dy$$

$\subset \mathbb{R}^2$

$$\int_0^1 \int_0^1$$

$P(0 \le X \le 1,\ 0 \le Y \le 1)$

$$f(x|y) = \frac{f(x,y)}{f_Y(y)}$$ function of x

$$f(x|3) = \frac{f(x,3)}{f_Y(3)}$$