Asymptotic Notation.

(Numericals)

Example 1: $f(n) = 2n+3$.

$$2n+3 \leq c \cdot g(n)$$

$$2n+3 \leq c \cdot (n)$$

$$2n+3 \leq \underset{c}{\underset{\downarrow}{5}} \cdot n \quad , \quad n \geq 1$$

$$\therefore f(n) = O(n)$$

For, $f(n) = O(n^2)$ is also true.

$f(n) = O(2^n)$ is also true.

$f(n) = O(\lg n)$ ✗

$$1 < \lg n < \sqrt{n} < n < n^2 < n^3 < \cdots < 2^n < 3^n \cdots < n^n$$

Lower bound Ω

Upper bound O

Average bound (θ)

Q2:

$f(n) = 2n^2 + 3n + 4$

As a practice, we ~~make~~ write everything in terms of n^2. To obtain the value of constant.

$2n^2+3n+4 \leq 2n^2+3n^2+4n^2$

$2n^2+3n+4 \leq \underbrace{9}_{c}n^2 \quad , n \geq 1$

$\boxed{f(n) = O(n^2)}$

$\Rightarrow \quad f(n) = 2n^2+3n+4$

$2n^2+3n+4 \geq 1 * n^2$

$\boxed{f(n) = \Omega(n^2)}$ where, $c=1$

$\Rightarrow \quad \underbrace{1}_{c_1}n^2 \leq 2n^2+3n+4 \leq \underbrace{9}_{c_2}n^2$

$\boxed{f(n) = \theta(n^2)}$

Q. $\boxed{f(n) = n!} = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$

$$1 \times 1 \times 1 \times \cdots \times 1 \le 1 \times 2 \times 3 \times \cdots \times n \le n \times n \times n \times n \times \cdots \times n$$

{As a practice, we make f(n) value as n.}

{As a practice, we make 1×1×1×...×1}.

Lower bound.

Upper bound

$$\boxed{1 \le n! \le n^n}$$

Here, we are not getting same thing i.e. n^n in both side of $f(n)$, so we cannot use Θ notation.

$$\boxed{\Omega(1) \cancel{\le} f(n) = O(n^n)}$$

upper bound.

Lower bound.

~~so, f(n)~~

So, for smaller values $f(n)$ is 1 and for larger of n, $f(n)$ is $O(n^n)$.

Note: we can not fixed any value for factorial

Example: $n^{10} < n! < n^{11}$ we cannot say any thing.

There is no tight bound for this function.

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Q $f(n) = 2n$, $g(n) = n^2$, prove that
$f(n) = o(g(n))$

Proof:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$$

$$\lim_{n\to\infty} \frac{2n}{n^2} = 0$$

$$\lim_{n\to\infty} \frac{2}{n} = 0$$

$\therefore$ True $\boxed{\therefore f(n) = o(n^2)}$

Q:

$f(n) = 2n^2 + 16$, $g(n) = n^2$, show that

$$\boxed{f(n) = \omega(g(n))}$$

$$\boxed{\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty}$$

Proof:

$$\lim_{n \to \infty} \frac{2n^2 + 16}{n^2}$$

$$\lim_{n \to \infty} \frac{\cancel{n^2}(2 + 16/n^2)}{\cancel{n^2}}$$

$$2 \neq \infty$$

$\therefore$ False

Numerical

1 check whether $2^{n+1} = O(2^n)$

$\rightarrow$ Use limit formula,

$$\boxed{\lim_{n\to\infty} \frac{f(n)}{g(n)} \leq C < \infty}$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \leq C$$

$$\lim_{n\to\infty} \frac{2^{n+1}}{2^n} \leq C$$

$$\frac{2 \cdot \cancel{2^n}}{\cancel{2^n}} \leq C.$$

$$\boxed{2 \leq C}$$

$\therefore$ True.

(2) Prove that $3^{n+3} = O(3^n)$

$$\lim_{n\to\infty} \frac{3^{n+3}}{3^n} \leq C$$

$$\frac{3^3 \cdot \cancel{3^n}}{\cancel{3^n}} \leq C$$

$\therefore$ True. $\boxed{27 \leq C.}$

(3) Check whether $2^{2n} = O(2^n)$

$$\lim_{n\to\infty} \frac{2^{2n}}{2^n} \leq C$$

$$\lim_{n\to\infty} \frac{\cancel{2^n} \cdot 2^n}{\cancel{2^n}} \leq C$$

$$\boxed{\infty \leq C.}$$

$\therefore$ False.

(7) $f(n) = 3n+2$, $g(n) = n^2$, show that $f(n) = \Omega(g(n))$

$$\boxed{\lim_{n\to\infty} \frac{f(n)}{g(n)} > 0}$$

$$\lim_{n\to\infty} \frac{3n+2}{n^2} > 0$$

$$\lim_{n\to\infty} \frac{3+2/n}{n} > 0$$

$$0 > 0 \quad \times$$

$\therefore$ False

(5) Show that the following equations are correct:

$$n! = O(n^n)$$

Here, $f(n) = n!$ and $g(n) = n^n$.

we know $f(n) = O(g(n))$

if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c < \infty$

where $0 \leq c < \infty$,

Now,

$$\lim_{n\to\infty} \frac{n!}{n^n}$$

$$= \lim_{n\to\infty} \frac{n(n-1)(n-2)\ldots 3.2.1}{n^n}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^n}(1-\frac{1}{n})(1-\frac{2}{n})\cdots\frac{3}{n}\cdot\frac{2}{n}\cdot\frac{1}{n}}{\cancel{n^n}}$$

$$= 0$$

Hence, $n! = O(n^n)$ is correct.

Numerical.

Q6: Compare the functions –

$n^{\log n}$ and 2^n

log method:

Taking log of both the functions

$$n^{\log n} = \log n \log n$$
$$= (\log n)^2$$

$$2^n = n \log_2 2$$
$$= n$$

Now, we can put large value of $n = 1024$.

$(\log n)^2$	n
$(\log_2 1024)^2$	1024
$(\log_2 2^{10})^2$	
$(10)^2$	$\therefore$ 2^n is bigger.
100	

Q7: Consider the following functions:-

$f(n) = 2^n$, $h(n) = n^{\log n}$ and $g(n) = \lfloor n$

ⓐ $f(n) = O(g(n))$ & $g(n) = O(h(n))$

ⓑ $f(n) = \Omega(g(n))$ & $g(n) = O(h(n))$

ⓒ $g(n) = O(f(n))$ & $h(n) = O(f(n))$

✓ ⓓ $g(n) = \Omega(f(n))$ & $h(n) = O(f(n))$

$g(n) > f(n) > h(n)$

Q8: Consider the following function:

$f(n) = 3n^{\sqrt{n}}$, $g(n) = 2^{\sqrt{n}\lg n}$, $h(n) = n!$

ⓐ $h(n) = o(f(n))$

ⓑ $h(n) = O(g(n))$

ⓒ $g(n) \neq O(f(n))$

ⓓ $f(n) = O(g(n))$.