# Insertion Sort

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insertion_sort(A)                                   cost        time
(1) for j = 2 to length(A)                          c1          n
(2)     key = A[j]                                  c2          n-1
(3)     i = j-1                                     c3          n-1
(4)     while(i>0 and A[i]>key)                     c4          Σ_{j=2}^{n} tj
(5)         A[i+1] = A[i]                           c5          Σ_{j=2}^{n} (tj-1)
(6)         i = i-1                                 c6          Σ_{j=2}^{n} (tj-1)
(7)     A[i+1] = key                                c7          n-1
```

T(n) = Sum of the product of the cost and time columns.

$$T(n) = c_1 n + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7 (n-1)$$

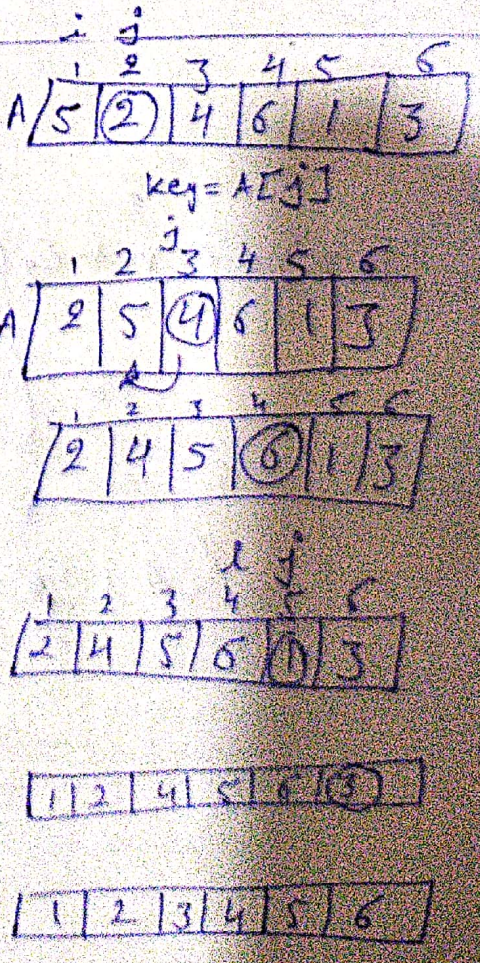
$\sum_{j=2}^{n} t_j$ → $j = 3$

$i = j-1 = 2$.

Inner loop, $i = 2, 1, 0$ i.e.

$t_j = j$

Example:



$$\sum_{j=2}^{n} t_j = \sum_{j=2}^{n} j = \boxed{2+3++4+--+n+1} - 1$$

$$\left[\frac{n(n+1)}{2} - 1\right]$$

$$\sum_{j=2}^{n}(t_j - 1) = \sum_{j=2}^{n}(j-1) = \underbrace{1+2+3+--+n-1}$$

Total terms are $n-1$.

$$\frac{(n-1)(n+1-1)}{2} = \frac{n(n-1)}{2}$$

$$T(n) = c_1 n + c_2(n-1) + c_3(n-1) + c_4 \cdot \left[\frac{n(n+1)}{2} - 1\right] +$$

$$c_5 \cdot \left(\frac{n(n-1)}{2}\right) + c_6 \cdot \left(\frac{n(n-1)}{2}\right) + c_8(n-1)$$

$$= \left(\frac{c_4}{2} + \frac{c_5}{2} + \frac{c_6}{2}\right) n^2 + \left(c_1 + c_2 + c_3 + \frac{c_4}{2} + \frac{c_5}{2} - \frac{c_6}{2} + c_8\right) n$$

$$- (c_2 + c_3 + c_4 + c_7).$$

We can express this worst-case running time as $an^2 + bn + c$ for constants $a$, $b$ and $c$.

$$T(n) = O(n^2)$$

.

Coff: The RAM model contains instructions commonly found in real computers: arithmetic (such as add, subtract, multiply, divide, remainder, floor, ceiling), data movement (load, store, copy) and control (conditional and unconditional branch), subroutine call & return. Each such instruction takes a constant amount of time.

A constant amount of time is required to execute each line of pseudocode, $c_i$.