1 Problem 6

Table 1: Problem 6 Equilibrium Points Table

Part	Equilibrium Points	Eigenvalues	Type
1	$(0,0)$ $(-\sqrt{6},0)$ $(\sqrt{6},0)$	$-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$ $-2,1$ $-2,1$	Asym. stable focus Unstable saddle Unstable saddle
2	$(-7.4494,-7.4494)$ $(-2.5505,-2.5505)$ $(0,0)$	$-1.5\pm1.1830i$ $-3.3706,0.3706$ $-2.0916,-0.9083$	Stable focus Unstable saddle Stable node
3	$(-3,-4)$ $(0,0)$ $(0,2)$ $(1,0)$	$\frac{7}{2}\pm\frac{\sqrt{73}}{2}$ $1,2$ $-3,-2$ $-1,2$	Unstable node Unstable node Stable node Unstable saddle
4	$(0,0)$	$\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$	Unstable focus
5	$(0,0)$	$-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$	Asym. stable focus
6	$(-\sqrt{1-x_2^2},x_2\leq1)$ $(+\sqrt{1-x_2^2},x_2\leq1)$ $(0,0)$ $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$ $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$	$0,2$ $0,2$ $-1\pm i$ $0,2$ $0,2$	Critically stable arc Critically stable arc Asym. stable focus Critically stable node Critically stable node
7	$(-1,-1)$ $(0,0)$ $(1,1)$ $(-i,i)$ $(i,-i)$ $(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i,+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i)$ $(+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i,-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i)$ $(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i,+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i)$ $(+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i,-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i)$	$-4,-2$ $-1,1$ $-4,-2$ $2,4$ $2,4$ $\pm\sqrt{8}i$ $\pm\sqrt{8}i$ $\pm\sqrt{8}i$ $\pm\sqrt{8}i$	Stable node Unstable saddle Stable node Unstable node Unstable node Critically stable center Critically stable center Critically stable center Critically stable center

2 Problem 7

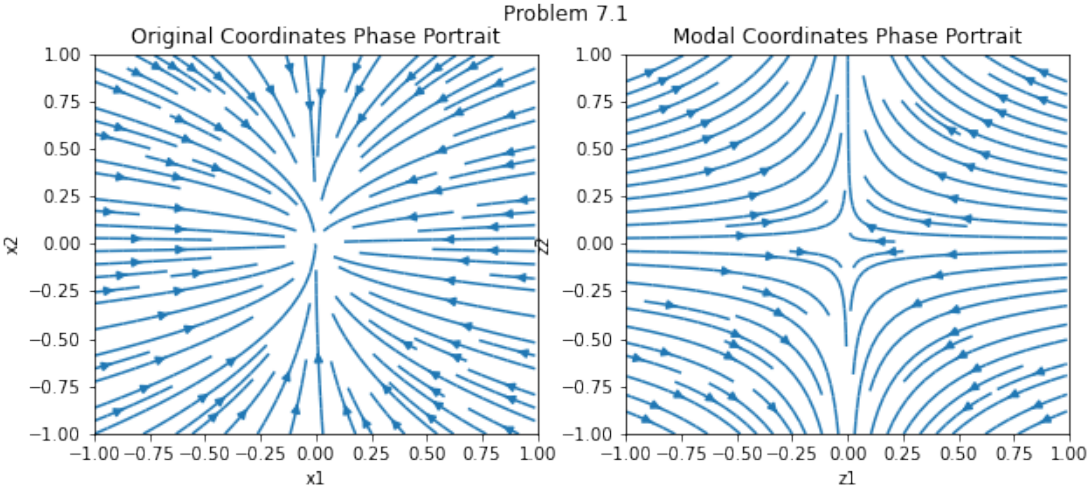

Figure 1: Problem 7.1 Phase Portraits

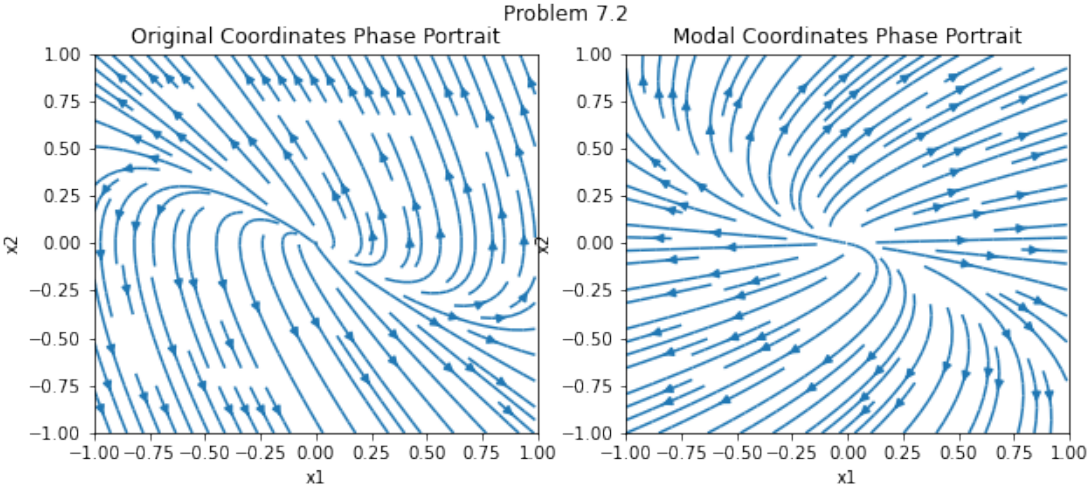

Figure 2: Problem 7.2 Phase Portraits

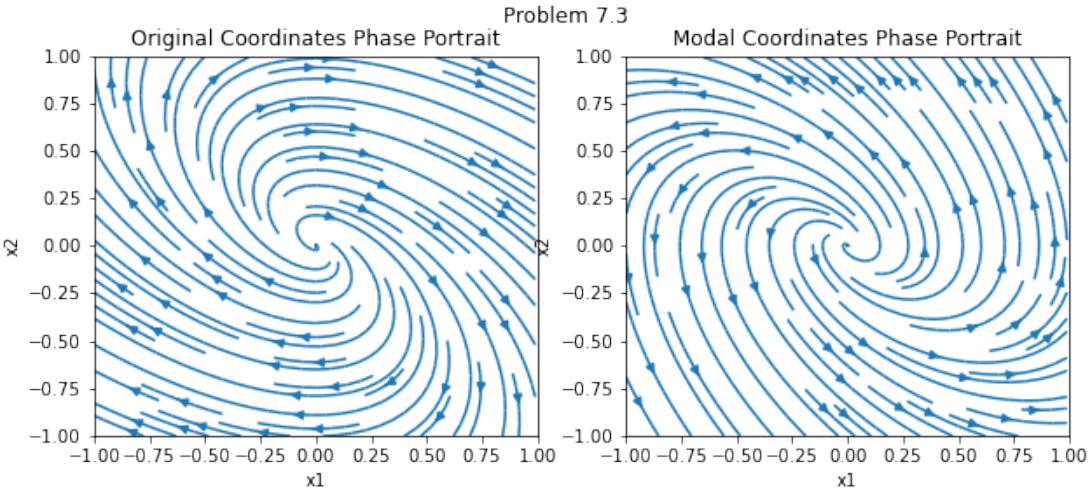

Figure 3: Problem 7.3 Phase Portraits

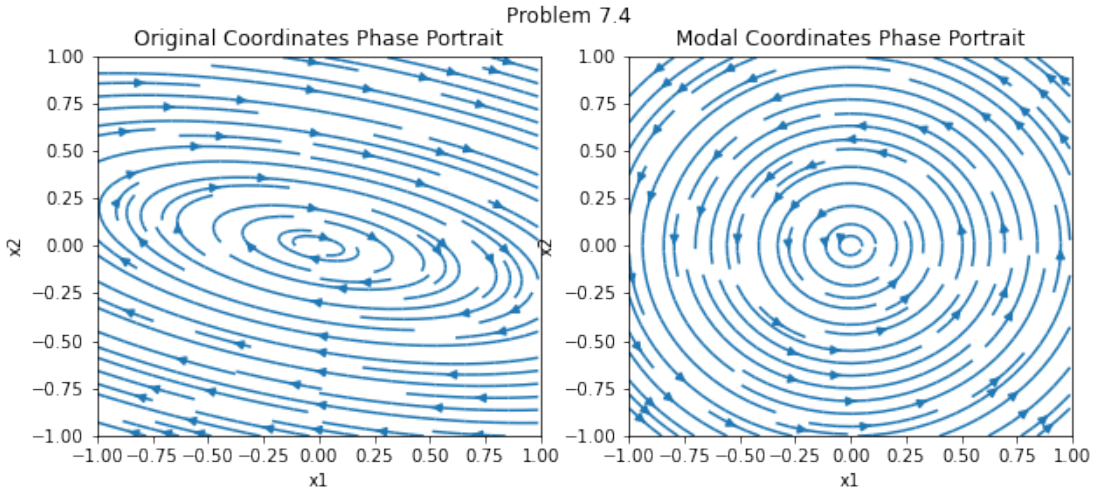

Figure 4: Problem 7.4 Phase Portraits

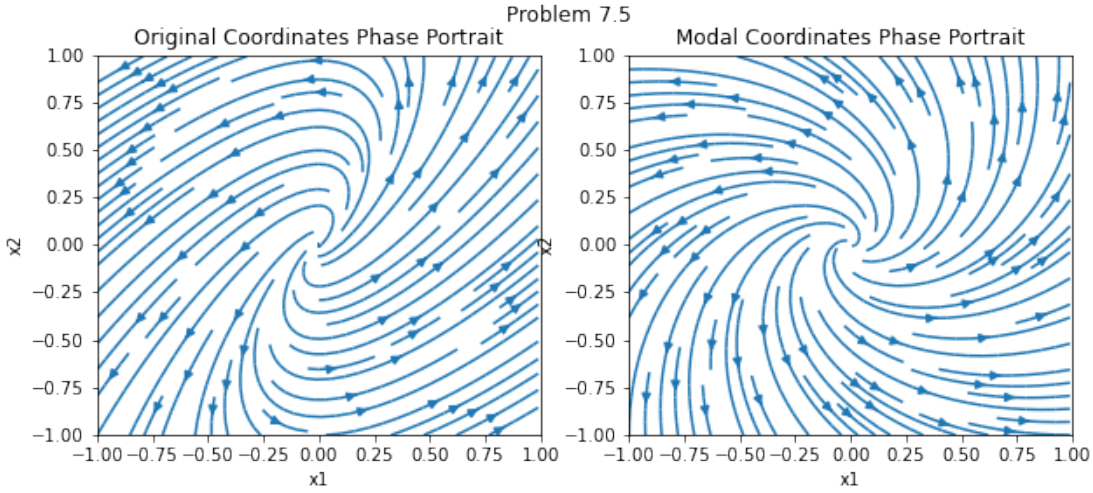

Figure 5: Problem 7.5 Phase Portraits

3 Problem 9

3.1 9.1

The system has a stable node at the origin.

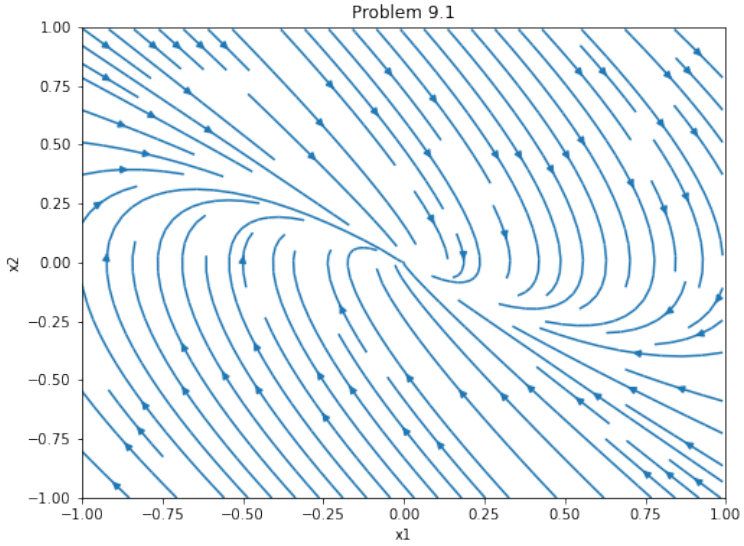

Figure 6: Problem 9.1 Phase Portraits

3.2 9.2

The system appears to converge to a single stable equilibrium trajectory centered at the origin.

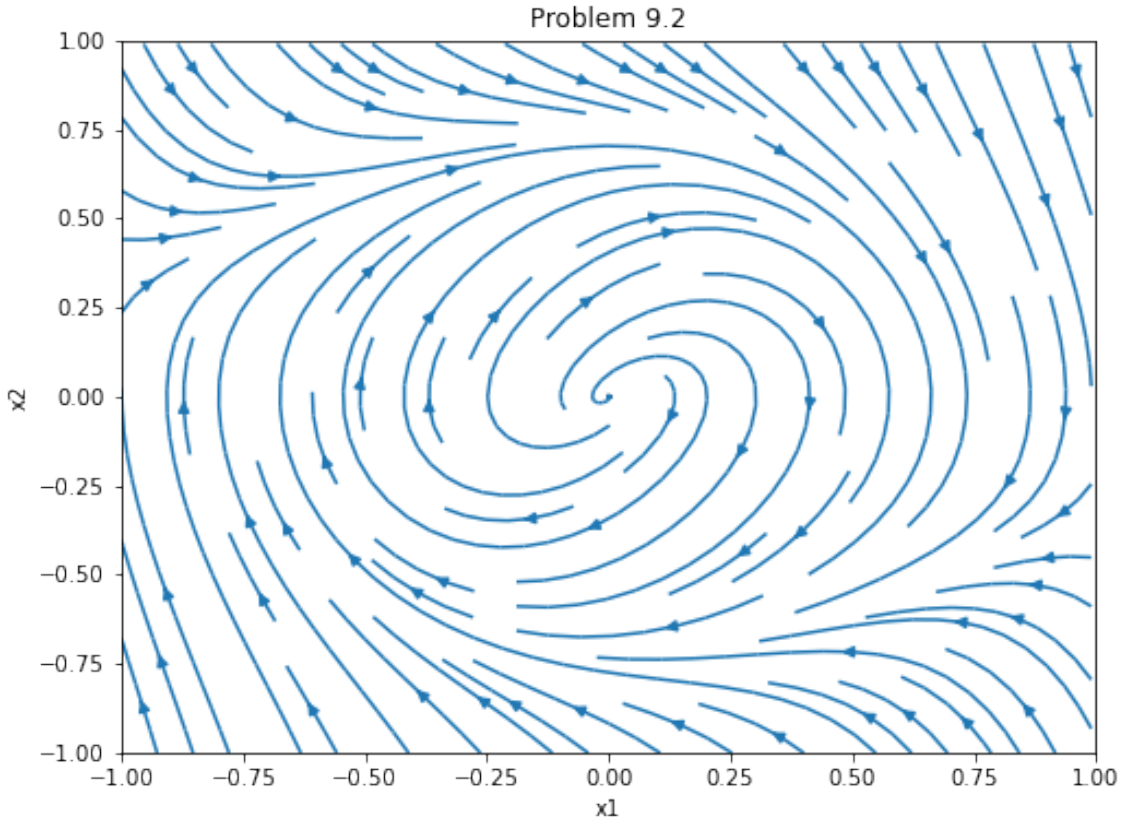

Figure 7: Problem 9.2 Phase Portraits

3.3 9.3

The system has an unstable equilibrium point at the origin and two asymptotically stable equilibrium points around $(\pm\frac{1}{2}, 1)$.

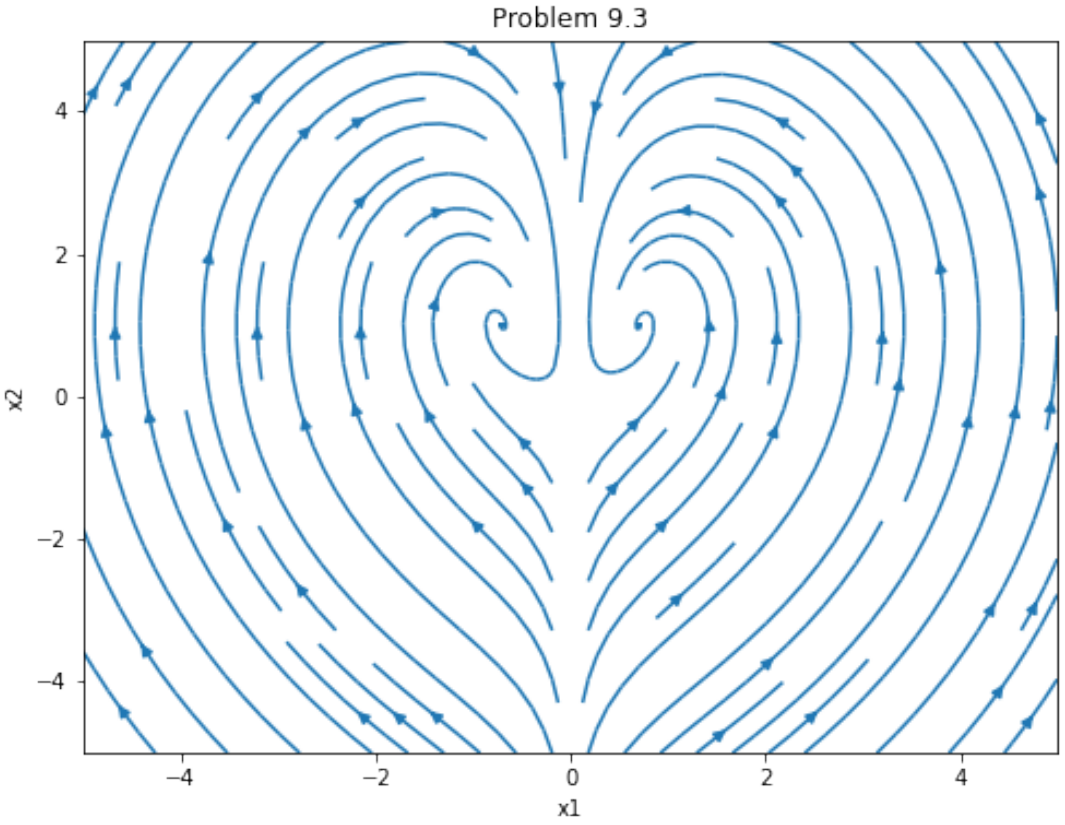

Figure 8: Problem 9.3 Phase Portraits

3.4 9.4

The system appears to converge to a single stable equilibrium trajectory centered at the origin.

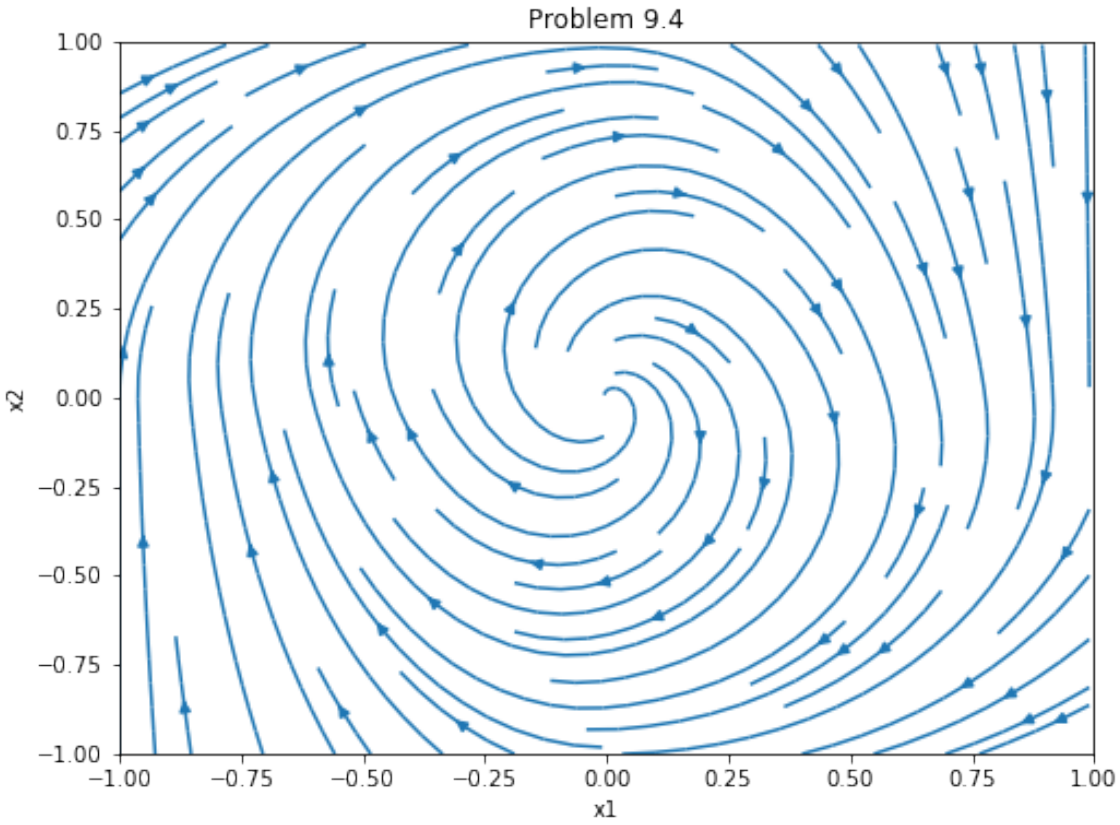

Figure 9: Problem 9.4 Phase Portraits

4 Problem 10

4.1 10.1

The arrows point in the correct direction. The system has an asymptotically stable equilibrium at the origin.

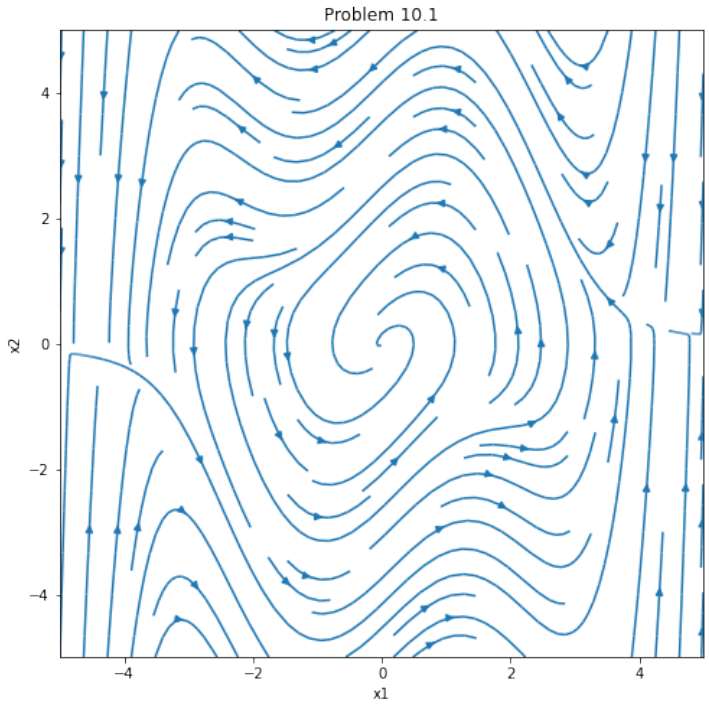

Figure 10: Problem 10.1 Phase Portrait

4.2 10.2

The arrows point in the correct direction. The system has an asymptotically stable equilibrium point at the origin with a finite region of convergence.

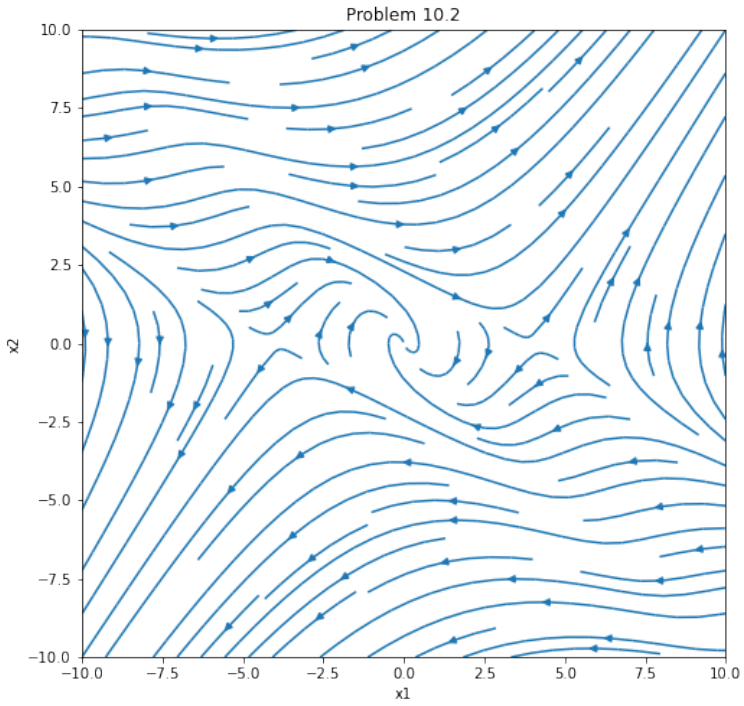

Figure 11: Problem 10.2 Phase Portrait

4.3 10.3

The arrows point in the correct direction. The system has a stable center at the origin.

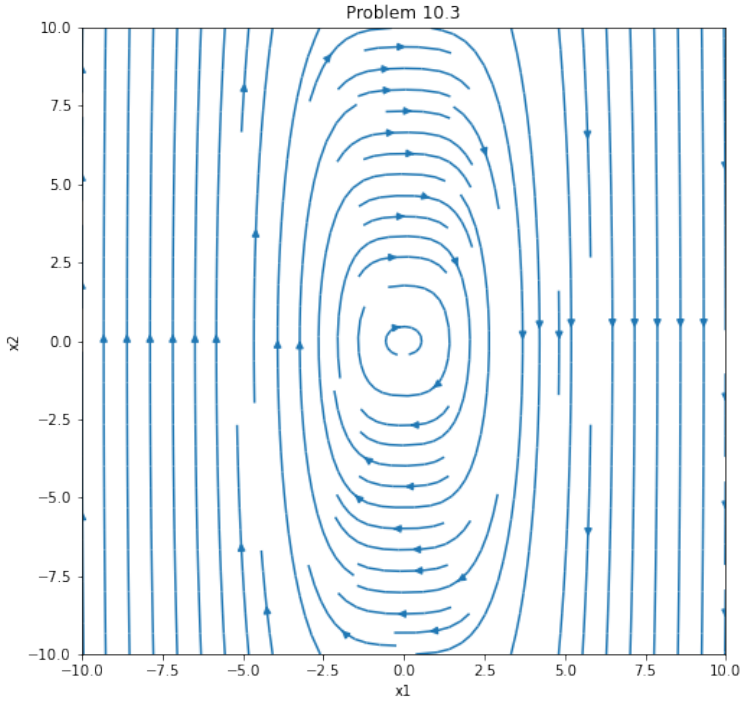

Figure 12: Problem 10.3 Phase Portrait

4.4 10.4

The arrows point in the correct direction. The system has an unstable focus and spirals outwards.

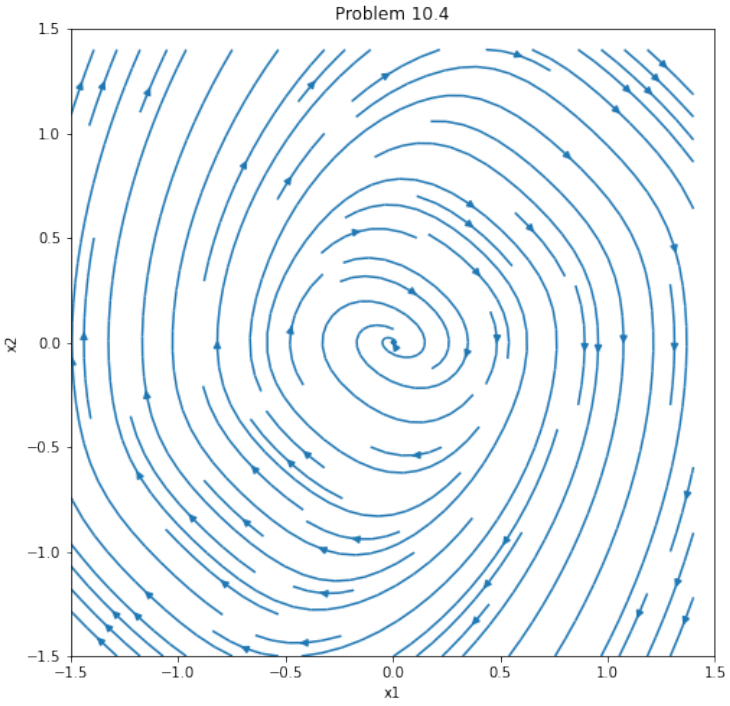

Figure 13: Problem 10.4 Phase Portrait

5 Problem 22

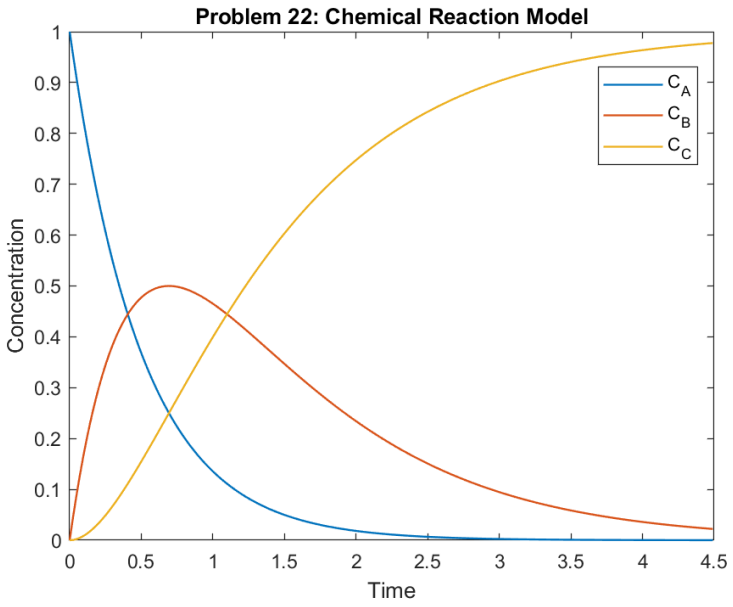

Figure 14: Chemical reaction model simulation