# Bond Maths

This presentation is for learning purpose to clear understanding of Bond Maths & Implementation in Python.

# Report Contents

# Zero Coupon Bond

- A Zero Coupon bond a debt instrument, that trade at discount of it's face value.
- The Difference between Purchase price & Face value of a bond, indicate investor's return.

$$Price = M \div (1 + r)^n$$

where:

- M = Maturity value or face value of the bond
- r = required rate of interest
- n = number of years until maturity

```python
# Simple Calculation of ZCB.
def zcb(fv,y,t):
    """
    To Calculate Price of Zero Coupon Bond = Face Value / (1+y)^t
    fv: Face Value of Bond
    y: Annual Yield or Rate
    t: Time to Maturity
    """
    return fv/(1+y)**t

# Calculate 2 Years ZCB Price.
print(zcb(100,0.02,2))
```

```
96.11687812379854
```

- The amount of 96.11 invested for 2 years compounded annually @ 2% interest will return 100 at maturity.

# Yield to Maturity (YTM)

- YTM is total return one can expect when all coupons and principal received and bond held till maturity.
- YTM is basically internal rate of return (IRR) if held till maturity.

$$Bond\ Price = \frac{Coupon\ 1}{(1+YTM)^1} + \frac{Coupon\ 2}{(1+YTM)^2} + \cdots + \frac{Coupon\ n}{(1+YTM)^n} + \frac{Face\ Value}{(1+YTM)^n}$$

- With the help of Scipy optimization function in python, we can solve for YTM to get desired output.

```
import scipy.optimize as optimize

def b_ytm(price, fv, T, coup, freq=2, guess=0.05): #Semi Annual Coupon payment, hence Freq = 2.
    freq = float(freq) # Convert into Float variable
    periods = T*freq # total number of coupun payment
    coupon = coup/100*fv/freq # coupon value
    dt = [(i+1)/freq for i in range(int(periods))] # calculation of dt.
    ytm_func = lambda y: sum([coupon/(1+y/freq)**(freq*t) for t in dt]) + fv/(1+y/freq)**(freq*max(dt)) - price #
    return optimize.newton(ytm_func, guess) # Solving equation using newton optimization to arrive at final value.
```

```
print(b_ytm(95.0428,100,1.5,5.75,2))
```

```
0.09369155345239477
```

- YTM of bond is 9.37% if the face value of bond is 100, maturity is 1.5 years with 5.75% coupon payment semi annually.

# Price of Bond

- Bond price is the present discounted value of future cash stream generated by a bond.
- When we know YTM we can plug in and get the value of bond.

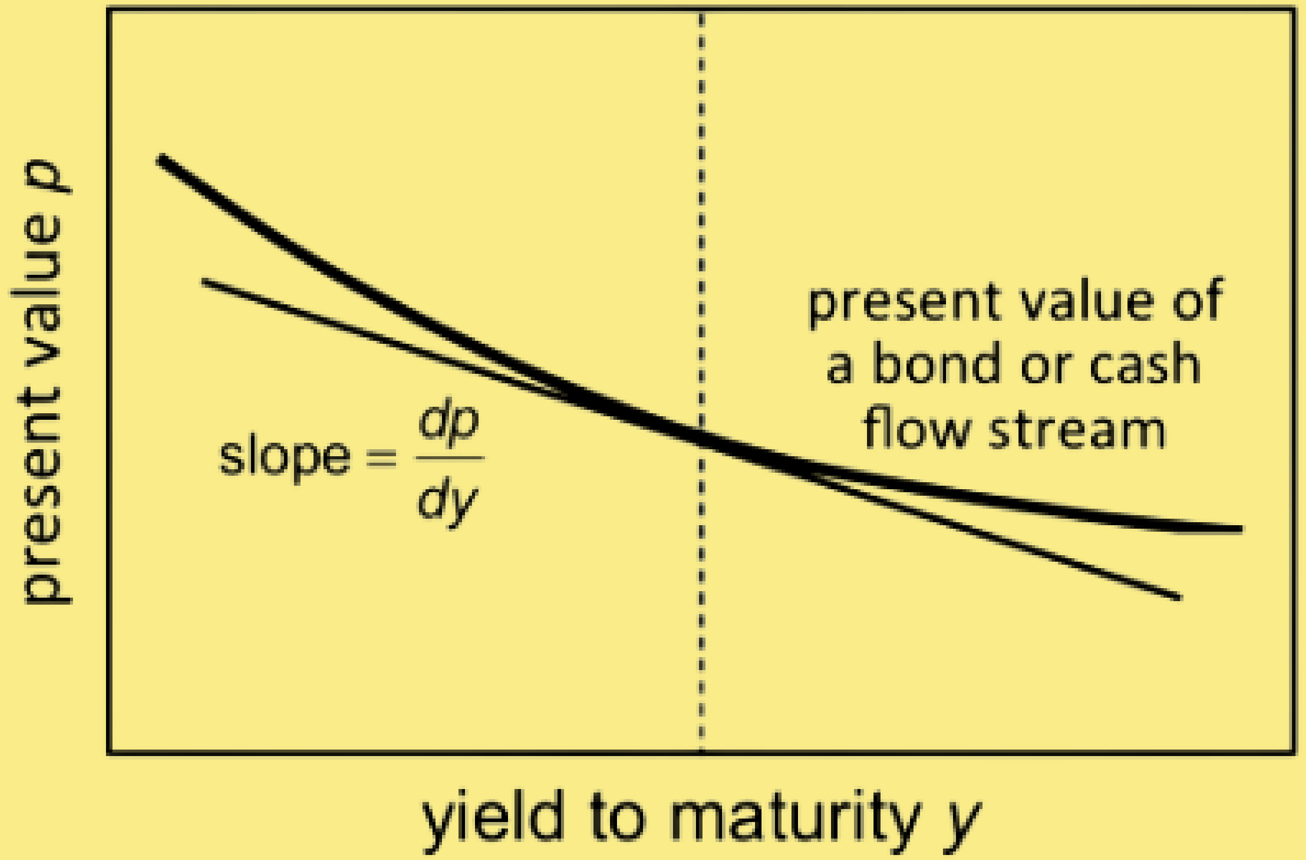
```
""" Get bond price from YTM """
def b_price(fv, T, ytm, coup, freq=2):
    freq = float(freq)
    periods = T*freq
    coupon = coup/100*fv/freq
    dt = [(i+1)/freq for i in range(int(periods))]
    price = sum([coupon/(1+ytm/freq)**(freq*t) for t in dt]) + fv/(1+ytm/freq)**(freq*T)
    return price
```

```
print(b_price(100,1.5,0.09369155345239477,5.75,2))
```

```
95.04280000000004
```

# Bond Duration

- Duration measures a bond's price sensitivity to change in interest rate.
- Modified duration measures the price change in a bond given a 1% change in interest rate.
- The modified duration of a bond can be thought of as the first derivative of the relationship between price and yield



```python
""" Calculate modified duration of a bond """
def mod_duration(price, par, T, coup, freq, dy=0.01):
    ytm = b_ytm(price, par, T, coup, freq)
    ytm_minus = ytm - dy
    price_minus = b_price(par, T, ytm_minus, coup, freq)
    ytm_plus = ytm + dy
    price_plus = b_price(par, T, ytm_plus, coup, freq)
    mduration = (price_minus-price_plus)/(2*price*dy)
    return mduration
```

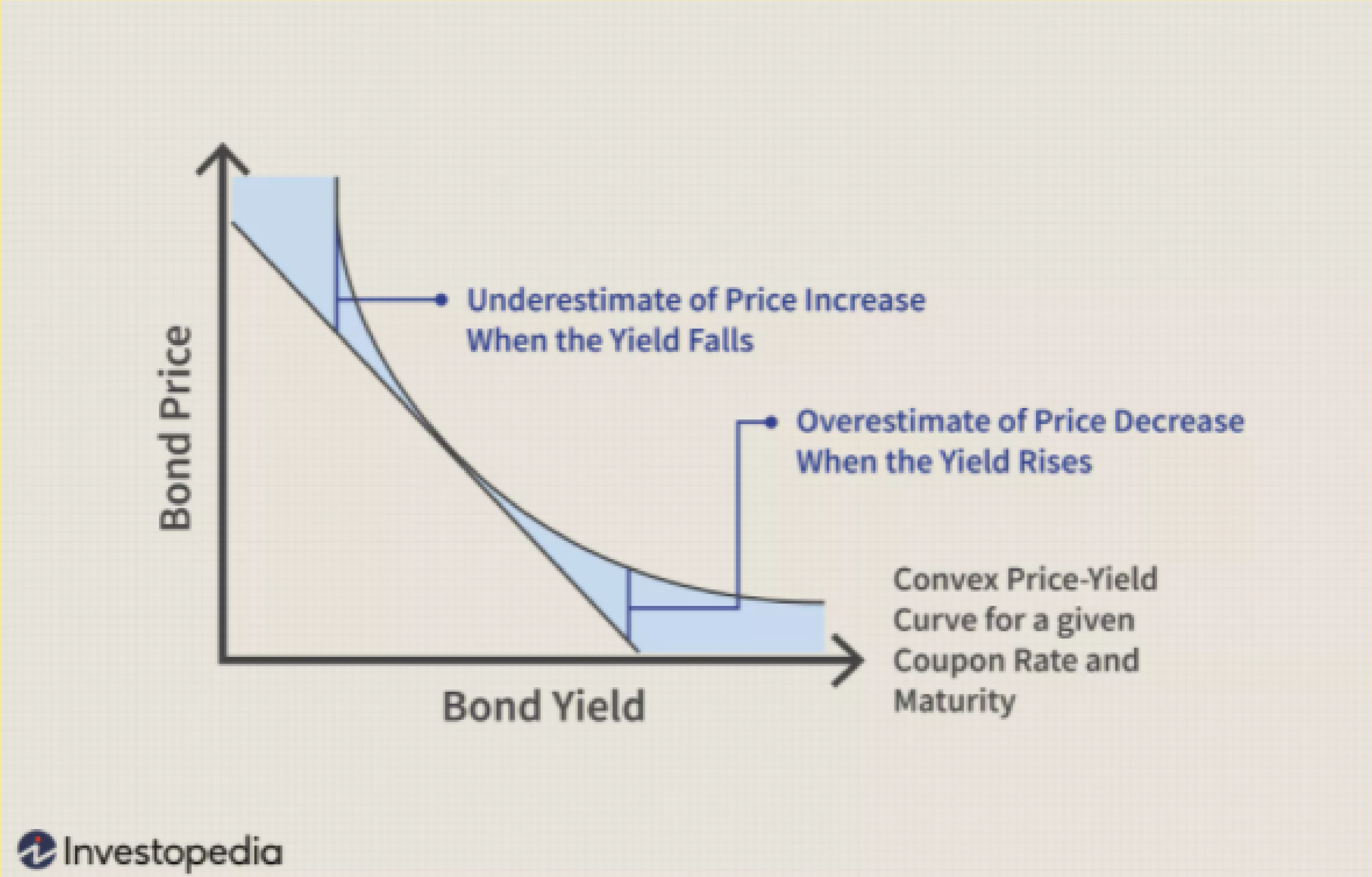
```python
print(mod_duration(95.04,100,1.5,5.75,2,0.01))
```

```
1.3921788121706968
```

- In above scenario the modified duration is 1.39 years.

# Bond Convexity

- Convexity is the sensitivity measure of the duration of a bond to yield changes.
- Convexity is the second derivative of the relationship between the price and yield.



$$\text{Effective convexity} = \frac{PV_- + PV_+ - 2PV_0}{(\Delta Curve)^2 PV_0}$$

```python
""" Calculate convexity of a bond """
def b_convexity(price, par, T, coup, freq, dy=0.01):
    ytm = b_ytm(price, par, T, coup, freq)
    ytm_minus = ytm - dy
    price_minus = b_price(par, T, ytm_minus, coup, freq)
    ytm_plus = ytm + dy
    price_plus = b_price(par, T, ytm_plus, coup, freq)
    convexity = (price_minus+price_plus-2*price)/(price*dy**2)
    return convexity
```

```python
print(b_convexity(95.0428, 100, 1.5, 5.75, 2))
```

```
2.6339593903428367
```

- In above scenario Convexity of bond is 2.63

# Acknowledgement

- https://www.investopedia.com/
- https://theintactone.com/2019/05/18/saim-u1-topic-9-yield-to-maturity/
- https://analystprep.com/cfa-level-1-exam/fixed-income/calculate-interpret-convexity/
- Mastering Python for Finance - Book by James Ma Weiming.