VOLATILITY MODELS

PROJECT OUTLINE

- Background
- Implied Volatility
- Local Volatility
- Heston Volatility
- Heston Calibration in Python

IMPLIED VOLATILITY

BACKGROUND

- The very intuitive definition of Volatility is nothing but standard deviation.
- In financial world, different products need different ways to model volatility, as financial product have lots of noise which leads to randomness.
- Also, Volatility modelling evolved over a period of time as need emerged to model them differently. We'll brodly look at below three models.

Implied Volatility	Local Volatility	Heston Volatility

IMPLIED VOLATILITY

- Historical data helps us to get one number for volatility based on past data.

- However, The Implied Volatility is not based on the historical data of stocks.

- Implied volatility is derived from observed market quote, using Black–Scholes model.

- In the Black–Scholes, the asset's price modeled as log-normal random variable, which states that asset's log returns are nomrally distributed.

IMPLIED VOLATILITY

- It was realized during 1987 crash that BS model provide constant volatility and it doesn't capture "Skew".

- Skew indicates that tail event occurs more often than the normal distribution would imply.

- So, traders started to price especially lower strike options with higher volatility.

- There is volatility skew for most options, which means the volatility is not constant across strikes.

LOCAL VOLATILITY

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LOCAL VOLATILITY

- The Local Volatility is deterministic function of asset price and time t

- When volatility is function of asset price, There is source of randomness from the stock price Wt.

$$\mathrm{d}S_t = \mu S_t\,\mathrm{d}t + \sigma(S_t, t)\mathrm{d}W_t$$

LOCAL VOLATILITY

- The implied volatility surface can be transformed to LV surface, which is known as calibration of LV model of Dupire.

- Observed options' prices under the assumption that they can be interpolated across continuous strikes and maturities.

$$\sigma_{Local}(K,T) = \sqrt{\frac{\frac{\partial C}{\partial T}}{\frac{1}{2}K^2\frac{\partial^2 C}{\partial K^2}}}$$

LOCAL VOLATILITY

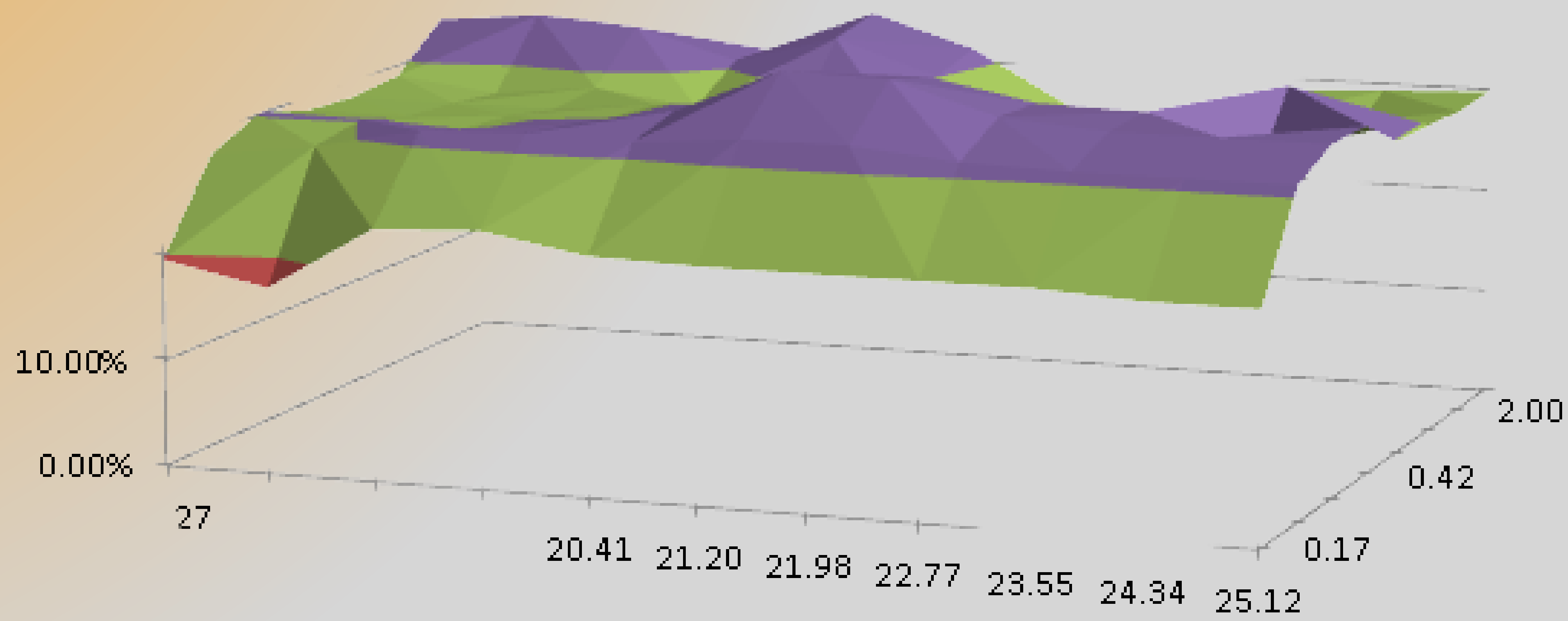
- Steps to be followed to calculate Local Volatility:
  - First, use the available quoted price to calculate the implied volatilities.
  - Apply interpolation method to produce a smooth implied volatility surface.
  - Calculate the local volatility according to Dupire formula. To avoid taking derivatives, we could use finite differences to approximate the derivative.

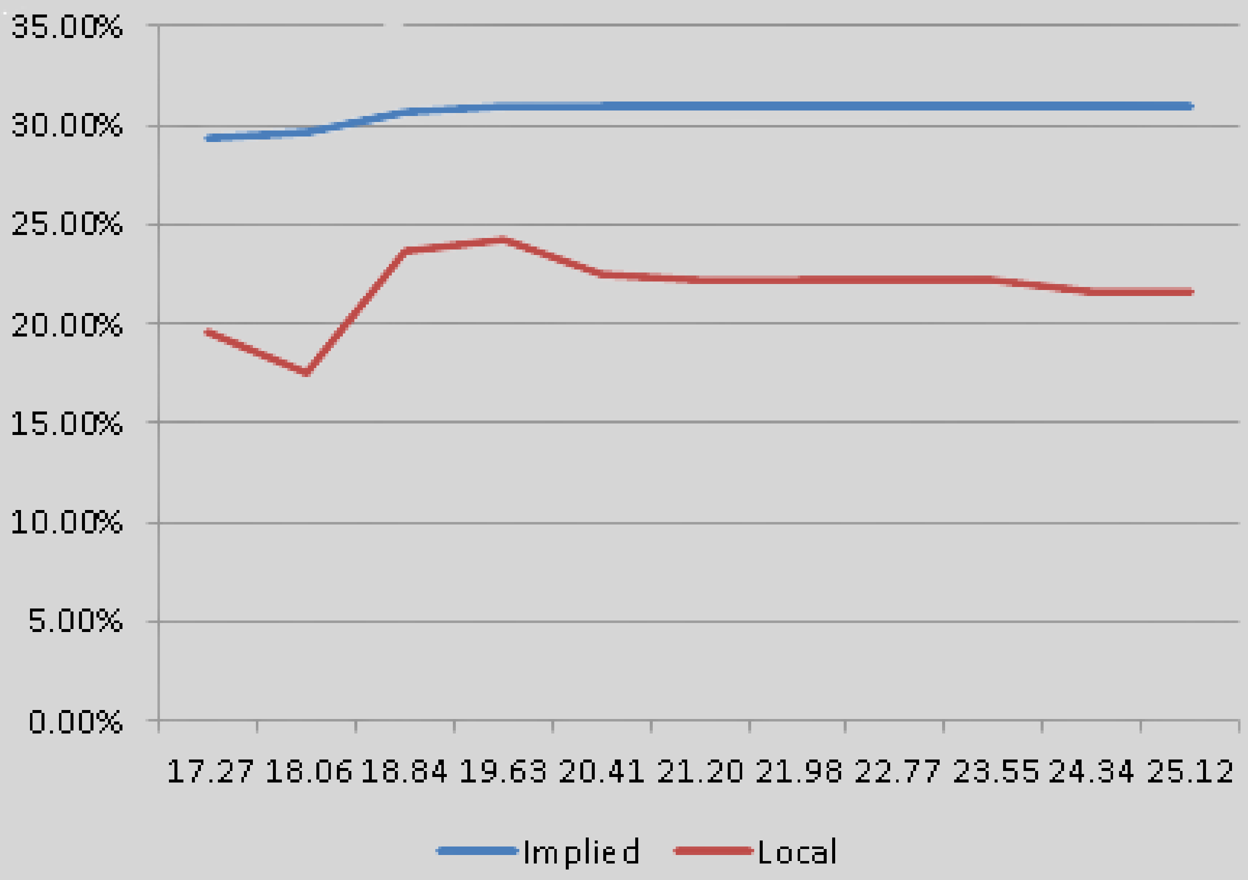
$$\frac{\partial C}{\partial T} \approx \frac{C(K, T+\Delta T) - C(K, T-\Delta T)}{2\Delta t}$$

$$\frac{\partial^2 C}{\partial K^2} \approx \frac{C(K-\Delta K, T) - 2C(K, T) + C(K+\Delta K, T)}{(\Delta K)^2}$$

LOCAL VOLATILITY IMPLEMENTATION

https://github.com/abhaydd22/LVSV/Local Vol.xlsx





LOCAL VOLATILITY

- Few Limitations with Local Volatility Model.
    - Model use current price and there are no assumption about behavior over a period of time. This doesn't provide us with Forward Volatility.
    - It becomes very difficult to price exotic options with such flat volatility.
    - Few exotic options like barrier option can be priced by adjusting Local Volatlity model by introducing jumps.

HESTON VOLATILITY

HESTON VOLATILITY

- Few Limitations with Local Volatility Model, can be addressed by SV (Stochastic Volatility) models.

- In SV Model both Asset Price and Volatility both can change over a time as both of them will have random component.

- Heston Model is two factor model, where there is a separate dynamics for both stock price and instantaneous volatility.

- The instantaneous variance of the stock price itself is stochastic process.

HESTON VOLATILITY

$$dS(t,S) = \mu S dt + \sqrt{v} S dW_1$$
$$dv(t,S) = \kappa(\theta - v)dt + \sigma\sqrt{v} dW_2$$
$$dW_1 dW_2 = \rho dt$$

- kappa (k): It's mean reverting parameter, speed at which it revert to it's long run variance.
- Theta: The long run variance.
- Initial Variance: Starting point initial variance.
- There is correlation between two random wieneer process.
- Vol of Vol parameter estimates volatility of volatility.

HESTON VOLATILITY

- In order to get various parameters highlighted in previous slide, we need to estimate them using calibration.
- The calibration process is performed using Python.
- We have conducted study on "SPY" stock option data for Oct'21 expiry for various strikes.

Strike	Expiry	Type	Symbol	Last	Bid	Ask	Chg	PctChg	Vol	Open_Int	IV	Root	IsNonstandard	Underlying	Underlying_Price
190.0	2021-10-15	call	SPY211015C00190000	219.60	224.79	225.29	0.0	0.0	1.0	0.0	0.000010	SPY	False	SPY	433.72
195.0	2021-10-15	call	SPY211015C00195000	185.09	205.29	207.39	0.0	0.0	3.0	0.0	0.000010	SPY	False	SPY	433.72

HESTON VOLATILITY

- Thanks to Quantlib open source library, which helps to calibrate parameters efficiently.
- We have performed calibration on SPY Options data & finally found following parameter estimations.

```
lm = ql.LevenbergMarquardt(1e-8, 1e-8, 1e-8)
model.calibrate(heston_helpers, lm, ql.EndCriteria(500, 50, 1.0e-8,1.0e-8, 1.0e-8))
long_term_var, rate_reversion, vol_of_vol, corr, initial_var = model.params()
print("long_term_var = %f, rate_reversion = %f, vol_of_vol = %f, corr = %f, initial_var = %f" % (long_term_var, rate_reversi

long_term_var = 0.195237, rate_reversion = 0.000002, vol_of_vol = 0.218177, corr = -0.836078, initial_var = 0.199178
```

https://github.com/abhaydd22/LVSV/blob/main/Heston.ipynb

ACKNOWLEDGEMENT

Various Paper and webpages helped to complete this project.

- https://www.quantconnect.com/tutorials/introduction-to-options/local-volatility-and-stochastic-volatility
- https://towardsdatascience.com/which-one-is-your-volatility-constant-local-or-stochastic-61508ef560c1
- http://gouthamanbalaraman.com/blog/volatility-smile-heston-model-calibration-quantlib-python.html (Thanks for Python Calibration Code).
- https://en.wikipedia.org/wiki/Heston_model