Chomsky classification:

Note: Here, we consider Grammar
$G \equiv (V_N, \Sigma, P, S)$

Type 0: (unrestricted form)

$\alpha \rightarrow \beta$

where
$\alpha \in (V_N \cup \Sigma)^* V_N (V_N \cup \Sigma)^*$
$\beta \in (V_N \cup \Sigma)^*$

Type 1: (Context sensitive)

$\alpha \rightarrow \beta$

where
$\alpha \in (V_N \cup \Sigma)^* V_N (V_N \cup \Sigma)^*$
$\beta \in (V_N \cup \Sigma)^+$ and $|\alpha| \leq |\beta|$

Note: in the Type 1 (Context sensitive grammar) Null production of the form $S \rightarrow \epsilon$ is allowed, however, then S should not occur in the R.H.S of any production.

Alternatively, in Type - 1, a production should be viewed as

Right context

$$\varphi A \psi \rightarrow \varphi \alpha \psi$$

where $|\alpha| \neq \epsilon \Rightarrow$ length non-contracting or increasing

$$|\varphi A \psi| \leq |\varphi \alpha \psi|$$

Left context

Type 2 (Context free):

$$\alpha \rightarrow \beta$$

where, $\alpha \in V_N$, $\beta \in (V_N \cup \Sigma)^*$.

Type 3 (Regular):

Productions of the following forms are allowed.

$A \rightarrow a$ where $A \in V_N$

$A \rightarrow bB$ $a, b \in \Sigma$

$A \rightarrow \epsilon$

Note: In Type-3, productions can also be viewed as

$$A \to w$$
$$A \to wB, \text{ where } w \in \Sigma^*$$

because

$A \to a_1 a_2 a_3 \cdots a_K B$ can be split into

$$A \to a_1 B_1,\ B_1 \to a_2 B_2, \ldots\ B_{K-1} \to a_K B$$

by using few more non-terminals $B_1, B_2, \ldots, B_{K-1}$

Also Note:

$$\underbrace{A \to wB \mid w}_{\text{Right linear}}$$

linear = Single Non-terminal in the R.H.S

Right = Extreme right here, $w \in \Sigma^*$.

$$\underbrace{A \to Bw \mid w}_{\text{Left linear}}$$

Note: this left linear can be viewed as Right linear

$$A \to w^R B \mid w^R$$

One more clarification regarding Type-3

Do not mix Right linear with left linear, such a grammar may not be a Regular grammar

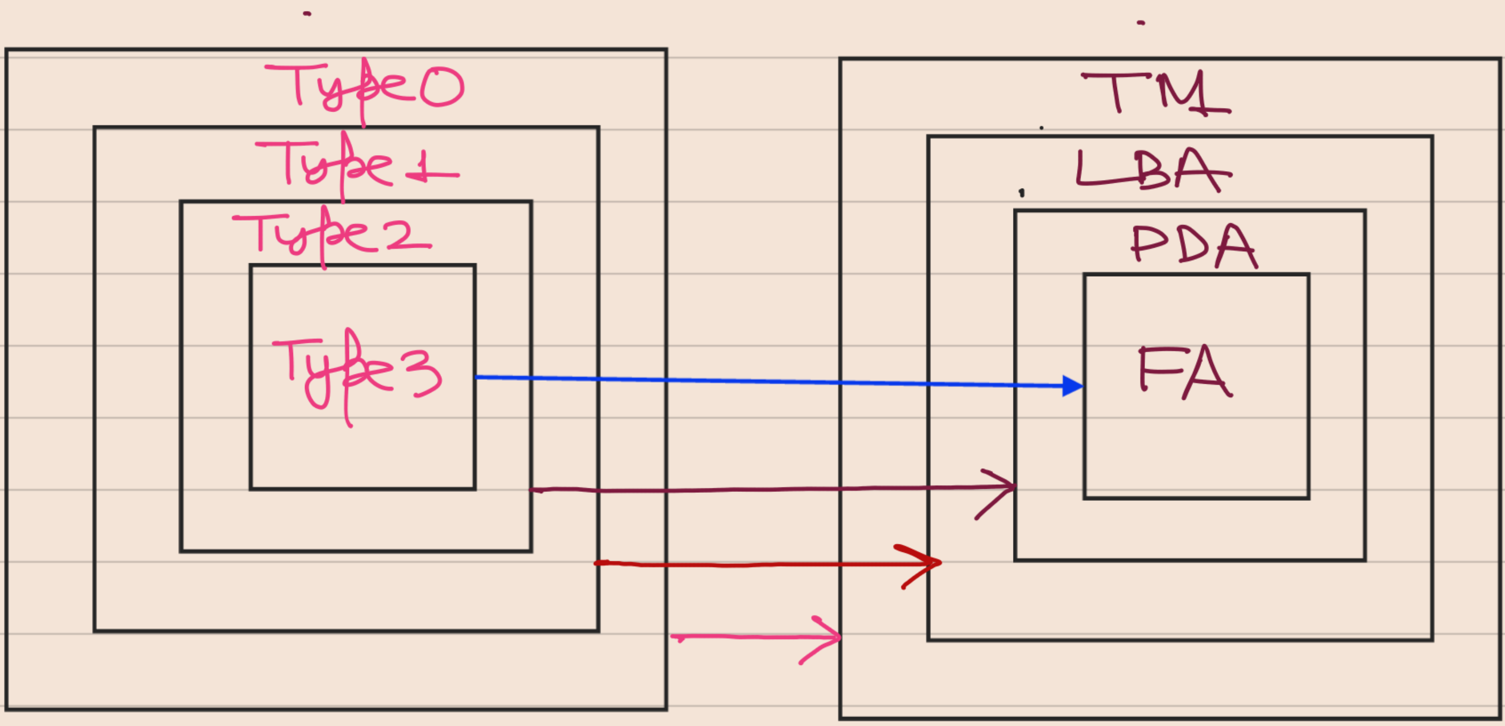
<u>Ex:</u>

$S \to 0A$
$S \to 1B$
$S \to \epsilon$
$A \to S0$
$B \to S1$

This grammar generate language

$L = \{ww^R \mid w \in \{0,1\}^*\}$

which is <u>NOT</u> regular



Type 3: Regular grammar
Type 2: Context free grammar
Type 1: Context sensitive grammar
Type 0: Unrestricted

FA: Finite Automata
PDA: Push down Automata
LBA: Linear Bounded Automata
TM: Turing M/C

Ex: 1 $S \rightarrow abA$
$A \rightarrow baB$
$B \rightarrow aA$
$B \rightarrow bb$
$\Rightarrow$ Regular grammar

Ex: 2 $S \rightarrow abB$
$A \rightarrow aaBb$
$B \rightarrow bbAa$
$A \rightarrow \epsilon$
$\Rightarrow$ Context free grammar

Ex: 3 $S \rightarrow aSb$
$S \rightarrow SS$
$S \rightarrow \epsilon$
$\Rightarrow$ Context free grammar

Ex: 4 $S \rightarrow aSa$
$S \rightarrow bSb$
$S \rightarrow \epsilon$
context free grammar

Ex: 5
$S \rightarrow aTb \mid ab$
$aT \rightarrow aaTb \mid ac$
context sensitive grammar