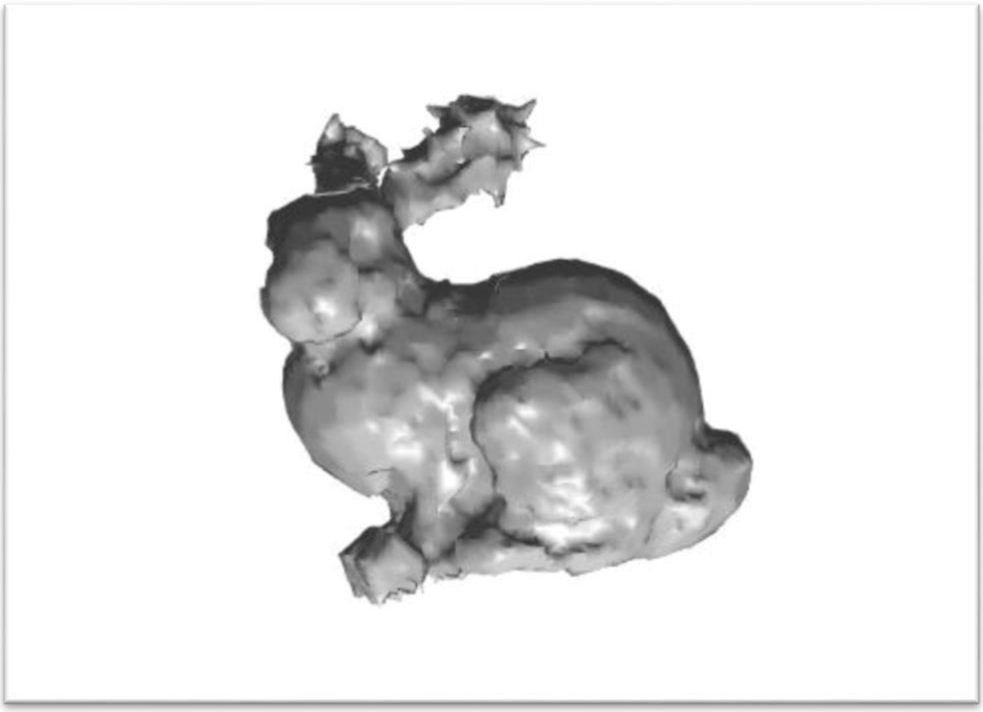
Abhay Doke

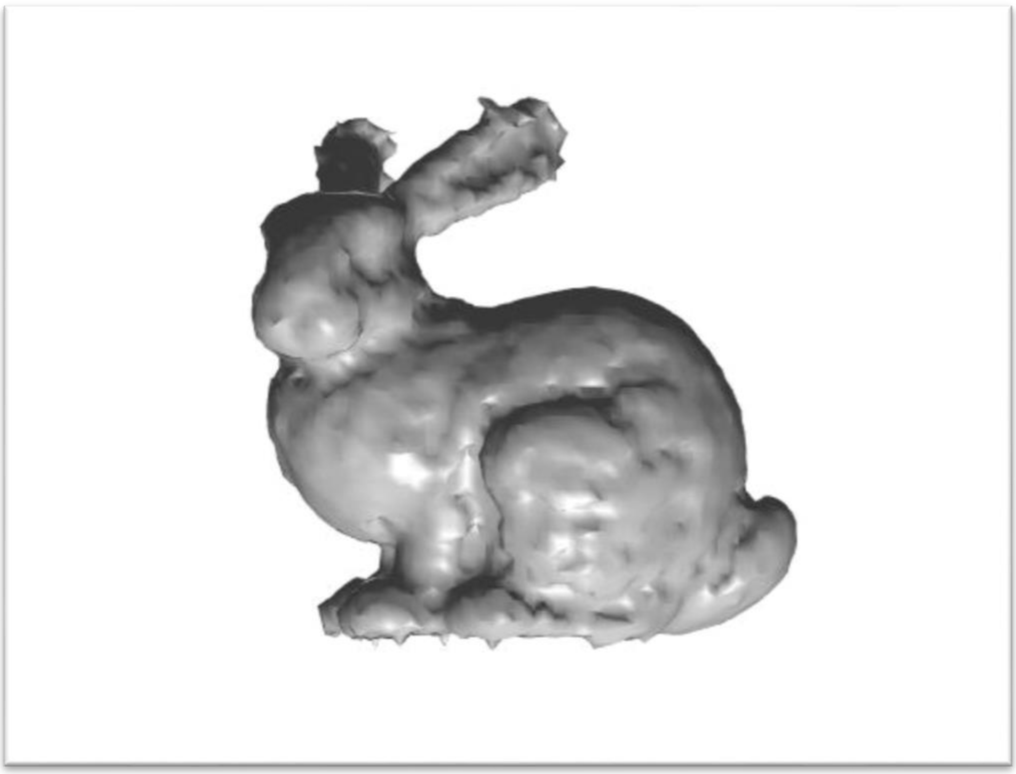
Intelligent Visual Computing

Assignment 4

1. Signed distance to tangent plane



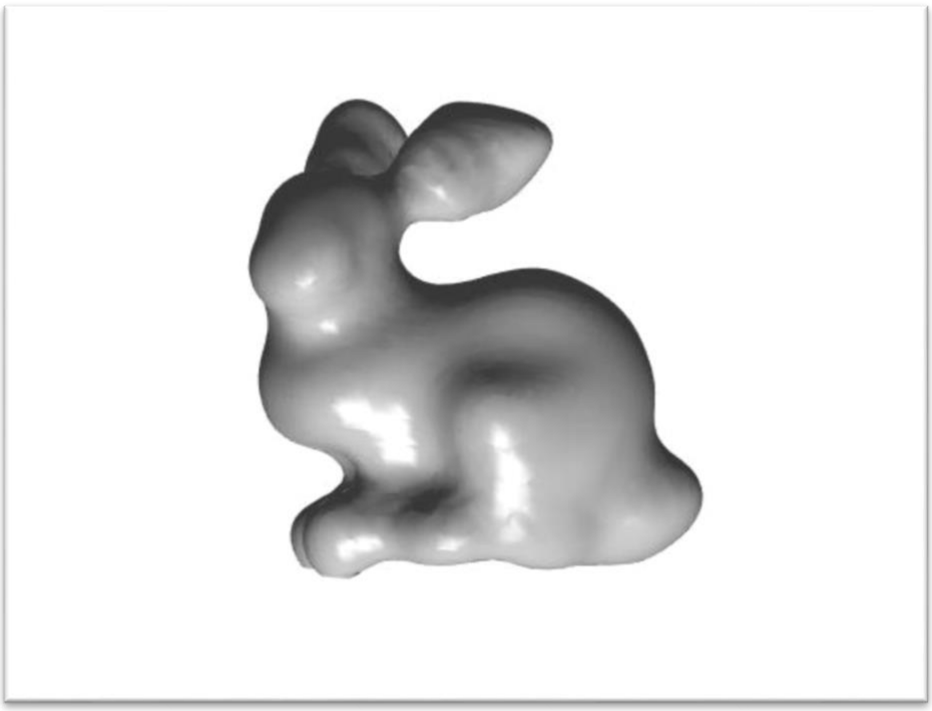
Bunny_500



Bunny_1000

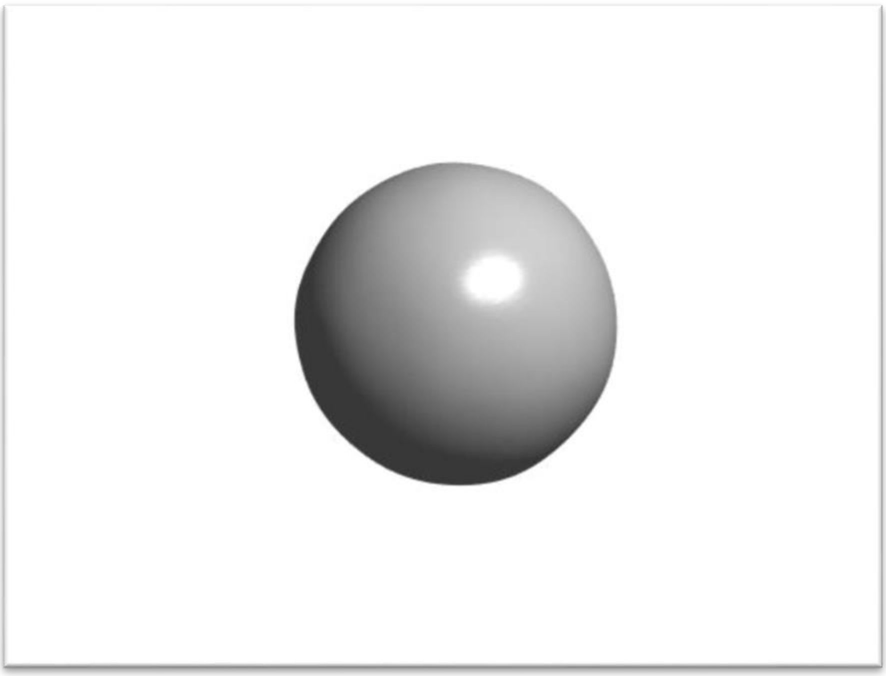
Sphere

2. Moving least squares distance to tangent



Bunny_500

Bunny_1000



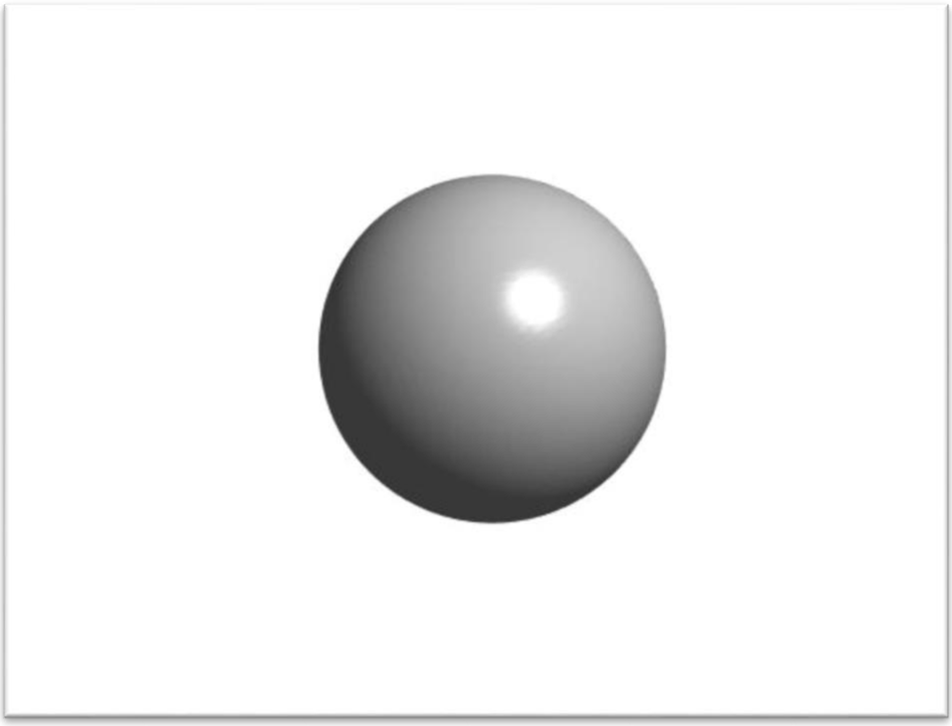
Sphere

3. Radial Basis Function (RBF) interpolation for approximating the signed distance

Epsilon = 0.003

Bunny_500

Bunny_1000



Sphere

Derivation for the optimal scale

Q> Derivation for the optimal scales.

Scale-symmetrical error function:

$$f_{q \Rightarrow p}(R, s) = \sum_i \| \frac{1}{s} p_i' - s R q_i' \|^2$$

To calculate optimal scale, equate the function derivative to zero.

$$\frac{\partial f}{\partial s} = 0$$

$$= \frac{\partial}{\partial s} \sum_i \| \frac{1}{s} p_i' - s R q_i' \|^2 = 0$$

$$= \sum_i 2 \left[\frac{1}{s} p_i' - s R q_i' \right] \cdot \left[-\frac{p_i'}{s^2} - R q_i' \right]$$

$$= \sum_i 2 \left[-\frac{1}{s^3} (p_i')^2 - \frac{R q_i' p_i'}{s} + \frac{p_i' R q_i'}{s} + s R q_i' R q_i' \right]$$

For any orthogonal matrix M

$$M M^T = I$$

Hence $R R^T = I$

$$= \sum_i 2\left[\frac{-\|p_i'\|^2 + s^4\|q_i'\|^2}{s^4}\right] = 0$$

$$= \sum_i \left[-\|p_i'\|^2 + \|q_i'\|^2 \cdot s^4\right] = 0$$

$$\sum_i \|q_i'\|^2 \cdot s^4 = \sum_i \|p_i'\|^2$$

$$\therefore s^4 = \frac{\sum_i \|p_i'\|^2}{\sum \|q_i'\|^2}$$

$$\boxed{s = \sqrt[4]{\frac{\sum \|p_i'\|^2}{\sum \|q_i'\|^2}}}$$