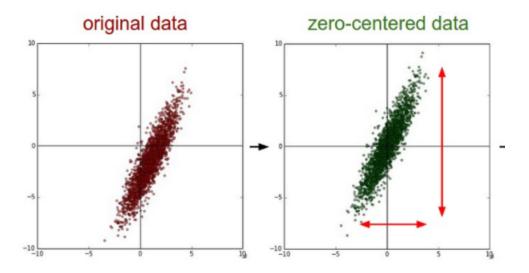
1. Pixel features

For the pixel features I experimented with two methods where two sets of different normalizations were applied on the data.

Method 1: Making the data zero centered by mean-subtraction. Data is made zero centered by subtracting the mean across every individual feature in the data, and results in centering the cloud of data around the origin along every dimension.



Method 2: Applying the L2-normalization followed by the square-root scaling.

Following are the accuracy result scores obtained by these two methods on the validation set and the test set.

```
Model Parameters:
function model = trainModel(x, y)
param.lambda = 0.001;  % Regularization term
param.maxiter = 1000;  % Number of iterations
param.eta = 0.001;  % Learning rate
```

```
Results:
Method 1
Normalization used: Zero Centered Data
Validation set accuracy:
```

+++ Accuracy Table [trainSet=1, testSet=2]

```
dataset pixel
_____
digits-normal.mat 85.60
digits-scaled.mat 82.80
digits-jitter.mat 21.20
Test set accuracy:
+++ Accuracy Table [trainSet=1, testSet=3]
dataset pixel
_____
digits-normal.mat 83.00
digits-scaled.mat 79.40
digits-jitter.mat 19.00
Method 2
Normalization used: L2 normalization followed by square root scaling
Validation set accuracy:
+++ Accuracy Table [trainSet=1, testSet=2]
_____
dataset pixel
-----
digits-normal.mat 87.60
digits-scaled.mat 73.00
digits-jitter.mat 20.80
Test set accuracy:
+++ Accuracy Table [trainSet=1, testSet=3]
-----
dataset pixel
_____
digits-normal.mat 84.20
digits-scaled.mat 68.00
digits-jitter.mat 18.80
```

For the normal data, the L2-normalization followed by the square-root scaling performed the best with 87.6% accuracy on the validation set. For the scaled data and the jittered data, mean-subtraction(method 1) worked better on both the validation as well as test dataset.

2.HoG features

I used the 4 x 4 patches in the images to calculate the weighted histogram of oriented gradients. Image size is 28 x 28, so there are total 7 x 7 = 49 patches in each image. Orientation values ranged from 0 - 360 degree and these values are binned into 18 bins viz. 0-20, 21-40, 41-60,...., 341-360. So total number of features per image are 49*18 = 882.

Method 1: No normalization

Method 2: L2-normalization followed by the square-root scaling. Following results shows the validation set and the test set accuracies.

```
{\tt HoG} features are calculated over 4 x 4 patches in the images.
Image size is 28 \times 28, so there are total 7 \times 7 = 49 patches in each image.
Orientation values ranged from {\tt O} - {\tt 360} degree and these values are binned into
18 bins viz. 0-20, 21-40, 41-60,..., 341-360.
So total number of features per image are 49*18 = 882
Method 1: No normalization
Validation set accuracy:
+++ Accuracy Table [trainSet=1, testSet=2]
dataset hog
_____
digits-normal.mat 91.20
digits-scaled.mat 91.20
digits-jitter.mat 39.00
Test set accuracy:
+++ Accuracy Table [trainSet=1, testSet=3]
dataset hog
-----
digits-normal.mat 91.20
digits-scaled.mat 90.40
digits-jitter.mat 33.80
Method 2: L2-normalization followed by the square-root scaling
+++ Accuracy Table [trainSet=1, testSet=2]
_____
dataset hog
_____
digits-normal.mat 93.60
digits-scaled.mat 93.60
digits-jitter.mat 46.40
+++ Accuracy Table [trainSet=1, testSet=3]
_____
dataset hog
_____
digits-normal.mat 95.80
digits-scaled.mat 96.00
digits-jitter.mat 39.80
```

Hyperarameters:

```
numOri = 0 - 360
histogramSize(binSize) = 18

param.lambda = 0.001;  % Regularization term
param.maxiter = 1000;  % Number of iterations
param.eta = 0.001;  % Learning rate
```

HoG features work very well for the normal data and the scaled data. L2-normalization followed by the square-root scaling worked better on all of the datasets as compared to the no normalization strategy.

3.LBP features

For the LBP features I used two methods for calculating the LBP features. The main observation of the LBP feature experiment was the exclusion of LBP feature with value 255 from the feature vector really improved the accuracies for all three datasets. LBP feature with value 255 represents either all surrounding pixels are same or greater than the current pixel. Since most part of the images are either black or white, this feature is not representing any significant information. Also the scaling the pixel values in the images do not alter the LBP accuracies i.e. the accuracies on the normal data and the scaled data with LBP features are almost the same.

Method 1: Calculating a single histogram of LBP features for the whole image. The resulting histogram size was 255 and hence the feature vector had 255 features.

Method 2: Each image was divided in two 4 parts and LBP feature histogram were calculated independently for all 4 patches. All of the four histograms were concatenated to obtain the final feature vector. The resulting feature vector had 255*4 = 1020 features.

```
Method 1: Calculating LBP features on the whole image.
***** Hyperparameters for the Method 1 *******
param.lambda = 0.001;  % Regularization term
param.maxiter = 2000; % Number of iterations
****************
Validation set accuracy:
+++ Accuracy Table [trainSet=1, testSet=2]
_____
dataset 1bp
_____
digits-normal.mat 65.20
digits-scaled.mat 65.20
digits-jitter.mat 63.40
Test set accuracy:
+++ Accuracy Table [trainSet=1, testSet=3]
dataset lbp
digits-normal.mat 62.40
digits-scaled.mat 62.40
digits-jitter.mat 59.80
Method 2: Calculating LBP features by dividing an image into 4 parts,
calculating LBP histogram for all four parts and then concatenating all four
***** Hyperparameters for the Method 2 *******
histograms to obtain a feature vector of size 1020.
param.lambda = 0.001;  % Regularization term
param.maxiter = 1000; % Number of iterations
***************
Validation set accuracy:
+++ Accuracy Table [trainSet=1, testSet=2]
```

LBP features are representing the local patterns in images. Also they are invariant to the rotation and scaling to some extent. Hence the classification accuracy for the jittered data is better than other two feature types.

Below is the table showing the best accuracies achieved for each of the three datasets.

```
BEST Validation set accuracies:
```

+++ Accuracy Table [trainSet=1, testSet=2]

dataset pixe	l hog	1bp	
digits-normal.mat 87.6d digits-scaled.mat 82.8d digits-jitter.mat 21.2d	93.40	91.20 91.40 63.40	

BEST Test set accuracies:

+++ Accuracy Table [trainSet=1, testSet=3]

dataset	pixel	hog	lbp	
digits-normal.mat	84.20	95.80	90.80	
digits-scaled.mat	68.00	96.00	91.60	
digits-jitter.mat	19.00	39.80	59.80	

Which feature works the best on normal digits? Why? – HoG features works the best for the normal digits. HoG features captures the local gradient patterns over the many patches in the images. Digits have fixed gradient patterns over the different areas and if the data is not jittered like rotated or skewed these features will capture the most information from the data.

Which feature works the best on scaled digits? Why? – Again data is scaled i.e. pixel values are scaled and as digits images are almost black and white, the local gradient patterns in the images still remains the same. Hence HoG features works the best followed by LBP features.

Which feature works the best on jittered digits? Why? – In the jittered data images are translated. As images are translated we need an image descriptor which is invariant to small translations and LBP are the best features to handle such data. LBP feature histogram remains the same if the image is translated. For example in an image number of pixels with value 10 are 100 and now if we translate the image, only pixel

positions are shifted but the local patterns remains the same and hence number of pixels with value 10 will still be around 10.

Code for extractDigitFeatures.m

```
function features = extractDigitFeatures(x, featureType)
% EXTRACTDIGITFEATURES extracts features from digit images
  features = extractDigitFeatures(x, featureType) extracts FEATURES from images
   images X of the provided FEATURETYPE. The images are assumed to the of
  size [W H 1 N] where the first two dimensions are the width and height.
\% The output is of size [D N] where D is the size of each feature and N
  is the number of images.
switch featureType
    case 'pixel'
        features = zeroFeatures(x);
        features = reshape(x, [784,2000]);
        %features = features - mean(features,2);
        %features = features./sum((features.^2),1);
        %features = features ./ std(features,0,2);
        %features = (features - min(features,[],2))./(max(features,[], 2) - min(features,[],2));
        %features = sqrt(features);
        features = features./sqrt(sum((features.^2),1));
        features = sqrt(features);
        disp(size(features));
    case 'hog'
       %features = zeroFeatures(x);
        [W,H,C,N] = size(x);
        blockSize = 4;
        overlap = 0;
        stepSize = int16(blockSize*(1-overlap));
        binAngle = 20;
        histogramSize = int16(360 / binAngle);
        features = [];
        for imageNumber = 1:N
            image = x(:,:,1,imageNumber);
            image = reshape(image,[W,H]);
            [h,w] = size(image);
            [gx, gy] = imgradientxy(image);
            [gMag, gDir] = imgradient(gx,gy);
            totalSteps = int16(h/stepSize);
            imageHistogram = [];
            for i = 1:totalSteps
                for j = 1:totalSteps
                    startRow = (i-1)*stepSize;
                    startColumn = (j-1)*stepSize;
                    tempMag = gMag(startRow+1:startRow+blockSize,startColumn+1:startColumn+blockSize);
                    tempDir = gDir(startRow+1:startRow+blockSize,startColumn+1:startColumn+blockSize);
                    negativeDirIndices = find(tempDir<0);</pre>
                    tempDir(negativeDirIndices) = tempDir(negativeDirIndices)+360;
                    tempHistogram = zeros(1,18);
```

```
for blockH = 1:blockSize
                         for blockW = 1:blockSize
                             %disp(tempDir(blockH,blockW));
                             binNumber = int16(floor(mod(tempDir(blockH,blockW),360)/binAngle)+1);
                             tempHistogram(binNumber) = tempHistogram(binNumber) + tempMag(blockH,blockW
                         end
                    end
                    imageHistogram = cat(2,imageHistogram,tempHistogram);
                end
            end
            features = cat(1,features,imageHistogram);
        end
        features = features';
        features = features./sqrt(sum((features.^2),1));
        features = sqrt(features);
        disp(size(features));
      case 'lbp'
%
          %features = zeroFeatures(x);
%
          mask = [1,2,4;128,0,8;64,32,16];
%
          [W,H,C,N] = size(x);
%
          features = [];
%
          x = reshape(x, [784,2000]);
%
          %x = x - mean(x,2);
%
          x = x./sum((x.^2),1);
%
          %x = sqrt(x);
%
          x = reshape(x, [28, 28, 1, 2000]);
%
          for imageNumber = 1:N
%
              image = x(:,:,1,imageNumber);
%
              image = reshape(image, [W,H]);
%
              lbpVector = zeros(1,255);
%
              for pixelH = 2:H-1
%
                  for pixelW = 2:W-1
%
                      pixelVal = image(pixelH, pixelW);
%
                       patch = image(pixelH-1:pixelH+1,pixelW-1:pixelW+1);
%
                       %disp(patch)
%
                       lessThanIndices = find(patch<pixelVal);</pre>
%
                       greaterThanIndices = find(patch>=pixelVal);
%
                      patch(lessThanIndices) = 0;
%
                      patch(greaterThanIndices) = 1;
%
                       %disp(patch);
%
%
                      patch(2,2) = 0;
%
                      patch = patch.*mask;
%
%
                      LBP = sum(sum(patch));
%
                       %disp(patch);
%
                      %disp(LBP);
%
                       if(LBP~=255)
%
                           %lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
%
                           lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
%
                       end
```

```
%
%
                   end
%
               end
%
               %disp(features);
%
%
               features = cat(1,features,lbpVector);
%
%
          end
%
          %disp(features(1,:));
%
          %disp(size(features));
%
          %features = features - mean(features,2);
%
          features = features';
%
%
          %disp(size(features));
%
      case 'lbp'
%
          zero = [0];
%
                        2
                               4
                                     8
                                                 32
                                                        64
                                                             128];
          one = [1]
                                           16
%
          two = [3]
                        6
                              12
                                    24
                                           48
                                                 96
                                                       192
                                                             129];
%
          three = [7]
                         14
                                28
                                      56
                                            112
                                                  224
                                                         193
                                                               131];
%
          four = [15]
                                                  225
                                                         195
                         30
                                60
                                     120
                                            240
                                                               135];
%
          five = [31]
                          62
                               124
                                     248
                                            241
                                                  227
                                                         199
                                                               143];
%
          six = [63]
                       126
                              252
                                    249
                                           243
                                                 231
                                                        207
                                                              159];
%
          seven = [127]
                          254
                                 253
                                       251
                                                    239
                                              247
                                                           223
                                                                  191];
%
          eight = [255];
%
%
          mask = [1,2,4;128,0,8;64,32,16];
%
          [W,H,C,N] = size(x);
%
          features = zeros(N,9);
%
          %x = reshape(x, [784,2000]);
%
          %x = x - mean(x,2);
%
          %x = x./sum((x.^2),1);
%
          %x = reshape(x, [28, 28, 1, 2000]);
%
          for imageNumber = 1:N
%
               image = x(:,:,1,imageNumber);
%
               image = reshape(image, [W,H]);
%
               lbpVector = zeros(1,255);
%
               for pixelH = 2:H-1
%
                   for pixelW = 2:W-1
%
                       pixelVal = image(pixelH, pixelW);
%
                       patch = image(pixelH-1:pixelH+1,pixelW-1:pixelW+1);
%
                       %disp(patch)
%
                       lessThanIndices = find(patch<pixelVal);</pre>
%
                       greaterThanIndices = find(patch>=pixelVal);
%
                       patch(lessThanIndices) = 0;
%
                       patch(greaterThanIndices) = 1;
%
                       %disp(patch);
%
%
                       patch(2,2) = 0;
%
%
                       patch = patch.*mask;
%
%
                       LBP = sum(sum(patch));
%
                       %disp(patch);
%
                       %disp(LBP);
```

```
%
                      if LBP==0
%
                           features(imageNumber,1) = features(imageNumber,1)+1;
%
                      elseif ismember(LBP,one)
%
                           features(imageNumber,2) = features(imageNumber,2)+1;
%
                      elseif ismember(LBP,two)
%
                           features(imageNumber,3) = features(imageNumber,3)+1;
%
                      elseif ismember(LBP,three)
%
                           features(imageNumber,4) = features(imageNumber,4)+1;
%
                      elseif ismember(LBP,four)
%
                           features(imageNumber,5) = features(imageNumber,5)+1;
%
                      elseif ismember(LBP,five)
%
                           features(imageNumber,6) = features(imageNumber,6)+1;
%
                      elseif ismember(LBP,six)
%
                           features(imageNumber,7) = features(imageNumber,7)+1;
%
                      elseif ismember(LBP,seven)
%
                           features(imageNumber,8) = features(imageNumber,8)+1;
%
                      elseif LBP==255
%
                           features(imageNumber,9) = features(imageNumber,9)+1;
%
                      else
%
                           features(imageNumber,9) = features(imageNumber,9)+1;
%
%
                      end
%
%
                  end
%
              end
%
              %disp(features);
%
%
              %features = cat(1,features,lbpVector);
%
%
          end
%
          disp(features(1,:));
%
          %disp(size(features));
%
          %features = features - mean(features,2);
%
          features = features';
%
%
          %disp(size(features));
%
case 'lbp'
        %features = zeroFeatures(x);
        mask = [1,2,4;128,0,8;64,32,16];
        [W,H,C,N] = size(x);
        features = [];
        %x = reshape(x, [784,2000]);
        %x = x - mean(x,2);
        x = x./sqrt(sum((x.^2),1));
        %x = sqrt(x);
        %x = reshape(x, [28, 28, 1, 2000]);
        for imageNumber = 1:N
            image = x(:,:,1,imageNumber);
            image = reshape(image, [W,H]);
            fullVector = [];
            lbpVector = zeros(1,255);
            for pixelH = 2:int16((H-1)/2)
                for pixelW = 2:int16((W-1)/2)
```

```
pixelVal = image(pixelH, pixelW);
        patch = image(pixelH-1:pixelH+1,pixelW-1:pixelW+1);
        %disp(patch)
        lessThanIndices = find(patch<pixelVal);</pre>
        greaterThanIndices = find(patch>=pixelVal);
        patch(lessThanIndices) = 0;
        patch(greaterThanIndices) = 1;
        %disp(patch);
        patch(2,2) = 0;
        patch = patch.*mask;
        LBP = sum(sum(patch));
        %disp(patch);
        %disp(LBP);
        if(LBP~=255)
            %lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
            lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
        end
    end
end
fullVector = cat(2,fullVector,lbpVector);
lbpVector = zeros(1,255);
for pixelH = 2:int16((H-1)/2)
    for pixelW = int16((W-1)/2):W-1
        pixelVal = image(pixelH, pixelW);
        patch = image(pixelH-1:pixelH+1,pixelW-1:pixelW+1);
        %disp(patch)
        lessThanIndices = find(patch<pixelVal);</pre>
        greaterThanIndices = find(patch>=pixelVal);
        patch(lessThanIndices) = 0;
        patch(greaterThanIndices) = 1;
        %disp(patch);
        patch(2,2) = 0;
        patch = patch.*mask;
        LBP = sum(sum(patch));
        %disp(patch);
        %disp(LBP);
        if(LBP~=255)
            %lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
            lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
        end
    end
end
%disp(features);
fullVector = cat(2,fullVector,lbpVector);
lbpVector = zeros(1,255);
for pixelH = int16((H-1)/2):H-1
    for pixelW = 2:int16((W-1)/2)
```

```
pixelVal = image(pixelH, pixelW);
        patch = image(pixelH-1:pixelH+1,pixelW-1:pixelW+1);
        %disp(patch)
        lessThanIndices = find(patch<pixelVal);</pre>
        greaterThanIndices = find(patch>=pixelVal);
        patch(lessThanIndices) = 0;
        patch(greaterThanIndices) = 1;
        %disp(patch);
        patch(2,2) = 0;
        patch = patch.*mask;
        LBP = sum(sum(patch));
        %disp(patch);
        %disp(LBP);
        if(LBP~=255)
            %lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
            lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
        end
    end
end
%disp(features);
fullVector = cat(2,fullVector,lbpVector);
lbpVector = zeros(1,255);
for pixelH = int16((H-1)/2):H-1
    for pixelW = int16((W-1)/2):W-2
        pixelVal = image(pixelH, pixelW);
        patch = image(pixelH-1:pixelH+1,pixelW-1:pixelW+1);
        %disp(patch)
        lessThanIndices = find(patch<pixelVal);</pre>
        greaterThanIndices = find(patch>=pixelVal);
        patch(lessThanIndices) = 0;
        patch(greaterThanIndices) = 1;
        %disp(patch);
        patch(2,2) = 0;
        patch = patch.*mask;
        LBP = sum(sum(patch));
        %disp(patch);
        %disp(LBP);
        if(LBP~=255)
            %lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
            lbpVector(LBP+1) = lbpVector(LBP+1) + 1;
        end
    end
end
%disp(features);
fullVector = cat(2,fullVector,lbpVector);
```

```
features = cat(1,features,fullVector);

end
    %disp(features(1,:));
    %disp(size(features));
    %features = features - mean(features,2);
    %features = sqrt(features);
    %features = features./sqrt(sum((features.^2),1));
    features = sqrt(features);
    features = features';

disp(size(features));
end

function features = zeroFeatures(x)
features = zeros(10, size(x,4));
```

For the LBP features, I tried 2-3 methods. One was from http://www.springer.com/cda/content/document/cda_downloaddocument/9780857297471-c2.pdf?SGWID=0-0-45-1153741-p174122174 "Local Binary Patterns for Still Images". One which I tried was, Mappings of the LBP Labels: Uniform Patterns. This method generates LBP histogram with 9 bins only. Only LBP values representing uniform patterns such as Spot, Spot/Flat, Line End, Edge and Corner are kept as LBP features.

4. Extension – Random forest classifier

```
NumTrees = 100
Validation set accuracy:
+++ Accuracy Table [trainSet=1, testSet=2]
_____
dataset pixel hog lbp
_____
digits-normal.mat 84.60 90.00 79.00
digits-scaled.mat 80.60 90.00 79.40
digits-jitter.mat 25.00 35.40 40.60
Test set accuracy:
+++ Accuracy Table [trainSet=1, testSet=3]
-----
        pixel hog lbp
dataset
-----
digits-normal.mat 78.20 84.80 77.00
digits-scaled.mat 76.00 85.80 76.20
digits-jitter.mat 24.20 36.00 40.80
Code of randomForest.m
\% There are three versions of MNIST dataset
dataTypes = {'digits-normal.mat', 'digits-scaled.mat', 'digits-jitter.mat'};
% You have to implement three types of features
%featureTypes = {'pixel', 'hog', 'lbp'};
featureTypes = {'lbp'};
% Accuracy placeholder
accuracy = zeros(length(dataTypes), length(featureTypes));
trainSet = 1;
testSet = 3; % 2 = validation, 3 = test
for i = 1:length(dataTypes),
   dataType = dataTypes{i};
   % load data
   load(fullfile('...', 'data', dataType));
   fprintf('+++ Loading digits of dataType: %s\n', dataType);
   % Optionally montage the digits in the val set
   % montageDigts(data.x(:,:,:,data.set==2));
```

for j = 1:length(featureTypes),

featureType = featureTypes{j};

```
% Extract features
       tic;
       features = extractDigitFeatures(data.x, featureType);
       fprintf(' %.2fs to extract %s features for %i images\n', toc, featureType, size(features,2));
       % Train model
       tic;
       model = TreeBagger(20,features(:, data.set==trainSet)', data.y(data.set==trainSet), 'Method', '
       fprintf(' %.2fs to train model\n', toc);
       % Test the model
       ypred = model.predict(features(:, data.set==testSet)');
       ypred = str2double(ypred);
       y = data.y(data.set==testSet);
       % Measure accuracy
       %disp(size(ypred));
       %disp(size(y));
       ypred = ypred';
       [acc, conf] = evaluateLabels(y, ypred, false);
       fprintf(' Accuracy [testSet=%i] %.2f%%\n\n', testSet, acc*100);
       accuracy(i,j) = acc;
   end
end
% Print the results in a table
fprintf('+++ Accuracy Table [trainSet=%i, testSet=%i]\n', trainSet, testSet);
fprintf('----\n');
fprintf('dataset\t\t\');
for j = 1:length(featureTypes),
   fprintf('%s\t',featureTypes{j});
end
fprintf('\n');
fprintf('----\n');
for i = 1:length(dataTypes),
   fprintf('%s\t', dataTypes{i});
   for j = 1:length(featureTypes),
       fprintf('%.2f\t', accuracy(i,j)*100);
   end
   fprintf('\n');
end
% Once you have optimized the hyperparameters, you can report test accuracy
% by setting testSet=3. You should not optimize your hyperparameters on the
\% test set. That would be cheating.
```