Computer Vision *CS 670, Fall 2016*

HW04: Image processing

Hand in via moodle at: `https://moodle.umass.edu/course/view.php?id=33024`. Remember that only PDF submissions are accepted. We encourage using LaTeX to produce your writeups. See `hw00.tex` for an example of how to do so. You can make a `.pdf` out of the `.tex` by running "`pdflatex hw00.tex`".

1. Show that filtering an image with a seperable 2D filter kernel is equivalent to filtering the with two 1D filter kernels.

Consider any separable filter X with its component filters X1 and X2 as

$$X = \begin{pmatrix} x1y1 & x1y2 & x1y3 \\ x2y1 & x2y2 & x2y3 \\ x3y1 & x3y2 & x3y3 \end{pmatrix}, X1 = \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}, X2 = \begin{pmatrix} y1 & y2 & y3 \end{pmatrix}$$

Consider any 3X3 image or image patch I as:

$$I = \begin{pmatrix} p11 & p12 & p13 \\ p21 & p22 & p23 \\ p31 & p32 & p33 \end{pmatrix}$$

For convolving image I with our 2D filter X we will center the filter at the pixel p22. This will give us the resultant image as

$$I.X = \begin{pmatrix} x1.y1.p11 & x1.y2.p12 & x1.y3.p13 \\ x2.y1.p21 & x2.y2.p22 & x2.y3.p23 \\ x3.y1.p31 & x3.y2.p32 & x3.y3.p33 \end{pmatrix}$$

Now let's look at the result of applying convolution on image I by X1 and then by X2. Convolving the image I with X1 will give us

$$I.X1 = \begin{pmatrix} x1.p11 & x1.p12 & x1.p13 \\ x2.p21 & x2.p22 & x2.p23 \\ x3.p31 & x3.p32 & x3.p33 \end{pmatrix}$$

and then convolving above result with the X2 will give us

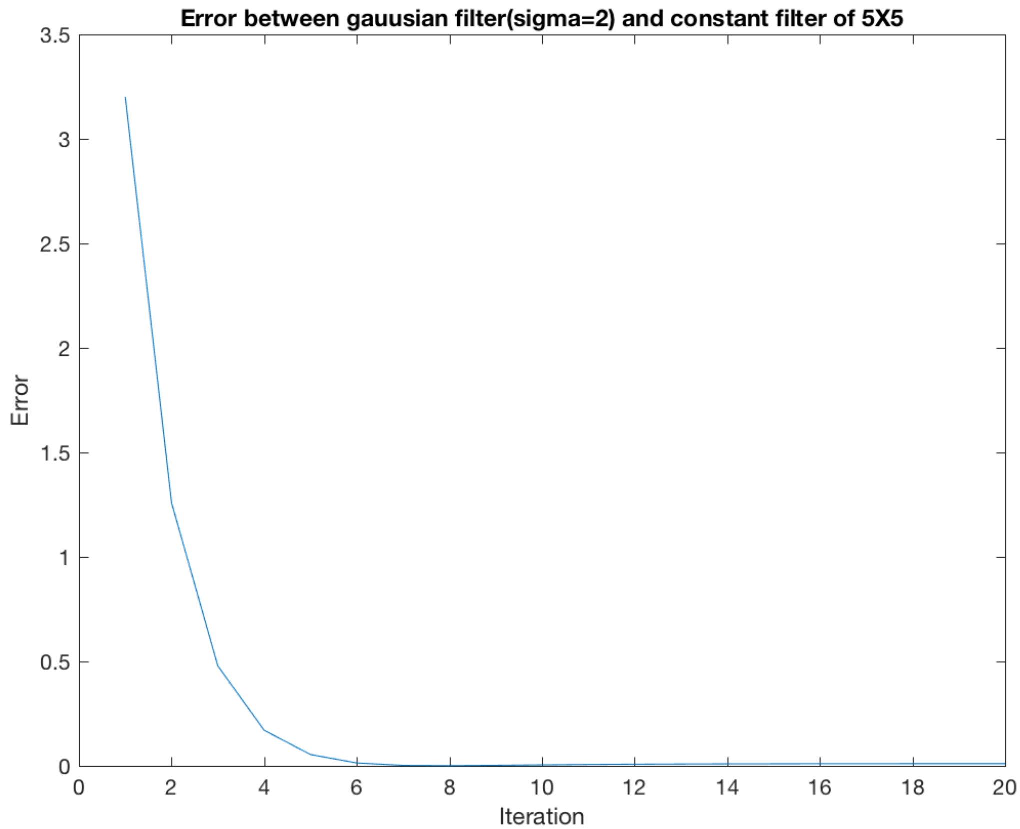
$$I.X1.X2 = \begin{pmatrix} x1.y1.p11 & x1.y2.p12 & x1.y3.p13 \\ x2.y1.p21 & x2.y2.p22 & x2.y3.p23 \\ x3.y1.p31 & x3.y2.p32 & x3.y3.p33 \end{pmatrix}$$

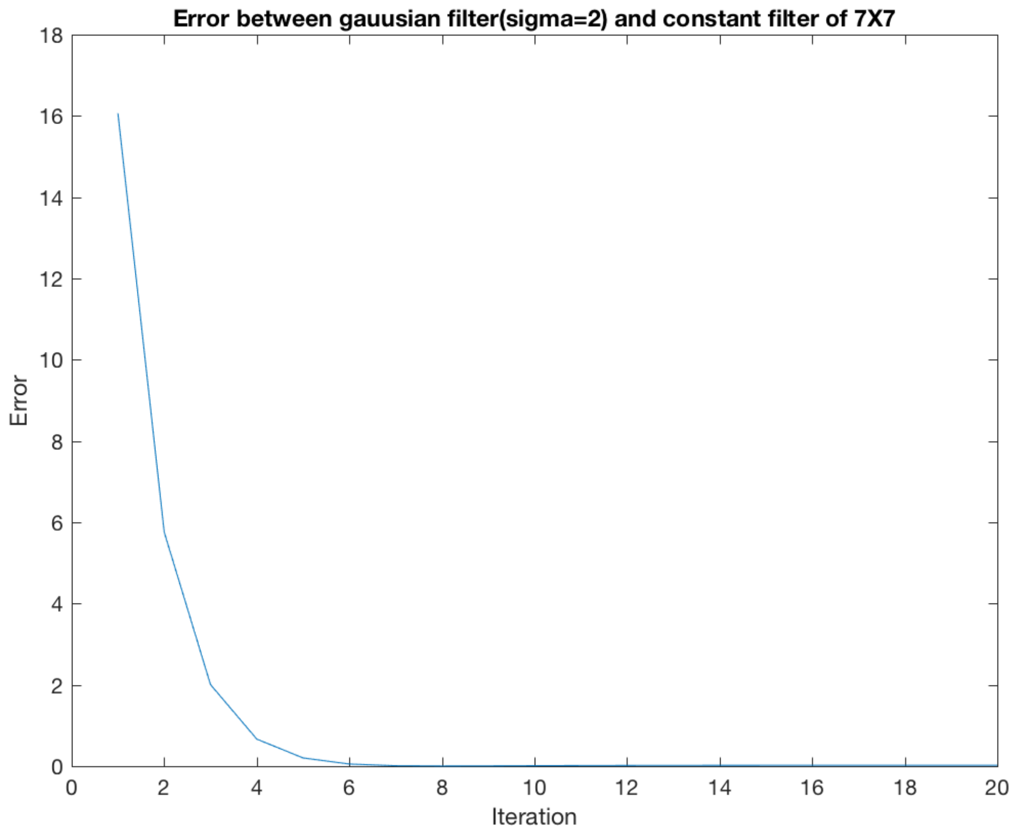
Which is equal to I.X. So filtering an image with a seperable 2D filter kernel is equivalent to filtering the with two 1D filter kernels

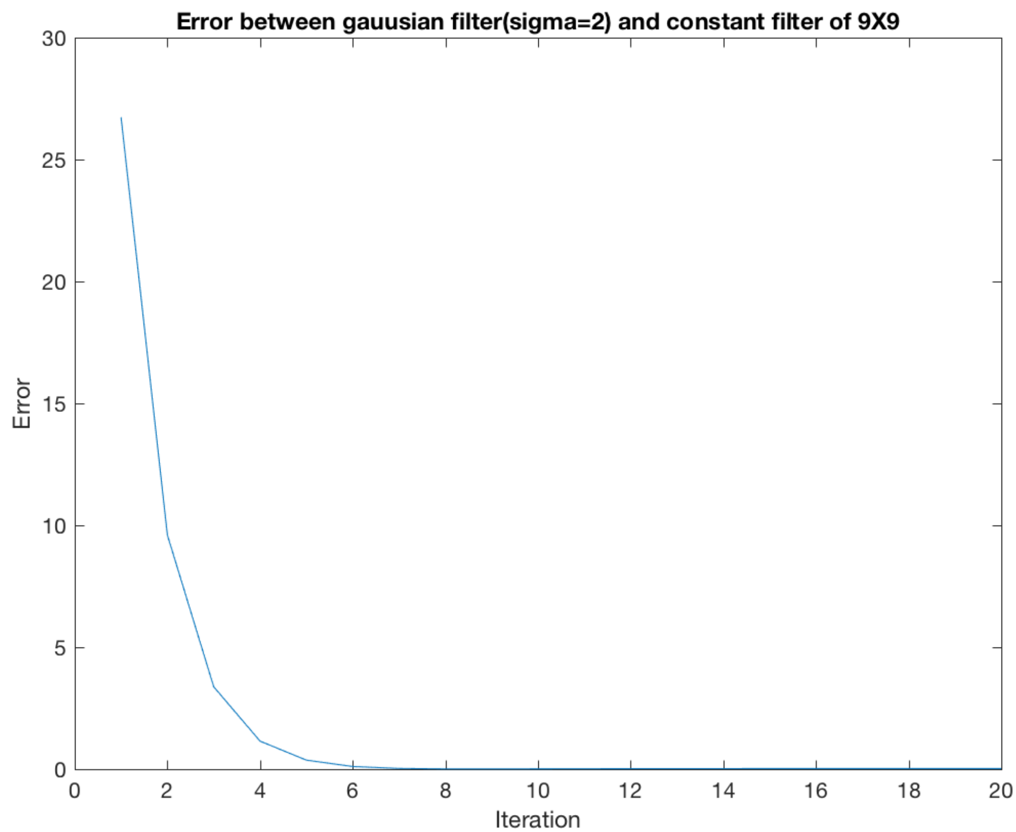
2. One way to obtain a Gaussian kernel is to filter a constant kernel with itself many times. Compare this strategy with evaluating a Guassain kernel.

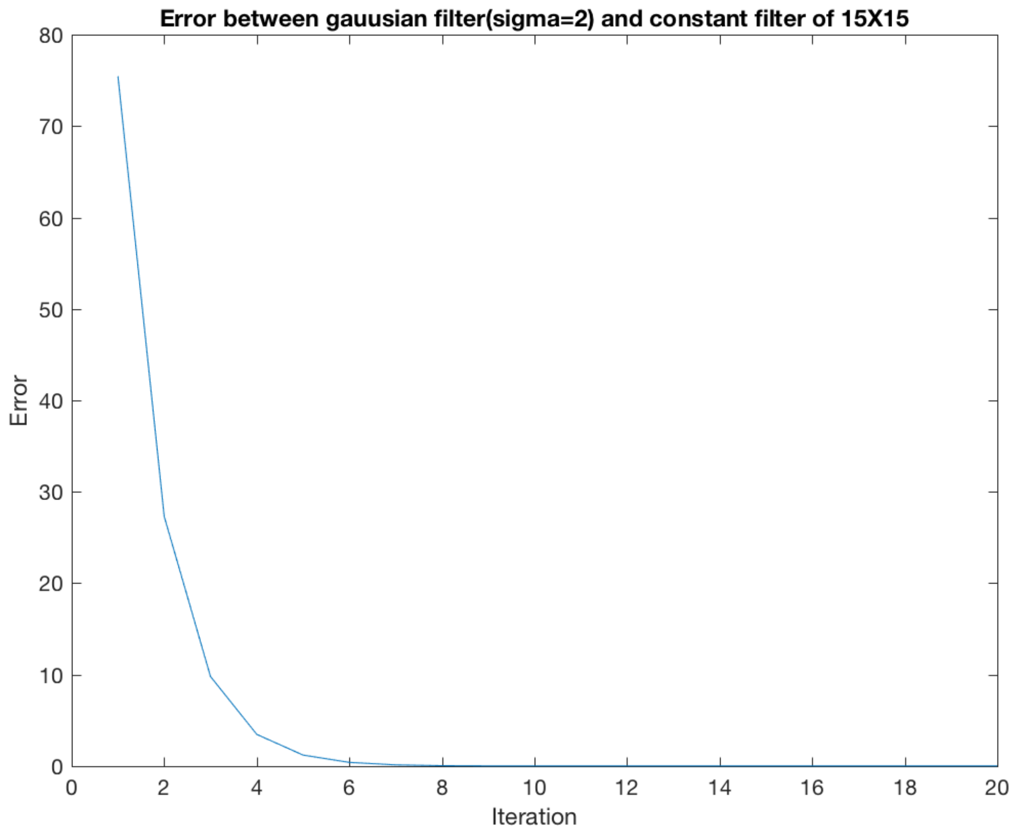
   (a) How many repeated filterings do you need to get a reasonable approximation? You need to establish what a reasonable approximation is; You might plot the quality of approximation against the number of repeated convolutions.

```
sigma = 2;
N = 5;

[x,y]=meshgrid(-floor(N/2) : floor(N/2),-floor(N/2) : floor(N/2));

Exp_comp = -(x.^2+y.^2)/(2*sigma*sigma);
Kernel= exp(Exp_comp)/(2*pi*sigma*sigma)
%size(Kernel);

%A = [1 1 1 1 1;1 1 1 1 1;1 1 1 1 1;1 1 1 1 1;1 1 1 1 1];
%B = [1 1 1 1 1;1 1 1 1 1;1 1 1 1 1;1 1 1 1 1;1 1 1 1 1];
A = ones(N);
B = ones(N);

x = ones(20,1);
kernals = zeros(N,N,20);
for i = 1:20
   A = conv2(A,B,'same');
   A = A./(N*N);

   d = sum(sum((A-Kernel).^2));
   x(i) = d;
   kernals(:,:,i) = A;
end
y = linspace(0,50,50);
[M,I] = min(x);
disp(kernals(:,:,I));
plot(x);

xlabel('Iteration');
ylabel('Error');
title('Error between gauusian filter(sigma=2) and constant filter of 5X5')
saveas(gcf,'Error5.png')
```


Error between gauusian filter(sigma=2) and constant filter of 5X5
Error
Iteration
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12
14
16
18
20


Error between gauusian filter(sigma=2) and constant filter of 7X7
Error
Iteration
0
2
4
6
8
10
12
14
16
18
20


Error between gauusian filter(sigma=2) and constant filter of 9X9
Error
Iteration
0
5
10
15
20
25
30
0
2
4
6
8
10
12
14
16
18
20



For different sizes of the filter, error is minimum at the iteration around 6-10. So iterating a constant filter for 6-10(depends on the size of the filter) times with itself gives a close enough approximation of the gaussian filter.

(b) Are there any benefits that can be obtained like this?

Generating a gaussian filter dynamically requires complex operations such as exponential terms. If we don't want to do these exponential operations, we can achieve the approximate version by simple operations on a constant filter. If we can achieve a good approximation in just few iterations, then it can be useful.