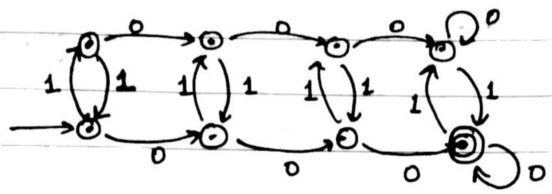
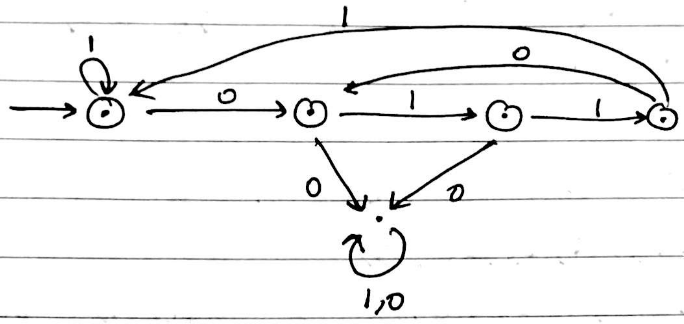
ToC : Problem set 1.

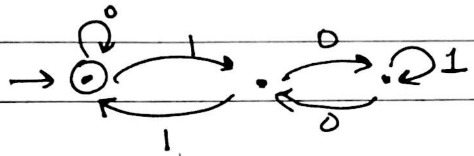
Q1. a)



b)

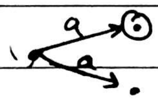


Q2.



A' will be same as A. and because it is symmetric, so yes it will accept the set of binary whose reverse's decimal is divisible by 3, ∵ reverse ∈ A.

Q-3. By definition of acceptance, x is accepted if $\delta(s,x) \in F$.
Since accepting and non-accepting states are interchanged
Implies all x that were accepted earlier, will not be accepted now
and all that were not accepted will be accepted now, Hence language is complemented.
For NFA,

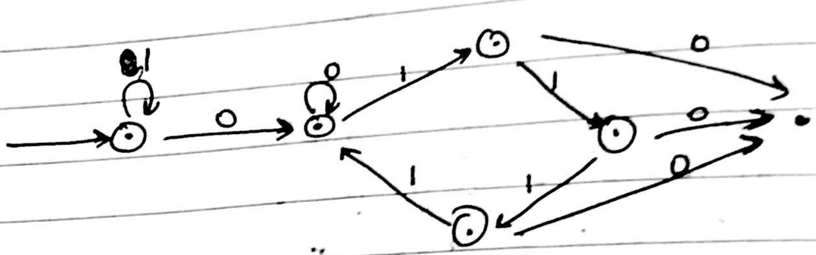


Due to this case, it is false
∵ a will be accepted in both cases

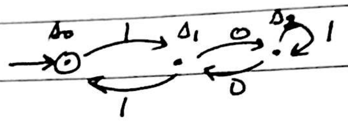
Q-4. Atleast one 1 and after first 1, every ~~00000~~ is followed immed. by atleast one 1.

occurrence of ~~max~~ ≥ 5 0's is followed by atleast one 1.

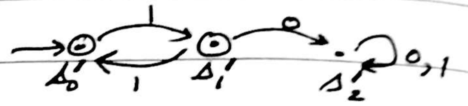
Q-7.



Q-8. M_1: (L_1)



M_2:



M_3 s.t. $L(M_3) = L_1 \cap L_2$

$\hookrightarrow (Q_3, \Sigma, \delta_3, s_3, F_3)$

$Q_3 = Q_1 \times Q_2$, $\delta_3((p,q), a) = (\delta_1(p,a), \delta_2(q,a))$

$s_3 = \{s_0, s_0'\}$, $F_3 = F_1 \times F_2 = \{(s_0, s_0'), (s_0, s_1')\}$.

Q- 5, 6, 9, 10

Problem set 2

Q-1. a) $(1^+ + 01^+)^*(00(1^+ + 01^+0)^* + \epsilon)$

or $1^*(011^*)^*(\epsilon + 00)\cdot(11^*0)^*1^*$

b) $(0+1)^*(0(0+1)^*1 + 1(0+1)^*0)(0+1)^*$

or $(0+1)^*01(0+1)^* + (0+1)^*10(0+1)^*$

Q-2. a) Set of all binary string with atleast one 11 ~~and repeating pattern after a given 11.~~

~~No consecutive~~ 11.

b) (Every 1 is followed by atleast one 0). or null string.

c) ~~Atleast 2 zeroes~~ 3n+2 zeroes, $n \geq 0$.

Q-3. a) $A = \{0^{n^2} \mid n \geq 0\}$

$w = 0^{n^2} \Rightarrow |w| \geq n \qquad xyz = w, \ |xy| \leq n$

$|xy^2z| = |xyz| + |y| = n^2 + |y| \leq n^2 + n < (n+1)^2$

$\Rightarrow xy^2z \notin A$

b) $A = \{0^{n!}, n \geq 0\}$

$w = 0^{n!} \Rightarrow |w| = n! \geq n, \quad xyz = w \quad |xy| \leq n$

$|xy^2z| = |xyz| + |y| \leq n! + n < (n+1)!$

$\Rightarrow xy^2z \notin A.$

Q-8. a) $A = \{0^n 1^m \mid m \leq n\}$ In the end, $B = \{0^k 1^k \mid k \geq 0\}$

$A \cap B = B \rightarrow$ not regular

$\Rightarrow A \,\&\, B$ are not regular (Closure property of ~~regular~~ Regular sets under Intersection)

$\Rightarrow$ A is not regular.

b) $A = \{0^n 1^m \mid m \neq n\}$. $B = \{0^k 1^k \mid k \geq 0\}$

~~$\bar{A} \cap B = \emptyset$~~ $\bar{A} \cap \underbrace{0^*1^*}_{\text{regular}} = B \rightarrow$ not regular $\Rightarrow \bar{A}$ is not regular

$\Downarrow$

A is not regular

Q7. $1^* 0 (11^* 0)^* 0 (0+1)^*$

Q6.

	0	1
q_0	q_1	q_3
q_1	q_0	q_2
F q_2	q_4	q_5
F q_3	q_4	q_5
q_4	q_5	q_4
q_5	q_5	q_5

	q_0	q_1	q_2	q_3	q_4
q_1	$\boxed{\ }$				
q_2	✓	✓			
q_3	✓	✓	$\boxed{\ }$		
q_4	✓	✓	✓	✓	
q_5	✓	✓	✓	✓	$\boxed{\ }$

$q_0 \approx q_1$, $q_2 \approx q_3$, $q_4 \approx q_5$

	0	1
q_0	q_1	q_2
F q_1	q_1	q_4
q_2	q_1	q_2
F q_3	q_1	q_2
q_4	q_1	q_3

	q_0	q_1	q_2	q_3
q_1	✓			
q_2	$\boxed{\ }$	✓		
q_3	✓	✓	✓	
q_4	✓	✓	✓	✓

$q_0 \approx q_2$

Q8. (a) $A = \{0^n 1^m \mid m \geq n \geq 0\}$.

Assumption: Assume it to be regular.

Construct its DFA (M) and reverse all edges. ~~Let it be A'~~ Let it be M'. Then since M is DFA, M' is also DFA. Let A' be ~~regular~~ set associated with M'.

So, $A' = \{1^m 0^n \mid m \geq n \geq 0\}$.

Construct a homomorphism with $h(0)=1$ & $h(1)=0$.

$\therefore A'' = h(A) = \{0^m 1^n \mid m \geq n\}$ will also be regular.

Since A and A'' are regular $\Rightarrow A \cap A''$ is also regular (Closure Property).

$A \cap A'' = \{0^n 1^n \mid n \geq 0\} = B$.

But B is not regular $\Rightarrow$ Assumption is false $\Rightarrow$ A is not regular.

Q5 b)

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Tab

To show

Q5 b) lets DFA M be associated with L. s.t. $\mathcal{L}(M) = L$

Construct DFA M' s.t ~~for every edge $a_i a_j$ in M, insert a node in between and make it $a_i s a_j$ and change final node it~~

Every edge a_i in M is replaced by $a_i s$ (by inserting a state in between). It can be easily argued that M' is also a DFA. ~~$\Rightarrow L_1$ is~~ and $L_1 = \mathcal{L}(M') \Rightarrow L_1$ is regular.

a) Using Pumping's Lemma:

$$L_1 = \{w \mid w = a_1 s a_2 ss a_3 sss \ldots a_n s^n \text{ where } a_1 a_2 \ldots a_n \in L\}.$$

Take $w = a_1 s a_2 s^2 a_3 s^3 \ldots a_n s^n$

$|w| \geq n. \quad (\exists n)$

<u>To show</u>: $\forall x, y, z$ with $xyz = w$. $\exists k$ s.t. $xy^k z \notin L_1$

Let $y = a_i$ then $xy^2 z \notin L_1$

$y = s$, $xy^2 z \notin L_1$

$y = a_i s$ $xy^2 z \notin L_1$

$y = a_i s^k$ $xy^2 z \notin L_1$

So, $\forall y$ $xy^k z \notin L_1$. for some k

Q-4

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Problem set 3

Q1. (ii) $L = \{w \in \{0,1\}^* \mid |w| \text{ is odd and middle symbol is } 0\}$.

CFG, $G = (N, \Sigma, P, S)$

$N = \{S\}$, $\Sigma = \{0,1\}$.

P include $S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0$

(i) ~~$S \rightarrow 0S \mid 0S1 \mid 1S0 \mid \epsilon$. $\rightarrow \#_0(w) >= \#_1(w)$.~~

~~$S \rightarrow 0S \mid 00S1 \mid 01S0 \mid 0S10 \mid 1S00 \mid \epsilon$ $\rightarrow \#_0(w) > \#_1(w)$.~~ $\times$.

$S \rightarrow 0A \mid A0 \mid 0$

$A \rightarrow 0A \mid 0A1 \mid 1A0 \mid \epsilon$. $\rightarrow \#_0(A) \geq \#_1(A)$

Q2. (i) $S \rightarrow AC' \mid A'C \mid \epsilon$

$A \rightarrow aAb \mid \epsilon$

$C \rightarrow bCc \mid \epsilon$

$C' \rightarrow cC' \mid \epsilon$

$A' \rightarrow aA' \mid \epsilon$.

(ii) $S \rightarrow aaaA \mid \epsilon$

$A \rightarrow aAb \mid Ab \mid \epsilon$.

(iii) $S \rightarrow aSc \mid bBc \mid \epsilon$

$B \rightarrow bBc \mid \epsilon$.

Q-3. $L(G) = \{w \mid \#_a(w) = \#_b(w)\}$.

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