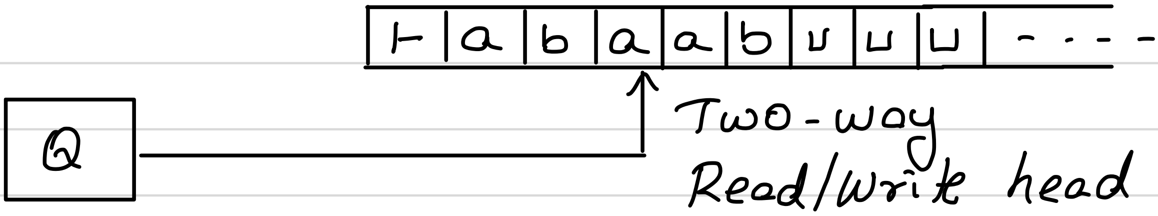
Turing machines - equivalent models.

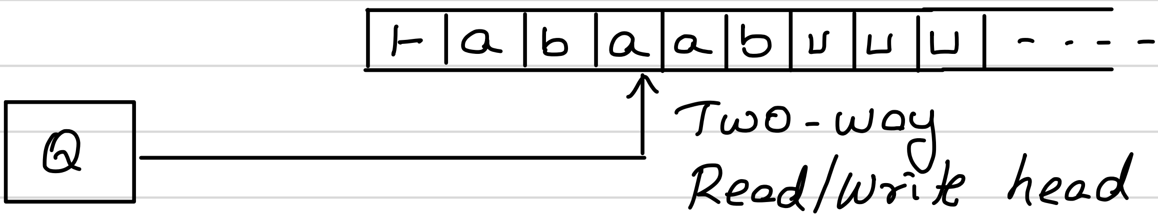


Tape with multiple tracks.

	⊢	a	a	b	a	b	⊔	⊔
⊢	⊢	b	b	b	a	b	b	⊔
	⊢	b	a	a	a	b	a	⊔

Tape symbol is a triple $(c, d, e) \mapsto \begin{array}{|c|}\hline c \\ \hline d \\ \hline e \\ \hline \end{array}$

Turing machines - equivalent models.



Tape with multiple tracks.

	⊢	a	a	b	a	b	⊔	⊔
⊢	⊢	b	b	b	a	b	b	⊔
	⊢	b	a	a	a	b	a	⊔

Tape symbol is a triple $(c, d, e) \mapsto \begin{array}{|c|} \hline c \\ \hline d \\ \hline e \\ \hline \end{array}$

Claim. For $A \subseteq \Sigma^*$, if A is r.e. and $\overline{A}$ is r.e. then A is recursive.

Proof. Let $L(M_1) = A$ and $L(M_2) = \overline{A}$.

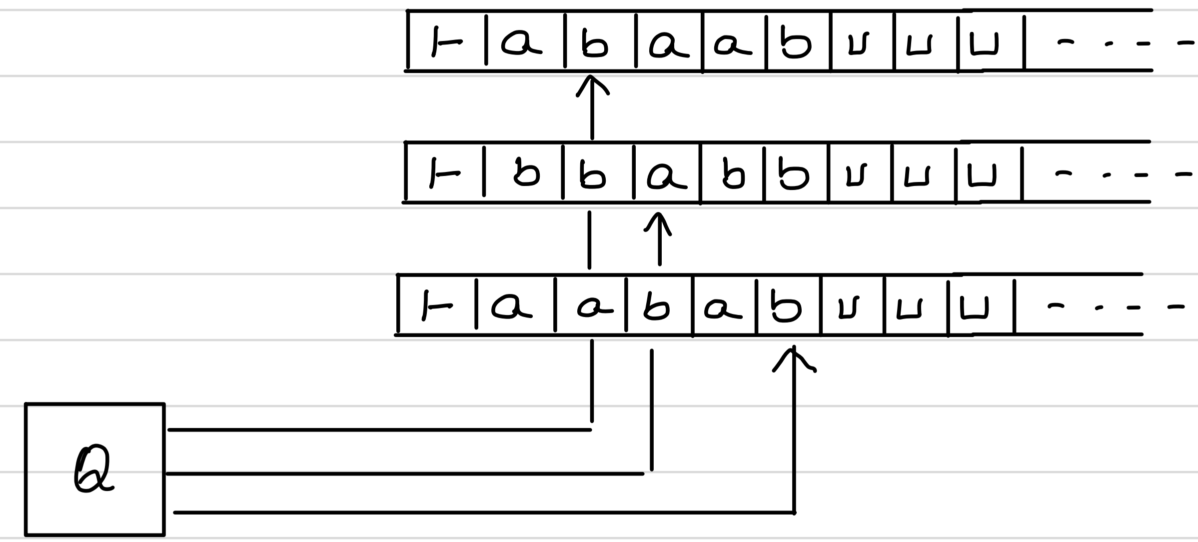
Construct M that on input x runs both M_1 & M_2 simultaneously on two tracks of its tape.

⊢	a	a	b	a	b	⊔	⊔
⊢	b	b	b	a	b	b	⊔

if M_1 accepts then M accepts. if M_2 accepts then M rejects

$$\left.\begin{array}{l} x \in A \Rightarrow x \in L(M_1) \Rightarrow M_1 \text{ accepts} \Rightarrow M \text{ accepts.} \\ x \notin A \Rightarrow x \in L(M_2) \Rightarrow M_2 \text{ accepts} \Rightarrow M \text{ rejects} \end{array}\right\} \text{Total TM}$$

Multiple tapes



$$\delta : Q \times \Gamma^3 \to Q \times \Gamma^3 \times \{L, R\}^3$$

	$\vdash$	a	b	a	a	b	$\sqcup$	$\sqcup$	
$\vdash$	$\vdash$	b	b	a	b	b	$\sqcup$	$\sqcup$	
	$\vdash$	a	a	b	a	b	$\sqcup$	$\sqcup$	

Take the tape alphabet to be

$$\Sigma \cup \{\vdash\} \cup (\Gamma \cup \Gamma')^3 \quad \text{where } \Gamma' = \{\hat{a} \mid a \in \Gamma\}$$

Two way infinite tape

-----	a	b	a	a	b	a	b	b	a	b	-----

*

⊢	b	a	a	b	a	- - -
	a	b	b	a	b	- - -

Simulate top track when head is on the left of * and simulate bottom track when head is on the right of *.

Two stacks.

Claim. A finite state machine with a two-way, read only input head and two stacks is as powerful as a Turing machine.

Power of the model – Universal Turing machine.

Fix an encoding scheme of TM over some alphabet – say $\{0,1\}$.

Any encoding is fine as long as it is possible for another TM to take as input the encoded string and decode the description.

An example encoding scheme.

$$0^n 1 0^m 1 0^k 1 0^s 1 0^t 1 0^r 1 0^u 1 0^v 1$$

M has n states

m tape symbols of which first k are input symbols

Similar encoding possible for transitions.

Important properties of the encoding scheme

- Able to encode all TMs upto isomorphism.
- Easy to interpret.

Universal Turing machine.

$$L(U) = \{ \underline{M} \# \underline{x} \mid x \in L(M) \}.$$

encoding of M

Symbol in U's input alphabet to delimit M and x.

encoding of input string x
x is over the input alphabet of M.

How does U work?

1. Check if M and x are valid encodings. if not, reject.

2. U does a step-by-step simulation of M on x.

Description of M
Contents of M's tape
State of M and position of tape head