



Compiler Design

Parse Table Construction

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Constructing parse table

Augment the grammar

- G is a grammar with start symbol S
- The augmented grammar G' for G has a new start symbol S' and an additional production $S' \rightarrow S$
- When the parser reduces by this rule it will stop with accept

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Production to Use for Reduction

- How do we know which production to apply in a given configuration
- We can guess!
 - May require backtracking
- Keep track of “ALL” possible rules that can apply at a given point in the input string
 - But in general, there is no upper bound on the length of the input string
 - Is there a bound on the number of applicable rules?

Some hands on!

1. $E' \rightarrow E$
2. $E \rightarrow E + T$
3. $E \rightarrow T$
4. $T \rightarrow T * F$
5. $T \rightarrow F$
6. $F \rightarrow (E)$
7. $F \rightarrow id$

Strings to Parse

- $id + id + id + id$
- $id * id * id * id$
- $id * id + id * id$
- $id * (id + id) * id$

Parser states

- Goal is to know the valid reductions at any given point
- Summarize all possible stack prefixes α as a parser state
- Parser state is defined by a DFA state that reads in the stack α
- Accept states of DFA are unique reductions

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Viable prefixes

- α is a viable prefix of the grammar if
 - $\exists w$ such that αw is a right sentential form
 - $\langle \alpha, w \rangle$ is a configuration of the parser
- As long as the parser has viable prefixes on the stack no parser error has been seen
- The set of viable prefixes is a regular language
- We can construct an automaton that accepts viable prefixes

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LR(0) items

- An LR(0) item of a grammar G is a production of G with a special symbol “.” at some position of the right side
- Thus production $A \rightarrow XYZ$ gives four LR(0) items
 - $A \rightarrow .XYZ$
 - $A \rightarrow X.YZ$
 - $A \rightarrow XY.Z$
 - $A \rightarrow XYZ.$

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LR(0) items

- An item indicates how much of a production has been seen at a point in the process of parsing
 - Symbols on the left of “.” are already on the stacks
 - Symbols on the right of “.” are expected in the input

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Start state

- Start state of DFA is an empty stack corresponding to $S' \rightarrow .S$ item
- This means no input has been seen
- The parser expects to see a string derived from S

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Closure of a state

- **Closure** of a state adds items for all productions whose LHS occurs in an item in the state, just after “.”
 - Set of possible productions to be reduced next
 - Added items have “.” located at the beginning
 - No symbol of these items is on the stack as yet

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Example

For the grammar If I is $\{ E' \rightarrow .E \}$ then
closure(I) is

| | |
|-------------------------------|------------------------|
| $E' \rightarrow E$ | $E' \rightarrow .E$ |
| $E \rightarrow E + T \mid T$ | $E \rightarrow .E + T$ |
| $T \rightarrow T * F \mid F$ | $E \rightarrow .T$ |
| $F \rightarrow (E) \mid id$ | $T \rightarrow .T * F$ |
| | $T \rightarrow .F$ |
| | $F \rightarrow .id$ |
| | $F \rightarrow .(E)$ |

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Closure operation

- Let I be a set of items for a grammar G
- closure(I) is a set constructed as follows:
 - Every item in I is in closure(I)
 - If $A \rightarrow \alpha.B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production then $B \rightarrow .\gamma$ is in closure(I)
- Intuitively $A \rightarrow \alpha.B\beta$ indicates that we expect a string derivable from $B\beta$ in input
- If $B \rightarrow \gamma$ is a production then we might see a string derivable from γ at this point

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Goto operation

- $\text{Goto}(I, X)$, where I is a set of items and X is a grammar symbol,
 - is closure of set of item $A \rightarrow \alpha X \beta$
 - such that $A \rightarrow \alpha X \beta$ is in I
- Intuitively if I is a set of items for some valid prefix α then $\text{goto}(I, X)$ is set of valid items for prefix αX

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Goto operation

If I is $\{ E' \rightarrow E., E \rightarrow E. + T \}$ then $\text{goto}(I, +)$ is

$E \rightarrow E + .T$
 $T \rightarrow .T * F$
 $T \rightarrow .F$
 $F \rightarrow .(E)$
 $F \rightarrow .id$

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Sets of items

C : Collection of sets of LR(0) items for grammar G'

$C = \{ \text{closure}(\{ S' \rightarrow .S \}) \}$

repeat

 for each set of items I in C

 for each grammar symbol X

 if $\text{goto}(I, X)$ is not empty and not in C

 ADD $\text{goto}(I, X)$ to C

until no more additions to C

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Example

Grammar:

$E' \rightarrow E$
 $E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$

I_0 : $\text{closure}(E' \rightarrow .E)$

$E' \rightarrow .E$
 $E \rightarrow .E + T$
 $E \rightarrow .T$
 $T \rightarrow .T * F$
 $T \rightarrow .F$
 $F \rightarrow .(E)$
 $F \rightarrow .id$

I_1 : $\text{goto}(I_0, E)$

$E' \rightarrow E.$
 $E \rightarrow E. + T$

I_2 : $\text{goto}(I_0, T)$

$E \rightarrow T.$
 $T \rightarrow T. * F$

I_3 : $\text{goto}(I_0, F)$

$T \rightarrow F.$

I_4 : $\text{goto}(I_0, ($

$F \rightarrow (.E)$
 $E \rightarrow .E + T$
 $E \rightarrow .T$
 $T \rightarrow .T * F$
 $T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

I_5 : $\text{goto}(I_0, id)$

$F \rightarrow id.$

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$l_6: \text{goto}(l_1, +)$
 $E \rightarrow E + .T$
 $T \rightarrow .T * F$
 $T \rightarrow .F$
 $F \rightarrow .(E)$
 $F \rightarrow .id$

$l_9: \text{goto}(l_6, T)$
 $E \rightarrow E + T.$
 $T \rightarrow T * .F$

$\text{goto}(l_6, F)$ is l_3
 $\text{goto}(l_6, ($ is l_4
 $\text{goto}(l_6, id)$ is l_5

$l_7: \text{goto}(l_2, *)$
 $T \rightarrow T * .F$
 $F \rightarrow .(E)$
 $F \rightarrow .id$

$l_{10}: \text{goto}(l_7, F)$
 $T \rightarrow T * F.$

$\text{goto}(l_7, ($ is l_4
 $\text{goto}(l_7, id)$ is l_5

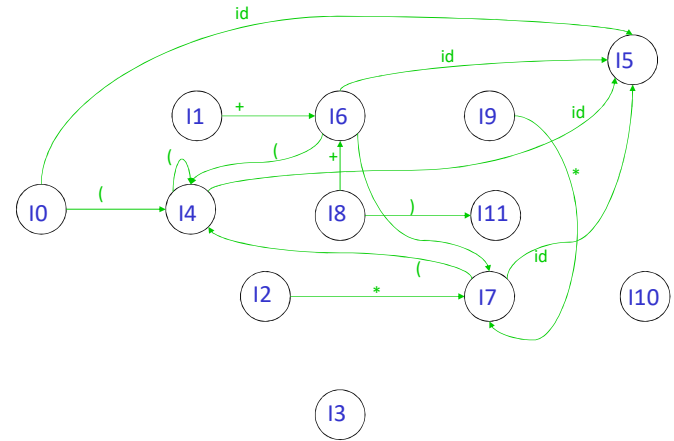
$l_8: \text{goto}(l_4, E)$
 $F \rightarrow (E.)$
 $E \rightarrow E + T$

$l_{11}: \text{goto}(l_8,)$
 $F \rightarrow (E).$

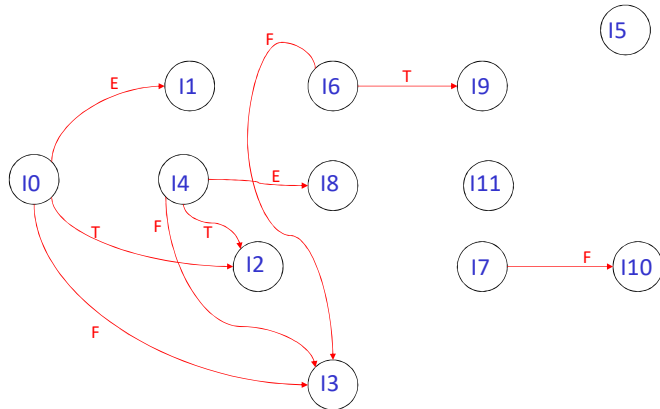
$\text{goto}(l_4, T)$ is l_2
 $\text{goto}(l_4, F)$ is l_3
 $\text{goto}(l_4, ($ is l_4
 $\text{goto}(l_4, id)$ is l_5

$\text{goto}(l_8, +)$ is l_6
 $\text{goto}(l_8, *)$ is l_7

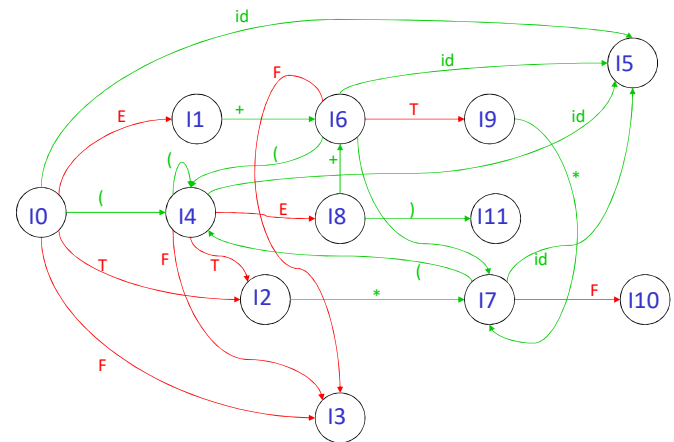
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LR(0) (?) Parse Table

- The information is still not sufficient to help us resolve shift-reduce conflict.

For example the state:

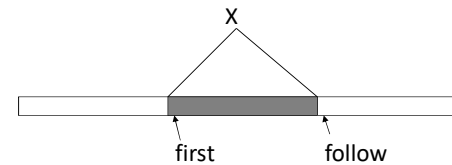
$$I_1: E' \rightarrow E.$$

$$E \rightarrow E. + T$$

- We need some more information to make decisions.

Constructing parse table

- First(α)** for a string of terminals and non terminals α is
 - Set of symbols that might begin the fully expanded (made of only tokens) version of α
- Follow(X)** for a non terminal X is
 - set of symbols that might follow the derivation of X in the input stream



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Compute first sets

- If X is a terminal symbol then $\text{first}(X) = \{X\}$
- If $X \rightarrow \epsilon$ is a production then ϵ is in $\text{first}(X)$
- If X is a non terminal and $X \rightarrow Y_1 Y_2 \dots Y_k$ is a production, then
 - if for some i , a is in $\text{first}(Y_i)$
 - and ϵ is in all of $\text{first}(Y_j)$ (such that $j < i$)
 - then a is in $\text{first}(X)$
- If ϵ is in $\text{first}(Y_1) \dots \text{first}(Y_k)$ then ϵ is in $\text{first}(X)$
- Now generalize to a string α of terminals and non-terminals

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Example

- For the expression grammar

$$\begin{aligned} E &\rightarrow T E' & E' &\rightarrow + T E' \mid \epsilon \\ T &\rightarrow F T' & T' &\rightarrow * F T' \mid \epsilon \\ F &\rightarrow (E) \mid \text{id} \end{aligned}$$

$$\begin{aligned} \text{First}(E) &= \text{First}(T) = \text{First}(F) \\ &= \{ (, \text{id} \} \end{aligned}$$

$$\begin{aligned} \text{First}(E') &= \{ +, \epsilon \} \end{aligned}$$

$$\begin{aligned} \text{First}(T') &= \{ *, \epsilon \} \end{aligned}$$

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Compute follow sets

1. Place $\$$ in $\text{follow}(S)$ // S is the start symbol
 2. If there is a production $A \rightarrow \alpha B \beta$ then everything in $\text{first}(\beta)$ (except ϵ) is in $\text{follow}(B)$
 3. If there is a production $A \rightarrow \alpha B \beta$ and $\text{first}(\beta)$ contains ϵ then everything in $\text{follow}(A)$ is in $\text{follow}(B)$
 4. If there is a production $A \rightarrow \alpha B$ then everything in $\text{follow}(A)$ is in $\text{follow}(B)$
- Last two steps have to be repeated until the follow sets converge.

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Example

- For the expression grammar
$$\begin{aligned} E &\rightarrow T E' \\ E' &\rightarrow + T E' \mid \epsilon \\ T &\rightarrow F T' \\ T' &\rightarrow * F T' \mid \epsilon \\ F &\rightarrow (E) \mid \text{id} \end{aligned}$$

$\text{follow}(E) = \text{follow}(E') = ?$
 $\text{follow}(T) = \text{follow}(T') = ?$
 $\text{follow}(F) = ?$

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Construct SLR parse table

- Construct $C = \{I_0, \dots, I_n\}$ the collection of sets of LR(0) items
- If $A \rightarrow \alpha.a\beta$ is in I_i and $\text{goto}(I_i, a) = I_j$ then $\text{action}[i, a] = \text{shift } j$
- If $A \rightarrow \alpha.$ is in I_i then $\text{action}[i, a] = \text{reduce } A \rightarrow \alpha$ for all a in $\text{follow}(A)$
- If $S' \rightarrow S.$ is in I_i then $\text{action}[i, \$] = \text{accept}$
- If $\text{goto}(I_i, A) = I_j$ then $\text{goto}[i, A] = j$ for all non terminals A
- All entries not defined are errors

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Practice Assignment

Construct SLR parse table for following grammar

$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid (E) \mid \text{digit}$

Show steps in parsing of string
 $9*5+(2+3*7)$

- Steps to be followed
 - Augment the grammar
 - Construct set of LR(0) items
 - Construct the parse table
 - Show states of parser as the given string is parsed

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Notes

- This method of parsing is called SLR (Simple LR)
- LR parsers accept LR(k) languages
 - L stands for left to right scan of input
 - R stands for rightmost derivation
 - k stands for number of lookahead token
- SLR is the simplest of the LR parsing methods. SLR is too weak to handle most languages!
- If an SLR parse table for a grammar does not have multiple entries in any cell then the grammar is unambiguous
- All SLR grammars are unambiguous
- Are all unambiguous grammars in SLR?

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Does Conflict => Ambiguity?

Example

- Consider following grammar and its SLR parse table:

$S' \rightarrow S$

$S \rightarrow L = R$

$S \rightarrow R$

$L \rightarrow *R$

$L \rightarrow id$

$R \rightarrow L$

$I_1: \text{goto}(I_0, S)$

$S' \rightarrow S.$

$I_2: \text{goto}(I_0, L)$

$S \rightarrow L = R$

$R \rightarrow L.$

$I_0: S' \rightarrow .S$

$S \rightarrow .L = R$

$S \rightarrow .R$

$L \rightarrow .*R$

$L \rightarrow .id$

$R \rightarrow .L$

Assignment (not to be submitted):
Construct rest of the items and the parse table.

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SLR parse table for the grammar

| | = | * | id | \$ | S | L | R |
|---|----|-------|----|-----|---|---|---|
| 0 | | s4 | s5 | | 1 | 2 | 3 |
| 1 | | | | acc | | | |
| 2 | | s6,r6 | | r6 | | | |
| 3 | | | | r3 | | | |
| 4 | | s4 | s5 | | | 8 | 7 |
| 5 | r5 | | | r5 | | | |
| 6 | | s4 | s5 | | | 8 | 9 |
| 7 | r4 | | | r4 | | | |
| 8 | r6 | | | r6 | | | |
| 9 | | | | r2 | | | |

The table has multiple entries in action[2,=]

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- There is both a shift and a reduce entry in action[2,=]. Therefore state 2 has a shift-reduce conflict on symbol "=", However, the grammar is not ambiguous.
- Parse id=id assuming reduce action is taken in [2,=]

| Stack | input | action |
|--------|-------|------------------------------|
| 0 | id=id | shift 5 |
| 0 id 5 | =id | reduce by $L \rightarrow id$ |
| 0 L 2 | =id | reduce by $R \rightarrow L$ |
| 0 R 3 | =id | error |

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- if shift action is taken in [2,=]

| Stack | input | action |
|----------------|---------|-------------------------------|
| 0 | id=id\$ | shift 5 |
| 0 id 5 | =id\$ | reduce by $L \rightarrow id$ |
| 0 L 2 | =id\$ | shift 6 |
| 0 L 2 = 6 | id\$ | shift 5 |
| 0 L 2 = 6 id 5 | \$ | reduce by $L \rightarrow id$ |
| 0 L 2 = 6 L 8 | \$ | reduce by $R \rightarrow L$ |
| 0 L 2 = 6 R 9 | \$ | reduce by $S \rightarrow L=R$ |
| 0 S 1 | \$ | ACCEPT |

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Another look at the grammar

- | | |
|-----------------------|--------------------------------------------------------------------------------------------|
| $S' \rightarrow S$ | • No sentential form of this grammar can start with $R=...$ |
| $S \rightarrow L = R$ | |
| $S \rightarrow R$ | • However, the reduce action in action[2,=] generates a sentential form starting with $R=$ |
| $L \rightarrow *R$ | |
| $L \rightarrow id$ | |
| $R \rightarrow L$ | • Therefore, the reduce action is incorrect |

Problems in SLR parsing

- In SLR parsing method state i calls for reduction on symbol "a", by rule $A \rightarrow \alpha$ if I_i contains $[A \rightarrow \alpha.]$ and "a" is in follow(A)
- However, when state i appears on the top of the stack, the viable prefix $\beta\alpha$ on the stack may be such that βA can not be followed by symbol "a" in any right sentential form
- Thus, the reduction by the rule $A \rightarrow \alpha$ on symbol "a" is invalid
- SLR parsers cannot remember the left context

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