

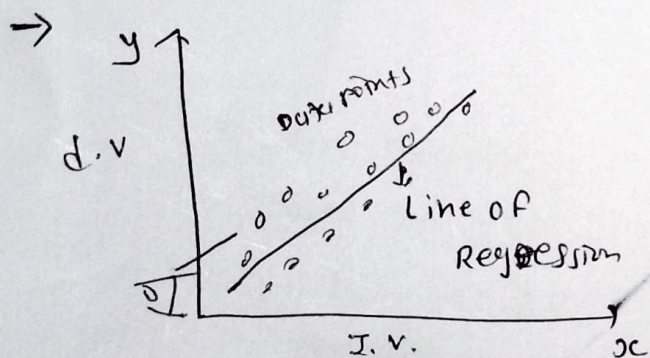
① ML

Unit-2

⇒ Linear Regression → supervised Learning base on

- Simple Linear Regression:-
- Multiple Linear Regression

- It is a statistical method that is used for predictive analysis.
- L.R makes predictions for continuous / real or numeric variables such as sales, salary, age, product, price etc.
- L.R Algo show a linear relationship between a dependent variable and one or more independent variable its called L.R.



$$y = \alpha_0 + \alpha_1 x + \epsilon$$

$$y = mx + c$$

* types of L.R

- simple:- If a single independent variable is used to predict the value of numerical dependent ~~the~~ ~~data~~ variable, then such a L.R is called simple

$$y = \alpha_0 + \alpha_1 x + \epsilon$$

α_i = Reg. coefficient

x_i = Independent

y_i = dependent

* Multi 1.2

→ If more than one independent variable is used to predict the value of numerical dependent variable

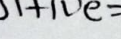
$$y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_m x_m$$

* L.R Line

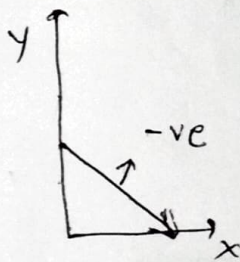
A linear line showing the relationship between the dependent and independent variable is called α -line.

two type relationship:-

- Positive L.R Line
- Negative L.R line

Positive \Rightarrow 

- dependent variable increases on y axis
- independent on x axis



- dependent decreases only
- independent increases only

* eqn of L.R

Line = $y = mx + c$ | $y = ax + b$
m-slop

$$u = \left(\frac{y(x - \bar{x}) \cdot (y - \bar{y})}{\sum (x - \bar{x})^2} \right)$$

$$a = \frac{n(\sum xy) - (\sum x)(\sum y)}{n \sum (x^2) - (\sum x)^2}$$

$$b = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2} \quad \Bigg| \quad b = \frac{1}{n} [\sum(y) - \sum(x) \cdot a]$$

* ~~ME~~

* MAE (mean absolute error)

$$= \frac{1}{N} \sum_{i=1}^N |y_i - \hat{y}|$$

* MSE (mean squared error)

$$= \frac{1}{N} \sum_{i=1}^N [(y_i - \hat{y})^2]$$

$$\rightarrow \bar{x} = \frac{\sum x}{n} \quad \bar{y} = \frac{\sum (y)}{n}$$

(mean)

\hat{y}^A (predicted)

* RMSE (Root Mean Squared Error)

$$= \sqrt{\text{MSE}}$$

$$= \sqrt{\frac{1}{N} (\sum (y - \hat{y})^2)}$$

* R² (coefficient determined) or g

$$R^2 = 1 - \frac{\sum (y_i - \hat{y})^2}{\sum (y_i - \bar{y})^2}$$

$$\sqrt{\frac{\sum (\hat{y} - \bar{y})^2}{\sum (y - \bar{y})^2}} = R^2$$

$$R^2 = \frac{\text{SSM}}{\text{SSM} + \text{SSE}}$$

* Example

X	Y	X.Y	X ²	\hat{y}	Error
1	3	3	1	2.8	0.2
2	4	8	4	4.1	0.1
3	5	15	9	5.4	0.4
4	7	28	16	6.7	0.3
$\sum x = 10$		$\sum y = 19$	54	30	1

$n = 4$ $\sigma = 4$

$y = ax + b$

$a =$ ~~2.8~~

$$a = \frac{n(\sum x.y) - \sum(x).(\sum(y))}{n(\sum x^2) - (\sum x)^2}$$

$$= \frac{4(54) - (10)(19)}{4(30) - (10)^2} = \frac{216 - 190}{120 - 100}$$

$$= \frac{26}{20} = \frac{13}{10} \boxed{1.3}$$

$$b = \frac{1}{n} [\sum(y) - \sum(x) \cdot \bar{y}]$$

$$= \frac{1}{4} [(19) - (10)(1.3)]$$

$$= \frac{1}{4} [19 - 13]$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

$$= 1.5$$

$$\underline{y = ax + b}$$

$$\therefore \boxed{y = 1.3x + 1.5} \text{ Line eq or Linear eq}$$

* Prediction x or y

$$x=1$$

$$y = (1.3)(1) + 1.5$$

$$y = 2.8$$

$$x=3$$

$$y = (1.3)(3) + 1.5$$

$$= 5.4$$

$$x=4$$

$$y = (1.3)(4) + 1.5$$

$$= 6.7$$

$$x=2$$

$$y = (1.3)(2) + 1.5$$

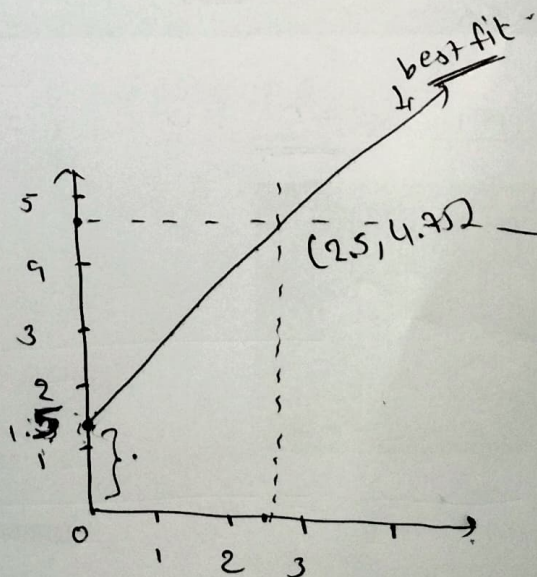
$$= 4.1$$

$$* \rightarrow \text{Error} \Rightarrow \underline{x-p} = x$$

\rightarrow best fit of given data

$$(\bar{x}, \bar{y}), \quad \bar{x} = \frac{10}{4} = 2.5$$

$$\bar{y} = \frac{19}{4} = 4.75$$



* Standard error of estimate

$$= \sqrt{\frac{\sum (\hat{y}_i - y_i)^2}{n-2}}$$

$$\begin{array}{r} (y - \hat{y})^2 \\ 0.04 \\ 0.01 \\ 0.16 \\ 0.09 \\ \hline 0.3 \end{array}$$

$$= \sqrt{\frac{0.3}{4-2}}$$

$$= \sqrt{0.15}$$

$$= \sqrt{0.0225}$$

$$= \sqrt{0.15} < 1$$

$$\sqrt{\frac{0.3}{4-2}}$$

$$\sqrt{\frac{0.3}{2}}$$

$$= \sqrt{0.15}$$

* MAE (Mean Absolute Error)

$$= \frac{1}{4} [1]$$

$$= \frac{1}{4}$$

$$= 0.25$$

$$* MSE = \frac{1}{4} [0.3]$$

$$= 0.075$$

$$* RMSE = \sqrt{0.075} \\ = 0.2738$$

$$R^2 = \frac{\sum (y_i - \bar{y})^2}{\sum (y_i - \bar{y})^2}$$

$$\begin{array}{r} -1.75 \\ -0.75 \\ +0.25 \\ +0.25 \\ \hline 0 \end{array}$$

$$\frac{\sum (\hat{y}_i - \bar{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

Roll.no	X Maths	Y ML	x.y	x ²	\hat{y}	(y- \hat{y})	(y- \bar{y})	(y- \bar{y}) ²
1	95	85	8075	9025	82.9	2.1	8	64
2	85	95	8075	7225	79.46	15.54	18	324
3	80	70	5600	6400	77.72	-7.7	-7	49
4	70	65	4550	4900	74.17	-4.17	-12	144
5	60	70	4200	3600	70.64	-0.64	-7	49
	390	385	30,500	31,750		5.13	0	630

$$\bar{y} = \frac{385}{5} = 77$$

$$a = \frac{5(30500) - (390)(385)}{5(31750) - (390)^2}$$

$$= \frac{152500 - 150150}{158750 - 152100}$$

$$= \frac{2350}{6650}$$

$$= \frac{235}{665}$$

$$= 0.353$$

$$y = ax + b$$

$$y = 0.353(x) + 49.46$$

$$R^2 = \frac{\sum (y_i - \hat{y})^2}{\sum (y_i - \bar{y})^2}$$

$$= \frac{(322.97)}{630}$$

$$= 0.5126$$

$$R^2 = 0.4874$$

$$b = \frac{1}{5} [385 - (390)(0.353)]$$

$$= \frac{1}{5} [385 - 137.67]$$

$$= \frac{247.33}{5}$$

$$= 49.466$$

($\hat{y} - \bar{y}$)	($\hat{y} - \bar{y}$) ²	(y - \hat{y}) ²
5.9	34.81	4.41
2.46	6.05	241.49
0.7	0.49	59.29
-2.83	8	17.38
-6.36	40.4	0.4096
89.75		322.9796

$$= \frac{89.75}{630}$$

$$= 0.1422$$

$$\frac{\sum (\hat{y} - \bar{y})^2}{\sum (y - \bar{y})^2}$$

* MSE

$$= \frac{1}{5} [322.97]$$

$$= 64.59$$

* RMSE

$$= \sqrt{64.59}$$

$$= 8.036$$

* Standard error estimate Residuals

$$= \sqrt{\frac{\sum (\hat{y} - y)^2}{(5-2)}}$$

$$= \sqrt{\cancel{89.75} \frac{89.75}{3}}$$

$$= \sqrt{29.91}$$

$$\boxed{= 5.46}$$