and denominator k sub 1 factorial (1 factorial) super k sub 1 ellipses k sub n factorial (n factorial) super k sub n end of fraction (D super 1 sub x u) super k sub 1 ellipses (D super n sub x u) super k sub n .

4. The resulting formatted output 21. *[HM25]* (Faa de Bruno's formula.)

Let $D_x^k u$ represent the kth derivative of a function u with respect to x. The "chain rule" states that $D_x^1 w = D_u^1 w D_x^1 u$. If we apply this to second derivatives, we find $D_x^2 w = D_u^2 w (D_x^1 u)^2 + D_u^1 w D_x^2 u$. Show that the *general formula* is

$$D_x^n w = \sum_{0 \le j \le n} \sum_{\substack{k_1 + k_2 + \cdots + k_n = j \\ k_1 + 2k_2 + \cdots + nk_n = n \\ k_1, k_2, \ldots, k_n \ge 0}} D_u^j w \frac{n!}{k_1!(1!)^{k_1} \cdots k_n!(n!)^{k_n}} (D_x^1 u)^{k_1} \cdots (D_x^n u)^{k_n}.$$