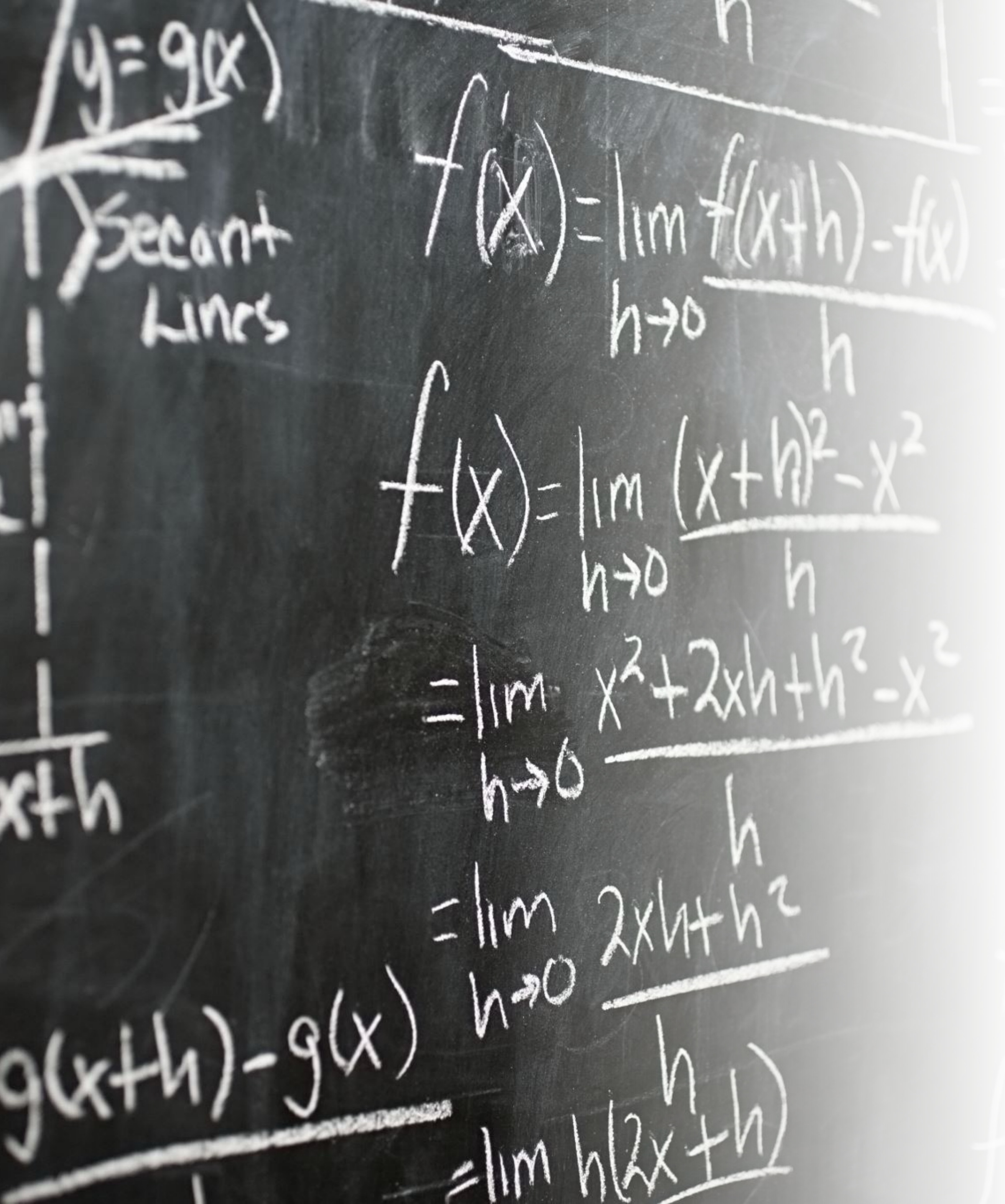


Fundamental Mathematics for DL

CSE 5172 – Deep Learning & Applications

Overview

- Tensors
- Vectors
- Matrices
- Notational convention

Tensors

Tensors are a specialized data structure that are very similar to arrays and matrices.

In PyTorch, tensors are used to encode the inputs and outputs of a model, as well as the model's parameters.

Tensors are similar to NumPy's ndarrays, except that tensors can run on GPUs.

Tensors are also optimized for automatic differentiation.

https://pytorch.org/tutorials/beginner/basics/tensorqs_tutorial.html

Vectors, Matrices, and Tensors

Scalar

(rank-0 tensor)

$x \in \mathbb{R}$

e.g.,

$x = 1$

Vector

(rank-1 tensor)

$\mathbf{x} \in \mathbb{R}^n$

but in this lecture, we will assume

$\mathbf{x} \in \mathbb{R}^{n \times 1}$

e.g.,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Matrix

(rank-2 tensor)

$\mathbf{X} \in \mathbb{R}^{m \times n}$

e.g.,

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,n} \\ x_{2,1} & x_{2,2} & \dots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \dots & x_{m,n} \end{bmatrix}$$

$$\mathbf{x}^\top = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}, \text{ where } \quad \mathbf{x}^\top \in \mathbb{R}^{1 \times n}$$

Vectors, Matrices, and Tensors

We will often use $\mathbf{X}$ as a special convention to refer to the "design matrix." That is, the matrix containing the training examples and features (inputs)

and assume the structure $\mathbf{X} \in \mathbb{R}^{n \times m}$

because n is often used to refer to the number of examples in literature across many disciplines.

$$\mathbf{X} = \begin{bmatrix} x_1^{[1]} & x_2^{[1]} & \dots & x_m^{[1]} \\ x_1^{[2]} & x_2^{[2]} & \dots & x_m^{[2]} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{[n]} & x_2^{[n]} & \dots & x_m^{[n]} \end{bmatrix}$$

E.g.,

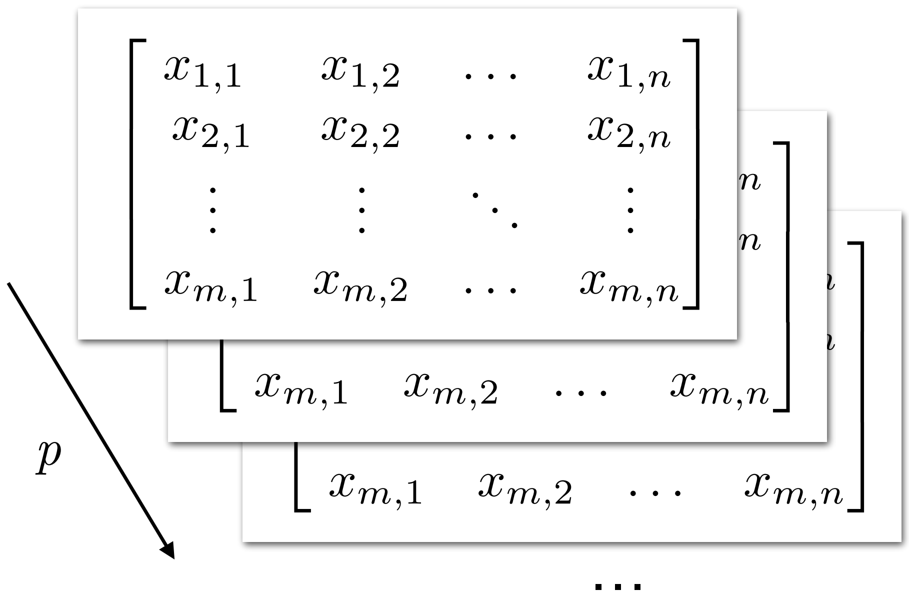
$x_2^{[1]}$ = 2nd feature value of the 1st training example

Vectors, Matrices, and Tensors

3D Tensor

(rank-3 tensor)

$$\mathrm{X} \in \mathbb{R}^{m \times n \times p}$$



Example of 3D Tensor

Single color image

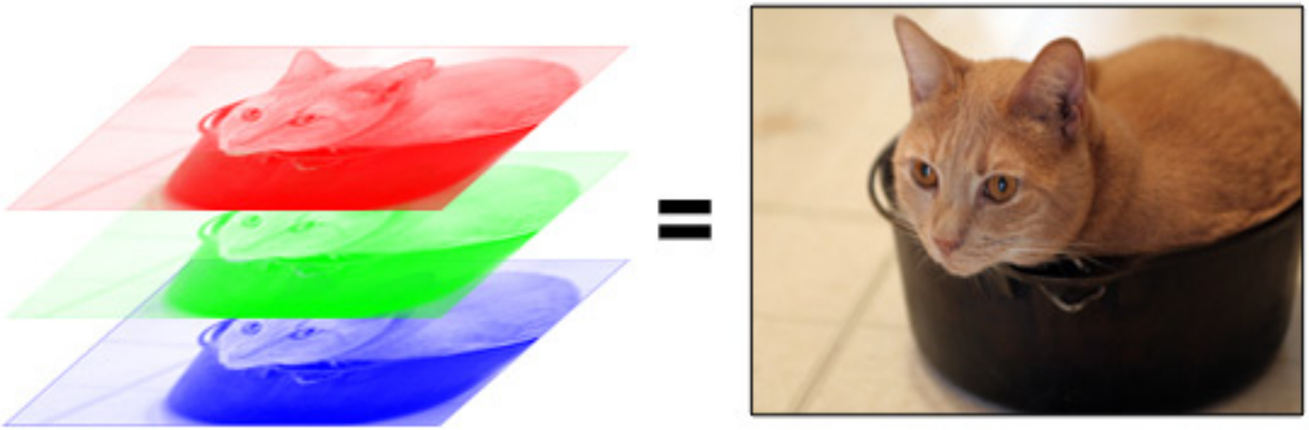


Image Source: https://code.tutsplus.com/tutorials/create-a-retro-crt-distortion-effect-using-rgb-shifting--active-3359