MANIPAL ACADEMY of HIGHER EDUCATION
(Institution of Eminence Deemed to be University)

Reg.No.									

I SEMESTER M. Tech. (CSE/CSIS)
END SEMESTER EXAMINATION, November 2024
Computational Methods and Stochastic Processes [MAT 5128]

Time: 09:30 to 12:30 PM (3 Hours) Date: 27 November 2024 MAX. MARKS: 50

Note (i) Answer ALL questions
(ii) Draw diagrams, and write equations wherever necessary

Q.1A If A and B are independent events then test whether A and B^c are independent events. Assuming that a year has 365 days, what is the probability that in a room with five people there are two of them with the same birthday?

(3 Marks; CO: 2; AEHP LO: 14; BL: 3)

Q.1B Express the following matrix A as product of elementary matrices and then describe the geometric effect of multiplication of a vector by A.

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

(3 Marks; CO: 1; AEHP LO: 14; BL: 3)

Q.1C Find the Singular Value Decomposition (SVD) of the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

(4 Marks; CO: 1; AEHP LO: 14; BL: 4)

Q.2A Find the nth power of the following matrix:

$$A = \begin{bmatrix} 1 & \text{-2} \\ \text{-2} & 1 \end{bmatrix}$$

(3 Marks; CO: 1;AEHP LO: 14;BL: 3)

Q.2B A number X is selected from $\{1, 2, 3, \ldots, 2n-1\}$. Find $E(X)$ and $V(X)$.

(3 Marks; CO: 2; AEHP LO: 14; BL: 4)

Q.2C A random variable (X, Y) is uniformly distributed over a square with vertices $(2, 0), (0, 2), (-2, 0), (0, -2)$. Find the covariance matrix and the correlation matrix.

(4 Marks; CO: 2; AEHP LO: 14;BL: 4)

Q.3A With the explicit use of matrices, fit a regression line $y = a + bx$ given the following data:

X	0	1	2	3
Y	-1	2	5	8

(3 Marks; CO: 2; AEHP LO: 14; BL: 3)

Q.3B Find the least squares solution to the inconsistent system of equations given by $AX = b$ and the error in the solution if:

$$A = \begin{bmatrix} \text{-1} & 1 \\ 1 & \text{-1} \\ 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$b = \begin{bmatrix} 8 \\ 4 \\ 16 \end{bmatrix}$$

(3 Marks; CO: 5; AEHP LO: 14; BL: 4)

Q.3C Draw the Markov chain and find the stationary distribution for the Markov chain using the graph theoretic method, given the following transition probability matrix:

$$\begin{pmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

Validate your answer with another technique.

(4 Marks; CO: 3; AEHP LO: 14; BL: 5)

Q.4A The round-off error to the first decimal place of a computer program has uniform distribution in the interval $[0.05, 0.05]$. What is the probability that the absolute error in the sum of 1500 numbers is greater than 1.5? Use the closest tabular value from the following list.

$Tabular\ \ Values : \Phi(1.24) = 0.8925; \Phi(1.34) = 0.9099; \Phi(1.44) = 0.9251.$

(3 Marks; CO: 3; AEHP LO: 14; BL: 3)

Q.4B Suppose that a server is accessed according to a Poisson stochastic process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes, the server is accessed:

(i) exactly 4 times.

(ii) more than 4 times.

(3 Marks; CO: 5; AEHP LO: 14; BL: 4)

Q.4C Consider a stochastic process with $X(t) = A\cos 5t + B\sin 5t$ where A, B are uncorrelated random variables with mean 0 and variance 1. Find

(i) $V(X(t))$.

(ii) Auto correlation coefficient $r(s,t)$.

(iii) Is the stochastic process wide sense stationary?

(4 Marks; CO: 3; AEHP LO: 14; BL: 4)

Q.5A Solve the following games given the payoff matrix:

$$\text{i)} \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

$$\text{ii)} \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$$

In each case of the above games, mention whether you would like to be the row player or the column player.

(3 Marks; CO: 5; AEHP LO: 14; BL: 4)

Q.5B With step size $h = \frac{1}{3}$, solve $u_{xx} + u_{yy} = -54xy$; $0 \leq x \leq 1$, $0 \leq y \leq 1$; $u(0,y) = u(x,0) = 0$; $u(1,y) = u(x,1) = 10$.

(3 Marks; CO: 4; AEHP LO: 14; BL: 3)

Q.5C Solve the following linear programming problem using the Simplex Method:

$$\text{Maximize } Z = 6x + 4y \text{ subject to}$$
$$x - y \leq 4;$$
$$x + y \leq 8;$$
$$x \geq 0; y \geq 0.$$

Validate your answer using the graphical method.

(4 Marks; CO: 4; AEHP LO: 14; BL: 4)