MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
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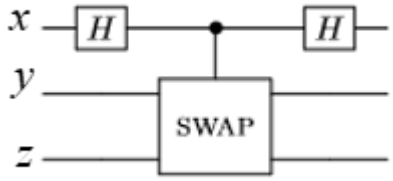
I SEMESTER M. TECH (Computer Science & Engineering)
END SEMESTER EXAMINATION, NOVEMBER 22, 2024
SUBJECT: QUANTUM COMPUTING (CSE 5115)
REVISED CREDIT SYSTEM

Time: 3 Hours (9.30 AM-12.30 AM) **MAX. MARKS: 50**
Note: Answer ALL the questions.

1A	Consider the following two qubit quantum state: $\|\psi\rangle = \frac{1}{\sqrt{30}}\left(\|00\rangle + 2i\|01\rangle - 3\|10\rangle - 4i\|11\rangle\right)$ The first qubit is measured. What is the probability that the result is 0? What is the probability that the result is 1? For each possible result, write down the post-measurement state, and calculate the probability that a measurement of the second qubit will give 0 and 1. Write down the states after the second measurement.	**5**
1B	Given $\|v\rangle = \frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$ *and* $\|w\rangle = \frac{1}{\sqrt{5}}\|0\rangle + \frac{2}{\sqrt{5}}\|1\rangle$, compute $\langle v\|w\rangle$.	**3**
1C	Given $\|\psi_2\rangle = \frac{1}{\sqrt{3}}\|0\rangle + \sqrt{\frac{2}{3}}\|1\rangle$ *and* $\|\psi_3\rangle = \frac{1}{2}\|0\rangle + \frac{\sqrt{3}}{2}\|1\rangle$, compute $\psi_2 \otimes \psi_3$.	**2**
2A	Verify with explicit calculation whether the following two qubit quantum state is entangled or not. $\|\phi\rangle = \frac{1}{\sqrt{6}}\left(\|00\rangle + i\|01\rangle + 2\|10\rangle\right)$	**4**
2B	Let H and I are Hadamard and Identity gates respectively and $\|\psi\rangle = \frac{1}{2}\left(\|00\rangle + \|01\rangle + \|10\rangle + \|11\rangle\right)$. Compute $\|\psi_1\rangle = (H \otimes I)\|\psi\rangle$	**3**
2C	Explain four postulates of quantum mechanics.	**3**
3A	Define Z gate. Design Controlled Z quantum circuit and construct its matrix representation.	**3**
3B	Define Toffoli gate. Design its quantum circuit and give its matrix representation. Implement OR gate using Toffoli gate.	**3**
3C	Imagine we can define a Unitary operator U that can copy the qubit states: $\|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(\|0\rangle + \|1\rangle\right)$ *and* $\|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(\|0\rangle - \|1\rangle\right)$: $U\|\psi_1\rangle\|0\rangle = \|\psi_1\rangle\|\psi_1\rangle$ *and* $U\|\psi_2\rangle\|0\rangle = \|\psi_2\rangle\|\psi_2\rangle$ Can U to be used to copy $\|0\rangle$. Justify your answer.	**4**
4A	Consider the following quantum circuit (H is Hadamard gate),	**4**

	 Where x denotes the first qubit, y denotes the second qubit and z denotes the third qubit respectively. Let swap denote the 2-qubit gate that swaps its input qubits (i.e., swap(y z) = z y for any y, z ∈ {0, 1}) irrespective of the value of qubit x. What are the outputs for the following inputs? 000 , 001 , 010 , 011 , 100 , 101 , 110 and 111 Hence find the matrix representation of the above quantum circuit.	
4B	Alice wishes to send Bob a message via a secure protocol. She chooses to use a private key encryption technique and decides to use the BB84 protocol to generate a provably secure private encryption key. Alice's first step is to generate a random binary string. The string she generates is *b* = 0101011101101000. Alice then encodes this as a string of quantum qubits as per the BB84 protocol, using the encoding H H H I I H I H H I I I H I I H. What is the quantum string that she generates? Alice then sends the string to Bob, who decodes using I H H I H I H H H I H I I I H I H. Alice and Bob announce their encodings publicly and retain the qubits for which they chose the same encoding. Compute the string that they retain?	**4**
4C	Suppose Alice transmits the two-bit string '00' using the superdense coding protocol and an evesdropper, Charlie, intercepts the qubit transmitted by Alice, measures it and then re-transmits to Bob. Calculate the probability that Bob correctly receives '00'.	**2**
5A	Suppose now that Alice has two qubits in a state: $\|\phi\rangle = \alpha_0\|00\rangle + \alpha_1\|01\rangle + \alpha_2\|10\rangle + \alpha_3\|11\rangle$ In addition, Alice and Bob each possess one qubit of an entangled pair of state: $\|\Phi^+\rangle = \frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$ such that Alice in possession of first qubit of the pair, while Bob in possession of the second. Alice now uses the quantum teleportation protocol to transmit to Bob the first qubit of $\|\phi\rangle$. What is the resulting joint state of the two qubit system composed of the second of Alice qubits and the qubit in Bob's possession?	**5**
5B	Let H is Hadamard gate, compute $H^{\otimes n}\|0\rangle^{\otimes n}$ and express result in summation form.	**3**
5C	Show that $\frac{1}{\sqrt{2}}\left(\|01\rangle + \|10\rangle\right)$ is a stabilizer state. and write down its stabilizer.	**2**