MANIPAL
ACADEMY of HIGHER EDUCATION
(Institution of Eminence Deemed to be University)

Reg.No.									

I SEMESTER M. Tech. (CSE/CSIS)
MID TERM EXAMINATION, October 2024
Computational Methods and Stochastic Processes [MAT 5128]

Time: 10:00 AM to 11:30 AM Date: 07 October, 2024 MAX. MARKS: 30

Note (i) Answer ALL questions
(ii) Draw diagrams, and write equations wherever necessary

Q.1 If $P(A) = \frac{1}{5}$, $P(B) = \frac{3}{5}$ and A and B are independent then find $P(A \cap B^c)$.

(2 Marks; CO: 2; BL: 2)

Q.2 Given a square matrix A, explain the geometric meaning of Eigenvalue of A and the corresponding Eigenvector of A.

(2 Marks; CO: 1; BL: 2)

Q.3 Express the following matrix A as product of elementary matrices and then describe the geometric effect of multiplication by A.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

(3 Marks; CO: 1; BL: 3)

Q.4 Show that $T(x, y) = (x + 6y, 6x + y)$ is a linear transformation. Find the matrix of linear transformation T and $T^{-1}(x, y)$.

(3 Marks; CO: 1; BL: 3)

Q.5 Find the least squares solution to the inconsistent system of equations given by $AX = b$ if

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(3 Marks; CO: 2; BL: 4)

Q.6 A two dimensional random variable (X, Y) is uniformly distributed over the region R given as follows:
$R = \{(x, y) \mid 0 < x < y < 1\}$.
Find the covariance matrix and the correlation matrix.

(3 Marks; CO: 2; BL: 4)

Q.7 Find the nth power of the following matrix:

$$\begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$$

(3 Marks; CO: 2; BL: 3)

Q.8 Fit a regression line $y = a + bx$ given the following data:

X	0	1	2	3
Y	3	5	7	9

(3 Marks; CO: 3; BL: 3)

Q.9 Find the singular value decomposition of the following matrix:

$$\begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

(4 Marks; CO: 1; BL: 4)

Q.10 A bag contains 75 fair coins (P(H) = 0.5 = P(T)) and 25 unfair coins which flip with P(H) = 0.9, P(T)= 0.1. A coin is picked at random and tossed n times and each one of the n tosses were heads.

(i) What is the probability that the picked coin is an unfair coin?

(ii) Find the least value of n that gives probability that the picked coin is unfair to be at least 80 percent.

(4 Marks; CO: 2; BL: 4)