Graph Theory-Class 1

Recap

A graph G consists of:

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(ii) a set E consisting of q number of **unordered pairs** of **distinct** elements from set V.

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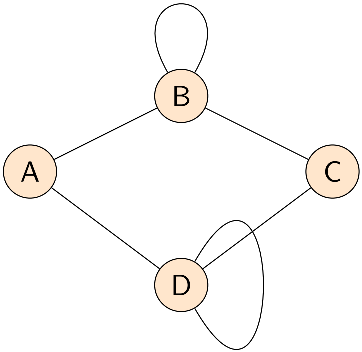
The $(1, 0)$ graph is called a **trivial graph**.

Example:



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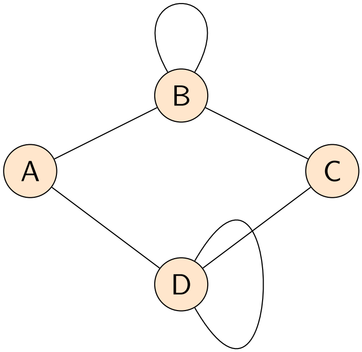

It has two loops.

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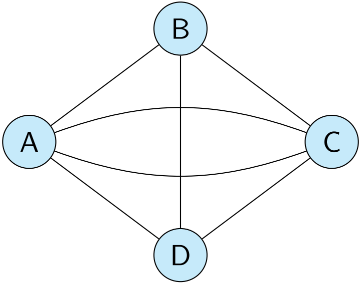

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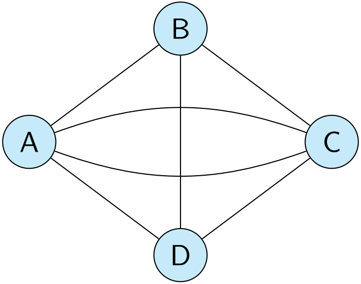

Has Multiple Edges, No Loops- We call them as **Multigraph**.

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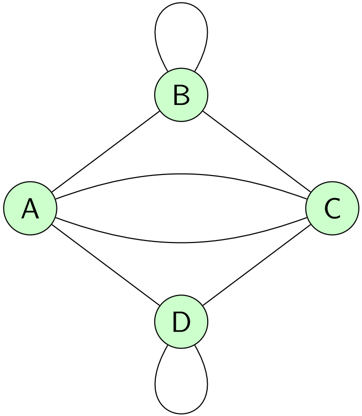

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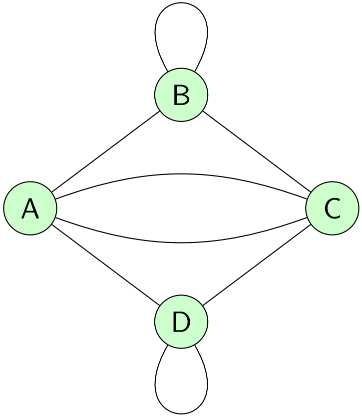

Has Loops and Multiple Edges- We call them as **Pseudograph.**

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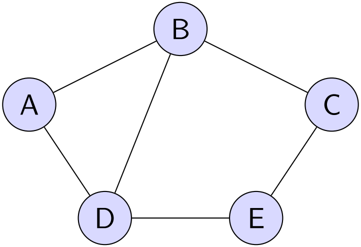
A graph with a finite number of vertices is called a **finite graph**.

A finite graph which has no loops or parallel edges is called a **simple graph**.

Example: Simple Finite Graph



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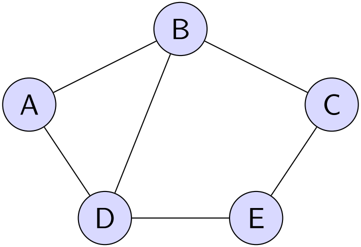

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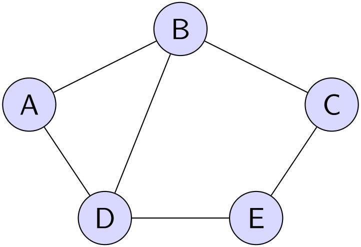

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The definition of graph we gave is actually a simple finite graph

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a. Draw a graph, count the degree of each vertex, and compute their sum.
b. Count the total number of edges.
c. Is there any relation?

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Theorem

Let $G(V, E)$ be a (p, q) graph. The sum of the degrees of the vertices of G is twice the number of edges, i.e.,

$$\sum_{v \in V} \deg(v) = 2q.$$

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Proof.

Since every edge is incident with two vertices, each edge contributes 2 to the sum of degrees of the vertices. Hence the result. $\square$

Basic Definitions

Walk: A *walk* in a graph G is

- an alternating sequence of vertices and edges:

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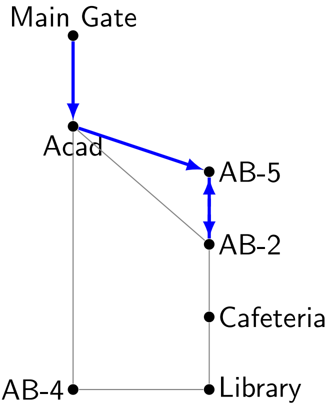
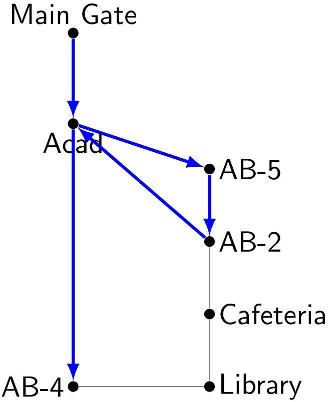
Path: A walk in which all vertices are distinct.

Cycle: A closed walk in which all vertices (except the initial and final vertex) are distinct.

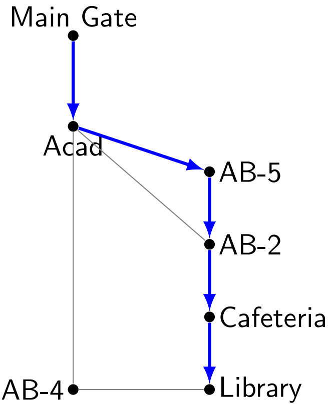
Walk



Trial



Path



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- **Length of a walk:** The number of occurrences of edges in the walk.

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- **Distance between two vertices:** The distance between vertices u and v in a graph G is the length of a shortest path joining them. If no such path exists, the distance is defined to be ∞.