Graph Theory-Class 2

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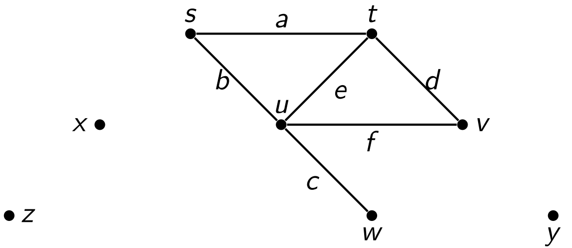

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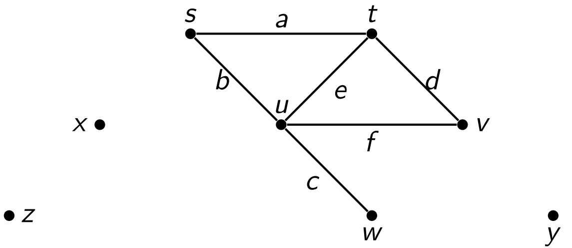

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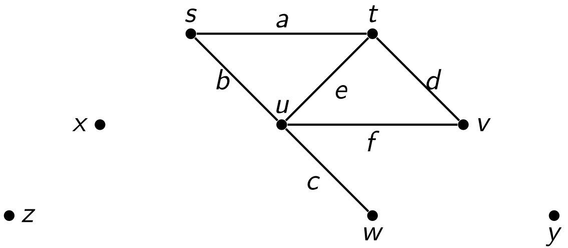

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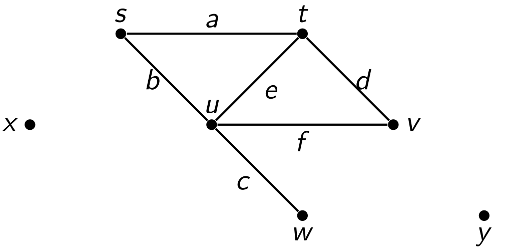

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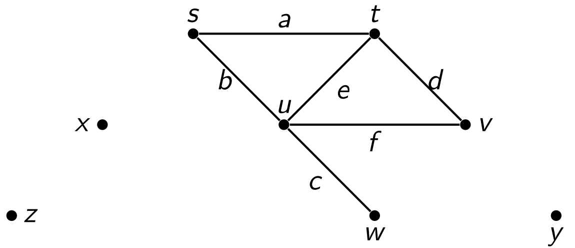

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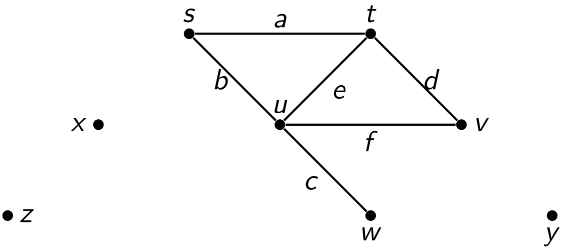

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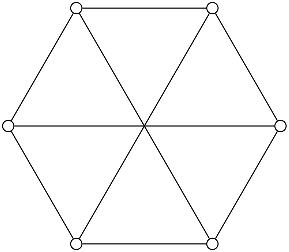

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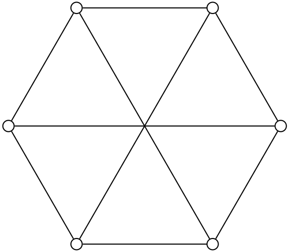

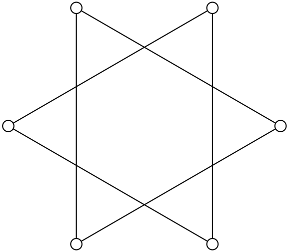

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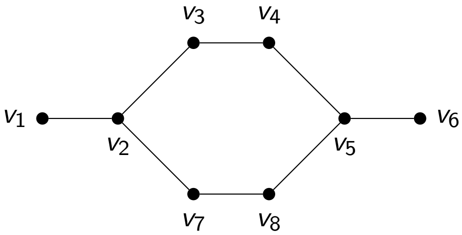

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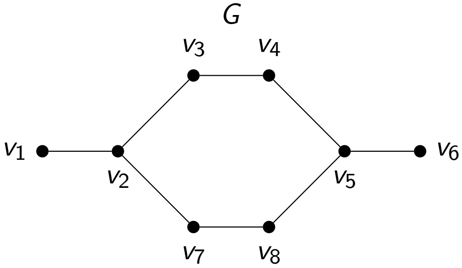

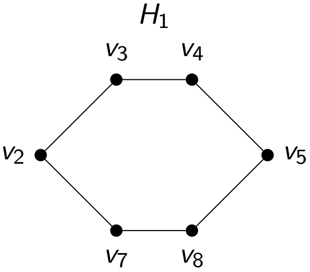

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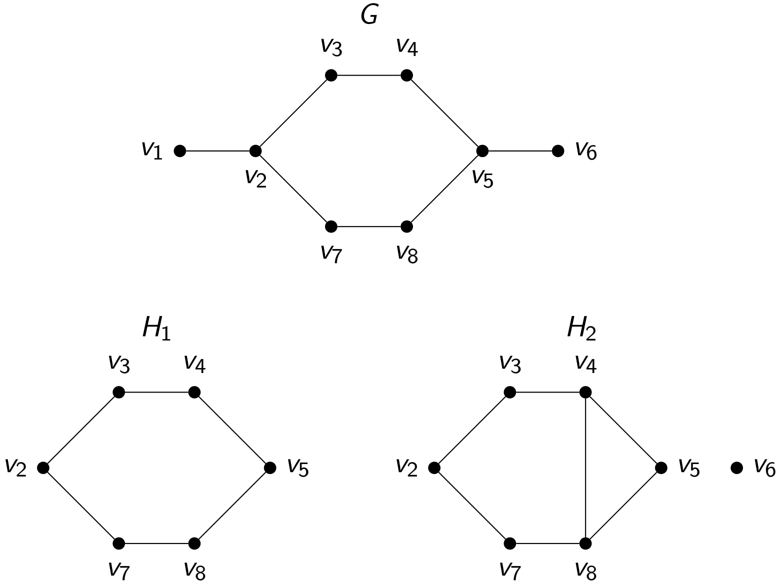

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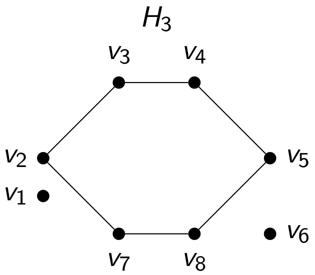

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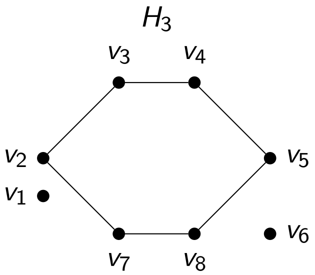
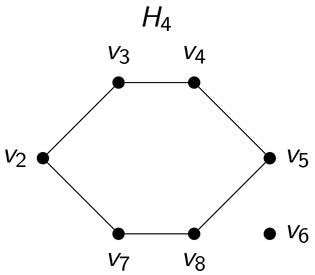
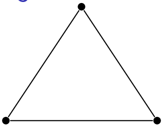




Spanning Subgraphs of C_3



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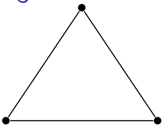

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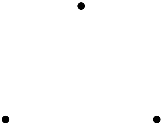

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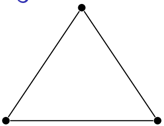

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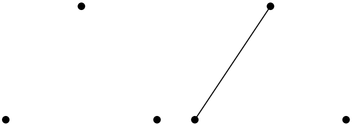

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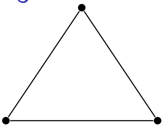

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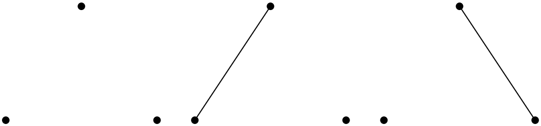

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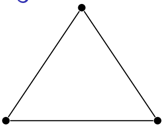

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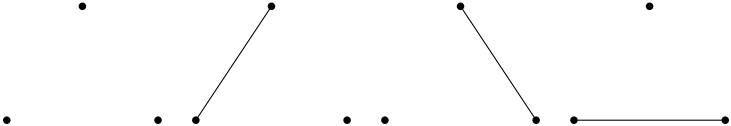

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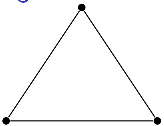

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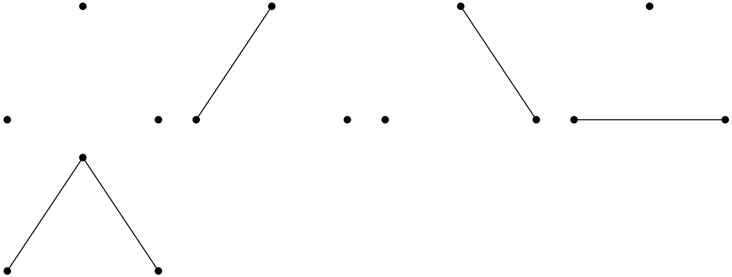

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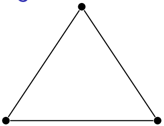

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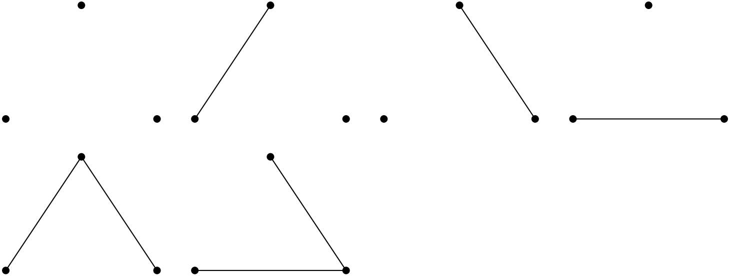

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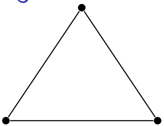

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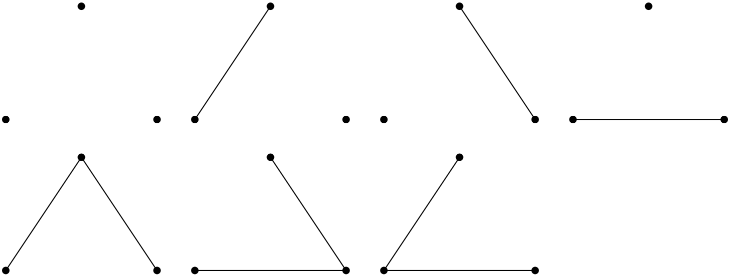

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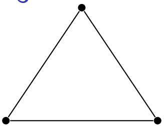

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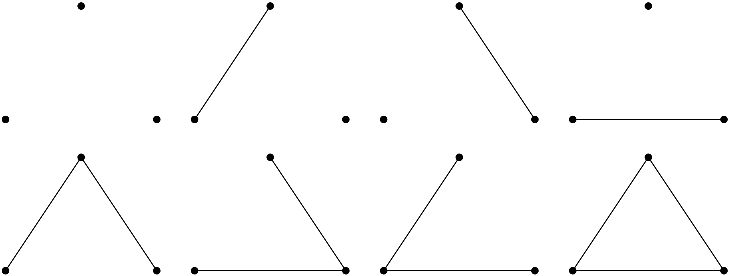

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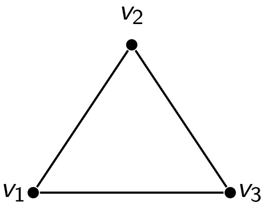
Acyclic graph: A graph with no cycles.

An example



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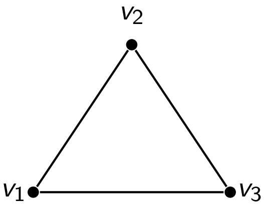

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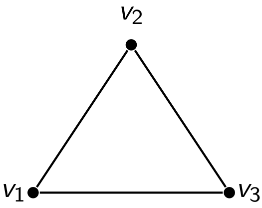

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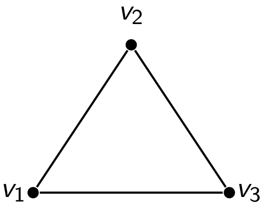

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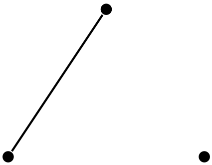

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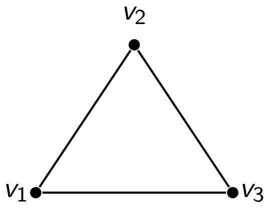

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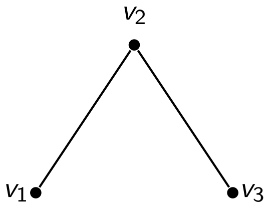

Maximal Acyclic Subgraph of C_3

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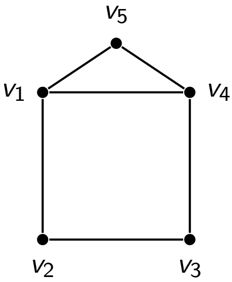

Graph G

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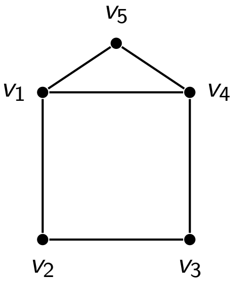

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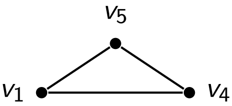

Task

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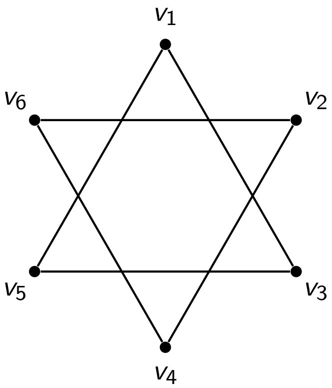
- Based on connectedness of vertices can you separate/divide the vertex set V of the following graph?



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- If G has exactly one component, G is connected; otherwise G is disconnected.

An equivalence relation on vertex set

Let $G(V, E)$ be a graph. Define a relation on $V(G)$ as follows: $u \sim v$ (u is related to v) if u and v are connected.

- This relation is an **equivalence relation** on the vertex set $V(G)$.
- Thus, there is a partition of $V(G)$ into non empty subsets $V_1, V_2, ..., V_k$ such that two vertices u and v are connected if and only if both u and v belongs to the same set V_i.
- The induced subgraphs $< V_1 >, < V_2 >, \ldots, < V_k >$ are called the (connected) components of G.
- If G has exactly one component, G is connected; otherwise G is disconnected.
- We denote the number of components of G by $\omega(G)$.