In[227]:=

Kinect Energy

Kinect Energy Link 1

In[228]:=
```
p1 = { L1 Cos[θ1[t]],
       L1 Sin[θ1[t]],
        0}
v1 = D[ p1, {t, 1}]

I1 = {{Ixx1, 0, 0},
      {0, Iyy1, 0},
      {0, 0, Izz1}}

ω1 = { 0,
       0,
       D[θ1[t], {t, 1}]}

KE1 = Simplify[1 / 2 m1 Dot [v1, v1]] + Simplify[ 1 / 2 Dot [ω1, I1 .ω1]]
```

Out[228]= $\{\text{Cos}[\theta_1[t]]\, L_1, \text{Sin}[\theta_1[t]]\, L_1, 0\}$

Out[229]= $\{-\text{Sin}[\theta_1[t]]\, L_1\, \theta_1'[t], \text{Cos}[\theta_1[t]]\, L_1\, \theta_1'[t], 0\}$

Out[230]= $\{\{Ixx_1, 0, 0\}, \{0, Iyy_1, 0\}, \{0, 0, Izz_1\}\}$

Out[231]= $\{0, 0, \theta_1'[t]\}$

Out[232]= $\frac{1}{2} Izz_1\, \theta_1'[t]^2 + \frac{1}{2} L_1^2\, m_1\, \theta_1'[t]^2$

Kinect Energy Link 2

In[233]:=
```
p2 = { L1 Cos[θ1[t]] + L2 Cos[θ1[t] + θ2[t]],
       L1 Sin[θ1[t]] + L2 Sin[θ1[t] + θ2[t]],
      0}

v2 = D[ p2, {t, 1}]

I2 = {{Ixx2, 0, 0}, {0, Iyy2, 0}, {0, 0, Izz2}}

ω2 = { 0, 0, D[θ1[t] + θ2[t], {t, 1}]}

KE2 = Simplify[1 / 2 m2 Dot [v2, v2]] + Simplify[ 1 / 2 Dot [ω2, I2 .ω2]]
```

Out[233]= $\{\text{Cos}[\theta_1[t]]\, L_1 + \text{Cos}[\theta_1[t] + \theta_2[t]]\, L_2,\ \text{Sin}[\theta_1[t]]\, L_1 + \text{Sin}[\theta_1[t] + \theta_2[t]]\, L_2,\ 0\}$

Out[234]= $\{-\text{Sin}[\theta_1[t]]\, L_1\, \theta_1'[t] - \text{Sin}[\theta_1[t] + \theta_2[t]]\, L_2\, (\theta_1'[t] + \theta_2'[t]),\ \text{Cos}[\theta_1[t]]\, L_1\, \theta_1'[t] + \text{Cos}[\theta_1[t] + \theta_2[t]]\, L_2\, (\theta_1'[t] + \theta_2'[t]),\ 0\}$

Out[235]= $\{\{\text{Ixx}_2, 0, 0\}, \{0, \text{Iyy}_2, 0\}, \{0, 0, \text{Izz}_2\}\}$

Out[236]= $\{0, 0, \theta_1'[t] + \theta_2'[t]\}$

Out[237]=
$$\frac{1}{2}\,\text{Izz}_2\,(\theta_1'[t] + \theta_2'[t])^2 + \frac{1}{2}\,\text{m}_2\left((\text{Cos}[\theta_1[t]]\, L_1\, \theta_1'[t] + \text{Cos}[\theta_1[t] + \theta_2[t]]\, L_2\, (\theta_1'[t] + \theta_2'[t]))^2 + (\text{Sin}[\theta_1[t]]\, L_1\, \theta_1'[t] + \text{Sin}[\theta_1[t] + \theta_2[t]]\, L_2\, (\theta_1'[t] + \theta_2'[t]))^2\right)$$

Kinect Energy Link 3

In[238]:=
```
p3 = { L1 Cos[θ1[t]] + L2 Cos[θ1[t] + θ2[t]] + L3 Cos[θ1[t] + θ2[t] + θ3[t]],
       L1 Sin[θ1[t]] + L2 Sin[θ1[t] + θ2[t]] + L3 Sin[θ1[t] + θ2[t] + θ3[t]],
       0}

v3 = D[ p3, {t, 1}]

I3 = {{Ixx3, 0, 0},
      {0, Iyy3, 0},
      {0, 0, Izz3}}

ω3 = { 0,
       0,
       D[θ1[t] + θ2[t] + θ3[t], {t, 1}]}

KE3 = Simplify[1 / 2 m3 Dot [v3, v3]] + Simplify[ 1 / 2 Dot [ω3, I3 .ω3]]
```

Out[238]=
$$\{\text{Cos}[\theta_1[t]]\, L_1 + \text{Cos}[\theta_1[t] + \theta_2[t]]\, L_2 + \text{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]]\, L_3,\ \text{Sin}[\theta_1[t]]\, L_1 + \text{Sin}[\theta_1[t] + \theta_2[t]]\, L_2 + \text{Sin}[\theta_1[t] + \theta_2[t] + \theta_3[t]]\, L_3,\ 0\}$$

Out[239]=
$$\{-\text{Sin}[\theta_1[t]]\, L_1\, \theta_1'[t] - \text{Sin}[\theta_1[t] + \theta_2[t]]\, L_2\, (\theta_1'[t] + \theta_2'[t]) - \text{Sin}[\theta_1[t] + \theta_2[t] + \theta_3[t]]\, L_3\, (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]),\ \text{Cos}[\theta_1[t]]\, L_1\, \theta_1'[t] + \text{Cos}[\theta_1[t] + \theta_2[t]]\, L_2\, (\theta_1'[t] + \theta_2'[t]) + \text{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]]\, L_3\, (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]),\ 0\}$$

Out[240]=
$$\{\{Ixx_3, 0, 0\}, \{0, Iyy_3, 0\}, \{0, 0, Izz_3\}\}$$

Out[241]=
$$\{0, 0, \theta_1'[t] + \theta_2'[t] + \theta_3'[t]\}$$

Out[242]=
$$\frac{1}{2} Izz_3\, (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \frac{1}{2} m_3 \Big((\text{Cos}[\theta_1[t]]\, L_1\, \theta_1'[t] + \text{Cos}[\theta_1[t] + \theta_2[t]]\, L_2\, (\theta_1'[t] + \theta_2'[t]) + \text{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]]\, L_3\, (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2 + (\text{Sin}[\theta_1[t]]\, L_1\, \theta_1'[t] + \text{Sin}[\theta_1[t] + \theta_2[t]]\, L_2\, (\theta_1'[t] + \theta_2'[t]) + \text{Sin}[\theta_1[t] + \theta_2[t] + \theta_3[t]]\, L_3\, (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2\Big)$$

Total Kinect Energy

In[243]:=
```
KE = KE1 + KE2 + KE3
```

Out[243]=
$$\frac{1}{2} Izz_1 \theta_1'[t]^2 + \frac{1}{2} L_1^2 m_1 \theta_1'[t]^2 + \frac{1}{2} Izz_2 (\theta_1'[t] + \theta_2'[t])^2 + \frac{1}{2} m_2 \left((\text{Cos}[\theta_1[t]] L_1 \theta_1'[t] + \text{Cos}[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))^2 + (\text{Sin}[\theta_1[t]] L_1 \theta_1'[t] + \text{Sin}[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))^2\right) + \frac{1}{2} Izz_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \frac{1}{2} m_3 \left((\text{Cos}[\theta_1[t]] L_1 \theta_1'[t] + \text{Cos}[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \text{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2 + (\text{Sin}[\theta_1[t]] L_1 \theta_1'[t] + \text{Sin}[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \text{Sin}[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2\right)$$

Potential Energy

Potential Energy Link 1

In[244]:=
```
PE1 = m1 g L1 Sin[θ1[t]]
```

Out[244]=
$$g \, \text{Sin}[\theta_1[t]] \, L_1 \, m_1$$

Potential Energy Link 2

In[245]:=
```
PE2 = m2 g (L1 Sin[θ1[t]] + L2 Sin[θ1[t] + θ2[t]])
```

Out[245]=
$$g \, (\text{Sin}[\theta_1[t]] \, L_1 + \text{Sin}[\theta_1[t] + \theta_2[t]] \, L_2) \, m_2$$

Potential Energy Link 3

In[246]:=
```
PE3 = m3 g (L1 Sin[θ1[t]] + L2 Sin[θ1[t] + θ2[t]] + L3 Sin[θ1[t] + θ2[t] + θ3[t]])
```

Out[246]=
$$g \, (\text{Sin}[\theta_1[t]] \, L_1 + \text{Sin}[\theta_1[t] + \theta_2[t]] \, L_2 + \text{Sin}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \, L_3) \, m_3$$

Total Potential Energy

In[247]:=
```
PE = PE1 + PE2 + PE3
```

Out[247]=
$$g \, \text{Sin}[\theta_1[t]] \, L_1 \, m_1 + g \, (\text{Sin}[\theta_1[t]] \, L_1 + \text{Sin}[\theta_1[t] + \theta_2[t]] \, L_2) \, m_2 + g \, (\text{Sin}[\theta_1[t]] \, L_1 + \text{Sin}[\theta_1[t] + \theta_2[t]] \, L_2 + \text{Sin}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \, L_3) \, m_3$$

Lagrangian Partial Derivatives

In[248]:=
```
(*Partial first order derivative of L with respect to θᵢ'*)
DLθ1' = D[KE - PE, {θ1', 1}]
DLθ2' = D[KE - PE, {θ2', 1}]
DLθ3' = D[KE - PE, {θ3', 1}]

(*Partial first order derivative of DLθᵢ' with respect to t, which is time*)
DLT1 = D[DLθ1', {t, 1}]
DLT2 = D[DLθ2', {t, 1}]
DLT3 = D[DLθ3', {t, 1}]

(*Partial first order derivative of L with respect to θᵢ*)
DLθ1 = D[KE - PE, {θ1, 1}]
DLθ2 = D[KE - PE, {θ2, 1}]
DLθ3 = D[KE - PE, {θ3, 1}]

(*τᵢ as defined by the required derivatives of the Lagrangian*)
τ1 = DLT1 - DLθ1
τ2 = DLT2 - DLθ2
τ3 = DLT3 - DLθ3
```

Out[248]=
$$1[t]\,Izz_1\,\theta_1'[t] + 1[t]\,L_1^2\,m_1\,\theta_1'[t] + 1[t]\,Izz_2\,(\theta_1'[t]+\theta_2'[t]) + \frac{1}{2}m_2\,(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2)\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])) + 2\,(1[t]\,\text{Sin}[\theta_1[t]]\,L_1 + 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2)\,(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))) + 1[t]\,Izz_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) + \frac{1}{2}m_3\,(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) + \text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])) + 2\,(1[t]\,\text{Sin}[\theta_1[t]]\,L_1 + 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2 + 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\,(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) + \text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])))$$

Out[249]=
$$1[t]\,Izz_2\,(\theta_1'[t]+\theta_2'[t]) + \frac{1}{2}m_2\,(2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])) + 2\times 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))) + 1[t]\,Izz_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) + \frac{1}{2}m_3\,(2\,(1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) + \text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])) + 2\,(1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2 + 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\,(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) + \text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])))$$

Out[250]=
$$1[t]\,Izz_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])+\frac{1}{2}m_3\,(2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+2\times 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])))$$

Out[251]=
$$1[t]\,Izz_1\,\theta_1''[t]+1[t]\,L_1^2\,m_1\,\theta_1''[t]+1[t]\,Izz_2\,(\theta_1''[t]+\theta_2''[t])+\frac{1}{2}m_2\,\Big(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))\,(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))+2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))\,(-1[t]\,\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]-1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))+2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1+1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2)\,\Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2-\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2+\text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])\Big)+2\,(1[t]\,\text{Sin}[\theta_1[t]]\,L_1+1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2)\,\Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2+\text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])\Big)\Big)+1[t]\,Izz_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\frac{1}{2}m_3\,\Big(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\,(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\,(-1[t]\,\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]-1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])-1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1+1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2+1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\,\Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2-\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2-\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)+2\,(1[t]\,\text{Sin}[\theta_1[t]]\,L_1+1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2+1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\,\Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2+\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)\Big)$$

Out[252]=
$$
\begin{aligned}
&1[t]\,Izz_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\frac{1}{2}m_2\Big(-2\,1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))+2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\\
&\quad(\theta_1'[t]+\theta_2'[t])\,(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))+\\
&\quad 2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,\Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2-\\
&\quad\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2+\text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t]+\\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])\Big)+2\times 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\\
&\quad L_2\,\Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2+\\
&\quad\text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])\Big)\Big)+\\
&1[t]\,Izz_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\\
&\frac{1}{2}\\
&\quad m_3\\
&\quad\Big(2\,(1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad\quad 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\quad\quad(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+\\
&\quad\quad 2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\quad\quad(-1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])-\\
&\quad\quad 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+\\
&\quad\quad 2\,(1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2+1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\\
&\quad\quad\Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2-\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2-\\
&\quad\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\\
&\quad\quad\text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\quad\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)+\\
&\quad\quad 2\,(1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2+1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\\
&\quad\quad\Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2+\\
&\quad\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\\
&\quad\quad\text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\quad\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)\Big)
\end{aligned}
$$

Out[253]=
$$
\begin{aligned}
&1[t]\,Izz_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\\
&\frac{1}{2}m_3\Big(-2\,1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])\\
&\quad(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+\\
&\quad 2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])\\
&\quad(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+\\
&\quad 2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,\Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2-\text{Sin}[\theta_1[t]+\theta_2[t]]\\
&\quad\quad L_2\,(\theta_1'[t]+\theta_2'[t])^2-\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\\
&\quad\text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)+\\
&\quad 2\times 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,\Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2+\text{Cos}[\theta_1[t]+\theta_2[t]]\\
&\quad\quad L_2\,(\theta_1'[t]+\theta_2'[t])^2+\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\\
&\quad\text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)\Big)
\end{aligned}
$$

Out[254]=
$$-g\,1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,m_1 - g\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2)\,m_2 -$$
$$g\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)$$
$$m_3 + \frac{1}{2}\,m_2\,(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))$$
$$(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])) +$$
$$2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))$$
$$(-1[t]\,\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] - 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))) +$$
$$\frac{1}{2}\,m_3\,(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +$$
$$1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))$$
$$(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +$$
$$\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])) +$$
$$2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +$$
$$\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))$$
$$(-1[t]\,\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] - 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) -$$
$$1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])))$$

Out[255]=
$$-g\,1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,m_2 -$$
$$g\,(1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\,m_3 +$$
$$\frac{1}{2}\,m_2\,(-2\,1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])$$
$$(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])) +$$
$$2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])$$
$$(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))) +$$
$$\frac{1}{2}\,m_3\,(2\,(1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +$$
$$1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))$$
$$(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +$$
$$\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])) +$$
$$2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +$$
$$\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))$$
$$(-1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) -$$
$$1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])))$$

Out[256]=
$$-g\,1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,m_3 +$$
$$\frac{1}{2}\,m_3\,(-2\,1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])$$
$$(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +$$
$$\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])) +$$
$$2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])$$
$$(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +$$
$$\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])))$$

Out[257]=

$$
\begin{aligned}
&g\,1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,m_1 + g\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2)\,m_2 + \\
&g\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\\
&\quad m_3 - \frac{1}{2}\,m_2\,(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))\\
&\qquad (\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])) +\\
&\qquad 2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))\\
&\qquad (-1[t]\,\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] - 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))) -\\
&\frac{1}{2}\,m_3\,(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +\\
&\qquad 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\qquad (\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +\\
&\qquad \text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])) +\\
&\qquad 2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +\\
&\qquad \text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\qquad (-1[t]\,\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] - 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) -\\
&\qquad 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))) +\\
&1[t]\,Izz_1\,\theta_1''[t] + 1[t]\,L_1^2\,m_1\,\theta_1''[t] + 1[t]\,Izz_2\,(\theta_1''[t]+\theta_2''[t]) +\\
&\frac{1}{2}\,m_2\\
&\quad \Big(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))\\
&\qquad (\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])) +\\
&\qquad 2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))\\
&\qquad (-1[t]\,\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] - 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])) +\\
&\qquad 2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2)\\
&\qquad \Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2 - \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2 +\\
&\qquad \text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])\Big) +\\
&\qquad 2\,(1[t]\,\text{Sin}[\theta_1[t]]\,L_1 + 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2)\\
&\qquad \Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2 + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2 +\\
&\qquad \text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])\Big)\Big) +\\
&1[t]\,Izz_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t]) +\\
&\frac{1}{2}\\
&\quad m_3\\
&\quad \Big(2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +\\
&\qquad 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\qquad (\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +\\
&\qquad \text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])) +\\
&\qquad 2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) +\\
&\qquad \text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\qquad (-1[t]\,\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t] - 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]) -\\
&\qquad 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])) +\\
&\qquad 2\,(1[t]\,\text{Cos}[\theta_1[t]]\,L_1 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2 + 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\\
&\qquad\quad L_3)\,\Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2 - \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2 -\\
&\qquad \text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2 +\\
&\qquad \text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t] + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t]) +\\
&\qquad \text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big) + 2\,(1[t]\,\text{Sin}[\theta_1[t]]\,L_1 +\\
&\qquad 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2 + 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\\
&\qquad \Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2 + \text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2 +\\
&\qquad \text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2 +\\
&\qquad \text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t] + \text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t]) +\\
&\qquad \text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)\Big)
\end{aligned}
$$

Out[258]=

$$
\begin{aligned}
&g\,1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,m_2+\\
&g\,(1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2+1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\,m_3-\\
&\frac{1}{2}\,m_2\,(-2\,1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])\\
&\quad(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))+\\
&\quad 2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])\\
&\quad(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])))-\\
&\frac{1}{2}\,m_3\,(2\,(1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\quad(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+\\
&\quad 2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\quad(-1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])-\\
&\quad 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])))+\\
&1[t]\,Izz_2\,(\theta_1''[t]+\theta_2''[t])+\frac{1}{2}\,m_2\,\Big(-2\,1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])\\
&\quad(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))+\\
&\quad 2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])\\
&\quad(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t]))+\\
&\quad 2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,\Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2-\\
&\quad\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2+\text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t]+\\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])\Big)+2\times 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\\
&\quad L_2\,\Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2+\\
&\quad\text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])\Big)\Big)+\\
&1[t]\,Izz_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\\
&\frac{1}{2}\,m_3\,\Big(2\,(1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\quad(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+\\
&\quad 2\,(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\\
&\quad(-1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])-\\
&\quad 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+\\
&\quad 2\,(1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2+1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\\
&\quad\Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2-\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2-\\
&\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\\
&\quad\text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)+\\
&\quad 2\,(1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2+1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3)\\
&\quad\Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])^2+\\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\\
&\quad\text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)\Big)
\end{aligned}
$$

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$$
\begin{aligned}
&g\,1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,m_3 - \\
&\frac{1}{2}\,m_3\,(-2\,1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) \\
&\quad(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+ \\
&\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+ \\
&\quad 2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) \\
&\quad(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+ \\
&\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])))+ \\
&1[t]\,Izz_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+ \\
&\frac{1}{2} \\
&\quad m_3 \\
&\quad\Big(-2\,1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) \\
&\quad\quad(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+ \\
&\quad\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+ \\
&\quad\quad 2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) \\
&\quad\quad(\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1'[t]+\theta_2'[t])+ \\
&\quad\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))+ \\
&\quad\quad 2\times 1[t]\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,\Big(-\text{Sin}[\theta_1[t]]\,L_1\,\theta_1'[t]^2-\text{Sin}[\theta_1[t]+\theta_2[t]] \\
&\quad\quad\quad L_2\,(\theta_1'[t]+\theta_2'[t])^2-\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+ \\
&\quad\quad\quad\text{Cos}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Cos}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+ \\
&\quad\quad\quad\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)+ \\
&\quad\quad 2\times 1[t]\,\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,\Big(\text{Cos}[\theta_1[t]]\,L_1\,\theta_1'[t]^2+\text{Cos}[\theta_1[t]+\theta_2[t]] \\
&\quad\quad\quad L_2\,(\theta_1'[t]+\theta_2'[t])^2+\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+ \\
&\quad\quad\quad\text{Sin}[\theta_1[t]]\,L_1\,\theta_1''[t]+\text{Sin}[\theta_1[t]+\theta_2[t]]\,L_2\,(\theta_1''[t]+\theta_2''[t])+ \\
&\quad\quad\quad\text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,L_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)\Big)
\end{aligned}
$$