Kinect Energy

Kinect Energy Link I

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\begin{split} & \text{In}[228] = \text{ } \textbf{p}_1 = \{ \text{ } \textbf{L}_1 \text{ } \text{Cos}[\theta_1[t]] \text{ }, \\ & \text{ } \textbf{L}_1 \text{ } \text{Sin}[\theta_1[t]] \text{ }, \\ & 0 \} \\ & \textbf{v}_1 = \textbf{D}[\textbf{p}_1, \{\textbf{t}, 1\}] \\ & \textbf{I}_1 = \{ \{ \textbf{Ixx}_1, \ 0, \ 0 \}, \\ & \{ 0, \ \textbf{Iyy}_1, \ 0 \}, \\ & \{ 0, \ 0, \ \textbf{Izz}_1 \} \} \\ & \boldsymbol{\omega}_1 = \{ 0, \\ & 0, \\ & D[\theta_1[t], \{\textbf{t}, 1\}] \} \\ & \textbf{KE}_1 = \textbf{Simplify}[1/2 \ \textbf{m}_1 \text{ Dot}[\textbf{v}_1, \textbf{v}_1]] + \textbf{Simplify}[1/2 \text{ Dot}[\boldsymbol{\omega}_1, \textbf{I}_1, \boldsymbol{\omega}_1]] \\ & \text{Out}[228] = \{ \textbf{Cos}[\theta_1[t]] \ \textbf{L}_1, \ \textbf{Sin}[\theta_1[t]] \ \textbf{L}_1, \ 0 \} \\ & \text{Out}[229] = \{ -\textbf{Sin}[\theta_1[t]] \ \textbf{L}_1 \ \theta_1'[t], \ \textbf{Cos}[\theta_1[t]] \ \textbf{L}_1 \ \theta_1'[t], \ 0 \} \\ & \text{Out}[230] = \{ \{ \textbf{Ixx}_1, \ 0, \ 0 \}, \ \{ 0, \ \textbf{Iyy}_1, \ 0 \}, \ \{ 0, \ 0, \ \textbf{Izz}_1 \} \} \\ & \text{Out}[231] = \{ 0, \ 0, \ \theta_1'[t] \}^2 + \frac{1}{2} \ \textbf{L}_1^2 \ \textbf{m}_1 \ \theta_1'[t]^2 \\ \end{split}
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Kinect Energy Link 2

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ln[233] = p_2 = \{ L_1 Cos[\theta_1[t]] + L_2 Cos[\theta_1[t]] + \theta_2[t] \},
                                                                                               L_1 Sin[\theta_1[t]] + L_2 Sin[\theta_1[t] + \theta_2[t]],
                                            v_2 = D[p_2, \{t, 1\}]
                                              I_2 = \{ \{Ixx_2, 0, 0\}, \{0, Iyy_2, 0\}, \{0, 0, Izz_2\} \}
                                            \omega_2 = \{ \; \mathsf{0} \; , \; \; \mathsf{0} \; , \; \; \mathsf{D} [\theta_1[\mathsf{t}] \; + \; \theta_2[\mathsf{t}] \; , \; \{\mathsf{t} \; , \; 1\}] \; \}
                                            \texttt{KE}_2 = \texttt{Simplify} [\texttt{1/2} \ \texttt{m}_2 \ \texttt{Dot} \ [\texttt{v}_2 \,,\, \texttt{v}_2] \,] \ + \\ \texttt{Simplify} [\texttt{1/2} \ \texttt{Dot} \ [\omega_2 \,,\, \texttt{I}_2 \,.\omega_2] \,]
  \text{Out} [233] = \left\{ \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \right. \\ \left. \text{L}_1 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] + \theta_2 \left[ t \right] \right] \right. \\ \left. \text{L}_2, \right. \\ \left. \text{Sin} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_1 + \text{Sin} \left[ \theta_1 \left[ t \right] + \theta_2 \left[ t \right] \right] \right. \\ \left. \text{L}_2, \right. \\ \left. \text{Otherwise} \right] \\ \left. \text{L}_2, \right. \\ \left. \text{Sin} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_1 + \text{Sin} \left[ \theta_1 \left[ t \right] + \theta_2 \left[ t \right] \right] \right. \\ \left. \text{L}_2, \right. \\ \left. \text{Otherwise} \right. \\ \left. \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_2 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_3 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_4 + \text{Cos} \left[ \theta_1 \left[ t \right] \right] \right] \right. \\ \left. \text{L}_5 + \text{L}_5
 \text{Out} [234] = \left\{-\sin\left[\theta_1\left[t\right]\right] \; \text{L}_1 \; \theta_1{}'\left[t\right] \; - \\ \sin\left[\theta_1\left[t\right] \; + \; \theta_2\left[t\right]\right] \; \text{L}_2 \; \left(\theta_1{}'\left[t\right] \; + \; \theta_2{}'\left[t\right]\right) \text{,} \right\}
                                                       \texttt{Cos}[\theta_{1}[\texttt{t}]] \; \texttt{L}_{1} \; \theta_{1}{'}[\texttt{t}] \; + \; \texttt{Cos}[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]] \; \texttt{L}_{2} \; (\theta_{1}{'}[\texttt{t}] \; + \; \theta_{2}{'}[\texttt{t}]) \; \text{, 0} \}
 Out[235]= \{\{Ixx_2, 0, 0\}, \{0, Iyy_2, 0\}, \{0, 0, Izz_2\}\}
 Out[236]= \{0, 0, \theta_1'[t] + \theta_2'[t]\}
Out[237]= \frac{1}{2} Izz<sub>2</sub> (\theta_1'[t] + \theta_2'[t])^2 +
                                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))^2)
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Kinect Energy Link 3

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\ln[238] = p_3 = \{ L_1 \cos[\theta_1[t]] + L_2 \cos[\theta_1[t]] + \theta_2[t] \} + L_3 \cos[\theta_1[t]] + \theta_2[t] + \theta_3[t] \},
                          L_1 \sin[\theta_1[t]] + L_2 \sin[\theta_1[t]] + \theta_2[t]] + L_3 \sin[\theta_1[t]] + \theta_2[t] + \theta_3[t]],
            v_3 = D[p_3, \{t, 1\}]
             I_3 = \{ \{Ixx_3, 0, 0\}, 
                         {0, Iyy3, 0},
                         {0, 0, Izz<sub>3</sub>}}
            \omega_3 = \{ 0,
                          D[\theta_1[t] + \theta_2[t] + \theta_3[t], \{t, 1\}]
            KE_3 = Simplify[1/2 m_3 Dot[v_3, v_3]] + Simplify[1/2 Dot[\omega_3, I_3.\omega_3]]
 \text{Out} [238] = \left\{ \text{Cos} \left[\theta_1 \left[\texttt{t}\right]\right] \; \mathsf{L}_1 + \text{Cos} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \; \mathsf{L}_2 + \text{Cos} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right] + \theta_3 \left[\texttt{t}\right]\right] \; \mathsf{L}_3 \text{, } \right\} 
               Sin[\theta_1[t]] L_1 + Sin[\theta_1[t] + \theta_2[t]] L_2 + Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3, 0
Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]),
               Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                 \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right) \text{, 0} \}
Out[240]= \{\{Ixx_3, 0, 0\}, \{0, Iyy_3, 0\}, \{0, 0, Izz_3\}\}
Out[241]= \{0, 0, \theta_1'[t] + \theta_2'[t] + \theta_3'[t]\}
Out[242]= \frac{1}{2} Izz<sub>3</sub> (\Theta_1'[t] + \Theta_2'[t] + \Theta_3'[t])^2 +
               \frac{1}{2} m_3 \left( (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \frac{1}{2} m_3 \left( (\cos[\theta_1[t]] L_1 \theta_1'[t] + \frac{1}{2} \cos[\theta_1[t]] L_2 (\theta_1'[t]) + \frac{1}{2} \cos[\theta_1[t]] \right) \right)
                              Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))^{2} +
                       (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                              Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2
```

Total Kinect Energy

$$\begin{aligned} & \text{In}[243] = \ \ \textbf{KE} \ = \ \textbf{KE}_1 + \ \textbf{KE}_2 + \ \textbf{KE}_3 \\ & \text{Out}[243] = \ \ \frac{1}{2} \ \text{Izz}_1 \ \theta_1{}'[t]^2 + \frac{1}{2} \ \text{L}_1^2 \ \text{m}_1 \ \theta_1{}'[t]^2 + \frac{1}{2} \ \text{Izz}_2 \ (\theta_1{}'[t] + \theta_2{}'[t])^2 + \\ & \frac{1}{2} \ \text{m}_2 \ \left((\text{Cos}[\theta_1[t]] \ \text{L}_1 \ \theta_1{}'[t] + \text{Cos}[\theta_1[t]] + \theta_2[t]) \ \text{L}_2 \ (\theta_1{}'[t] + \theta_2{}'[t]))^2 + \\ & (\text{Sin}[\theta_1[t]] \ \text{L}_1 \ \theta_1{}'[t] + \text{Sin}[\theta_1[t]] + \theta_2[t]) \ \text{L}_2 \ (\theta_1{}'[t] + \theta_2{}'[t]))^2 \right) + \\ & \frac{1}{2} \ \text{Izz}_3 \ (\theta_1{}'[t] + \theta_2{}'[t] + \theta_3{}'[t])^2 + \\ & \frac{1}{2} \ \text{m}_3 \ \left((\text{Cos}[\theta_1[t]] \ \text{L}_1 \ \theta_1{}'[t] + \text{Cos}[\theta_1[t]] + \theta_2[t]] \ \text{L}_2 \ (\theta_1{}'[t] + \theta_2{}'[t]) + \\ & \text{Cos}[\theta_1[t]] + \theta_2[t] + \theta_3[t]] \ \text{L}_3 \ (\theta_1{}'[t] + \theta_2{}'[t] + \theta_3{}'[t]))^2 + \\ & (\text{Sin}[\theta_1[t]] \ \text{L}_1 \ \theta_1{}'[t] + \text{Sin}[\theta_1[t] + \theta_2[t]] \ \text{L}_2 \ (\theta_1{}'[t] + \theta_2{}'[t]) + \\ & \text{Sin}[\theta_1[t]] + \theta_2[t] + \theta_3[t]] \ \text{L}_3 \ (\theta_1{}'[t] + \theta_2{}'[t] + \theta_3{}'[t]))^2 \right) \end{aligned}$$

Potential Energy

Potential Energy Link I

```
\ln[244]:= \mathbf{PE_1} = \mathbf{m_1} \mathbf{g} \mathbf{L_1} \mathbf{Sin} [\theta_1[\mathbf{t}]]
Out[244]= g Sin[\theta_1[t]] L_1 m_1
```

Potential Energy Link 2

```
\ln[245] := PE_2 = m_2 g (L_1 Sin[\theta_1[t]] + L_2 Sin[\theta_1[t] + \theta_2[t]])
Out[245]= g (Sin[\theta_1[t]] L<sub>1</sub> + Sin[\theta_1[t] + \theta_2[t]] L<sub>2</sub>) m<sub>2</sub>
```

Potential Energy Link 3

```
\ln[246] = PE_3 = m_3 g \left( L_1 \sin[\theta_1[t]] + L_2 \sin[\theta_1[t]] + \theta_2[t] \right) + L_3 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t] \right)
\texttt{Out}[246] = \texttt{g} \left( \texttt{Sin} \left[ \theta_1 \left[ \mathsf{t} \right] \right] \, \mathsf{L}_1 \, + \, \texttt{Sin} \left[ \theta_1 \left[ \mathsf{t} \right] \, + \, \theta_2 \left[ \mathsf{t} \right] \right] \, \mathsf{L}_2 \, + \, \texttt{Sin} \left[ \theta_1 \left[ \mathsf{t} \right] \, + \, \theta_2 \left[ \mathsf{t} \right] \, + \, \theta_3 \left[ \mathsf{t} \right] \right] \, \mathsf{L}_3 \right) \, \, \mathsf{m}_3 \right) \, \, \mathsf{m}_3 \, \, \, \mathsf{m}_3 \, \, \, \mathsf{m}_3 \, \, \mathsf{m}_3
```

Total Potential Energy

```
ln[247]:= PE = PE_1 + PE_2 + PE_3
\text{Out} [247] = \text{ g} \sin \left[\theta_1\left[\text{t}\right]\right] \text{ L}_1 \text{ m}_1 + \text{g} \left(\sin \left[\theta_1\left[\text{t}\right]\right] \text{ L}_1 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2 + \sin \left[\theta_1\left[\text{t}\right] + \theta_2\left[\text{t}\right]\right] \text{ L}_2\right) \text{ m}_2
                                                                               \texttt{g} \; (\texttt{Sin}[\theta_1[\texttt{t}]] \; \texttt{L}_1 + \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \; \texttt{L}_2 + \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]] \; \texttt{L}_3) \; \texttt{m}_3
```

Lagrangian Partial Derivatives

```
\ln[248]:= (*Partial first order derivative of L with respect to \theta_{i}'*)
                  DL_{\theta_1}' = D[KE - PE, \{\theta_1', 1\}]
                   DL_{\theta_2} = D[KE - PE, \{\theta_2', 1\}]
                  DL_{\theta_3}' = D[KE - PE, \{\theta_3', 1\}]
                    (*Partial first order derivative of DLa, with respect to t, which is time*)
                  DLT_1 = D[DL_{\theta_1}, \{t, 1\}]
                  DLT_2 = D[DL_{\theta_2}, \{t, 1\}]
                  DLT_3 = D[DL_{\theta_3}, \{t, 1\}]
                   (*Partial first order derivative of L with respect to \theta_i \star)
                  DL_{\theta_1} = D[KE - PE, \{\theta_1, 1\}]
                   DL_{\theta_2} = D[KE - PE, \{\theta_2, 1\}]
                  DL_{\theta_3} = D[KE - PE, \{\theta_3, 1\}]
                   (*\tau_i] as defined by the required derivatives of the Lagrangian*)
                   \tau_1 = DLT_1 - DL_{\theta_1}
                   \tau_2 = DLT_2 - DL_{\theta_2}
                   \tau_3 = DLT_3 - DL_{\theta_3}
\text{Out} [248] = \ 1 \ [t] \ \text{Izz}_1 \ \theta_1{}'[t] \ + \ 1 \ [t] \ \text{L}_1^2 \ \text{m}_1 \ \theta_1{}'[t] \ + \ 1 \ [t] \ \text{Izz}_2 \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ \text{Izz}_2 \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ \text{Izz}_2 \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ \text{Izz}_2 \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ \text{Izz}_2 \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ \text{Izz}_2 \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ \text{Izz}_2 \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ \text{Izz}_2 \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ \text{Izz}_2 \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ + \ 1 \ [t] \ (\theta_1{}'[t] \ + \ \theta_2{}'[t]) \ +
                       - m_2 (2 (1[t] Cos[\theta_1[t]] L_1 + 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2)
                                       (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                  2 (1[t] Sin[\theta_1[t]] L_1 + 1[t] Sin[\theta_1[t] + \theta_2[t]] L_2)
                                       (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) +
                      1[t] Izz_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) + \frac{1}{2} m_3
                           (2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                \cos \left[ \theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t] \right] L_{3} \left( \theta_{1}^{\prime}[t] + \theta_{2}^{\prime}[t] + \theta_{3}^{\prime}[t] \right) ) + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] L_{1} + \theta_{2}^{\prime}[t] \right) 
                                             \texttt{1[t]} \; \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \; \texttt{L}_2 + \texttt{1[t]} \; \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]] \; \texttt{L}_3)
                                       \left(\text{Sin}\left[\theta_{1}\left[\mathtt{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathtt{t}\right] + \text{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right]\right) \; + \right.
                                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
Out[249]= 1[t] Izz_2 (\theta_1'[t] + \theta_2'[t]) +
                       \frac{1}{2} m_2 (2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]]
                                                 L_2(\theta_1'[t] + \theta_2'[t])) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]] L_2
                                       \left( \text{Sin}[\theta_{1}[t]] \ \text{L}_{1} \ \theta_{1}{'}[t] + \text{Sin}[\theta_{1}[t] + \theta_{2}[t]] \ \text{L}_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \right) \right) + \\
                       1[t] Izz_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) +
                       -m_3
                           (2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                       \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}] \; + \; \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right) \; + \;
                                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                  2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                       (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
```

```
Out[250]= 1[t] Izz<sub>3</sub> (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) +
                                                            \frac{1}{2} \, m_3 \, (2 \times 1[t] \, \text{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \, L_3 \, (\text{Cos}[\theta_1[t]] \, L_1 \, \theta_1'[t] + \text{Cos}[\theta_1[t] + \theta_2[t]] \, L_2
                                                                                                                                 (\theta_1'[t] + \theta_2'[t]) + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                        2\times1[t]\,\sin[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,\,\mathtt{L}_3\,\left(\sin[\theta_1[t]]\,\,\mathtt{L}_1\,\,\theta_1{}'[t]+\sin[\theta_1[t]+\theta_2[t]\right)
                                                                                                                            L_{2}(\theta_{1}'[t] + \theta_{2}'[t]) + Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3}(\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])))
\text{Out}[251] = 1[t] \text{Izz}_1 \Theta_1''[t] + 1[t] \text{L}_1^2 \text{m}_1 \Theta_1''[t] + 1[t] \text{Izz}_2 (\Theta_1''[t] + \Theta_2''[t]) + 1[t] \text{Izz}_2 (\Theta_1''[t] + \Theta_1''[t] + \Theta_1''[t]) + 1[t] \text{Izz}_2 (\Theta_1''[t] + \Theta_1''[t] + \Theta_1''[t]) + 1[t] \text{Izz}_2 (\Theta_1''[t] + \Theta_1''[t] + \Theta_1''[t] + \Theta_1''[t]) + 1[t] \text{Izz}_2 (\Theta_1''[t] + \Theta_1''[t] + 
                                                            \frac{1}{2} \, \mathrm{m_2} \, \left( 2 \, \left( 1 [\mathsf{t}] \, \mathsf{Cos}[\theta_1[\mathsf{t}]] \, \mathrm{L_1} \, \theta_1{}'[\mathsf{t}] + 1 [\mathsf{t}] \, \mathsf{Cos}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \, \mathrm{L_2} \, \left( \theta_1{}'[\mathsf{t}] + \theta_2{}'[\mathsf{t}] \right) \right)
                                                                                                   (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                                        2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))
                                                                                               (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                                        2 (1[t] Cos[\theta_1[t]] L_1 + 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2)
                                                                                                   \left(-\sin\left[\theta_{1}[t]\right]\right. \left.L_{1}\right. \theta_{1}{'}[t]^{2} - \sin\left[\theta_{1}[t]\right. + \left.\theta_{2}[t]\right]\right. \left.L_{2}\left.\left(\theta_{1}{'}[t]\right. + \left.\theta_{2}{'}[t]\right)\right)^{2} + \left.\left(\theta_{1}\right)^{2}\right] + \left.\left(\theta_{1}\right)^{2}\right] + \left.\left(\theta_{1}\right)^{2}\left[\theta_{1}\right]\right] + \left.\left(\theta_{1}\right)^{2}\left[\theta_{1}\right] + \left(\theta_{1}\right)^{2}\left[\theta_{1}\right]\right] + \left(\theta_{1}\right)^{2}\left[\theta_{1}\right] +
                                                                                                                     Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                        2 (1[t] Sin[\theta_1[t]] L_1 + 1[t] Sin[\theta_1[t] + \theta_2[t]] L_2)
                                                                                                   (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                                    Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) +
                                                           1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                            1
                                                               2
                                                                  m_3
                                                                      \left(2\;(1[t]\;\mathsf{Cos}\,[\theta_1[t]]\;\mathsf{L}_1\;\theta_1{}'[t]\;+\;1[t]\;\mathsf{Cos}\,[\theta_1[t]\;+\;\theta_2[t]]\;\mathsf{L}_2\;(\theta_1{}'[t]\;+\;\theta_2{}'[t])\;+\;\theta_2{}'[t]\right)\;+\;\theta_2{}'[t]
                                                                                                                     1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                   (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                    Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                        2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t]] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                   (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                                                                     1[t] \, \operatorname{Sin}[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \, \operatorname{L}_{3} \, (\theta_{1}{}'[t] + \theta_{2}{}'[t] + \theta_{3}{}'[t])) \, + \,
                                                                                        2 \ (1[t] \ Cos[\theta_1[t]] \ L_1 + 1[t] \ Cos[\theta_1[t]] + \theta_2[t]] \ L_2 + 1[t] \ Cos[\theta_1[t]] + \theta_2[t] + \theta_3[t]]
                                                                                                                            L_3) \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 + \cos[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 + \cos[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin
                                                                                                                    Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                                                    \texttt{Cos}[\theta_1[\texttt{t}]] \; \texttt{L}_1 \; \theta_1^{\prime\prime\prime}[\texttt{t}] \; + \; \texttt{Cos}[\theta_1[\texttt{t}] \; + \; \theta_2[\texttt{t}]] \; \texttt{L}_2 \; (\theta_1^{\prime\prime\prime}[\texttt{t}] \; + \; \theta_2^{\prime\prime\prime}[\texttt{t}]) \; + \;
                                                                                                                    \cos \left[ \theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t] \right] L_{3} \left( \theta_{1}''[t] + \theta_{2}''[t] + \theta_{3}''[t] \right) \right) + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] L_{1} + \theta_{2}''[t] \right) + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] L_{1} + \theta_{2}''[t] \right) + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{1} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{2} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right] \right) L_{3} + 2 \left( 1[t] \cos \left[ \theta_{1}[t] \right
                                                                                                                     1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                                                                   (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                                    \mathsf{Cos} \, [\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_3[\mathsf{t}]] \, \, \mathsf{L}_3 \, \, (\theta_1{}'[\mathsf{t}] + \theta_2{}'[\mathsf{t}] + \theta_3{}'[\mathsf{t}])^{\, 2} + \\
                                                                                                                    Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                                    Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
```

```
Out[252]= 1[t] Izz_2 (\theta_1''[t] + \theta_2''[t]) +
                                      \frac{1}{2} m_2 \left( -21[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\cos[\theta_1[t]] L_1 \theta_1'[t] + \theta_2'[t]) \right)
                                                                         Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t])) + 2 \times 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2
                                                               (\theta_{1}'[t] + \theta_{2}'[t]) (Sin[\theta_{1}[t]] L_{1} \theta_{1}'[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])) +
                                                       2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                                                         Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                                                         Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t]) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]]
                                                             L_2 \left( Cos[\theta_1[t]] L_1 \theta_1'[t]^2 + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_2 (\theta_1'[t])^2 + Cos[\theta_1[t]
                                                                         Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]))) +
                                     1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                      1
                                       2
                                          m_3
                                             (2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                          \texttt{l[t]}\;\mathsf{Cos}[\theta_1[\texttt{t}]+\theta_2[\texttt{t}]+\theta_3[\texttt{t}]]\;\mathtt{L_3}\;(\theta_1{'}[\texttt{t}]+\theta_2{'}[\texttt{t}]+\theta_3{'}[\texttt{t}]))
                                                               (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                         Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])) +
                                                       2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                          Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                               (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                           1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                       2 \ (1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t]] \ \mathtt{L}_2 + 1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \ \mathtt{L}_3)
                                                               \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - (\theta_{1}'[t])^{2} + (\theta_{
                                                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                         Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                          Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                                       2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                               \left( \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}]^{2} + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right)^{2} \; + \right.
                                                                         \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]\right] \; \texttt{L}_{3} \; \left(\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}]\right)^{2} + \\
                                                                         Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
Out[253]= 1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                     \frac{1}{2} m_3 \left( -21[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t] \right) L_3 \left( \theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right)
                                                               (\cos[\theta_{1}[t]] L_{1} \theta_{1}'[t] + \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]) +
                                                                         \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                       2 \times 1[t] \; \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; \mathsf{L_3} \; (\theta_1{}'[t] + \theta_2{}'[t] + \theta_3{}'[t])
                                                               (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                          Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                       2 \times 1[t] \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} \left( -\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] \right)
                                                                               L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])^{2} + \theta_{3}'[t] + \theta_{3
                                                                         \texttt{Cos}\left[\theta_{1}[\texttt{t}]\right] \; \texttt{L}_{1} \; \theta_{1}^{\prime\prime}[\texttt{t}] \; + \; \texttt{Cos}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \texttt{L}_{2} \; \left(\theta_{1}^{\prime\prime}[\texttt{t}] \; + \; \theta_{2}^{\prime\prime}[\texttt{t}]\right) \; + \;
                                                                         Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                       2 \times 1[t] \sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} \left(\cos[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} + \cos[\theta_{1}[t] + \theta_{2}[t]]\right)
                                                                               L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                         Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
```

```
\text{Out}[254] = -\text{g1[t]} \; \text{Cos}[\theta_1[\text{t}]] \; \text{L}_1 \; \text{m}_1 - \text{g} \; (\text{1[t]} \; \text{Cos}[\theta_1[\text{t}]] \; \text{L}_1 + \text{1[t]} \; \text{Cos}[\theta_1[\text{t}]] + \theta_2[\text{t}]] \; \text{L}_2) \; \text{m}_2 - \text{m}_2[\text{t}] \; \text{Cos}[\theta_1[\text{t}]] \; \text{Cos}[\theta_1[\text{t}]
                                    g (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t]] + \theta_2[t] L_2 + 1[t] \cos[\theta_1[t]] + \theta_2[t] + \theta_3[t] L_3)
                                         m_{3} + \frac{1}{2} m_{2} (2 (1[t] Cos[\theta_{1}[t]] L_{1} \theta_{1}'[t] + 1[t] Cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]))
                                                             (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                      2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))
                                                             (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t]] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) +
                                     \frac{-}{2} m_3 (2 (1[t] Cos[\theta_1[t]] L_1 \theta_1'[t] + 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                        1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                             (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                        \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right)) \; + \;
                                                      2 (Cos[\theta_1[t]] L<sub>1</sub> \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L<sub>2</sub> (\theta_1'[t] + \theta_2'[t]) +
                                                                        Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                             (-1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_1 \, \theta_1{}'[t] \, -1[t] \, \operatorname{Sin}[\theta_1[t] \, + \, \theta_2[t]] \, \operatorname{L}_2 \, (\theta_1{}'[t] \, + \, \theta_2{}'[t]) \, - \, \theta_2{}'[t]) \, - \, \theta_2{}'[t]) \, - \, \theta_2{}'[t] 
                                                                        1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
Out[255]= -g1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 m_2 -
                                    g (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 + 1
                                      \frac{1}{2} m_2 (-21[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                             (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                      2\times \texttt{1[t]}\; \texttt{Cos}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \; \texttt{L}_2 \; (\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}])
                                                             (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) +
                                      -m_3 (2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L<sub>2</sub> (\theta_1'[t] + \theta_2'[t]) +
                                                                        1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                             (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                        \sin \left[ \theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t] \right] \, L_{3} \, \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t] \right)) \, + \,
                                                      2 \left( \text{Cos} \left[ \theta_{1}[t] \right] \right] \text{L}_{1} \ \theta_{1}{'}[t] \ + \ \text{Cos} \left[ \theta_{1}[t] \ + \ \theta_{2}[t] \right] \right) \text{L}_{2} \ \left( \theta_{1}{'}[t] \ + \ \theta_{2}{'}[t] \right) \ + \ \theta_{2}{'}[t] 
                                                                        Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                             (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                        1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
Out[256]= -g1[t] Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 +
                                     \frac{1}{2} m_3 (-21[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                             (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                        Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                      2\times1\texttt{[t]}\;\texttt{Cos}\left[\theta_{1}\texttt{[t]}+\theta_{2}\texttt{[t]}+\theta_{3}\texttt{[t]}\right]\;\texttt{L}_{3}\;\left(\theta_{1}{'}\texttt{[t]}+\theta_{2}{'}\texttt{[t]}+\theta_{3}{'}\texttt{[t]}\right)
                                                             \left(\text{Sin}\left[\theta_{1}\left[\texttt{t}\right]\right] \; \text{L}_{1} \; \theta_{1}{'}\left[\texttt{t}\right] + \text{Sin}\left[\theta_{1}\left[\texttt{t}\right] + \theta_{2}\left[\texttt{t}\right]\right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[\texttt{t}\right] + \theta_{2}{'}\left[\texttt{t}\right]\right) \; + \right.
                                                                        Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
```

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\text{Out} \text{(257)= g 1[t] Cos} \left[\theta_1 \left[t\right]\right] \text{ L}_1 \text{ m}_1 + \text{g (1[t] Cos} \left[\theta_1 \left[t\right]\right] \text{ L}_1 + \text{1[t] Cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2 + \text{cos} \left[\theta_1 \left[t\right] + \theta_2 \left[t\right]\right] \text{ L}_2\right) \text{ m}_2
                                         g (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t]] + \theta_2[t] L_2 + 1[t] \cos[\theta_1[t]] + \theta_2[t] + \theta_3[t] L_3)
                                              m_{3} - \frac{1}{2} m_{2} (2 (1[t] Cos[\theta_{1}[t]] L_{1} \theta_{1}'[t] + 1[t] Cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]))
                                                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                              2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))
                                                                      (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t]] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) -
                                           \frac{-}{2} m_3 (2 (1[t] Cos[\theta_1[t]] L_1 \theta_1'[t] + 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                  1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                  Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \; L_{3} \; (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t])) \; + \;
                                                              2 \; (\text{Cos}[\theta_{1}[t]] \; \text{L}_{1} \; \theta_{1}{}'[t] \; + \; \text{Cos}[\theta_{1}[t] \; + \; \theta_{2}[t]] \; \text{L}_{2} \; (\theta_{1}{}'[t] \; + \; \theta_{2}{}'[t]) \; + \;
                                                                                  Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                      (-1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_1 \, \theta_1{}'[t] \, -1[t] \, \operatorname{Sin}[\theta_1[t] \, + \, \theta_2[t]] \, \operatorname{L}_2 \, (\theta_1{}'[t] \, + \, \theta_2{}'[t]) \, - \, \theta_2{}'[t]) \, - \, \theta_2{}'[t]) \, - \, \theta_2{}'[t] 
                                                                                   1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
                                         1[t] Izz_1 \theta_1''[t] + 1[t] L_1^2 m_1 \theta_1''[t] + 1[t] Izz_2 (\theta_1''[t] + \theta_2''[t]) +
                                          1
                                           - m<sub>2</sub>
                                                  \left(2\;(1[t]\;\cos[\theta_1[t]]\;L_1\;\theta_1{}'[t]+1[t]\;\cos[\theta_1[t]+\theta_2[t]]\;L_2\;(\theta_1{}'[t]+\theta_2{}'[t])\right)
                                                                      \left( \operatorname{Sin}\left[\theta_{1}[t]\right] \operatorname{L}_{1}\theta_{1}{'}[t] + \operatorname{Sin}\left[\theta_{1}[t] + \theta_{2}[t]\right] \operatorname{L}_{2}\left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) \right) + \\
                                                              2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))
                                                                      (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                              2 (1[t] Cos[\theta_1[t]] L_1 + 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2)
                                                                      \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} + \theta_{1}'[t] + \theta_{2}'[t]\right)^{2}
                                                                                   Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                              2 (1[t] Sin[\theta_1[t]] L_1 + 1[t] Sin[\theta_1[t] + \theta_2[t]] L_2)
                                                                      (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                  Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) +
                                         1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                          1
                                            _
2
                                              m_3
                                                  (2 (1[t] \cos[\theta_1[t]] L_1 \theta_1'[t] + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + 
                                                                                   1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                  Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                              2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                  \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                                                      (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t]] + \theta_2[t]] L_2 (\theta_1'[t]] + \theta_2'[t]) - \frac{1}{2} (\theta_1'[t]) - \frac{1}{2} (\theta_1'[t]) + \frac
                                                                                   1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                              2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t]] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t]] + \theta_2[t] + \theta_3[t]]
                                                                                        L_3) \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin[\theta_1'[t]] L_3 (\theta_1'[t
                                                                                  Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                  \texttt{Cos}\left[\theta_{1}[\texttt{t}]\right] \; \texttt{L}_{1} \; \theta_{1}{''}[\texttt{t}] \; + \; \texttt{Cos}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \texttt{L}_{2} \; \left(\theta_{1}{''}[\texttt{t}] \; + \; \theta_{2}{''}[\texttt{t}]\right) \; + \;
                                                                                  \cos \left[ \theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t] \right] L_{3} \left( \theta_{1}''[t] + \theta_{2}''[t] + \theta_{3}''[t] \right) + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] L_{1} + \theta_{2}''[t] \right) + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] L_{1} + \theta_{2}''[t] \right)
                                                                                   1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                                      (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                  \texttt{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]] \; \texttt{L}_{3} \; (\theta_{1}{}'[\texttt{t}] + \theta_{2}{}'[\texttt{t}] + \theta_{3}{}'[\texttt{t}])^{2} + \\
                                                                                  Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                  Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
```

```
Out[258]= g1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 m_2 +
                      g (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 -
                       -m_2 (-21[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                      (\cos[\theta_{1}[t]] L_{1} \theta_{1}'[t] + \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])) +
                                 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) -
                       - m_3 (2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                            1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                            \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right)) \; + \;
                                 2 \; (\text{Cos}[\theta_{1}[t]] \; \text{L}_{1} \; \theta_{1}{}'[t] \; + \; \text{Cos}[\theta_{1}[t] \; + \; \theta_{2}[t]] \; \text{L}_{2} \; (\theta_{1}{}'[t] \; + \; \theta_{2}{}'[t]) \; + \;
                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                      (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                             1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                      1[t] Izz_{2} (\theta_{1}''[t] + \theta_{2}''[t]) + \frac{1}{2} m_{2} (-21[t] Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])
                                      (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t])
                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                            Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                            Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]]
                                     L_2 \left( Cos[\theta_1[t]] L_1 \theta_1'[t]^2 + Cos[\theta_1[t]] + \theta_2[t] \right) L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 +
                                            Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) +
                      1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                       2
                           (2 (1[t] Cos[\theta_1[t] + \theta_2[t]) L_2 (\theta_1'[t] + \theta_2'[t]) +
                                            1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                            \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right)) \; + \;
                                 2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                      (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                             1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                 2 \ (1[t] \ \mathsf{Cos} \ [\theta_1[t] \ + \theta_2[t] \ ] \ \mathtt{L}_2 \ + \ 1[t] \ \mathsf{Cos} \ [\theta_1[t] \ + \theta_2[t] \ + \ \theta_3[t] \ ] \ \mathtt{L}_3)
                                      \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{1} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{2} (\theta_{1}'[t])^{2} - \sin[\theta_{1}[t]] L_{2} (\theta_{1}'[t]
                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                            Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                 2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                      (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                            \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]\right] \; \texttt{L}_{3} \; \left(\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}]\right)^{2} + \\
                                            Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
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Out[259]= g1[t] Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 -
                                        \frac{-}{2} m_3 (-21[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                  \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\left.\theta_{1}{'}[\mathsf{t}]\right. + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right. + \theta_{2}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{2}\left.\left(\theta_{1}{'}[\mathsf{t}]\right. + \theta_{2}{'}[\mathsf{t}]\right)\right. + \\ \left.\left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\left.\theta_{1}{'}\right[\mathsf{t}]\right. + \left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left[\mathsf{t}\right]\right]\right. \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left(\theta_{1}{'}[\mathsf{t}]\right)\right. + \\ \left.\mathsf{L}_{2}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{3}\left(\theta_{1}{'}[\mathsf{t}]\right)\right. + \\ \left.\mathsf{L}_{3}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{3}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{3}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{3}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{4}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{5}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{5}\left(\theta_{1
                                                                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                          2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                  (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                             \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right))) \; + \;
                                      1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                         2
                                            m_3
                                              (-21[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                  (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t]] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                             \texttt{Cos} \left[ \theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}] \right] \, \texttt{L}_{3} \, \left( \theta_{1}{}'[\texttt{t}] + \theta_{2}{}'[\texttt{t}] + \theta_{3}{}'[\texttt{t}] \right) ) \, + \,
                                                          2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                  (Sin[\theta_{1}[t]] \; L_{1} \; \theta_{1}{'}[t] \; + \; Sin[\theta_{1}[t] \; + \; \theta_{2}[t]] \; L_{2} \; (\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]) \; + \;
                                                                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                          2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]]\right)
                                                                                   L_2(\theta_1'[t] + \theta_2'[t])^2 - Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                             \texttt{Cos}[\theta_{1}[\texttt{t}]] \; \texttt{L}_{1} \; \theta_{1}^{\, \prime \prime}[\texttt{t}] \; + \; \texttt{Cos}[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]] \; \texttt{L}_{2} \; (\theta_{1}^{\, \prime \prime}[\texttt{t}] \; + \; \theta_{2}^{\, \prime \prime}[\texttt{t}]) \; + \;
                                                                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                          2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]]
                                                                                   L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} + Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])^{2} +
                                                                              \sin[\theta_{1}[t]] \; L_{1} \; \theta_{1}^{\prime\prime}[t] \; + \; \sin[\theta_{1}[t] \; + \; \theta_{2}[t]] \; L_{2} \; (\theta_{1}^{\prime\prime}[t] \; + \; \theta_{2}^{\prime\prime}[t]) \; + \;
                                                                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
```