

Kinect Energy

Kinect Energy Link I

```
In[1]:= p1 = { L1 Cos[θ1[t]],  
             L1 Sin[θ1[t]],  
             0}  
v1 = D[p1, {t, 1}]  
  
I1 = {{Ixx1, 0, 0},  
      {0, Iyy1, 0},  
      {0, 0, Izz1}}  
  
ω1 = { 0,  
       0,  
       D[θ1[t], {t, 1}]}  
  
KE1 = Simplify[1 / 2 m1 Dot [v1, v1]] + Simplify[ 1 / 2 Dot [ω1, I1 . ω1]]  
  
Out[1]= {Cos[θ1[t]] L1, Sin[θ1[t]] L1, 0}  
Out[2]= {-Sin[θ1[t]] L1 θ1'[t], Cos[θ1[t]] L1 θ1'[t], 0}  
Out[3]= {{Ixx1, 0, 0}, {0, Iyy1, 0}, {0, 0, Izz1}}  
Out[4]= {0, 0, θ1'[t]}  
Out[5]=  $\frac{1}{2} I_{zz1} \theta_1'^2[t] + \frac{1}{2} L_1^2 m_1 \theta_1'^2[t]$ 
```

Kinect Energy Link 2

```
In[6]:= p2 = { L1 Cos[θ1[t]] + L2 Cos[θ1[t] + θ2[t]],
              L1 Sin[θ1[t]] + L2 Sin[θ1[t] + θ2[t]],
              0 }
```

```
v2 = D[ p2, {t, 1} ]
```

```
I2 = {{Ixx2, 0, 0}, {0, Iyy2, 0}, {0, 0, Izz2}}
```

```
ω2 = { 0, 0, D[θ1[t] + θ2[t], {t, 1}] }
```

```
KE2 = Simplify[1 / 2 m2 Dot [v2, v2]] + Simplify[ 1 / 2 Dot [ω2, I2 . ω2]]
```

```
Out[6]= {Cos[θ1[t]] L1 + Cos[θ1[t] + θ2[t]] L2, Sin[θ1[t]] L1 + Sin[θ1[t] + θ2[t]] L2, 0}
```

```
Out[7]= {-Sin[θ1[t]] L1 θ1'[t] - Sin[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]),
          Cos[θ1[t]] L1 θ1'[t] + Cos[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]), 0}
```

```
Out[8]= {{Ixx2, 0, 0}, {0, Iyy2, 0}, {0, 0, Izz2}}
```

```
Out[9]= {0, 0, θ1'[t] + θ2'[t]}
```

```
Out[10]= 1/2 Izz2 (θ1'[t] + θ2'[t])^2 +
          1/2 m2 ((Cos[θ1[t]] L1 θ1'[t] + Cos[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]))^2 +
                (Sin[θ1[t]] L1 θ1'[t] + Sin[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]))^2)
```

Kinect Energy Link 3

```
In[16]:= p3 = { L1 Cos[θ1[t]] + L2 Cos[θ1[t] + θ2[t]] + L3 Cos[θ1[t] + θ2[t] + θ3[t]],
               L1 Sin[θ1[t]] + L2 Sin[θ1[t] + θ2[t]] + L3 Sin[θ1[t] + θ2[t] + θ3[t]],
               0 }
```

```
v3 = D[ p3, {t, 1} ]
```

```
I3 = {{Ixx3, 0, 0},
      {0, Iyy3, 0},
      {0, 0, Izz3}}
```

```
ω3 = { 0,
        0,
        D[θ1[t] + θ2[t] + θ3[t], {t, 1}]} 
```

```
KE3 = Simplify[1 / 2 m3 Dot [v3, v3]] + Simplify[ 1 / 2 Dot [ω3, I3 . ω3]]
```

```
Out[16]= {Cos[θ1[t]] L1 + Cos[θ1[t] + θ2[t]] L2 + Cos[θ1[t] + θ2[t] + θ3[t]] L3,
          Sin[θ1[t]] L1 + Sin[θ1[t] + θ2[t]] L2 + Sin[θ1[t] + θ2[t] + θ3[t]] L3, 0}
```

```
Out[17]= {-Sin[θ1[t]] L1 θ1'[t] - Sin[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]) -
          Sin[θ1[t] + θ2[t] + θ3[t]] L3 (θ1'[t] + θ2'[t] + θ3'[t]),
          Cos[θ1[t]] L1 θ1'[t] + Cos[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]) +
          Cos[θ1[t] + θ2[t] + θ3[t]] L3 (θ1'[t] + θ2'[t] + θ3'[t]), 0}
```

```
Out[18]= {{Ixx3, 0, 0}, {0, Iyy3, 0}, {0, 0, Izz3}}
```

```
Out[19]= {0, 0, θ1'[t] + θ2'[t] + θ3'[t]}
```

```
Out[20]= 1/2 Izz3 (θ1'[t] + θ2'[t] + θ3'[t])^2 +
          1/2 m3 ( (Cos[θ1[t]] L1 θ1'[t] + Cos[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]) +
                  Cos[θ1[t] + θ2[t] + θ3[t]] L3 (θ1'[t] + θ2'[t] + θ3'[t]))^2 +
                  (Sin[θ1[t]] L1 θ1'[t] + Sin[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]) +
                  Sin[θ1[t] + θ2[t] + θ3[t]] L3 (θ1'[t] + θ2'[t] + θ3'[t]))^2 )
```

Total Kinect Energy

$$\text{In[21]:= } \mathbf{KE} = \mathbf{KE}_1 + \mathbf{KE}_2 + \mathbf{KE}_3$$

$$\begin{aligned} \text{Out[21]= } & \frac{1}{2} I_{zz_1} \theta_1'[t]^2 + \frac{1}{2} L_1^2 m_1 \theta_1'[t]^2 + \frac{1}{2} I_{zz_2} (\theta_1'[t] + \theta_2'[t])^2 + \\ & \frac{1}{2} m_2 \left((\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))^2 + \right. \\ & \quad \left. (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))^2 \right) + \\ & \frac{1}{2} I_{zz_3} (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\ & \frac{1}{2} m_3 \left((\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \right. \\ & \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2 + \\ & \quad \left. (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \right. \\ & \quad \left. \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2 \right) \end{aligned}$$

Potential Energy

Potential Energy Link 1

$$\text{In[22]:= } \mathbf{PE}_1 = m_1 g L_1 \sin[\theta_1[t]]$$

$$\text{Out[22]= } g \sin[\theta_1[t]] L_1 m_1$$

Potential Energy Link 2

$$\text{In[23]:= } \mathbf{PE}_2 = m_2 g (L_1 \sin[\theta_1[t]] + L_2 \sin[\theta_1[t] + \theta_2[t]])$$

$$\text{Out[23]= } g (\sin[\theta_1[t]] L_1 + \sin[\theta_1[t] + \theta_2[t]] L_2) m_2$$

Potential Energy Link 3

$$\text{In[24]:= } \mathbf{PE}_3 = m_3 g (L_1 \sin[\theta_1[t]] + L_2 \sin[\theta_1[t] + \theta_2[t]] + L_3 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]])$$

$$\text{Out[24]= } g (\sin[\theta_1[t]] L_1 + \sin[\theta_1[t] + \theta_2[t]] L_2 + \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3$$

Total Potential Energy

$$\text{In[25]:= } \mathbf{PE} = \mathbf{PE}_1 + \mathbf{PE}_2 + \mathbf{PE}_3$$

$$\begin{aligned} \text{Out[25]= } & g \sin[\theta_1[t]] L_1 m_1 + g (\sin[\theta_1[t]] L_1 + \sin[\theta_1[t] + \theta_2[t]] L_2) m_2 + \\ & g (\sin[\theta_1[t]] L_1 + \sin[\theta_1[t] + \theta_2[t]] L_2 + \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 \end{aligned}$$

Lagrangian Partial Derivatives

$$\begin{aligned} & \left\{ \left\{ g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 + g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \right. \right. \\ & \quad \left. \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) \right\} \end{aligned}$$

$$\begin{aligned}
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 1[t] Izz_2 (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + \\
& 1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 \\
& (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))), \\
& \{-g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3) m_3 - \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3
\end{aligned}$$

$$\begin{aligned}
& (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 1[t] Izz_2 \\
& (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + \\
& 1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 \\
& (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))), \\
& \left\{ -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \right. \\
& \theta_3[t]] L_3) m_3 - \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \\
& \theta_2'[t]) + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] Izz_2 \\
& (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
\end{aligned}$$

[illegible]

[illegible]

$$\begin{aligned} & \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])) + \\ & 1[t] Izz_3 (\theta''_1[t] + \theta''_2[t] + \theta''_3[t]) + \frac{1}{2} m_3 \\ & (-2 \cdot 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t]) \\ & (\cos[\theta_1[t]] L_1 \theta'_1[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t]) + \\ & \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])) + \\ & 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t]) \\ & (\sin[\theta_1[t]] L_1 \theta'_1[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t]) + \\ & \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])) + \\ & 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (-\sin[\theta_1[t]] L_1 \theta'_1[t]^2 - \\ & \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t])^2 - \\ & \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])^2 + \\ & \cos[\theta_1[t]] L_1 \theta''_1[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta''_1[t] + \theta''_2[t]) + \\ & \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta''_1[t] + \theta''_2[t] + \theta''_3[t])) + \\ & 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta'_1[t]^2 + \\ & \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t])^2 + \\ & \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])^2 + \\ & \sin[\theta_1[t]] L_1 \theta''_1[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta''_1[t] + \theta''_2[t]) + \\ & \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta''_1[t] + \theta''_2[t] + \theta''_3[t])))) \Big\}_2, \\ g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \frac{1}{2} \\ m_3 \\ (-2 \\ \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\ L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t]) \\ (\cos[\theta_1[t]] L_1 \theta'_1[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t]) + \\ \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])) + \\ 2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t]) \\ (\sin[\theta_1[t]] L_1 \theta'_1[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t]) + \\ \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t]))) + \\ 1[t] Izz_3 (\theta''_1[t] + \theta''_2[t] + \theta''_3[t]) + \\ \frac{1}{2} \\ m_3 \\ (-2 \cdot 1[t] \\ \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\ L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t]) \\ (\cos[\theta_1[t]] L_1 \theta'_1[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t]) + \\ \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])) + \\ 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t]) \\ (\sin[\theta_1[t]] L_1 \theta'_1[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t]) + \\ \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])) + \\ 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (-\sin[\theta_1[t]] L_1 \theta'_1[t]^2 - \\ \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t])^2 - \\ \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])^2 + \\ \cos[\theta_1[t]] L_1 \theta''_1[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta''_1[t] + \theta''_2[t]) + \\ \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta''_1[t] + \theta''_2[t] + \theta''_3[t])) + \\ 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta'_1[t]^2 + \\ \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta'_1[t] + \theta'_2[t])^2 + \\ \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta'_1[t] + \theta'_2[t] + \theta'_3[t])^2 + \\ \sin[\theta_1[t]] L_1 \theta''_1[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta''_1[t] + \theta''_2[t]) + \end{aligned}$$

$$\begin{aligned} & \frac{1}{2} [t] \text{Izz}_2 (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} \\ & m_2 \\ & (-2 [t] \sin[\theta_1[t] + \theta_2[t]] \\ & \quad L_2 (\theta_1'[t] + \theta_2'[t]) \\ & \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\ & \quad 2 \times [t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\ & \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\ & \quad 2 \times [t] \cos[\theta_1[t] + \theta_2[t]] L_2 \\ & \quad (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\ & \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + \\ & \quad 2 \times [t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \\ & \quad \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \sin[\theta_1[t]] L_1 \theta_1''[t] + \\ & \quad \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + \\ & \frac{1}{2} [t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\ & \frac{1}{2} \\ & m_3 \\ & (2 ([t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\ & \quad [t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\ & \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\ & \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\ & \quad 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\ & \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\ & \quad (-[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\ & \quad [t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\ & \quad 2 ([t] \cos[\theta_1[t] + \theta_2[t]] L_2 + [t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\ & \quad (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\ & \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\ & \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\ & \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\ & \quad 2 ([t] \sin[\theta_1[t] + \theta_2[t]] L_2 + [t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\ & \quad (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \end{aligned}$$

$$\begin{aligned}
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) \Big), \\
& \left\{ -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \right. \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \\
& \quad \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \quad \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \\
& \quad (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] Izz_2 \\
& \quad (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \quad \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \quad \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& \quad L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) \Big) + \\
& \quad 1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 \\
& \quad (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& \quad 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) \Big), \\
& \left\{ -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \right. \\
& \quad \theta_3[t]] L_3) m_3 - \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
\end{aligned}$$

$$\begin{aligned}
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \\
& \theta_2'[t]) + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] Izz_2 \\
& (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] \\
& L_2 (-\sin[\theta_1[t]] L_1 \theta_1''[t] - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times \\
& 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] \\
& L_2 (\theta_1'[t] + \theta_2'[t])^2 + \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] \\
& L_2 (\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} m_3 (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + 1[t] \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (-\sin[\theta_1[t]] L_1 \theta_1''[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))), \\
& -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \\
& \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\cos[\theta_1[t]] L_1 \theta_1'[t] + \\
& \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] +
\end{aligned}$$

[illegible]

$$\begin{aligned}
& \left. \begin{aligned}
& L_3 \left(\theta_1''[t] + \theta_2''[t] + \theta_3''[t] \right) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 \left(\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right)^2 + \right. \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right)^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1''[t] + \theta_2''[t] \right) + \\
& \left. \left. \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1''[t] + \theta_2''[t] + \theta_3''[t] \right) \right) \right) \Bigg\}_2, \\
& -g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \frac{1}{2} m_3 \left(-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \right. \\
& L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \\
& \left(\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right) + \right. \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \Bigg) + \\
& 2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \\
& \left(\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right) + \right. \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \Bigg) \Bigg) + \\
& 1[t] \text{Izz}_3 \left(\theta_1''[t] + \theta_2''[t] + \theta_3''[t] \right) + \frac{1}{2} m_3 \\
& \left(-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \right. \\
& \left(\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right) + \right. \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \Bigg) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \\
& \left(\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right) + \right. \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \Bigg) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \right. \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right)^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right)^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1''[t] + \theta_2''[t] \right) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1''[t] + \theta_2''[t] + \theta_3''[t] \right) \Bigg) + \\
& 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \right. \\
& \cos[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right)^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right)^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1''[t] + \theta_2''[t] \right) + \\
& \left. \left. \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1''[t] + \theta_2''[t] + \theta_3''[t] \right) \right) \right) \Bigg\}_2, \\
& g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \frac{1}{2} \\
& m_3 \\
& (-2 \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \\
& \left(\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right) + \right. \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \Bigg) + \\
& 2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \\
& \left(\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right) + \right. \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \Bigg) \Bigg) + \\
& 1[t] \text{Izz}_3 \left(\theta_1''[t] + \theta_2''[t] + \theta_3''[t] \right) + \\
& \frac{1}{2} \\
& m_3 \\
& (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 \left(\theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right) \\
& \left(\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t] \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))), \\
& \left\{ -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \right. \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \\
& \quad \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \quad \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \\
& \quad (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] \text{Izz}_2 \\
& \quad (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \quad \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \quad \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& \quad L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + \\
& \quad 1[t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 \\
& \quad (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
\end{aligned}$$

$$\begin{aligned}
& \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \right. \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \left. \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) \right) + \\
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \left(\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \right. \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \left. \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) \right) \Big), \\
& \left\{ -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \right. \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \\
& \quad \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\cos[\theta_1[t]] L_1 \theta_1'[t] + \\
& \quad \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \\
& \quad \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) - \\
& \quad \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& \quad L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \\
& \quad \theta_2'[t]) + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 1[t] Izz_2 \\
& \quad (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \\
& \quad \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] \\
& \quad L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] \\
& \quad L_2 (\theta_1'[t] + \theta_2'[t])^2 + \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] \\
& \quad L_2 (\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \quad \frac{1}{2} m_3 (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + 1[t] \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& \quad 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
\end{aligned}$$

$$\begin{aligned}
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))), \\
& -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \\
& \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\cos[\theta_1[t]] L_1 \theta_1'[t] + \\
& \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \\
& \theta_2'[t]) + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] Izz_2 \\
& (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] \\
& L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times \\
& 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] \\
& L_2 (\theta_1'[t] + \theta_2'[t])^2 + \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] \\
& L_2 (\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} m_3 (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + 1[t] \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))), \\
& -g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \frac{1}{2} m_3 (-2 \sin[\theta_1[t] + \theta_2[t] + \\
& \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
\end{aligned}$$

[illegible]

$$\begin{aligned}
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 1[t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} \\
& m_3 \\
& \left(-2 \right. 1[t] \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& \quad L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \right. \\
& \quad \quad \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \quad \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& \quad 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \right. \\
& \quad \quad \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \quad \left. \left. \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) \right) \right) \left. \right\}_3
\end{aligned}$$

$$\begin{aligned}
\text{Out[159]} = & 1[t] \text{Izz}_1 \theta_1'[t] + 1[t] L_1^2 m_1 \theta_1'[t] + 1[t] \text{Izz}_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \frac{1}{2} m_2 (2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 (1[t] \sin[\theta_1[t]] L_1 + 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) + \\
& 1[t] \text{Izz}_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) + \frac{1}{2} m_3 \\
& (2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& \quad L_3) (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (1[t] \sin[\theta_1[t]] L_1 + \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])))
\end{aligned}$$

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Out[160]= 1[t] Izz2 (θ1'[t] + θ2'[t]) +
          1
          2 m2 (2 × 1[t] Cos[θ1[t] + θ2[t]] L2 (Cos[θ1[t]] L1 θ1'[t] + Cos[θ1[t] + θ2[t]]
              L2 (θ1'[t] + θ2'[t])) + 2 × 1[t] Sin[θ1[t] + θ2[t]] L2
              (Sin[θ1[t]] L1 θ1'[t] + Sin[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]))) +
          1[t] Izz3 (θ1'[t] + θ2'[t] + θ3'[t]) +
          1
          2 m3
          (2 (1[t] Cos[θ1[t] + θ2[t]] L2 + 1[t] Cos[θ1[t] + θ2[t] + θ3[t]] L3)
              (Cos[θ1[t]] L1 θ1'[t] + Cos[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]) +
                  Cos[θ1[t] + θ2[t] + θ3[t]] L3 (θ1'[t] + θ2'[t] + θ3'[t]))) +
          2 (1[t] Sin[θ1[t] + θ2[t]] L2 + 1[t] Sin[θ1[t] + θ2[t] + θ3[t]] L3)
              (Sin[θ1[t]] L1 θ1'[t] + Sin[θ1[t] + θ2[t]] L2 (θ1'[t] + θ2'[t]) +
                  Sin[θ1[t] + θ2[t] + θ3[t]] L3 (θ1'[t] + θ2'[t] + θ3'[t])))

Out[161]= 1[t] Izz3 (θ1'[t] + θ2'[t] + θ3'[t]) +
          1
          2 m3 (2 × 1[t] Cos[θ1[t] + θ2[t] + θ3[t]] L3 (Cos[θ1[t]] L1 θ1'[t] + Cos[θ1[t] + θ2[t]] L2
              (θ1'[t] + θ2'[t]) + Cos[θ1[t] + θ2[t] + θ3[t]] L3 (θ1'[t] + θ2'[t] + θ3'[t]))) +
          2 × 1[t] Sin[θ1[t] + θ2[t] + θ3[t]] L3 (Sin[θ1[t]] L1 θ1'[t] + Sin[θ1[t] + θ2[t]]
              L2 (θ1'[t] + θ2'[t]) + Sin[θ1[t] + θ2[t] + θ3[t]] L3 (θ1'[t] + θ2'[t] + θ3'[t])))

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$$\begin{aligned}
\text{Out[162]} = & 1[t] \text{Izz}_1 \theta_1''[t] + 1[t] L_1^2 m_1 \theta_1''[t] + 1[t] \text{Izz}_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \frac{1}{2} m_2 \left(2 (1[t] \cos[\theta_1[t]] L_1 \theta_1'[t] + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) \right. \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) \\
& \quad (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2) \\
& \quad (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + \\
& \quad 2 (1[t] \sin[\theta_1[t]] L_1 + 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) \Big) + \\
& 1[t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} \\
& m_3 \\
& \left(2 (1[t] \cos[\theta_1[t]] L_1 \theta_1'[t] + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \right. \\
& \quad 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& \quad L_3) (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + 2 (1[t] \sin[\theta_1[t]] L_1 + \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) \Big)
\end{aligned}$$

$$\begin{aligned}
\text{Out[163]} = & 1[t] \text{Izz}_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \frac{1}{2} m_2 \left(-2 \cdot 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\cos[\theta_1[t]] L_1 \theta_1'[t] + \right. \\
& \quad \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 \\
& \quad (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \quad \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \quad \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& \quad L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) \Big) + \\
& 1[t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} m_3 \\
& \left(2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \right. \\
& \quad 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& \quad 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) \Big) \\
\text{Out[164]} = & 1[t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} m_3 \left(-2 \cdot 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \right. \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] \\
& \quad L_2 (\theta_1'[t] + \theta_2'[t])^2 - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& \quad 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] \\
& \quad L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) \Big)
\end{aligned}$$

$$\begin{aligned}
\text{Out[165]} = & -g \cos[\theta_1[t]] L_1 m_1 - g (\cos[\theta_1[t]] L_1 + \cos[\theta_1[t] + \theta_2[t]] L_2) m_2 - \\
& g (\cos[\theta_1[t]] L_1 + \cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 + \\
& \frac{1}{2} m_2 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t] - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) + \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t] - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) \\
\text{Out[166]} = & -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 + \\
& \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) + \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \\
& (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) \\
\text{Out[167]} = & -g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 + \\
& \frac{1}{2} m_3 (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) + \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])))
\end{aligned}$$

$$\begin{aligned}
\text{Out[168]} = & g \cos[\theta_1[t]] L_1 m_1 + g (\cos[\theta_1[t]] L_1 + \cos[\theta_1[t] + \theta_2[t]] L_2) m_2 + \\
& g (\cos[\theta_1[t]] L_1 + \cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \\
& \frac{1}{2} m_2 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t] - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t] - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + \\
& 1[t] \text{Izz}_1 \theta_1''[t] + 1[t] L_1^2 m_1 \theta_1''[t] + 1[t] \text{Izz}_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \frac{1}{2} m_2 \\
& (2 (1[t] \cos[\theta_1[t]] L_1 \theta_1'[t] + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2) \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + \\
& 2 (1[t] \sin[\theta_1[t]] L_1 + 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + \\
& 1[t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} \\
& m_3 \\
& (2 (1[t] \cos[\theta_1[t]] L_1 \theta_1'[t] + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3) (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + 2 (1[t] \sin[\theta_1[t]] L_1 + \\
& 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
\end{aligned}$$

$$\begin{aligned}
\text{Out[169]} = & g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 + g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \\
& \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \\
& (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + \\
& 1[t] \text{Izz}_2 (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + \\
& 1[t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} \\
& m_3 \\
& (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
\end{aligned}$$

$$\begin{aligned}
\text{Out}[170] = & g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \\
& \frac{1}{2} m_3 (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + \\
& 1[t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} m_3 \\
& (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] \\
& L_2 (\theta_1'[t] + \theta_2'[t])^2 - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] \\
& L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) \\
\text{Out}[171] = & \left\{ g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 + g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \right. \\
& \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \\
& (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + \\
& 1[t] \text{Izz}_2 (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) \\
& \left. + 1[t] \text{Izz}_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \right\}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} m_3 \\
& \left(2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \right. \\
& \quad 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& \quad 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \right. \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& \quad 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& \quad \left(\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \right. \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) \left. \right), \\
& \left\{ -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \right. \\
& \quad m_3 - \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \quad \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& \quad L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + \\
& \quad 1[t] I_{zz2} (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \quad 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \quad \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \quad \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& \quad L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \quad \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) \left. \right) + 1[t] I_{zz3} \\
& \quad (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& \quad (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \quad \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& \quad 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
\end{aligned}$$

$$\begin{aligned}
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))), \\
& \left\{ -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] \right. \\
& L_3) m_3 - \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \\
& (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] Izz_2 \\
& (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 1[t] \\
& Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \\
& \theta_2'[t]) + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
\end{aligned}$$

$$\begin{aligned}
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \sin[\theta_1[t]] L_1 \\
& \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \sin[\theta_1[t] + \theta_2[t] + \\
& \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))), -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - \\
& g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \\
& \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) - \\
& \frac{1}{2} m_3 (2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \\
& (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (\cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] Izz_2 \\
& (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-2 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \\
& \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \cos[\theta_1[t]] L_1 \theta_1''[t] + \\
& \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] \\
& L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]))) + 1[t] \\
& Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \\
& \theta_2'[t]) + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \\
& (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \\
& 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 (1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))) , \\
& -g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \frac{1}{2} m_3 (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \\
& L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} m_3 \\
& (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 1[t] I_{zz_3} (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \\
& \frac{1}{2} m_3 \\
& (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \\
& 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) \\
& (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \cos[\theta_1[t]] L_1 \theta_1''[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + \\
& 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \\
& \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + \\
& \sin[\theta_1[t]] L_1 \theta_1''[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) + \\
& \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) \}
\end{aligned}$$

Out[172]/MatrixForm=

$$\left\{ -g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 - \right.$$