# Kinetic Energy

In[236]:=
```
KE1 = 1/2 Izz1 θ1'[t]^2 + 1/2 m1 l1^2 θ1'[t]^2
```

Out[236]=
$$\frac{1}{2}\,Izz_1\,\theta_1'[t]^2+\frac{1}{2}\,l_1^2\,m_1\,\theta_1'[t]^2$$

In[237]:=
```
KE2 = 1 / 2 m2 (l1^2 θ2'[t]^2 + l2^2 (θ1'[t] + θ2'[t])^2 +
      2 l1 l2 θ1'[t] (θ1'[t] + θ2'[t]) Cos[θ2[t]]) + 1 / 2 Izz2 (θ1'[t] + θ2'[t])^2
```

Out[237]=
$$\frac{1}{2}\,Izz_2\,(\theta_1'[t]+\theta_2'[t])^2+\frac{1}{2}\,m_2\left(l_1^2\,\theta_2'[t]^2+2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t])+l_2^2\,(\theta_1'[t]+\theta_2'[t])^2\right)$$

In[238]:=
```
KE3 =
 1 / 2 m3 (l1^2 θ2'[t]^2 + l2^2 (θ1'[t] + θ2'[t])^2 + l3^2 (θ1'[t] + θ2'[t] + θ3'[t])^2 +
    2 (l1 l2 θ1'[t] (θ1'[t] + θ2'[t]) Cos[θ2[t]] +
       l1 l3 θ1'[t] (θ1'[t] + θ2'[t] + θ3'[t]) Cos[θ2[t] + θ3[t]] + l2 l3 (θ1'[t] + θ2'[t])
        (θ1'[t] + θ2'[t] + θ3'[t]) Cos[θ3[t]])) + 1 / 2 Izz3 (θ1'[t] + θ2'[t] + θ3'[t])^2
```

Out[238]=
$$\frac{1}{2}\,Izz_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\frac{1}{2}\,m_3\left(l_1^2\,\theta_2'[t]^2+l_2^2\,(\theta_1'[t]+\theta_2'[t])^2+l_3^2\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+2\,(\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t])+\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])+\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t])\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\right)$$

In[239]:=
```
KE = KE1 + KE2 + KE3
```

Out[239]=
$$\frac{1}{2}\,Izz_1\,\theta_1'[t]^2+\frac{1}{2}\,l_1^2\,m_1\,\theta_1'[t]^2+\frac{1}{2}\,Izz_2\,(\theta_1'[t]+\theta_2'[t])^2+\frac{1}{2}\,m_2\left(l_1^2\,\theta_2'[t]^2+2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t])+l_2^2\,(\theta_1'[t]+\theta_2'[t])^2\right)+\frac{1}{2}\,Izz_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+\frac{1}{2}\,m_3\left(l_1^2\,\theta_2'[t]^2+l_2^2\,(\theta_1'[t]+\theta_2'[t])^2+l_3^2\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2+2\,(\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t])+\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])+\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t])\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\right)$$

# Potential Energy

In[240]:=
```
PE1 = m1 g l1 Sin[θ1[t]]
```

Out[240]=
$$g\,\text{Sin}[\theta_1[t]]\,l_1\,m_1$$

In[241]:=
```
PE2 = m2 g (l1 Sin[θ1[t]] + l2 Sin[θ1[t] + θ2[t]])
```

Out[241]=
$$g\,(\text{Sin}[\theta_1[t]]\,l_1 + \text{Sin}[\theta_1[t]+\theta_2[t]]\,l_2)\,m_2$$

In[242]:=
```
PE3 = m3 g (l1 Sin[θ1[t]] + l2 Sin[θ1[t] + θ2[t]] + l3 Sin[θ1[t] + θ2[t] + θ3[t]])
```

Out[242]=
$$g\,(\text{Sin}[\theta_1[t]]\,l_1 + \text{Sin}[\theta_1[t]+\theta_2[t]]\,l_2 + \text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,l_3)\,m_3$$

In[243]:=
```
PE = PE1 + PE2 + PE3
```

Out[243]=
$$g\,\text{Sin}[\theta_1[t]]\,l_1\,m_1 + g\,(\text{Sin}[\theta_1[t]]\,l_1 + \text{Sin}[\theta_1[t]+\theta_2[t]]\,l_2)\,m_2 + g\,(\text{Sin}[\theta_1[t]]\,l_1 + \text{Sin}[\theta_1[t]+\theta_2[t]]\,l_2 + \text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,l_3)\,m_3$$

# Lagrangian

In[244]:=
```
la = KE - PE
```

Out[244]=
$$-g\,\text{Sin}[\theta_1[t]]\,l_1\,m_1 - g\,(\text{Sin}[\theta_1[t]]\,l_1 + \text{Sin}[\theta_1[t]+\theta_2[t]]\,l_2)\,m_2 - g\,(\text{Sin}[\theta_1[t]]\,l_1 + \text{Sin}[\theta_1[t]+\theta_2[t]]\,l_2 + \text{Sin}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,l_3)\,m_3 + \frac{1}{2}\,\text{Izz}_1\,\theta_1'[t]^2 + \frac{1}{2}\,l_1^2\,m_1\,\theta_1'[t]^2 + \frac{1}{2}\,\text{Izz}_2\,(\theta_1'[t]+\theta_2'[t])^2 + \frac{1}{2}\,m_2\left(l_1^2\,\theta_2'[t]^2 + 2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t]) + l_2^2\,(\theta_1'[t]+\theta_2'[t])^2\right) + \frac{1}{2}\,\text{Izz}_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2 + \frac{1}{2}\,m_3\left(l_1^2\,\theta_2'[t]^2 + l_2^2\,(\theta_1'[t]+\theta_2'[t])^2 + l_3^2\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])^2 + 2\,(\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t]) + \text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) + \text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t])\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\right)$$

# Lagrangian Partial Derivatives

In[245]:=
```
(*Partial first order derivative of l with respect to θi'*)
```

In[246]:=
```
Dlθ1' = D[la, {θ1'[t], 1}]
```

Out[246]=
$$\text{Izz}_1\,\theta_1'[t] + l_1^2\,m_1\,\theta_1'[t] + \text{Izz}_2\,(\theta_1'[t]+\theta_2'[t]) + \frac{1}{2}\,m_2\left(2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t] + 2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,(\theta_1'[t]+\theta_2'[t]) + 2\,l_2^2\,(\theta_1'[t]+\theta_2'[t])\right) + \text{Izz}_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) + \frac{1}{2}\,m_3\left(2\,l_2^2\,(\theta_1'[t]+\theta_2'[t]) + 2\,l_3^2\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) + 2\,(\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t] + \text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t] + \text{Cos}[\theta_2[t]]\,l_1\,l_2\,(\theta_1'[t]+\theta_2'[t]) + \text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t]) + \text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]) + \text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\right)$$

In[247]:=
```
Dl_θ2' = D[la, {θ2'[t], 1}]
```

Out[247]=
$$\begin{aligned}
&Izz_2\,(\theta_1'[t]+\theta_2'[t])+\\
&\frac{1}{2}m_2\left(2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]+2\,l_1^2\,\theta_2'[t]+2\,l_2^2\,(\theta_1'[t]+\theta_2'[t])\right)+\\
&Izz_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])+\\
&\frac{1}{2}m_3\left(2\,l_1^2\,\theta_2'[t]+2\,l_2^2\,(\theta_1'[t]+\theta_2'[t])+2\,l_3^2\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])+\right.\\
&\quad 2\,(\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]+\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]+\\
&\quad\left.\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t])+\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\right)
\end{aligned}$$

In[248]:=
```
Dl_θ3' = D[la, {θ3'[t], 1}]
```

Out[248]=
$$\begin{aligned}
&Izz_3\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])+\\
&\frac{1}{2}m_3\left(2\,(\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]+\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t]))+\right.\\
&\quad\left.2\,l_3^2\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])\right)
\end{aligned}$$

In[249]:=
```
(*Partial first order derivative of Dl_θi' with respect to t, which is time*)
DlT1 = D[Dl_θi', {t, 1}]
```

Out[249]=
$$\begin{aligned}
&Izz_1\,\theta_1''[t]+l_1^2\,m_1\,\theta_1''[t]+Izz_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\frac{1}{2}m_2\left(-2\,\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,\theta_2'[t]-2\,\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_2'[t]\,(\theta_1'[t]+\theta_2'[t])+\right.\\
&\quad 2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1''[t]+2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\quad\left.2\,l_2^2\,(\theta_1''[t]+\theta_2''[t])\right)+Izz_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\\
&\frac{1}{2}m_3\left(2\,l_2^2\,(\theta_1''[t]+\theta_2''[t])+2\,l_3^2\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\right.\\
&\quad 2\,(-\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,\theta_2'[t]-\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_2'[t]\,(\theta_1'[t]+\theta_2'[t])-\\
&\quad\text{Sin}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t])\,\theta_3'[t]-\text{Sin}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]\\
&\quad(\theta_2'[t]+\theta_3'[t])-\text{Sin}[\theta_3[t]]\,l_2\,l_3\,\theta_3'[t]\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])-\\
&\quad\text{Sin}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,(\theta_2'[t]+\theta_3'[t])\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])+\\
&\quad\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1''[t]+\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1''[t]+\text{Cos}[\theta_2[t]]\,l_1\,l_2\\
&\quad(\theta_1''[t]+\theta_2''[t])+\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1''[t]+\theta_2''[t])+\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\\
&\quad\left.l_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t]))\right)
\end{aligned}$$

In[250]:=
```
DlT2 = D[Dl_θ2', {t, 1}]
```

Out[250]=
$$\begin{aligned}
&Izz_2\,(\theta_1''[t]+\theta_2''[t])+\\
&\frac{1}{2}m_2\left(-2\,\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,\theta_2'[t]+2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1''[t]+\right.\\
&\quad\left.2\,l_1^2\,\theta_2''[t]+2\,l_2^2\,(\theta_1''[t]+\theta_2''[t])\right)+Izz_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\\
&\frac{1}{2}m_3\left(2\,l_1^2\,\theta_2''[t]+2\,l_2^2\,(\theta_1''[t]+\theta_2''[t])+2\,l_3^2\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\right.\\
&\quad 2\,(-\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,\theta_2'[t]-\text{Sin}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t])\,\theta_3'[t]-\\
&\quad\text{Sin}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]\,(\theta_2'[t]+\theta_3'[t])-\\
&\quad\text{Sin}[\theta_3[t]]\,l_2\,l_3\,\theta_3'[t]\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])+\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1''[t]+\\
&\quad\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1''[t]+\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1''[t]+\theta_2''[t])+\\
&\quad\left.\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t]))\right)
\end{aligned}$$

In[251]:=
```
DlT3 = D[Dlθ3', {t, 1}]
```

Out[251]=
$$\begin{aligned}&\text{Izz}_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\\&\frac{1}{2}\,m_3\,\Big(2\,(-\text{Sin}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t])\,\theta_3'[t]-\text{Sin}[\theta_2[t]+\theta_3[t]]\\&\qquad l_1\,l_3\,\theta_1'[t]\,(\theta_2'[t]+\theta_3'[t])+\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1''[t]+\\&\qquad\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1''[t]+\theta_2''[t]))+2\,l_3^2\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])\Big)\end{aligned}$$

In[252]:=
```
(*Partial first order derivative of l with respect to θi*)
Dlθ1 = D[la, {θ1[t], 1}]
```

Out[252]=
$$\begin{aligned}&-g\,\text{Cos}[\theta_1[t]]\,l_1\,m_1-g\,(\text{Cos}[\theta_1[t]]\,l_1+\text{Cos}[\theta_1[t]+\theta_2[t]]\,l_2)\,m_2-\\&g\,(\text{Cos}[\theta_1[t]]\,l_1+\text{Cos}[\theta_1[t]+\theta_2[t]]\,l_2+\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,l_3)\,m_3\end{aligned}$$

In[253]:=
```
Dlθ2 = D[la, {θ2[t], 1}]
```

Out[253]=
$$\begin{aligned}&-g\,\text{Cos}[\theta_1[t]+\theta_2[t]]\,l_2\,m_2-g\,(\text{Cos}[\theta_1[t]+\theta_2[t]]\,l_2+\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,l_3)\,m_3-\\&\text{Sin}[\theta_2[t]]\,l_1\,l_2\,m_2\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t])+\\&m_3\,(-\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t])-\\&\qquad\text{Sin}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\end{aligned}$$

In[254]:=
```
Dlθ3 = D[la, {θ3[t], 1}]
```

Out[254]=
$$\begin{aligned}&-g\,\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,l_3\,m_3+\\&m_3\,(-\text{Sin}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])-\\&\qquad\text{Sin}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t])\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t]))\end{aligned}$$

In[255]:=
```
(*τi as defined by the required derivatives of the lagrangian*)
τ1 = DlT1 - Dlθ1
```

Out[255]=
$$\begin{aligned}&g\,\text{Cos}[\theta_1[t]]\,l_1\,m_1+g\,(\text{Cos}[\theta_1[t]]\,l_1+\text{Cos}[\theta_1[t]+\theta_2[t]]\,l_2)\,m_2+\\&g\,(\text{Cos}[\theta_1[t]]\,l_1+\text{Cos}[\theta_1[t]+\theta_2[t]]\,l_2+\text{Cos}[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,l_3)\,m_3+\\&\text{Izz}_1\,\theta_1''[t]+l_1^2\,m_1\,\theta_1''[t]+\text{Izz}_2\,(\theta_1''[t]+\theta_2''[t])+\\&\frac{1}{2}\,m_2\,\Big(-2\,\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,\theta_2'[t]-2\,\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_2'[t]\,(\theta_1'[t]+\theta_2'[t])+\\&\qquad2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1''[t]+2\,\text{Cos}[\theta_2[t]]\,l_1\,l_2\,(\theta_1''[t]+\theta_2''[t])+\\&\qquad2\,l_2^2\,(\theta_1''[t]+\theta_2''[t])\Big)+\text{Izz}_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\\&\frac{1}{2}\,m_3\,\Big(2\,l_2^2\,(\theta_1''[t]+\theta_2''[t])+2\,l_3^2\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\\&\qquad2\,(-\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_1'[t]\,\theta_2'[t]-\text{Sin}[\theta_2[t]]\,l_1\,l_2\,\theta_2'[t]\,(\theta_1'[t]+\theta_2'[t])-\\&\qquad\text{Sin}[\theta_3[t]]\,l_2\,l_3\,(\theta_1'[t]+\theta_2'[t])\,\theta_3'[t]-\text{Sin}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1'[t]\\&\qquad\quad(\theta_2'[t]+\theta_3'[t])-\text{Sin}[\theta_3[t]]\,l_2\,l_3\,\theta_3'[t]\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])-\\&\qquad\text{Sin}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,(\theta_2'[t]+\theta_3'[t])\,(\theta_1'[t]+\theta_2'[t]+\theta_3'[t])+\\&\qquad\text{Cos}[\theta_2[t]]\,l_1\,l_2\,\theta_1''[t]+\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\,l_3\,\theta_1''[t]+\text{Cos}[\theta_2[t]]\,l_1\,l_2\\&\qquad\quad(\theta_1''[t]+\theta_2''[t])+\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1''[t]+\theta_2''[t])+\text{Cos}[\theta_2[t]+\theta_3[t]]\,l_1\\&\qquad\quad l_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t])+\text{Cos}[\theta_3[t]]\,l_2\,l_3\,(\theta_1''[t]+\theta_2''[t]+\theta_3''[t]))\Big)\end{aligned}$$

In[257]:=
$$\tau_2 = \text{DlT}_2 - \text{Dl}_{\theta_2}$$

Out[257]=
$$g \cos[\theta_1[t] + \theta_2[t]]\, l_2\, m_2 + g\,(\cos[\theta_1[t] + \theta_2[t]]\, l_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]]\, l_3)\, m_3 + \sin[\theta_2[t]]\, l_1\, l_2\, m_2\, \theta_1'[t]\,(\theta_1'[t] + \theta_2'[t]) - m_3\,(-\sin[\theta_2[t]]\, l_1\, l_2\, \theta_1'[t]\,(\theta_1'[t] + \theta_2'[t]) - \sin[\theta_2[t] + \theta_3[t]]\, l_1\, l_3\, \theta_1'[t]\,(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \text{Izz}_2\,(\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 \left(-2 \sin[\theta_2[t]]\, l_1\, l_2\, \theta_1'[t]\, \theta_2'[t] + 2 \cos[\theta_2[t]]\, l_1\, l_2\, \theta_1''[t] + 2\, l_1^2\, \theta_2''[t] + 2\, l_2^2\,(\theta_1''[t] + \theta_2''[t])\right) + \text{Izz}_3\,(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 \left(2\, l_1^2\, \theta_2''[t] + 2\, l_2^2\,(\theta_1''[t] + \theta_2''[t]) + 2\, l_3^2\,(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + 2\,(-\sin[\theta_2[t]]\, l_1\, l_2\, \theta_1'[t]\, \theta_2'[t] - \sin[\theta_3[t]]\, l_2\, l_3\,(\theta_1'[t] + \theta_2'[t])\, \theta_3'[t] - \sin[\theta_2[t] + \theta_3[t]]\, l_1\, l_3\, \theta_1'[t]\,(\theta_2'[t] + \theta_3'[t]) - \sin[\theta_3[t]]\, l_2\, l_3\, \theta_3'[t]\,(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) + \cos[\theta_2[t]]\, l_1\, l_2\, \theta_1''[t] + \cos[\theta_2[t] + \theta_3[t]]\, l_1\, l_3\, \theta_1''[t] + \cos[\theta_3[t]]\, l_2\, l_3\,(\theta_1''[t] + \theta_2''[t]) + \cos[\theta_3[t]]\, l_2\, l_3\,(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))\right)$$

In[258]:=
$$\tau_3 = \text{DlT}_3 - \text{Dl}_{\theta_3}$$

Out[258]=
$$g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]]\, l_3\, m_3 - m_3\,(-\sin[\theta_2[t] + \theta_3[t]]\, l_1\, l_3\, \theta_1'[t]\,(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) - \sin[\theta_3[t]]\, l_2\, l_3\,(\theta_1'[t] + \theta_2'[t])\,(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + \text{Izz}_3\,(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 \left(2\,(-\sin[\theta_3[t]]\, l_2\, l_3\,(\theta_1'[t] + \theta_2'[t])\, \theta_3'[t] - \sin[\theta_2[t] + \theta_3[t]]\, l_1\, l_3\, \theta_1'[t]\,(\theta_2'[t] + \theta_3'[t]) + \cos[\theta_2[t] + \theta_3[t]]\, l_1\, l_3\, \theta_1''[t] + \cos[\theta_3[t]]\, l_2\, l_3\,(\theta_1''[t] + \theta_2''[t])) + 2\, l_3^2\,(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])\right)$$