Kinect Energy

Kinect Energy Link I

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 \begin{aligned} & \text{In}\{i\} = \ p_1 = \{ \ L_1 \cos[\theta_1[t]] \ , & L_1 \sin[\theta_1[t]] \ , & 0 \} \\ & v_1 = D[\ p_1, \{t, 1\}] \\ & I_1 = \{ \{ Ixx_1, \ 0, \ 0 \}, \\ & \{ 0, \ Iyy_1, \ 0 \}, \\ & \{ 0, \ 0, \ Izz_1 \} \} \\ & \omega_1 = \{ 0, \\ & 0, \\ & D[\theta_1[t], \{t, 1\}] \} \\ & KE_1 = Simplify[1/2 \ m_1 \ Dot \ [v_1, v_1]] + Simplify[1/2 \ Dot \ [\omega_1, \ I_1 . \omega_1]] \\ & \text{Out}[i] = \{ \cos[\theta_1[t]] \ L_1, \ \sin[\theta_1[t]] \ L_1, \ 0 \} \\ & \text{Out}[2] = \{ -\sin[\theta_1[t]] \ L_1 \theta_1'[t], \ \cos[\theta_1[t]] \ L_1 \theta_1'[t], \ 0 \} \\ & \text{Out}[3] = \{ \{ Ixx_1, \ 0, \ 0 \}, \ \{0, \ Iyy_1, \ 0 \}, \ \{0, \ 0, \ Izz_1 \} \} \\ & \text{Out}[4] = \{ 0, \ 0, \ \theta_1'[t] \} \end{aligned}
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Kinect Energy Link 2

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\ln[6] = p_2 = \{ L_1 \cos[\theta_1[t]] + L_2 \cos[\theta_1[t]] + \theta_2[t] \},
                                                                                                  L_1 Sin[\theta_1[t]] + L_2 Sin[\theta_1[t] + \theta_2[t]],
                                           v_2 = D[p_2, \{t, 1\}]
                                             I_2 = \{ \{Ixx_2, 0, 0\}, \{0, Iyy_2, 0\}, \{0, 0, Izz_2\} \}
                                           \omega_2 = \{ \; \mathsf{0} \; , \; \; \mathsf{0} \; , \; \; \mathsf{D} [\theta_1[\mathsf{t}] \; + \; \theta_2[\mathsf{t}] \; , \; \{\mathsf{t} \; , \; 1\}] \; \}
                                           \texttt{KE}_2 = \texttt{Simplify} [\texttt{1/2} \ \texttt{m}_2 \ \texttt{Dot} \ [\texttt{v}_2 \,,\, \texttt{v}_2] \,] \ + \\ \texttt{Simplify} [\texttt{1/2} \ \texttt{Dot} \ [\omega_2 \,,\, \texttt{I}_2 \,.\omega_2] \,]
        \text{Out[G]= } \left\{ \text{Cos}\left[\theta_1[\texttt{t}]\right] \text{ L}_1 + \text{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]\right] \text{ L}_2 \text{, } \text{Sin}\left[\theta_1[\texttt{t}]\right] \text{ L}_1 + \text{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]\right] \text{ L}_2 \text{, } 0 \right\} 
       \text{Out}[7] = \left\{-\sin\left[\theta_1\left[t\right]\right] \; L_1 \; \theta_1{'}[t] \; - \sin\left[\theta_1\left[t\right] + \theta_2\left[t\right]\right] \; L_2 \; \left(\theta_1{'}[t] + \theta_2{'}[t]\right) \text{,} \right.
                                                      \texttt{Cos}[\theta_{1}[\texttt{t}]] \; \texttt{L}_{1} \; \theta_{1}{'}[\texttt{t}] \; + \; \texttt{Cos}[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]] \; \texttt{L}_{2} \; (\theta_{1}{'}[\texttt{t}] \; + \; \theta_{2}{'}[\texttt{t}]) \; \text{, 0} \}
       Out[8]= \{\{Ixx_2, 0, 0\}, \{0, Iyy_2, 0\}, \{0, 0, Izz_2\}\}
       Out[9]= \{0, 0, \theta_1'[t] + \theta_2'[t]\}
Out[10]= \frac{1}{2} \text{Izz}_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                      \frac{1}{2}\, \mathrm{m_2}\, \left( \, \left( \, \mathrm{Cos} \left[ \theta_1 \left[ \, \mathrm{t} \, \right] \, \right] \, \mathrm{L_1} \, \theta_1{}' \left[ \, \mathrm{t} \, \right] \, + \, \mathrm{Cos} \left[ \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2 \left[ \, \mathrm{t} \, \right] \, \right] \, \mathrm{L_2} \, \left( \, \theta_1{}' \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right) \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \theta_2{}' \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right]^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right]^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right)^2 + \, \mathrm{Cos} \left[ \, \theta_1 \left[ \, \mathrm{t} \, \right] \, \right)^2 + \,
                                                                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))^2)
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Kinect Energy Link 3

```
\ln[16] = p_3 = \{ L_1 \cos[\theta_1[t]] + L_2 \cos[\theta_1[t]] + \theta_2[t] \} + L_3 \cos[\theta_1[t]] + \theta_2[t] + \theta_3[t] \},
                                                          L_1 \sin[\theta_1[t]] + L_2 \sin[\theta_1[t]] + \theta_2[t]] + L_3 \sin[\theta_1[t]] + \theta_2[t] + \theta_3[t]],
                        v_3 = D[p_3, \{t, 1\}]
                          I_3 = \{ \{ Ixx_3, 0, 0 \}, \}
                                                     {0, Iyy3, 0},
                                                      {0, 0, Izz<sub>3</sub>}}
                        \omega_3 = \{ 0,
                                                         D[\theta_1[t] + \theta_2[t] + \theta_3[t], \{t, 1\}]
                         KE_3 = Simplify[1/2 m_3 Dot[v_3, v_3]] + Simplify[1/2 Dot[\omega_3, I_3.\omega_3]]
\text{Out[16]= } \left\{ \text{Cos}\left[\theta_1[\texttt{t}]\right] \text{ L}_1 + \text{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]\right] \text{ L}_2 + \text{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \text{ L}_3 \text{, } \right\}
                               Sin[\theta_1[t]] L_1 + Sin[\theta_1[t] + \theta_2[t]] L_2 + Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3, 0
\text{Out}[17] = \left\{-\sin\left[\theta_{1}[t]\right] \text{ L}_{1} \; \theta_{1}{'}[t] \; -\sin\left[\theta_{1}[t] + \theta_{2}[t]\right] \text{ L}_{2} \; \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) \; -\right\}
                                     Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]),
                                Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                    \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right) \text{, 0} \}
Out[18]= \{\{Ixx_3, 0, 0\}, \{0, Iyy_3, 0\}, \{0, 0, Izz_3\}\}
Out[19]= \{0, 0, \theta_1'[t] + \theta_2'[t] + \theta_3'[t]\}
Out[20]= \frac{1}{2} Izz<sub>3</sub> (\Theta_1'[t] + \Theta_2'[t] + \Theta_3'[t])^2 +
                               \frac{1}{2} m_3 \left( (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \frac{1}{2} m_3 (\cos[\theta_1[t]] L_2 (\theta_1'[t] + \theta_1'[t]) + \frac{1}{2} m_3 (\cos[\theta_1[t]] L_2 (\theta_1'[t] + \theta_1'[t]) +
                                                                  Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))^{2} +
                                                  (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                 Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2
```

Total Kinect Energy

```
ln[21]:= KE = KE_1 + KE_2 + KE_3
Out[21]= \frac{1}{2} \operatorname{Izz}_1 \Theta_1'[t]^2 + \frac{1}{2} \operatorname{L}_1^2 \operatorname{m}_1 \Theta_1'[t]^2 + \frac{1}{2} \operatorname{Izz}_2 (\Theta_1'[t] + \Theta_2'[t])^2 + \frac{1}{2} \operatorname{Izz}_2 (\Theta_1'[t] + \Theta_1'[t])^2 + \frac{
                                                                                                              \frac{1}{2} m_2 \left( \left( \text{Cos}\left[\theta_1[t]\right] \text{L}_1 \theta_1'[t] + \text{Cos}\left[\theta_1[t] + \theta_2[t]\right] \text{L}_2 \left(\theta_1'[t] + \theta_2'[t]\right) \right)^2 + \frac{1}{2} m_2 \left( \left( \text{Cos}\left[\theta_1[t]\right] \text{L}_1 \theta_1'[t] + \frac{1}{2} \text{Cos}\left[\theta_1[t] + \frac{1}{2} \text{Cos}\left[\theta_1[t]\right] + \frac{1}{2} \text{Cos}\left[\theta_1[t] + \frac
                                                                                                                                                                            (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))^2) +
                                                                                                                   \frac{1}{2} \, {\rm Izz}_3 \, \left( \theta_1{'}[{\rm t}] + \theta_2{'}[{\rm t}] + \theta_3{'}[{\rm t}] \right)^2 +
                                                                                                                \frac{1}{2}\,{\rm m}_{3}\,\left(\,\left({\rm Cos}\left[\theta_{1}\left[{\rm t}\right]\right]\,{\rm L}_{1}\,\theta_{1}{'}\left[{\rm t}\right]\,+{\rm Cos}\left[\theta_{1}\left[{\rm t}\right]\,+\theta_{2}\left[{\rm t}\right]\right]\,{\rm L}_{2}\,\left(\theta_{1}{'}\left[{\rm t}\right]\,+\theta_{2}{'}\left[{\rm t}\right]\right)\,+\right.
                                                                                                                                                                                                                                      Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2 +
                                                                                                                                                                               \left( \operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right]\right] \, \operatorname{L}_{1} \, \theta_{1}{}'\left[\mathtt{t}\right] + \operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right] \, \operatorname{L}_{2} \, \left(\theta_{1}{}'\left[\mathtt{t}\right] + \theta_{2}{}'\left[\mathtt{t}\right]\right) \, + \right.
                                                                                                                                                                                                                                    Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))^2
```

Potential Energy

Potential Energy Link I

```
\ln[22]:= \mathbf{PE_1} = \mathbf{m_1} \mathbf{g} \mathbf{L_1} \mathbf{Sin} [\theta_1[\mathbf{t}]]
Out[22]= g Sin[\theta_1[t]] L_1 m_1
```

Potential Energy Link 2

```
\ln[23] := PE_2 = m_2 g (L_1 Sin[\theta_1[t]] + L_2 Sin[\theta_1[t]] + \theta_2[t]])
Out[23]= g (Sin[\theta_1[t]] L<sub>1</sub> + Sin[\theta_1[t] + \theta_2[t]] L<sub>2</sub>) m<sub>2</sub>
```

Potential Energy Link 3

```
\ln[24] = \mathbf{PE}_3 = \mathbf{m}_3 \mathbf{g} \left( \mathbf{L}_1 \operatorname{Sin}[\theta_1[t]] + \mathbf{L}_2 \operatorname{Sin}[\theta_1[t] + \theta_2[t]] + \mathbf{L}_3 \operatorname{Sin}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \right)
\texttt{Out}[24] = \texttt{g} \left( \texttt{Sin} \left[ \theta_1 \left[ \texttt{t} \right] \right] \texttt{L}_1 + \texttt{Sin} \left[ \theta_1 \left[ \texttt{t} \right] + \theta_2 \left[ \texttt{t} \right] \right] \texttt{L}_2 + \texttt{Sin} \left[ \theta_1 \left[ \texttt{t} \right] + \theta_2 \left[ \texttt{t} \right] + \theta_3 \left[ \texttt{t} \right] \right] \texttt{L}_3 \right) \, \texttt{m}_3
```

Total Potential Energy

```
ln[25]:= PE = PE_1 + PE_2 + PE_3
\text{Out} \text{[25]= g Sin} \left[\theta_1 \left[\texttt{t}\right]\right] \text{ L}_1 \text{ m}_1 + \text{g (Sin} \left[\theta_1 \left[\texttt{t}\right]\right] \text{ L}_1 + \text{Sin} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2) \text{ m}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{constant} \left[\theta_1 \left[\texttt{t}\right] + \theta_2 \left[\texttt{t}\right]\right] \text{ L}_2 + \text{
                                                                                                                                                                                    \texttt{g} \; (\texttt{Sin}[\theta_1[\texttt{t}]] \; \texttt{L}_1 + \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \; \texttt{L}_2 + \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]] \; \texttt{L}_3) \; \texttt{m}_3
```

Lagrangian Partial Derivatives

```
\left\{ g \cos \left[\theta_{1}[t] + \theta_{2}[t]\right] \right] L_{2} m_{2} + g \left(\cos \left[\theta_{1}[t] + \theta_{2}[t]\right] L_{2} + \cos \left[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]\right] L_{3}\right) m_{3} - \left[\cos \left[\theta_{1}[t] + \theta_{2}[t]\right] L_{2} + \cos \left[\theta_{1}[t] + \theta_{2}[t]\right] L_{3}\right] m_{3} - \left[\cos \left[\theta_{1}[t] + \theta_{2}[t]\right] m_{3}\right] m_{3} - \left[\cos \left[\theta_{1}[t] + \theta_{2}[
                                                                                                                                       \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
```

```
(\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                 2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                             \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}\left[t\right] \; L_{2} 
         \frac{1}{2}\,\mathrm{m_3}\,\left(2\,\left(\mathrm{Cos}\left[\theta_1[\mathrm{t}]\right]\,\mathrm{L}_1\,\theta_1{'}[\mathrm{t}] + \mathrm{Cos}\left[\theta_1[\mathrm{t}] + \theta_2[\mathrm{t}]\right]\,\mathrm{L}_2\,\left(\theta_1{'}[\mathrm{t}] + \theta_2{'}[\mathrm{t}]\right) + \right.
                                                            \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                             (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                    L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2(\cos[\theta_1[t] + \theta_2[t]) L_2(\theta_1'[t] + \theta_2'[t]) +
                                                            \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right))
                                             \left( \text{Sin} \left[ \theta_{1}[t] \right] \right. \left. \text{L}_{1} \right. \theta_{1}{'}[t] + \text{Sin} \left[ \theta_{1}[t] + \theta_{2}[t] \right] \right. \left. \text{L}_{2} \left. \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{1} \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{2} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{2} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{3} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{4} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{5} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{5} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{6} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{5} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{6} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left.
                                                              \sin \left[ \theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t] \right] \, L_{3} \, \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t] \right))) \, + \,
      1[t] Izz_{2} (\theta_{1}''[t] + \theta_{2}''[t]) + \frac{1}{2} m_{2} (-21[t] Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])
                                             \left( \text{Cos} \left[ \theta_{1}[t] \right] \right. \left. \text{L}_{1} \right. \theta_{1}{'}[t] + \text{Cos} \left[ \theta_{1}[t] + \theta_{2}[t] \right] \right. \left. \text{L}_{2} \left. \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) \right) + \\
                                 2\times1\texttt{[t]}\;\texttt{Cos}\left[\theta_{1}\texttt{[t]}+\theta_{2}\texttt{[t]}\right]\;\texttt{L}_{2}\;\left(\theta_{1}{'}\texttt{[t]}+\theta_{2}{'}\texttt{[t]}\right)
                                             (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                                             Sin[\theta_1[t] + \theta_2[t]] \ L_2 \ (\theta_1{'}[t] + \theta_2{'}[t])^2 + Cos[\theta_1[t]] \ L_1 \ \theta_1{''}[t] +
                                                            Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t]) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]]
                                        L_2 \left( \cos \left[ \theta_1[t] \right] L_1 \theta_1'[t]^2 + \cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right]^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 + Co
                                                             Sin[\theta_1[t]] L_1 \theta_1^{\prime\prime\prime}[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1^{\prime\prime\prime}[t] + \theta_2^{\prime\prime\prime}[t]))) +
       1[t] Izz<sub>3</sub> (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3
                  (2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                            1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                             (Sin[\theta_{1}[t]] \ L_{1} \ \theta_{1}{'}[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] \ L_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \ +
                                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                  2 \left( \text{Cos}[\theta_{1}[t]] \; \text{L}_{1} \; \theta_{1}{'}[t] + \text{Cos}[\theta_{1}[t] + \theta_{2}[t]] \; \text{L}_{2} \; (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \; + \right. \\
                                                            \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                             (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                             1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                  2 \; (1[t] \; \mathsf{Cos}[\theta_1[t] \; + \; \theta_2[t]] \; \mathsf{L}_2 \; + \; 1[t] \; \mathsf{Cos}[\theta_1[t] \; + \; \theta_2[t] \; + \; \theta_3[t]] \; \mathsf{L}_3)
                                             \left(-\sin[\theta_{1}[t]] \; L_{1} \; \theta_{1}{'}[t]^{2} \; - \; \sin[\theta_{1}[t] \; + \; \theta_{2}[t]] \; L_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{1} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \; \theta_{2}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin[\theta_{1}[t]] \; L_{2} \;
                                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                            Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                 2\;(1[t]\;Sin[\theta_{1}[t]+\theta_{2}[t]]\;L_{2}+1[t]\;Sin[\theta_{1}[t]+\theta_{2}[t]+\theta_{3}[t]]\;L_{3})
                                             (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                            \texttt{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]] \; \texttt{L}_{3} \; (\theta_{1}{}'[\texttt{t}] + \theta_{2}{}'[\texttt{t}] + \theta_{3}{}'[\texttt{t}])^{2} + \\
                                                            \operatorname{Sin}[\theta_1[\mathsf{t}]] \ \operatorname{L}_1 \theta_1^{\prime\prime\prime}[\mathsf{t}] + \operatorname{Sin}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime\prime}[\mathsf{t}] + \theta_2^{\prime\prime\prime}[\mathsf{t}]) + \\
                                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
\left\{-g\cos\left[\theta_{1}[t]+\theta_{2}[t]\right] \right. L_{2} \left. m_{2}-g\left(\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]\right] \right. L_{2} + \cos\left[\theta_{1}[t]+\theta_{2}[t]\right] + \theta_{3}[t]\right\}
                                                          L_3) m_3 - \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                             \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \;
                                                   2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                              \frac{1}{2} m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                              (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3
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(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                  Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                            (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                    Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] Izz_2
                       (\theta_{1}''[t] + \theta_{2}''[t]) + \frac{1}{2} m_{2} \left(-21[t] \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]) + \frac{1}{2} m_{2} (\theta_{
                                                            \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{L}_{2} \; \mathsf{L}_{3} \; \mathsf{L}_{4} \; \mathsf{L}_{5} \; \mathsf{L}
                                              2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t])
                                                            \left( \text{Sin} \left[ \theta_{1} \left[ t \right] \right] \; \text{L}_{1} \; \theta_{1}{'} \left[ t \right] \; + \; \text{Sin} \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \right] \; \text{L}_{2} \; \left( \theta_{1}{'} \left[ t \right] \; + \; \theta_{2}{'} \left[ t \right] \right) \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \right] \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \right] \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \right] \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \; \right) \; \right) \; + \; \left( \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ 
                                              2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                                                                  Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                                                                  Cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}''[t] + \theta_{2}''[t])) + 2 \times 1[t] Sin[\theta_{1}[t] + \theta_{2}[t]]
                                                         L_2 \left( Cos[\theta_1[t]] L_1 \theta_1'[t]^2 + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_2 (\theta_1'[t])^2 + Cos[\theta_1[t]
                                                                                  Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) +
          1[t] Izz<sub>3</sub> (\theta_1'''[t] + \theta_2'''[t] + \theta_3'''[t]) + \frac{1}{2} m_3
                          (2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                  1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                            (Sin[\theta_{1}[t]] \ L_{1} \ \theta_{1}{'}[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] \ L_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \ +
                                                                                 \sin \left[ \theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t] \right] \, L_{3} \, \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t] \right)) \, + \,
                                               2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                 \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                                           (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                                  \texttt{1[t]} \; \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]] \; \texttt{L}_3 \; (\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}])) \; + \\
                                              2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                           \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{1} \theta_{1}'[t] + \theta_{2}'[t] + \theta_{2}'[t] + \theta_{2}'[t] + \theta_{2}'[t] + \theta_{2}'[t] + \theta_{2}'[t] 
                                                                                  Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                 \texttt{Cos}\left[\theta_{1}[\texttt{t}]\right] \; \texttt{L}_{1} \; \theta_{1}^{\prime\prime}[\texttt{t}] \; + \; \texttt{Cos}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \texttt{L}_{2} \; \left(\theta_{1}^{\prime\prime}[\texttt{t}] \; + \; \theta_{2}^{\prime\prime}[\texttt{t}]\right) \; + \;
                                                                                 \texttt{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]] \; \texttt{L}_{3} \; (\theta_{1}{''}[\texttt{t}] + \theta_{2}{''}[\texttt{t}] + \theta_{3}{''}[\texttt{t}]) \, \big) \; + \\
                                              2 \; (1[t] \; Sin[\theta_1[t] + \theta_2[t]] \; L_2 + 1[t] \; Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; L_3)
                                                            (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                  Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                  Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                 Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
\left\{-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2\operatorname{m}_2-\operatorname{g} \; (\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]\right\} \right\} + \left\{-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]\right\} \right\} + \left\{-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]\right\} \right\} + \left\{-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]
                                                                                                     \theta_3[t] L<sub>3</sub>) m<sub>3</sub> - \frac{1}{2} m<sub>2</sub> (-2 Sin[\theta_1[t] + \theta_2[t]] L<sub>2</sub> (\theta_1'[t] + \theta_2'[t])
                                                                                    (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                       2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \theta_2'[t])
                                                                                                         Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) -
                                  \frac{1}{2} m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                        Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                     (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                                                                     \mathtt{L_{3}}\ ({\theta_{1}}'[\texttt{t}] + {\theta_{2}}'[\texttt{t}] + {\theta_{3}}'[\texttt{t}])) + 2\ (\mathsf{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}]]\ \mathtt{L_{2}}\ ({\theta_{1}}'[\texttt{t}] +
                                                                                                                                            \theta_{2}'[t]) + \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                                                                                      \left(\operatorname{Sin}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \operatorname{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \operatorname{Sin}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \operatorname{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \; + \right.
                                                                                                         \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right] + \theta_3\left[\texttt{t}\right]\right] \; \texttt{L}_3 \; \left(\theta_1{'}\left[\texttt{t}\right] + \theta_2{'}\left[\texttt{t}\right] + \theta_3{'}\left[\texttt{t}\right]\right)\right) ) \; + \; \texttt{1}\left[\texttt{t}\right] \; \texttt{Izz}_2
                                                 (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-21[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                      \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \;
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2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] + \theta_2'[t])
                                                           \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \; \texttt{L}_2 \; (\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}])) \; + \; 2 \times \texttt{1}[\texttt{t}] \; \texttt{Cos}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]]
                                         L_2 \left( -\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \right)
                                                           Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times
                                         1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]]
                                                                    L_2 (\theta_1'[t] + \theta_2'[t])^2 + Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]]
                                                                   L_2(\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
       \frac{1}{2} m_3 \left( 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + 1[t] \right)
                                                                    Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                                           (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                  2 \left( \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \right. \left. \mathsf{L}_{1} \right. \theta_{1}{'}[\mathsf{t}] + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}]\right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \left. \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. \right. + \\ \left. \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. + \\ \left. \mathsf{L}_{4} \left.
                                                           \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]\right] \texttt{L}_{3} \left(\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}]\right))
                                            (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                           1[t] \sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])) +
                                  2 \ (1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t]] \ \mathtt{L}_2 + 1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \ \mathtt{L}_3)
                                            \left(-\sin\left[\theta_{1}[\mathsf{t}]\right]\,L_{1}\,\theta_{1}{'}[\mathsf{t}]^{\,2}-\sin\left[\theta_{1}[\mathsf{t}]\right]+\theta_{2}[\mathsf{t}]\right]\,L_{2}\,\left(\theta_{1}{'}[\mathsf{t}]\right]+\theta_{2}{'}[\mathsf{t}]\right)^{\,2}-
                                                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                           \texttt{Cos}[\theta_{1}[\texttt{t}]] \ \texttt{L}_{1} \ \theta_{1} "[\texttt{t}] + \texttt{Cos}[\theta_{1}[\texttt{t}]] + \theta_{2}[\texttt{t}]] \ \texttt{L}_{2} \ (\theta_{1} "[\texttt{t}] + \theta_{2} "[\texttt{t}]) + \theta_{2} "[\texttt{t}]) + \theta_{3} "[\texttt{t}] + \theta_{4} "[\texttt{t}] + \theta_{5} "[\texttt{t}] + \theta_{5} "[\texttt{t}]) + \theta_{5} "[\texttt{t}] + \theta_{5} "
                                                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                  2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                            \left( \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}]^{2} + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right)^{2} + \right.
                                                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                           \operatorname{Sin}[\theta_1[\mathsf{t}]] \ \operatorname{L}_1 \theta_1^{\prime\prime\prime}[\mathsf{t}] + \operatorname{Sin}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime\prime}[\mathsf{t}] + \theta_2^{\prime\prime\prime}[\mathsf{t}]) + \\
                                                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
-g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 +
                                 Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 -
       \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\cos[\theta_1[t]] L_1 \theta_1'[t] + \theta_2'[t])
                                                           Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                  2 \cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t]) (\sin[\theta_1[t]] L_1 \theta_1'[t] +
                                                           Sin[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t])) -
       \frac{1}{2}\,\mathrm{m_3}\,\left(2\,\left(\mathrm{Cos}\left[\theta_1[\mathsf{t}]\right]\,\mathrm{L}_1\,\theta_1{'}[\mathsf{t}] + \mathrm{Cos}\left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]\right]\,\mathrm{L}_2\,\left(\theta_1{'}[\mathsf{t}] + \theta_2{'}[\mathsf{t}]\right) + \right.
                                                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                            (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                    \mathtt{L_{3}} \ (\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}])) + 2 \ (\mathtt{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}]] \ \mathtt{L_{2}} \ (\theta_{1}{'}[\texttt{t}] + \theta_{2}[\texttt{t}]) + \theta_{2}(\theta_{1}{'}[\texttt{t}]) 
                                                                                     \theta_{2}'[t]) + \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                                            \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; \text{L}_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right) \; + \right.
                                                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] Izz_2
                (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 \left(-21[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])\right)
                                            \left( \text{Cos} \left[ \theta_{1}[t] \right] \right. \right. \left. \text{L}_{1} \left. \theta_{1}{}'[t] + \text{Cos} \left[ \theta_{1}[t] + \theta_{2}[t] \right] \right. \left. \text{L}_{2} \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right) \right. + \\ \left. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right) \right. + \\ \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right) \right. + \\ \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right) \right. + \\ \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. + \\ \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] \right) \right] \right. \left. \left( \theta_{1}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] \right) \right. \left. \left( \theta_{1}{}'[t] \right) \right] \right. \left. \left
                                 2\times1\texttt{[t]}\;\texttt{Cos}\left[\theta_{1}\texttt{[t]}+\theta_{2}\texttt{[t]}\right]\;\texttt{L}_{2}\;\left(\theta_{1}'\texttt{[t]}+\theta_{2}'\texttt{[t]}\right)\;\left(\texttt{Sin}\left[\theta_{1}\texttt{[t]}\right]\;\texttt{L}_{1}\;\theta_{1}'\texttt{[t]}+\theta_{2}'\texttt{[t]}\right)
                                                           \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \; \texttt{L}_2 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}]\right)) \; + \; 2 \times \texttt{1}[\texttt{t}] \; \texttt{Cos}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]]
                                         L_2 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \right)
                                                           Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times
                                          1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (Cos[\theta_1[t]] L_1 \theta_1'[t]^2 + Cos[\theta_1[t] + \theta_2[t]]
                                                                    L_2 (\theta_1'[t] + \theta_2'[t])^2 + Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]]
                                                                    L_2(\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
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\frac{1}{2} m_3 \left( 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + 1[t] \right)
                                                                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                        (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                      Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \ L_{3} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t])) \ +
                                               2 \left( \cos \left[ \theta_{1}[t] \right] \right] L_{1} \theta_{1}{'}[t] + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) + \\
                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                        (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                      1[t] \, \operatorname{Sin}[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \, \operatorname{L}_{3} \, (\theta_{1}{}'[t] + \theta_{2}{}'[t] + \theta_{3}{}'[t])) \, + \,
                                               2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                        \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{1} \theta_{1}'[t] + \theta_{2}'[t] + \theta_{2}'[t] \right)^{2} - \theta_{1}'[t] + \theta_{2}'[t] + \theta_{
                                                                      Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                      \mathsf{Cos}[\theta_{1}[\mathsf{t}]] \; \mathsf{L}_{1} \; \theta_{1}{''}[\mathsf{t}] \; + \; \mathsf{Cos}[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]] \; \mathsf{L}_{2} \; (\theta_{1}{''}[\mathsf{t}] \; + \; \theta_{2}{''}[\mathsf{t}]) \; + \;
                                                                      Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                               2 \; (1[t] \; Sin[\theta_1[t] + \theta_2[t]] \; L_2 + 1[t] \; Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; L_3)
                                                        (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                      \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                      \operatorname{Sin}[\theta_1[\mathsf{t}]] \ \operatorname{L}_1 \theta_1^{\prime\prime\prime}[\mathsf{t}] + \operatorname{Sin}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime\prime}[\mathsf{t}] + \theta_2^{\prime\prime\prime}[\mathsf{t}]) + \\
                                                                      Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
                -\operatorname{g} \operatorname{Cos} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_3[\mathsf{t}]\right] \operatorname{L}_3 \operatorname{m}_3 - \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]\right] + \frac{1}{2} \operatorname{m}_3 \left(-2 \operatorname{Sin} \left[\theta_1[\mathsf{t}
                                                                      \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                        \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\right. \left.\theta_{1}{'}[\mathsf{t}]\right. + \left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right. + \left.\theta_{2}[\mathsf{t}]\right.\right]\right. \left.\mathsf{L}_{2}\right. \left.\left(\theta_{1}{'}[\mathsf{t}]\right. + \left.\theta_{2}{'}[\mathsf{t}]\right.\right) + \\
                                                                      Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                               2 \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                        \left(\text{Sin}\left[\theta_{1}\left[\texttt{t}\right]\right] \; L_{1} \; \theta_{1}{'}\left[\texttt{t}\right] + \text{Sin}\left[\theta_{1}\left[\texttt{t}\right] + \theta_{2}\left[\texttt{t}\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[\texttt{t}\right] + \theta_{2}{'}\left[\texttt{t}\right]\right) \; + \right.
                                                                      \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right))) \; + \;
                       1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3
                                 \left(-2\,\mathbf{1}[\mathsf{t}]\,\operatorname{Sin}[\theta_1[\mathsf{t}]+\theta_2[\mathsf{t}]+\theta_3[\mathsf{t}]\right)\,L_3\,\left(\theta_1{}'[\mathsf{t}]+\theta_2{}'[\mathsf{t}]+\theta_3{}'[\mathsf{t}]\right)
                                                        \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \; + \right.
                                                                     \texttt{Cos} \left[ \theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}] \right] \, \texttt{L}_{3} \, \left( \theta_{1}{}'[\texttt{t}] + \theta_{2}{}'[\texttt{t}] + \theta_{3}{}'[\texttt{t}] \right) ) \, + \,
                                               2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                        \left( \operatorname{Sin}[\theta_1[t]] \right. \left. \operatorname{L}_1 \theta_1{}'[t] + \operatorname{Sin}[\theta_1[t] + \theta_2[t]] \right. \left. \operatorname{L}_2 \left( \theta_1{}'[t] + \theta_2{}'[t] \right) \right. + \\
                                                                      \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]] \; \texttt{L}_3 \; (\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}])) \; + \;
                                               2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_1 \theta_1' [t]^2 - \cos[\theta_1[t]] L_1 \theta_1' [t]^2 - \cos[\theta_1[t]] L_2 \theta_1' [t]^2 - \cos[\theta_1[t]] L_3 \theta_1' - \cos[\theta_1[t]] L_3 \theta_
                                                                      Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                            L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                                                      \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}]\right] \; \texttt{L}_{2} \; \left(\theta_{1}^{\;\prime\prime}[\texttt{t}] + \theta_{2}^{\;\prime\prime}[\texttt{t}]\right) \; + \; \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]\right]
                                                                             L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                       L_{3} \left( \cos \left[ \theta_{1}[t] \right] L_{1} \theta_{1}{'}[t]^{2} + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right)^{2} + \right. 
                                                                      Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                      \operatorname{Sin}[\theta_1[\mathtt{t}]] \ \operatorname{L}_1 \theta_1^{\prime\prime\prime}[\mathtt{t}] + \operatorname{Sin}[\theta_1[\mathtt{t}] + \theta_2[\mathtt{t}]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime\prime}[\mathtt{t}] + \theta_2^{\prime\prime\prime}[\mathtt{t}]) + \\
                                                                      Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])))
-g \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} m_{3} - \frac{1}{2} m_{3} (-2 \sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]])
                                      L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                       \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] \; + \; \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] \; + \; \theta_{2}{'}\left[\mathsf{t}\right]\right) \; + \\
                                                       Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                               2 \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                        (Sin[\theta_{1}[t]] \ L_{1} \ \theta_{1}{'}[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] \ L_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \ +
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Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
                    1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3
                             (-21[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                 \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\left.\theta_{1}{'}[\mathsf{t}]\right. + \left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right. + \theta_{2}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{2}\left.\left(\theta_{1}{'}[\mathsf{t}]\right. + \theta_{2}{'}[\mathsf{t}]\right)\right. + \\ \left.\left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\left.\theta_{1}{'}[\mathsf{t}]\right. + \left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. + \left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{2}\left.\left(\theta_{1}{'}[\mathsf{t}]\right)\right. + \left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \right] \right. \\ \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right]\right. \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. \right] \right. \\ \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. \right] \right] \right. \\ \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. \right] \right) \right. \\ \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. \right] \right] \right. \\ \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\right\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{
                                                            Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])) +
                                         2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                 (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                             \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right] + \theta_3\left[\texttt{t}\right]\right] \; \texttt{L}_3 \; \left(\theta_1{'}\left[\texttt{t}\right] + \theta_2{'}\left[\texttt{t}\right] + \theta_3{'}\left[\texttt{t}\right]\right)\right) \; + \;
                                         2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_1 \theta_1'[t]
                                                            Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} -
                                                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                             Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                         2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 +
                                                             Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                             Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
g Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L<sub>3</sub> m<sub>3</sub> - \frac{1}{2}
             m_3
              (-2)
                                Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                 L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                   \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}] \; + \; \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right) \; + \;
                                               \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right)) \; + \;
                           2 \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                    (Sin[\theta_{1}[t]] \ L_{1} \ \theta_{1}{'}[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] \ L_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \ +
                                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
       1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
         2
            m_3
               (-21[t]
                                 Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                   \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \; + \right.
                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                           2 \times 1[t] \; \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; \mathsf{L}_3 \; (\theta_1{}'[t] + \theta_2{}'[t] + \theta_3{}'[t])
                                    (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                               \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right] + \theta_3\left[\texttt{t}\right]\right] \; \texttt{L}_3 \; \left(\theta_1{'}\left[\texttt{t}\right] + \theta_2{'}\left[\texttt{t}\right] + \theta_3{'}\left[\texttt{t}\right]\right)\right) \; + \\
                           2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                               Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                               \texttt{Cos}\left[\theta_{1}[\texttt{t}]\right] \; \texttt{L}_{1} \; \theta_{1}^{\,\prime\prime}[\texttt{t}] \; + \; \texttt{Cos}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \texttt{L}_{2} \; \left(\theta_{1}^{\,\prime\prime}[\texttt{t}] \; + \; \theta_{2}^{\,\prime\prime}[\texttt{t}]\right) \; + \;
                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                           2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 +
                                               Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                               Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
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Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 + g (\cos[\theta_1[t] + \theta_2[t]] L_2 +
                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 -
              \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                 \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \, \mathsf{L}_{1}\left.\theta_{1}{'}[\mathsf{t}]\right. + \left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right. + \left.\theta_{2}[\mathsf{t}]\right.\right] \, \mathsf{L}_{2}\left.\left(\theta_{1}{'}[\mathsf{t}]\right. + \left.\theta_{2}{'}[\mathsf{t}]\right.\right)\right) \, + \\
                           2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                 (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) -
              \frac{-}{2} m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                          Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                                 (-Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                          Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                           2 (Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                          Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                 \left( \text{Sin}\left[\theta_{1}\left[\mathtt{t}\right]\right] \; \mathbf{L}_{1} \; \theta_{1}{'}\left[\mathtt{t}\right] + \text{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right] \; \mathbf{L}_{2} \; \left(\theta_{1}{'}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right]\right) \; + \right.
                                          Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
            1[t] Izz<sub>2</sub> (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2}
                 m_2
                   \left(-21[t] \sin[\theta_1[t] + \theta_2[t]]\right)
                                L_2 (\theta_1'[t] + \theta_2'[t])
                                 \left( \text{Cos} \left[ \theta_{1}[t] \right] \right. \left. \text{L}_{1} \right. \theta_{1}{'}[t] + \text{Cos} \left[ \theta_{1}[t] + \theta_{2}[t] \right] \right. \left. \text{L}_{2} \left. \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) \right) + \\
                           2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t])
                                (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                           2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2
                                 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                          Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                           2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 +
                                          Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t])^2 + Sin[\theta_1[t]] L_1\theta_1''[t] +
                                          Sin[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t])) +
             1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
              2
                 m_3
                   (2 (1[t] \cos[\theta_1[t] + \theta_2[t]) L_2 (\theta_1'[t] + \theta_2'[t]) +
                                          1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                 (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                          Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                            2 \left( \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \right. \left. \mathsf{L}_{1} \right. \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right. + \\ \left. \left. \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right] \right) \right. \\ \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right. \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right] \right] \right. \\ \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \right] \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \right] \right. \\ \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \right] \right. \\ \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \right] \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \right. \\ \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \right] \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \right] \right. \\ \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \left. \mathsf{L}_{5} \left(\theta_{1}{'}\left[\mathsf{t}\right] \right] \right
                                         Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                 (-1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                          1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                            2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                                          Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                          Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                          Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                           2 \; (1[t] \; Sin[\theta_1[t] + \theta_2[t]] \; L_2 + 1[t] \; Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; L_3)
                                 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
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Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                        \operatorname{Sin}[\theta_1[\mathtt{t}]] \ \operatorname{L}_1 \theta_1^{\prime\prime}[\mathtt{t}] + \operatorname{Sin}[\theta_1[\mathtt{t}] + \theta_2[\mathtt{t}]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime}[\mathtt{t}] + \theta_2^{\prime\prime}[\mathtt{t}]) + \\
                                                                        Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
\left\{-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2\operatorname{m}_2-\operatorname{g} \; (\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\right. \\ \left. + \left. \left(\operatorname{cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\left(\operatorname{cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]\right) \right] \right\} \right\} \\ \left\{-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2\operatorname{m}_2-\operatorname{g} \; (\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2+\left(\operatorname{cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]\right) \right] \right\} \\ \left\{-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2\operatorname{m}_2-\operatorname{g} \; (\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]]) \; \operatorname{L}_2+\left(\operatorname{cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]]\right) \right\} \right\} \\ \left\{-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2\operatorname{m}_2-\operatorname{g} \; (\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]]) \; \operatorname{L}_2+\left(\operatorname{cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]]\right) \right\} \\ \left\{-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2\operatorname{m}_2-\operatorname{g} \; (\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]]) \; \operatorname{L}_2+\left(\operatorname{cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]]\right) \right\} \\ \left[-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2\operatorname{m}_2-\operatorname{g} \; (\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]]) \; \operatorname{L}_2+\left(\operatorname{cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]]\right) \right] \\ \left[-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]] \; \operatorname{L}_2\operatorname{m}_2-\operatorname{g} \; (\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]]) \; \operatorname{L}_2+\left(\operatorname{cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]]\right) \right\} \\ \left[-\operatorname{g}\operatorname{Cos}[\theta_1[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]+\theta_2[\operatorname{t}]
                                                              Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3}) m_{3} -
                                 \frac{1}{-} m_2 \left(-2 \operatorname{Sin}[\theta_1[t] + \theta_2[t]] \operatorname{L}_2 \left(\theta_1'[t] + \theta_2'[t]\right)\right)
                                                                            (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                              2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                           \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; L_{2} \; \left(\theta_{1}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] 
                                 \frac{1}{2} m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                           (-\sin[\theta_1[t]+\theta_2[t]] \; L_2 \; (\theta_1{}'[t]+\theta_2{}'[t]) \; -\sin[\theta_1[t]+\theta_2[t]+\theta_3[t]] \; L_3
                                                                                                           (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                           \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \right. \left. \text{L}_{1} \right. \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \right. \left. \text{L}_{2} \left. \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right) \right. + \\
                                                                                              \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right] + \theta_3\left[\texttt{t}\right]\right] \; \texttt{L}_3 \; \left(\theta_1{'}\left[\texttt{t}\right] + \theta_2{'}\left[\texttt{t}\right] + \theta_3{'}\left[\texttt{t}\right]\right)\right)) \; + \; \texttt{1}\left[\texttt{t}\right] \; \texttt{Izz}_2
                                           (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 \left(-21[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])\right)
                                                                            \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] \; + \; \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] \; + \; \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Im}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\mathsf{Los}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Los}\left[\theta_{1}\left[\mathsf{Lo
                                                              2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                           \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \operatorname{L}_{1} \theta_{1}{'}[t] + \operatorname{Sin}\left[\theta_{1}[t] + \theta_{2}[t]\right] \operatorname{L}_{2} \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right)\right) + \\
                                                              2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                                                                              Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                                                                              Cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}''[t] + \theta_{2}''[t])) + 2 \times 1[t] Sin[\theta_{1}[t] + \theta_{2}[t]]
                                                                       L_{2} \left( \cos \left[ \theta_{1}[t] \right] L_{1} \theta_{1}'[t]^{2} + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} \left( \theta_{1}'[t] + \theta_{2}'[t] \right)^{2} + \right.
                                                                                              Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) +
                              1[t] Izz<sub>3</sub> (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m<sub>3</sub>
                                            (2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                              1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                           (Sin[\theta_{1}[t]] \; L_{1} \; \theta_{1}{'}[t] \; + \; Sin[\theta_{1}[t] \; + \; \theta_{2}[t]] \; L_{2} \; (\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]) \; + \;
                                                                                             Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])) +
                                                              2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                           (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                                              1[t] \, \operatorname{Sin}[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \, \operatorname{L}_{3} \, (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t])) \, + \,
                                                              2 \ (1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t]] \ \mathtt{L}_2 + 1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \ \mathtt{L}_3)
                                                                           \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - (\theta_{1}'[t] + \theta_{2}'[t])^{2} 
                                                                                              Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                             Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                                               2 \; (\texttt{1[t]} \; \texttt{Sin}[\theta_1[\texttt{t}] \; + \; \theta_2[\texttt{t}]] \; \texttt{L}_2 \; + \; \texttt{1[t]} \; \texttt{Sin}[\theta_1[\texttt{t}] \; + \; \theta_2[\texttt{t}] \; + \; \theta_3[\texttt{t}]] \; \texttt{L}_3) 
                                                                           \left( \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}]^{2} + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right)^{2} + \right.
                                                                                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                             \operatorname{Sin}[\theta_1[\mathsf{t}]] \ \operatorname{L}_1\theta_1{''}[\mathsf{t}] + \operatorname{Sin}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \ \operatorname{L}_2\left(\theta_1{''}[\mathsf{t}] + \theta_2{''}[\mathsf{t}]\right) + \\
                                                                                              Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
                     \left\{-g\cos\left[\theta_{1}\left[\mathtt{t}\right]+\theta_{2}\left[\mathtt{t}\right]\right]\,L_{2}\,\mathsf{m}_{2}-g\,\left(\cos\left[\theta_{1}\left[\mathtt{t}\right]+\theta_{2}\left[\mathtt{t}\right]\right]\,L_{2}+\cos\left[\theta_{1}\left[\mathtt{t}\right]+\theta_{2}\left[\mathtt{t}\right]+\theta_{2}\left[\mathtt{t}\right]\right]\right\}\right\}
                                                                                                                \theta_3[t] L<sub>3</sub>) m<sub>3</sub> - \frac{1}{2} m<sub>2</sub> (-2 Sin[\theta_1[t] + \theta_2[t]] L<sub>2</sub> (\theta_1'[t] + \theta_2'[t])
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(\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                               2 \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]) (\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t] +
                                                       Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) -
       \frac{1}{2} \, \mathsf{m}_3 \, \left( 2 \, \left( \mathsf{Cos}[\theta_1[\mathsf{t}]] \, \mathsf{L}_1 \, \theta_1'[\mathsf{t}] + \mathsf{Cos}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \, \mathsf{L}_2 \, \left( \theta_1'[\mathsf{t}] + \theta_2'[\mathsf{t}] \right) + \right. \right.
                                                       \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                         (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                               L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2[t])
                                                                              \theta_{2}'[t]) + \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                                         \left( \text{Sin} \left[ \theta_{1}[t] \right] \right. \left. \text{L}_{1} \right. \theta_{1}{'}[t] + \text{Sin} \left[ \theta_{1}[t] + \theta_{2}[t] \right] \right. \left. \text{L}_{2} \left. \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{1} \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{2} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{2} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{3} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{4} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{5} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{6} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta
                                                       Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))) + 1[t] Izz_{2}
              (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-21[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                         (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                              2\times1\texttt{[t]}\;\texttt{Cos}\left[\theta_{1}\texttt{[t]}+\theta_{2}\texttt{[t]}\right]\;\texttt{L}_{2}\;\left(\theta_{1}'\texttt{[t]}+\theta_{2}'\texttt{[t]}\right)\;\left(\texttt{Sin}\left[\theta_{1}\texttt{[t]}\right]\;\texttt{L}_{1}\;\theta_{1}'\texttt{[t]}+\theta_{2}'\texttt{[t]}\right)
                                                        \sin[\theta_{1}[t] + \theta_{2}[t]] \ L_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t])) + 2 \times 1[t] \ \cos[\theta_{1}[t] + \theta_{2}[t]] 
                                      L_2 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \right)
                                                       Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times
                                      1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (Cos[\theta_1[t]]) L_1 \theta_1'[t]^2 + Cos[\theta_1[t]] + \theta_2[t]]
                                                               L_2 (\theta_1'[t] + \theta_2'[t])^2 + Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]]
                                                               L_2(\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
       \frac{1}{2}\,\mathrm{m_3}\,\left(2\,\left(1[\mathrm{t}]\,\mathrm{Cos}\left[\theta_1[\mathrm{t}]+\theta_2[\mathrm{t}]\right]\,\mathrm{L}_2\,\left(\theta_1{'}[\mathrm{t}]+\theta_2{'}[\mathrm{t}]\right)+1[\mathrm{t}]\right.
                                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                        \left( \text{Sin} \left[ \theta_1 \left[ \mathsf{t} \right] \right] \; \mathsf{L}_1 \; \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \text{Sin} \left[ \theta_1 \left[ \mathsf{t} \right] + \theta_2 \left[ \mathsf{t} \right] \right] \; \mathsf{L}_2 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_2 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_3 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] + \theta_2^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left( \theta_1^{\, \prime} \left[ \mathsf{t} \right] \right) \; + \; \mathsf{L}_4 \; \left
                                                       \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]] \; \texttt{L}_3 \; (\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}])) \; + \\
                               2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                      Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                         (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                       1[t] \, \operatorname{Sin}[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \, \operatorname{L}_{3} \, (\theta_{1}{}'[t] + \theta_{2}{}'[t] + \theta_{3}{}'[t])) \, + \,
                               2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                         (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                                                       Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                       \texttt{Cos}[\theta_1[\texttt{t}]] \; \texttt{L}_1 \; \theta_1^{\prime\prime}[\texttt{t}] \; + \; \texttt{Cos}[\theta_1[\texttt{t}] \; + \; \theta_2[\texttt{t}]] \; \texttt{L}_2 \; (\theta_1^{\prime\prime}[\texttt{t}] \; + \; \theta_2^{\prime\prime}[\texttt{t}]) \; + \; \theta_2^{\prime\prime}[\texttt{t}] 
                                                       Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                              2 \; (1[t] \; Sin[\theta_1[t] + \theta_2[t]] \; L_2 + 1[t] \; Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; L_3)
                                         (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                       Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                       \operatorname{Sin}[\theta_1[\mathsf{t}]] \ \operatorname{L}_1 \theta_1^{\prime\prime\prime}[\mathsf{t}] + \operatorname{Sin}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime\prime}[\mathsf{t}] + \theta_2^{\prime\prime\prime}[\mathsf{t}]) + \\
                                                       Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
-g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]) L_2 +
                              Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3}) m_{3} -
       \frac{-}{2} m_2 \left(-2 \sin[\theta_1[t] + \theta_2[t]\right] L_2 \left(\theta_1'[t] + \theta_2'[t]\right) \left(\cos[\theta_1[t]] L_1 \theta_1'[t] + \theta_2'[t]\right)
                                                       Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                              2 \cos [\theta_1[t] + \theta_2[t]] \ L_2 \ (\theta_1{'}[t] + \theta_2{'}[t]) \ (\sin [\theta_1[t]] \ L_1 \ \theta_1{'}[t] + \theta_2{'}[t])
                                                       \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]\right] \; \texttt{L}_2 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}]\right))) \; - \;
      \frac{1}{-} \, \mathrm{m_3} \, \left( 2 \, \left( \mathrm{Cos}[\theta_1[\mathtt{t}]] \, \mathrm{L_1} \, \theta_1{}'[\mathtt{t}] + \mathrm{Cos}[\theta_1[\mathtt{t}] + \theta_2[\mathtt{t}]] \, \mathrm{L_2} \, \left( \theta_1{}'[\mathtt{t}] + \theta_2{}'[\mathtt{t}] \right) + \right.
                                                       Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                         (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                               L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2[t])
```

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\theta_{2}'[t]) + \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                       (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                              \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]] \; \texttt{L}_3 \; (\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]))) \; + \; \texttt{1}[\texttt{t}] \; \texttt{Izz}_2
         (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-21[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                      \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \\
                2\times1[\texttt{t}]\;\texttt{Cos}[\theta_1[\texttt{t}]+\theta_2[\texttt{t}]]\;\texttt{L}_2\;(\theta_1{'}[\texttt{t}]+\theta_2{'}[\texttt{t}])\;(\texttt{Sin}[\theta_1[\texttt{t}]]\;\texttt{L}_1\;\theta_1{'}[\texttt{t}]+\theta_2{'}[\texttt{t}])
                              Sin[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t])) + 2 \times 1[t] Cos[\theta_1[t] + \theta_2[t]]
                      L_{2} \left( - Sin[\theta_{1}[t]] L_{1} \theta_{1}{}'[t]^{2} - Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}{}'[t] + \theta_{2}{}'[t])^{2} + \right. 
                              Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times
                     1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]]
                                   L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} + Sin[\theta_{1}[t]] L_{1} \theta_{1}''[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] 
                                  L_2(\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
   \frac{1}{2}\,{\rm m_3}\,\left(2\,\left(1[{\rm t}]\,\cos{[\theta_1[{\rm t}]}+\theta_2[{\rm t}]\right)\,{\rm L_2}\,\left(\theta_1{'}[{\rm t}]+\theta_2{'}[{\rm t}]\right)+1[{\rm t}]\right.
                                  \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                      (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                              Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \ L_{3} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t])) \ +
                 2 \left( \cos \left[ \theta_{1}[t] \right] \right] L_{1} \theta_{1}{'}[t] + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) + \\
                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                      (-1[t] \, \operatorname{Sin}[\theta_1[t] + \theta_2[t]] \, \operatorname{L}_2 \, (\theta_1{}'[t] + \theta_2{}'[t]) \, - \,
                              1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                 2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                      (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                              Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                              Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                 2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                      \left( \cos \left[ \theta_{1}[t] \right] \right] L_{1} \theta_{1}'[t]^{2} + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} \left( \theta_{1}'[t] + \theta_{2}'[t] \right)^{2} +
                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                              Sin[\theta_1[t]] \ L_1 \ \theta_1^{\prime\prime}[t] + Sin[\theta_1[t] + \theta_2[t]] \ L_2 \ (\theta_1^{\prime\prime}[t] + \theta_2^{\prime\prime}[t]) \ +
                              Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
-g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \frac{1}{2} m_3 (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t])
                              \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                      \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \; + \right.
                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                 2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                      \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; \text{L}_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right) \; + \right.
                              \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right))) \; + \;
   1[t] Izz<sub>3</sub> (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3
         \left(-21[t] \sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]\right] L_{3} \left(\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]\right)
                      \left(\text{Cos}\left[\theta_{1}\left[t\right]\right] \text{ L}_{1} \; \theta_{1}{'}\left[t\right] + \text{Cos}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \text{ L}_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right) + \right.
                             \texttt{Cos} \left[ \theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}] \right] \, \texttt{L}_{3} \, \left( \theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}] \right) ) \, + \,
                 2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                      \left(\text{Sin}\left[\theta_{1}\left[\texttt{t}\right]\right] \; \text{L}_{1} \; \theta_{1}{'}\left[\texttt{t}\right] + \text{Sin}\left[\theta_{1}\left[\texttt{t}\right] + \theta_{2}\left[\texttt{t}\right]\right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[\texttt{t}\right] + \theta_{2}{'}\left[\texttt{t}\right]\right) \; + \right.
                              Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]\right) L_3 \left(-\sin[
                              Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                  L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                              Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t]) + Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]]
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\mathtt{L}_{3}\left(\theta_{1}^{\,\prime\prime}[\mathtt{t}]+\theta_{2}^{\,\prime\prime}[\mathtt{t}]+\theta_{3}^{\,\prime\prime}[\mathtt{t}]\right)\right)+2\times1[\mathtt{t}]\,\,\mathtt{Sin}\left[\theta_{1}[\mathtt{t}]+\theta_{2}[\mathtt{t}]+\theta_{3}[\mathtt{t}]\right]
                            L_{3} \left( \cos \left[ \theta_{1}[t] \right] L_{1} \theta_{1}{'}[t]^{2} + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right)^{2} + \right. 
                                 Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                 \texttt{Sin}[\theta_1[\texttt{t}]] \; \texttt{L}_1 \; \theta_1{''}[\texttt{t}] \; + \; \texttt{Sin}[\theta_1[\texttt{t}] \; + \; \theta_2[\texttt{t}]] \; \texttt{L}_2 \; \left(\theta_1{''}[\texttt{t}] \; + \; \theta_2{''}[\texttt{t}]\right) \; + \;
                                 Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])))\Big\}_{2},
      -g \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \frac{1}{2} m_3 (-2 \sin [\theta_1[t] + \theta_2[t] + \theta_3[t]]
                     L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                     \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] \; + \; \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] \; + \; \theta_{2}{'}\left[\mathsf{t}\right]\right) \; + \\
                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                  2 \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                     (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                           Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \ L_{3} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t]))) \ +
         1[t] Izz<sub>3</sub> (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m<sub>3</sub>
             (-21[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                     \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] \; + \; \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] \; + \; \theta_{2}{'}\left[\mathsf{t}\right]\right) \; + \\
                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                  2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                     (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                  2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right)
                           Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                           \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \, \mathsf{L}_{1} \, \theta_{1}^{\prime\prime}[\mathsf{t}] \, + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \, + \theta_{2}[\mathsf{t}]\right] \, \mathsf{L}_{2} \, \left(\theta_{1}^{\prime\prime}[\mathsf{t}] \, + \theta_{2}^{\prime\prime}[\mathsf{t}]\right) \, + \\
                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                  2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 +
                           Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                           \texttt{Sin}[\theta_1[\texttt{t}]] \ \texttt{L}_1 \ \theta_1{''}[\texttt{t}] + \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \ \texttt{L}_2 \ (\theta_1{''}[\texttt{t}] + \theta_2{''}[\texttt{t}]) \ +
                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
g Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L<sub>3</sub> m<sub>3</sub> - \frac{1}{2}
      m_3
      (-2)
               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
               L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
               \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}] \; + \; \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right) \; + \;
                     Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])) +
            2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                     Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
   1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
    1
    2
     m_3
      (-21[t]
               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
               L_3 (\Theta_1'[t] + \Theta_2'[t] + \Theta_3'[t])
               (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
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Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                                2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                                                    (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                      \sin \left[\theta_1[t] + \theta_2[t] + \theta_3[t]\right] \perp_3 \left(\theta_1{}'[t] + \theta_2{}'[t] + \theta_3{}'[t]\right)) + \\
                                                                                                2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]\right) L_3 \left(-\sin[
                                                                                                                                                   Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                                                                                                                                                   Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                                                                                 \texttt{Cos}\left[\theta_{1}\left[\texttt{t}\right]\right] \; \texttt{L}_{1} \; \theta_{1}{''}\left[\texttt{t}\right] \; + \; \texttt{Cos}\left[\theta_{1}\left[\texttt{t}\right] \; + \; \theta_{2}\left[\texttt{t}\right]\right] \; \texttt{L}_{2} \; \left(\theta_{1}{''}\left[\texttt{t}\right] \; + \; \theta_{2}{''}\left[\texttt{t}\right]\right) \; + \;
                                                                                                                                                   \texttt{Cos}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]] \ \texttt{L}_3 \ (\theta_1{''}[\texttt{t}] + \theta_2{''}[\texttt{t}] + \theta_3{''}[\texttt{t}]) \, \big) + \\
                                                                                                2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 +
                                                                                                                                                   Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                                                                   Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                                                                                   Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                                                                   Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
\left\{ \texttt{gCos}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \; \texttt{L}_2\; \texttt{m}_2 + \texttt{g} \; (\texttt{Cos}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \; \texttt{L}_2 + \texttt{m}_2[\texttt{t}] \right\} \right\} = 0
                                                                                                Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3}) m_{3} -
                                                 \frac{-}{2} m_2 \left(-2 \sin \left[\theta_1[t] + \theta_2[t]\right] L_2 \left(\theta_1'[t] + \theta_2'[t]\right)
                                                                                                                      \left( \text{Cos} \left[ \theta_{1}[t] \right] \right. \left. \text{L}_{1} \right. \theta_{1}{}'[t] + \text{Cos} \left[ \theta_{1}[t] + \theta_{2}[t] \right] \right. \left. \text{L}_{2} \left. \left( \theta_{1}{}'[t] + \theta_{2}{}'[t] \right) \right) + \\
                                                                                                2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                                                      (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) -
                                                   \frac{1}{2} \, \mathbf{m}_{3} \, \left( 2 \, \left( \mathsf{Cos} \left[ \theta_{1} \left[ \mathbf{t} \right] \right] \, \mathbf{L}_{1} \, \theta_{1}{}' \left[ \mathbf{t} \right] + \mathsf{Cos} \left[ \theta_{1} \left[ \mathbf{t} \right] + \theta_{2} \left[ \mathbf{t} \right] \right] \, \mathbf{L}_{2} \, \left( \theta_{1}{}' \left[ \mathbf{t} \right] + \theta_{2}{}' \left[ \mathbf{t} \right] \right) \, + \, \mathbf{H}_{2} \, \mathbf{H}_{
                                                                                                                                                   Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                                    (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                                                                                                   Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                                2 (Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                   Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                                      \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}]\right] \; \operatorname{L}_{1} \; \theta_{1}{'}[\mathsf{t}] \; + \; \operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \operatorname{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \operatorname{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \operatorname{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \operatorname{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \theta_{2}[\mathsf{t}]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; + 
                                                                                                                                                   Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
                                             1[t] Izz_2 (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2}
                                                             m_2
                                                                   \left(-21[t] \operatorname{Sin}\left[\theta_{1}[t] + \theta_{2}[t]\right]\right)
                                                                                                              L_2 (\Theta_1'[t] + \Theta_2'[t])
                                                                                                                    \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\left.\theta_{1}^{\,\prime}[\mathsf{t}]\right. + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right. + \theta_{2}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{2}\left.\left(\theta_{1}^{\,\prime}[\mathsf{t}]\right. + \theta_{2}^{\,\prime}[\mathsf{t}]\right)\right) + \left.\mathsf{L}_{2}\left(\theta_{1}^{\,\prime}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{3}\left(\theta_{1}^{\,\prime}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{4}\left(\theta_{1}^{\,\prime}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{5}\left(\theta_{1}^{\,\prime}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{5}\left(\theta_{1}^{\,\prime}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{5}\left(\theta_{1}^{\,\prime}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{6}\left(\theta_{1}^{\,\prime}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{7}\left(\theta_{1}^{\,\prime}[\mathsf{t}]\right)\right] \left.\mathsf{L}
                                                                                                2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t])
                                                                                                                    \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right]\right] \; \operatorname{L}_{1} \; \theta_{1}{'}\left[\mathtt{t}\right] + \operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right] \; \operatorname{L}_{2} \; \left(\theta_{1}{'}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right]\right)\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{1}{'}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{1}{'}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{1}{'}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right] + \theta_{2}{'}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right] \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}
                                                                                                2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2
                                                                                                                    \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} + \theta_{1}(\theta_{1}[t])^{2} + \theta_{2}(\theta_{1}[t])^{2} + \theta_{2}(\theta_{1}[
                                                                                                                                                   Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                2 \times 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 +
                                                                                                                                                   \mathsf{Cos}[\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}]] \; \mathsf{L}_{2} \; (\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}])^{2} + \mathsf{Sin}[\theta_{1}[\mathsf{t}]] \; \mathsf{L}_{1} \; \theta_{1}{''}[\mathsf{t}] + \mathsf{L}_{2} \; (\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}])^{2} + \mathsf{Sin}[\theta_{1}[\mathsf{t}]] \; \mathsf{L}_{3} \; \theta_{1}{''}[\mathsf{t}] + \mathsf{L}_{3} \; (\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}])^{2} + \mathsf{Sin}[\theta_{1}[\mathsf{t}]] \; \mathsf{L}_{4} \; \theta_{1}{''}[\mathsf{t}] + \mathsf{L}_{5} \; (\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}])^{2} + \mathsf{Sin}[\theta_{1}[\mathsf{t}]] \; \; (\theta_{1}{'}[\mathsf{t}] + \theta_{2}[\mathsf{t}])^{2} + \mathsf{Sin}[\theta_{1}[\mathsf{t}]] \; (\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}])^{2} + \mathsf{Sin}[\theta_{1}[\mathsf{t}]] \; (\theta_{1}[\mathsf{t}] +
                                                                                                                                                   Sin[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t])) +
                                               1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                   1
                                                   2
                                                           mз
                                                                   (2 (1[t] \cos[\theta_1[t] + \theta_2[t]) L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                   1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
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(Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                 2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                      (-1[t] \, \operatorname{Sin}[\theta_1[t] + \theta_2[t]] \, \operatorname{L}_2 \, (\theta_1{}'[t] + \theta_2{}'[t]) \, - \,
                               1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                 2\;\left(\text{1[t]}\;\text{Cos}\left[\theta_{1}[\text{t}]+\theta_{2}[\text{t}]\right]\;\text{L}_{2}+\text{1[t]}\;\text{Cos}\left[\theta_{1}[\text{t}]+\theta_{2}[\text{t}]+\theta_{3}[\text{t}]\right]\;\text{L}_{3}\right)
                       \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{2} (\theta_{1}'[t])^{2} - \sin[\theta_{1}[t]] L_{2} (\theta_{1}'[t]
                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                               \texttt{Cos}\left[\theta_{1}[\texttt{t}]\right] \; \texttt{L}_{1} \; \theta_{1}^{\prime\prime}[\texttt{t}] \; + \; \texttt{Cos}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \texttt{L}_{2} \; \left(\theta_{1}^{\prime\prime}[\texttt{t}] \; + \; \theta_{2}^{\prime\prime}[\texttt{t}]\right) \; + \;
                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                 2 \; (1[t] \; Sin[\theta_1[t] + \theta_2[t]] \; L_2 + 1[t] \; Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; L_3)
                      (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                               \operatorname{Sin}[\theta_1[\mathsf{t}]] \ \operatorname{L}_1 \ \theta_1^{\prime\prime}[\mathsf{t}] + \operatorname{Sin}[\theta_1[\mathsf{t}]] + \theta_2[\mathsf{t}]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime}[\mathsf{t}] + \theta_2^{\prime\prime}[\mathsf{t}]) + \\
                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
\left\{-g\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]\right. L_{2} m_{2} - g\left(\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]\right. L_{2} + \left. \left(\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]\right]\right\} L_{2} + \left. \left(\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]\right]\right\} L_{2} + \left. \left(\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]\right)\right\} L_{2} + \left. \left(\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]\right)\right]
                          Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 -
             \frac{-}{2} m<sub>2</sub> (-2 Sin[\theta_1[t] + \theta_2[t]] L<sub>2</sub> (\theta_1'[t] + \theta_2'[t])
                                (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                          2\,\text{Cos}\left[\theta_1\left[\mathsf{t}\right]+\theta_2\left[\mathsf{t}\right]\right]\,\mathrm{L}_2\,\left(\theta_1{}'\left[\mathsf{t}\right]+\theta_2{}'\left[\mathsf{t}\right]\right)
                                (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) -
             \frac{1}{2} m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                        \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                (-\sin[\theta_1[t]+\theta_2[t]] \; L_2 \; (\theta_1'[t]+\theta_2'[t]) \; -\sin[\theta_1[t]+\theta_2[t]+\theta_3[t]] \; L_3
                                              (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])) + 2 (\cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]) +
                                        Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                        \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right] + \theta_3\left[\texttt{t}\right]\right] \; \texttt{L}_3 \; \left(\theta_1{'}\left[\texttt{t}\right] + \theta_2{'}\left[\texttt{t}\right] + \theta_3{'}\left[\texttt{t}\right]\right)\right)) \; + \; \texttt{1}\left[\texttt{t}\right] \; \texttt{Izz}_2
                  (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 \left(-21[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])\right)
                                (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                          2\times \texttt{1[t]}\; \texttt{Cos}\left[\theta_{\texttt{1}}[\texttt{t}] + \theta_{\texttt{2}}[\texttt{t}]\right] \; \texttt{L}_{\texttt{2}}\; \left(\theta_{\texttt{1}}{'}[\texttt{t}] + \theta_{\texttt{2}}{'}[\texttt{t}]\right)
                               (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                          2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                        Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                        Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]]
                              L_{2} \left( \cos \left[ \theta_{1}[t] \right] L_{1} \theta_{1}{'}[t]^{2} + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right)^{2} + \right.
                                        Sin[\theta_1[t]] \ L_1 \ \theta_1^{\prime\prime\prime}[t] + Sin[\theta_1[t]] + \theta_2[t]] \ L_2 \ (\theta_1^{\prime\prime\prime}[t] + \theta_2^{\prime\prime\prime}[t])))) + \\
            1[t] Izz<sub>3</sub> (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3
                  (2 (1[t] Cos[\theta_1[t] + \theta_2[t]) L_2 (\theta_1'[t] + \theta_2'[t]) +
                                        1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                (Sin[\theta_{1}[t]] \ L_{1} \ \theta_{1}{'}[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] \ L_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \ +
                                        Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                          2 \left( \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \right. \left. \mathsf{L}_{1} \right. \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right. + \\
                                        Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                        1[t] \, \operatorname{Sin}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \, \operatorname{L}_3 \, (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \, + \,
                           2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
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\left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{1} \theta_{1}'[t] + \theta_{2}'[t] \theta_{2}'[t] + \theta_{2}'[t] \theta_{1}'[t] + \theta_{2}'[t] + \theta_{2}'[t] + \theta_{2}'[t] + \theta_{2}'[t] + \theta_{2}'[t] + \theta_{2}'[t] 
                                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                         \cos[\theta_{1}[t]] \ L_{1} \ \theta_{1}^{\, \prime \prime}[t] \ + \cos[\theta_{1}[t]] \ + \theta_{2}[t]] \ L_{2} \ (\theta_{1}^{\, \prime \prime}[t] \ + \theta_{2}^{\, \prime \prime}[t]) \ +
                                                        Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                2\;(1[t]\;Sin[\theta_{1}[t]+\theta_{2}[t]]\;L_{2}+1[t]\;Sin[\theta_{1}[t]+\theta_{2}[t]+\theta_{3}[t]]\;L_{3})
                                          \left( \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}]^{2} + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right)^{2} + \right.
                                                         \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])^{2} +
                                                          \sin[\theta_1[t]] \; L_1 \; \theta_1^{\prime\prime}[t] \; + \; \sin[\theta_1[t] \; + \; \theta_2[t]] \; L_2 \; \left(\theta_1^{\prime\prime}[t] \; + \; \theta_2^{\prime\prime}[t]\right) \; + \;
                                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3}) m_{3} -
                         \frac{-}{2} m_2 \left(-2 \sin \left[\theta_1[t] + \theta_2[t]\right] L_2 \left(\theta_1'[t] + \theta_2'[t]\right) \left(\cos \left[\theta_1[t]\right] L_1 \theta_1'[t] + 2\right)
                                                                         Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                2\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]L_{2}\left(\theta_{1}'[t]+\theta_{2}'[t]\right)\left(\sin\left[\theta_{1}[t]\right]L_{1}\left(\theta_{1}'[t]+\theta_{2}'[t]\right)\right)
                                                                         Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])) -
                         \frac{1}{2} m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                        Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                          \left(-\sin[\theta_1[t]+\theta_2[t]] \mathrel{L}_2 \left(\theta_1{}'[t]+\theta_2{}'[t]\right) - \sin[\theta_1[t]+\theta_2[t]+\theta_3[t]\right]
                                                                                  \texttt{L}_{3} \ (\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}]) ) + 2 \ (\texttt{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}]] \ \texttt{L}_{2} \ (\theta_{1}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}]) + \theta_{3}{'}[\texttt{t}] + \theta_
                                                                                                 \theta_{2}'[t]) + \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                                                           \left( \text{Sin} \left[ \theta_{1}[t] \right] \right. \left. \text{L}_{1} \right. \theta_{1}{'}[t] + \text{Sin} \left[ \theta_{1}[t] + \theta_{2}[t] \right] \right. \left. \text{L}_{2} \left. \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{1} \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{2} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{2} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{3} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{4} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{5} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{6} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta
                                                                         \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right] + \theta_3\left[\texttt{t}\right]\right] \; \texttt{L}_3 \; \left(\theta_1{}'\left[\texttt{t}\right] + \theta_2{}'\left[\texttt{t}\right] + \theta_3{}'\left[\texttt{t}\right]\right)\right)) \; + \; \texttt{1}\left[\texttt{t}\right] \; \texttt{Izz}_2
                                 (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 \left(-21[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])\right)
                                                           \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}] \; + \; \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right)\right) \; + \;
                                                 2\times1[t]\;\mathsf{Cos}\left[\theta_{1}[t]+\theta_{2}[t]\right]\;\mathsf{L}_{2}\;\left(\theta_{1}^{\;\prime}[t]+\theta_{2}^{\;\prime}[t]\right)\;\left(\mathsf{Sin}\left[\theta_{1}[t]\right]\;\mathsf{L}_{1}\;\theta_{1}^{\;\prime}[t]+\theta_{2}^{\;\prime}[t]\right)
                                                                         \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])) + 2 \times 1[t] \cos[\theta_{1}[t] + \theta_{2}[t]]
                                                        L_2 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \right]
                                                                         Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times
                                                         1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]]
                                                                                 L_2 (\theta_1'[t] + \theta_2'[t])^2 + Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]]
                                                                                 L_2(\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                        \frac{1}{2} \, \mathrm{m_3} \, \left( 2 \, \left( 1[t] \, \mathrm{Cos}[\theta_1[t] + \theta_2[t]] \, \mathrm{L_2} \, \left( \theta_1{}'[t] + \theta_2{}'[t] \right) + 1[t] \right)
                                                                                 Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                          (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                 2 \left( \cos \left[ \theta_{1}[t] \right] \right] L_{1} \theta_{1}{'}[t] + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) + \\
                                                                         Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                          (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                         1[t] \; \text{Sin}[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \; L_{3} \; (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t])) \; + \;
                                                 2 \ (1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t]] \ \mathtt{L}_2 + 1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \ \mathtt{L}_3)
                                                          \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{1} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{2} (\theta_{1}'[t])^{2} - \sin[\theta_{1}[t]] L_{2} (\theta_{1}'[t])^{2} - \sin[\theta_
                                                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                         Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                         Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                                 2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                           (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                         Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
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\operatorname{Sin}[\theta_1[\mathtt{t}]] \ \operatorname{L}_1 \theta_1^{\prime\prime}[\mathtt{t}] + \operatorname{Sin}[\theta_1[\mathtt{t}] + \theta_2[\mathtt{t}]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime}[\mathtt{t}] + \theta_2^{\prime\prime}[\mathtt{t}]) + \\
                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
-g \cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - g (\cos[\theta_1[t] + \theta_2[t]] L_2 +
                Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3}) m_{3} -
    \frac{1}{2} m_2 \left(-2 \sin \left[\theta_1[t] + \theta_2[t]\right] L_2 \left(\theta_1'[t] + \theta_2'[t]\right) \left(\cos \left[\theta_1[t]\right] L_1 \theta_1'[t] + \theta_2'[t]\right)
                             Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                2 \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]) (\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t] +
                             Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) -
    \frac{1}{2} \, \mathsf{m}_3 \, \left( 2 \, \left( \mathsf{Cos}[\theta_1[\mathsf{t}]] \, \mathsf{L}_1 \, \theta_1{}'[\mathsf{t}] + \mathsf{Cos}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \, \mathsf{L}_2 \, \left( \theta_1{}'[\mathsf{t}] + \theta_2{}'[\mathsf{t}] \right) + \right. \\
                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                      (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                 \mathtt{L_{3}}\;(\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}])) + 2\;(\mathtt{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}]]\;\mathtt{L_{2}}\;(\theta_{1}{'}[\texttt{t}] +
                                         \theta_{2}'[t]) + \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                      (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) + 1[t] Izz_2
        (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 \left(-21[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])\right)
                      \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \;
                2\times1[\texttt{t}]\;\mathsf{Cos}\left[\theta_{1}[\texttt{t}]+\theta_{2}[\texttt{t}]\right]\;\mathsf{L}_{2}\;\left(\theta_{1}{'}[\texttt{t}]+\theta_{2}{'}[\texttt{t}]\right)\;\left(\mathsf{Sin}\left[\theta_{1}[\texttt{t}]\right]\right)\;\mathsf{L}_{1}\;\theta_{1}{'}[\texttt{t}]+
                             \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right]\right] \; \texttt{L}_2 \; \left(\theta_1{'}\left[\texttt{t}\right] + \theta_2{'}\left[\texttt{t}\right]\right)\right) \; + \; 2 \times 1\left[\texttt{t}\right] \; \texttt{Cos}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right]\right]
                    L_2 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \right)
                             Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times
                     1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (Cos[\theta_1[t]] L_1 \theta_1'[t]^2 + Cos[\theta_1[t]] + \theta_2[t]]
                                 L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} + Sin[\theta_{1}[t]] L_{1} \theta_{1}''[t] + Sin[\theta_{1}[t] + \theta_{2}[t]]
                                 L_2(\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
    \frac{1}{2}\,\mathrm{m_3}\,\left(2\,\left(1[\mathrm{t}]\,\mathrm{Cos}\left[\theta_1[\mathrm{t}]+\theta_2[\mathrm{t}]\right]\,\mathrm{L}_2\,\left(\theta_1'[\mathrm{t}]+\theta_2'[\mathrm{t}]\right)+1[\mathrm{t}]\right.
                                 Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                     (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                             \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right)) \; + \;
                2 \; (\text{Cos}[\theta_1[t]] \; \text{L}_1 \; \theta_1{}'[t] \; + \; \text{Cos}[\theta_1[t] \; + \; \theta_2[t]] \; \text{L}_2 \; (\theta_1{}'[t] \; + \; \theta_2{}'[t]) \; + \;
                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                      (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                             1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                2 \; (1[t] \; \mathsf{Cos}[\theta_1[t] + \theta_2[t]] \; \mathtt{L}_2 + 1[t] \; \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; \mathtt{L}_3)
                     \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{1} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \sin[\theta_{1}[t]] L_{2} (\theta_{1}'[t])^{2} - \sin[\theta_{1}[t]] L_{2} (\theta_{1}'[t])^{2} - \sin[\theta_
                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                             \texttt{Cos}[\theta_{1}[\texttt{t}]] \; \texttt{L}_{1} \; \theta_{1}{''}[\texttt{t}] \; + \; \texttt{Cos}[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]] \; \texttt{L}_{2} \; (\theta_{1}{''}[\texttt{t}] \; + \; \theta_{2}{''}[\texttt{t}]) \; + \;
                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                      \left(\cos\left[\theta_{1}[t]\right] L_{1} \theta_{1}'[t]^{2} + \cos\left[\theta_{1}[t] + \theta_{2}[t]\right] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} + \right.
                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                             Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
-g \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} m_{3} - \frac{1}{2} m_{3} (-2 \sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t])]
                             \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                      \left(\text{Cos}\left[\theta_{1}\left[t\right]\right] \text{ L}_{1} \; \theta_{1}{'}\left[t\right] + \text{Cos}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \text{ L}_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right) + \right.
                             \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right)) \, + \,
                2 \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
```

```
\left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \text{ L}_{1} \; \theta_{1}^{\prime}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \text{ L}_{2} \; \left(\theta_{1}^{\prime}\left[t\right] + \theta_{2}^{\prime}\left[t\right]\right) + \left(\theta_{1}^
                                                             \texttt{Sin}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]\right] \; \texttt{L}_{3} \; \left(\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}]\right))) \; + \;
                            1[t] Izz<sub>3</sub> (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3
                                   \left(-2 \ 1 \ [t] \ \text{Sin} \left[\theta_1 \ [t] \ + \theta_2 \ [t] \ + \theta_3 \ [t] \right] \ L_3 \ \left(\theta_1' \ [t] \ + \theta_2' \ [t] \ + \theta_3' \ [t] \right)
                                                   \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \; + \right.
                                                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                            2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                   (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                             \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right)) \; + \;
                                             2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_1 \theta_1'[t]
                                                             Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                  L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                                            \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}]\right] \, \texttt{L}_{2} \, \left(\theta_{1}^{\,\prime\prime}[\texttt{t}] + \theta_{2}^{\,\prime\prime}[\texttt{t}]\right) + \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]\right]
                                                                  L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + 2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                 L_{3} \left( \cos \left[ \theta_{1}[t] \right] L_{1} \theta_{1}'[t]^{2} + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} \left( \theta_{1}'[t] + \theta_{2}'[t] \right)^{2} + \right.
                                                             Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                             \texttt{Sin}[\theta_1[\texttt{t}]] \; \texttt{L}_1 \; \theta_1{''}[\texttt{t}] \; + \; \texttt{Sin}[\theta_1[\texttt{t}] \; + \; \theta_2[\texttt{t}]] \; \texttt{L}_2 \; \left(\theta_1{''}[\texttt{t}] \; + \; \theta_2{''}[\texttt{t}]\right) \; + \;
                                                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
           -g \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \frac{1}{2} m_3 (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t])
                                       L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                       (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                  \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right)) \; + \;
                                 2\cos\left[\theta_{1}\left[\mathtt{t}\right]+\theta_{2}\left[\mathtt{t}\right]+\theta_{3}\left[\mathtt{t}\right]\right]\,\mathtt{L}_{3}\,\left(\theta_{1}{'}\left[\mathtt{t}\right]+\theta_{2}{'}\left[\mathtt{t}\right]+\theta_{3}{'}\left[\mathtt{t}\right]\right)
                                        (Sin[\theta_{1}[t]] \ L_{1} \ \theta_{1}{'}[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] \ L_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \ +
                                                  \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right] + \theta_3\left[\texttt{t}\right]\right] \; \texttt{L}_3 \; \left(\theta_1{'}\left[\texttt{t}\right] + \theta_2{'}\left[\texttt{t}\right] + \theta_3{'}\left[\texttt{t}\right]\right)\right)) \; + \;
                1[t] Izz<sub>3</sub> (\theta_1'''[t] + \theta_2'''[t] + \theta_3'''[t]) + \frac{1}{2} m_3
                       (-21[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                        (\cos[\theta_{1}[t]] L_{1} \theta_{1}'[t] + \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]) +
                                                 \texttt{Cos} \left[ \theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}] \right] \, \texttt{L}_{3} \, \left( \theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}] \right) ) \, + \,
                                 2\times \texttt{1[t]}\,\, \texttt{Cos}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]] \,\, \texttt{L}_3 \,\, (\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}])
                                        (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                  Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                 2\times1\texttt{[t]}\;\mathsf{Cos}\,[\theta_{1}\texttt{[t]}+\theta_{2}\texttt{[t]}+\theta_{3}\texttt{[t]}\texttt{]}\;\mathtt{L}_{3}\;\left(-\,\mathsf{Sin}\,[\theta_{1}\texttt{[t]}\texttt{]}\;\mathtt{L}_{1}\;\theta_{1}{'}\texttt{[t]}^{2}\,-\,\mathsf{L}_{1}^{2}\,\theta_{1}^{2}\mathsf{[t]}\right)
                                                  Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                                                  Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                  \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \, \mathsf{L}_{1} \, \theta_{1}^{\,\prime\prime}[\mathsf{t}] \, + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \, + \theta_{2}[\mathsf{t}]\right] \, \mathsf{L}_{2} \, \left(\theta_{1}^{\,\prime\prime}[\mathsf{t}] \, + \theta_{2}^{\,\prime\prime}[\mathsf{t}]\right) \, + \\
                                                  Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                 2\times1\texttt{[t]}\,\,\texttt{Sin}\,[\theta_{1}\texttt{[t]}+\theta_{2}\texttt{[t]}+\theta_{3}\texttt{[t]}\texttt{]}\,\,\texttt{L}_{3}\,\,\big(\texttt{Cos}\,[\theta_{1}\texttt{[t]}\texttt{]}\,\,\texttt{L}_{1}\,\,\theta_{1}{}'\texttt{[t]}^{2}\,+
                                                  Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                  Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                  Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                   Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
g Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L<sub>3</sub> m<sub>3</sub> - \frac{1}{2}
          m_3
            (-2)
```

 $Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]$

```
L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                                                                                                                   (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                                                                              Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                                                                                                   2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                                                                                                                     (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                                                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                                                           1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                                                                                              2
                                                                                                                                     m_3
                                                                                                                                           (-21[t]
                                                                                                                                                                                Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                                                                                                                              L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                                                                                                                   \left( \text{Cos}\left[\theta_{1}[\texttt{t}]\right] \; \text{L}_{1} \; \theta_{1}{'}[\texttt{t}] \; + \; \text{Cos}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \text{L}_{2} \; \left(\theta_{1}{'}[\texttt{t}] \; + \; \theta_{2}{'}[\texttt{t}]\right) \; + \; \theta_{2}{'}[\texttt{t}] \; 
                                                                                                                                                                                                            \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]\right] \; \texttt{L}_{3} \; \left(\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}]\right)) \; + \;
                                                                                                                                                                   2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                                                                                                                     \left( \text{Sin} \left[ \theta_{1}[t] \right] \right. \left. \text{L}_{1} \right. \theta_{1}{'}[t] + \text{Sin} \left[ \theta_{1}[t] + \theta_{2}[t] \right] \right. \left. \text{L}_{2} \left. \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{1} \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{2} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{2} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{3} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{4} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{5} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{5} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{6} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{5} \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right] \left. \text{L}_{7} \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. + \\ \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right] \left. \left( \theta_{1}[t] \right) \right. \left.
                                                                                                                                                                                                            \texttt{Sin}\left[\theta_{1}\left[\texttt{t}\right]+\theta_{2}\left[\texttt{t}\right]+\theta_{3}\left[\texttt{t}\right]\right]\,\texttt{L}_{3}\,\left(\theta_{1}{'}\left[\texttt{t}\right]+\theta_{2}{'}\left[\texttt{t}\right]+\theta_{3}{'}\left[\texttt{t}\right]\right)\right)\,+
                                                                                                                                                                   2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]\right)
                                                                                                                                                                                                            Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                                                                                                                                                                                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                                                                                                                                            \texttt{Cos}[\theta_1[\texttt{t}]] \ \texttt{L}_1 \ \theta_1^{\prime\prime\prime}[\texttt{t}] + \texttt{Cos}[\theta_1[\texttt{t}]] + \theta_2[\texttt{t}]] \ \texttt{L}_2 \ (\theta_1^{\prime\prime\prime}[\texttt{t}] + \theta_2^{\prime\prime\prime}[\texttt{t}]) + \theta_2^{\prime\prime\prime}[\texttt{t}]) + \theta_2^{\prime\prime\prime}[\texttt{t}] + \theta_2^{\prime\prime\prime}[\texttt{t}] + \theta_2^{\prime\prime\prime}[\texttt{t}]) + \theta_2^{\prime\prime\prime}[\texttt{t}] + \theta_2^{\prime\prime\prime\prime}[\texttt{t}] + \theta_2^{\prime\prime\prime}[\texttt{t}] + \theta_2^{\prime
                                                                                                                                                                                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                                                                                                                                                   2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \theta_2[t] \sin[\theta_1[t]] + \theta_2[t] + \theta_3[t]
                                                                                                                                                                                                            Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                                                                                                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                                                                                                                                            Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                                                                                                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
 \text{Out} [159] = 1[t] \text{ Izz}_1 \ \theta_1{}'[t] + 1[t] \text{ L}_1^2 \ \text{m}_1 \ \theta_1{}'[t] + 1[t] \text{ Izz}_2 \ (\theta_1{}'[t] + \theta_2{}'[t]) + 1[t] \text{ Izz}_2 \ (\theta_1{}'[t
                                                                                   \frac{1}{2} \, \mathrm{m_2} \, \left( 2 \, \left( 1 [\mathsf{t}] \, \mathrm{Cos} \left[ \theta_1 [\mathsf{t}] \right] \, \mathrm{L_1} + 1 [\mathsf{t}] \, \mathrm{Cos} \left[ \theta_1 [\mathsf{t}] + \theta_2 [\mathsf{t}] \right] \, \mathrm{L_2} \right)
                                                                                                                                           \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \;
                                                                                                                           2 (1[t] Sin[\theta_1[t]] L_1 + 1[t] Sin[\theta_1[t] + \theta_2[t]] L_2)
                                                                                                                                           (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) +
                                                                                 1[t] Izz_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) + \frac{1}{2} m_3
                                                                                                   (2 (1[t] Cos[\theta_1[t]] L_1 + 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                                                                                                                              L_3) (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + \theta_2'[t]
                                                                                                                                                                   \cos \left[ \theta_{1}\left[ t \right] + \theta_{2}\left[ t \right] + \theta_{3}\left[ t \right] \right] \, L_{3} \, \left( \theta_{1}{'}\left[ t \right] + \theta_{2}{'}\left[ t \right] + \theta_{3}{'}\left[ t \right] \right) ) \, + \, 2 \, \left( 1 \left[ t \right] \, \sin \left[ \theta_{1}\left[ t \right] \right] \, L_{1} \, + \, 2 \, \left( 1 \left[ t \right] \, \sin \left[ \theta_{1}\left[ t \right] \right] \, L_{2} \, + \, 2 \, \left( 1 \left[ t \right] \, \sin \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \cos \left[ \theta_{1}\left[ t \right] \right] \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{3} \, + \, 2 \, \left( 1 \left[ t \right] \, \right) \, L_{
                                                                                                                                                                      1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                                                                                                           (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                                   Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
```

```
Out[160]= 1[t] Izz_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                              \frac{1}{2} \, \mathbf{m}_2 \, \left( 2 \times \mathbf{1}[\mathsf{t}] \, \mathsf{Cos}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \, \mathbf{L}_2 \, \left( \mathsf{Cos}[\theta_1[\mathsf{t}]] \, \mathbf{L}_1 \, \theta_1^{\, \prime}[\mathsf{t}] + \mathsf{Cos}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \right)
                                                                                                                                                                 L_2(\theta_1'[t] + \theta_2'[t])) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]] L_2
                                                                                                                                \left( \text{Sin} \left[ \theta_{1} \left[ t \right] \right] \; \text{L}_{1} \; \theta_{1}{'} \left[ t \right] \; + \; \text{Sin} \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2} \left[ t \right] \right] \; \text{L}_{2} \; \left( \theta_{1}{'} \left[ t \right] \; + \; \theta_{2}{'} \left[ t \right] \right) \right) \right) \; + \; \left( \text{Sin} \left[ \theta_{1} \left[ t \right] \; + \; \theta_{1}{'} \left[ t \right] \; + \; \theta_{2}{'} \left[ t \right] \right) \right) \; + \; \left( \text{Sin} \left[ \theta_{1} \left[ t \right] \; + \; \theta_{1}{'} \left[ t \right] \; + \; \theta_{2}{'} \left[ t \right] \right) \right) \; + \; \left( \text{Sin} \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2}{'} \left[ t \right] \; + \; \theta_{2}{'} \left[ t \right] \right] \; \right) \; + \; \left( \text{Sin} \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2}{'} \left[ t \right] \; + \; \theta_{2}{'} \left[ t \right] \right) \; \right) \; + \; \left( \text{Sin} \left[ \theta_{1} \left[ t \right] \; + \; \theta_{2}{'} \left[
                                                                           1[t] Izz_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) +
                                                                              -m_3
                                                                                          (2 \ (1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t]] \ \mathsf{L}_2 + 1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \ \mathsf{L}_3)
                                                                                                                                \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\left.\theta_{1}{'}[\mathsf{t}]\right. + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right. + \theta_{2}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{2}\left.\left(\theta_{1}{'}[\mathsf{t}]\right. + \theta_{2}{'}[\mathsf{t}]\right)\right. + \\ \left.\left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\left.\theta_{1}{'}\right[\mathsf{t}]\right. + \left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left[\mathsf{t}\right]\right]\right. \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left(\theta_{1}{'}[\mathsf{t}]\right)\right. + \\ \left.\mathsf{L}_{2}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{3}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{4}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{5}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{5}\left(\theta_{1}{'}
                                                                                                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                                                 2 \; (1[t] \; Sin[\theta_1[t] + \theta_2[t]] \; L_2 + 1[t] \; Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; L_3)
                                                                                                                              ( \sin[\theta_{1}[t]] \; \mathbb{L}_{1} \; \theta_{1}{'}[t] \; + \; \sin[\theta_{1}[t] \; + \; \theta_{2}[t]] \; \mathbb{L}_{2} \; (\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]) \; + \;
                                                                                                                                                     Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
Out[161]= 1[t] Izz_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]) +
                                                                              \frac{1}{2}\,\mathrm{m_3}\,\left(2\times1[\mathrm{t}]\,\mathrm{Cos}\left[\theta_1[\mathrm{t}]+\theta_2[\mathrm{t}]+\theta_3[\mathrm{t}]\right]\,\mathrm{L_3}\,\left(\mathrm{Cos}\left[\theta_1[\mathrm{t}]\right]\,\mathrm{L_1}\,\theta_1{'}[\mathrm{t}]+\mathrm{Cos}\left[\theta_1[\mathrm{t}]+\theta_2[\mathrm{t}]\right]\,\mathrm{L_2}\right)
                                                                                                                                                                    (\theta_{1}'[t] + \theta_{2}'[t]) + \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])) +
                                                                                                                 2\times1[t]\,\sin[\theta_1[t]+\theta_2[t]+\theta_3[t]]\,\,\mathtt{L}_3\,\left(\sin[\theta_1[t]]\,\,\mathtt{L}_1\,\,\theta_1{}'[t]+\sin[\theta_1[t]+\theta_2[t]\right)
                                                                                                                                                                L_{2}\left(\theta_{1}'[t] + \theta_{2}'[t]\right) + Sin\left[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]\right]L_{3}\left(\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]\right)))
```

```
\text{Out}_{1}(2) = 1[t] \text{ Izz}_{1} \theta_{1}^{"}[t] + 1[t] \text{ L}_{1}^{2} \text{ m}_{1} \theta_{1}^{"}[t] + 1[t] \text{ Izz}_{2} (\theta_{1}^{"}[t] + \theta_{2}^{"}[t]) + \theta_{2}^{"}[t] 
                                                                         \frac{1}{2}\,\mathrm{m_2}\,\left(2\,\left(1[\mathrm{t}]\,\cos[\theta_1[\mathrm{t}]]\,\,\mathrm{L_1}\,\theta_1{'}[\mathrm{t}] + 1[\mathrm{t}]\,\cos[\theta_1[\mathrm{t}] + \theta_2[\mathrm{t}]]\,\,\mathrm{L_2}\,\left(\theta_1{'}[\mathrm{t}] + \theta_2{'}[\mathrm{t}]\right)\right)
                                                                                                                         (Sin[\theta_{1}[t]] \; L_{1} \; \theta_{1}{'}[t] \; + \; Sin[\theta_{1}[t] \; + \; \theta_{2}[t]] \; L_{2} \; (\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]) \; ) \; + \;
                                                                                                          2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t]] + \theta_2[t] L_2 (\theta_1'[t]] + \theta_2'[t])
                                                                                                                       (-1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_1 \, \theta_1{}'[t] \, -1[t] \, \operatorname{Sin}[\theta_1[t] \, + \, \theta_2[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right)) \, + \, \theta_2{}'[t] \, + \, \theta_2{}'[t]
                                                                                                          2 (1[t] Cos[\theta_1[t]] L_1 + 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2)
                                                                                                                         \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} + \right)
                                                                                                                                                Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                          2 (1[t] Sin[\theta_1[t]] L_1 + 1[t] Sin[\theta_1[t] + \theta_2[t]] L_2)
                                                                                                                         (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                                                              Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) +
                                                                        1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                                           2
                                                                                m_3
                                                                                     \left(2\;(1[t]\;\text{Cos}[\theta_{1}[t]]\;\text{L}_{1}\;\theta_{1}{}'[t]\;+\;1[t]\;\text{Cos}[\theta_{1}[t]\;+\;\theta_{2}[t]]\;\text{L}_{2}\;(\theta_{1}{}'[t]\;+\;\theta_{2}{}'[t])\;+\;\theta_{2}{}'[t]\;\right)\;+\;\theta_{2}{}'[t]\;
                                                                                                                                              1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                                         (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                                          2 \; (\text{Cos}[\theta_{1}[\texttt{t}]] \; \text{L}_{1} \; \theta_{1}{}'[\texttt{t}] \; + \; \text{Cos}[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]] \; \text{L}_{2} \; (\theta_{1}{}'[\texttt{t}] \; + \; \theta_{2}{}'[\texttt{t}]) \; + \; \theta_{2}{}'[\texttt{t}]) \; + \; \theta_{2}{}'[\texttt{t}] \; + \;
                                                                                                                                             \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                                                                                                         (-1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_1 \, \theta_1{}'[t] \, -1[t] \, \operatorname{Sin}[\theta_1[t] \, + \, \theta_2[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right) \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right) \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right) \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right) \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right) \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right) \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right) \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right) \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, \operatorname{L}_2 \, \left(\theta_1{}'[t] \, + \, \theta_2{}'[t]\right) \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, -1[t] \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, -1[t] \, -1[t] \, \operatorname{Sin}[\theta_1[t]] \, -1[t] 
                                                                                                                                                1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                                          2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t]] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t]] + \theta_2[t] + \theta_3[t]]
                                                                                                                                                       L_3) \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t] + \theta_1'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin[\theta_1'[t]] L
                                                                                                                                             Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])^{2} +
                                                                                                                                             \texttt{Cos} \left[ \theta_{1}[\texttt{t}] \right] \, \texttt{L}_{1} \, \theta_{1}^{\prime \prime}[\texttt{t}] \, + \texttt{Cos} \left[ \theta_{1}[\texttt{t}] \, + \theta_{2}[\texttt{t}] \right] \, \texttt{L}_{2} \, \left( \theta_{1}^{\prime \prime}[\texttt{t}] \, + \theta_{2}^{\prime \prime}[\texttt{t}] \right) \, + \\
                                                                                                                                             \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + 2 (1[t] \sin [\theta_1[t]] L_1 + \theta_3''[t])
                                                                                                                                              1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                                                                                         (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                                                             \texttt{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]] \; \texttt{L}_{3} \; (\theta_{1}{}'[\texttt{t}] + \theta_{2}{}'[\texttt{t}] + \theta_{3}{}'[\texttt{t}])^{2} + \\
                                                                                                                                             Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                                                             Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
```

```
Out[163]= 1[t] Izz_2 (\theta_1''[t] + \theta_2''[t]) +
                                       \frac{1}{2} m_2 \left( -21[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) (\cos[\theta_1[t]] L_1 \theta_1'[t] + \theta_2'[t]) \right)
                                                                           Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1'[t] + \theta_2'[t])) + 2 \times 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2
                                                                (\theta_{1}'[t] + \theta_{2}'[t]) (Sin[\theta_{1}[t]] L_{1} \theta_{1}'[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])) +
                                                        2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                                                           Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                                                           Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]]
                                                              L_2 \left( Cos[\theta_1[t]] L_1 \theta_1'[t]^2 + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_2 (\theta_1'[t])^2 + Cos[\theta_1[t]
                                                                           Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]))) +
                                      1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                       1
                                       2
                                          m_3
                                              (2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                           \texttt{1[t]}\;\mathsf{Cos}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]]\;\mathtt{L}_3\;(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]))
                                                                (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                        2 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                            1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                        2 \ (1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t]] \ \mathtt{L}_2 + 1[t] \ \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \ \mathtt{L}_3)
                                                                \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - (\theta_{1}'[t])^{2} + (\theta_{
                                                                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                           Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                                        2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                                \left( \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}]^{2} + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right)^{2} \; + \right.
                                                                           \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]\right] \; \texttt{L}_{3} \; \left(\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}]\right)^{2} + \\
                                                                           Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
Out[164]= 1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                      \frac{1}{2} m_3 \left( -21[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t] \right) L_3 \left( \theta_1'[t] + \theta_2'[t] + \theta_3'[t] \right)
                                                                (\cos[\theta_{1}[t]] L_{1} \theta_{1}'[t] + \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]) +
                                                                           \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                        2 \times 1[t] \; \mathsf{Cos}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; \mathsf{L_3} \; (\theta_1{}'[t] + \theta_2{}'[t] + \theta_3{}'[t])
                                                                \left( \operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right]\right] \, \operatorname{L}_{1} \, \theta_{1}{}'\left[\mathtt{t}\right] + \operatorname{Sin}\left[\theta_{1}\left[\mathtt{t}\right] + \theta_{2}\left[\mathtt{t}\right]\right] \, \operatorname{L}_{2} \, \left(\theta_{1}{}'\left[\mathtt{t}\right] + \theta_{2}{}'\left[\mathtt{t}\right]\right) \, + \right. \\
                                                                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                        2 \times 1[t] \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} \left( -\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] \right)
                                                                                L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])^{2} + \theta_{3}'[t] + \theta_{3
                                                                           \texttt{Cos}\left[\theta_{1}[\texttt{t}]\right] \; \texttt{L}_{1} \; \theta_{1}{''}[\texttt{t}] \; + \; \texttt{Cos}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \texttt{L}_{2} \; \left(\theta_{1}{''}[\texttt{t}] \; + \; \theta_{2}{''}[\texttt{t}]\right) \; + \;
                                                                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                        2 \times 1[t] \sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} \left(\cos[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} + \cos[\theta_{1}[t] + \theta_{2}[t]]\right)
                                                                                L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                           Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                           Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
```

```
\text{Out} [\text{165}] = -g \cos \left[\theta_1\left[t\right]\right] \text{ L}_1 \text{ m}_1 - g \left(\cos \left[\theta_1\left[t\right]\right] \text{ L}_1 + \cos \left[\theta_1\left[t\right] + \theta_2\left[t\right]\right] \text{ L}_2\right) \text{ m}_2 - \left[\cos \left[\theta_1\left[t\right]\right] \text{ m}_2\right] + \left[\cos \left[\theta_1\left[t\right]\right] + \left[\cos \left[\theta_1\left[t\right]\right] \text{ m}_2\right] + \left[\cos \left[\theta_1\left[t\right]\right] + \left[\cos \left[\theta_1\left[t\right]\right]\right] + \left[\cos \left[\theta_1\left[t\right]\right]
                                                                                     g (\cos[\theta_1[t]] L_1 + \cos[\theta_1[t] + \theta_2[t]] L_2 + \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3) m_3 + \cos[\theta_1[t]] L_2 + \cos[\theta_1[t]] + \cos
                                                                                        - m_2 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) 
                                                                                                                                               (-\sin[\theta_1[t]] L_1 \theta_1'[t] - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                                                                              2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))
                                                                                                                                               (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) +
                                                                                       \frac{-}{2}\,\mathrm{m_3}\,\left(2\,\left(\mathrm{Cos}\left[\theta_1[\mathrm{t}]\right]\,\mathrm{L}_1\,\theta_1{}'[\mathrm{t}] + \mathrm{Cos}\left[\theta_1[\mathrm{t}] + \theta_2[\mathrm{t}]\right]\,\mathrm{L}_2\,\left(\theta_1{}'[\mathrm{t}] + \theta_2{}'[\mathrm{t}]\right) + \right)
                                                                                                                                                                       Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                                                               (-\sin[\theta_1[t]] L_1 \theta_1'[t] - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                                                                                                                       Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \ L_{3} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t])) \ +
                                                                                                                              2 \left( \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \right. \left. \mathsf{L}_{1} \right. \theta_{1}{'}[\mathsf{t}] + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}]\right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. \\ \left. \mathsf{L}_{5} \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{
                                                                                                                                                                       Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                                                               (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                                       Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
 \text{Out} [166] = -g \cos [\theta_1[t] + \theta_2[t]] \text{ L}_2 \text{ m}_2 - g (\cos [\theta_1[t] + \theta_2[t]] \text{ L}_2 + \cos [\theta_1[t] + \theta_2[t]] + \theta_3[t]] \text{ L}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m}_3) \text{ m}_3 + (\cos [\theta_1[t] + \theta_2[t]) \text{ m}_3) \text{ m
                                                                                       \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                                                                               \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{3} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{3} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{3} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \; \mathsf{L}_{4} \; \mathsf{L}_{5} \; 
                                                                                                                              2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                                                                               (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) +
                                                                                        - m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + 2 
                                                                                                                                                                           Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                                                               (-\sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) - \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3
                                                                                                                                                                                        (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2(\cos[\theta_1[t] + \theta_2[t]) L_2(\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                                       \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                                                                                                                               \left(\text{Sin}\left[\theta_{1}\left[\texttt{t}\right]\right] \; \text{L}_{1} \; \theta_{1}^{\prime}\left[\texttt{t}\right] + \text{Sin}\left[\theta_{1}\left[\texttt{t}\right] + \theta_{2}\left[\texttt{t}\right]\right] \; \text{L}_{2} \; \left(\theta_{1}^{\prime}\left[\texttt{t}\right] + \theta_{2}^{\prime}\left[\texttt{t}\right]\right) \; + \right.
                                                                                                                                                                       \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right)))
Out[167]= -g \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 +
                                                                                       \frac{1}{-} \, \mathbf{m}_3 \, \left( -2 \, \mathrm{Sin} \left[ \boldsymbol{\theta}_1[\mathtt{t}] + \boldsymbol{\theta}_2[\mathtt{t}] + \boldsymbol{\theta}_3[\mathtt{t}] \right] \, \mathbf{L}_3 \, \left( \boldsymbol{\theta}_1{}'[\mathtt{t}] + \boldsymbol{\theta}_2{}'[\mathtt{t}] + \boldsymbol{\theta}_3{}'[\mathtt{t}] \right) \right.
                                                                                                                                               (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                                       Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                                                              2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                                                                               (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                                                                       Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
```

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\texttt{Out[168]= g Cos}\left[\theta_1[\texttt{t}]\right] \texttt{L}_1 \; \texttt{m}_1 + \texttt{g} \; \left(\texttt{Cos}\left[\theta_1[\texttt{t}]\right] \; \texttt{L}_1 + \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]\right] \; \texttt{L}_2\right) \; \texttt{m}_2 + \texttt{m}_2[\texttt{t}] \; \texttt{L}_2) \; \texttt{m}_2 + \texttt{L}_2[\texttt{t}] \; \texttt{L}_2
                                                        g \left( \cos \left[ \theta_{1}[t] \right] \right] L_{1} + \cos \left[ \theta_{1}[t] + \theta_{2}[t] \right] L_{2} + \cos \left[ \theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t] \right] L_{3} \right) m_{3} - m_{3} + m_{3} 
                                                          - \underset{2}{\text{m}_2} \left( 2 \left( \text{Cos}[\theta_1[\texttt{t}]] \text{L}_1 \, \theta_1'[\texttt{t}] + \text{Cos}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \right) \text{L}_2 \left( \theta_1'[\texttt{t}] + \theta_2'[\texttt{t}] \right) \right)
                                                                                               (-\sin[\theta_1[t]] L_1 \theta_1'[t] - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                                   2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))
                                                                                               (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) -
                                                           - m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + 
                                                                                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                               (-\sin[\theta_1[t]] L_1 \theta_1'[t] - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                                                               Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \ L_{3} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t])) \ +
                                                                                   2 \left( \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \right. \left. \mathsf{L}_{1} \right. \theta_{1}{'}[\mathsf{t}] + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}]\right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{L}_{3} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \left. \mathsf{L}_{4} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right. + \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{5} \left. \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right] \right. \\ \left. \mathsf{L}_{5} \left(\theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}]\right) \right] \right. \\ \left. \mathsf{L}_{
                                                                                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                               (Sin[\theta_{1}[t]] \ L_{1} \ \theta_{1}{'}[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] \ L_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \ +
                                                                                                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
                                                        1[t] Izz_1 \Theta_1''[t] + 1[t] L_1^2 m_1 \Theta_1''[t] + 1[t] Izz_2 (\Theta_1''[t] + \Theta_2''[t]) +
                                                         1
                                                          — m<sub>2</sub>
                                                                    \left(2\;(1[t]\;\cos[\theta_1[t]]\;L_1\;\theta_1{}'[t]+1[t]\;\cos[\theta_1[t]+\theta_2[t]]\;L_2\;(\theta_1{}'[t]+\theta_2{}'[t])\right)
                                                                                               \left( \text{Sin}\left[\theta_{1}[\texttt{t}]\right] \; \text{L}_{1} \; \theta_{1}{}'[\texttt{t}] \; + \; \text{Sin}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \text{L}_{2} \; \left(\theta_{1}{}'[\texttt{t}] \; + \; \theta_{2}{}'[\texttt{t}]\right) \right) \; + \;
                                                                                   2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))
                                                                                               (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                                   2 (1[t] Cos[\theta_1[t]] L_1 + 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2)
                                                                                               \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} + \right.
                                                                                                                Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                   2 (1[t] Sin[\theta_1[t]] L_1 + 1[t] Sin[\theta_1[t] + \theta_2[t]] L_2)
                                                                                               (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                               Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) +
                                                        1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                         1
                                                            2
                                                               m_3
                                                                     (2 (1[t] Cos[\theta_1[t]] L_1 \theta_1'[t] + 1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) + 
                                                                                                                1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                               (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                   2 (Cos[\theta_1[t]] L<sub>1</sub> \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L<sub>2</sub> (\theta_1'[t] + \theta_2'[t]) +
                                                                                                               \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                                                                               (-1[t] \sin[\theta_1[t]] L_1 \theta_1'[t] - 1[t] \sin[\theta_1[t]] + \theta_2[t]] L_2 (\theta_1'[t]] + \theta_2'[t]) - \frac{1}{2} (\theta_1'[t]) - \frac{1}{2} (\theta_1'[t]) + \frac
                                                                                                                1[t] \, \operatorname{Sin}[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \, \operatorname{L}_{3} \, (\theta_{1}{}'[t] + \theta_{2}{}'[t] + \theta_{3}{}'[t])) + \\
                                                                                   2 (1[t] \cos[\theta_1[t]] L_1 + 1[t] \cos[\theta_1[t]] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t]] + \theta_2[t] + \theta_3[t]]
                                                                                                                        L_3) \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - \sin[\theta_1[t]] L_3 (\theta_1'[t])^2 - \sin[\theta_1'[t]] L_3 (\theta_1'[t
                                                                                                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                                               \texttt{Cos}\left[\theta_{1}[\texttt{t}]\right] \; \texttt{L}_{1} \; \theta_{1}{''}[\texttt{t}] \; + \; \texttt{Cos}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \texttt{L}_{2} \; \left(\theta_{1}{''}[\texttt{t}] \; + \; \theta_{2}{''}[\texttt{t}]\right) \; + \;
                                                                                                               \cos \left[ \theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t] \right] L_{3} \left( \theta_{1}''[t] + \theta_{2}''[t] + \theta_{3}''[t] \right) + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] L_{1} + \theta_{2}''[t] \right) + 2 \left( 1[t] \sin \left[ \theta_{1}[t] \right] L_{1} + \theta_{2}''[t] \right)
                                                                                                                1[t] \sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                                                               (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                               \texttt{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]] \; \texttt{L}_{3} \; (\theta_{1}{}'[\texttt{t}] + \theta_{2}{}'[\texttt{t}] + \theta_{3}{}'[\texttt{t}])^{2} + \\
                                                                                                               Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
```

```
\texttt{Out[169]= g Cos}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; \texttt{L}_{2} \; \texttt{m}_{2} + \texttt{g} \; \left(\texttt{Cos}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; \texttt{L}_{2} + \texttt{Cos}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right] + \theta_{3}\left[t\right]\right] \; \texttt{L}_{3}\right) \; \texttt{m}_{3} - \texttt{m}_{3} + \texttt{m}_{3}
                                                           \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                                 (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                                     2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                                 (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) -
                                                            \frac{-}{2} m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                    Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                 (-\sin[\theta_1[t]+\theta_2[t]] \ L_2 \ (\theta_1{}'[t]+\theta_2{}'[t]) \ -\sin[\theta_1[t]+\theta_2[t]+\theta_3[t]] \ L_3
                                                                                                                             (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2(\cos[\theta_1[t] + \theta_2[t]) L_2(\theta_1'[t] + \theta_2'[t]) +
                                                                                                                  \mathsf{Cos} \, [\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}] + \theta_3[\mathsf{t}]] \, \, \mathsf{L}_3 \, \, (\theta_1{}'[\mathsf{t}] + \theta_2{}'[\mathsf{t}] + \theta_3{}'[\mathsf{t}]) \, )
                                                                                                 \left( \text{Sin}\left[\theta_{1}\left[t\right]\right] \text{ L}_{1} \; \theta_{1}^{\prime}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \text{ L}_{2} \; \left(\theta_{1}^{\prime}\left[t\right] + \theta_{2}^{\prime}\left[t\right]\right) + \left(\theta_{1}
                                                                                                                    Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
                                                        1[t] \; \mathsf{Izz}_2 \; (\theta_1{''}[t] + \theta_2{''}[t]) \; + \; \frac{1}{2} \; \mathsf{m}_2 \; \left( - \; 2 \; 1[t] \; \mathsf{Sin}[\theta_1[t] + \theta_2[t]] \; \mathsf{L}_2 \; (\theta_1{'}[t] + \theta_2{'}[t]) \; \mathsf{L}_3 \; (\theta_1{''}[t] + \theta_2{''}[t]) \; \mathsf{L}_4 \; (\theta_1{''}[t] + \theta_2{''}[t]) \; \mathsf{L}_5 \; (\theta_1{''}[t] + \theta_2{''}[t]) \; \mathsf{L}_6 \; (\theta_1{''}[t] + \theta_2{''}[t]) \; \mathsf{L}_7 \; (\theta_1{''}[t] + \theta_2{''}[t]) \; \mathsf{L}_8 \; (\theta_1{''}[t] + \theta_2{''}[t]) \; \mathsf{L}_8 \; (\theta_1{''}[t] + \theta_2{''}[t]) \; \mathsf{L}_9 \; 
                                                                                                 \left( \text{Cos} \left[ \theta_{1} \left[ \mathsf{t} \right] \right] \; \mathsf{L}_{1} \; \theta_{1}{'} \left[ \mathsf{t} \right] + \text{Cos} \left[ \theta_{1} \left[ \mathsf{t} \right] + \theta_{2} \left[ \mathsf{t} \right] \right] \; \mathsf{L}_{2} \; \left( \theta_{1}{'} \left[ \mathsf{t} \right] + \theta_{2}{'} \left[ \mathsf{t} \right] \right) \right) \; + \; \mathsf{L}_{2} \; \mathsf{L}_{3} \; \mathsf{L}_{4} \; \mathsf{L}_{5} 
                                                                                     2\times1[\texttt{t}]\;\texttt{Cos}\left[\theta_{1}[\texttt{t}]+\theta_{2}[\texttt{t}]\right]\;\texttt{L}_{2}\;\left(\theta_{1}{'}[\texttt{t}]+\theta_{2}{'}[\texttt{t}]\right)
                                                                                                 (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                                     2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                                                                                                  Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                                                                                                 Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]]
                                                                                               L_2 \left( Cos[\theta_1[t]] L_1 \theta_1'[t]^2 + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \right)
                                                                                                                 Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) +
                                                          1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                             2
                                                                     (2 (1[t] Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                    1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                 (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                 Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                     2 (Cos[\theta_1[t]] L<sub>1</sub> \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L<sub>2</sub> (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                 \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right))
                                                                                                 (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                                                                    1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                     2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                                                                \left(-\sin[\theta_{1}[t]] L_{1} \theta_{1}'[t]^{2} - \sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - \right)
                                                                                                                 Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])^{2} +
                                                                                                                 \texttt{Cos}\left[\theta_{1}[\texttt{t}]\right] \; \texttt{L}_{1} \; \theta_{1}^{\,\prime\prime}[\texttt{t}] \; + \; \texttt{Cos}\left[\theta_{1}[\texttt{t}] \; + \; \theta_{2}[\texttt{t}]\right] \; \texttt{L}_{2} \; \left(\theta_{1}^{\,\prime\prime}[\texttt{t}] \; + \; \theta_{2}^{\,\prime\prime}[\texttt{t}]\right) \; + \;
                                                                                                                  Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                                                                     2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                                                                 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                                 \texttt{Cos}\left[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}]\right] \; \texttt{L}_{3} \; \left(\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}]\right)^{2} + \\
                                                                                                                 Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                                 Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
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Out[170]= g \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 -
                                                          \frac{-}{2} m_3 (-2 \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                               \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\left.\theta_{1}{'}[\mathsf{t}]\right. + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right. + \theta_{2}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{2}\left.\left(\theta_{1}{'}[\mathsf{t}]\right. + \theta_{2}{'}[\mathsf{t}]\right)\right. + \\ \left.\left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. \left.\mathsf{L}_{1}\left.\theta_{1}{'}\right[\mathsf{t}]\right. + \left.\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right]\right. + \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left[\mathsf{t}\right]\right]\right. \left.\mathsf{L}_{2}\left.\mathsf{L}_{2}\left(\theta_{1}{'}[\mathsf{t}]\right)\right. + \\ \left.\mathsf{L}_{2}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{3}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{4}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{5}\left(\theta_{1}{'}[\mathsf{t}]\right)\right] \left.\mathsf{L}_{5}\left(\theta_{1}{'}
                                                                                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                    2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                               (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                               \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right))) \; + \;
                                                        1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                           2
                                                               m_3
                                                                   (-21[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                               (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t]] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                                    2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                               (Sin[\theta_{1}[t]] \; L_{1} \; \theta_{1}{'}[t] \; + \; Sin[\theta_{1}[t] \; + \; \theta_{2}[t]] \; L_{2} \; (\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]) \; + \;
                                                                                                               \texttt{Sin}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right)) \; + \;
                                                                                    2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]]\right)
                                                                                                                        L_{2} (\theta_{1}'[t] + \theta_{2}'[t])^{2} - Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])^{2} + \theta_{3}'[t] + \theta_{3
                                                                                                               \cos [\theta_1[t]] L_1 \theta_1''[t] + \cos [\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                                                    2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]]
                                                                                                                        \text{L}_{2} \, \left( \theta_{1}{}'[\texttt{t}] + \theta_{2}{}'[\texttt{t}] \right)^{2} + \text{Cos} \left[ \theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}] + \theta_{3}[\texttt{t}] \right] \, \text{L}_{3} \, \left( \theta_{1}{}'[\texttt{t}] + \theta_{2}{}'[\texttt{t}] + \theta_{3}{}'[\texttt{t}] \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{3} \, \left( \theta_{1}{}'(\texttt{t}) + \theta_{2}{}'(\texttt{t}) + \theta_{3}{}'(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{3} \, \left( \theta_{1}{}'(\texttt{t}) + \theta_{2}{}'(\texttt{t}) + \theta_{3}{}'(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{3} \, \left( \theta_{1}{}'(\texttt{t}) + \theta_{2}{}'(\texttt{t}) + \theta_{3}{}'(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{3} \, \left( \theta_{1}{}'(\texttt{t}) + \theta_{2}{}'(\texttt{t}) + \theta_{3}{}'(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{3} \, \left( \theta_{1}{}'(\texttt{t}) + \theta_{2}{}'(\texttt{t}) + \theta_{3}{}'(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{3} \, \left( \theta_{1}{}'(\texttt{t}) + \theta_{2}{}'(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{3} \, \left( \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{3} \, \left( \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{4} \, \left( \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{4} \, \left( \theta_{1}(\texttt{t}) + \theta_{2}(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{5} \, \left( \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{5} \, \left( \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{5} \, \left( \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{5} \, \left( \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right] \, \text{L}_{5} \, \left( \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_{3}(\texttt{t}) \right)^{2} + \text{Cos} \left[ \theta_{1}(\texttt{t}) + \theta_{3}(\texttt{t}) + \theta_
                                                                                                               Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
 \text{Out} [171] = \left\{ g \cos \left[ \theta_1 \left[ t \right] + \theta_2 \left[ t \right] \right] \right. \\  \left. L_2 \right. \\  \left. m_2 + g \left( \cos \left[ \theta_1 \left[ t \right] + \theta_2 \left[ t \right] \right] \right. \\  \left. L_2 + \cos \left[ \theta_1 \left[ t \right] + \theta_2 \left[ t \right] \right] + \theta_3 \left[ t \right] \right] \right. \\  \left. L_3 \right) \right. \\  \left. m_3 - m_3 + m
                                                                  \frac{1}{2} m_2 (-2 \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                                        \left( \text{Cos} \left[ \theta_{1}[t] \right] \right. \left. \text{L}_{1} \right. \theta_{1}{'}[t] + \text{Cos} \left[ \theta_{1}[t] + \theta_{2}[t] \right] \right. \left. \text{L}_{2} \left. \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] \right) \right) + \\
                                                                                             2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                                        \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; L_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; d_{1}\right] + \theta_{2}\left[t\right]\right) \; d_{1} \; d_{2} \; d_{
                                                                    \frac{-}{2} m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                                         Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                        (-\sin[\theta_1[t]+\theta_2[t]] \ L_2 \ (\theta_1{}'[t]+\theta_2{}'[t]) \ -\sin[\theta_1[t]+\theta_2[t]+\theta_3[t]] \ L_3
                                                                                                                                    (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])) + 2 (Cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]) +
                                                                                                                         Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                                        \left(\text{Sin}\left[\theta_{1}\left[\texttt{t}\right]\right] \; \text{L}_{1} \; \theta_{1}{'}\left[\texttt{t}\right] + \text{Sin}\left[\theta_{1}\left[\texttt{t}\right] + \theta_{2}\left[\texttt{t}\right]\right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[\texttt{t}\right] + \theta_{2}{'}\left[\texttt{t}\right]\right) \; + \right.
                                                                                                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
                                                                1[t] Izz_{2} (\theta_{1}''[t] + \theta_{2}''[t]) + \frac{1}{2} m_{2} (-21[t] Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])
                                                                                                        (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                                             2\times1[\texttt{t}]\;\texttt{Cos}\left[\theta_{1}[\texttt{t}]+\theta_{2}[\texttt{t}]\right]\;\texttt{L}_{2}\;\left(\theta_{1}{'}[\texttt{t}]+\theta_{2}{'}[\texttt{t}]\right)
                                                                                                        (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                                                                             2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                                                                                                         Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                                                                                                         Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]]
                                                                                                      L_2 \left( Cos[\theta_1[t]] L_1 \theta_1'[t]^2 + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + \right)
                                                                                                                         Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) +
                                                                  1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
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\frac{1}{2}
                     m_3
                        (2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                        1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                               \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right. \left. \operatorname{L}_{1} \right. \theta_{1}{'}[t] + \operatorname{Sin}\left[\theta_{1}[t] + \theta_{2}[t]\right] \right. \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{1} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t] + \theta_{2}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}{'}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \right. \left(\theta_{1}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \left. \left(\theta_{1}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \right] \left. \operatorname{L}_{2} \left(\theta_{1}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right] \right) \left. \operatorname{L}_{2} \left(\theta_{1}[t]\right) + \left(\operatorname{Sin}\left[\theta_{1}[t]\right]
                                                                                      Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                2 \; (\text{Cos}[\theta_{1}[t]] \; \text{L}_{1} \; \theta_{1}{}'[t] \; + \; \text{Cos}[\theta_{1}[t] \; + \; \theta_{2}[t]] \; \text{L}_{2} \; (\theta_{1}{}'[t] \; + \; \theta_{2}{}'[t]) \; + \;
                                                                                      Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                               (-1[t] \, \operatorname{Sin}[\theta_1[t] + \theta_2[t]] \, \operatorname{L}_2 \, (\theta_1{}'[t] + \theta_2{}'[t]) \, - \,
                                                                                      1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 
                                                2~(1\texttt{[t]}~\mathsf{Cos}[\theta_1\texttt{[t]} + \theta_2\texttt{[t]}]~\mathtt{L}_2 + 1\texttt{[t]}~\mathsf{Cos}[\theta_1\texttt{[t]} + \theta_2\texttt{[t]} + \theta_3\texttt{[t]}]~\mathtt{L}_3)
                                                               \left(-\sin\left[\theta_{1}[t]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[t]^{2} \; - \; \sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[t] \; + \; \theta_{2}{'}[t]\right)^{2} \; - \; \left(-\sin\left[\theta_{1}[t] \; + \; \theta_{2}[t]\right] \; + \; \left(-\sin\left[\theta_{1}[t] 
                                                                                      Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                     \texttt{Cos}[\theta_1[\texttt{t}]] \; \texttt{L}_1 \; \theta_1{''}[\texttt{t}] \; + \; \texttt{Cos}[\theta_1[\texttt{t}] \; + \; \theta_2[\texttt{t}]] \; \texttt{L}_2 \; (\theta_1{''}[\texttt{t}] \; + \; \theta_2{''}[\texttt{t}]) \; + \;
                                                                                      Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                                2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                               (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                      Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                      \texttt{Sin}\left[\theta_{1}\left[\texttt{t}\right]\right] \; \texttt{L}_{1} \; \theta_{1}^{\prime\prime}\left[\texttt{t}\right] + \texttt{Sin}\left[\theta_{1}\left[\texttt{t}\right] + \theta_{2}\left[\texttt{t}\right]\right] \; \texttt{L}_{2} \; \left(\theta_{1}^{\prime\prime}\left[\texttt{t}\right] + \theta_{2}^{\prime\prime}\left[\texttt{t}\right]\right) \; + \\
                                                                                     Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
\left\{-g\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]L_{2}m_{2}-g\left(\cos\left[\theta_{1}[t]+\theta_{2}[t]\right]L_{2}+\cos\left[\theta_{1}[t]+\theta_{2}[t]+\theta_{3}[t]\right]L_{3}\right\}
                                           m_3 - \frac{1}{2} m_2 \left( -2 \sin[\theta_1[t] + \theta_2[t]] L_2 \left( \theta_1'[t] + \theta_2'[t] \right) \right)
                                                                                        \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \;
                                                                        2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                          (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) -
                                      \frac{1}{2} m_3 (2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                          \left(-\sin\left[\theta_{1}[t]+\theta_{2}[t]\right]\right] \, L_{2} \, \left(\theta_{1}{}'[t]+\theta_{2}{}'[t]\right) \, - \sin\left[\theta_{1}[t]+\theta_{2}[t]+\theta_{3}[t]\right]
                                                                                                                             \texttt{L}_{3} \ (\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}] + \theta_{3}{'}[\texttt{t}])) \ + \ 2 \ (\texttt{Cos}[\theta_{1}[\texttt{t}] + \theta_{2}[\texttt{t}]] \ \texttt{L}_{2} \ (\theta_{1}{'}[\texttt{t}] + \theta_{2}{'}[\texttt{t}]) \ + \ \theta_{2}{'}[\texttt{t}]) \ + \ \theta_{2}{'}[\texttt{t}] \ + \ \theta_{2}{'}[\texttt{t}]) \ + \ \theta_{2}{'}[\texttt{t}] \ + \ \theta_{2}{'}[\texttt{t}] \ + \ \theta_{2}{'}[\texttt{t}]) \ + \ \theta_{2}{'}[\texttt{t}] \ + \ \theta_{2}{'}[\texttt
                                                                                                               Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                                                          \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; \text{L}_{1} \; \theta_{1}^{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; \text{L}_{2} \; \left(\theta_{1}^{'}\left[t\right] + \theta_{2}^{'}\left[t\right]\right) \; + \right.
                                                                                                                 \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right] + \theta_3\left[\texttt{t}\right]\right] \; \texttt{L}_3 \; \left(\theta_1{'}\left[\texttt{t}\right] + \theta_2{'}\left[\texttt{t}\right] + \theta_3{'}\left[\texttt{t}\right]\right)\right)) \; + \;
                                  1[t] Izz_{2} (\theta_{1}''[t] + \theta_{2}''[t]) + \frac{1}{2} m_{2} (-21[t] Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t])
                                                                                          \left(\mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}] \; + \; \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right)\right) \; + \;
                                                                        2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                                                        \left(\operatorname{Sin}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \operatorname{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \operatorname{Sin}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \operatorname{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right]\right) \; + \; \left(\operatorname{Sin}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{
                                                                        2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_1 \theta_1'[t]
                                                                                                                 Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                                                                                               Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t]) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]]
                                                                                     L_2 \left( \cos \left[ \theta_1[t] \right] L_1 \theta_1'[t]^2 + \cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t
                                                                                                                 Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 1[t] Izz_3
                                                   (\theta_{1}''[t] + \theta_{2}''[t] + \theta_{3}''[t]) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}'[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}'[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}'[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}''[t]] L_{2} (\theta_{1}'[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] \cos[\theta_{1}[t] + \theta_{2}''[t]) + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] + \theta_{2}'[t] + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] + \theta_{2}'[t] + \theta_{2}''[t])\right) + \frac{1}{2} m_{3} \left(2 (1[t] + \theta_{2}'[t] + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] + \theta_{2}'[t] + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] + \theta_{2}'[t] + \theta_{2}''[t]\right) + \frac{1}{2} m_{3} \left(2 (1[t] + \theta_{2}'[t] + \theta_{2}'[t]\right) + \frac
                                                                                                               \texttt{1[t]}\; \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \; \texttt{L}_3 \; \left(\theta_1{'}[\texttt{t}] + \theta_2{'}[\texttt{t}] + \theta_3{'}[\texttt{t}]\right))
                                                                                        (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                               Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                          2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
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Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                       (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                     1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                             2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                      (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                                                    Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                    Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                    Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                            2 \; (1[t] \; Sin[\theta_1[t] + \theta_2[t]] \; L_2 + 1[t] \; Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] \; L_3)
                                       (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                    Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                    Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                    Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
\left\{-g\cos[\theta_{1}[t] + \theta_{2}[t]]\right\} L<sub>2</sub> m<sub>2</sub> - g (Cos[\theta_{1}[t] + \theta_{2}[t]] L<sub>2</sub> + Cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]]
                                                  L_3) m_3 - \frac{1}{2} m_2 \left(-2 \sin[\theta_1[t] + \theta_2[t]] L_2 \left(\theta_1'[t] + \theta_2'[t]\right)\right)
                                                      \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \\
                                             2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                      (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]))) -
                      \frac{1}{2} \, m_3 \, \left( 2 \, \left( \text{Cos} \left[ \theta_1[t] \right] \, L_1 \, \theta_1{}'[t] + \text{Cos} \left[ \theta_1[t] + \theta_2[t] \right] \, L_2 \, \left( \theta_1{}'[t] + \theta_2{}'[t] \right) + \right) \right) \, dt + 2 \, dt +
                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                      (-\sin[\theta_1[t]+\theta_2[t]] \ L_2 \ (\theta_1{}'[t]+\theta_2{}'[t]) \ -\sin[\theta_1[t]+\theta_2[t]+\theta_3[t]] \ L_3
                                                                             (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                      \left(\text{Sin}\left[\theta_{1}\left[\texttt{t}\right]\right] \; \text{L}_{1} \; \theta_{1}{'}\left[\texttt{t}\right] + \text{Sin}\left[\theta_{1}\left[\texttt{t}\right] + \theta_{2}\left[\texttt{t}\right]\right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[\texttt{t}\right] + \theta_{2}{'}\left[\texttt{t}\right]\right) \; + \right.
                                                                      \sin \left[\theta_1\left[\mathtt{t}\right] + \theta_2\left[\mathtt{t}\right] + \theta_3\left[\mathtt{t}\right]\right] \, \mathtt{L}_3 \, \left(\theta_1{}'\left[\mathtt{t}\right] + \theta_2{}'\left[\mathtt{t}\right] + \theta_3{}'\left[\mathtt{t}\right]\right)\right)) \, + \, \mathtt{1}\left[\mathtt{t}\right] \, \mathtt{Izz}_2 
                             (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 (-21[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                      \left( \text{Cos} \left[ \theta_{1} \left[ t \right] \right] \; \text{L}_{1} \; \theta_{1}{'} \left[ t \right] + \text{Cos} \left[ \theta_{1} \left[ t \right] + \theta_{2} \left[ t \right] \right] \; \text{L}_{2} \; \left( \theta_{1}{'} \left[ t \right] + \theta_{2}{'} \left[ t \right] \right) \right) \; + \; \left( \text{Cos} \left[ \theta_{1} \left[ t \right] \right] \; \text{Cos} \left[ \theta_{1} \left[ t \right] \right] \; \text{L}_{2} \; \left( \theta_{1}{'} \left[ t \right] + \theta_{2}{'} \left[ t \right] \right) \right) \; + \; \left( \text{Cos} \left[ \theta_{1} \left[ t \right] \right] \; \text{L}_{2} \; \left( \theta_{1}{'} \left[ t \right] \right) \; \text{Cos} \left[ \theta_{1} \left[ t \right] \right] \; \text{Cos} \left[ \theta_{1} \left[ t \right] \; \text{Cos} \left[ \theta_{1} \left[ t \right] \right] \; \text{L}_{2} \; \left( \theta_{1}{'} \left[ t \right] \right) \; \text{Cos} \left[ \theta_{1} \left[ t \right] \; \text{Cos} \left[ \theta_{1} \left[ t \right] \; \text{Cos} \left[ \theta_{1} \left[ t \right] \right] \; \text{Cos} \left[ \theta_{1} \left[ t \right] \; \text
                                             2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                      \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; \text{L}_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right) \; + \\
                                             2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                                                      \sin \left[\theta_1[t] + \theta_2[t]\right] \, L_2 \, \left(\theta_1{}'[t] + \theta_2{}'[t]\right)^2 + \cos \left[\theta_1[t]\right] \, L_1 \, \theta_1{}''[t] + \left[\theta_1{}''[t] + \theta_2{}''[t]\right] \, L_2 \, \left(\theta_1{}''[t] + \theta_2{}''[t]\right)^2 + \cos \left[\theta_1[t]\right] \, L_1 \, \theta_1{}''[t] + \left[\theta_1{}''[t] + \theta_2{}''[t]\right] \, L_2 \, \left(\theta_1{}''[t] + \theta_2{}''[t]\right)^2 + \cos \left[\theta_1[t]\right] \, L_1 \, \theta_1{}''[t] + \left[\theta_1{}''[t] + \theta_2{}''[t]\right] \, L_2 \, \left(\theta_1{}''[t] + \theta_2{}''[t]\right)^2 + \cos \left[\theta_1[t]\right] \, L_1 \, \theta_1{}''[t] + \left[\theta_1{}''[t] + \theta_2{}''[t]\right] \, L_2 \, \left(\theta_1{}''[t] + \theta_2{}''[t]\right)^2 + \cos \left[\theta_1[t]\right] \, L_1 \, \theta_1{}''[t] + \left[\theta_1{}''[t] + \theta_2{}''[t]\right] \, L_2 \, \left(\theta_1{}''[t] + \theta_2{}''[t]\right)^2 + \cos \left[\theta_1[t]\right] \, L_1 \, \theta_1{}''[t] + \left[\theta_1{}''[t] + \theta_2{}''[t]\right] \, L_2 \, \left(\theta_1{}''[t] + \theta_2{}''[t]\right)^2 + \cos \left[\theta_1[t]\right] \, L_1 \, \theta_1{}''[t] + \left[\theta_1{}''[t] + \theta_2{}''[t]\right] \, L_2 \, \left(\theta_1{}''[t] + \theta_2{}''[t]\right)^2 + \left[\theta_1{}''[t] + \theta_2{}''[t]\right] \, L_2 \, \left(\theta_1{}''[t] + \theta_2{}''[t]\right)^2 + \left[\theta_1{}''[t] + \theta_2{}''[t]\right]^2 + \left[\theta_1{}''[t]\right]^2 + \left[\theta_1{}''[t]\right]
                                                                    Cos[\theta_1[t] + \theta_2[t]] L_2(\theta_1''[t] + \theta_2''[t]) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t]]
                                                    L_2 \left( Cos[\theta_1[t]] L_1 \theta_1'[t]^2 + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 + Cos[\theta_1[t]] L_2 (\theta_1'[t])^2 + Cos[\theta_1[t]
                                                                     Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 1[t]
                              Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3 (2 (1[t] Cos[\theta_1[t] + \theta_2[t]) L_2 (\theta_1'[t] + \theta_2[t]))
                                                                                            \theta_{2}'[t]) + 1[t] \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                                                      (Sin[\theta_{1}[t]] \ L_{1} \ \theta_{1}{'}[t] + Sin[\theta_{1}[t] + \theta_{2}[t]] \ L_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) \ +
                                                                     Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                             2 \left( \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \right. \left. \mathsf{L}_{1} \right. \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \right. \left. \mathsf{L}_{2} \left. \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right) \right. + \\
                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                      (-1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) -
                                                                     1[t] \, \operatorname{Sin}[\theta_1[t] + \theta_2[t] + \theta_3[t]] \, \operatorname{L}_3 \, (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) \, + \,
                                             2 \; (1[t] \; \mathsf{Cos}[\theta_1[t] \; + \; \theta_2[t]] \; \mathsf{L}_2 \; + \; 1[t] \; \mathsf{Cos}[\theta_1[t] \; + \; \theta_2[t] \; + \; \theta_3[t]] \; \mathsf{L}_3)
                                                      \left(-\sin\left[\theta_{1}\left[\mathtt{t}\right]\right]\,L_{1}\,\theta_{1}^{\,\prime}\left[\mathtt{t}\right]^{\,2}-\sin\left[\theta_{1}\left[\mathtt{t}\right]+\theta_{2}\left[\mathtt{t}\right]\right]\,L_{2}\,\left(\theta_{1}^{\,\prime}\left[\mathtt{t}\right]+\theta_{2}^{\,\prime}\left[\mathtt{t}\right]\right)^{\,2}-\right.
                                                                     Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                     Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
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2\;(1[t]\;Sin[\theta_{1}[t]+\theta_{2}[t]]\;L_{2}+1[t]\;Sin[\theta_{1}[t]+\theta_{2}[t]+\theta_{3}[t]]\;L_{3})
                                                        (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + Sin[\theta_1[t]] L_1
                                                                                     \theta_1^{\prime\prime}[\texttt{t}] + \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}]] \; \texttt{L}_2 \; (\theta_1^{\prime\prime}[\texttt{t}] + \theta_2^{\prime\prime}[\texttt{t}]) \; + \; \texttt{Sin}[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] \; + \; \texttt{Sin}[\theta_1[\texttt{t}]] \; + \; \texttt{Sin}[\theta_1[\texttt{t}
                                                                                                          \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])), -g Cos[\theta_1[t] + \theta_2[t]] L_2 m_2 - \theta_3[t]
          g (Cos[\theta_1[t] + \theta_2[t]] L<sub>2</sub> + Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L<sub>3</sub>) m<sub>3</sub> -
             - m_2 \left(-2 \sin \left[\theta_1[t] + \theta_2[t]\right] L_2 \left(\theta_1'[t] + \theta_2'[t]\right)
                                                        \left(\mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{1}{'}\left[\mathsf{t}\right] + \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right)\right) \; + \; \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right] + \theta_{2}\left[\mathsf{t}\right]\right] \; \mathsf{L}_{3} \; \left(\theta_{1}{'}\left[\mathsf{t}\right] + \theta_{2}{'}\left[\mathsf{t}\right]\right]\right) \; \mathsf{L}_{3} \;
                                           2 \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])
                                                       \left(\text{Sin}\left[\theta_{1}\left[t\right]\right] \; \text{L}_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; \right] \; \text{L}_{1} \; \theta_{1}{'}\left[t\right] + \text{Sin}\left[\theta_{1}\left[t\right] \; \right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)\right)\right) \; - \; \left(\text{Sin}\left[\theta_{1}\left[t\right] \; \right] \; \text{L}_{2} \; \left(\theta_{1}{'}\left[t\right] \; \right] \; \text{L}_{3} \; \left(\theta_{1}{'}\left[t\right] \; \right) \; \text{L}_{4} \; \left(\theta_{1}{'}\left[t\right] \; \right) \; \text{L}_{5} \; \left(\theta_{1}{'}\left[t\right] \; \right) \; \text{L}_{6} \; \left(\theta_{1}{'}\left[t\right] \; \right) \; \text{L}_{7} \; \left(\theta_{1}{'}\left[t\right] \; \left(\theta_{1}{'}\left[t\right] \; \right) \; \text{L}_{7} \; \left(\theta_{1}{'}\left[t\right] \; \left(\theta_{1}{'}\left[t\right] \; \right) \; \text{L}_{7} \; \left(\theta_{1}{'}\left[t\right] \; \left(\theta_{1}{'}\left[t\right] \; \left(\theta_{1}{'}\left[t\right] \; \right) \; \text{L}_{7} \; \left(\theta_{1}{'}\left[t\right] \; \left(\theta_{1}{'}\left[t\right
           \frac{-}{2} \, \mathrm{m_3} \, \left( 2 \, \left( \mathrm{Cos} \left[ \theta_1[\mathtt{t}] \right] \, \mathrm{L_1} \, \theta_1{}'[\mathtt{t}] + \mathrm{Cos} \left[ \theta_1[\mathtt{t}] + \theta_2[\mathtt{t}] \right] \, \mathrm{L_2} \, \left( \theta_1{}'[\mathtt{t}] + \theta_2{}'[\mathtt{t}] \right) + \right. \right) \, 
                                                                            \mathsf{Cos}\left[\theta_{1}\left[\mathsf{t}\right]+\theta_{2}\left[\mathsf{t}\right]+\theta_{3}\left[\mathsf{t}\right]\right]\,\mathsf{L}_{3}\,\left(\theta_{1}{'}\left[\mathsf{t}\right]+\theta_{2}{'}\left[\mathsf{t}\right]+\theta_{3}{'}\left[\mathsf{t}\right]\right)\right)
                                                        (-\sin[\theta_1[t]+\theta_2[t]] \; L_2 \; (\theta_1{}'[t]+\theta_2{}'[t]) \; - \\ \sin[\theta_1[t]+\theta_2[t]+\theta_3[t]] \; L_3
                                                                                         (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) + 2 (Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))
                                                        \left(\operatorname{Sin}[\theta_{1}[t]] \ \operatorname{L}_{1} \theta_{1}{'}[t] + \operatorname{Sin}[\theta_{1}[t] + \theta_{2}[t]] \ \operatorname{L}_{2} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t]) + \right.
                                                                            \texttt{Sin}\left[\theta_1\left[\texttt{t}\right] + \theta_2\left[\texttt{t}\right] + \theta_3\left[\texttt{t}\right]\right] \; \texttt{L}_3 \; \left(\theta_1{'}\left[\texttt{t}\right] + \theta_2{'}\left[\texttt{t}\right] + \theta_3{'}\left[\texttt{t}\right]\right)\right)) \; + \; \texttt{1}\left[\texttt{t}\right] \; \texttt{Izz}_2
                       (\theta_1''[t] + \theta_2''[t]) + \frac{1}{2} m_2 \left(-21[t] \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])\right)
                                                        (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                           2\times \texttt{1[t]}\; \texttt{Cos}\left[\theta_{\texttt{1}}[\texttt{t}] + \theta_{\texttt{2}}[\texttt{t}]\right] \; \texttt{L}_{\texttt{2}}\; \left(\theta_{\texttt{1}}{'}[\texttt{t}] + \theta_{\texttt{2}}{'}[\texttt{t}]\right)
                                                       (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])) +
                                           2 \times 1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 -
                                                                             \sin \left[\theta_1[t] + \theta_2[t]\right] \, \mathrm{L}_2 \, \left(\theta_1{}'[t] + \theta_2{}'[t]\right)^2 + \cos \left[\theta_1[t]\right] \, \mathrm{L}_1 \, \theta_1{}''[t] + \\
                                                                            Cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}^{"}[t] + \theta_{2}^{"}[t])) + 2 \times 1[t] Sin[\theta_{1}[t] + \theta_{2}[t]]
                                                    L_2 \left( \cos \left[ \theta_1[t] \right] L_1 \theta_1'[t]^2 + \cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2'[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right]^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1'[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right] L_2 \left( \theta_1[t] + \theta_2[t] \right)^2 + Cos \left[ \theta_1[t] + \theta_2[t] \right)^2 +
                                                                            Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t])) + 1[t]
                      Izz_{3} (\theta_{1}^{"}[t] + \theta_{2}^{"}[t] + \theta_{3}^{"}[t]) + \frac{1}{2} m_{3} (2 (1[t] Cos[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}^{'}[t] + \theta_{2}^{"}[t]))
                                                                                                          \theta_{2}'[t]) + 1[t] \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t]))
                                                        (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                            2 (Cos[\theta_1[t]] L_1 \theta_1'[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                          \texttt{Cos}\left[\theta_1[\texttt{t}] + \theta_2[\texttt{t}] + \theta_3[\texttt{t}]\right] \, \texttt{L}_3 \, \left(\theta_1{}'[\texttt{t}] + \theta_2{}'[\texttt{t}] + \theta_3{}'[\texttt{t}]\right) \right)
                                                        (-1[t] \; \mathtt{Sin}[\theta_1[t] + \theta_2[t]] \; \mathtt{L}_2 \; (\theta_1{}'[t] + \theta_2{}'[t]) \; - \;
                                                                          1[t] \; \text{Sin} \left[ \theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t] \right] \; L_{3} \; \left( \theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t] \right) ) \; + \;
                                           2 (1[t] \cos[\theta_1[t] + \theta_2[t]] L_2 + 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                        \left(-\sin\left[\theta_{1}\left[t\right]\right] \; L_{1} \; \theta_{1}{'}\left[t\right]^{2} - \sin\left[\theta_{1}\left[t\right] + \theta_{2}\left[t\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[t\right] + \theta_{2}{'}\left[t\right]\right)^{2} - \left(-\sin\left[\theta_{1}\left[t\right]\right] \; L_{1} \; \theta_{1}{'}\left[t\right]\right)^{2} - \left(-\sin\left[\theta_{1}\left[t\right]\right] \; L_{2} \; \theta_{2}{'}\left[t\right]\right)^{2} - \left(-\cos\left[\theta_{1}\left[t\right]\right] \; L_{2} \; 
                                                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                            \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \, \mathsf{L}_{1} \, \theta_{1}^{\,\prime\prime}[\mathsf{t}] \, + \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \, + \theta_{2}[\mathsf{t}]\right] \, \mathsf{L}_{2} \, \left(\theta_{1}^{\,\prime\prime}[\mathsf{t}] \, + \theta_{2}^{\,\prime\prime}[\mathsf{t}]\right) \, + \\
                                                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                           2 (1[t] Sin[\theta_1[t] + \theta_2[t]] L_2 + 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3)
                                                        (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                            Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                            \operatorname{Sin}[\theta_1[t]] \ \operatorname{L}_1 \theta_1^{\prime\prime}[t] + \operatorname{Sin}[\theta_1[t] + \theta_2[t]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime}[t] + \theta_2^{\prime\prime}[t]) + \\
                                                                            Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])),
-g \cos[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} m_{3} - \frac{1}{2} m_{3} (-2 \sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]])
                                                     L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                        \left( \text{Cos} \left[ \theta_{1} \left[ t \right] \right] \right. \left. \text{L}_{1} \right. \theta_{1}{'} \left[ t \right] + \text{Cos} \left[ \theta_{1} \left[ t \right] + \theta_{2} \left[ t \right] \right] \right. \left. \text{L}_{2} \right. \left( \theta_{1}{'} \left[ t \right] + \theta_{2}{'} \left[ t \right] \right) + \\
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Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                              2 \cos [\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])
                                  (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
                  1[t] Izz<sub>3</sub> (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) + \frac{1}{2} m_3
                       (-21[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                  (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                         Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                              2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                  (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                         Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] L_{3} (\theta_{1}'[t] + \theta_{2}'[t] + \theta_{3}'[t])) +
                              2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]\right)^2 - \cos[\theta_1[t]] L_1 \theta_1'[t]
                                         Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 - Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                             L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 + Cos[\theta_1[t]] L_1 \theta_1''[t] +
                                         \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}]\right] \, \mathsf{L}_{2} \, \left(\theta_{1}^{\prime\prime}[\mathsf{t}] + \theta_{2}^{\prime\prime}[\mathsf{t}]\right) \, + \, \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] + \theta_{2}[\mathsf{t}] + \theta_{3}[\mathsf{t}]\right]
                                             L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) + 2 \times 1[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3
                                  (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                         Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                         Sin[\theta_1[t]] L_1 \theta_1''[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                         Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
       -g \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 - \frac{1}{2} m_3 (-2 \sin [\theta_1[t] + \theta_2[t] + \theta_3[t]]
                          L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                          \left( \mathsf{Cos}\left[\theta_{1}[\mathsf{t}]\right] \; \mathsf{L}_{1} \; \theta_{1}{'}[\mathsf{t}] \; + \; \mathsf{Cos}\left[\theta_{1}[\mathsf{t}] \; + \; \theta_{2}[\mathsf{t}]\right] \; \mathsf{L}_{2} \; \left(\theta_{1}{'}[\mathsf{t}] \; + \; \theta_{2}{'}[\mathsf{t}]\right) \; + \;
                                 Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                      2 \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                           (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                  Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t]))) +
           1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
            2
               (-21[t] Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                           (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                      2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                           \left(\text{Sin}\left[\theta_{1}\left[\texttt{t}\right]\right] \; L_{1} \; \theta_{1}{'}\left[\texttt{t}\right] + \text{Sin}\left[\theta_{1}\left[\texttt{t}\right] + \theta_{2}\left[\texttt{t}\right]\right] \; L_{2} \; \left(\theta_{1}{'}\left[\texttt{t}\right] + \theta_{2}{'}\left[\texttt{t}\right]\right) \; + \right.
                                 Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \ L_{3} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t])) \ +
                      2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \cos[\theta_1[t]]\right) L_3 \left(-\sin[\theta_1[t]] L_1 \theta_1'[t]\right)^2 - \cos[\theta_1[t]] L_1 \theta_1'[t]
                                 Sin[\theta_{1}[t] + \theta_{2}[t]] L_{2} (\theta_{1}{'}[t] + \theta_{2}{'}[t])^{2} -
                                 Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                 Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                 Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                      2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 +
                                 Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                 Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                 \operatorname{Sin}[\theta_1[\mathsf{t}]] \ \operatorname{L}_1 \theta_1^{\prime\prime\prime}[\mathsf{t}] + \operatorname{Sin}[\theta_1[\mathsf{t}] + \theta_2[\mathsf{t}]] \ \operatorname{L}_2 \ (\theta_1^{\prime\prime\prime}[\mathsf{t}] + \theta_2^{\prime\prime\prime}[\mathsf{t}]) + \\
                                 Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])))\Big]_{2},
g \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 m_3 -
```

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1
                                                           _
2
                                                               m_3
                                                                                       Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                                        (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                        \left( \mathsf{Cos}\left[ \theta_{1}[\mathsf{t}] \right] \right. \left. \mathsf{L}_{1} \right. \theta_{1}{'}[\mathsf{t}] + \mathsf{Cos}\left[ \theta_{1}[\mathsf{t}] \right. + \theta_{2}[\mathsf{t}] \right] \left. \mathsf{L}_{2} \right. \left. \left( \theta_{1}{'}[\mathsf{t}] + \theta_{2}{'}[\mathsf{t}] \right) + \left. \mathsf{Holemore of the properties of the properti
                                                                                                    Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                               2 \cos [\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                        (Sin[\theta_1[t]] L_1 \theta_1'[t] + Sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                      Sin[\theta_{1}[t] + \theta_{2}[t] + \theta_{3}[t]] \ L_{3} \ (\theta_{1}{'}[t] + \theta_{2}{'}[t] + \theta_{3}{'}[t]))) + \\
                                                         1[t] Izz_3 (\theta_1''[t] + \theta_2''[t] + \theta_3''[t]) +
                                                           _
2
                                                              m_3
                                                                  (-2)
                                                                                       1[t]
                                                                                       Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                                        (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                        (\cos[\theta_1[t]] L_1 \theta_1'[t] + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                               2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                                      L_3
                                                                                        (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])
                                                                                        (\sin[\theta_1[t]] L_1 \theta_1'[t] + \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t]) +
                                                                                                     Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1'[t] + \theta_2'[t] + \theta_3'[t])) +
                                                                               2 \times 1[t] \cos[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                                      L_3
                                                                                        (-\sin[\theta_1[t]] L_1 \theta_1'[t]^2 - \sin[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 -
                                                                                                     Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                                     Cos[\theta_1[t]] L_1 \theta_1''[t] + Cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1''[t] + \theta_2''[t]) +
                                                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t])) +
                                                                               2 \times 1[t] \sin[\theta_1[t] + \theta_2[t] + \theta_3[t]]
                                                                                        (\cos[\theta_1[t]] L_1 \theta_1'[t]^2 + \cos[\theta_1[t] + \theta_2[t]] L_2 (\theta_1'[t] + \theta_2'[t])^2 +
                                                                                                     Cos[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3 (\theta_1'[t] + \theta_2'[t] + \theta_3'[t])^2 +
                                                                                                     \text{Sin}[\theta_1[\texttt{t}]] \; L_1 \; \theta_1{''}[\texttt{t}] \; + \; \text{Sin}[\theta_1[\texttt{t}] \; + \; \theta_2[\texttt{t}]] \; L_2 \; (\theta_1{''}[\texttt{t}] \; + \; \theta_2{''}[\texttt{t}]) \; + \;
                                                                                                      Sin[\theta_1[t] + \theta_2[t] + \theta_3[t]] L_3(\theta_1''[t] + \theta_2''[t] + \theta_3''[t]))
Out[172]//MatrixForm=
                                                    \left\{-g \cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right. \left. L_{2} \right. m_{2} - g \left. \left(\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right. L_{2} + \cos \left[\theta_{1}[t]+\theta_{2}[t]\right] + \theta_{3}[t]\right] \right. L_{3} \right) \\ \left. m_{3} - \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \left. L_{2} \right. m_{2} - g \left. \left(\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right. L_{2} \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right. L_{2} + \left. \left(\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right. L_{3} \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right. L_{3} \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right. L_{3} \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right. L_{3} \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]\right] \right] \right] \\ \left. + \left[\cos \left[\theta_{1}[t]+\theta_{2}[t]+\theta_{2}[t]\right] \right] \\ \left. + \left[\cos \left[
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