

Practice Questions

Wednesday, January 5, 2022
8:49 AM

- (1) Evaluate the value of determinant of A if 10×10
 $a_{ii} = 1 \forall i$
 $a_{ij} = 2$ if $i+j = 11$
 else $a_{ij} = 0$

Solution

$$A_{10 \times 10} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|A| = ?$$

D.O

$A_{n \times n}$ then
 $|A| = (-3)^{n/2}$

$$\begin{aligned} D_{10} &= 1 \times 1 \times D_8 - 2 \times 2 \times D_8 = D_8 - 4D_8 \\ D_{10} &= -3D_8, D_8 = -3D_6, D_6 = -3D_4 \\ D_4 &= -3D_2, D_2 = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \\ &= 1 - 4 = -3 \end{aligned}$$

$$|A| = D_{10} = (-3)^5$$

- (2) If $S = \left\{ A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \mid |A| \neq 0 \right\}$ then $|S| = ?$

- (a) 0 (b) 1 (c) 2 (d) 3

Skew-Symmetric

$$A^T = -A$$

$$a_{ij} = -a_{ji}$$

Every skew-symmetric matrix of odd order $|A| = 0$

- (3) If $A = \begin{bmatrix} 1 & 2 & 3 & a & b & c \\ 0 & 5 & 6 & d & e & f \\ 0 & 0 & 11 & h & i & k \end{bmatrix}$ then Row Rank, Column Rank

Row Rank = 3
 Column Rank = 3
 matrix Rank = 3

$$\begin{aligned} \text{Rank}(A) &\leq \min(m, n) \\ \text{Rank}(A) &= r_1 = 2 \text{ \& Rank}(A) = r_2 = 3 \\ \text{Rank}(AB) &\leq \min(2, 3) = 2 \end{aligned}$$

$$13 + 17 = 30$$

$$c_1 + c_2 = 13$$

④ If $A = \begin{bmatrix} 13 & + & 17 & = & 30 \\ 19 & + & 31 & = & 50 \\ 42 & + & 61 & = & 103 \end{bmatrix}$ and $\lambda \neq 0$ then find other Eigen values?
 $c_1 + c_2 = 13$
 $\lambda_1 + \lambda_2 + \lambda_3 = \text{Trace}(A) = 147$
 $\lambda_1 \lambda_2 \lambda_3 = |A| = 0, \lambda_3 = 147 - \lambda_1 - \lambda_2$
 $|A| = 0 \checkmark$
 \hookrightarrow Singular
 \hookrightarrow at least one of the Eigen value ($\lambda_1 = 0$)
 $\text{Rank}(A) < 3$
 $[0, \lambda_2, 147 - \lambda_2]$

⑤ If 'A' is involutory matrix of order 2022 and $\text{Trace}(A) = 1800$ then what is $|A| = ?$

Solution

(A) 1 (B) -1 (C) 2022 (D) 1800
 $A^{-1} = I \rightarrow$ Eigen Values -1 or 1
 $|A| = 1$ or -1

$A^{5 \times 5}$
 $\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix}$
 $\dots \dots \dots |A| = 1$

$A_{4 \times 4}$
 $\begin{matrix} -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \end{matrix}$
 $\dots \dots \dots$
 $4 - 6 = -2$

+	-ve	Trace
n	0	n
1	n-1	2-n
2	n-2	4-n
3	n-3	6-n
...
0	n	-n

$n, 2-n, 2-n, \dots, n$

$$\frac{\text{order}(A) - \text{Trace}(A)}{2}$$

$$|A| = (-1)$$

$$\frac{2022 - 1800}{2}$$

$$|A| = (-1)$$

$$|A| = (-1)^{111} = -1$$

2022

$$\text{Trace}(A) = 1800 \checkmark$$

$$1901 \text{ +ve Eigenvaly} = 2022 - 1800 = 222$$

111 -ve Eigen values

111 +ve

$\sim \sqrt{u}$

$$= \times =$$

Solution

$$\underline{\check{A}} = \begin{bmatrix} \check{1} & \check{1} & \check{1} & \check{1} \\ \check{1} & \check{1} & \check{1} & \check{1} \\ \check{1} & \check{1} & \check{1} & \check{1} \\ \check{1} & \check{1} & \check{1} & \check{1} \end{bmatrix}$$

$$\begin{pmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & 1 & 1 \\ 1 & 1 & 1-\lambda & 1 \\ 1 & 1 & 1 & 1-\lambda \end{pmatrix}$$

$$= (-1)^n \lambda^{n-1} (\lambda - n)$$

$$\lambda^3 (\lambda - 4) = ch(\lambda)$$

$$\Rightarrow \lambda = 1 - 10 = -9$$

$$|A| = (-1)^{2022} (-9)^{2022-1} \cdot (-9-2022) \checkmark$$

$$\checkmark (-1)^n \lambda^{n-1} (\lambda - n) \checkmark$$

$$1 - \lambda = 2$$

② How many real Eigen values exist for the following matrix $\begin{bmatrix} 7 & 1 & 2 & 3 \\ & & - & 4 \end{bmatrix}$

(2)

following matrix

$$A^T = A$$

$$A = \begin{bmatrix} 7 & 1 & 2 & 3 \\ 1 & 11 & 5 & 4 \\ 2 & 5 & 9 & 6 \\ 3 & 4 & 6 & 12 \end{bmatrix}$$

Ans:- All four Eigen values are real

∴ Given A is a real symmetric Matrix

$$\Rightarrow \text{It will have only real Eigen values}$$

(3)

$$A =$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 0 & 7+i & i & 9 \\ -7+i & 2i & 5 & 7i \\ i & -5 & 3i & 6 \\ -9 & 7i & -6 & 5i \end{bmatrix}$$

Eigenvalues

then how many

$$\begin{matrix} 4 \\ 0 \\ 1 \end{matrix}$$

non-zero real Eigen values will exist!

$$9 \pm i5$$

$$0 \pm i$$

(i) Skew-Hermitian matrix $(\bar{a})^T = -a$

$$|A^T| = |-A|$$

$$|A| \neq 0$$

$$A \rightarrow \lambda$$

$$|\bar{A}| = (-1)^n \cdot |A|$$

Complex value

$$(\bar{\lambda})^T = -\lambda$$

$$\bar{\lambda} = -\lambda$$

* Conjugate will not effect real values

* It effects only for Sign Complex values

∴ possible Eigen values of a Skew-Hermitian matrix are either 0 or imaginary

(*)

2f

$$A =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A^2 = I$$

$$AA^T = I$$

Ans: $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ 7×7

possible Eigen values

$AA^T = I \Rightarrow A^2 = I$ $\Rightarrow A^T = A$ Symmetric also
 $\lambda^2 = 1 \Rightarrow \lambda = \pm 1$

Trace (A) = 1 = 1 + 1 + 1 + (-1) + (-1) + (-1) = 1
 $|A| = -1$

Ex: - Given matrix A is a nilpotent matrix of order 3
 (a) $A^2 = 0$ (b) $A^3 = 0$ (c) $A^{50} = 0$ (d) $A^6 = 0$

Ans: - Nilpotent matrix maximum index is its order $A^3 = 0 \Rightarrow A^6 = 0$
 $A^{50} = (A^6)^8 \cdot A^2 = 0$

Find characteristic polynomial and minimal polynomial of

$\lambda^2 = 0$
 $\lambda^2 = 0$

$A \neq 0$

$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
 $B = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$

$\Rightarrow \text{ch}(A) = \lambda^4$
 $\text{min}(A) = \lambda^2$

$\text{ch}(A) = \lambda^4$

λ

$$\frac{B^2}{\lambda^2} = 0$$

$B \neq 0$

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$ch(A) = \lambda^4$$

$$minimal(A) = \lambda^2$$

λ is a minimal polynomial only if $m(\lambda) = \lambda$
 $m(\lambda) = \lambda \Rightarrow m(\lambda) = \lambda = 0$

⑦

$$\begin{cases} P(x, y, z, w) = x^2 + y^2 + z^2 + bw^2 \\ Q(x, y, z, w) = x^2 + y^2 + czw \end{cases}$$

$$x^2 + y^2 + z^2 + bw^2 = [x \ y \ z \ w] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$x^2 + y^2 + czw = [x \ y \ z \ w] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c/2 & c/2 \\ 0 & 0 & c/2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Eigen values (Q_1) = 1, 1, 1, b

Eigen values (Q_2) = 1, 1, $\frac{c}{2}$, $-\frac{c}{2}$
 $\lambda^2 - (\frac{c}{2})^2 = 0$
 $\lambda = \pm \frac{c}{2}$

$A = \begin{bmatrix} 1 & \dots & \dots \\ \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots \end{bmatrix}$ Symmetric matrix \rightarrow Diagonalisable
 $A \simeq I$ on complex domain
 $A = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & -1 \end{bmatrix} = I$
 $i \cdot i = -1$