Programme 5: Matrices

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Matrices – definitions

A matrix is a set of real or complex numbers (called *elements*) arranged in rows and columns to form a rectangular array.

A matrix having m rows and n columns is called an $m \times n$ matrix.

For example:

$$\begin{pmatrix} 5 & 7 & 2 \\ 6 & 3 & 8 \end{pmatrix}$$

is a 2×3 matrix.

Matrices – definitions

Row matrix

A row matrix consists of a single row. For example:

$$\begin{pmatrix} 4 & 3 & 7 & 2 \end{pmatrix}$$

Column matrix

A column matrix consists of a single column. For example:

$$\begin{pmatrix} 6 \\ 3 \\ 8 \end{pmatrix}$$

Matrices – definitions

Double suffix notation

Each element of a matrix has its own address denoted by double suffices, the first indicating the row and the second indicating the column. For example, the elements of 3 × 4 matrix can be written as:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$$

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Matrix notation

Where there is no ambiguity a matrix can be represented by a single general element in brackets or by a capital letter in bold type.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix} \text{ can be denoted by } \left(a_{ij}\right) \text{ or by } \mathbf{A}$$

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Equal matrices

Two matrices are equal if corresponding elements throughout are equal.

$$\mathbf{A}=\mathbf{B} \text{ that is } \left(a_{ij}\right)=\left(b_{ij}\right) \text{ if } a_{ij}=b_{ij} \text{ for all values of } i \text{ and } j$$

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Addition and subtraction of matrices

Two matrices are added (or subtracted) by adding (or subtracting) corresponding elements. For example:

$$\begin{pmatrix} 4 & 2 & 3 \\ 5 & 7 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 8 & 9 \\ 3 & 5 & 4 \end{pmatrix} = \begin{pmatrix} 4+1 & 2+8 & 3+9 \\ 5+3 & 7+5 & 6+4 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 10 & 12 \\ 8 & 12 & 10 \end{pmatrix}$$

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Multiplication of matrices

Scalar multiplication

Multiplication of two matrices

Multiplication of matrices

Scalar multiplication

To multiply a matrix by a single number (a scalar), each individual element of the matrix is multiplied by that number. For example:

$$4\times\begin{pmatrix}3 & 2 & 5\\ 6 & 1 & 7\end{pmatrix}=\begin{pmatrix}12 & 8 & 20\\ 24 & 4 & 28\end{pmatrix}$$

That is:

$$k\left(a_{ij}\right)=\left(ka_{ij}\right)$$

Multiplication of matrices

Multiplication of two matrices

Two matrices can only be multiplied when the number of columns in the first matrix equals the number of rows in the second matrix.

The *ij*th element of the product matrix is obtained by multiplying each element in the *i*th row of the first matrix by the corresponding element in the *j*th column of the second matrix and the element products added.

For example:

Multiplication of matrices

Multiplication of two matrices

If $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \end{pmatrix}$

then $\mathbf{A.B} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} . \begin{pmatrix} b_{11} \\ b_{21} \\ b_{23} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} \end{pmatrix}$

Multiplication of matrices

Multiplication of two matrices

If $\mathbf{A} = \left(a_{ij}\right)$ is an $n \times m$ matrix and

$\mathbf{B} = \left(b_{ij}\right)$ is an $m \times q$ matrix then

$\mathbf{C} = \mathbf{A.B} = \left(c_{ij}\right)$ is an $n \times q$ matrix where

$$c_{ij} = \sum_{k=1}^{m} a_{ik} b_{kj}$$

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Transpose of a matrix

If a new matrix is formed by interchanging rows and columns the new matrix is called the *transpose* of the original matrix. For example, if:

$$\mathbf{A}=\begin{pmatrix}4 & 6\\ 7 & 9\\ 2 & 5\end{pmatrix} \text{ then } \mathbf{A}^{\mathbf{T}}=\begin{pmatrix}4 & 7 & 2\\ 6 & 9 & 5\end{pmatrix}$$

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Square matrix

Diagonal matrix

Unit matrix

Null matrix

Special matrices

Square matrix

A square matrix is of order $m \times m$.

A square matrix is *symmetric* if $a_{ij} = a_{ji}$. For example:

$$\begin{pmatrix} 1 & 2 & 5 \\ 2 & 8 & 9 \\ 5 & 9 & 4 \end{pmatrix}$$

A square matrix is *skew-symmetric* if $a_{ij} = -a_{ji}$. For example:

$$\begin{pmatrix} 0 & 2 & 5 \\ -2 & 0 & 9 \\ -5 & -9 & 0 \end{pmatrix}$$

Special matrices

Diagonal matrix

A diagonal matrix is a square matrix with all elements zero except those on the leading diagonal. For example:

$$\begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

Special matrices

Unit matrix

A unit matrix is a diagonal matrix with all elements on the leading diagonal being equal to unity. For example:

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The product of matrix **A** and the unit matrix is the matrix **A**:

$$\mathbf{A}.\mathbf{I} = \mathbf{A}$$

Special matrices

Null matrix

A null matrix is one whose elements are all zero.

$$\mathbf{0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Notice that

$$\mathbf{A.0} = \mathbf{0}$$

But that if $\mathbf{A.B} = \mathbf{0}$ we cannot deduce that $\mathbf{A} = \mathbf{0}$ or $\mathbf{B} = \mathbf{0}$

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Singular matrix

Cofactors

Adjoint of a square matrix

Determinant of a square matrix

Singular matrix

Every square matrix has its associated determinant. For example, the determinant of

$$\begin{pmatrix} 5 & 2 & 1 \\ 0 & 6 & 3 \\ 8 & 4 & 7 \end{pmatrix} \text{ is } \begin{vmatrix} 5 & 2 & 1 \\ 0 & 6 & 3 \\ 8 & 4 & 7 \end{vmatrix} = 150$$

The determinant of a matrix is equal to the determinant of its transpose.

A matrix whose determinant is zero is called a *singular* matrix.

Determinant of a square matrix

Cofactors

Each element a_{ij} of a square matrix has a *minor* which is the value of the determinant obtained from the matrix after eliminating the *i*th row and *j*th column to which the element is common.

The cofactor of element a_{ij} is then given as the minor of a_{ij} multiplied by

$$(-1)^{i+j}$$

Determinant of a square matrix

Adjoint of a square matrix

Let square matrix **C** be constructed from the square matrix **A** where the elements of **C** are the respective cofactors of the elements of **A** so that if:

$$\mathbf{A} = \left(a_{ij}\right) \text{ and } A_{ij} \text{ is the cofactor of } a_{ij} \text{ then } \mathbf{C} = \left(A_{ij}\right)$$

Then the transpose of **C** is called the adjoint of **A**, denoted by adj**A**.

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Inverse of a square matrix

If each element of the adjoint of a square matrix **A** is divided by the determinant of **A** then the resulting matrix is called the inverse of **A**, denoted by $\mathbf{A}^{-1}$.

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}}\left(\operatorname{adj}\mathbf{A}\right)$$

Note: if $\det \mathbf{A} = 0$ then the inverse does not exist

Inverse of a square matrix

Product of a square matrix and its inverse

The product of a square matrix and its inverse is the unit matrix:

$$\mathbf{A}.\mathbf{A}^{-1} = \mathbf{A}^{-1}.\mathbf{A} = \mathbf{I}$$

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Solution of a set of linear equations

The set of n simultaneous linear equations in n unknowns

$$\begin{array}{c} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \qquad \vdots \qquad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n = b_n \end{array}$$

can be written in matrix form as:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \ldots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \quad \text{that is} \quad \mathbf{A.x = b}$$

Solution of a set of linear equations

Since:

$$\mathbf{A.x} = \mathbf{b} \quad \text{then}$$

$$\mathbf{A}^{-1}\mathbf{.Ax} = \mathbf{A}^{-1}\mathbf{.b} \quad \text{that is}$$

$$\mathbf{I.x} = \mathbf{A}^{-1}\mathbf{.b} \quad \text{and} \quad \mathbf{I.x} = \mathbf{x}$$

The solution is then:

$$\boxed{\mathbf{x} = \mathbf{A}^{-1}\mathbf{.b}}$$

Solution of a set of linear equations

Gaussian elimination method for solving a set of linear equations

Given:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

Create the augmented matrix **B**, where:

$$\mathbf{B} = \left(\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} & b_n \end{array}\right)$$

Solution of a set of linear equations

Gaussian elimination method for solving a set of linear equations

$$\mathbf{B} = \left(\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} & b_n \end{array} \right)$$

Eliminate the elements other than a_{11} from the first column by subtracting a_{21}/a_{11} times the first row from the second row, a_{31}/a_{11} times the first row from the third row, etc. This gives a new matrix of the form:

Solution of a set of linear equations

Gaussian elimination method for solving a set of linear equations

$$\left(\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ 0 & c_{22} & c_{23} & \cdots & c_{2n} & d_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & c_{n2} & c_{n3} & \cdots & c_{nn} & d_n \end{array}\right)$$

This process is repeated to eliminate the c_{i2} from the third and subsequent rows until a matrix of the following form is arrived at:

$$\left(\begin{array}{ccccc|c} a_{11} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1n} & b_1 \\ 0 & \cdots & p_{n-3,n-2} & p_{n-2,n-1} & p_{n-2,n} & q_2 \\ 0 & \cdots & 0 & p_{n-1,n-1} & p_{n-1,n} & \vdots \\ 0 & \cdots & 0 & 0 & p_{nn} & q_n \end{array}\right)$$

Solution of a set of linear equations

Gaussian elimination method for solving a set of linear equations

$$\left(\begin{array}{ccccc|c} a_{11} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1n} & b_1 \\ 0 & \cdots & p_{n-3,n-2} & p_{n-2,n-1} & p_{n-2,n} & q_2 \\ 0 & \cdots & 0 & p_{n-1,n-1} & p_{n-1,n} & \vdots \\ 0 & \cdots & 0 & 0 & p_{nn} & q_n \end{array}\right)$$

From this augmented matrix we revert to the product:

$$\begin{pmatrix} a_{11} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1n} \\ 0 & \cdots & p_{n-3,n-2} & p_{n-2,n-1} & p_{n-2,n} \\ 0 & \cdots & 0 & p_{n-1,n-1} & p_{n-1,n} \\ 0 & \cdots & 0 & 0 & p_{nn} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix}$$

Solution of a set of linear equations

Gaussian elimination method for solving a set of linear equations

$$\begin{pmatrix} a_{11} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1n} \\ 0 & \cdots & p_{n-3,n-2} & p_{n-2,n-1} & p_{n-2,n} \\ 0 & \cdots & 0 & p_{n-1,n-1} & p_{n-1,n} \\ 0 & \cdots & 0 & 0 & p_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix}$$

From this product the solution is derived by working backwards from the bottom starting with:

$$p_{nn}x_n = q_n \quad \text{so} \quad x_n = \frac{q_n}{p_{nn}}$$

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Eigenvalues and eigenvectors

Equations of the form:

$$\mathbf{A}.\mathbf{x} = \lambda \mathbf{x}$$

where **A** is a square matrix and λ is a number (scalar) have non-trivial solutions ($\mathbf{x} \neq \mathbf{0}$) for **x** called *eigenvectors* or *characteristic vectors* of **A**. The corresponding values of λ are called *eigenvalues*, *characteristic values* or *latent roots* of A.

Eigenvalues and eigenvectors

Expressed as a set of separate equations:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ : & : & : & & : \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ : \\ x_n \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \\ : \\ x_n \end{pmatrix}$$

That is:

$$\begin{array}{c} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = \lambda x_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = \lambda x_2 \\ : \quad\quad : \quad\quad : \quad\quad\quad\quad : \quad\quad : \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n = \lambda x_n \end{array}$$

Eigenvalues and eigenvectors

These can be rewritten as:

$$\begin{pmatrix} a_{11}-\lambda & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn}-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

That is:

$$\left(\mathbf{A}-\lambda\mathbf{I}\right).\mathbf{x}=\mathbf{0}$$

Which means that, for non-trivial solutions:

$$\left|\mathbf{A}-\lambda\mathbf{I}\right|=0$$

Eigenvalues and eigenvectors

Eigenvalues

To find the eigenvalues of: $\mathbf{A}=\begin{pmatrix}4 & 1\\ 3 & 2\end{pmatrix}$

solve the characteristic equation: $\begin{vmatrix}4-\lambda & 1\\ 3 & 2-\lambda\end{vmatrix}=0$

That is: $(\lambda-1)(\lambda-5)=0$

This gives eigenvalues $\boxed{\lambda_1=1;\ \lambda_2=5}$

Eigenvalues and eigenvectors

Eigenvectors

To find the eigenvectors of $\mathbf{A}=\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ solve the equation $\mathbf{A.x}=\lambda\mathbf{x}$

For the eigenvalues $\lambda = 1$ and $\lambda = 5$

For $\lambda=1$

$$\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=1\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \text{ and so } x_2=-3x_1 \text{ giving eigenvector } \begin{pmatrix} k \\ -3k \end{pmatrix}$$

For $\lambda=5$

$$\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=5\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \text{ and so } x_2=x_1 \text{ giving eigenvector } \begin{pmatrix} k \\ k \end{pmatrix}$$

Learning outcomes

- ✓Define a matrix
- ✓Understand what is meant by the equality of two matrices
- ✓Add and subtract two matrices
- ✓Multiply a matrix by a scalar and multiply two matrices together
- ✓Obtain the transpose of a matrix
- ✓Recognize the special types of matrix
- ✓Obtain the determinant, cofactors and adjoint of a square matrix
- ✓Obtain the inverse of a non-singular matrix
- ✓Use matrices to solve a set of linear equations using inverse matrices
- ✓Use the Gaussian elimination method to solve a set of linear equations
- ✓Evaluate eigenvalues and eigenvectors