# LU decomposition:-

A ~~n×m~~ n×m matrix which is reduced to a row-echelon form by Gauss-elimination method used certain square of elementary row operations.

- A factorization of a square matrix A as $\boxed{A = LU}$, where L is lower triangular and U is the upper triangular matrix is called an LU decomposition (or) triangular decomposition of A

## Applications:-

1. A solution of a system of equations, which itself is an integral part of many applications such as ~~finding cu~~
   - finding current in a circuit
   - solution of discrete dynamical system problems
2. finding inverse of a matrix
3. finding determinant of the matrix

## Advantages:-

1. Once A is decomposed, we can solve $AX = b$ for as many constant vector b as we pleas [different values of b for the same A]

2. The cost of each additional solution is relatively small, because the forward and backward substitution operations are much less time consuming then the decomposition process

## Disadvantages:

1. It requires forward and backward substituti
2. Solving requires storing in memory the LU factor's

# Doolittle's decomposition method:

Decomposition phase: Doolittle's decomposition is closely related to Gauss elimination.

1. Create matrices A, X and B, where A is the augmented matrix, X constitutes the variable vectors and B are the constants.

2. Let $A = LU$, where L is the lower triangular and U is the upper triangular ~~assume~~; Diagonal terms in the lower triangular should be equal one

$$A = LU$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

# Gauss Jacobi iteration Method:

This method is applicable to ~~determine~~ the system of equation in which leading diagonal elements of co-efficient matrix are dominant (large in magnitude) in their respective rows.

**Working rule:** Consider the system of eqns.

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

**Note:** Diagonal dominance property must be satisfied

i.e $|a_{11}| \geq |a_{12}| + |a_{13}|$

$|a_{22}| \geq |a_{21}| + |a_{23}|$

$|a_{33}| \geq |a_{31}| + |a_{32}|$

- Rewriting the eqns for $x, y, z$ respectively

$$x = \frac{1}{a_{11}}(b_1 - a_{12}y - a_{13}z)$$

$$y = \frac{1}{a_{22}}(b_2 - a_{21}x - a_{23}z)$$

$$z = \frac{1}{a_{33}}(b_3 - a_{31}x - a_{32}y)$$

Iteration 1: Put $x = x_0$, $y = y_0$, $z = z_0$

$$x_1 = \frac{1}{a_{11}}(b_1 - a_{12}y_0 - a_{13}z_0)$$

$$y_1 = \frac{1}{a_{22}}(b_2 - a_{21}x_0 - a_{23}z_0)$$

$$z_1 = \frac{1}{a_{33}}(b_3 - a_{31}x_0 - a_{32}y_0)$$

The above iteration process is continued until two successive approximations are equal

Advantages:

1. It can be done in parallel
2. It is highly desirable for many applications

Disadvantages.

1. It is verylengthy and time taking
2. Inflexible

Appilication

Jacobi method is an iterative algorithm for determining the solutions of a strictly diagonally dominant system of linear equation

Convergence:

It converges if A is strictly diagonally dominant

# Jacobi method in matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$A \qquad X \qquad B$$

$$L = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & a_{32} & 0 \end{bmatrix}, \quad D = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & 0 & a_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{A = (L + D + U)}$$

$$\boxed{X^{k+1} = \frac{1}{D}\left[B - (L+U)X^{k}\right]}$$

Stoping criteria :- $\boxed{\|X^{k} - X^{k+1}\| \leq \epsilon, \quad \epsilon > 0}$

$$X^{k} \cong X^{k+1}$$

# Gauss - Siedel Method :-

This method is also known as the Liebmann method or the method of of successive displacement is an iterative method to solve a system of linear equations

- Here immidiate updation will be taken place

Working rule :-

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

Note :- Diagonal dominance property must be satisfied

$$|a_{11}| \geq |a_{12}| + |a_{13}|$$
$$|a_{22}| \geq |a_{21}| + |a_{23}|$$
$$|a_{33}| \geq |a_{31}| + |a_{32}|$$

$$x_1 = \frac{1}{a_{11}}(b_1 - (a_{12}x_2 + a_{13}x_3))$$

$$x_2 = \frac{1}{a_{22}}(b_2 - (a_{21}x_1 + a_{23}x_3))$$

$$x_3 = \frac{1}{a_{33}}(b_3 - (a_{31}x_1 + a_{32}x_2))$$

Initial approximations

$x_1^{(0)} = 0, \quad x_2^{(0)} = 0, \quad x_3^{(0)} = 0$

I$^{st}$ iteration

$$x_1^{1} = \frac{1}{a_{11}} \left( b_1 - \left( a_{12} x_2^{(0)} + a_{13} x_3^{(0)} \right) \right)$$

$$x_2^{1} = \frac{1}{a_{22}} \left( b_2 - \left( a_{21} x_1^{(1)} + a_{23} x_3^{(0)} \right) \right)$$

$$x_3^{1} = \frac{1}{a_{33}} \left( b_3 - \left( a_{31} x_1^{(1)} + a_{32} x_2^{(1)} \right) \right)$$

II$^{nd}$ iteration

$$x_1^{(2)} = \frac{1}{a_{11}} \left( b_1 - \left( a_{12} x_2^{(1)} + a_{13} x_3^{(1)} \right) \right)$$

$$x_2^{(2)} = \frac{1}{a_{22}} \left( b_2 - \left( a_{21} x_1^{(2)} + a_{23} x_3^{(1)} \right) \right)$$

$$x_3^{(2)} = \frac{1}{a_{33}} \left( b_3 - \left( a_{31} x_1^{(2)} + a_{32} a_2^{(2)} \right) \right)$$

The above iteration process is continued until two successive approximations are equal

Advantages:

1) It requires less iteration
2) Relatively easy to program

Disadvantages:

1) Not suitable for large systems
2) Required more no. of iterations to reach convergence
3) Convergence time increases with size of the system

Applications:

1. It is applicable to strictly diagonally dominant or symmetric positive definite matrices because only in this case convergence is possible

2.

Convergence:

The solu^n of linear eq^n by iterative methods requires for convergence that the absolute magnitude of all the eigenvalues of the iteration matrix should be less than unity alue

2. G-S method converges if the no. of roots inside the unit circle is equal to the order of the iteration matrix

General form:-

$$x_1^{k+1} = \frac{1}{a_{11}}[b_1 - (a_{12}x_2^k + a_{13}x_3^k)]$$

$$x_2^{k+1} = \frac{1}{a_{22}}[b_2 - (a_{21}x_1^{k+1} + x_{23}x_3^k)]$$

$$x_3^{k+1} = \frac{1}{a_{33}}[b_3 - (a_{31}x_1^{k+1} + a_{32}x_2^{k+1})]$$

$a_{33}$ ↓ D, B, L + U

$$\boxed{X^{k+1} = \frac{1}{D}[B - (LX^{k+1} + UX^k)]}$$

$$||X^{k+1} - X^k|| \leq 0.001$$

$$\boxed{X^{k+1} = (D+L)^{-1}B - (D+L)^{-1}U\cancel{X}^k}$$