Wednesday, January 12, 2022
8:25 AM

(1) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}_{\underline{4\times 4}}$, find ch(A) and Eigenvalues of A?

$A_{n\times n}$

$$ch(A) = (-1)^n \lambda^{n-1} \cdot (\lambda - n)$$

$$ch(A) = (-1)^4 \lambda^{4-1} (\lambda - 4) = \boxed{\lambda^3 (\lambda - 4)}$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & 1 & 1 \\ 1 & 1 & 1-\lambda & 1 \\ 1 & 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$= \lambda^3 (\lambda - 4) = 0$$

(1) let A square matrix of order $\underline{2022}$ and

$\underline{a_{ii} = 10} \ \forall i$

$\underline{a_{ij} = 1}$ otherwise then find $|A| = ?$

$$\underset{4\times 4}{A} = \begin{bmatrix} 10 & 1 & 1 & 1 \\ 1 & 10 & 1 & 1 \\ 1 & 1 & 10 & 1 \\ 1 & 1 & 1 & 10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 10 & 1 & 1 & 1 \\ 1 & 10 & 1 & 1 \\ 1 & 1 & 10 & 1 \\ 1 & 1 & 1 & 10 \end{vmatrix} = \begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & 1 & 1 \\ 1 & 1 & 1-\lambda & 1 \\ 1 & 1 & 1 & 1-\lambda \end{vmatrix}_{4\times 4}$$

$$1 - \lambda = 10 \Rightarrow \underline{\lambda = -9}$$

$$= \lambda^3(\lambda - 4)$$

$$|A| = (-9)^3(-9-4)$$

$$A_{2022\times 2022} = |A| = \lambda^{2021}(\lambda - 2022)$$

$$= (-9)^{2021}(-9-2022)$$

$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = \lambda^3(\lambda - 4), \quad 1-\lambda = 2$$

$$\lambda = -1$$

$$\lambda = ? \quad = -1 \checkmark$$

$$= (-1)^3(-1-4)$$

(2) How many real Eigenvalues Exist for the following matrix

$$\begin{bmatrix} 7 & 1 & 2 & 3 \\ 1 & 11 & 5 & 4 \\ 2 & 5 & 9 & 6 \\ 3 & 4 & 6 & 12 \end{bmatrix}_{4\times 4}.$$

How many real Eigenvalues are possible?

(a) 0 (b) 1 (c) 2 (D) 3 (E) 4 ✓

(i) $A^T = A$ Symmetric matrix

(ii) All the entries are real entries

Every real symmetric matrix will have real Eigenvalues only

(3) If A is Skew - Hermitian matrix then what are the possible Eigen values

(a) real (b) Non-zero (c) imaginary

(b) zero or purly imaginary.

$$A^{\theta} = -A \Rightarrow (\bar{A})^T = -A$$

$\bar{0} = -0$ if λ is the Eigen value of A

$\bar{i} = -i$

$-\bar{i} = i$, $\overline{2i} = -2i$

$$(\bar{\lambda})^T = -\lambda$$

$$\underline{\bar{\lambda} = -\lambda}$$

0 or purly imaginary

(3) $A^2 = \begin{bmatrix} 0 & 7+i & i & 9 \\ -7+i & 2i & 5 & 7i \\ i & -5 & 3i & 6 \\ -9 & 7i & -6 & 5i \end{bmatrix}$, then possible Eigen values of A?

$(\bar{A})^T = -A$

Skew Hermitian ~~matrix~~ matrix

zero or purlyimaginary Eigen values

(2) If $A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. possible Eigen values of (A)?

Solution

i) $\underline{A^T = A}$

ii) $AA^T = I$ orthogonal

iii) $\underline{A^2 = I}$ Involutary matrix

$\lambda^2 = 1 \Rightarrow \underline{\lambda = \pm 1}$

$A_{7\times 7} \Rightarrow \underline{\text{Trace}(A) = 1}$

7 ... Eigen values

$\hookrightarrow$ '7' eigen values

$$\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7=1$$
$$1+1+1+1-1-1-1=1$$

(*) If 'A' is a nilpotent matrix of order 3 and $A^{100}=0$ then which of the following statements are true?

(a) $A^2=0$ (b) $A^3=0$ ✓ (c) $A^{50}=0$ ✓ (d) $A^6=0$ ✓

$\Rightarrow$ $A^7=0$, A is nilpotent, 7 is its index

(*) Maximum possible index of any nilpotent matrix is its order. $A^3=0$ ✓ $A^{50}=A^3\cdot A^{47}=0$ ✓

(*) Find the possible characteristic polynomial and minimal polynomials of the following matrices.

$$\begin{bmatrix} a & b & c \\ 0 & e & f \\ 0 & 0 & d \end{bmatrix}$$
$\hookrightarrow$ only diagonal entries

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$ch(A)=\lambda^4$

minimal polynomial $(A)=\lambda^2$

$\deg\min(A)\le\deg ch(A)$

$\cancel{\lambda}, \lambda^2, \lambda^3, \lambda^4$

$\boxed{A=0}$ ✗

$A^2=0$ ✓

$A^3=0$ ✓

$A^4=0$ ✓

$$B=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$ch(B)=\lambda^4$

$\min(B)=\lambda^2$

$\cancel{\lambda}, \lambda^2, \lambda^3, \lambda^4$ ✓

$A^2=0$ ✓

$A=0$ ✗

$B^2=0$

① $p(x,y,z,w) = x^2+y^2+z^2+bw^2$

$Q(x,y,z,w) = x^2+y^2+czw$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & -5 \end{bmatrix}$$

$D = PAP^T$

$A = I.A.I^T$

elementary operations.

$D = \{1, 1, --1, -1, -1, -\cdot, 0, 0\}$

$$\begin{bmatrix} 1 & 0 \\ & \sim -3 \\ 0 & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1, \quad C_2 \to C_2 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & -5 \\ 3 & -5 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to 13R_1 - R_3$

$C_3 \to 3C_1 - C_3$

$D = PAP^T$

on Real domain

$$\begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & -1 & \\ & & & -1 \\ & & & & -1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 5 \\ 0 & 5 & 14 \end{bmatrix}$$

$C_2 \to 14C_3 - 5C_2$

$-1 = \sqrt{-1}\cdot\sqrt{-1}$

$R_2 \to \frac{R_2}{\sqrt{-1}}$

$C_2 \to \frac{C_2}{\sqrt{-1}}$

Complex domain

you can change the sign also

$i^2 = -1$

$i = \sqrt{-1}$

$-1 = i \cdot i$

$A = I.A.I$

symmetric

$D = PAP^T$ ✓

$$p(x,y,z,w) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & b \end{bmatrix} = x^2+y^2+z^2+bw^2$$

Eigenvalues $= 1, 1, 1, b$

$$Q(x,y,z,w) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & c/2 \\ 0 & 0 & c/2 & 0 \end{bmatrix} = x^2+y^2+czw$$

P_1 P_2

Eig $1, 1, c/2, -c/2$

$\lambda^2 = (c/2)^2$

$\lambda = \pm c/2$

$b<0$ & $c>0$

$1, 1, 1, -b = 1, 1, 1, -1$

$1, 1, c/2, -c/2 = 1, 1, 1, -1$

on real both are similar

$b \neq 0$, $c \neq 0$

then on complex both are similar