

## Gauss Jacobi and Gauss Siedel Method

Friday, December 24, 2021  
11:12 AM

### Indirect Methods

Gauss Jacobi iteration Method

Gauss Siedel Method

$$\left\{ \begin{array}{l} -a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{array} \right. \rightarrow AX = B$$

Gauss Jacobi Method :-

Arrange the given system of equations as diagonally dominant system

$$\begin{aligned} &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \rightarrow (1) \\ & a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \rightarrow (2) \\ & a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \rightarrow (3) \end{aligned}$$

Diagonally dominant

$$|a_{11}| \geq |a_{12}| + |a_{13}|$$

$$|a_{22}| \geq |a_{21}| + |a_{23}|$$

$$|a_{33}| \geq |a_{31}| + |a_{32}|$$

From (1), (2) & (3)

$$a_{11}x_1 = b_1 - (a_{12}x_2 + a_{13}x_3)$$

$$\Rightarrow x_1 = \frac{1}{a_{11}} [b_1 - (a_{12}x_2 + a_{13}x_3)]$$

$$x_2 = \frac{1}{a_{22}} [b_2 - (a_{21}x_1 + a_{23}x_3)]$$

$$x_3 = \frac{1}{a_{33}} [b_3 - (a_{31}x_1 + a_{32}x_2)]$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{aligned} x_0 &= \begin{bmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{bmatrix} \\ x^0 \rightarrow x^1 &= \begin{bmatrix} x_1^1 \\ x_2^1 \\ x_3^1 \end{bmatrix} \end{aligned}$$

$$k=1$$

$$x_1^1 = \frac{1}{a_{11}} [b_1 - (a_{12}x_2^0 + a_{13}x_3^0)]$$

$$x_2^1 = \frac{1}{a_{22}} [b_2 - (a_{21}x_1^0 + a_{23}x_3^0)]$$

$$x_3^1 = \frac{1}{a_{33}} [b_3 - (a_{31}x_1^0 + a_{32}x_2^0)]$$

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$$x_3 = -\bar{a}_{33}^{-1} x^3$$

$x^1 \rightarrow x^2$

K=2

$$\bar{x}_1^2 = \frac{1}{a_{11}} [b_1 - (a_{12}\bar{x}_2^1 + a_{13}\bar{x}_3^1)]$$

$$\bar{x}_2^2 = \frac{1}{a_{22}} [b_2 - (a_{21}\bar{x}_1^1 + a_{23}\bar{x}_3^1)]$$

$$\bar{x}_3^2 = \frac{1}{a_{33}} [b_3 - (a_{31}\bar{x}_1^1 + a_{32}\bar{x}_2^1)]$$

K=1

$$x^{k+1} = \frac{1}{D} [B - (L+U)x^k]$$

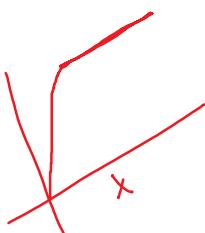
$$\begin{aligned} x_1^{k+1} &= \frac{1}{a_{11}} [b_1 - (a_{12}\bar{x}_2^k + a_{13}\bar{x}_3^k)] \\ x_2^{k+1} &= \frac{1}{a_{22}} [b_2 - (a_{21}\bar{x}_1^k + a_{23}\bar{x}_3^k)] \\ x_3^{k+1} &= \frac{1}{a_{33}} [b_3 - (a_{31}\bar{x}_1^k + a_{32}\bar{x}_2^k)] \end{aligned}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

A                            X                            B

$$A = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & a_{32} & 0 \end{bmatrix}, D = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}, U = \begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & 0 & a_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = (L + D + U) \quad \checkmark$$



$$x^1 \rightarrow x^2 \rightarrow x^3 \rightarrow \dots \rightarrow x^k \approx x^{k+1}$$

$x^1, x^2, x^3, \dots, x^k \approx x^{k+1} \quad \text{solution}$

$$s_n = \{y_n\} \Rightarrow 1, y_2, y_3, y_4, y_5, \dots, y_n \rightarrow 0 \quad \checkmark$$

$$x^3 \approx x^4 \approx x^5$$

$$\|x^k - x^{k+1}\| \leq \epsilon$$

Stoping criteria :-  $\|x^k - x^{k+1}\| \leq \epsilon, \epsilon \geq 0$

$$x^k \approx x^{k+1}$$

Gauss Seidel :-  $x^i = (x_1^i, x_2^i, x_3^i)$

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Gauss Siedel :-  $X^1 = (x_1^1, x_2^1, x_3^1)$

$$\begin{aligned}x_1^1 &= \frac{1}{a_{11}} [b_1 - (a_{12}x_2^1 + a_{13}x_3^1)] \\x_2^1 &= \frac{1}{a_{22}} [b_2 - (a_{21}x_1^1 + a_{23}x_3^1)] \\x_3^1 &= \frac{1}{a_{33}} [b_3 - (a_{31}x_1^1 + a_{32}x_2^1)]\end{aligned}$$

$$\left\{ \begin{array}{l} x_1^{k+1} = \frac{1}{a_{11}} [b_1 - (a_{12}x_2^k + a_{13}x_3^k)] \\ x_2^{k+1} = \frac{1}{a_{22}} [b_2 - (a_{21}x_1^{k+1} + a_{23}x_3^k)] \\ x_3^{k+1} = \frac{1}{a_{33}} [b_3 - (a_{31}x_1^k + a_{32}x_2^{k+1})] \end{array} \right.$$

$$B \quad L+U$$

$$x^{k+1} = \frac{1}{D} [B - (Lx^{k+1} + Ux^k)]$$

for  $k = 1 \text{ to } 10$   
 $x^{k+1} = (D+L)^{-1} B - (D+L)^{-1} U x^k$   
 end  
 $\|x^{k+1} - x^k\| \leq 0.001$

$$\begin{aligned}DX^{k+1} + Lx^{k+1} &= B - UX^k \\(D+L)x^{k+1} &= B - UX^k \\x^{k+1} &= (D+L)^{-1} B - (D+L)^{-1} UX^k\end{aligned}$$

E: - Solve the following system of equations by  
 @ Gauss Jacobi  
 ⑤ Gauss Siedel

$$\begin{aligned}6x + 2y - z &= 4 \\2x + y + 4z &= 3 \\x + 2y + z &= 27\end{aligned} \quad X^1 = (0, 0, 0)$$

Gauss Jacobi :-

$$\begin{cases} 6x + 2y - z = 4 \\ x + 3y + z = 27 \\ 2x + y + 4z = 3 \end{cases}$$

$$\begin{cases} x + 3y + z = 1 \\ 2x + y + 4z = 3 \end{cases}$$

$$\left\{ \begin{array}{l} x = 2/3 \\ y = 1/3(2x - 1/3 - 0) \\ z = 1/4(3 - 2(x/3) - x/3) \end{array} \right.$$

$$x = \frac{1}{6}(4 - (2y - z))$$

$$y = \frac{1}{3}(2x - x - z)$$

$$z = \frac{1}{4}(3 - 2x - y)$$

$$x=0, y=0, z=0$$

$$x = \frac{1}{6}[4 - 0] = 4/6 = 2/3$$

$$y = \frac{1}{3}[2x - 0] = 2x/3 = 9$$

$$z = \frac{1}{4}(3) = 3/4$$

$$x^2 = \left\{ \frac{2}{3}, 9, \frac{3}{4} \right\}, \quad x = \frac{2}{3}, y = 9, z = \frac{3}{4}$$

$$x^3 =$$

$$y^3 =$$

$$z^3 =$$