

Indirect Methods

Gauss Jacobi iteration Method

Gauss Siedel Method

$$\begin{cases}
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\
 \vdots \\
 a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n
 \end{cases}$$

$AX = B$

Gauss Jacobi Method:-

Arrange the given system of equations as diagonally dominant system

$$\begin{cases}
 a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \rightarrow (1) \\
 a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \rightarrow (2) \\
 a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \rightarrow (3)
 \end{cases}$$

Diagonally dominant

$$|a_{11}| \geq |a_{12}| + |a_{13}|$$

$$|a_{22}| \geq |a_{21}| + |a_{23}|$$

$$|a_{33}| \geq |a_{31}| + |a_{32}|$$

from (1), (2) & (3)

$$a_{11}x_1 = b_1 - (a_{12}x_2 + a_{13}x_3)$$

$$\Rightarrow x_1 = \frac{1}{a_{11}} [b_1 - (a_{12}x_2 + a_{13}x_3)]$$

$$x_2 = \frac{1}{a_{22}} [b_2 - (a_{21}x_1 + a_{23}x_3)]$$

$$x_3 = \frac{1}{a_{33}} [b_3 - (a_{31}x_1 + a_{32}x_2)]$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{bmatrix}$$

$$X^0 \rightarrow X^1 = \begin{bmatrix} x_1^1 \\ x_2^1 \\ x_3^1 \end{bmatrix}$$

$$X_1$$

$$x_1^1 = \frac{1}{a_{11}} [b_1 - (a_{12}x_2^0 + a_{13}x_3^0)]$$

$$x_2^1 = \frac{1}{a_{22}} [b_2 - (a_{21}x_1^1 + a_{23}x_3^0)]$$

$$x_3^1 = \frac{1}{a_{33}} [b_3 - (a_{31}x_1^1 + a_{32}x_2^1)]$$

$$x^1 \rightarrow x^2$$

$$x_3 = -a_{33}^{-1} \cdot 3$$

k=2

$$x_1^k = \frac{1}{a_{11}} [b_1 - (a_{12}x_2^1 + a_{13}x_3^1)]$$

$$x_2^k = \frac{1}{a_{22}} [b_2 - (a_{21}x_1^1 + a_{23}x_3^1)]$$

$$x_3^k = \frac{1}{a_{33}} [b_3 - (a_{31}x_1^1 + a_{32}x_2^1)]$$

k=1

$$x^{k+1} = \frac{1}{D} [B - (L+U)x^k]$$

$$x^{k+1} = \frac{1}{D} [B - (L+U)x^k]$$

$$x_1^{k+1} = \frac{1}{a_{11}} [b_1 - (a_{12}x_2^k + a_{13}x_3^k)]$$

$$x_2^{k+1} = \frac{1}{a_{22}} [b_2 - (a_{21}x_1^k + a_{23}x_3^k)]$$

$$x_3^{k+1} = \frac{1}{a_{33}} [b_3 - (a_{31}x_1^k + a_{32}x_2^k)]$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

A X B

$$A = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & a_{32} & 0 \end{bmatrix}, D = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}, U = \begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & 0 & a_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = (L + D + U) \checkmark$$



$$x^1 \rightarrow x^2 \rightarrow x^3 \rightarrow \dots \rightarrow x^k \approx x^{k+1}$$

$x^1, x^2, x^3 \dots \approx \text{solution}$

$$s_n = \left\{ \frac{1}{n} \right\} \Rightarrow \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \dots \rightarrow \frac{1}{n} \rightarrow 0 \checkmark$$

$$x^3 \approx x^4 \approx x^5$$

$$\|x^k - x^{k+1}\| \leq \epsilon$$

Stopping criteria :-

$$\|x^k - x^{k+1}\| \leq \epsilon, \quad \epsilon \geq 0$$

$x^k \approx x^{k+1}$

Gauss Seidel :- $x^1 = (x_1^1, x_2^1, x_3^1)$

Gauss Seidel :- $X' = (x'_1, x'_2, x'_3)$

$$\begin{aligned} x'_1 &= \frac{1}{a_{11}} [b_1 - (a_{12}x'_2 + a_{13}x'_3)] \\ x'_2 &= \frac{1}{a_{22}} [b_2 - (a_{21}x'_1 + a_{23}x'_3)] \\ x'_3 &= \frac{1}{a_{33}} [b_3 - (a_{31}x'_1 + a_{32}x'_2)] \end{aligned}$$

$$\begin{aligned} x^{k+1}_1 &= \frac{1}{a_{11}} [b_1 - (a_{12}x^k_2 + a_{13}x^k_3)] \\ x^{k+1}_2 &= \frac{1}{a_{22}} [b_2 - (a_{21}x^{k+1}_1 + a_{23}x^k_3)] \\ x^{k+1}_3 &= \frac{1}{a_{33}} [b_3 - (a_{31}x^{k+1}_1 + a_{32}x^{k+1}_2)] \end{aligned}$$

$$X^{k+1} = \frac{1}{D} [B - (LX^{k+1} + UX^k)]$$

$$\begin{aligned} DX^{k+1} + LX^{k+1} &= B - UX^k \\ (D+L)X^{k+1} &= B - UX^k \end{aligned}$$

$$X^{k+1} = (D+L)^{-1} B - (D+L)^{-1} UX^k$$

For $k=1$ to 10
 $X^{k+1} = (D+L)^{-1} B - (D+L)^{-1} UX^k$
 end
 || $X^{k+1} - X^k$ || < 0.001

Ex:- Solve the following system of equations by
 (a) Gauss Jacobi
 (b) Gauss Seidel

$$6x + 2y - z = 4$$

$$2x + y + 4z = 3$$

$$x + 2y + z = 27$$

$$X' = (0, 0, 0)$$

Gauss Jacobi :-

$$\begin{cases} 6x + 2y - z = 4 \\ x + 3y + z = 27 \\ 2x + y + 4z = 3 \end{cases}$$

$$\begin{cases} x + 3y + z = 11 \\ 2x + y + 4z = 3 \end{cases}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{aligned} x^2 &= 2/3 \\ y^2 &= 1/3 (2x - y - z) \\ z^2 &= 1/4 (3 - 2x - y) \end{aligned}$$

$$x = \frac{1}{6} (4 - (2y - z))$$

$$y = \frac{1}{3} (2x - x - z)$$

$$z = \frac{1}{4} (3 - 2x - y)$$

$$x=0, y=0, z=0$$

$$x^2 = \frac{1}{6} [4 - 0] = 4/6 = 2/3$$

$$y^2 = \frac{1}{3} [2x - 0] = 2x/3 = 9$$

$$z^2 = \frac{1}{4} [3] = 3/4$$

$$x^2 = (2/3, 9, 3/4), \quad x=2/3, y=9, z=3/4$$

$$x^3 =$$

$$y^3 =$$

$$z^3 =$$