

Continuity:

A function of two variable is said to be continuous at (a, b) if

$$\lim_{(x, y) \rightarrow (a, b)} f(x, y) = \underbrace{f(a, b)}$$

must be defined



$$\Downarrow \\ \text{Limit} = L$$

\neq f is discontinuous at (a, b)

limit:

$$\lim_{(x,y) \rightarrow (a,b)}$$

$$\underbrace{f(x,y)}_Z = L \text{ (number)}$$

$$Z = f(x,y)$$

SVC

$$\lim_{x \rightarrow a} f(x) = L$$

$$\begin{aligned} (\Rightarrow) \lim_{x \rightarrow a^+} f(x) &= L \\ &= \lim_{x \rightarrow a^-} f(x) \end{aligned}$$

Jacobians.

If u and v are functions of two independent variables x and y , then the determinant

$$\begin{vmatrix} \frac{du}{dx} & \frac{du}{dy} \\ \frac{dv}{dx} & \frac{dv}{dy} \end{vmatrix} \text{ is called Jacobian of } u, v \text{ with}$$

respect to x, y and is denoted by the

$$\text{Symbol } J \left[\begin{matrix} u, v \\ x, y \end{matrix} \right] \text{ or } \frac{d(u, v)}{d(x, y)}$$

Properties

If u, v are functions of r, s where r, s are functions of x, y then

$$\frac{d(u, v)}{d(x, y)} = \frac{d(u, v)}{d(r, s)} \times \frac{d(r, s)}{d(x, y)}$$

If J_1 is the Jacobian of u, v with respect to x, y and J_2 is the Jacobian of x, y with respect to u, v then $J_1 J_2 = 1$ i.e. $\frac{d(u, v)}{d(x, y)} \cdot \frac{d(x, y)}{d(u, v)} = 1$

Jacobian of implicit function

$$\frac{f(u_1, u_2, u_3, \dots, u_n)}{J(x_1, x_2, \dots, x_n)} = \frac{(-1)^n \frac{d(f_1, f_2, \dots, f_n)}{d(x_1, x_2, \dots, x_n)}}{\frac{d(f_1, f_2, \dots, f_n)}{d(u_1, u_2, \dots, u_n)}}$$

Taylor's theorem

Taylor's theorem about $f(a, b)$ for $f(x, y)$

$$f(x, y) = f(a, b) + \frac{1}{1!} [(x-a)f_x(a, b) + (y-b)f_y(a, b)]$$

$$+ \frac{1}{2!} [(x-a)^2 f_{xx}(a, b) + 2(x-a)(y-b)f_{xy}(a, b) + (y-b)^2 f_{yy}(a, b)]$$

$$+ \frac{1}{3!} [(x-a)^3 f_{xxx}(a, b) + 3(x-a)^2(y-b)f_{xxy}(a, b) + 3(x-a)(y-b)^2 f_{xyy}(a, b) + (y-b)^3 f_{yyy}(a, b) + \dots]$$

$$+ \frac{1}{3!} [(x-a)^3 f_{xxx}(a, b) + 3(x-a)^2(y-b)f_{xxy}(a, b) + 3(x-a)(y-b)^2 f_{xyy}(a, b) + (y-b)^3 f_{yyy}(a, b) + \dots]$$

$$+ \frac{1}{4!} [(x-a)^4 f_{xxxx}(a, b) + 6(x-a)^3(y-b)f_{xxx}(a, b) + 6(x-a)^2(y-b)^2 f_{xxyy}(a, b) + 6(x-a)(y-b)^3 f_{xyyy}(a, b) + (y-b)^4 f_{yyyy}(a, b) + \dots]$$

If

$$(x, y) = (a, b) = (0, 0)$$

~~this whole is known as~~
[Maclaurin's theorem for two variables]

Partial derivatives

const

$$f(x, y), \quad \left. \frac{df}{dx} \right|_{x=x_0} = \lim_{\substack{x \rightarrow x_0 \\ h \rightarrow 0}} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h}$$

here y is const

$$\left. \frac{df}{dy} \right|_{y=y_0} = \lim_{\substack{y \rightarrow y_0 \\ h \rightarrow 0}} \frac{f(x_0, y_0 + h) - f(x_0, y_0)}{h}$$

const \leftarrow

here x is const
existy