

Continuity:

A function of two variable is said to be continuous at (a,b) if

$$\lim_{(x,y) \rightarrow (a,b)} f(x,y) = f(a,b)$$

\sim
must be defined



$$\text{Limit} = L$$

\neq f is discontinuous
at (a,b)

limit:

$$\lim_{(x,y) \rightarrow (a,b)} f(x,y)$$

$$\underset{z}{\text{f}(x,y) = L \text{ (number)}}$$

$$z = f(x, y)$$

SVC

$$\lim_{x \rightarrow a} f(x) = L$$

$$\Rightarrow \lim_{x \rightarrow a+} f(x) = L$$

$$= \lim_{x \rightarrow a^-} f(x)$$

Jacobians

If u and v are functions of two independent variables x and y , then the determinant

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$
 is called Jacobian of u, v with respect to x, y and is denoted by the

symbol $J \left[\begin{matrix} u, v \\ x, y \end{matrix} \right]$ or $\frac{d(u, v)}{d(x, y)}$

Properties

If u, v are functions of r, s where r, s are functions of x, y then

$$\frac{d(u, v)}{d(x, y)} = \frac{d(u, v)}{d(r, s)} \times \frac{d(r, s)}{d(x, y)}$$

If J_1 is the Jacobian of u, v with respect to x, y and J_2 is the Jacobian of x, y with respect to u, v then $J_1 J_2 = 1$ i.e. $\frac{d(u, v)}{d(x, y)} \cdot \frac{d(x, y)}{d(u, v)} = 1$

Jacobian of implicit function

$$\frac{f(u_1, u_2, u_3, \dots, u_n)}{d(x_1, x_2, \dots, x_n)} = (-1)^n \frac{d(f_1, f_2, \dots, f_n)}{d(x_1, x_2, \dots, x_n)} \frac{d(x_1, x_2, \dots, x_n)}{d(f_1, f_2, \dots, f_n)} \frac{d(f_1, f_2, \dots, f_n)}{d(u_1, u_2, \dots, u_n)}$$

Taylor's theorem

Taylor's theorem about $f(x, y)$ for

$$f(x, y) = f(a, b) + \frac{1}{1!} [(x-a)f_x(a, b) + (y-b)f_y(a, b)] \\ + \frac{1}{2!} \left[(x-a)^2 f_{xx}(a, b) + 2(x-a)(y-b) f_{xy}(a, b) \right. \\ \left. + (y-b)^2 f_{yy}(a, b) \right] \\ + \frac{1}{3!} \left[(x-a)^3 f_{xxx}(a, b) + 3(x-a)^2 (y-b) f_{xxy}(a, b) \right. \\ \left. + (y-b)^3 f_{yyy}(a, b) + \dots \right]$$

if

$$(x, y) = (a, b)$$

this whole is known

[maclauria's theorem
for two variables]

Partial derivatives

cont

$$f(x, y), \frac{\partial f}{\partial x} \Big|_{x=x_0} = \lim_{\substack{x \rightarrow x_0 \\ h \rightarrow 0}} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h}$$

here y is const

$$\frac{\partial f}{\partial y} \Big|_{y=y_0} = \lim_{\substack{y \rightarrow y_0 \\ h \rightarrow 0}} \frac{f(x_0, y_0 + h) - f(x_0, y_0)}{h}$$

here x is const
exists