

The following notations will be followed in this report. These are basic notations, same as those done in high school or above, by most of the people.

Sets

A *set* is a collection of objects. The following sets (among others) will be used in this report:

\mathbb{R} : the set of real numbers.

\mathbb{Z} : the set of integers (positive, negative, or zero).

\mathbb{Z}_n : $\{0, 1, 2, \dots, n-1\}$

The symbols \emptyset , \in , \notin , \cup , \cap , \subseteq and \supseteq have their usual meanings. If S and T are sets and, $S \cap T = \emptyset$, then S and T are said to be *disjoint*.

If S is a set and P a property (or a combination of properties), we can define a new set with the notation

$$\{x \in S \mid P(x)\}$$

which denotes ‘set of all elements of S which have property P ’.

The *order* or *cardinality* of a finite set S is the number of elements in S and is denoted by $|S|$. For example, $|\mathbb{Z}_n| = n$.

The *Cartesian Product* of two sets S and T is given by

$$S \times T = \{(s, t) \mid s \in S, t \in T\}.$$

If S and T are finite sets, then $|S \times T| = |S| \cdot |T|$.

In general,

$$S_1 \times S_2 \times \dots \times S_n = \{(s_1, s_2, \dots, s_n) \mid s_i \in S_i, i = 1, 2, \dots, n\},$$

is the *Cartesian Product* (a set of *ordered n -tuples*) of n sets S_1, S_2, \dots, S_n .

In this report, an ordered n -tuple (x_1, x_2, \dots, x_n) will be denoted simply as $x_1 x_2 \dots x_n$.

Combinatorics

Number of ways of choosing m distinct objects from n distinct objects

or

the coefficient of x^m in $(1+x)^n$

are both given by

$$\binom{n}{m} = \frac{n!}{m! (n-m)!}$$

where $p! = p(p-1) \dots 3 \cdot 2 \cdot 1$ for $m > 0$ and $0! = 1$.

This bracket notation will be used throughout the report.

A *permutation* of a set $S = \{x_1, x_2, \dots, x_n\}$ is a one-to-one mapping from set S to itself. It is denoted by

$$\begin{pmatrix} x_1 & x_2 & \dots & x_n \\ \downarrow & \downarrow & & \downarrow \\ f(x_1) & f(x_2) & \dots & f(x_n) \end{pmatrix}$$

Modular Arithmetic

Let m be a fixed positive integer. Two integers a and b are written as

$$a \equiv b \pmod{m}$$

if $a - b$ is divisible by m .

It can be noted that if $a \equiv a' \pmod{m}$ and $b \equiv b' \pmod{m}$ then

$$(i) \ a + b \equiv a' + b' \pmod{m}$$

$$(ii) \ ab \equiv a'b' \pmod{m}$$

Fermat's Little Theorem: Let p be a prime, and a be any integer, then $a^p \equiv a \pmod{p}$