

The following notations will be followed in this report. These are basic notations, same as those done in high school or above, by most of the people.

## Sets

A *set* is a collection of objects. The following sets (among others) will be used in this report:

$\mathbb{R}$  : the set of real numbers.

$\mathbb{Z}$  : the set of integers (positive, negative, or zero).

$\mathbb{Z}_n$  :  $\{0, 1, 2, \dots, n-1\}$

The symbols  $\emptyset$ ,  $\in$ ,  $\notin$ ,  $\cup$ ,  $\cap$ ,  $\subseteq$  and  $\supseteq$  have their usual meanings. If  $S$  and  $T$  are sets and,  $S \cap T = \emptyset$ , then  $S$  and  $T$  are said to be *disjoint*.

If  $S$  is a set and  $P$  a property (or a combination of properties), we can define a new set with the notation

$$\{x \in S \mid P(x)\}$$

which denotes ‘set of all elements of  $S$  which have property  $P$ ’.

The *order* or *cardinality* of a finite set  $S$  is the number of elements in  $S$  and is denoted by  $|S|$ . For example,  $|\mathbb{Z}_n| = n$ .

The *Cartesian Product* of two sets  $S$  and  $T$  is given by

$$S \times T = \{(s, t) \mid s \in S, t \in T\}.$$

If  $S$  and  $T$  are finite sets, then  $|S \times T| = |S| \cdot |T|$ .

In general,

$$S_1 \times S_2 \times \dots \times S_n = \{(s_1, s_2, \dots, s_n) \mid s_i \in S_i, i = 1, 2, \dots, n\},$$

is the *Cartesian Product* (a set of *ordered  $n$ -tuples*) of  $n$  sets  $S_1, S_2, \dots, S_n$ .

In this report, an ordered  $n$ -tuple  $(x_1, x_2, \dots, x_n)$  will be denoted simply as  $x_1 x_2 \dots x_n$ .

## Combinatorics

Number of ways of choosing  $m$  distinct objects from  $n$  distinct objects

or

the coefficient of  $x^m$  in  $(1+x)^n$

are both given by

$$\binom{n}{m} = \frac{n!}{m! (n-m)!}$$

where  $p! = p(p-1) \dots 3 \cdot 2 \cdot 1$  for  $m > 0$  and  $0! = 1$ .

This bracket notation will be used throughout the report.

A *permutation* of a set  $S = \{x_1, x_2, \dots, x_n\}$  is a one-to-one mapping from set  $S$  to itself. It is denoted by

$$\begin{pmatrix} x_1 & x_2 & \dots & x_n \\ \downarrow & \downarrow & & \downarrow \\ f(x_1) & f(x_2) & \dots & f(x_n) \end{pmatrix}$$

## Modular Arithmetic

Let  $m$  be a fixed positive integer. Two integers  $a$  and  $b$  are written as

$$a \equiv b \pmod{m}$$

if  $a - b$  is divisible by  $m$ .

It can be noted that if  $a \equiv a' \pmod{m}$  and  $b \equiv b' \pmod{m}$  then

$$(i) \ a + b \equiv a' + b' \pmod{m}$$

$$(ii) \ ab \equiv a'b' \pmod{m}$$

*Fermat's Little Theorem*: Let  $p$  be a prime, and  $a$  be any integer, then  $a^p \equiv a \pmod{p}$