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PLATFORM COMPETITION WITH ENDOGENOUS HOMING*

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We consider two-sided markets in which consumers and firms endogenously determine whether they single-home, multi-home, or exit the market. We find that the competitive bottleneck allocation in which consumers single-home and firms multi-home is always an equilibrium. In addition, we find equilibria with multi-homing and single-homing on each side of the market. However, unlike the standard pricing result where the side that multi-homes faces higher prices, we find that lower prices coincide with multi-homing: agents find multi-homing more attractive when faced with lower prices. We also show that endogenous homing can induce straddle pricing which deters price undercutting between platforms.

1. INTRODUCTION

The market structures in which platforms operate vary considerably in terms of the participation decisions that agents make. When several platforms offer competing services, participants of the market may join only one platform (called single-homing), or they may patronize several platforms (called multi-homing). For the smartphone industry, the majority of consumers single-home and only own one platform although most content (apps, videos, and music) are available on all platforms. With video game consoles, many consumers own more than one console and thus multi-home, whereas others single-home and own only one—resulting in a mixed-homing configuration. Similarly, there are many games that are available on a single console, whereas many others are available across consoles.[1]

Despite the prevalence of mixed-homing configurations where single-homing and multi-homing agents exist on each side of the market, the literature on platform competition generally abstracts from the issue of endogenous homing by exogenously fixing agent homing decisions prior to platform pricing decisions. This seminal literature has then largely focussed on other critical issues in these markets. Many, for instance, consider the potential coordination issues across the two sides of the platform. Caillaud and Jullien (2003) assume that coordination favors the incumbent platform; otherwise platforms may fail to gain a critical mass, that is, "fail to launch." They argue this solves the "chicken and egg" problem of each side's action depending on the other side's action. Hagiu (2006) shows the chicken and egg problem does not occur when sides join platforms sequentially; and Jullien (2011) investigates this further over a broader class of multi-sided markets. Ambrus and Argenziano (2009) show how prices can

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[1] For example, Bungie, Sega, and Valve Corporation have all developed some video games that are exclusive to a particular console; whereas Blizzard, Electronic Arts, and Ubisoft primarily develop games that are available for all consoles.

endogenize heterogeneity and steer agents to asymmetric allocation configurations; and Karle et al. (forthcoming) consider how the structure of competition within the firm side of the market determines whether all agents tip to one platform or whether the market is segmented between the two platforms.[2]

When exogenous homing decisions are assumed, allocation-specific pricing decisions occur: exogenously fixed multi-homers face high prices as platforms do not compete for them (due to the fact that, by assumption, they join both platforms), whereas exogenously fixed single-homers (who must be dislodged from a rival before they can be acquired as new customers) face low prices. Moreover, postulating universal single- or multi-homing on one side precludes observed allocations in which there is a mix of single- and multi-homing agents on the same side of the platform. To understand how prices relate to equilibrium homing decisions requires a model where platforms set prices prior to endogenous homing decisions made by consumers and firms.

We consider platform competition in which consumers and firms observe platform pricing before endogenously deciding whether and how many platforms to join. We show that different allocations of consumers and firms emerge in equilibrium, including the mixed-homing allocation not seen in the previous literature. In addition, we find that sufficiently low prices induce multi-homing and that certain allocations with multi-homing on one side require that the price on that side be sufficiently low. These results highlight how lower prices coincide with multi-homing and higher prices coincide with single-homing. We also identify conditions that are required for certain allocations to exist as equilibria, and we show that industries where a particular allocation occurs match the equilibrium conditions for that allocation. We show this alignment between equilibrium conditions and real world industries for several platform markets including those for smartphones, game consoles, video streaming, and sharing economy services.

Finally, we also show how two homogeneous platforms might avoid the Bertrand Paradox. This is possible in platform markets, contrary to traditional markets, because undercutting certain prices that earn profit might not be optimal in a two-side market. In particular, platforms can both garner participation using straddle prices (two-sided prices where one price is greater than its corresponding marginal cost whereas the other price is below its marginal cost) that discourage undercutting. In this case, an undercutting price deviation, where the undercutting platform tips the market and garners all participation, results in additional revenues on the side of the market with a positive margin side but also results in additional costs from the side of the market with a negative margin. It is possible that the additional costs outweigh the additional revenues so that price undercutting is not desirable and profitable price constellations are attainable.[3]

Previous literature that has considered endogenous homing includes Armstrong and Wright (2007), who extend Armstrong (2006) and show how the common "competitive bottleneck" equilibrium, in which all consumers single-home and all firms multi-home, can endogenously arise. The result stems from a model with horizontally differentiated platforms (i.e., the Hotelling framework) in which for high transportation costs all agents single-home, whereas for low transportation costs all agents multi-home. Although these allocations are observed in

[2] In addition to the primary participation decision, the role of beliefs and information play an important role in determining the equilibria as examined by Hagiu and Hałaburda (2014), who consider "passive" price expectations on one side in contrast to complete information about prices on the second side, and Gabszewicz and Wauthy (2014) who also consider active and passive beliefs in determining platform allocations, or Hałaburda and Yehezkel (2013), who show how multi-homing alleviates coordination issues tied to asymmetric information. In a normative analysis, Weyl (2010) and White and Weyl (2016) fully mitigate coordination issues through insulating tariffs in which prices are contingent upon participation, thus resolving failure-to-launch and multiple equilibrium concerns.

[3] Ambrus and Argenziano (2009) also discover the possibility of straddle pricing equilibria in the form of orthogonal straddle prices. We find that, in addition to orthogonal straddle prices, symmetric straddle pricing equilibria are also possible with endogenous homing.

platform industries, a common feature of many markets is that there are a mix of single-homers and multi-homers on either or both sides.

It is also important to note that Armstrong and Wright (2007) generate the same pricing results as Armstrong (2006) where the side that single-homes faces competitive prices whereas the side that multi-homes faces monopoly pricing by both platforms. As Armstrong (2006) points outs, these pricing results follow because platforms do not compete over multi-homers but instead compete over single-homers who can switch to the competing platform. However, this pricing result stems from the homing decision being made prior to platform pricing decisions. Instead, we find that lower prices drive multi-homing when prices are set prior to homing decisions.[4]

Another paper that allows for endogenous homing is Rochet and Tirole (2003), where buyers and sellers engage in a matched transaction that takes place on a platform. The model is best illustrated by the credit card market: It is assumed that card-issuers only charge per transaction and do not charge card-users or merchants any membership fees, so all agents can costlessly multi-home. However, because card-users choose which card to use when they make a purchase, merchants may wish to single-home in order to limit the customers' options of which card to use. In contrast to per-transaction fees, many platform markets—including smartphones and gaming systems—are characterized by access or membership fees. Although we allow for both usage and stand-alone membership benefits, we consider platforms that compete by setting membership/access fees, as this relates more closely to the markets that we are concerned with.[5]

In a treatment similar to endogenous homing, recent papers have considered endogenous side decisions and multi-siders in two-sided markets. For example, Choi and Zennyo (2019) consider a model where agents decide which side to join (e.g., agents decide to be either a consumer or a firm), and analyze how this side decision process impacts platform competition. However, their analysis does not allow agents to be mixed-siders (i.e., agents as both a consumer and a firm). In contrast, Gao (2018) develops a model with single-siders and mixed-siders and considers how a monopoly platform uses bundle pricing to price discriminate. Gao's analysis is particularly important for sharing economy markets where this mix of users is not uncommon.[6] These papers highlight how the nature of agent participation decisions has become increasingly important for analyzing platform pricing and competition in the face of more diverse and complex platform markets.

[4] One paper that does consider mixed-homing in the Armstrong framework is Belleflamme and Peitz (2019). They consider the question of who benefits from multi-homing. The customary answer is to compare the model of exogenous single-homing on both sides of the market with the competitive bottleneck model. This comparison suggests that multi-homing harms the side which multi-homes. Belleflamme and Peitz instead consider how the competitive bottleneck model compares with a model of buyers that single-home and sellers who can either single- or multi-home. They find that the possibility of seller multi-homing can generate a benefit to the mixed-homing seller side through greater participation. Their results highlight how greater endogeneity of agent homing decisions can produce results that differ from those generated by the conventional framework. However, their results also coincide with the classic relationship between multi-homing and higher prices.

[5] There is also a nascent literature on endogenous multi-homing in media markets. There the focus is on determining the pricing structure on the advertising-side of the platform. Thus, in Anderson et al. (2018), consumers are not charged to join a platform, so the only cost is a nuisance cost of advertising. However, as consumers do not observe the prices platforms charge to advertisers, platforms cannot affect consumer participation. In Athey et al. (2018), the focus is on endogenous homing on the ad-side, while assuming that consumer allocations are exogenously fixed; and in a related model, Ambrus et al. (2016) allow platform pricing to affect consumer participation; however, participation on any given platform does not affect demand on other platforms, so there is no competition between platforms for consumers. In contrast to these studies of media markets, we explicitly model the competition that takes place between platforms to attract agents on both sides of the market and we allow agents from both sides to endogenously make their homing decisions.

[6] For example, in homesharing where some users are renters, others are hosts, and others are both; or, for example, in ridesharing where some users are riders, others are drivers, and others participate as drivers and riders.

2. THE MODEL

2.1. *Platforms.* Two groups of agents can benefit from interaction, but require an intermediary in order to do so. The benefits from the interaction to an agent in one group depends on the number of agents of the other group that are made available through the intermediary. This intermediary—the platform—brings these two groups together by charging agents in each group a price to participate on the platform. We consider two platforms, indexed by $X \in \{A, B\}$.

The sequence of play is as follows: First the platforms simultaneously (and non-cooperatively) set prices to each of the two groups. We refer to this stage as the platform pricing game, or simply the pricing game. Thereafter, upon observing all platform prices, agents on each side simultaneously make participation decisions. This stage is referred to as the allocation subgame, which follows the pricing game.

Agents on each side of the platform are described by continuous variables. Agents on Side 1 are consumers or buyers, and agents on Side 2 are firms or sellers. The number of consumers that join Platform X is $n_1^X \in [0, \overline{N}_1]$, and the number of firms on Platform X is $n_2^X \in [0, \overline{N}_2]$.

The cost to the platform of accommodating an agent on Side $i \in \{1, 2\}$ who joins the platform is $f_i \geq 0$, and there are no fixed costs. Platform X has profits of

$$\Pi^X = n_1^X\left(p_1^X - f_1\right) + n_2^X\left(p_2^X - f_2\right), \tag{1}$$

where p_i^X is the (uniform) price that Platform X charges to the agents on Side i.

2.2. *Side 1: Consumers.* Consumers on Side 1 draw their type θ_1 from the distribution F_1 on the support $[0, 1]$. All consumers' outside options are valued at 0, whereas the utility for a consumer of type θ_1 from single-homing on Platform X is

$$u_1^X(\theta_1) = v + \alpha_1(\theta_1) \cdot n_2^X - p_1^X. \tag{2}$$

Here $v \geq 0$ is the membership value every consumer receives from joining the platform. This is the stand-alone utility of being a member of the platform that one gets even if no firms join the platform. Note that it is possible for $v = 0$, but for smartphones and video game consoles $v > 0$. For smartphones, v is the utility from using a smartphone as a phone, including the preloaded features, and for video game consoles, v is the utility from using the console to watch video content. Consumers are homogeneous in their membership benefit to the platform, so v does not depend on consumer type θ_1, and the stand-alone value of joining a platform is the same regardless of which platform is joined.

Consumers are heterogeneous in their marginal benefit from firms. The network effect or the marginal benefit to a consumer of type θ_1 for an additional firm on the platform is constant and given by $\alpha_1(\theta_1)$, and the number of firms that join the platform is n_2^X. We focus on the case when network effects are positive so $\alpha_1(\theta_1) \geq 0$ for all θ_1, with $\alpha_1(\cdot)$ decreasing and continuous. Since $\alpha_1(\theta_1)$ is decreasing, it orders consumers by their marginal benefits. Consumers whose type θ_1 is close to zero have marginal benefits that are high relative to those consumers whose type is located far from zero. Without loss of generality, we normalize F_1 to be the uniform distribution over $[0, 1]$ so that the mass of type θ_1 consumers is given by $\theta_1 \cdot \overline{N}_1$.[7]

With there being two platforms in the market, consumers who elect to join a platform can either join just one platform (single-home) or join both platforms (multi-home). A consumer who multi-homes has utility

$$u_1^M(\theta_1) = (1 + \delta)v + \alpha_1(\theta_1) \cdot N_2 - p_1^A - p_1^B. \tag{3}$$

[7] This simplifies notation so that $F_1 \circ \alpha_1^{-1}(\cdot)$ reduces to $\alpha_1^{-1}(\cdot) \times \overline{N}_1$.

Notice that if a consumer participates on two platforms, then the intrinsic benefit from membership to the second platform diminishes, so that the total stand-alone membership benefit from the two platforms is $(1+\delta)v$ with $\delta \in [0, 1]$. If $\delta = 0$, then there is no additional membership benefit from joining the second platform, and when $\delta = 1$ the membership benefit is unaffected by being a member of another platform.[8]

Apart from the positive membership value of being on a second platform, the main gain to joining a second platform is access to additional firms. Letting n_2^M denote the number of multi-homing firms, a consumer who multi-homes has access to $N_2 := n_2^A + n_2^B - n_2^M$ distinct firms; these are all the firms that join at least one platform. The above utility function implies that a multi-homing firm provides only a one-time gain to a consumer who multi-homes. Having a firm available on both platforms to which the consumer has access provides no added benefit.

2.3. *Side 2: Firms.* On the other side of the platform, Side 2, are firms that draw their type θ_2 according to distribution F_2 on the support [0, 1]. Similar to Side 1, without loss of generality, we normalize F_2 to be the uniform distribution over [0, 1] so that the mass of type θ_2 firms is given by $\theta_2 \cdot \overline{N}_2$. All firms' outside options are 0, whereas a firm's payoff from single-homing on Platform X is

$$u_2^X(\theta_2) = \alpha_2(\theta_2) \cdot n_1^X - c - p_2^X. \tag{4}$$

Similar to consumers, firms are heterogeneous in their marginal benefit from consumers. The network effect or the marginal benefit to a firm of type θ_2 for an additional consumer on the platform is constant and given by $\alpha_2(\theta_2)$, and the number of consumers that join the platform is n_1^X.[9] We focus on the case when network effects are positive, so $\alpha_2(\theta_2) \geq 0$ for all θ_2, with $\alpha_2(\cdot)$ decreasing and continuous. Since $\alpha_2(\theta_2)$ is decreasing, it orders firms by their marginal benefits. Firms whose type θ_2 is close to zero have marginal benefits that are high relative to those firms whose type is located far from zero.

Firms incur a cost of $c > 0$ to join the platform. This cost reflects development and synchronization costs associated with programming and formatting their product to fit the platform. Firms are homogeneous with respect to their development and synchronization costs. A firm that multi-homes has payoff

$$u_2^M(\theta_2) = \alpha_2(\theta_2) \cdot N_1 - (1+\sigma) \cdot c - p_2^A - p_2^B, \tag{5}$$

where $N_1 := n_1^A + n_1^B - n_1^M$ is the number of distinct consumers to which the firm gains access; these are all the consumers that join at least one platform. As noted above, when a firm's product is available to the multi-homing consumer on both platforms, a consumer will only purchase the product at most once. Therefore, a firm only cares about the number of distinct consumers that are available to it through the platforms.

When a firm participates on two platforms, its development and synchronization cost for joining the second platform diminishes to σc with $\sigma \in [0, 1]$. Thus, σ represents the amount of "duplication economies" that exist when synchronizing an app or game to a second platform. If $\sigma = 1$, then there are no economies of duplication and as σ decreases, there exists economies of duplication.

[8] One can also consider the possibility that owning a second platform is tedious so that $\delta < 0$, but this does not affect the main results. Also, depreciation in network benefits, α_1, is possible, as studied in Ambrus et al. (2016).

[9] Note that we abstract from the transaction costs that may occur between consumers and firms. Thus, the benefits that accrue to consumers from interacting with firms—apps and games—can be viewed as being net of prices paid to firms. Deltas and Jeitschko (2007) consider auctioneers setting optimal reserves on an auction hosting platform, Reisinger (2014) generalizes Armstrong (2006) and considers tariff-pricing with heterogeneous trading, and Karle et al. (forthcoming) consider the impact of different competition structures within the firm side. See Tremblay (2016) for a more detailed analysis of pricing across the platform in a framework that is similar to the current setting.

2.4. *Strategies and Equilibrium Concept.* We focus exclusively on pure strategies and solve for subgame-perfect Nash equilibria using backward induction. Applying backward induction requires that equilibrium allocations for platform price constellations be derived. To reduce the expositional burden, we restrict our focus to prices for which platforms have not priced themselves out of the market.[10]

Because there are strategic complementarities across the two sides of the platform, for any given set of prices chosen by the platform, multiple allocation equilibrium configurations can exist in the allocation subgame. This problem does not arise in much of the platform literature, which assumes a strong horizontal differentiation between platforms (e.g., platforms located at the endpoints of a Hotelling line) and assumes exogenously given homing decisions on each side of the market. To reduce some of the multiplicity in our model, we restrict the set of allocation equilibrium constellations by ruling out two strong forms of coordination failure in the allocation subgame. First, the no-trade subgame equilibrium where no consumers and no firms join any platform in the allocation subgame is ruled out on and off path, whenever there concurrently exists an allocation equilibrium with positive participation by at least some consumers or firms.[11]

Second, agent beliefs about allocations (after observing prices) are generally important determinants of equilibria. However, in the case where the observed prices do not impact agent beliefs about allocations, then a price advantage of one platform on both sides of the market need not rule out the allocation where all agents tip to the higher priced alternative platform as an equilibrium. Conversely, if agents believe that a platform with a higher price on Side i will earn no participation on that side, then price undercutting on both sides always generates a unique equilibrium allocation where agents tip to the lower priced platform.

We make the following assumption: If Platform Y has prices that are strictly better on one side of the market and no worse on the other ($p_i^Y < p_i^X$ and $p_j^Y \leq p_j^X$ for $j \neq i$), then Platform Y has $n_k^Y > n_k^X$ for some $k = 1, 2$. In other words, we preclude discoordinated allocation configurations in which despite having better (i.e., lower) prices a platform fails to attract more agents than its rival on at least one side of the platform. Although this assumption generates an advantage for a lower priced platform, it is important to note that this assumption does not imply that lower prices always result in the tipping allocation to the lower priced platform (as shown in Lemma 1). It is also important to note that this assumption says nothing about which side has greater participation—either i or j is consistent with the assumption, and, importantly, it says nothing about the case of orthogonal pricing where $p_i^Y < p_i^X$ and $p_j^Y > p_j^X$ for $i \neq j$.

3. EQUILIBRIUM

We now investigate the allocation subgame of consumers and firms in joining platforms for arbitrary (undominated) prices, and then we determine the pricing equilibria for the entire game by considering price competition between the two platforms, in light of the continuation equilibrium from the allocation subgame.

Solving the allocation subgame is cumbersome because each side has multiple thresholds that impact homing allocation decisions. Fortunately, our main results focus on the pricing game, which is more streamlined. Thus, to ease exposition we divide our equilibrium analysis into two sections: Section 3.1 solves the technical allocation subgame and Section 3.2 focuses on the main results of the pricing equilibria and connects these findings to a variety of platform industries.

[10] Absent this restriction, the allocation equilibria would require making a distinction between prices in ranges in which agents participate and ranges under which agents do not join a platform. As the latter are dominated strategies in the platform pricing game that are not used in equilibrium, including them in the discussion of the allocation subgames leads to a needless distraction.

[11] If we allow these no-trade equilibria to exist in the allocation subgames, then there exists a continuum of trivial equilibria of the entire game where an arbitrary price constellation is an equilibrium because no-trade occurs off path for any other price constellation.

TABLE 1
SUMMARY OF RESULTS

Price Constellations	Allocation Subgame Equilibria	Equilibrium Prices and Profit
Strictly Lower Prices by Platform X (Lemma 1) $p_1^X < p_1^Y$ and $p_2^X < p_2^Y$	(i) Tip to Platform X	(i) Does Not Occur in Equilibrium
Weakly Lower Prices by Platform X (Lemma 1) $p_i^X < p_i^Y$ and $p_j^X \leq p_j^Y$	(i) Tip to Platform X (ii) Asymmetric Allocations	(i) Does Not Occur in Equilibrium (ii) Do Not Occur in Equilibrium
Orthogonal Pricing (Lemma 2) $p_1^X < p_1^Y$ and $p_2^X > p_2^Y$	(i) Tipping to A or B (ii) Asymmetric Allocations	(i) $f_i > p_i^Y > p_i^X, f_j < p_j^Y < p_j^X$ with $\Pi = 0$ or $p_i^Y > f_i > p_i^X$, $p_j^X > f_j > p_j^Y$ with $\Pi = 0$ (Proposition 2) (ii) $f_i > p_i^Y > p_i^X, f_j < p_j^Y < p_j^X$ with $\Pi \geq 0$ or $p_i^Y > f_i > p_i^X$, $p_j^X > f_j > p_j^Y$ with $\Pi \geq 0$ (Proposition 2)
Symmetric Pricing (Lemma 3) $p_1 = p_1^X = p_1^Y, p_2 = p_2^X = p_2^Y$	(i) Tipping to A or B (ii) Competitive Bottleneck (iii) Mixed-homing (iv) Asymmetric Allocations	(i) $p_1 = f_1, p_2 = f_2$ with $\Pi = 0$ (Prop. 1) or $p_i < f_i, p_j > f_j$ with $\Pi \geq 0$ (Prop. 2) (ii) $p_1 = f_1, p_2 = f_2$ with $\Pi = 0$ (Prop. 1) or $p_i < f_i, p_j > f_j$ with $\Pi \geq 0$ (Prop. 2) (iii) $p_1 = f_1, p_2 = f_2$ with $\Pi = 0$ (Prop. 1) or $p_i < f_i, p_j > f_j$ with $\Pi \geq 0$ (Prop. 2) (iv) Do Not Occur in Equilibrium
Sufficiently Low Price Constellations (Lemma 4) p_1^X or $p_1^Y < \delta v$ or p_2^X or $p_2^Y < -\sigma c$	(i) Entire Side Multi-homes	(i) $p_1 = p_1^A = p_1^B = \delta v$, $p_2 = p_2^A = p_2^B = f_2$ with $n_1^M = N_1, n_2^M = 0$, and $\Pi \geq 0$ (Prop. 3)

NOTES: The third column provides the conditions that must be satisfied for the particular price constellation to be supported as an equilibrium.

To bridge the two sections, we summarize their overlap by offering a preview of our results in Table 1. In the first column we see that there are five types of relevant price constellations: lower prices by one platform, orthogonal pricing, symmetric pricing, and price constellations where at least one price is sufficiently low to induce uniform multi-homing. These price constellations produce a variety of subgame equilibrium allocations, as shown in the second column of Table 1 and characterized explicitly in the Lemmas found in Section 3.1. The third column in Table 1, which corresponds to the Propositions in Section 3.2, highlights the price constellations and allocations that survive in equilibrium. There, we see that endogenous homing produces several equilibrium allocations in addition to the competitive bottleneck equilibrium; those include tipping equilibria where one platform captures all participation, asymmetric allocations where platforms capture different market shares and charge different prices on both sides of the market, and mixed-homing equilibria where a mix of single-homing and multi-homing agents exist on both sides of the market.

3.1. *The Allocation Equilibrium for Arbitrary Prices.* In this section we formalize the results found in the second column of Table 1. To start, we define several thresholds that play important roles in determining the allocation equilibrium. Define $s_1 := \delta v$ and let $s_2 := -\sigma c$ so that s_i denotes the threshold utility for joining a second platform when that platform has no participation on the other side of the market. In addition, let $w_1 := \delta v + \alpha_1(0) \cdot (\overline{N}_2/2) > s_1$ and $w_2 := -\sigma c + \alpha_2(0) \cdot (\overline{N}_1/2) > s_2$ so that w_i captures the maximum benefit to a Side i agent from joining a second platform when the platforms split participation equally on the other side of the market.

LEMMA 1 (ALLOCATIONS WITH A LOWER PRICED PLATFORM). *Suppose that* $s_i < p_i^X$ *and* $p_i^Y \leq p_i^X$ *for* $i = 1, 2$ *with at least one inequality being strict. If* $p_i^Y < p_i^X$ *for* $i = 1, 2$ *or if* $p_i^Y < w_i < p_i^X$ *and* $p_j^Y = p_j^X$ *for* $i = 1, 2$ *and* $j \neq i$*, then there exists a unique allocation equilibrium in which all active agents join Platform Y exclusively:* $n_i^Y = N_i$ *and* $n_i^X = 0$*, for* $i = 1, 2$*.*

If $p_i^Y < p_i^X < w_i$ *and* $p_j^Y = p_j^X$ *for* $i = 1, 2$ *and* $j \neq i$*, then tipping to Platform Y is an equilibrium. In addition, there exist equilibrium allocations where all active Side i agents multi-home* ($n_i^X = n_i^Y = n_i^M$) *and the Side j agents single-home so that* $n_j^Y > n_j^X$*.*

Lemma 1 implies that if prices are sufficiently low (a $p_i^X < w_i$), then Platform Y must have lower prices on *both* sides of the market for the tipping to Platform Y allocation to be the unique equilibrium. The reason why successful price undercutting—defined as the case where lower prices guarantee that all active participation occurs exclusively on the lower priced platform—can fail is that with endogenous homing, multi-homing becomes a more attractive option when δv, $-\sigma c$, or the α_i are sufficiently large relative to prices, and this willingness to multi-homing can generate additional equilibrium allocations, or prevent tipping allocations entirely, when the prices are equal on one of the two sides. However, if prices are undercut on both sides of the platform, then agents joining the lower priced platform is the unique equilibrium allocation.

Now consider possible allocations for orthogonal price constellations:

LEMMA 2 (ALLOCATIONS UNDER ORTHOGONAL PRICING). *When* $p_1^X > p_1^Y > s_1$ *and* $p_2^Y > p_2^X > s_2$*, there exists as many as three types of equilibria:*

(1) Tipping equilibria in which all active participation takes place on one platform. Tipping to Platform Y is always an equilibrium. A sufficient condition for tipping to Platform X to be an equilibrium is that $p_1^Y > v$*.*
(2) There exists an equilibrium in which Platform Y only attracts multi-homing firms and Platform X also attracts single-homing firms, $n_2^M = n_2^Y < n_2^X$*, and high-valued consumers single-home on Platform X, that is,* $\theta_1 \in [0, \theta_1^X]$ *where* $n_1^X = \theta_1^X \cdot \overline{N}_1$*, and low-valued consumers single-home on Platform Y,* $\theta_1 \in [\theta_1^X, \theta_1']$ *where* $N_1 = \theta_1' \cdot \overline{N}_1$ *and* $n_1^Y = N_1 - n_1^X > n_1^X$*. This allocation only exists when the consumer price difference is bounded and when the firm price difference is sufficiently large.*
(3) Similarly, for firm prices sufficiently close together, there exists an equilibrium in which Platform X only attracts multi-homing consumers whereas Platform Y also attracts single-homing consumers, $n_1^M = n_1^X < n_1^Y$*; and high-value firms single-home on Platform Y, that is,* $\theta_2 \in [0, \theta_2^Y]$*, where* $n_2^Y = \theta_2^Y \cdot \overline{N}_2$*, and low-value firms single-home on Platform X,* $\theta_2 \in [\theta_2^Y, \theta_2']$*, where* $N_2 = \theta_2' \cdot \overline{N}_2$ *and* $n_2^X = N_2 - n_2^Y > n_2^Y$*. This allocation only exists when the firm price difference is bounded and* $p_2^Y > w_2$*.*

The tipping outcomes are equilibria with orthogonal prices. In addition to the tipping equilibria, asymmetric participation allocations can also arise with orthogonal prices where the higher priced platform on Side i offers the Side i agents more Side j users to compensate for the higher price. This asymmetric allocation coincides with the orthogonal pricing allocation in Ambrus and Argenziano (2009) in terms of total participation asymmetries, but differs in terms of homing.

Focusing on the second allocation of Lemma 2, we see that all active firms join Platform X and some also join Platform Y—so Platform Y has no firms that are exclusive to it. Consumers

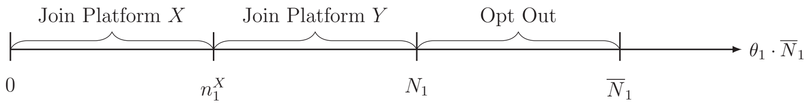

FIGURE 1

HOMING DECISIONS BY CONSUMERS IN SECOND TYPE OF EQUILIBRIUM

single-home across the two platforms, with high-valued consumers being willing to pay the higher price at Platform X in order to have access to all firms, whereas low-valued consumers are willing to forgo access to some firms in return for a lower price at Platform Y (see Figure 1). The price difference on the consumer side is limited by the degree of differentiation that is possible to attain from differential firm participation. Hence, if the difference in the consumer prices is too large or if p_1^X is too high, then only a tipping outcome is supported. The third case under orthogonal pricing is analogous to the second case, but has firms splitting participation across platforms.

For constellations where prices are symmetric between the two platforms, we have the following:

LEMMA 3 (ALLOCATIONS UNDER SYMMETRIC PRICING). *If $p_i^A = p_i^B = p_i > s_i$, then there exist three types of equilibria:*

(1) Tipping equilibria in which all participation takes place exclusively on one platform.

(2) Symmetric participation equilibria with $n_i^A = n_i^B = n_i$, $i = 1, 2$.
In a symmetric participation equilibrium, there exist multiple equilibrium allocations in which the distribution of multi-homers and single-homers on each side depends on the distribution of multi-homers and single-homers on the other side.
In particular, the set of multi-homing consumers is given by $[0, n_1^M]$, and the set of single-homing consumers is given by $(n_1^M, N_1]$, with

$$n_1^M = \min\left\{\alpha_1^{-1}\left(\frac{p_1 - \delta v}{n_2 - n_2^M}\right) \cdot \overline{N}_1,\ \alpha_1^{-1}\left(\frac{\max\{2p_1 - (1+\delta)v, 0\}}{N_2}\right) \cdot \overline{N}_1\right\}, \tag{6}$$

$$N_1 = \max\left\{\alpha_1^{-1}\left(\frac{\max\{p_1 - v, 0\}}{n_2}\right) \cdot \overline{N}_1,\ n_1^M\right\}, \tag{7}$$

where the max*-operators in $\alpha_1^{-1}(\cdot)$ are due to the possible full participation corner solution. The set of multi-homing firms is given by $[0, n_2^M]$, and the set of single-homing firms is given by $(n_2^M, N_2]$, with*

$$n_2^M = \min\left\{\alpha_2^{-1}\left(\frac{p_2 + \sigma c}{n_1 - n_1^M}\right) \cdot \overline{N}_2,\ \alpha_2^{-1}\left(\frac{2p_2 + (1+\sigma)c}{N_1}\right) \cdot \overline{N}_2\right\}, \tag{8}$$

$$N_2 = \max\left\{\alpha_2^{-1}\left(\frac{p_2 + c}{n_1}\right) \cdot \overline{N}_2,\quad n_2^M\right\}. \tag{9}$$

(3) Asymmetric participation equilibria can only exist when $p_i \leq w_i$ for some $i = 1, 2$.

Lemma 3 says that when platforms set equal prices, then either a tipping allocation in which all agents join only one of the platforms occurs, the platforms split both sides of the market

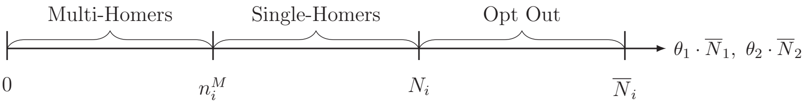

FIGURE 2

HOMING DECISIONS BY CONSUMERS AND FIRMS ACCORDING TO TYPE θ_1, θ_2

equally, or (provided that prices are sufficiently low) platforms split the market asymmetrically. A tipping equilibrium generates greater welfare due to the network effects and it is stable, but as there are two configurations, it requires agents on each side, and also across sides, to coordinate to one of the otherwise symmetric platforms. In contrast, the symmetric allocation equilibrium is susceptible to tipping, but may be natural in that when agents only observe prices, and prices are the same, then the symmetric equal division across the two platforms arises with agents' equilibrium indifference between the two platforms. Naturally, the participation equilibria admit a richer analysis of endogenous homing. In fact, the division of agents across platforms does not uniquely determine the extent to which consumers and firms multi-home in equilibrium, as the allocation of one side of the market depends on the allocation on the other side in both the symmetric and asymmetric cases. That is, allocation multiplicity arises because if consumers expect more firms to multi-home, then single-homing becomes more attractive to consumers. While this give-and-take generates homing multiplicity, we can say, however, that agent homing across types will follow Figure 2 when there is a mix of single-homing and multi-homing agents on each side of the market.

Consider specifically the allocation of consumers. Consumers always obtain an added benefit from joining a second platform, namely δv. Hence, if prices to consumers are above this threshold, but still below the stand-alone utility from a single platform membership (i.e., $\delta v < p_1 < v$), then all consumers will join at least one platform, $N_1 = \overline{N}_1$, but whether any consumer joins a second platform and multi-homes depends on whether firms multi-home. In particular, if the number of multi-homing firms is large ($n_2 - n_2^M$ is small), then consumers have access to many firms when joining the first platform and so the number of multi-homing consumers is small, or even zero. For even higher consumer prices, consumers with large values of θ_1 even refrain from joining a single platform, $N_1 < \overline{N}_1$.

Unlike consumers, firms do not obtain a stand-alone benefit from joining a platform. However, they experience duplication economies in production when joining a second platform. This implies that a firm will multi-home only when the marginal gain from joining a second platform and the total payoff from being on two platforms are both positive. Hence, the set of multi-homing firms depends on the number of consumers that multi-home. Like consumers, the firms that choose to multi-home instead of single-home are the firms with sufficiently high benefits from network effects (θ_2 close to zero). That is, as the network effect gets larger, the marginal gain from having access to additional consumers outweighs the marginal cost of joining another platform. Hence, for firms with lower network effects, θ_2 farther from zero, it is too costly to join more than one platform. Note finally that if few consumers multi-home (n_1^M is small) and there are strong duplication economies (small σ), then it is possible that no firms single-home and all firms multi-home.

Finally, consider the case of prices that inherently generate multi-homing.

LEMMA 4 (PRICES GUARANTEEING MULTI-HOMING). *If $p_i^X, p_i^Y \leq s_i$ for $i = 1, 2$, then all agents multi-home in equilibrium. Similarly, if $p_i^X, p_i^Y \leq s_i$ and $p_j^Y \leq p_j^X$ with $p_j^X > s_j$ for $j \neq i$, then all Side i agents multi-home and all active Side j agents single-home on Platform Y so that $n_j^Y = N_j$ is an equilibrium allocation; furthermore, if $p_j^Y = p_j^X$, then the Side j agents are indifferent between the two platforms, since $p_j^Y = p_j^X$ and $n_i^X = n_i^Y = n_i^M$, and a continuum of single-homer distributions on Side j are possible in equilibrium.*

3.2. *Equilibria of the Pricing Game.* Prior to the allocation subgame, the platforms simultaneously select their prices to consumers and firms. Notice that by considering Lemma 4, we see that successful price undercutting is not possible when platform prices are sufficiently low. Naturally, this has important implications for the platform pricing game. In particular, if platform marginal costs are low ($f_i \leq s_i$ for $i = 1, 2$), then profitable prices cannot be undercut. However, as $s_2 = -\sigma c < 0$, only the marginal cost on the consumer side is of concern. That is, if $f_1 < s_1 = \delta v$, then there are consumer prices greater than marginal costs where price undercutting does not result in a tipping allocation to the lower priced platform and so platform competition is dampened in this case. Otherwise, if $f_1 > s_1$, then platform competition is stronger, since price undercutting can be effective for prices greater than platform marginal costs (Lemma 1).

To determine pricing equilibria, we utilize the following off-path beliefs for the continuation game that follows a price deviation: (i) a platform that deviates to lower prices ($p_i^X \leq p_i^Y$ for $i = 1, 2$ with at least one inequality being strict) tips the market in their favor; (ii) a platform that deviates to higher prices ($p_i^X \geq p_i^Y$ for $i = 1, 2$ with at least one inequality being strict) is tipped out of the market; and (iii) a platform that deviates to orthogonal price constellations is tipped out of the market.

Now consider the case of strong competition.

Proposition 1 (Strong Competition). *If $f_1 > s_1$, then symmetric marginal cost pricing ($p_1 = p_1^A = p_1^B = f_1$ and $p_2 = p_2^A = p_2^B = f_2$) with at least one and possibly three types of participation allocations occur in equilibrium:*

(1) *(The Competitive Bottleneck) All active consumers single-home and all active firms multi-home: $n_1^M = 0$, $n_2^M = N_2$. This is always an equilibrium.*
(2) *(Mixed-Homing) A mix of multi-homing and single-homing consumers with multi-homing and single-homing firms: There exists $\overline{n}_1^M \in (0, N_1)$ so that $n_1^M \in (\overline{n}_1^M, N_1)$ and $n_2^M \in (0, N_2)$. Existence requires that $p_1 < \delta v + \alpha_1(0)N_2$, where N_2 is given in Lemma 3.*
(3) *(The Inverse Competitive Bottleneck) All active firms single-home and all active consumers multi-home: $n_1^M = n_1$, $n_2^M = 0$. Existence requires that $p_1 \leq w_1$.*

In addition, tipping allocations also produce equilibria with symmetric marginal cost pricing.

First note that the asymmetries between consumers and firms (through c and v) play an important role in homing decisions. Namely, this asymmetry results in the competitive bottleneck allocation as an equilibrium, always, whereas the inverse competitive bottleneck allocation requires consumer prices to be sufficiently low. This observation ties to a second point, equilibrium allocations where consumers multi-home require that the consumer price be sufficiently low.

In terms of equilibrium allocations, Proposition 1 states that tipping equilibria are generated. Examples of tipping equilibria in platform markets are abundant. The ultimate success of VHS over Betamax in the videotape format war can be seen as an example of tipping in a platform market that was otherwise poised to have consumers single-home (purchase either one or the other system) and distributors multi-home (provide films in both formats). More recently, Blu-ray similarly tipped out HD DVD in the high-resolution video market.

In addition to the tipping configurations, there exist three other allocations. However, only one equilibrium with participation on both platforms always exists. Importantly, this equilibrium produces the allocation where all consumers single-home and all firms multi-home (Allocation 1). The fact that endogenous homing always generates this allocation as an equilibrium appears to vindicate its recurring usage in the literature. Note, however, that in the extant literature, the homing decisions are exogenously assumed rather than endogenously derived. As an example of this allocation, the smartphone market with Apple and Android mirrors Allocation 1: Almost all consumers single-home, they own only one phone; and almost all firms multi-home, the vast majority of apps are available across all types of smartphones. Similarly, Allocation 1 occurs in the personal computer market where consumers single-home (they use Windows or a Mac, but not both) and developers multi-home by making their software available on both operating systems.[12]

The second allocation, Allocation 2, which we refer to as the mixed-homing allocation, also resembles current allocations seen in many two-sided markets, including those for game consoles. For video game platforms, there exist consumers who multi-home—buying several game consoles—and others who single-home, and there exists game designers whose games are available across platforms (i.e., they multi-home), whereas others are available on only one system (i.e., they single-home). We also observe the mixed-homing allocation in the video streaming market where there is a mix of single- and multi-homing by both video providers and consumers across HBO, Netflix, and Hulu. Finally, the mixed-homing allocation also occurs in online marketplace industries. For example, some products are offered across Amazon, eBay, and Walmart.com, whereas others are only available on one platform; at the same time, some consumers shop across platforms, whereas others only purchase products from a single platform. Similarly, in the sharing economy, there is mixed-homing between Uber and Lyft by riders and drivers in the ridesharing market and there is mixed-homing by hosts and renters across homesharing platforms like Airbnb, Homeaway, and VRBO.[13]

The last allocation, which we refer to as the inverse competitive bottleneck allocation, is Allocation 3. Here, all active firms single-home and all active consumers multi-home. It should be noted that we do not observe platform industries where such an allocation exists. This lack of observation suggests that two-sided markets exhibit parameters so that $p_1 > w_1$ (the condition under which Allocation 3 does not exist as an equilibrium allocation). With such parameters, we then have that $p_1 = f_1 > w_1 > s_1$ so that platform competition is always strong and only the competitive bottleneck and mixed-homing allocations survive as non-tipping, marginal cost pricing equilibria.

The results from Proposition 1 also imply that Allocation 1 must occur instead of Allocation 2 when $p_1 > \delta v + \alpha_1(0)N_2$ and must occur instead of Allocation 3 when $p_1 > \delta v + \alpha_1(0) \cdot (\overline{N}_2/2) = w_1$. In the smartphone market, δ is close to zero whereas α is relatively small compared to the price of a smartphone.[14] Similarly, δ is close to zero whereas N_2 is relatively small compared to the price of a personal computer. This suggests that both $p_1 > \delta v + \alpha_1(0)N_2$ and $p_1 > \delta v + \alpha_1(0) \cdot (\overline{N}_2/2) = w_1$ occur in the market for smartphones and the market for personal computers. Given such parameters, our results from Proposition 1 imply that Allocations 2 and 3 are not possible in these markets, and indeed Allocation 1 is observed in both the smartphone and personal computer markets. Unfortunately, these parameter conditions do not imply that Allocation 1 is the unique equilibrium since the tipping equilibria also exist. However, multiple equilibria might be reasonable for many platform industries. For example, Research-in-Motion's Blackberry was initially a monopoly in the smartphone market and was then effectively tipped out by Apple's iPhone.[15]

For the mixed-homing allocation, we find that industries which map into the parameter requirements given in Allocation 2 indeed exhibit mixed-homing. For example, note that $p_1 < \delta v + \alpha_1(0)N_2$ is likely true for the video game market, since video games generate significant gains to consumers, although consoles are usually subsidized to maintain a low price to consumers. Thus, the existence of Allocation 2 aligns with the mixed-homing allocation that we observe in the video game market. As another example, $p_1 < \delta v + \alpha_1(0)N_2$ should also hold for the video streaming industry, since consumer membership prices are low relative to the amount

[12] Shopping malls and traditional newspapers are also examples of platforms where Allocation 1 occurs: In both markets, consumers often single-home and retailers or advertisers multi-home.

[13] Dating apps are another example of a platform market with a mix of single- and multi-homing men on one side and a mix of single- and multi-homing women on the other side.

[14] For example, retail price for the iPhone X started at $999.

[15] Equilibrium multiplicity is often present in our model because of the unrestrictive nature of agent homing decisions. This allows us to investigate endogenous homing, but also generates either multiple participation equilibria or tipping equilibria.

of content consumers receive. In the video streaming industry, we observe mixed-homing across HBO, Netflix, and Hulu by both the consumer side and the video provider side.

It is also worth noting that the equilibrium allocation configurations are exhaustive for the case of marginal cost pricing. In particular, although many papers on platform competition assume exclusive single-homing, there does not exist an equilibrium in our model where all active agents single-home on both sides of the market. An example of such an exclusively single-homing constellation occurred in the early home-computer market in which consumers either purchased a DOS-based or a Macintosh machine and software was not available across the two systems. However, when all consumers single-home, then firms optimally multi-home in order to reach all consumers, which is the equilibrium that was eventually reached in the home-computer market.

In addition to the marginal cost pricing equilibria, other (potentially profitable) equilibria can exist.

PROPOSITION 2 (STRADDLE PRICING). *If $f_1 > s_1$, then symmetric prices that straddle marginal costs ($f_i > p_i = p_i^X = p_i^Y > s_i$ and $f_j < p_j = p_j^X = p_j^Y$) occur in equilibrium when $(p_i - f_i) \cdot (n_i^T - n_i) + (p_j - f_j) \cdot (n_j^T - n_j) \equiv \Delta\Pi^T < 0$, where n_i, n_j are the equilibrium participation levels on each platform (the equilibrium allocations follow the allocations in Proposition 1) and n_i^T, n_j^T are the participation levels for a platform that undercuts both prices and tips the market.*

Similarly, if $f_1 > s_1$, then orthogonal prices that straddle marginal costs [either (a) $f_i > p_i^Y > p_i^X > s_i$ with $f_j < p_j^Y < p_j^X$ or (b) $p_i^Y > f_i > p_i^X > s_i$ with $p_j^X > f_j > p_j^Y > s_j$] occur in equilibrium when both (i) $(p_i^Y - f_i) \cdot n_i^{TX} - (p_i^X - f_i) \cdot n_i^X + (p_j^Y - f_j) \cdot n_j^{TX} - (p_j^X - f_j) \cdot n_j^X \equiv \Delta\Pi^{TX} < 0$, where n_i, n_j are the equilibrium participation levels on each platform (the equilibrium allocations follow the allocations in Lemma 2) and n_i^{TX}, n_j^{TX} are the participation levels when Platform X undercuts both of Platform Y's prices and tips the market, and (ii) $(p_i^X - f_i) \cdot n_i^{TY} - (p_i^Y - f_i) \cdot n_i^Y + (p_j^X - f_j) \cdot n_j^{TY} - (p_j^Y - f_j) \cdot n_j^Y \equiv \Delta\Pi^{TY} < 0$, where n_i^{TY}, n_j^{TY} are the participation levels when Platform Y undercuts both of Platform X's prices and tips the market.

These pricing equilibria can result in positive profit for at least one of the platforms and, combined with the equilibria in Proposition 1, are exhaustive for the case of $f_1 > s_1$.

Prices that straddle marginal costs (either symmetric prices or orthogonal prices) can generate equilibria where platforms earn profit. Equilibrium profits are possible since price undercutting makes the winning platform worse off when the greater costs from the subsidized side of the market outweigh the greater revenues from the markup side of the market. Such cases are reflected in the conditions above. It is important to note that the orthogonal prices that generate straddle pricing equilibria give the allocations from Lemma 2 so that platforms endogenously differentiate themselves in terms of serving high valuation agents on Side i and low valuation agents on Side $j \neq i$. In contrast, symmetric prices that generate straddle pricing equilibria give the symmetric pricing allocations found in Proposition 1.

The results in Proposition 2 complement results found in Ambrus and Argenziano (2009) who consider an alternative model where agents single-home on both sides. They find that orthogonal straddle prices can result in equilibrium platform profits. In addition to orthogonal straddle pricing equilibria, we find that symmetric straddle pricing equilibria can also be profitable with endogenous homing.[16]

Unfortunately, equilibrium participation levels in Proposition 2 are implicitly defined so we are unable to obtain conditions that are explicitly characterized by model parameters.[17] However, the intuition behind when a straddle pricing equilibrium can exist is straight forward:

[16] For orthogonal straddle pricing outcomes to exist as equilibria in Ambrus and Argenziano (2009), they show that agent network effects must differ discretely within a single side of the market. Given that our $\alpha_i(\cdot)$ are continuous, this requirement is not possible in our model; so, the fact that straddle pricing equilibria exist in our setting suggests that profitable straddle pricing equilibria can be more easily sustained with endogenous homing.

[17] See the proof of Proposition 2 for the implicit expressions.

If the negative margin is small, then the positive margin must also be relatively small (all else equal). Instead, if the positive margin is large relative to the negative margin (all else equal), then a platform can undercut its rival to increase profit and the straddle pricing outcome breaks down. To see this scenario explicitly, we provide the following example:

Example 1 (A Straddle Pricing Equilibrium[18]). Let $\alpha_i(\theta_i) = 1 - \theta_i$ and $\overline{N}_i = 1$ for $i = 1, 2$, and consider a symmetric straddle pricing constellation where $f_1 = 0.21 > 0.2 = p_1$, $p_2 = 0.05$, $v = 0.1$, $c = 0.1$, and $\sigma = 0.5$; leaving $f_2 < p_2$ as a free parameter. By Lemma 3, Allocation 1 in Proposition 1 and tipping allocations are possible. In Allocation 1: $n_1 = {}^{N_1}\!/_2 \approx 0.43$ and $n_2 = N_2 = n_2^M \approx 0.71$. In a tipping allocation: $n_1^T \approx 0.88$ and $n_2^T \approx 0.83$. This implies that the difference in profit is given by: $\Delta\Pi^T \approx 0.45(-0.01) + 0.12 \cdot (0.05 - f_2)$, so that undercutting is not profitable when $f_2 \geq 0.0127$. At the same time, platforms' profit from Allocation 1 is non-negative when $f_2 \leq 0.0439$. Thus, this symmetric straddle pricing constellation generates a profitable equilibrium when $f_2 \in [0.0127, 0.0439]$. Otherwise, if $f_2 > 0.0439$, then the price constellation does not produce an equilibrium due to negative profits, and if $f_2 < 0.0127$, then the price constellation is not an equilibrium because platforms undercut.

In this example, symmetric straddle prices with $f_1 > p_1$ and $f_2 < p_2$ result in an equilibrium where consumers single-home and firms multi-home and both platforms earn profit. This profitable equilibrium survives undercutting because the new consumers that join the undercutting platform generate additional costs that outweigh the additional revenues from the new firms.

The existence of profitable straddle pricing equilibria has important implications for platform pricing strategies in emerging industries where competition does not yet exist. In particular, if a monopoly platform chooses a straddle pricing strategy instead of charging markups to both sides, then the platform insulates itself against price undercutting competitors. It is also important to note that this straddle pricing strategy differs from the commonly seen pricing strategy of cross-subsidization. That is, platforms often internalize the network externalities between the two sides by subsidizing one side of the market while extracting surplus from the other side of the market. While the cross-subsidization pricing strategy can result in prices that straddle marginal costs, the straddle pricing strategy is a result of platforms strategically setting straddle prices to reduce competition. Thus, the straddle pricing strategy is not used to internalize the network externalities, but is instead used to prevent price undercutting between platforms.

While the straddle pricing strategy offers a new reason for charging a price below marginal costs in platform markets, it is also worth noting that such strategies are often observed in platform industries. For example, based on recent findings from the Supreme Court ruling on American Express,[19] we see that American Express charges merchants higher fees while offering better rewards to high-end consumers. This corresponds to the equilibrium where orthogonal prices straddle marginal costs: $p_2^{AmEx} > p_2^{Visa} > f_2$ and $p_1^{AmEx} < p_1^{Visa} < 0 \leq f_1$. In contrast, consider competition between Visa and Mastercard, where both platforms charge similar merchant fees and offer similar rewards to consumers: $p_2^{MC} = p_2^{Visa} > f_2$ and $p_1^{MC} = p_1^{Visa} < 0 \leq f_1$.[20]

A recurring theme throughout the allocation subgame is that multi-homing becomes more attractive with lower prices, and this increased willingness to multi-home can allow for different allocation equilibria when prices are low enough (e.g., for $p_i < s_i$ in Lemma 4). This setting becomes relevant specifically for industries where $f_1 < s_1$, and our results from Lemma 1, Lemma 4, and Proposition 2 provide guidance for platform outcomes in this case. From Proposition 2, we see that a variety of equilibria can exist where platforms earn profit when f_1 is greater than s_1.

[18] We provide the solution to the system of equations for these equilibria in the appendix.

[19] See "Supreme Court Sides With American Express on Merchant Fees," by the New York Times on June 25, 2018.

[20] The video game market offers another example of an industry with symmetric prices that straddle marginal costs. In this case, consoles are often sold to consumers at a loss to the platform but positive fees are taken on the video game side (see Lee (2013) for details).

In addition, Lemmas 1 and 4 highlight how price undercutting is possible only when prices are greater than the s_i. Combined, these results suggest that it is easier for platforms to earn profit when f_1 is less than s_1, since now there exist prices $p_1 \in (f_1, s_1)$ that provide a positive markup and cannot be undercut. To better understand this case of weak competition, we present the following result:

PROPOSITION 3 (WEAK COMPETITION). *If $f_1 \leq s_1$, then there exist equilibria with $p_1^A = p_1^B = s_1$ and $p_2^A = p_2^B = f_2$ such that all consumers multi-home, $n_1^M = \overline{N}_1$, and all active firms single-home, $n_2^M = 0$ with $n_2^A, n_2^B > 0$. Platform profits are $\Pi^A = \Pi^B = \overline{N}_1(s_1 - f_1) \geq 0$. These equilibria are exhaustive.*

It is worth noting that when competition is weak, platforms do not need to use a straddle pricing strategy to earn profit; a markup on the consumer side of $s_1 - f_1$ and marginal cost pricing on the firm side generates profit for both platforms. Recall that this equilibrium allocation, where all consumers multi-home and all firms single-home, is the inverse competitive bottleneck allocation (Allocation 3) in Proposition 1. Although this allocation is possible in either strong or weak competition, it is unobserved in platform industries. Combining our results from Propositions 1 and 3, this suggests that $f_1 > w_1 > s_1$ is likely for most platform industries and in that case the only possible equilibria are the symmetric marginal cost pricing with the competitive bottleneck, symmetric pricing with mixed-homing, or straddle pricing equilibria.

4. DISCUSSION AND EXTENSIONS

Before concluding, we briefly address additional issues that the model can account for. First, we consider a welfare comparison between Allocations 1 and 2, entry beyond two platforms, exclusive deals, and then we consider other issues addressed in the literature, in particular the case of one platform having an inherent advantage by being "focal."

4.1. *Welfare.* A well known result in platform markets is that due to network effects greater platform competition does not necessarily improve welfare. In our model, competition results in lower prices and additional stand-alone membership benefits to consumers who multi-home. However, competition can increase synchronization costs, as well as destroy network surplus by fragmenting the market. Moreover, competition that results in marginal cost pricing may also undermine welfare-increasing cross-subsidization that takes place with a monopoly platform.

Although comparing welfare across different levels of platform competition is important,[21] we aim to consider welfare across allocations. From Proposition 1, we know that marginal cost pricing results in either a tipping allocation, the allocation where all consumers single-home whereas all firms multi-home, or an allocation with mixed-homing on each side of the market. Not surprisingly, due to network effects, a tipping allocation generates the greatest welfare, since the prices are the same in both the tipping and non-tipping allocations. Specifically, the single platform garners greater participation than the total participation between the two platforms that divide the network effects, diminishing welfare.

For the non-tipping allocations, a welfare comparison is less clear. We would like to know when the welfare generated with Allocation 2 dominates the welfare generated with Allocation 1. To compare welfare between the two allocations, consider the change in welfare at the margin where some of the single-homing consumers become multi-homers and some of the multi-homing firms become single-homers. In this case, the consumers that become multi-homers result in lost surplus of $p_1 - \delta v$, whereas the firms that become single-homers reduce their costs by σc. Thus, if the stand-alone gains from consumer multi-homing are large (close to p_1) or the firm multi-homing costs are high (σc), then the welfare gains favor the mixed-homing allocation. In the video game market where the mixed-homing allocation occurs, costs are high

[21] See Jeitschko and Tremblay (2016) for a detailed discussion of this.

for game developers who port their game across multiple platforms.[22] Similarly, a driver needs a second phone with cellular service to fully multi-home in the ridesharing market.[23] This corresponds to a high σc and we observe mixed-homing in ridesharing. These examples suggest that the welfare improving allocation perhaps occur in markets where both allocations are possible.

4.2. *Entry.* If additional platforms enter the market, then multi-homing becomes more complex, as agents may join any number of platforms. However, welfare implications are unambiguously worse for entry beyond two platforms for the case where the cost of accommodating another consumer is greater than the marginal value of the second platform, $f_1 \geq \delta v$. If tipping occurs, nothing is gained, and otherwise with even just two platforms, prices can be competed down to marginal costs, and so there are no additional beneficial price effects from entry. However, if consumers fragment across platforms, then firms' additional replication costs destroy welfare, and network effects that accrue to firms are also diminished.

In terms of potential allocations with more than two platforms, if a third platform enters so that three potential platforms exist, then the non-tipping allocations in Proposition 1 (Allocations 1 and 2) where prices equal marginal costs will persist. That is, the third platform can be tipped out of the market and agents allocate according to Proposition 1 onto two platforms. Such an outcome is consistent with the smartphone market where several platforms have been largely tipped out the market (Blackberry, Windows Phone, and Amazon's Fire Phone) whereas Allocation 1 has persisted between Apple and Android.

Additional platforms might generate greater surplus for the case where $f_1 < \delta v$. In this case, if all prices offered to consumers are less than δv, then all consumers multi-home. and so more platforms generates greater surplus on the consumer side. However, this case requires a great deal of platform differentiation—large δ that does not decrease (too much) upon consumer participation on additional platforms—in order to increase welfare in the market.

4.3. *Exclusive Deals.* In some instances, firms enter into exclusivity contracts with platforms postulating that they cannot offer their service on competing platforms. In our model this has two implications. First, there exist equilibrium configurations in which the market tips and only one platform survives. That is, one platform can use exclusive deals to lock in enough firms to have the market tip.

Exclusive deals can also be used to successfully enter into the platform market, as was studied by Lee (2013), for the case of the successful entry of the Xbox in the video game market. In our model, there also exists exclusive contract equilibrium configurations without market tipping. These are distortions of the Allocation 1 equilibrium where in Proposition 1 all firms multi-home, all consumers single-home, and prices are $p_i^A = p_i^B = f_i$. To see this, take these prices as given on Platform X and suppose that Platform Y sets $p_2^Y = p_2^X = f_2$, but also offers some of the firms an exclusive contract with $p_{2,\text{excl}}^Y < f_2$. The firms who are offered the exclusive deal will join Platform Y and the remaining firms will multi-home so long as there are the same number of consumers on each platform. This can be the case, although with $p_{2,\text{excl}}^Y < f_2$, Platform Y must charge $p_1^Y > f_1 = p_1^X$. Now, with $p_1^Y > f_1 = p_1^X$ and $n_2^Y = N_2 > n_2^X$, consumers are indifferent when $u_1^Y(\theta_1) = v + \alpha_1(\theta_1) \cdot N_2 - p_1^Y = v + \alpha_1(\theta_1) \cdot n_2^X - p_1^X = u_1^X(\theta_1)$, so $n_1^Y = n_1^X$, and both platforms have zero profits, $\Pi^Y = \Pi^X = 0$; and this characterizes a possible equilibrium.[24]

4.4. *Favorable Beliefs/Focal Platform.* In many settings, one platform may have an inherent advantage over another that is not tied to superior technology or pricing. This may be due

[22] In fact, the costs are high enough to generate a market for game porting. See "What exactly goes into porting a video game? Blit Works explains," in Gamasutra, August 13, 2014, for details.

[23] The driver multi-homing costs are high enough to generate the market for another app, Mystro, that allows drivers the ability to be available across Uber and Lyft on the same phone.

[24] It is worth pointing out that a similar asymmetric equilibrium can also be derived for the case that platforms differ in their stand-alone values, $v^A \lesseqgtr v^B$.

to incumbency, greater name recognition, successful marketing campaigns, or the like. We call Platform X *focal* when all agents believe that $n_1^X = N_1$, that is, all agents believe that all consumers that join at least one platform will join Platform X.[25]

Naturally, having Platform X be focal makes it harder for Platform Y to compete. Nevertheless, Platform Y may be able to compete by using a muted divide-and-conquer strategy. Caillaud and Jullien (2003) note that by subsidizing one side of the market a platform "divides" that side, and by subsequently "conquering" the other side of the market it can recover the loss it made through subsidization. Jullien (2011) demonstrates this in a model where a second mover platform uses divide-and-conquer to compete against a first mover platform. However, all agents are assumed to single-home.[26]

With endogenous homing decisions, the divide-and-conquer strategy is muted: the non-focal platform's ability to attract one side of the markets by pricing low is enhanced through the possibility of those agents multi-homing. However, this need not lead to a "divide" (because the multi-homers remain on the focal platform as well), and hence the ability to recover the costs of the subsidy may be harder to achieve as the "conquering" is less effective (focal platform agents that access multi-homers have no need to join the "conquering" platform). Thus, when agents make endogenous homing decisions, a focal platform cannot necessarily tip the market and become a monopoly platform, but competition can be strong between the focal and non-focal platforms because agents' abilities to either single-home or multi-home allow the non-focal platform to attract one side of the market to multi-home, generating competition with the focal platform.

5. CONCLUSION

In many markets in which platforms compete against each other, agents choose to join either one platform (single-home), no platform, or they join several platforms (multi-home). Although most of the previous literature on platform competition has assumed this decision to be given exogenously prior to platform pricing, we allow participants on both sides of the platform to endogenously make an optimal homing decision after observing platform prices.

With endogenous homing, we find that multiple allocations can arise in equilibrium and that agent multi-homing goes hand-in-hand with lower prices. In addition, it is possible that straddle price constellations generate equilibria (price constellations with one price above its marginal cost while the price on the other side is below its marginal cost). These constellations can persist as equilibria because successful price undercutting might not increase profit (the additional sales on the costly side might outweigh the gains on the earnings side). Thus, by using straddle pricing strategies, platforms effectively dampen competition.

Although these straddle pricing constellations need not exist as equilibria, marginal cost pricing equilibria always exist. In particular, marginal cost pricing with consumers single-homing and firms multi-homing (the competitive bottleneck allocation) is always an equilibrium. This mirrors the allocation in the market for smartphones, where virtually all consumers own only a single phone, but virtually all apps are available across competing smartphones. Another type of equilibrium allocation can also emerge where a mix of multi-homing and single-homing occurs within each side of the market. This constellation is found in the market for video game consoles: although many consumers have only one console, serious gamers often have more than one system, and although some games are available across gaming platforms, others are exclusive to one system. In addition, sharing economy markets (ridesharing and homesharing)

[25] Alternatively, we could have believed that $n_1^X > n_1^Y$ instead of $n_1^X = N_1$. Either framework is largely consistent with the models developed by Caillaud and Jullien (2003); Hagiu (2006), and Jullien (2011) on favorable beliefs and focal platforms.

[26] Similarly, favorable beliefs also play an important role in dynamic platform competition. For example, Hałaburda et al. (2019) show that a non-focal platform can implement a type of the divide-and-conquer strategy to beat a focal platform over time. However, their model considers only one-sided platforms and so it is unclear how endogenous homing might impact the divide-and-conquer strategy in their setting.

as well as the market for video streaming are additional examples of industries that fit our model and where the mixed-homing allocation occurs.

APPENDIX: OF PROOFS

PROOF OF LEMMA 1. If $p_i^Y < p_i^X$ for $i = 1, 2$, then no dis-coordination implies that $n_k^Y > n_k^X$ for some $k = 1$ or 2. Without loss of generality, suppose that $k = 1$. Thus, $n_1^Y > n_1^X$ and $p_2^Y < p_2^X$ imply that $u_2^Y(\theta_2) > u_2^X(\theta_2)$ for all θ_2 so that all active firms join Platform Y: $n_2^Y = N_2$. All active firms on Platform Y and $p_1^Y < p_1^X$ implies that $u_1^Y(\theta_1) > u_1^Y(\theta_1)$ for all θ_1 so that all active consumers join Platform Y: $n_1^Y = N_1$. Furthermore, all participation on Platform Y and $p_i^X > s_i$ for $i = 1, 2$ implies that no agent will multi-home.

If $p_i^Y < w_i < p_i^X$ and $p_j^Y = p_j^X$ for $i = 1, 2$ and $j \neq i$, then no dis-coordination implies that $n_k^Y > n_k^X$ for some $k = i$ or j. If $k = j$ so that $n_j^Y > n_j^X$, then $p_i^Y < p_i^X$ implies that $u_i^Y(\theta_i) > u_i^X(\theta_i)$ for all θ_i so that all active Side i agents join Platform Y and no single-homing Side i agent joins Platform X: $n_i^Y = N_i \geq n_i^M = n_i^X$. With $n_i^Y = N_i \geq n_i^M = n_i^X$ and $p_j^Y = p_j^X$, we have that $u_j^Y(\theta_j) \geq u_j^X(\theta_j)$ for all θ_j. If the inequality is strict, then no single-homing Side j agent joins Platform X so that $n_j^Y = N_j \geq n_j^M = n_j^X$, and then all participation on Platform Y and $p_i^X > s_i$ for $i = 1, 2$ implies that no agent will multi-home. Instead, if equality holds, then $n_i^Y = N_i = n_i^M = n_i^X$ so that all active Side i agents multi-home and all Side j agents are indifferent between single-homing across the two platforms so that any single-homing allocation with $n_j^Y > n_j^X$ is possible. However, in this case, the active Side i agents that all join Platform Y prefer to single-home to Platform Y instead of multi-homing because $u_i^Y(\theta_i) - u_i^M(\theta_i) = p_i^X - [s_i + \alpha_i(\theta_i)(n_j^X)] > p_i^X - [s_i + \alpha_i(0)(\overline{N}_j/2)] = p_i^X - w_i > 0$. This is a contradiction so that the inequality, $u_j^Y(\theta_i) \geq u_j^X(\theta_i)$, must be strict so that no single-homing Side j agent joins Platform X; then, all participation on Platform Y and $p_i^X > s_i$ for $i = 1, 2$ implies that no agent will multi-home. Similarly, if no dis-coordination has $k = i$ so that $n_i^Y > n_i^X$, then all agents join Platform Y exclusively.[27]

Now consider the case where $p_i^Y < p_i^X < w_i$ and $p_j^Y = p_j^X$ for $i = 1, 2$ and $j \neq i$. Naturally, if all active agents join Platform Y, then $p_i^X > s_i$ for $i = 1, 2$ implies that no agent has an incentive to deviate so that tipping to Platform Y is an equilibrium in this case. However, unlike the previous argument in the paragraph above, here we have that $u_i^Y(\theta_i) - u_i^M(\theta_i) = p_i^X - [s_i + \alpha_i(\theta_i)(n_j^X)] > p_i^X - [s_i + \alpha_i(0)(\overline{N}_j/2)] = p_i^X - w_i < 0$ so that our argument to eliminate the allocation where all active Side i agents multi-home and all Side j agents are indifferent between single-homing across the two platforms breaks down. This implies that such an allocation with single-homing on Side j that satisfies $n_j^Y > n_j^X$ generates additional allocation equilibria in this case. □

PROOF OF LEMMA 2. The tipping allocations are characterized as follows: For tipping to Platform Y, it must be that firms prefer to join Platform Y exclusively when all active consumers join Platform Y exclusively. If all active consumers join Platform Y exclusively, then $u_2^Y(\theta_2) = \alpha_2(\theta_2)N_1 - c - p_2^Y > \alpha_2(\theta_2)N_1 - (1+\sigma)c - p_2^X - p_2^Y = u_2^M(\theta_2)$ since $p_2^X > s_2$. For all firms to prefer Platform Y over Platform X it must be that $u_2^Y(\theta_2) = \alpha_2(\theta_2)N_1 - c - p_2^Y > -c - p_2^X = u_2^X(\theta_2)$. This occurs since $p_2^X > s_2 = -\sigma c > -c$ so that all consumers joining Platform Y exclusively implies that all firms join Platform Y exclusively. For tipping to Platform Y to be an equilibrium, it must also be that consumers prefer to join Platform Y exclusively when all active firms join Platform Y exclusively and this follows directly since $p_1^Y < p_1^X$ and $p_1^X > s_1$.

[27] In this case, $n_i^Y > n_i^X$ and $p_j^Y = p_j^X$ imply that $u_j^Y(\theta_j) > u_j^X(\theta_j)$ for all θ_j so that all active Side j agents join Platform Y and no single-homing Side j agent joins Platform X: $n_j^Y = N_j \geq n_j^M = n_j^X$. With $n_j^Y = N_j \geq n_j^M = n_j^X$ and $p_i^Y < p_i^X$, we have that $u_i^Y(\theta_i) > u_i^X(\theta_i)$ for all θ_i so that all active Side i agents join Platform Y and no single-homing Side i agent joins Platform X: $n_i^Y = N_i \geq n_i^M = n_i^X$. Finally, all participation on Platform Y and $p_i^X > s_i$ for $i = 1, 2$ implies that no agent will multi-home.

For tipping to Platform X, it must be that firms prefer to join Platform X exclusively when all active consumers join Platform X exclusively. This follows directly since $p_2^X < p_2^Y$ and $p_2^Y > s_2$. For tipping to Platform X, it must also be that consumers prefer to join Platform X exclusively when all active firms join Platform X exclusively. If all active firms join Platform X exclusively, then $u_1^X(\theta_1) = v + \alpha_1(\theta_1)N_2 - p_1^X > (1+\delta)v + \alpha_1(\theta_1)N_2 - p_1^X - p_1^Y = u_1^M(\theta_1)$ since $p_1^Y > s_1$ and active consumers prefer to join Platform X exclusively over multi-homing. For all consumers to prefer Platform X over Platform Y, it must be that $u_1^X(\theta_1) = v + \alpha_1(\theta_1)N_2 - p_1^X > v - p_1^Y = u_1^Y(\theta_1)$. Note that this holds if $p_1^Y > v$. Alternatively, if $v > p_1^Y$, then we require that $p_1^X \leq p_1^Y + \alpha_1(1)N_2$ holds. Note that if this condition holds for a consumer of type $\theta_1 = 1$, then the condition holds for all consumer types. Thus, if $p_1^X \leq p_1^Y + \alpha_1(1)N_2$ or if $p_1^Y > v$, then tipping to Platform X is an allocation equilibrium.

The second allocation is characterized as follows: Using Equation (2), the marginal consumer who is indifferent between Platforms X and Y is given by

$$\text{(A.1)} \qquad u_1^X(\theta_1^X) = v + \alpha_1(\theta_1^X)n_2^X - p_1^X = v + \alpha_1(\theta_1^X)n_2^Y - p_1^Y = u_1^Y(\theta_1^X),$$

where $\theta_1^X \cdot \overline{N}_1 = n_1^X$. The last consumer to join Platform Y, the θ_1' so that $N_1 = \theta_1' \cdot \overline{N}_1$, is characterized by $u_1^Y(\theta_1') = v + \alpha_1(\theta_1')n_2^Y - p_1^Y = 0$ with $n_1^Y = N_1 - n_1^X$. Firm side participation is given by setting Equations (4) and (5) equal to zero, which yields $n_2^X = \alpha_2^{-1}(p_2^X+c/n_1^X) \cdot \overline{N}_2$ and $n_2^M = n_2^Y = \alpha_2^{-1}(p_2^Y+\sigma\cdot c/n_1^Y) \cdot \overline{N}_2$. For this allocation to be an equilibrium, it must be that (i) $n_2^X > n_2^Y$ and (ii) $n_1^Y > n_1^X$.

Notice that (ii) requires $\theta_1^X < \theta_1'/2$. Let $\overline{\theta}_1 \equiv \theta_1'/2$. For (i), $\alpha_2(\cdot)$ decreasing implies that we require $p_2^X+c/n_1^X < p_2^Y+\sigma\cdot c/n_1^Y$ which occurs if and only if $p_2^X+c/p_2^Y+\sigma\cdot c < n_1^X/n_1^Y$, and since $n_1^Y > n_1^X$, we must have that $p_2^X+c/p_2^Y+\sigma\cdot c < 1$ which implies that we require $p_2^Y - p_2^X > (1-\sigma)c$. This provides the lower bound on the firm price difference. In addition, $n_1^Y = N_1 - n_1^X$ implies that $p_2^X+c/p_2^Y+\sigma\cdot c < n_1^X/n_1^Y$ occurs if and only if $p_2^X+c/p_2^Y+\sigma\cdot c < \theta_1^X/\theta_1'-\theta_1^X$. This occurs when $\theta_1^X > \theta_1' \cdot (p_2^X+c/p_2^X+p_2^Y+(1+\sigma)\cdot c)$. Let $\underline{\theta}_1 \equiv \theta_1' \cdot (p_2^X+c/p_2^X+p_2^Y+(1+\sigma)\cdot c)$ so that (i) and (ii) hold when $p_2^Y - p_2^X > (1-\sigma)c$ and $\theta_1^X \in (\underline{\theta}_1, \overline{\theta}_1)$.[28] Using Equation (A.1), we have that $\theta_1^X \in (\underline{\theta}_1, \overline{\theta}_1)$ if and only if $(p_1^X - p_1^Y) \in (\alpha_1(\overline{\theta}_1) \cdot (n_2^X - n_2^Y), \alpha_1(\underline{\theta}_1) \cdot (n_2^X - n_2^Y))$, which provides the bounds on the consumer price difference.

Finally, for this allocation to be an equilibrium, we must also show that no single-homing consumer wants to multi-home. This follows since $p_1^X, p_1^Y > s_1$ and $n_2^X = N_2$ imply that $u_1^X(\theta_1) > u_1^M(\theta_1)$ for all $\theta_1 \in [0, 1]$.

The third allocation is characterized using Equation (4) so that the marginal firm who is indifferent between X and Y is given by

$$\text{(A.2)} \qquad u_2^Y(\theta_2^Y) = \alpha_2(\theta_2^Y)n_1^Y - c - p_2^Y = \alpha_2(\theta_2^Y)n_1^X - c - p_2^X = u_2^Y(\theta_2^Y),$$

where $\theta_2^Y \cdot \overline{N}_2 = n_2^Y$. The last firm to join Platform X, the θ_2' so that $N_2 = \theta_2' \cdot \overline{N}_2$, is characterized by $u_2(\theta_2') = \alpha_2(\theta_2')n_1^X - c - p_2^X = 0$. This gives $n_2^X = N_2 - n_2^Y$. Consumer side participation is given by setting Equations (2) and (3) equal to zero, which yields $n_1^Y = \alpha_1^{-1}(p_1^Y-v/n_2^Y) \cdot \overline{N}_1$ and $n_1^M = n_1^X = \alpha_1^{-1}(p_1^X-\delta\cdot v/n_2^X) \cdot \overline{N}_1$. For this allocation to be an equilibrium, it must be that (i) $n_1^Y > n_1^X$ and (ii) $n_2^X > n_2^Y$.

Notice that (ii) requires $\theta_2^Y < \theta_2'/2$. Let $\overline{\theta}_2 \equiv \theta_2'/2$. For (i), $\alpha_1(\cdot)$ decreasing implies that we require $p_1^Y-v/n_2^Y < p_1^X-\delta v/n_2^X$ which occurs if and only if $n_2^X/n_2^Y < p_1^X-\delta v/p_1^Y-v$, and since $n_2^X > n_2^Y$ we must have that $p_1^X-\delta v/p_1^Y-v > 1$. This implies that we require $p_1^X - p_1^Y > \delta v - v$ which always holds since $p_1^X > p_1^Y$ and $\delta \leq 1$. Using $n_2^X = N_2 - n_2^Y$ we also have that $n_2^X/n_2^Y < p_1^X-\delta v/p_1^Y-v$ occurs if and only if $\theta_2'-\theta_2^Y/\theta_2^Y < p_1^X-\delta v/p_1^Y-v$. This occurs when $\theta_2^Y > \theta_2' \cdot (p_1^Y-v/p_1^X+p_1^Y-(1+\delta)\cdot v)$. Let $\underline{\theta}_2 \equiv$

[28] The set $(\underline{\theta}_1, \overline{\theta}_1)$ is nonempty when $p_2^Y - p_2^X > (1-\sigma)c$, which is already included as a requirement.

$\theta_2' \cdot ({}^{p_1^Y - v}/_{p_1^X + p_1^Y - (1+\delta)\cdot v})$ so that (i) and (ii) hold when $\theta_2^Y \in (\underline{\theta}_2, \overline{\theta}_2)$.[29] Using Equation (A.2), we have that $\theta_2^Y \in (\underline{\theta}_1, \overline{\theta}_1)$ if and only if $p_2^Y - p_2^X \in [\alpha_2(\overline{\theta}_2)\cdot(n_1^X - n_1^Y), \alpha_2(\underline{\theta}_2)\cdot(n_1^X - n_1^Y)]$, which provides the bounds on the firm price difference.

Finally, for this allocation to be an equilibrium, we must also show that no single-homing firm wants to multi-home. A firm on Platform Y will not multi-home since Platform Y already has all consumers and $p_2^X > s_2$. At the same time, a firm on Platform X does not also join Platform Y when $p_2^Y > w_2 \geq -\sigma c + \alpha_1(0)[n_1^Y - n_1^M]$. Thus, the prescribed allocation is an equilibrium allocation when $p_2^Y > w_2$.

To show that other allocations are not equilibria, first note that if both platforms have participation, then it cannot be that $n_2^Y \geq n_2^X > 0$. This is since $p_1^X > p_1^Y > s_1$ implies that $u_1^Y(\theta_1) > u_1^X(\theta_1)$ for all θ_1 so that all the consumers on Platform X deviate to Platform Y (without multi-homing since $p_1^X > s_1$), resulting in a tipping allocation where all agents join Platform Y so that $n_2^X = 0$, a contradiction. Similarly, it cannot be that $n_1^X \geq n_1^Y > 0$, since $p_2^Y > p_2^X > s_2$ implies that $u_2^X(\theta_2) > u_2^Y(\theta_2)$ for all θ_2 so that all firms on Platform Y deviate to Platform X (without multi-homing since $p_2^Y > s_2$), resulting in a tipping allocation where all agents join Platform X so that $n_1^Y = 0$, a contradiction. Combined, these results imply that a participation equilibrium must have $n_2^X > n_2^Y$ and $n_1^Y > n_1^X$, which implies that the only participation equilibrium are the second and third allocations in the lemma. □

PROOF OF LEMMA 3. First, we find the cases that rule out the asymmetric allocations. If $n_2^X > n_2^Y > 0$ so that the set of firms on Platform Y is a subset of the firms on Platform X, then $u_1^X(\theta_1) = v + \alpha_1(\theta_1)n_2^X - p_1 > v + \alpha_1(\theta_1)n_2^Y - p_1 = u_1^Y(\theta_1)$ for all θ_1. Now, if $p_1 > w_1$, then $u_1^X(\theta_1) = v + \alpha_1(\theta_1)n_2^X - p_1 > v + \alpha_1(\theta_1)n_2^X - p_1 + (w_1 - p_1) = (1+\delta)v + \alpha_1(\theta_1)[n_2^X + {}^{\overline{N}_2}/_2] - 2p_1 > (1+\delta)v + \alpha_1(\theta_1)N_2 - 2p_1 = u_1^M(\theta_1)$ for all θ_1. In this case, all consumers deviate to join Platform X exclusively which implies that $u_2^X(\theta_2) = \alpha_2(\theta_2)n_1^X - c - p_2 > \alpha_2(\theta_2)n_1^Y - c - p_2 = u_2^Y(\theta_2)$. This with $p_2 > s_2$ implies that $u_2^X(\theta_2) = \alpha_2(\theta_2)n_1^X - c - p_2 > \alpha_2(\theta_2)N_1 - (1+\sigma)c - 2p_2 = u_2^M(\theta_2)$ so that all firms join Platform X exclusively. This means that $n_2^Y = 0$, a contradiction (we supposed that $n_2^X > n_2^Y > 0$). Similarly, if $n_1^X > n_1^Y > 0$ so that the set of consumers on Platform Y is a subset of the firms on Platform X, then all firms join Platform X exclusively when $p_2 > w_2$ which implies that all consumers join Platform X exclusively when $p_1 > s_1$, and this implies that $n_1^Y = 0$, a contradiction. Thus, asymmetric allocations are only possible if $p_i \leq w_i$ for some $i = 1$ or 2.

Second, consider the symmetric allocation equilibria. Given symmetric participation across platforms, symmetric prices imply that $u_i^A(\theta_i) = u_i^B(\theta_i)$ for all θ_i for $i = 1, 2$. Also note that symmetry implies that for $i = 1, 2$, if $u_i^M(\theta_i') > u_i^X(\theta_i')$, then $u_i^M(\theta_i) > u_i^X(\theta_i)$ for all $\theta_i < \theta_i'$. Thus, to determine the set of multi-homers, note that an agent multi-homes when (i) $u_i^M(\theta_i) \geq u_i^X(\theta_i)$ and (ii) $u_i^M(\theta_i) \geq 0$. Consider the multi-homing consumers. If condition (i) binds while condition (ii) holds, then Equations (2) and (3) imply that the multi-homing consumers are given by: $\alpha_1^{-1}({}^{p_1 - \delta v}/_{n_2 - n_2^M}) \cdot \overline{N}_1$. Instead, if condition (ii) binds while condition (i) holds, then Equation (2) implies that the multi-homing consumers are given by: $\alpha_1^{-1}({}^{\max\{2p_1 - (1+\delta)v, 0\}}/_{N_2}) \cdot \overline{N}_1$ where the max-operator is necessary for possible corner solutions. Given that both conditions (i) and (ii) must hold, the set of multi-homers is given by the minimum of the two values: Equation (6). The multi-homing firms given by Equation (8) follows in a similar manner. The collection of all agents on Side i is either given by n_i^M, if all agents multi-home on Side i, or by $u_i^X(\theta_i) = 0$. Using Equation (2) this implies that the collection of consumers is given by the maximum of $\alpha_1^{-1}({}^{p_1 - v}/_{n_2}) \cdot \overline{N}_1$ or n_1^M which generates Equation (7). The collection of firms given by Equation (9) follows in a similar manner.

Third, the tipping allocations are equilibria since symmetric prices with all agents on a single platform implies that no participating consumer nor firm will deviate since $p_i > s_i$. □

[29] The set $(\underline{\theta}_2, \overline{\theta}_2)$ is nonempty when $p_1^X - p_1^Y > \delta v - v$ which always holds since $p_1^X > p_1^Y$ and $\delta \leq 1$.

PROOF OF LEMMA 4. If $p_i^Y, p_i^X \leq s_i$ for $i = 1, 2$, then clearly all agents multi-home. If $p_i^Y, p_i^X \leq s_i$, and $p_j^Y \leq p_j^X$ with $p_j^X > s_j$ for $j \neq i$, then $p_i^Y, p_i^X \leq s_i$ implies that all agents on Side i multi-home so that $\overline{N}_i = N_i = n_i^M = n_i^X = n_i^Y$. Note that if $p_i^Y < p_i^X$, then no dis-coordination is satisfied whenever $n_j^X < n_j^Y$, which produces a continuum of possible single-homing equilibrium allocations that must satisfy $n_j^X < n_j^Y$. Instead, if $p_i^Y \geq p_i^X$ and $p_j^Y = p_j^X$, then active Side j agents are indifferent between the two platforms and no Side j agent multi-homes, since $N_i = n_i^M$ and $p_j > s_j$. This implies that $n_j^M = 0$ with $N_j = n_j^X + n_j^Y$ generating a continuum of equilibrium allocations. Finally, if $p_i^Y \geq p_i^X$ and $p_j^Y < p_j^X$, then Side j agents prefer Platform Y so that all active Side j agents single-home to Platform Y ($n_j^Y = N_j > 0 = n_j^X$) and a unique equilibrium allocation is produced. □

PROOF OF PROPOSITION 1. For existence, notice that when $p_1 = p_1^A = p_1^B = f_1 > s_1$, $p_2 = p_2^A = p_2^B = f_2 > s_2$, a platform charging a higher price loses all customers (by Lemma 1) when such a deviation results in the higher priced platform being tipped out of the market. In addition, a platform charging a lower price looses money. Orthogonal price-deviations result in zero profit provided that the market tips to the platform charging marginal cost prices. Consider now the equilibrium allocations:

Allocation 1: If the system of Equations (6)–(9) have a solution when $n_1^M = 0$ and $n_2^M = N_2$, then Allocation 1 is an equilibrium. If $n_1^M = 0$ with $N_1 = 2n_1 + 0$ so that $n_1 = {}^{N_1}\!/_2$ and $\alpha_2(\cdot)$ is decreasing, then Equations (8) and (9) are satisfied with $n_2^M = N_2$ if and only if ${}^{2(p_2+\sigma\cdot c)}\!/_{N_1} < {}^{2p_2+(1+\sigma)c}\!/_{N_1}, {}^{2(p_2+c)}\!/_{N_1}$. Each of these inequalities are satisfied if and only if $\sigma < 1$ and so we have that $n_2^M = N_2$ when $n_1^M = 0$. For the other direction, if $n_2^M = N_2 = n_2$ and $\alpha_1(\cdot)$ is decreasing, then Equation (6) is only satisfied when $n_1^M = 0$ and Equation (7) is satisfied with $N_1 > 0$. Thus, we have that $n_1^M = 0$ and $N_1 > 0$ when $n_2^M = N_2$.

Allocation 2: First note that mixed-homing is not possible when all consumers strictly prefer to single-home instead of multi-home. The parameters that are required for single-homing to strictly dominate multi-homing for all consumers is given by $u_1^X(\theta_1 = 0, n_2^X = 0) > u_1^M(\theta_1 = 0, n_2^X = N_2)$. This occurs when $p_1 > \delta v + \alpha_1(0) \cdot N_2$. Thus, Allocation 2 requires that $p_1 < \delta v + \alpha_1(0) \cdot N_2$.

To prove existence of the mixed-homing equilibrium, the system of Equations (6)–(9) must have a solution when $N_1 > n_1^M > 0$ simultaneously with $N_2 > n_2^M > 0$. First we start by showing how $N_2 > n_2^M > 0$ occurs when $N_1 > n_1^M > 0$. In this case, if $N_1 > n_1^M > 0$ with $N_1 = 2n_1 - n_1^M$ so that $n_1 = {}^{N_1+n_1^M}\!/_2$ and $\alpha_1(\cdot)$ is decreasing, then Equations (8) and (9) are satisfied with $N_2 > n_2^M$ if and only if ${}^{2(p_2+c)}\!/_{N_1+n_1^M} < {}^{2(p_2+\sigma c)}\!/_{N_1-n_1^M}, {}^{2p_2+(1+\sigma)c}\!/_{N_1}$. Each of these inequalities are satisfied if and only if

$$n_1^M > \frac{(1-\sigma)c}{2p_2 + (1+\sigma)c} \cdot N_1 \equiv \overline{n}_1^M.$$

In addition, $\overline{n}_1^M \in (0, N_1)$ when $c > 0$. This implies that for the system of Equations (6)–(9) to have a solution, we must have $n_1^M \in (\overline{n}_1^M, N_1)$.

Now consider how $N_1 > n_1^M > 0$ occurs when $N_2 > n_2^M > 0$. In this case, if $N_2 > n_2^M > 0$ with $N_2 = 2n_2 - n_2^M$ so that $n_2 = {}^{N_2+n_2^M}\!/_2$ and $\alpha_2(\cdot)$ is decreasing, then Equations (6) and (7) are satisfied with $N_1 > n_1^M$ if and only if ${}^{2(p_1-v)}\!/_{N_2+n_2^M} < {}^{2(p_1-\delta v)}\!/_{N_2-n_2^M}, {}^{2p_1-(1+\delta)v}\!/_{N_2}$. Each of these inequalities are satisfied if and only if

$$\text{(A.3)} \qquad -(1-\delta)v \cdot N_2 < [2p_1 - (1+\delta)v] \cdot n_2^M.$$

If $f_1 = p_1 \geq {}^{(1+\delta)v}/_2$, then Equation (A.3) is satisfied for all $n_2^M \geq 0$. Instead, if $f_1 = p_1 < {}^{(1+\delta)v}/_2$, then Equation (A.3) is satisfied when $n_2^M < {}^{-(1-\delta)v}/_{2p_1-(1+\delta)v}$; however, ${}^{-(1-\delta)v}/_{2p_1-(1+\delta)v} > 1$ in this case. This implies that for the system of Equations (6)-(9) to have a solution we must have $n_2^M \in (0, N_2)$.

Altogether, these conditions imply that the system of Equations (6)-(9) have a solution so that mixed-homing allocations are equilibria when $n_1^M \in (\overline{n}_1^M, N_1)$ and $n_2^M \in (0, N_2)$.[30]

Allocation 3: To prove existence of the inverse competitive bottleneck equilibrium, the system of Equations (6)–(9) must have a solution when $n_1^M = N_1$ and $n_2^M = 0$. First note that if $n_1^M = N_1$ and $\alpha_2(\cdot)$ is decreasing, then Equation (8) is only satisfied when $n_2^M = 0$ and Equation (9) is satisfied with $N_2 > 0$. However, in the other direction, if $n_2^M = 0$ and $\alpha_2(\cdot)$ is decreasing, then Equations (6) and (7) are satisfied if $(1+\delta)v + \alpha_1(\theta_1)N_2 - 2f_1 > v + \alpha_1(\theta_1)^{N_2}/_2 - f_1$ which occurs if and only if $\delta v + \alpha_1(\theta_1)({}^{N_2}/_2) > f_1$, but if $f_1 > w_1 = \delta v + \alpha_1(0)({}^{\overline{N}_2}/_2) > \delta v + \alpha_1(\theta_1)({}^{N_2}/_2)$, then this breaks down so that $f_1 \leq w_1$ is necessary for this allocation to exist as an equilibrium.

To conclude, there exists at least one and potentially three non-tipping allocations that occur in equilibrium with $p_1^A = p_1^B = f_1$ and $p_2^A = p_2^B = f_2$, and Lemma 3 implies that the tipping allocations are also equilibria for symmetric marginal cost pricing.

□

PROOF OF PROPOSITION 2. To show what other equilibrium can exist when $f_1 > s_1$, consider allocations under (i) platforms charging prices above marginal costs ($p_i^X \geq p_i^Y \geq f_i$ for $i = 1, 2$, and at least one inequality being strict), (ii) symmetric prices not equal to marginal cost, and (iii) orthogonal prices.

(i) If $p_i^X \geq p_i^Y \geq f_i$ for $i = 1, 2$, with at least one inequality being strict, then Platform X has an incentive to undercut Platform Y and tip the market when $p_i^Y > f_i$ and Platform Y has an incentive to raise its price and continue to tip the market when $p_i^X > p_i^Y$. Tipping in each case follows from Lemma 1. Thus, platforms charging prices above marginal costs ($p_i^X \geq p_i^Y \geq f_i$ for $i = 1, 2$, and at least one inequality being strict) cannot constitute an equilibrium.

(ii) With symmetric prices both below marginal costs, profits are negative and a platform deviates by charging higher prices. Symmetric prices both above marginal costs is ruled out by (i) above. However, note that symmetric prices that straddle their marginal costs ($p_i < f_i$ and $p_j > f_j$) cannot be ruled out as equilibria. In this case, each platform might be made worse off by undercutting. This occurs when $(p_i - f_i) \cdot (n_i^T - n_i) + (p_j - f_j) \cdot (n_j^T - n_j) \equiv \Delta\Pi^T < 0$, where the n_1, n_2 are implicitly given by Equations (6)–(9) from Lemma 3, and the n_1^T, n_2^T are implicitly given by:

$$n_1^T = \alpha_1^{-1}\left(\frac{\max\{p_1 - v, 0\}}{n_2^T}\right) \cdot \overline{N}_1 \text{ and } n_2^T = \alpha_2^{-1}\left(\frac{p_2 + c}{n_1^T}\right) \cdot \overline{N}_2.$$

In this case, the proof for the equilibrium allocations follows the proof of Proposition 1.

(iii) If prices are orthogonal with one platform having both prices above marginal costs and both platforms earning participation ($p_i^X > p_i^Y > f_i$ and $p_j^Y > p_j^X, f_i$ with either the second or third allocations in Lemma 2), then Platform X can deviate to undercutting

[30] Note that the asymmetric allocations in Lemma 3 can also produce mixed-homing allocations that satisfy the derivations provided above. However, existence for an asymmetric mixed-homing allocation also requires that $f_i \leq w_i$ for some $i = 1, 2$ by Lemma 3.

both Y's prices which increases both its markup and its sales on Side j while increasing its sales on Side i but sacrificing its markup on Side i. Thus, orthogonal prices with one platform having both prices above marginal costs and both platforms earning participation is ruled out for prices sufficiently close together (see the proof of Lemma 2 for the explicit condition on price differences). If either platform has both prices below marginal costs while earning participation, then that platform loses money and would deviate.

Finally, consider orthogonal prices that straddle the marginal costs $[(a)\, f_i > p_i^Y > p_i^X > s_i$ with $f_j < p_j^Y < p_j^X$ or $(b)\, p_i^Y > f_i > p_i^X > s_i$ with $p_j^X > f_j > p_j^Y > s_j]$ and earn participation. As in case (ii), even if $p_i^X > s_i$ and $p_j^Y > s_j$ so that price undercutting is successful, platforms will still choose not to undercut when the additional earnings from the profitable side of the market do not outweigh the additional from the subsidized side of the market. For both (a) and (b), this occurs when $(p_i^Y - f_i)\cdot n_i^{TX} - (p_i^X - f_i)\cdot n_i^X + (p_j^Y - f_j)\cdot n_j^{TX} - (p_j^X - f_j)\cdot n_j^X \equiv \Delta\Pi^{TX} < 0$ and $(p_i^X - f_i)\cdot n_i^{TY} - (p_i^Y - f_i)\cdot n_i^Y + (p_j^X - f_j)\cdot n_j^{TY} - (p_j^Y - f_j)\cdot n_j^Y \equiv \Delta\Pi^{TY} < 0$, where $n_i^X, n_j^X, n_i^Y, n_j^Y$ are the orthogonal pricing equilibrium participation levels from allocations two and three in Lemma 2[31] and the n_1^{TZ}, n_2^{TZ} for $Z = X, Y$, are implicitly given by:

$$n_1^{TZ} = \alpha_1^{-1}\left(\frac{\max\{p_1 - v, 0\}}{n_2^{TZ}}\right)\cdot \overline{N}_1 \text{ and } n_2^{TZ} = \alpha_2^{-1}\left(\frac{p_2 + c}{n_1^{TZ}}\right)\cdot \overline{N}_2.$$

In this case, the proof for the equilibrium allocations follows the proof of Proposition 1. □

Deriving the Results in Example 1: First note that $f_i > s_i$ for $i = 1, 2$, since $f_1 = 0.21 > 0.1 = v \geq \delta v = s_1$ and $f_2 \geq 0 > -(0.5)(0.1) = -\sigma c = s_2$, so that price undercutting results in tipping by Lemma 1, Allocation 1 is an equilibrium with symmetric prices by Lemma 3, and straddle pricing equilibria are possible by Proposition 2. The Allocation 1 equilibrium is given by solving the following system of equations:

$$0 = v + (1 - N_1)N_2 - p_1 = 0.1 + (1 - N_1)N_2 - 0.2,$$
$$0 = (1 - N_2)N_1 - (1 + \sigma)c - 2p_2 = (1 - N_2)N_1 - (1 + 0.5)0.1 - 2(0.05),$$

where $n_1 = {}^{N_1}\!/_2$ and $N_2 = n_2 = n_2^M$. Solving the system of equations for $N_i \in [0, 1]$ implies that $n_1 = {}^{N_1}\!/_2 \approx 0.43$ and $n_2 = N_2 \approx 0.71$. Similarly, the tipping allocation is given by solving the following system of equations:

$$0 = v + \left(1 - n_1^T\right)n_2^T - p_1 = 0.1 + \left(1 - n_1^T\right)n_2^T - 0.2,$$
$$0 = \left(1 - n_2^T\right)n_1^T - c - p_2 = \left(1 - n_2^T\right)n_1^T - 0.1 - 0.05.$$

Solving the system of equations for $n_i^T \in [0, 1]$ implies that $n_1^T \approx 0.88$ and $n_2^T \approx 0.83$. Finally, the difference in profit is given by,

$$\Delta\Pi^T = (p_1 - f_1)\left(n_1^T - n_1^{AI}\right) + (p_2 - f_2)\left(n_2^T - n_2^{AI}\right) \approx 0.45(-0.01) + 0.12\cdot(0.05 - f_2).$$

□

[31] As shown in the proof of Lemma 2, the second allocation is implicitly given by Equation (A.1), $n_2^X = \alpha_2^{-1}({}^{p_2^X + c}\!/_{n_1^X})\cdot \overline{N}_2$, and $n_2^M = n_2^Y = \alpha_2^{-1}({}^{p_2^Y + \sigma\cdot c}\!/_{n_1^Y})\cdot \overline{N}_2$; and the third allocation is implicitly given by Equation (A.2), $n_1^Y = \alpha_1^{-1}({}^{p_1^Y - v}\!/_{n_2^Y})\cdot \overline{N}_1$, and $n_1^M = n_1^X = \alpha_1^{-1}({}^{p_1^X - \delta\cdot v}\!/_{n_2^X})\cdot \overline{N}_1$.

PROOF OF PROPOSITION 3. Given these prices, platform profits are $\Pi^A = \Pi^B = \overline{N}_1(s_1 - f_1) \geq 0$. Furthermore, the possible allocations follow from Lemma 4: all consumers multi-home since $p_1 = s_1$ which implies that all active firms single-home. For existence of the equilibrium prices, note that any deviation that reduces the price to consumers results in a lower price offered for the same number of sales and any deviation that reduces the price to firms generates greater losses since $n_2^A, n_2^B > 0$. Similarly, a price deviation to a higher consumer price or a higher firm price results in zero profit when participation tips to the lower priced platform. Thus, these prices constitute an equilibrium in the pricing game. □

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