Mathematics for Economists

Important Proofs and Theorems

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Outline

Logical Implication

Theorem (Transitivity of Implication)

For statements P, Q, and R, the following is a tautology:

$$((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$$

Proof.

Using Boolean algebra:

$$\begin{aligned}
&((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R) \\
&\equiv ((\neg P \vee Q) \wedge (\neg Q \vee R)) \Rightarrow (\neg P \vee R) \\
&\equiv \neg[(\neg P \vee Q) \wedge (\neg Q \vee R)] \vee (\neg P \vee R) \\
&\equiv (P \wedge \neg Q) \vee (Q \wedge \neg R) \vee \neg P \vee R \\
&\equiv \neg P \vee R \vee (P \wedge \neg Q) \vee (Q \wedge \neg R) \\
&\equiv (\neg P \vee (P \wedge \neg Q)) \vee (R \vee (Q \wedge \neg R)) \\
&\equiv (\neg P \vee \neg Q) \vee (R \vee Q)
\end{aligned}$$

Mean Value Theorem

Theorem (Mean Value Theorem)

Let f be continuous on $[a, b]$ and differentiable on (a, b). Then there exists $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Eigenvalues and Eigenvectors

Definition (Eigenvalue and Eigenvector)

Let A be an $n \times n$ matrix. A scalar λ is an eigenvalue of A if there exists a nonzero vector $\mathbf{v}$ such that

$$A\mathbf{v} = \lambda\mathbf{v}.$$

Example

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Then $\lambda = 3$ is an eigenvalue with eigenvector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Summary

- Covered key proofs in logic, calculus, and linear algebra.
- Used Boolean algebra for logical tautologies.
- Presented fundamental theorems with examples.