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ORIGINAL ARTICLE

Unequal connections

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Abstract Empirical work suggests that social and economic networks are characterized by an unequal distribution of connections across individuals. This paper explores the circumstances under which networks will or will not exhibit inequality. Two specific models of network formation are explored. The first is a playing the field game in which the aggregate payoffs of an individual depend only on the number of his links and the aggregate number of links of the rest of the population. The second is a local spillovers game in which the aggregate payoffs of an individual depend on the distribution of links of all players and the identity of neighbors. For both class of games we develop results on existence and characterize equilibrium networks under different combinations of externalities/spillovers. We also examine conditions under which having more connections implies a higher payoff.

1 Introduction

Connections seem to matter both at the individual level as well as in the aggregate. For example, it has been argued that better connected managers are higher achievers in organizations and that well-connected workers earn higher wages, firms use collaborations to gain competitive advantage vis-a-vis competitors,

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social connections shape returns from individual investments in human capital, and countries which are better linked may exploit their ties to bargain for better terms in particular contexts.[1] These arguments become specially relevant in view of recent empirical findings that in many social and economic networks, the distribution of connections is very unequal across individuals.[2]

Given the substantial advantages that accrue from having connections, it seems natural that individuals will invest in forming links with others. This leads us to examine the circumstances under which unequal distribution of connections will arise when rational individuals form links with each other. We are also interested in understanding the implications of such inequalities in connections for the utility of the individuals involved.

We develop a strategic model of network formation to study these issues. Strategic considerations become relevant because a link between two persons A and B has effects on returns from other links of A and B as well as on links between other individuals. Thus spillovers/externalities created by links are central to our study and we are specially interested in understanding the relationship between different types of externalities and the inequality in links in equilibrium networks.

The formulation of spillovers in a network context is complicated by the fact that indirect effects of links can travel across networks in a variety of different ways. We tackle this problem as follows: we focus on two statistics of the network, the number of links of the players who are forming a link and the total number of links of the rest of the players. This leads us then to define two classes of games. In the first set of games, the aggregate payoffs of an individual depend only on the number of his links and the aggregate number of links of the rest of the population. We refer to these games as *playing the field* games. We present two economic applications – Cournot firms forming cost-reducing collaboration links and firms sharing knowledge in a patent race – that satisfy these payoff conditions. In a playing the field game, the marginal returns to player i from a link with player j depend on the total number of links of all other players and are not sensitive to the number of links of j. We next examine games in which the aggregate payoffs of an individual depend on the distribution of the number of links of all the players and the identity of neighbors; however, marginal returns of player i from a link with player j depend only the number of links of i and j. We refer to these games as *local spillover* games. A variety of economic applications – examples include the production of a public good, market sharing agreements between firms not to enter each other's home market, and

[1] There is a large body of theoretical and empirical work on this subject; see e.g., Burt (1992) on careers of professional managers, Montgomery (1991) on wage inequality in labour markets, Delapierre and Mytelka (1998) and Hagedoorn and Schakenraad (1990) on collaborations among firms, Durlauf (2000) on the memberships approach to inequality, and Siedman (2001) on international trading ties. There is an extensive literature on sociology in issues of power and inequality in social networks; see e.g., Wasserman and Faust (1994).

[2] Barabasi (2001), Goyal et al. (2006).

free trade agreements between countries to eliminate tariffs – satisfy the local spillovers property.

We start with a study of playing the field games. We establish existence and provide a characterization of equilibrium networks for the different combinations of spillovers (Propositions 3.1–3.3). In case where aggregate payoffs are convex in own links, we show that an equilibrium network is either empty, complete, or has the dominant group architecture.[3] We then analyze games where aggregate payoffs are concave with respect to own links. In this setting the role of strategic complementarity versus substitutability of own links and the links of others emerges quite clearly. If strategic complementarity obtains, then equilibrium symmetric networks always exist and asymmetric networks with sharp inequality in the number of links, such as the star, cannot arise in equilibrium. In contrast, if strategic substitutability holds, then equilibrium symmetric networks may not exist and asymmetric networks such as the star can emerge in an equilibrium.

We then turn to an examination of local spillovers games. We prove existence of equilibrium networks for the different combinations of spillovers and our results provide a characterization of equilibrium network architectures (Propositions 4.1–4.4).[4] In case of convexity in own links and strategic complementarity with respect to the partner's links, the equilibrium network is either empty, complete, a dominant group network or an interlinked star.[5] We were unable to obtain a general characterization result in the two mixed spillovers cases. However, under a stronger monotonicity condition on payoffs (which essentially requires that the positive spillovers dominate the negative spillovers) we obtain the following characterization: an equilibrium network is either empty, complete, or has the exclusive groups architecture (in which there are distinct groups of completely connected players and some isolated players). Our final result covers games where payoffs are concave in own links and strategic substitutability obtains between own and partner's links. Here we find that symmetric networks as well as sharply asymmetric networks such as the star can arise in equilibrium.

We turn next to the payoff implications of unequal connections. Here we ask: do players with more links earn higher payoffs as compared to those with fewer links? We find that in playing the field games, this is always the case if the aggregate payoff of a player is decreasing in the number of links of the other players. Applications such as cost-reducing collaborations in oligopoly, and research collaboration in patent races satisfy this negative externality condition. A similar

[3] A dominant group architecture has two groups of players, one group which is mutually completely linked and a second group which consists of isolated players.

[4] We prove existence in three of the four possible cases; we have been unable to obtain an existence result in one of the mixed spillovers case, where aggregate payoffs are convex in own links while own links and the partner's links are strategic substitutes.

[5] An interlinked star network has a maximally connected group and a minimally connected group of players. In addition, the maximally connected players are connected to all players while the minimally connected group has links only with the players in the maximally connected set.

result also obtains under local spillovers. We show that international free trade agreements and friendship networks exhibit such negative spillovers. However, if the aggregate payoff of a player is increasing in the number of links of the other players, then payoff monotonicity may not obtain. In the public goods example (see section 4 below) members of a dominant group with many links earn less than isolated players who have no links. This is because players with no links can free-ride on the link formation activity of those in the dominant group and earn higher payoffs by avoiding the direct costs of link formation.

A characterization result in our paper has the following form: a network formation game which satisfies a particular set of spillover conditions will generate an equilibrium network within a particular set of architectures. This leaves open the issue of tightness: can all the network architectures identified arise in a particular application or for the same set of parameters in a given application? Our computations with different applications suggest that the characterization results are tight in the following sense: the architectural forms identified in a characterization result are all sustainable within a specific application that satisfies the hypotheses of the result. In case there is more than one architecture identified by the characterization result, the different forms typically arise in equilibrium for different costs of forming links.

Our paper is a contribution to the theory of network formation. This is currently a very active field of research; see e.g., Albert et al. (2000), Bala and Goyal (2000), Boorman (1975), Calvo (2002), Dutta et al. (1995), Kranton and Minehart (2001), Jackson and Watts (2002), Jackson and Wolinsky (1996), and Slikker and van den Nouweland (2001).[6] The present paper makes three contributions: the *first* contribution is a simple model of networks in which positive and negative spillovers can be modeled in a straightforward manner. We also illustrate how different models in the literature fit into this general taxonomy. The *second* contribution is a characterization of equilibrium network architectures, both symmetric as well as asymmetric networks, under these combinations of spillovers. The *third* contribution is to identify the conditions under which players with more links in an asymmetric network earn higher payoffs than those with fewer links as well provide an example when this payoff monotonicity may not obtain.

The paper is organized as follows. Section 2 presents the model. Section 3 studies playing the field games. Section 4 analyzes games of local spillovers. Section 5 concludes. All the proofs are given in the Appendix at the end of the paper.

[6] Traditionally, group formation has been studied with the help of coalition formation models; see Bloch (1997) for a survey of this literature, and Bloch (1996), Ray and Vohra (1997), and Yi (1997) for related papers on coalition formation with externalities. For a discussion on the relationship between networks and coalitions, refer to Jackson and Wolinsky (1996). In the coalition formation literature also there has been some interest in the relation between nature of spillovers and equilibrium coalitions structures (see e.g., Yi 1997). Our paper is similar in motivation and studies the relation between spillovers and equilibrium networks; it should be seen as being complementary to the work on coalition formation.

2 The model

Link formation game and networks Let $N = \{1,2,\ldots,n\}$ denote a finite set of ex-ante identical players. We shall assume that $n \geq 3$. Every player makes an announcement of intended links. An intended link $s_{i,j} \in \{0,1\}$, where $s_{i,j} = 1$ means that player i intends to form a link with player j, while $s_{i,j} = 0$ means that player i does not intend to form such a link. Thus a strategy of player i is given by $s_i = \{\{s_{i,j}\}_{j\in N\setminus\{i\}}\}$. Let S_i denote the strategy set of player i. A link between two players i and j is formed if and only if $s_{i,j} = s_{j,i} = 1$. We denote the formed link by $g_{i,j} = 1$ and the absence of a link by $g_{i,j} = 0$. A strategy profile $s = \{s_1,s_2,\ldots,s_n\}$ therefore induces a network $g(s)$. For expositional simplicity we shall often omit the dependence of the network on the underlying strategy profile. A *network* $g = \{(g_{i,j})\}$ is a formal description of the pair-wise links that exist between the players. We let $\mathcal{G}$ denote the set of all networks (the set of all undirected networks with n vertices). We also let $N_i(g) = \{j \in N : j \neq i, g_{i,j} = 1\}$ be the set of players with whom player i has a link in the network g, and let $\eta_i(g) = |N_i(g)|$ denote the cardinality of this set. We will occasionally refer to $\eta_i(g)$ as the *degree* of player i in network g.[7] We shall define g_{-i} as the network obtained by deleting player i and all his links from the network g and $L(g_{-i}) = \sum_{j\neq i}\eta_j(g_{-i})$ as the total number of links in g_{-i}.

Given a network g, $g+g_{i,j}$ denotes the network obtained by replacing $g_{i,j} = 0$ in network g by $g_{i,j} = 1$, while $g-g_{i,j}$ denotes the network obtained by replacing $g_{i,j} = 1$ in network g by $g_{i,j} = 0$. There exists a *path* between i and j in a network g if either $g_{i,j} = 1$ or if there is a distinct set of players $\{i_1,\ldots,i_n\}$ such that $g_{i,i_1} = g_{i_1,i_2} = g_{i_2,i_3} = \cdots = g_{i_n,j} = 1$. A network is *connected* if there exists a path between any pair $i,j \in N$. A network, $g' \subset g$, is a *component* of g if for all $i,j \in g'$, $i \neq j$, there exists a path in g' connecting i and j, and for all $i \in g'$ and $k \in g$, $g_{i,k} = 1$ implies $k \in g'$. A component $g' \subset g$ is *complete* if $g_{i,j} = 1$ for all $i,j \in g'$.

We shall say that a network is *symmetric* if every player has the same number of links, i.e., $\eta_i(g) = \eta\ \forall i \in N$. We refer to η as the *degree* of the network. It should be noted that if the number of players is even, then a symmetric graph of every degree is possible; this is not true when the number of players is odd. In some parts of the analysis, we explore symmetric networks then we will be (implicitly) assuming that the number of players in the game is even.[8] The *complete* network, g^c, is a symmetric network in which $\eta = n-1$, $\forall i \in N$, while the *empty* network, g^e, is a symmetric network in which $\eta = 0$, $\forall i \in N$. We shall say that a network is asymmetric if there is at least one pair of players who have different number of links.

[7] See Myerson (1991), and Dutta et al. (1995) for an earlier model of link announcements.

[8] Note for example that we cannot construct symmetric networks of degree 1 or 3 when $n = 5$. The fact that we cannot physically construct such symmetric networks has no relation to the substantive issue of existence or non-existence of such networks due to strategic considerations. By implicitly considering even n, we can focus on cases where symmetric networks are ruled out due to strategic behavior on the part of individuals rather than as an artifact of the physical construction of networks of a particular degree.

Let $\mathbf{N}_1(g), \mathbf{N}_2(g), \ldots, \mathbf{N}_m(g)$ be a partition of players corresponding to the number of links that players have, i.e., $i, j \in \mathbf{N}_k(g)$, $k = 1, 2, \ldots, m$, if and only if $\eta_i(g) = \eta_j(g)$. We note that k here refers to the order in the partition and not the precise number of links that players have. An *inter-linked stars* architecture has at least two members in the above partition, and the maximally and minimally linked groups satisfy the following two conditions: (i) $\eta_i(g) = n - 1$ for $i \in \mathbf{N}_m(g)$ and (ii) $N_i(g) = \mathbf{N}_m(g)$ for $i \in \mathbf{N}_1(g)$. The star network is a special case of such an architecture with $|\mathbf{N}_m(g)| = 1$ and $|\mathbf{N}_1(g)| = n - 1$. An *exclusive groups* architecture is characterized by $m + 1$ groups, a group of isolated players $D_1(g)$ and $m \geq 1$ distinct groups of completely connected players, $D_2(g), \ldots, D_{m+1}(g)$. Thus $\eta_i(g) = 0$, for $i \in D_1(g)$, while $\eta_j(g) = |D_x(g)| - 1$, for $j \in D_x(g)$, $x \in \{2, 3, \ldots, m + 1\}$. A special case of this architecture is the *dominant group* network in which there is one complete component with $1 < k < n$ players while $n - k > 0$ players are isolated.

Pairwise equilibrium network Given a strategy profile $s = \{s_1, s_2, \ldots, s_n\}$, the (net) payoffs to player i are given by:

$$\Pi_i(s_i, s_{-i}) = \pi_i(g(s)) - C(\eta_i(g(s))) \tag{1}$$

where $\pi_i(\cdot)$ denotes the gross payoff function and $C(\cdot)$ is the total cost on links incurred by player i.[9] A strategy profile $s^* = \{s_1^*, s_2^*, \ldots, s_n^*\}$ is said to be a Nash equilibrium if $\Pi_i(s_i^*, s_{-i}^*) \geq \Pi_i(s_i, s_{-i}^*)$, $\forall s_i \in S_i$, $\forall i \in N$. In our model a link requires that both players acquiesce in the formation of the link. It is then easy to see that an empty network is always a Nash equilibrium. More generally, for any pair i and j, it is always a mutual best response for the players to offer to form no link. To avoid this potential coordination problem we supplement the idea of Nash equilibrium with the requirement of pairwise stability (which is taken from Jackson and Wolinsky 1996). An equilibrium network is said to be pairwise stable if any pair of players have no incentive to form a link that does not exist in the network. This allows us to state the following definition of pairwise equilibrium, where c denotes the constant cost to each player of forming a link.

Definition 2.1 *A network g is a pairwise equilibrium network if the following conditions hold:*

1. *There is a Nash equilibrium strategy profile which supports g.*
2. *For $g_{i,j} = 0$, $\pi_i(g + g_{i,j}) - \pi_i(g) > c \implies \pi_j(g + g_{i,j}) - \pi_j(g) < c$.*

The notion of pairwise stability is clearly very mild. There are two main reasons for using this solution concept. The first is tractability and the second

[9] In this paper we assume that costs of link formation are constant across all links and focus on effects of network structure on the gross returns function. Clearly, an analogous analysis can be carried out if we assume that returns from links are constant while the costs of links vary as a function of the network.

is that it yields sharp restrictions on the set of permissible networks.[10] In our analysis we shall focus on the architecture of pairwise equilibrium networks. Two networks g and g' are said to have the same architecture if one network can be obtained from the other by a permutation of the players' labels.

3 Playing the field games

In this section we consider network formation games in which the aggregate gross returns from links for every player can be expressed in terms of the number of links of the player and the aggregate number of links of the rest of the players. Recall that g_{-i} is the network obtained by deleting player i and all his links from the network g and $L(g_{-i}) = \sum_{j \neq i} \eta_j(g_{-i})$. Given a strategy profile $s = \{s_1, s_2, \ldots, s_n\}$, the gross payoffs to a player i are given by:

$$\pi_i(g(s_i, s_{-i})) = \Phi(\eta_i(g), L(g_{-i})). \tag{2}$$

Throughout this section we will assume that for all $L(g_{-i})$, $\Phi(k, L(g_{-i}))$ is strictly increasing in own links k. Links are costly to form and the total cost is linearly increasing in the number of links. This allows us to write the net payoffs as follows:

$$\Pi_i(g(s_i, s_{-i})) = \Phi(\eta_i(g), L(g_{-i})) - \eta_i(g)c. \tag{3}$$

Broadly speaking, two types of externality effects arise in this context: an externality across links of the same player, and two, externality effects across links of different players. The analysis will focus on the case where these externality effects are either positive or negative. This motivates the following definitions.

Definition 3.1 *The payoff function Φ is convex (concave) in own links if the marginal returns, $\Phi(k+1, L) - \Phi(k, L)$, are strictly increasing (decreasing) in k.*

The next definition captures externality effects across players.

Definition 3.2 *Suppose $L' > L$. The payoff function satisfies the strategic substitutes property if $\Phi(k+1, L') - \Phi(k, L') < \Phi(k+1, L) - \Phi(k, L)$, while it satisfies the strategic complements property if $\Phi(k+1, L') - \Phi(k, L') > \Phi(k+1, L) - \Phi(k, L)$.*

The following two examples illustrate the scope of these games.

[10] We note that the pairwise stability requirement is different from the refinement of perfection. The latter rules out Nash equilibrium in weakly dominated strategies; the requirement of pairwise stability is a local condition and so in settings with negative spillovers, there can exist perfect equilibrium networks which are not pairwise stable.

Example 3.1 (Cost-reducing collaboration in oligopoly)[11] Consider a homogeneous product Cournot oligopoly consisting of n ex-ante symmetric firms who face the linear inverse demand: $p = \alpha - \sum_{i \in N} q_i, \alpha > 0$. The firms initially have zero fixed costs and identical constant returns-to-scale cost functions. Bilateral collaborations lower marginal costs, $C_i(g) = \gamma_0 - \gamma \eta_i(g), i \in N$, where γ_0 is a positive parameter representing a firm's marginal cost if it has no links. Given any network g, the Cournot equilibrium output can be written as:[12]

$$q_i(g) \;=\; \frac{(\alpha - \gamma_0) + (n-1)\gamma \eta_i(g) - \gamma L(g_{-i})}{(n+1)}, \quad i \in N. \tag{4}$$

The Cournot profits for firm $i \in N$ are given by $\pi_i(g) = q_i^2(g)$. The marginal gross returns to player i from an additional link with player j are:

$$\pi_i(g + g_{i,j}) - \pi_i(g) = \frac{(n-1)\gamma}{(n+1)^2}\left[\lambda(n) + 2(n-1)\gamma \eta_i(g) - 2\gamma L(g_{-i})\right] \tag{5}$$

where $\lambda(n) = 2(\alpha - \gamma_0) + (n-1)\gamma$. This shows cost-reducing collaboration is a playing the field game. Moreover, it can be checked that payoffs are increasing and convex in own links but decreasing in links of others. Moreover the strategic substitutes property holds.

Example 3.2 (Patent races)[13] Consider n firms who are racing to innovate a new product. The firm which wins the race is awarded a patent of value 1; the loser earns 0. All firms use the same discount rate ρ. Suppose that firms are endowed with one unit of R&D capability. Firms can speed their R&D by sharing their R&D capability. Let $\tau(\eta_i(g))$ denote the random time at which firm i innovates in a network g. We assume that τ has an exponential distribution:

$$\Pr\{\tau(\eta_i(g)) \le t\} \;=\; 1 - \mathrm{e}^{-\eta_i(g)t}. \tag{6}$$

Thus as firm i establishes more links it increases the probability of innovating successfully before time t. In addition to this technological uncertainty, there is also market uncertainty: any of the rival $n-1$ firms may successfully innovate before firm i. Assuming that the distribution of the time of innovation is stochastically independent across the firms, the expected gross payoff to a firm i in a network g is:

$$\pi_i(g) = \int_0^\infty \mathrm{e}^{-\rho t} \eta_i(g)\, \mathrm{e}^{-tL(g)} \mathrm{d}t \;=\; \frac{\eta_i(g)}{\rho + 2\eta_i(g) + L(g_{-i})}. \tag{7}$$

[11] This model is taken from Goyal and Joshi (2003).

[12] In order to ensure that each firm produces a strictly positive quantity in equilibrium, we will assume that $(\alpha - \gamma_0) - (n-1)(n-2)\gamma > 0$.

[13] We present a variation on the classical patent race model (see e.g., Dasgupta and Stiglitz 1980).

The marginal gross payoffs to firm i from an additional link are given by:

$$\pi_i(g + g_{i,j}) - \pi_i(g) = \frac{\rho + L(g_{-i})}{[\rho + 2\eta_i(g) + L(g_{-i})][\rho + 2\eta_i(g) + L(g_{-i}) + 2]}. \quad (8)$$

Collaborating in a patent race is thus a playing the field game. Payoffs are increasing and concave in own links. They also exhibit the strategic substitutes property (for suitably large ρ).

The first result provides a partial characterization of pairwise equilibrium architectures in games in which individual payoffs are convex in own links.

Proposition 3.1 *Suppose the payoffs of each player satisfy* (3) *and convexity in own links. A pairwise equilibrium network is empty, complete or has the dominant group architecture. There exists a pairwise equilibrium if payoffs satisfy either strategic complementarity or substitutability.*

In a game with five players there are 34 distinct network architectures [for e.g. see Harary (1972)]. However, only five architectures – which can be parameterized in terms of the size of the dominant group – can arise in equilibrium networks. Figure 1 presents these architectures.

The following transitivity property is central to the proof: in a pairwise equilibrium network g, if players i and j have any links then they must also be linked with each other. Since g is a pairwise equilibrium network the marginal return from the last link for both i and j must exceed the costs. It now follows from convexity in own links, that the marginal returns of linking with each other are strictly greater while the marginal costs are constant. Hence both players have a strict incentive to link with each other. This transitivity property immediately implies that every component in a pairwise equilibrium network must be complete. It also implies that there can be at most one non-singleton component

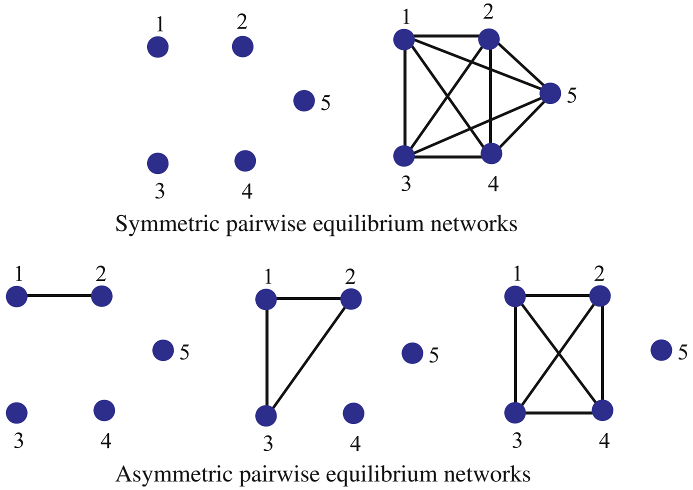

Fig. 1 Pairwise equilibrium networks in playing the field models: Payoffs are convex in own links

in a pairwise equilibrium network. Thus there are only three candidates for pairwise equilibrium networks: the empty network, the complete network and the dominant group network.

The networks in Fig. 1 exhibit very different levels of inequality in degrees, and this raises the question: what determines the relative likelihood of different networks? It turns out that the answer to this question is related to the nature of externality effects across links of players. If the payoffs exhibit strategic complementarity then players will be encouraged to form links as others form links and so the empty network or the complete network is always a pairwise equilibrium network. On the other hand, if payoffs satisfy the strategic substitutes property then, for a range of link formation costs, only intermediate size dominant groups – which exhibit considerable degree inequality – arise in a pairwise equilibrium network (e.g. Goyal and Joshi 2003).

The next result takes up the case where payoffs are concave in own links.

Proposition 3.2 *Suppose payoffs satisfy* (3)*, concavity in own links, and the strategic complementarity property holds. Then a pairwise equilibrium symmetric network always exists. In an asymmetric pairwise equilibrium network all non-maximal degree players are mutually linked.*

Figure 2 illustrates some pairwise equilibrium networks.

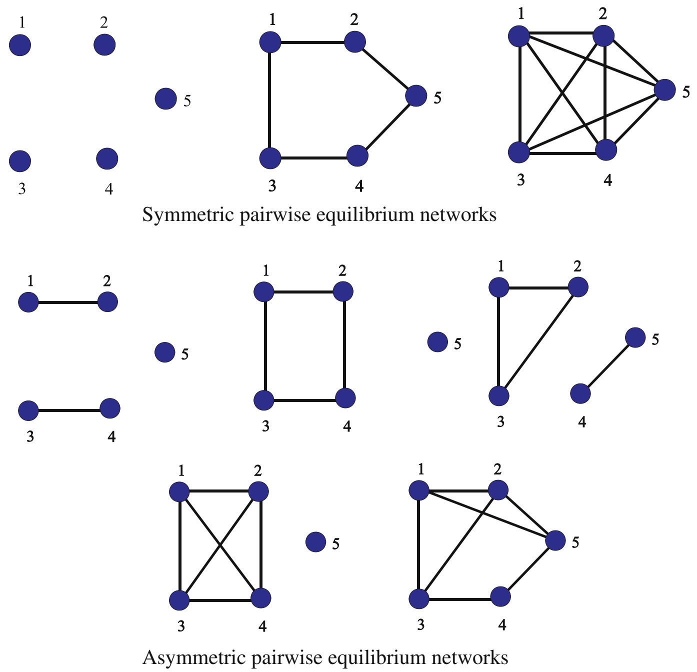

Fig. 2 Pairwise equilibrium networks in playing the field models: Payoffs satisfy concavity in own links and strategic complementarity

The intuition for the result is as follows. Start from the empty network. If it is a pairwise equilibrium network the statement follows. If it is not, then construct an 'improving path' of networks where links are formed by distinct pairs of players which strictly increase payoffs for the concerned players. From strategic complementarity it follows that along such an improving path every player strictly increases utility when others form links. This implies that the improving path will proceed to a symmetric network with a higher degree. Either this degree is a pairwise equilibrium network and the argument is done, or pairs of players have an incentive to form an additional link. In the latter case, repeat the argument and existence of pairwise equilibrium symmetric networks follows from the fact that the set of networks is finite.

Next consider asymmetric networks. Let i be a maximally linked player and let l and k be two non-maximally linked players. It follows from concavity in own links and the strategic complements property that the marginal returns to players l and k from forming a link with each other is strictly larger than the marginal returns to the last link formed by i. The statement follows.

The discussion now turns to the case where payoffs are concave in own links but satisfy strategic substitutability. Let $\phi(x, y) = \Phi(x + 1, y) - \Phi(x, y)$, for the second part of the result. The following result provides a partial characterization of pairwise equilibrium networks for this case.

Proposition 3.3 *Suppose payoffs of every player satisfy* (3)*, concavity in own links and the strategic substitutes property. If* $\phi(k, k(n-2)) < c < \phi(k-1, k(n-2))$ *for* $k \in \{1, 2, 3, \ldots, n-2\}$ *then a symmetric network with a degree k is a pairwise equilibrium network. A star is a pairwise equilibrium network if and only if* $\phi(1, 2(n-2)) < c < \min\{\phi(n-2, 0), \phi(0, 2(n-2))\}$.

The inequality in the first statement says that given $k(n-2)$ links of others, if marginal payoffs to a player from k link are higher than cost of forming links, while the marginal return from the $(k+1)$th link is lower than the costs of link formation, then a symmetric network with degree k is an equilibrium. The expressions in the second statement come out directly from the incentives of the central player and the peripheral players in the star network. Figure 3 illustrates some of the networks that arise with this structure of externality effects.

The patent race game (Example 3.2) can be used to illustrate Proposition 3.3. Simple computations show that with $n = 4$, the empty network is an equilibrium for high costs, the complete network is an equilibrium for low costs, while symmetric networks of a unique degree arise in equilibrium for intermediate values of costs. Moreover, a star network is an equilibrium network for $n = 3$.[14]

[14] Specifically, our computations with $n = 4$ reveal: the empty network is an equilibrium for $c > 1/(\rho+2)$ while the complete network is an equilibrium for $c \leq (\rho+6)/[(\rho+10)(\rho+12)]$. The symmetric network of degree 1 is an equilibrium when costs satisfy $(\rho+2)/[(\rho+4)(\rho+6)] < c < 1/(\rho+4)$ while the symmetric network of degree 2 is an equilibrium when $(\rho+4)/[(\rho+8)(\rho+10)] < c < (\rho+4)/[(\rho+6)(\rho+8)]$. There are also intermediate values of costs for which no symmetric network is an equilibrium: when $1/(\rho+4) < c < 1/(\rho+2)$, the dominant group with two firms is the unique equilibrium and when $(\rho+4)/[(\rho+6)(\rho+8)] < c < (\rho+2)/[(\rho+4)(\rho+6)]$ the dominant group

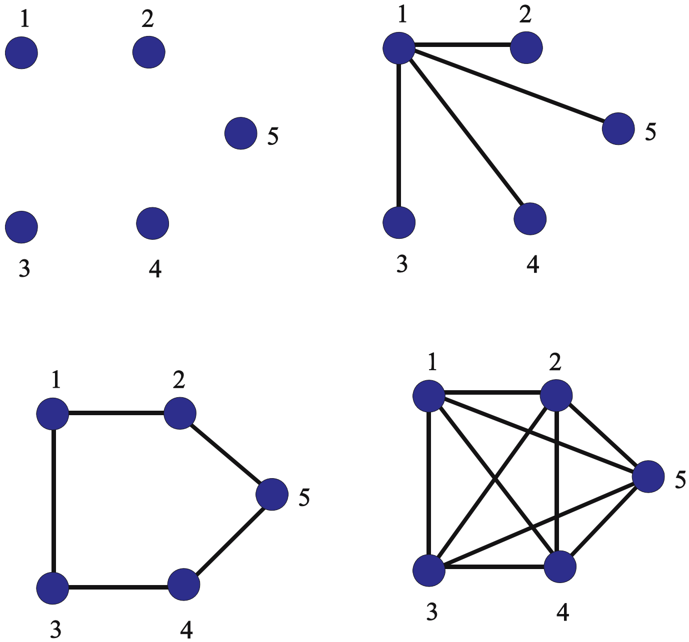

Fig. 3 Some pairwise equilibrium networks in playing the field models: Payoffs satisfy concavity in own links and strategic substitutability

3.1 Payoff distribution

The interest now turns to the payoff implications of degree inequality. In some cases, this relationship can be easily derived. Consider a pairwise equilibrium-dominant group network when payoffs are increasing and convex in own links and the strategic substitutes property holds. Suppose that payoff function $\Phi(k, l)$ is (strictly) decreasing in l. From the equilibrium property, it follows that the payoff of a player in a dominant group with, say, k players, $\Phi(k-1, (k-1)(k-2)) - (k-1)c \geq \Phi(0, (k-1)(k-2))$, the payoff from deleting all links. Since $\Phi(k, l)$ is decreasing in l, it follows that the payoff of the isolated player in the pairwise stable network $\Phi(0, k(k-1)) < \Phi(0, (k-1)(k-2))$ and so the player with many connections earns a higher payoff as compared to the isolated player. Similarly, consider the star pairwise equilibrium network when payoffs are increasing and concave in own links and the strategic substitutes property holds. Suppose again that individual payoffs are falling in links of others. Then it follows that $\Phi(n-1, 0) - (n-1)c \geq \Phi(0, 0) > \Phi(0, 2(n-2))$. The central player, with $n-1$ links earns a higher payoff as compared to the peripheral players, who have one link each.

with three firms and the line network are equilibria, respectively. We also note that the star network is not an equilibrium for $n \geq 4$; however, it is an equilibrium when $n = 3$ if $\rho > 2$ and costs satisfy $(\rho+2)/[(\rho+4)(\rho+6)] < c < \rho/[(\rho+2)(\rho+4)]$.

The following result summarizes these ideas.

Proposition 3.4 *Consider a playing the field game and suppose that $\Phi(\eta_i, L(g_{-i}))$, is strictly decreasing in $L(g_{-i})$. Then in a pairwise equilibrium network g a player with more connections earns a higher payoff, i.e., $\Pi_i(g) > \Pi_j(g)$, if $\eta_i(g) > \eta_j(g)$.*

What can we say about the relation between inequality in connections and payoffs in games with positive externalities. The following examples illustrate some difficulties in obtaining general results for this case.

Example 3.3 Consider a game with $n = 3$ and suppose that the payoffs are as follows: $\Pi_i(g^e) = 0$, for all i, $\Pi_i(g^c) = 10$, for all i. In the one link network, the linked players each get 10, while the isolated player gets 5. In the star network the center gets 9 while the spokes get 12 each. It can be checked that payoffs satisfy concavity in own links, positive externalities, and strategic substitutes properties. The single link network is the unique pairwise equilibrium network, and in this network the players with the link earn a higher payoff as compared to the player with zero links.

The example above shows a positive relation between number of links and payoffs. The following example shows that the reverse pattern is also possible under positive externalities.

Example 3.4 Suppose $n = 3$ and let the payoffs be as follows: $\Pi_i(g^e) = 0$, for all i, $\Pi_i(g^c) = 15$, for all i. In the one link network, the linked players each get 10, while the isolated player gets 8. In the star network the center gets 14 while the spokes get 16 each. It can be checked that payoffs satisfy concavity in own links, positive externalities, and strategic substitutes properties. The star network is the unique pairwise equilibrium network, and in this network the center of the star with two links earns less than the players with a single link each.

These examples suggest that it is difficult to say anything general about the relation between number of links and payoffs in games with positive externalities.

4 The local spillovers games

In this section, we will consider games where aggregate gross payoffs of player i can be written as:

$$\pi_i(g) = \Psi_1(\eta_i(g)) + \sum_{j \in N_i(g)} \Psi_2(\eta_j(g)) + \sum_{j \notin N_i(g)} \Psi_3(\eta_j(g)). \tag{9}$$

Therefore, in contrast to playing the field games, the aggregate payoff of player i depends on the identity of the players both inside and outside the neighbourhood of i. We call any game in which aggregate payoffs satisfy (9) as one of local spillovers. Note that in this class of games, for any $j \notin N_i(g)$:

$$\pi_i\left(g+g_{i,j}\right)-\pi_i(g) = \Psi_1(\eta_i(g)+1)-\Psi_1(\eta_i(g))+[\Psi_2(\eta_j(g)+1)-\Psi_3(\eta_j(g))] \tag{10}$$

In games with local spillovers, the marginal return to player i from a link with player j depends only on the number of links of i and j and is independent of the number of links of $k \neq i, j$. Therefore, the two types of externality effects that arise now operate through the own links of the player and through the links of the potential partner. Once again we will assume that these externalities are either positive or negative. We will say that aggregate payoffs are convex (concave) in own links if $\Psi_1(k+1) - \Psi_1(k)$ is strictly increasing (decreasing) in k. With regard to the links of some potential partner j, we can define:

Definition 4.1 *The payoff function satisfies the strategic substitutes (strategic complements) property if* $\Psi_2(k+2) - \Psi_3(k+1) < (>) \Psi_2(k+1) - \Psi_3(k)$.

We now present some economic applications to illustrate the scope of this class of games.

Example 4.1 (Provision of a pure public good)[15] There are n players, each of whom is deciding on a level of output, x_i, to produce of a pure public good. Given each player's output, the utility of player i is: $u_i(x) = x_i + \sum_{j \neq i} x_j$. A collaboration link between two players is an agreement to share knowledge about the production of the public good. Let $c > 0$ be the fixed investment required from each player in such a link. In any network g the cost of producing output x_i is given by $f_i(x_i; g) = \frac{1}{2}\left(\frac{x_i}{\eta_i(g)+1}\right)^2$. Given a network g, player i will choose output to maximize utility net of production costs. This yields an optimal output of $x_i(g) = (\eta_i(g)+1)^2$. Therefore, the reduced form gross payoff of player i is:

$$\pi_i(g) = \frac{1}{2}(\eta_i(g)+1)^2 + \sum_{j \neq i}(\eta_j(g)+1)^2. \tag{11}$$

The marginal gross returns to person i from an additional link $g_{i,j} = 1$ can be written as:

$$\pi_i(g + g_{i,j}) - \pi_i(g) = \frac{9}{2} + \eta_i(g) + 2\eta_j(g). \tag{12}$$

This is therefore a local spillovers game. It can be checked that aggregate gross payoffs are convex in own links and satisfy the strategic complements property.

Example 4.2 (Market sharing)[16] Consider n ex-ante symmetric firms and associate with each firm i a homogeneous product market i. Before engaging in competition in these markets the firms can form collaboration links. A link

[15] This is an extension to the network setting of a provision of public goods game discussed in Bloch (1997).

[16] This game is borrowed from Belleflamme and Bloch (2005). Their paper also provides several empirical examples of market-sharing agreements.

between i and j is an agreement between the two firms to stay out of each others market. If firm i has $\eta_i(g)$ links in a network g, then there are $n - \eta_i(g)$ active firms in market i who compete as Cournot oligopolists. The Cournot profits earned by a firm that is active in market k is given by $\lambda(n - \eta_k(g))$. Therefore, the gross payoff to firm i in a network g is given by:

$$\pi_i(g) \;=\; \lambda(n - \eta_i(g)) \;+\; \sum_{k \notin N_i(g)} \lambda(n - \eta_k(g)). \tag{13}$$

The marginal gross payoff to i from establishing a link with j is given by:

$$\pi_i(g + g_{i,j}) - \pi_i(g) = \left[\lambda(n - \eta_i(g) - 1) - \lambda(n - \eta_i(g))\right] - \lambda(n - \eta_j(g)). \tag{14}$$

Therefore, this game satisfies the local spillovers property. In particular, if $\lambda(.)$ is decreasing and convex in the number of active firms in a market then the aggregate gross payoffs are convex in own links and satisfy the strategic substitutes property.

Example 4.3 (Free trade agreements among countries)[17] Suppose there are n countries. In each country there is one firm producing a homogeneous good and competing as a Cournot oligopolist in all countries. We let the output of firm j in country i be denoted by Q_i^j. The total output in country i is given by $Q_i = \sum_{j \in N} Q_i^j$. In each country i, a firm faces an identical inverse linear demand given by $P_i \;=\; \alpha - Q_i,\; \alpha > 0$. All firms have a constant and identical marginal cost of production, $\gamma > 0$. We assume that $\alpha > \gamma$. Let the initial pre-agreement import tariff in each country be $T > \alpha$. Countries can form agreements which lower the tariff to 0. It the follows that for a network g, the number of active firms in country i is $\eta_i(g) + 1$. If firm i is active in market j, then its output is given by $Q_j^i = (\alpha - \gamma)/(\eta_j(g_{-i}) + 3)$. The (gross) social welfare of country i is given by:

$$\pi_i(g) \;=\; \frac{1}{2}\left[\frac{(\alpha - \gamma)(\eta_i(g) + 1)}{\eta_i(g) + 2}\right]^2 + \sum_{j \in N_i(g)} \left[\frac{\alpha - \gamma}{\eta_j(g) + 2}\right]^2. \tag{15}$$

The marginal gross return to country i from a free trade agreement with country j is given by:

$$\pi_i(g + g_{i,j}) - \pi_i(g) = (\alpha - \gamma)^2 \left[\frac{(2\eta_i^2(g) + 4\eta_i(g) - 3)}{2(\eta_i(g) + 3)^2(\eta_i(g) + 2)^2} + \frac{1}{(\eta_j(g) + 3)^2}\right] \tag{16}$$

The free trade agreements game satisfies local spillovers and shows concavity in own links for $\eta_i(g) \geq 2$ and the strategic substitutes property for $\eta_j(g) \geq 0$.

17 This game is taken from Goyal and Joshi (2005). Furusawa and Konishi (2002) study a closely related bilateral free trade agreements model with a continuum of differentiated goods.

Example 4.4 (Friendship networks) Consider n individuals who derive utility from social interaction. A link between two individuals is interpreted as a friendship relation. Individuals like more friends and their utility is increasing in the time they are able to spend with each friend. Each person has a fixed amount of total time available, and allocates this time equally among their different friends. Finally, each relationship involves an investment of time and resources. These considerations lead us to write the gross payoff to individual i from a network g as follows:

$$\pi_i(g) = \sqrt{\eta_i(g)} + \sum_{k \in N_i(g)} \frac{1}{\eta_k(g)}. \tag{17}$$

The marginal gross returns to player i from a link with player j are:

$$\pi_i(g + g_{i,j}) - \pi_i(g) = \sqrt{\eta_i(g) + 1} - \sqrt{\eta_i(g)} + \frac{1}{\eta_j(g) + 1}. \tag{18}$$

It follows that the friendship interaction is a local spillovers game exhibiting concavity in own links and the strategic substitutes property.

We start by considering games satisfying convexity with respect to own links and strategic complementarity. Recall that the pure public good game (Example 4.1 above) satisfies these properties. The following result proves existence and provides a partial characterization of pairwise equilibrium networks for this class.

Proposition 4.1 *Suppose the aggregate payoff function satisfies the local spillovers property, convexity with respect to own links, and strategic complementarity. A pairwise equilibrium network g always exists. If g is a symmetric network then it is either empty or complete. If g is asymmetric then it has at most one non-singleton component. This component is either complete or has the interlinked stars architecture.*[18]

Figure 4 illustrates all the equilibrium networks satisfying the conditions of Proposition 4.1.

We would like to note that our characterization result is tight: it can be shown that the different architectures identified – empty, complete, dominant group network and inter-linked stars – can all arise in equilibrium in the public goods game discussed.[19]

[18] This result can actually be strengthened slightly: in a equilibrium only dominant groups of size 3 or more can be sustained, and in an inter-linked star network if $\eta_i(g) \neq \eta_j(g)$, for any two players i and j, then $|\eta_i - \eta_j| \geq 2$. We also note that a network with a unique (non-singleton) complete component has the dominant group architecture.

[19] Consider an inter-linked stars with two groups of players, one maximally connected and the other minimally connected where $m = |N_2(g)|$ and $n - m = |N_1(g)|$. Define $x_m = (n+1)/2 + (m-1)(2n-1)/(n-1) + (n-m)(2m+1)/(n-1)$, $y_m = m/2 + 2n$, and $z_m = 3m + 9/2$. (a) An inter-linked

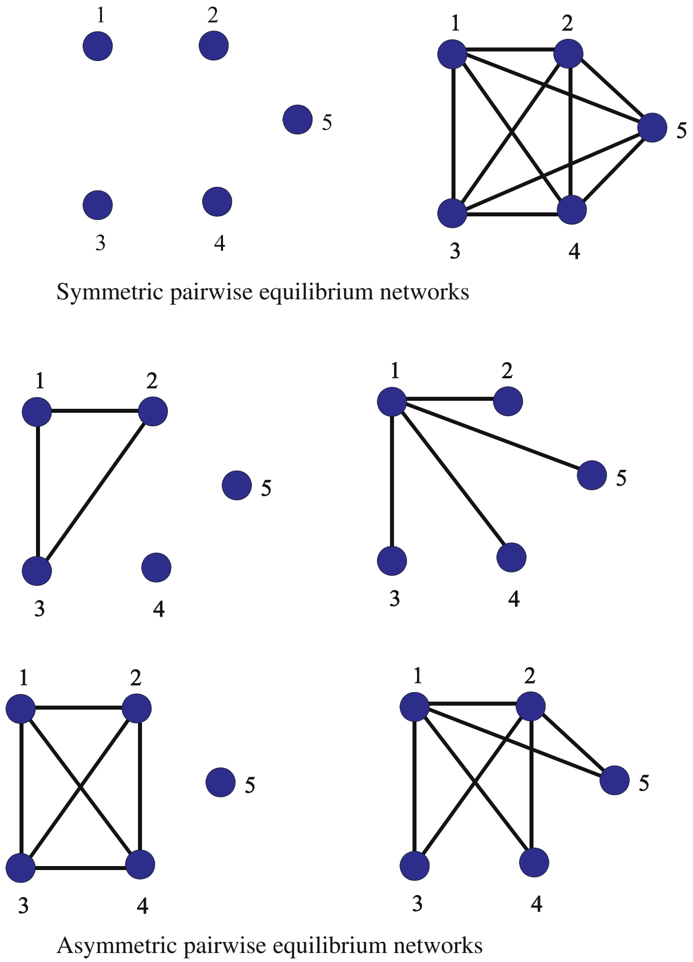

Fig. 4 Pairwise equilibrium networks with local spillovers: Payoffs satisfy convexity in own links and strategic complementarity

We next study the case where aggregate payoffs are convex in own links and satisfy the strategic substitutes property. The market sharing game (Example 4.2 above) satisfies these conditions. We first discuss some of the complications that arise in the analysis. This motivates a stronger monotonicity condition on gross returns and our main result proves existence and provides a complete characterization of pairwise equilibrium networks under this condition.

A wide variety of networks can arise under convexity and strategic substitutability. We illustrate this first for the symmetric networks by considering a mild

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star with $|N_2(g)| = m$, where $m \in \{1, 2, .., n-2\}$, is a equilibrium if and only if $z_m \leq c \leq \min\{x_m, y_m\}$ (b) A dominant-group network of size k, where $k \in \{2, \ldots, n-1\}$ is a equilibrium if and only if $7/2 + k < c < 5k/2 - 1/2$. The complete network is a equilibrium if $c < 5n/2 - 1/2$ while the empty network is a equilibrium if $c > 9/2$.

monotonicity condition on payoffs. In particular, for $\eta \in \{1, 2, \ldots, n-1\}$:

$$\begin{aligned} \Psi_1(\eta+1) - \Psi_1(\eta) + \ & [\Psi_2(\eta+1) - \Psi_3(\eta)] \\ & > \Psi_1(\eta) - \Psi_1(\eta-1) + [\Psi_2(\eta) - \Psi_3(\eta-1)]. \end{aligned} \tag{M}$$

$$\begin{aligned} \Psi_1(\eta+1) - \Psi_1(\eta) + \ & [\Psi_2(\eta+1) - \Psi_3(\eta)] \\ & < \Psi_1(\eta) - \Psi_1(\eta-1) + [\Psi_2(\eta) - \Psi_3(\eta-1)]. \end{aligned} \tag{M$'$}$$

It is easy to see that if (M) holds then a symmetric network of degree k with $0 < k < n-1$ cannot be sustained in equilibrium. Thus if g is a pairwise equilibrium symmetric network then it must be empty or complete. On the other hand, if (M′) holds, then for generic values of c, g has a unique degree. Thus the type of networks that arise in equilibrium are very much influenced by whether (M) or (M′) is satisfied.

What can we say about asymmetric pairwise equilibrium networks? The main structural restriction we can derive is that players with non-maximal number of links must be mutually linked. This restriction rules out networks like interlinked stars. To see why this is true we consider incentives to form links in a star where n is the central player and 1 and 2 are peripheral players. If players 1 and 2, who have zero links respectively, are willing to link with the central player n who has many links then surely they would be willing to link with each other since they now have more links (one link each as compared to zero before) and the other player (1 or 2 as the case may be) has fewer links as compared to the center of the star (since marginal returns satisfy convexity and strategic substitutability).

The main difficulty in the analysis is the combination of convexity in own links and strategic substitutability with the partner's links. Condition (M) imposes restrictions on the relative magnitude of these two effects for symmetric networks, but has limited power in the case of asymmetric networks. We now turn to the market sharing game (Example 4.2 above) to motivate stronger conditions. Belleflamme and Bloch (2005) show that if firm profits are decreasing and log-convex in number of firms then a pairwise equilibrium network consists of a group of isolated firms and distinct complete components. It can be checked that the log-convexity assumption implies that condition (M) is satisfied. This raises the question: can their result be generalized for all payoff functions that satisfy convexity in own links, strategic substitutability and (M). A closer inspection of their analysis reveals that they exploit the following property of payoffs: in an equilibrium network if two players i and j have a link then they must have the same number of links individually as well. Is this property present in general? We now construct an example of a network which satisfies the restrictions implied by convexity, strategic substitutability and (M) but which violates this equal links property. Consider a network with 6 players: players 1–4 are fully

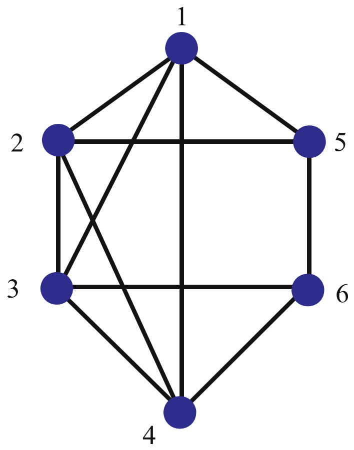

Fig. 5 Pairwise equilibrium network with local spillovers: Payoffs satisfy convexity in own links, strategic substitutability and (M)

linked with each other, player 5 is linked with 1, 2 and 6, while player 6 is linked with 3, 4 and 5, respectively. Figure 5 illustrates this network.

This network satisfies the above restrictions derived for asymmetric pairwise equilibrium networks. In this network, player 5 has an incentive to maintain current links while he does not have an incentive to link with players 3 or 4 since they have more links as compared to players 1, 2 and 6. A similar argument applies to the incentives of player 6. To move toward the characterization of equilibrium networks in terms of complete components, players 5 and 3 should have an incentive to form a link. This can be ensured through the following strong monotonicity (SM) condition that for all $k, l \in \{0, 1, 2, \ldots, n-2\}$:

$$\begin{aligned}\Psi_1(k+1) - \Psi_1(k) + [\Psi_2(l+1) \ &-\Psi_3(l)] \\ &> \Psi_1(k) - \Psi_1(k-1) + [\Psi_2(l) - \Psi_3(l-1)].\end{aligned} \tag{SM}$$

We have been unable to prove that an equilibrium always exists but the following result provides a characterization of equilibrium networks in this setting.

Proposition 4.2 *Suppose the aggregate payoff function satisfies the local spillovers property, convexity with respect to own links, strategic substitutability and* (SM). *A pairwise equilibrium network is either empty or complete or has the exclusive groups architecture (with the non-singleton groups having different sizes).*

We first note that since (SM) is stronger than (M) it follows from the discussion above that the only symmetric networks that can arise in equilibrium are the empty and the complete networks. The additional argument we use in the proof here pertains to the impact of (SM) on asymmetric components. We show that (SM) implies the following restriction: in an equilibrium network g if $g_{i,j} = 1$, then $g_{i,k} = 1$ for all players $k \neq i, j$ such that $\eta_i(g) \leq \eta_k(g) \leq \eta_j(g)$. The role of assumption (SM) can be understood with the help of the above example with six players. We argued that under convexity, strategic complementarity and (M) the above network is an equilibrium. If this is true, then player 5 is willing to form a link with player 1 which implies that $c < \Psi_1(3) - \Psi_1(2) + [\Psi_2(4) - \Psi_3(3)]$. However, if (SM) is satisfied, then $c < \Psi_1(4) - \Psi_1(3) + [\Psi_2(5) - \Psi_3(4)]$ and it follows that player 5 has an incentive to form a link with players 3 and 4 as well; similarly, it can be shown that 3 and 4 have an incentive to form a link with 5. Therefore, the above network is no longer an equilibrium. A general version of this argument shows that every component in a pairwise equilibrium network is complete. We then exploit (SM) to argue that players in distinct equal sized components, and hence with the same number of links, have an incentive to form a mutual link. Thus in equilibrium distinct components must be complete and of unequal size. Figure 6 illustrates pairwise equilibrium networks under the conditions of Proposition 4.2.[20]

We move now to the case where aggregate payoffs are concave with respect to own links and satisfy the property of strategic complementarity. We are able to prove existence of equilibrium in this setting, but the characterization of equilibrium is once again difficult due to the mixture of negative external effects through own links and positive externalities through the partner's links. This motivates us to impose a stronger monotonicity condition as in the previous result. The following result proves existence of pairwise equilibrium symmetric networks and provides a characterization of asymmetric pairwise equilibrium for this class of games.

Proposition 4.3 *Suppose aggregate payoffs satisfies the local spillovers property, concavity with respect to own links and strategic complementarity. Then there always exists a pairwise equilibrium symmetric network. If in addition* (SM) *holds then a pairwise equilibrium is either empty or complete or has the exclusive groups architecture (with the non-singleton groups having different sizes).*

We finally study the case where aggregate gross payoffs are concave in own links and satisfy the strategic substitutes property. The payoffs in the free trade agreements game (Example 4.3 above) and the friendship network game (Example 4.4 above) satisfy these properties. The following result establishes existence and uniqueness of symmetric equilibrium networks in this class of games.

Proposition 4.4 *Suppose that the aggregate payoff function satisfies the local spillovers property, concavity in own links and strategic substitutability. A pairwise equilibrium symmetric network always exists and for generic parameter values the degree of this network is unique.*

An analysis of friendship networks (Example 4.4 above) indicates that this characterization result is tight. For example, when $n = 4$, the empty network is an equilibrium if $c > 1$, a symmetric network of degree 1 is an equilibrium for $0.914 < c < 1$, a symmetric network of degree 2 is an equilibrium for

[20] Here we assume generic values of parameters, and this rules out the dominant group of size 2.

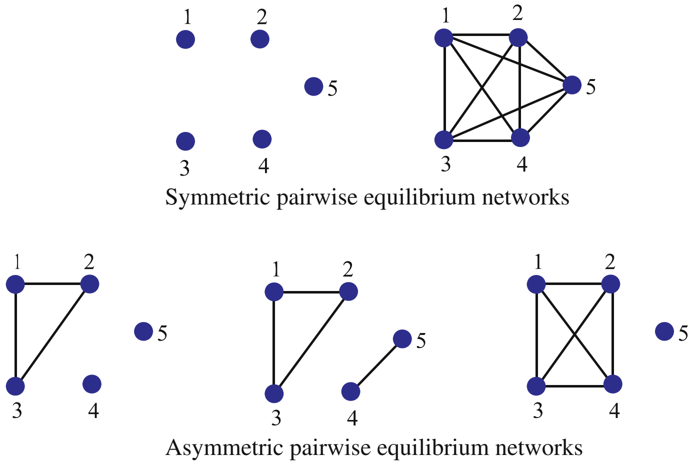

Fig. 6 Pairwise equilibrium network with local spillovers: Payoffs satisfy convexity in own links, strategic substitutability and (SM)

$0.65 < c < 0.914$, and the complete network is an equilibrium for $c \leq 0.65$. And, for generic values of c, a symmetric network of unique degree is an equilibrium.

We have been unable to obtain a complete characterization of asymmetric pairwise equilibrium architectures. However, concavity in own links and strategic substitutability do imply some interesting restrictions on the nature of asymmetric networks which we informally discuss. It can be shown that there can be at most one singleton player in a pairwise equilibrium network and that the rest of the network is a symmetric network with a unique degree. This property does rule out all dominant group networks with two or more isolated players. The uniqueness of degree property follows from considerations mentioned above. What about asymmetric networks with no singleton components? Here concavity and strategic substitutability jointly imply that if two players $i, j \in \boldsymbol{N}_q(g)$ have a link then any two less linked players must be mutually connected. Likewise if two player $i, j \in \boldsymbol{N}_q(g)$ do not have a link then all players with greater number of links are not mutually linked. This property rules out inter-linked stars with two or more central players but does allow a star to arise in equilibrium. We find moreover that a symmetric network and star can be equilibria over the same range of costs. In the friendship network, the star is an equilibrium for $0.914 < c < 1.32$; thus, when $0.914 < c < 1$, both the star and the symmetric network of degree 1 are equilibria.

4.1 Payoff distribution

We now turn to the relation between degree and payoffs in equilibrium networks. We start with a result on games which exhibit negative spillovers.

Proposition 4.5 *Consider the local spillovers model and suppose that the aggregate payoff of a player is strictly decreasing in the number of links of others.*

Let g be a pairwise equilibrium network in which $\eta_j(g) < \eta_k(g)$. If player j's neighborhood is included in the neighborhood of player k then $\Pi_k(g) > \Pi_j(g)$.

The intuition for this result is analogous to that for Proposition 3.4.

In the discussion following Proposition 4.4 we had noted that when aggregate payoffs are concave in own links and satisfy strategic substitutability, then a star network can be a pairwise equilibrium (although inter-linked stars with two or more central players are ruled out). We can directly verify that if aggregate payoffs are strictly decreasing in the links of other players, then the central player i in a star network will earn a strictly greater payoff than the peripheral players. To see this, let g be a pairwise equilibrium star network and let g' denote the network in which i deletes links with all players except j. Then, $\Pi_i(g) \geq \Pi_i(g') = \Psi_1(1)+\Psi_2(1)+(n-2)\Psi_3(0)-c > \Psi_1(1)+\Psi_2(n-1)+(n-2)\Psi_3(1)-c = \Pi_j(g)$.

Consider next the positive spillovers case. Intuitively, if a player's links have positive spillovers on others and links are costly, then a highly linked player can earn a lower payoff as compared to a less linked person. Indeed, this what happens in the public goods example discussed in section 4. This is a game with local spillovers which satisfies convexity in own links and strategic complements property vis-a-vis partners' links. Proposition 4.1 says that dominant groups and inter-linked stars can arise in equilibrium. For example, with $n = 5$ and $6.5 < c < 7$ the network with a dominant group of size 3 is a equilibrium. In this network the members of the dominant group earn $24.5 - 2c$, while isolated players earn a payoff of 28.5!

5 Concluding remarks

Empirical work shows that many social and economic networks exhibit an unequal distribution of connections across individuals. This empirical finding raises two important questions: one, what are the economic circumstances which give rise to unequal degree distributions, and two, what are the implications of such inequality for the distribution of payoffs? This paper develops a strategic model of link formation to address these questions.

Our results bring out the central role of externalities across players links in understanding these questions: if links are strategic complements then symmetric networks arise naturally, while if links are strategic substitutes then an unequal degree distribution is more likely to arise. Moreover, if links of a player have negative externalities on other players then a player with more connections earns a higher payoff than her less connected cohort. This positive relation between connections and payoffs can get reversed if a player's links generate positive externalities for others.

Appendix

For playing the field games, define $\pi_i(g+g_{i,j}) - \pi_i(g) = \Phi(\eta_i(g)+1, L(g_{-i})) - \Phi(\eta_i(g), L(g_{-i}))) = \phi(\eta_i, L(g_{-i}))$.

Proof of Proposition 3.1 We start by deriving a general implication of convexity in own links: for any two players $i,j \in N$ if $g_{i,k} = g_{j,l} = 1$ for some $k,l \in N\backslash\{i,j\}$, then $g_{i,j} = 1$. Suppose not and let $g_{i,j} = 0$. By the definition of a pairwise equilibrium:

$$\begin{aligned} \pi_i(g) - \pi_i(g - g_{i,k}) &= \phi(\eta_i(g) - 1, L(g_{-i})) \geq c \\ \pi_j(g) - \pi_j(g - g_{j,l}) &= \phi(\eta_j(g) - 1, L(g_{-j})) \geq c. \end{aligned} \tag{19}$$

Since each player's marginal payoffs are strictly increasing in own links:

$$\begin{aligned} \pi_i(g + g_{i,j}) - \pi_i(g) &= \phi(\eta_i(g), L(g_{-i})) > \phi(\eta_i(g) - 1, L(g_{-i})) \geq c \\ \pi_j(g + g_{i,j}) - \pi_j(g) &= \phi(\eta_j(g), L(g_{-j})) > \phi(\eta_j(g) - 1, L(g_{-j})) \geq c. \end{aligned} \tag{20}$$

This implies that players i and j have an incentive to form a link and, therefore, g is not a pairwise equilibrium. This transitivity property implies that any component must be complete and that there can be at most one non-singleton component in an pairwise equilibrium. These observations imply that an pairwise equilibrium network is either empty or complete or possesses the dominant group architecture.

Existence: (convexity in own links and strategic complements across others links) If the complete network is a pairwise equilibrium then we are done. If it is not a pairwise equilibrium then there is some player i who strictly prefers to delete some subset of his links. Given convexity in own links this implies that $\phi(0, (n-1)(n-2)) < c$. The marginal returns to player i from forming a link with some player j are given by $\phi(0, 0)$. It follows from the strategic complements property that $\phi(0, 0) < \phi(0, (n-1)(n-2)) < c$. It follows that the empty network is a pairwise equilibrium.

Existence (convexity in own links and strategic substitutes) Consider a dominant group network g^k where k refers to the size of the dominant group. Let $D(g^k)$ refer to the set of players in the dominant group. If g^k is a pairwise equilibrium then we are done. If not then: (a) there is some $i \in D(g^k)$, who wants to delete a subset of his links; (b) there is an $i \in D(g^k)$ and some player $j \notin D(g^k)$ who wish to form a link. We take up these cases in turn.

(a) Suppose some $i \in D(g^k)$ who prefers to delete links: Given convexity in own links, this implies that $\phi(0, (k-1)(k-2)) < c$. Consider then the network g^{k-1}. If it is a pairwise equilibrium then we are done. If not then there are two possible reasons for it: (i). There is some $l \in D(g^{k-1})$ and some $j \notin D(g^k)$ who wish to form a link. This means that for player j the marginal returns $\phi(0, (k-1)(k-2)) \geq c$, which contradicts the above inequality derived from incentives of player i. (ii). There is some player $l \in D(g^{k-1})$, who wishes to delete a subset of his links. In that case, consider network g^{k-2} and so on. This process will lead to either some dominant group network $g^{k'}$ or the empty network being a pairwise equilibrium.

(b) Suppose there is $i \in D(g^k)$ and $j \notin D(g^k)$, who wish to form a link. The for player i, $\phi(k-1,(k-1)(k-2)) > c$, while for player j, $\phi(0,k(k-1)) > c$. Next, consider the dominant group network g^{k+1}; if g^{k+1} is an pairwise equilibrium then we are done. If not then we first note that no player has an incentive to delete any links. If this were true then from convexity in own links it follows that $\phi(0,k(k-1)) < c$, which violates the above incentive condition for player j. Thus the only reason for g^{k+1} to not be a pairwise equilibrium is that some $i \in D(g^{k+1})$ wants to form a link with some $m \notin D(g^{k+1})$. Now, the above argument of a growing dominant group can be repeated and existence of equilibrium follows from finiteness of G. □

Proof of Proposition 3.2 The method of proof for the existence result here is borrowed from Jackson and Watts (2002, Theorem 1). Consider a symmetric network of degree $k, g^k, 0 \leq k \leq n-1$. If $\phi(k-1,k(n-2)) \leq c \,\forall\, k = 1,2,\ldots,n-1$ then the empty network is a pairwise equilibrium. Consider next the case that $\phi(k-1,k(n-2)) > c$ for some $k = 1,2,\ldots,n-1$. In a symmetric network of degree k, no one has an incentive to delete any link. If there is a player who has an incentive to form links then define an improving path: g' is an improvement over a (possibly asymmetric) network g if $g' = g + g_{i,j}$ for some pair of players i,j and these two players have a strict incentive to form a link, i.e. $\phi(\eta_i(g), L(g_{-i})) > c$ and $\phi(\eta_j(g), L(g_{-j})) > c$. An improving path from g to g', denoted by $g \to g'$, is a finite sequence of networks $g_1, g_2, \ldots, g_r, \ldots, g_R$ with $g_1 = g$ and $g_R = g'$ such that g_{r+1} is an improvement over g_r, $r = 1,2,\ldots,R-1$.

Define an improving path starting from g^k as follows: get every pair of players with k links to form an additional link. Since strategic complements property holds it follows that every pair of players also has an incentive to form links. This leads to a symmetric network with $k'+1$ links. If $\phi(k'+1,(k'+1)(n-2)) < c$ then the degree $k+1$ symmetric network is a pairwise equilibrium and we are done. Otherwise we proceed further by adding links and existence follows from the finiteness of $\mathcal{G}$.

Finally, we consider the case of asymmetric networks and show that in a pairwise equilibrium network g if $i,j \notin N_m(g)$, then $g_{i,j} = 1$. Take some player $k \in N_m(g)$. It follows from the pairwise equilibrium hypothesis that $\phi(\eta_k(g)-1, L(g_{-k})) \geq c$. Suppose $i,j \notin N_m(g)$, and $g_{i,j} = 0$. Since $\eta_i(g) < \eta_k(g)$, it follows that $L(g_{-i}) > L(g_{-k})$. Hence it follows that

$$\begin{aligned}\phi(\eta_i(g), L(g_{-i})) &> \phi(\eta_i(g), L(g_{-k})) \\ &\geq \phi(\eta_k(g)-1, L(g_{-k})) \geq c. \end{aligned} \tag{21}$$

The first inequality follows from strategic complements property, while the second inequality follows from noting that $\eta_k(g) > \eta_i(g)$ and applying concavity in own links. The final inequality follows from the pairwise equilibrium hypothesis. Similar reasoning establishes that $\phi(\eta_j(g), L(g_{-j})) > c$. Hence, i and j have a strict incentive to form a link, contradicting our hypothesis that g is a pairwise equilibrium. □

Proof of Proposition 3.3 Consider first the existence result. For any symmetric network of degree $k$is given by $\phi(k, k(n-2))$. This expression is declining in k under concavity in own links and the strategic substitutes property. Therefore, there are three possible cases:

(i) $\phi(0,0) < c$. The empty network is clearly a pairwise equilibrium in this case.

(ii) $\phi(k, k(n-2)) < c < \phi(k-1, (k-1)(n-2))$ for $k \in \{1,2,3,..,n-2\}$. We proceed 'upward' from a degree $k-1$ network by forming links between distinct pairs of players in sequence, such that no player has more than k links along the way. If $\phi(k-1, k(n-2)) > c$ then we proceed until the symmetric network with degree k which is a pairwise equilibrium since $\phi(k, k(n-2)) < c < \phi(k-1, k(n-2))$. If $\phi(k-1, k(n-2)) < c$ then no symmetric pairwise equilibrium is possible.

(iii) $c < \phi(n-2, (n-2)(n-2))$. Using arguments from case (ii) above, it follows that if $\phi(n-2, (n-1)(n-2)) < c$ then no pairwise equilibrium symmetric network is possible while if $\phi(n-2, (n-1)(n-2)) > c$ then the complete network is a pairwise equilibrium.

To prove the unique degree result suppose that networks g and g' of degree k and k' respectively are both pairwise equilibrium networks and that $k < k'$. Then it follows that,

$$\phi(k, L(g_{-i})) \leq c, \quad \phi(k-1, L(g_{-i})) \geq c \tag{SM}$$

and moreover that,

$$\phi(k'-1, L(g'_{-i})) \geq c. \tag{SM}$$

Combining these equations yields us the following:

$$c \leq \phi\left(k'-1, \sum_{j\neq i} \eta_j(g'_{-i})\right) < \phi\left(k'-1, \sum_{j\neq i} \eta_j(g_{-i})\right) \leq \phi\left(k, \sum_{j\neq i} \eta_j(g_{-i})\right) \leq c.$$

The first inequality follows from the hypothesis that g' is a pairwise equilibrium network, the second inequality follows from $k < k'$ and the strategic substitutes property, while the third inequality follows from the hypothesis that $k < k'$ and concavity in own links. The final inequality follows from the hypothesis that g is a pairwise equilibrium. This generates a contradiction which completes the proof. □

Proof of Proposition 3.4 Suppose that g is a pairwise equilibrium network and $\eta_1(g) = k_1 > \eta_2(g) = k_2$. Suppose player 1 deletes $k_1 - k_2$ links and let g' denote the resulting network. Then $\eta_1(g') = k_2$. Note next that since

$L(g'_{-i}) = L(g_{-i})$, and Φ is strictly decreasing in $L(g_{-i})$, it follows that:

$$\begin{aligned}\Pi_1(g') &= \Phi(k_2, L(g'_{-1})) - k_2 c \\ &= \Phi(k_2, L(g_{-1})) - k_2 c \\ &> \Phi(k_2, L(g_{-2})) - k_2 c \\ &= \Pi_2(g).\end{aligned}$$

Since g is a pairwise equilibrium, $\Pi_1(g) \geq \Pi_1(g')$, and the result follows. □

For the local spillovers game, for any network g with $g_{i,j} = 0$, it will be convenient to let:

$$\begin{aligned}\pi_i(g + g_{i,j}) - \pi_i(g) &= \Psi_1(\eta_i(g) + 1) - \Psi_1(\eta_i(g)) + [\Psi_2(\eta_j(g) + 1) - \Psi_3(\eta_j(g))] \\ &= \psi(\eta_i(g), \eta_j(g)).\end{aligned}$$

Proof of Proposition 4.1 First consider the existence of a pairwise equilibrium. If the complete network is an equilibrium then we are done. If it is not an equilibrium then there is some player who wishes to delete a subset of his links. This implies that $\psi(k, n-2) < c$ for some k; but then convexity in own links tells us that $\psi(0, n-2) < \psi(k, n-2) < c$. We now use the strategic complementarity property to note that $\psi(0, 0) < \psi(0, n-2) < c$ and so the empty network is a equilibrium. This argument also proves that one of the two networks – empty or complete – is always an equilibrium in this model.

In order prove to the characterization result, we will make repeated use of the following property implied by convexity in own links and strategic complementarity:

(P.1) If $k' > k''$ and $l' > l''$, then $\psi(k', l') > \psi(k'', l'')$.

This property follows by noting that $\psi(k', l') > \psi(k'', l')$ from convexity in own links and $\psi(k'', l') > \psi(k'', l'')$ from strategic complementarity. We now turn to the characterization result by first showing that no symmetric network with degree $k \in \{1, 2, \ldots, n-2\}$ can be a pairwise equilibrium. Consider such a symmetric network of degree k. Then there are players i and j with k links each but with $g_{i,j} = 0$. Since g is a pairwise equilibrium network it is true that $\psi(\eta_i(g) - 1, \eta_l(g) - 1) \geq c$ and $\psi(\eta_j(g) - 1, \eta_p(g) - 1) \geq c$, for some $l, p \neq i, j$. Since g is symmetric, $\eta_j(g) > k - 1$, and $\eta_i(g) > k - 1$. From (P.1) we can then infer that $\psi(\eta_i(g), \eta_j(g)) > \psi(k-1, k-1) \geq c$. A similar incentive holds for player j. Therefore g is not a pairwise equilibrium, a contradiction that completes the proof. We next prove that a pairwise equilibrium network can have at most one non-singleton component. Let g' and g'' be two non-singleton components in g. Suppose that $i, j \in g'$ with $g_{i,j} = 1$ and $l, m \in g''$, with $g_{l,m} = 1$. Moreover, let $\eta_i(g) \geq \eta_p(g), \forall p \in N$. Since g is pairwise equilibrium it follows that $\psi(\eta_l(g)-1, \eta_m(g)-1) \geq c$, $\psi(\eta_m(g)-1, \eta_l(g)-1) \geq c$. Since $\eta_i(g) \geq \eta_p(g)$, $\forall p$, it follows that $\eta_i(g) > \eta_m(g) - 1$. Therefore (P.1) implies that for player i, $\psi(\eta_i(g), \eta_l(g)) > \psi(\eta_m(g) - 1, \eta_l(g) - 1) \geq c$. A similar argument applies with regard to player l. This contradicts the hypothesis that the network g is a pairwise equilibrium proving that there can be at most one non-singleton component g' in a pairwise equilibrium network g.

Let $\boldsymbol{N}_1(g'), \ldots, \boldsymbol{N}_m(g')$ be a partition of players according to their number of links, in g'. The argument with regard to impossibility of intermediate level pairwise equilibrium symmetric networks implies that, if g' is symmetric, then it must be complete. Therefore consider a component g' which is asymmetric. We show that it must be an inter-linked star. We start by showing that $g_{i,j} = 1$ if $i \in \boldsymbol{N}_1(g')$ and $j \in \boldsymbol{N}_m(g')$. Suppose not. Note that player i has a link with some player z implying that $\psi(\eta_i(g) - 1, \eta_z - 1) \geq c$. Similarly, for player z, the pairwise equilibrium hypothesis implies that $\psi(\eta_z(g) - 1, \eta_i - 1) \geq c$. Note next that marginal returns to player i from linking with player j are $\psi(\eta_i(g), \eta_j) > \psi(\eta_i(g) - 1, \eta_z - 1) \geq c$ due to (P.1). Similarly, the marginal returns to player j from a link with player i are $\psi(\eta_j(g), \eta_i) > \psi(\eta_z(g) - 1, \eta_i - 1) \geq c$, since $\eta_j(g) > \eta_z(g) - 1$, $\eta_i(g) > \eta_i(g) - 1$, and (P.1) holds. It is now straightforward to show that for all $j \in \boldsymbol{N}_m(g')$ and for all $x \in \boldsymbol{N}_1(g')$, $g_{j,x} = 1$ and thus $\eta_j(g) = |g'| - 1$, for all $j \in \boldsymbol{N}_m(g')$. To complete the proof we need to show that $N_i(g) = \boldsymbol{N}_m(g')$ for all $i \in \boldsymbol{N}_1(g')$. Suppose there is some $i \in \boldsymbol{N}_1(g')$ such that $g_{i,y} = 1$ for $y \notin \boldsymbol{N}_m(g')$. Since g is a pairwise equilibrium, $\psi(\eta_i(g) - 1, \eta_y(g) - 1) \geq c$, $\psi(\eta_y(g) - 1, \eta_i(g) - 1) \geq c$. Since $y \notin \boldsymbol{N}_m(g')$, there exists a player $k \in g'$ such that $g_{y,k} = 0$. Note that $\eta_k(g) > \eta_i(g) - 1$. We now show that y and k have an incentive to form a link with each other. The marginal returns to player y are given by $\psi(\eta_y(g), \eta_k(g)) > \psi(\eta_y(g) - 1, \eta_i(g) - 1) \geq c$, due to (P.1). A similar argument shows that $\psi(\eta_k(g), \eta_y(g)) > \psi(\eta_i(g) - 1, \eta_y(g) - 1) \geq c$. This proves that g is not a pairwise equilibrium. □

Proof of Proposition 4.2 We first note that since (SM) is stronger than (M) the results on symmetric networks are immediate. We now derive a property of asymmetric networks which exploits (SM)

(P.2) If $g_{i,j} = 1$, $\eta_j > \eta_i$, then $g_{i,k} = 1$ if $\eta_i \leq \eta_k \leq \eta_j$.

In order to prove (P.2), first consider the incentives of player i:

$$\psi(\eta_i, \eta_k) > \psi(\eta_i - 1, \eta_k - 1) \geq \psi(\eta_i - 1, \eta_j - 1) \geq c. \tag{SM}$$

The first inequality follows from (SM), the second inequality follows from the hypothesis $\eta_k \leq \eta_j$ and strategic substitutability, while the third inequality follows from the pairwise equilibrium hypothesis. Next consider the incentives of player k:

$$\psi(\eta_k, \eta_i) > \psi(\eta_i - 1, \eta_i) \geq \psi(\eta_i - 1, \eta_j - 1) \geq c. \tag{SM}$$

The first inequality follows from convexity in own links, the second from strategic substitutability, while the final inequality follows from the pairwise equilibrium hypothesis. Thus players i and k have an incentive to form a link.

We now show using (P.2) that every non-singleton component g' is complete. Clearly a component with 2 players is complete; suppose therefore that $|g'| > 2$ and it is incomplete. From previous arguments we can restrict attention to cases where g' is asymmetric. Let player j be a maximally connected player in this component who has a link with a non-maximally connected player i (such players must exist), i.e. $g_{i,j} = 1$ and $\eta_i < \eta_j$. Then there must exist some player k such that $g_{j,k} = 1$ but $g_{i,k} = 0$. There are two possibilities: $\eta_i \leq \eta_k \leq \eta_j$ and $\eta_k \leq \eta_i \leq \eta_j$, and in both instances (P.2) implies that $g_{i,k} = 1$. Since k was arbitrary, this implies that $\eta_j \leq \eta_i$, a contradiction that proves the claim that g' is complete. Finally we note that a pairwise equilibrium cannot have two non-singleton components of equal size; this follows from (SM) as noted above. The proof is complete. □

Proof of Proposition 4.3 Consider a symmetric network of degree k, g^k, $0 \leq k \leq n-1$. For any given cost c of forming links, there are two possibilities:

(i) $\psi(k-1, k-1) \leq c \; \forall \; k = 1, 2, \ldots, n-1$.
In this case we are done because the empty network is a pairwise equilibrium. To verify this, note that there are no links to delete. Moreover, no pair of players will have an incentive to add a link since $\psi(0,0) \leq c$ which follows by substituting $k = 1$.

(ii) $\psi(k-1, k-1) > c$ for some $k = 1, 2, \ldots, n-1$.
No player in symmetric network g^k has an incentive to delete the last link. From concavity in own links, this also implies that no player has an incentive to delete any subset of links. Now consider the incentives of players to form links. If $\psi(k,k) < c$ then no one wishes to form a link then g^k is a airwise equilibrium. If $\psi(k,k) > c$ then let a a pair of players form links. We now construct an improving path from g as follows: at each stage a distinct pair of players forms a link. Due to strategic complementarity and exploiting an even number of players assumption we can then move to the next degree symmetric network, g^{k+1}. If $\psi(k+1, k+1) < c$ then the degree $k+1$ network is an pws-equilibrium. If not then we repeat the above step. Since there are only a finite number of degrees, this process has to stop at a symmetric network, which is a pairwise equilibrium.

We now turn to characterizing asymmetric networks. We start by showing that property (P.2) in the proof of Proposition 4.2 continues to hold. Since g is a ws-equilibrium, it follows that $\psi(\eta_i(g) - 1, \eta_j(g) - 1) \geq c$ and $\psi(\eta_j(g) - 1, \eta_i(g) - 1) \geq c$. However:

$$\psi(\eta_i(g), \eta_k(g)) \geq \psi(\eta_j(g) - 1, \eta_k(g)) > \psi(\eta_j(g) - 1, \eta_i(g) - 1) \geq c. \quad (26)$$

where the first inequality follows from the hypotheses that $\eta_i(g) \leq \eta_j(g) - 1$ and concavity in own links while the second inequality follows from the hypothesis $\eta_k(g) > \eta_i(g) - 1$ and strategic complementarity. Similarly, for player k, we note that

$$\psi(\eta_k(g), \eta_i(g)) > \psi(\eta_k(g) - 1, \eta_i(g) - 1) \geq \psi(\eta_j(g) - 1, \eta_i(g) - 1) \geq c. \quad (27)$$

where the first inequality follows from (SM) and the second inequality follows from the hypotheses that $\eta_j(g) - 1 \geq \eta_k(g) - 1$ and concavity of aggregate payoffs in own links. Thus players i and k have a strict incentive to form a link. The exclusive group architecture of equilibrium networks now follows the same argument as Proposition 4.2.

Proof of Proposition 4.4 We start by noting that $\psi(k,k)$ is declining in k under concavity in own links and strategic substitutability. If $c > \psi(0,0)$ then the empty network is clearly a pairwise equilibrium. If $\psi(n-2, n-2) > c$ then the complete network is a pairwise equilibrium; to see this note that there are no further links to be added and observe that concavity in own links implies that $\psi(k, n-2) > \psi(n-2, n-2) \geq c$ for all $k \leq n-2$. Hence no player has an incentive to delete any links either. Finally consider $\psi(k,k) < c < \psi(k-1, k-1)$ for some $k \in \{1, .., n-2\}$. Observe that since $\psi(k,k) < c$, no player has an incentive to add a link, while concavity in own links implies $\psi(l, k-1) > \psi(k-1, k-1) > c$ for all $l = 1, 2, \ldots, k-2$, which means that no player has an incentive to delete any links either. Thus a network of degree k is a pairwise equilibrium. The uniqueness of degree follows directly from noting that the inequalities $\psi(k,k) < c < \psi(k-1, k-1)$ and $\psi(k', k') < c < \psi(k'-1, k'-1)$ cannot be simultaneously satisfied for k and k', with $k > k'$. □

Proof of Proposition 4.5 Suppose g is a pairwise equilibrium network where $N_j(g) \subset N_k(g)$. Without loss of generality, let $N_j(g) = \{1, 2, \ldots, l\}$ and $N_k(g) = \{1, 2, \ldots, l, l+1, \ldots, m\}$. Now let player k delete the $m - l$ links with players $\{l+1, \ldots, m\}$ and let g' denote the resulting network. Then:

$$\begin{aligned}
\Pi_k(g') &= \Psi_1(l) + \sum_{r=1}^{l} \Psi_2(\eta_r(g')) + \sum_{r=l+1}^{m} \Psi_3(\eta_r(g')) + \sum_{p \notin N_k(g)} \Psi_3(\eta_p(g')) - lc \\
&> \Psi_1(l) + \sum_{r=1}^{l} \Psi_2(\eta_r(g)) + \sum_{r=l+1}^{m} \Psi_3(\eta_r(g)) + \sum_{p \notin N_k(g)} \Psi_3(\eta_p(g)) - lc \\
&= \Pi_j(g) \qquad (28)
\end{aligned}$$

where we have used the fact that:

$$\begin{aligned}
\eta_r(g') &= \eta_r(g), & r &= 1, 2, \ldots, l \\
\eta_r(g') &< \eta_r(g), & r &= l+1, \ldots, m \\
\eta_p(g') &= \eta_p(g), & p &\notin N_k(g).
\end{aligned}$$

Since g is a pairwise equilibrium, $\Pi_k(g) \geq \Pi_k(g')$ proving the result. □

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