

Endogenous Formation of Links Between Players and of Coalitions: An Application of the Shapley Value

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1 Introduction

Consider the coalitional game v on the player set $(1,2,3)$ defined by

$$v(S) = \begin{cases} 0 & \text{if } |S| = 1, \\ 60 & \text{if } |S| = 2, \\ 72 & \text{if } |S| = 3, \end{cases} \quad (1)$$

where $|S|$ denotes the number of players in S . Most cooperative solution concepts “predict” (or assume) that the all-player coalition $\{1, 2, 3\}$ will form and divide the payoff 72 in some appropriate way. Now suppose that P_1 (player 1) and P_2 happen to meet each other in the absence of P_3 . There is little doubt that they would quickly seize the opportunity to form the coalition $\{1, 2\}$ and collect a payoff of 30 each. This would happen in spite of its inefficiency. The reason is that if P_1 and P_2 were to invite P_3 to join the negotiations, then the three players would find themselves in effectively symmetric roles, and the expected outcome would be $\{24, 24, 24\}$. P_1 and P_2 would not want to risk offering, say, 4 to P_3 (and dividing the remaining 68 among themselves), because they would realize that once P_3 is invited to participate in the negotiations, the situation turns “wide open” – anything can happen.

All this holds if P_1 and P_2 “happen” to meet. But even if they do not meet by chance, it seems fairly clear that the players in this game would seek to form pairs for the purpose of negotiation, and not negotiate the all-player framework.

The preceding example is due to Michael Maschler (see Aumann and Dreze 1974, p. 235, from which much of this discussion is cited). Maschler’s example is particularly transparent because of its symmetry. Even in unsymmetric cases, though, it is clear that the framework of negotiations plays an important role in the outcome, so individual players and groups of players will seek frameworks that are advantageous to them. The phenomenon of seeking an advantageous

framework for negotiating is also well known in the real world at many levels – from decision making within an organization, such as a corporation or university, to international negotiations. It is not for nothing that governments think hard and often long-about “recognizing” or not recognizing other governments; that the question of whether, when, and under what conditions to negotiate with terrorists is one of the utmost substantive importance; and that at this writing the government of Israel is tottering over the question not of *whether* to negotiate with its neighbors, but of the *framework* for such negotiations (broad-base international conference or direct negotiations).

Maschler’s example has a natural economic interpretation in terms of S-shaped production functions. The first player alone can do nothing because of setup costs. Two players can produce 60 units of finished product. With the third player, decreasing returns set in, and all three together can produce only 72. The foregoing analysis indicates that the form of industrial organization in this kind of situation may be expected to be inefficient.

The simplest model for the concept “framework of negotiations” is that of a *coalition structure*, defined as a partition of the player set into disjoint coalitions. Once the coalition structure has been determined, negotiations take place only within each of the coalitions that constitute the structure; each such coalition B divides among its members the total amount $v(B)$ that it can obtain for itself. Exogenously given coalition structures were perhaps first studied in the context of the bargaining set (Aumann and Maschler 1964), and subsequently in many contexts; a general treatment may be found in Aumann and Dreze (1974). Endogenous coalition formation is implicit already in the von Neumann–Morgenstern (1944) theory of stable sets; much of the interpretive discussion in their book and in subsequent treatments of stable sets centers around which coalitions will “form”. However, coalition structures do not have a formal, explicit role in the von Neumann–Morgenstern theory. Recent treatments that consider endogenous coalition structures explicitly within the context of a formal theory include Hart and Kurz (1983), Kurz (1988), and others.

Coalition structures, however, are not rich enough adequately to capture the subtleties of negotiation frameworks. For example, diplomatic relations between countries or governments need not be transitive and, therefore, can not be adequately represented by a partition; thus both, Syria and Israel have diplomatic relations with the United States but not with each other. For another example, in salary negotiations within an academic department, the chairman plays a special role; members of the department cannot usually negotiate directly with each other, though certainly their salaries are not unrelated.

To model this richer kind of framework, Myerson (1977) introduced the notion of a *cooperation structure* (or *cooperation graph*) in a coalitional game. This graph is simply defined as one whose vertices are the players. Various interpretations are possible; the one we use here is that a link between two players (an edge of the graph) exists if it is possible for these two players to carry on meaningful direct negotiations with each other. In particular, ordinary coalition structures (B_1, B_2, \dots, B_k) (with disjoint B_j) may be modeled within