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A simple matching model with social networks

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Abstract

We provide a matching model à la Pissarides with social networks. We derive our matching function from an urn-ball model, showing that one can introduce network in an homogenous and thus simple framework. Then we derive the decentralized equilibrium.

Keywords: Search; Matching; Social networks

JEL classification: E24; J64; J68; Z13

1. Introduction

Sociological studies promote the idea that the economic theory of job search bypasses the specific role of social networks in recruitment. Particularly, employees in a firm generate applicants for jobs that are not advertised by that firm. For instance Waldinger (1997) claims that "social networks produce applicants for employers who don't yet have vacancies to fill". In the same way, Granovetter (1995), in his survey of the literature, argues that "if employers do not advertise vacancies, this may be in part because they know they can be filled by friends and relatives of existing employees". If job advertising is used by employers to produce matches, the existing employees can also find applicants. Generally speaking, between one third and two third of the jobs are found through networks (for a recent survey of the empirical and theoretical literature, see Ioannides and Loury, 2004). However, the introduction of social

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networks in models of the labor market often leads to complicated frameworks (e.g. Calvo-Armengol and Jackson, 2004, 2005) where most of the characteristics of the labor market are taken as given. We aim at providing an equilibrium matching model in the line of Pissarides (2000), with endogenized wages and vacancies, but that introduce social networks. It has to be simple enough to be easily extended and able to deal with various macroeconomic issues (aggregated unemployment rate, inequalities, economic policies…).

We begin by deriving our matching function from an urn-ball process that borrows from Albrecht et al. (2003, 2004). Such process offers microfoundations to the congestion externalities underlined by Pissarides (2000). Interestingly enough, we show that one can introduce social networks while keeping a homogenous matching function. This homogeneity makes the decentralized equilibrium simple to characterize. We point out that there is a substitution between the different search channels and demonstrate the uniqueness of the equilibrium. Finally, we show that the higher the efficiency of social networks, the higher the wages but the lower the unemployment rate.

2. The matching function

We consider an economy with V advertised vacancies, U unemployed workers and L employees. The labor force is divided in N groups indexed by i, $i=\{1, 2,\ldots, N\}$. Employers and unemployed workers are brought together in pairs through an imperfect matching process. Firms use different search methods. One is a high cost search method where each offer is sent and advertised at a cost h per unit of time. For example, firms put advertisements in newspapers and use employment agencies. Besides, firms benefit from the social capital of their employees. We assume that each employee produces applicants for his employer at a rate λ_i.

A job offers is represented by a ball which is sent into urns, i.e. to the unemployed (see Albrecht et al., 2003, 2004, for a similar framework[1]). Besides, at each period, each employee also contacts unemployed friends (send a ball into the urns) with a probability λ_i. To introduce the idea of social closure (Waldinger, 1997), we assume that an employee of a given group forwards job offers only to unemployed workers belonging to the same group. On the firms' side, we assume that they send their V offers at random to the U urns. V we will be endogenize in the next section.

We denote by C_i the probability for an unemployed worker to receive at least one job offer. It follows a binomial distribution:

$$C_i = 1-\left(1-\frac{1}{U}\right)^V \left(1-\frac{1}{U_i}\right)^{\lambda_i L_i}$$

For a large number of job offers and a large number of unemployed, this binomial distribution can be approximated by a Poisson distribution. Moreover, remark that the expected number of offers reaching

[1] Notice that, in their papers, the job offers are the urns and the job seekers the balls. We make the opposite symmetric assumption. Consequently, if in our their case, $\theta m(\theta)$ is the exit rate from unemployment, it is denoted $m(\theta)$ in their paper. This does not change neither the results nor the interpretation.

the members of the ith group reads $p_i\ V+\lambda_i L_i$, with $p_i=U_i/\sum_{j=1}^{N} U_j$. Consequently C_i simplifies, with $\theta_i=(p_i V+\lambda_i L_i)/U_i$ the labor market tightness faced by the members of the network i,

$$C_i \approx 1-\exp(1-\theta_i)$$

The probability for an offer received by an unemployed worker of the group i to be filled amounts to

$$m(\theta_i) = U_i C_i \frac{1}{p_i V + \lambda_i L_i} = \frac{1}{\theta_i}(1-\exp(1-\theta_i))$$

Consequently, the unconditional probability for an offer sent through the formal channel (V) to be filled reads

$$\sum_{i=1}^{N} p_i \frac{1}{\theta_i}(1-\exp(-\theta_i))$$

and the exit rate from unemployment for a worker of the ith group $\theta_i m(\theta_i)$.

Within each social network, the contact rate reads $(p_i V+\lambda_i L_i)m(\theta_i)$ and thus exhibits approximate constant return to scale. Finally, the overall matching rate is the sum of homogenous matching functions:

$$MG = \sum_{i=1}^{N} (p_i V + \lambda_i L_i)m(\theta_i) = \sum_{i=1}^{N} M(\lambda_i L_i + p_i V, U_i)$$

3. The homogenous case

For sake of simplicity, while similar results could be presented with multiple networks, we consider a submarket with a large and *unique* social network. Each employee produces y while each job is destroyed at a rate q.

We assumed that λ is an exogenous variable. While it would be interesting to understand why the employees are willing to help other workers to find a job, this is beyond the scope of this note. On the one hand, the expected payoff of cooperation could be related to future job search (for example in case of a job loss). On the other hand, it can also be related to something outside the labor market. For example, it could be affected by social conventions, social group's traditions or social prestige. Moreover, the firm could also try to influence λ (see the discussion in the next section).

Let us denote by r the exogenous discount rate and w the wage. The firm chooses the number of vacancies publicly advertised (V_j) and takes into account that its employees also produce applicants. The value function for the problem of the firm j solves the following Bellman equation:

$$r\Pi(L_j) = \max_{V_i \geq 0}\left[yL_j - \mathrm{w}L_j - \mathrm{h}V_j + \dot{\Pi}(L_j)\right] \tag{1}$$

subject to

$$\dot{L}_j = (\lambda L_j + V_j)m(\theta) - qL_j \tag{2}$$

The firm chooses its optimal number of employees, knowing that the positions can be filled using two methods of search. On the one hand, jobs can be advertised by the firm. On the other hand, social networks of employees produce applicants at a rate $\lambda L_j m(\theta)$. Since, at each period, the firm chooses the number of advertised vacancies (V_i), she controls the increase in the number of employees and, consequently, the subsequent number of applicants the social network will produce. Moreover, wages are the subject of bargaining and a firm can refuse to hire an applicant. Firms are identical. Using Eqs. (1), (2) and the Kuhn–Tucker conditions, ones gets, at the steady state,

$$\frac{h}{m(\theta)} = \frac{y-w}{r+q-\lambda m(\theta)} \Leftrightarrow V>0 \tag{3}$$

$$\frac{h}{m(\theta)} > \frac{y-w}{r+q-\lambda m(\theta)} \Leftrightarrow V=0 \tag{4}$$

Eq. (3) defines the optimal employment level when the optimal number of advertised vacancies is positive. In the second case (Eq. (4)) the expected cost of a job advertisement is higher than the expected profit. Consequently, the optimal number of advertised vacancies amounts to zero. In this paper, we focus on the non degenerate case where the formal number of vacancies is positive[2].

An unemployed worker benefits from an income flow z. Let us denote by U the value function of an unemployed worker and by E the value function of an employee. U and E satisfy at the symmetric steady state:

$$rU = z + \theta m(\theta)(E-U) \tag{5}$$

$$rE = w + q(U-E) \tag{6}$$

"The employers view workers' social connections as resources in which they can invest, and which might yield economic returns in form of better hiring outcomes" claim Fernandez et al. (2000). In our model, social connections are valuable since they increase the matching rate of the firm. This changes the value of jobs and influences wages through bargaining. In our model, wages are subject to bargaining between the firm and the worker and can be renegotiated each period at no cost. The surplus of each match is shared according to the Nash solution of the bargaining problem

$$\beta J = (1-\beta)(E-U) \tag{7}$$

with $\beta \in [0,1]$ the share that accrues to the worker. One gets

$$w = z + \beta \frac{r+q+\theta m(\theta)}{r+q+\beta\theta m(\theta) - (1-\beta)\lambda m(\theta)}(y-z) \tag{8}$$

Eq. (8) is very close to that of standard matching models (see for example Pissarides (2000)). Nevertheless, in our framework, wages are increasing functions of the efficiency of networks λ because firms directly take into account during wage bargaining that their employees produce applicants. A

[2] The alternative case has been studied in Fontaine (2004).

higher efficiency induces a higher job filling rate for the firm. Taking the market tightness θ as given, the higher the efficiency of the networks, the higher the wage.

We now consider the equilibrium value of the labor market tightness. Eq. (3) together with Eq. (8) imply that labor market tightness satisfies the following condition:

$$\frac{h}{m(\theta)} = \frac{(1-\beta)(y-z)}{r+q+\beta\theta m(\theta)-(1-\beta)\lambda m(\theta)} \tag{9}$$

In equilibrium, the expected cost of an advertised vacancy, represented by the left-hand side, equates to the expected profit of a filled position, represented by the right-hand side. It can easily be checked that Eq. (9) defines a unique equilibrium value of labor market tightness. The higher the number of employees, the higher the number of job offers transmitted by networks. However, this induces a substitution effect and the number of advertised jobs decrease. This substitution effect ensures the uniqueness of equilibrium[3].

At the steady sate, this induces a unique value of the unemployment rate, $u=q/(q+\theta m(\theta))$. The unemployment rate decreases with λ. Accordingly if one compare two submarkets that only differ with respect to the efficiency of their networks (same y, h, q but different λ), the market with the higher λ benefits from higher wages and a lower unemployment rate.

4. Discussion

The aim of this short note is to provide a way to introduce social networks in a flexible but microfounded matching framework. The existing literature on networks and labor market (e.g. Calvo-Armengol and Jackson, 2004, 2005 and Bramoullé and Saint-Paul, 2004) often insists on networks' structure. It offers a rich but complicated framework where wages and vacancies have to be taken exogenous. Hence, it isn't well-suited to deal with macroeconomic issues. Calvo-Armengol and Zenou (2005) have recently provided a nonhomogeneous matching model with social networks. Our model is a simpler alternative. Our homogenous matching function offers an uncostly way to introduce networks within an equilibrium model. The model can thus be easily extended to study various problems.

First, wages inequalities across different networks could be considered. Obviously, a distribution of λ_i induces a distribution of wage. However, λ_i could be chosen by the employees (λ_i could be seen as an investment in an "interpersonnal" insurance against unemployment) or influenced by firms (through monetary bonuses). The effect of economic policies could also been investigated. For example, some active labor market policies try to encourage the improvement of networks (McClure, 2000). In our model, an increase in the efficiency of the networks (λ) can lead to a decrease in the number of advertised vacancies, because of the substitution between search channels. Imagine that some workers are excluded from networks. A policy trying to stimulate social networks will hurt these workers by inducing a switch of the hiring policy from formal method to social networks.

[3] It is worth noting that the right hand side of Eq. (9) is only defined when the value of a marginal job is positive, that is if $r+q+\beta\theta m(\theta)-(1-\beta)\lambda m(\theta)>0$. We denote by $\overline{\theta}$, the value of the labor market tightness such that $r+q+\beta\overline{\theta}m(\overline{\theta})=(1-\beta)\lambda m(\overline{\theta})$. For $\theta \in]\overline{\theta},+\infty[$, the value of a marginal job is decreasing with θ, whereas the expected cost of a advertised vacancy increases with the labor market tightness. Consequently, the equilibrium value of θ when both methods are used is always greater that $\overline{\theta}$ and unique.

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