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# R&D networks

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*We develop a model of strategic networks that captures two distinctive features of interfirm collaboration: bilateral agreements and nonexclusive relationships. Our analysis highlights the relationship between market competition, firms' incentives to invest in R&D, and the architecture of collaboration networks. In the absence of firm rivalry, the complete network, where each firm collaborates with all others, is uniquely stable, industry-profit maximizing, and efficient. By contrast, under strong market rivalry the complete network is stable, but intermediate levels of collaboration and asymmetric networks are more attractive from a collective viewpoint. This suggests that competing firms may have excessive incentives to form collaborative links.*

## 1. Introduction

■ Many markets are characterized by a high level of interfirm collaboration in R&D activity. A significant proportion of such collaboration takes place between firms that are horizontally related, i.e., where firms exhibit some degree of market rivalry. For instance, Hagedoorn and Schakenraad (1990) report that during the 1980s, on average there were an additional 100 collaborative agreements every year in biotechnology, and over 200 every year in information technologies. Similarly, Burgers, Hill, and Kim (1993) find that the 23 largest firms in the world automobile market had been involved in 58 bilateral linkages by 1988. The growth in technological collaborations between firms in recent years has been documented by Delapierre and Mytelka (1998), Ghemawat, Porter, and Rawlinson (1986), Hagedoorn and Narula (1996), and Harrigan (1988). Collaboration agreements and alliances vary greatly in their form. They may range from loose and informal agreements (such as a "memorandum of understanding") all the way to the formation of common legal organizations with strong equity ties (such as a research joint venture or RJV).[1] Similarly, the nature of collaborative activity varies widely. In some cases, the collaboration

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[1] WISe and IVANS have recently signed a "letter of understanding" as the basis for a strategic alliance (see *National Underwriter*, Vol. 104 (2000), p. 9). Another example is the memorandum of understanding recently signed by Presstek and Xerox (see *Printing Impressions*, Vol. 42 (2000), p. 6). Hagedoorn and Schakenraad (1990) and Delapierre and Mytelka (1998) present evidence suggesting that non-RJV forms of collaboration have become more important in recent years.

involves the sharing of information (concerning new technologies and market conditions), while in others the focus is on sharing facilities (such as distribution channels and complementary assets) or on setting market standards. Hybrid types of collaboration are also observed (some firms share their information, while in exchange others share advertising expenses).

In this article we study the incentives for collaboration between horizontally related firms. We suppose that the form of the agreement is relatively loose and that the benefits arise from sharing knowledge about a cost-reducing technology. When firms collaborate, their individual R&D efforts lower the costs of their partners too. The impact of these spillovers is intimately related to the nature of market rivalry between the firms. Thus market competition has a bearing on the incentives for collaboration. By forming collaborations, however, firms alter the competitive position of different firms and in turn influence market structure and performance. This two-way flow of influence is central to our analysis. In particular, we address the following questions:

(i) What are the effects of collaborative activity on individual R&D and industry performance?

(ii) What are the incentives of firms to collaborate and what is the architecture of "incentive-compatible" networks?

(iii) Are individual incentives to collaborate adequate from a social welfare point of view? We also study how the answers to these questions depend on the nature of market competition.

A distinctive feature of collaboration agreements is that they are often *bilateral* and are embedded in a broader network of collaborations (Delapierre and Mytelka, 1998; Mody, 1993). This translates into situations where firms $i$ and $j$, and $j$ and $k$ have a collaborative relationship, respectively, while firms $i$ and $k$ do not have any collaborative arrangement, i.e., collaborative relationships are *nonexclusive* (Milgrom and Roberts, 1992).[2] These structural features lead us to formulate a strategic model of R&D networks.

We consider an oligopoly with (*ex ante*) identical firms. Prior to market interaction, each firm has an opportunity to form *pair-wise* collaborative links with other firms. The purpose of these ties is to share R&D knowledge about a cost-reducing technology. The collection of pair-wise links between the firms defines a *network* of collaboration. Our model allows for nonexclusive ties, and it generates a rich class of possible collaborative structures (with $n$ firms there are $2^{n(n-1)/2}$ possible networks of collaboration). Given a collaboration network, firms choose a (costly) level of effort in R&D *unilaterally*, aimed at reducing production costs. The level of effort of different firms and the network of collaboration define the *effective* costs of the different firms in the market. Given these costs, firms operate in the market by setting quantities. We consider two types of market interaction: in the first case, firms operate in independent markets, while in the second case, they compete in a homogeneous-product market.

We start with a consideration of *symmetric* networks, i.e., networks in which all firms maintain the same number of collaborative ties. For such networks, the level of collaborative activity is naturally measured in terms of the number of ties of a typical firm. Our first result pertains to the relationship between the level of collaboration and individual R&D. We show that if firms compete in a homogeneous-product market, individual R&D effort is declining in the level of collaborative activity. In contrast, when firms operate in independent markets, individual R&D effort increases monotonically in the level of collaborative activity. Thus, in the former case individual R&D attains its minimum, whereas in the latter case it attains its maximum under the complete network, i.e., a network in which every pair of firms has a collaborative agreement. This contrast highlights the influence of the competition effect.

We examine next the level of cost reduction under different levels of collaboration. For any given level of R&D effort, adding a collaboration link leads to lower costs for all firms. However,

[2] This pattern can be discerned in collaborative ties between pharmaceutical firms. For example, in the late 1980s and early 1990s, Merck and Ciba-Giegy had a series of collaborative ties. Similarly, Bristol-Mayers and Bayer had such bilateral collaborations. At the same time, these firms also had collaboration ties with nonoverlapping sets of firms. For example, Bayer had collaboration arrangements with Hoechst, while Bristol-Mayers had no such collaboration. Delapierre and Mytelka (1998) present information on a variety of such collaborative ties.

as mentioned above, in a homogeneous-product market a higher level of collaborative activity lowers individual research efforts. Thus we have to compare the relative magnitude of these two effects. We find that in a homogeneous-product setting, the level of cost reduction is initially increasing and then decreasing in the level of collaborative activity, i.e., it is nonmonotonic with respect to the number of collaborations. By contrast, when firms operate in independent markets, individual R&D effort—and hence, by implication, the cost reduction—is maximal under the complete network.

Our next result pertains to the incentives of firms to form collaborative alliances. We show that irrespective of the degree of market competition, firms have an incentive to form collaborative relations; in other words, the empty network is never incentive compatible. Further, we show that the incentives to form collaborations are quite large in both settings: the complete network is a strategically stable network, irrespective of the market setting.

We then examine aggregate industry performance. We show that if firms compete in a homogeneous-product market, industry performance both in terms of aggregate profits and social welfare is highest when firms have an intermediate level of collaboration with other firms. In other words, both the empty and the complete network are dominated by intermediate levels of collaboration. Thus, under market rivalry, the incentives of firms to form collaborative ties may be excessive both from an industry-profit-maximizing perspective as well as from a social welfare point of view. By contrast, if firms interact in independent markets, aggregate industry profits as well as social welfare are highest under the complete network.

Our results on welfare and profit properties of collaboration provide an explanation for why a large number of strategic alliances are unstable or are terminated early, and they also help explain why some alliances work well. In highly competitive environments, firms would "collectively" prefer not to form many collaborative ties, since in this way they could attain higher profits. However, a pair of individual firms gain competitive advantage over the rivals by forming a collaboration and thus increase their profits. This implies that firms may have incentives to form too many links, which would lead to poor overall performance. In the case of independent markets, this dilemma does not appear because firms' R&D effort is not declining with the number of links. Seen from another perspective, the results suggest that firms should be more successful in sustaining collaboration in independent markets.[3] Similar considerations should apply in highly differentiated product markets.

In the first part of the article we restrict attention to symmetric network structures. In such structures, every firm is *ex post* in a similar situation. It has been argued that one of the primary motivations for firms to form collaborative alliances is to gain competitive advantage vis-à-vis their rivals (see, e.g., Hagedoorn and Schakenraad, 1990). We next examine the role of collaborations in generating such competitive advantages and their influence on market structure and industry performance. This motivates an examination of *asymmetric networks*. A general analysis of asymmetric networks, however, turns out to be very complicated; we therefore work with an example of three firms and completely characterize its solution.

In this setting there are four possible network architectures: the complete network, the star network, the partially connected network, and the empty network.[4] We find that asymmetric networks such as the star or the partially connected network perform quite well from the social as well as the private point of view. Indeed, the star network always dominates the complete network from both perspectives; moreover, for some parameter values, the star is industry-profit maximizing.[5] We also find that asymmetric forms of collaboration may alter the market structure

[3] Podolny and Page (1998) present evidence that suggests that a large proportion of collaboration agreements between firms fail or do not meet expectations. See also Kogut (1988) for related empirical work. Our results are consistent with the empirical findings of Roller, Tombak, and Siebert (1998) on the relative success of research joint ventures between noncompeting firms.

[4] The star is a network in which there is a central firm directly linked to every other firm, while none of the other firms have a direct link with each other. In our three-firm example, the partially connected network refers to a configuration in which two firms have a link while the third firm is isolated (see Figure 3 below).

[5] These results may help explain the empirically observed pattern of dominant firms at the center of stars, with the other firms at the spokes (see, e.g., Delapierre and Mytelka, 1998).

by causing large disparities between firms, or even leading to the exit of firms, and that this is not necessarily detrimental from a social standpoint. Indeed, under certain circumstances, the partially connected network is strategically stable, and both industry-profit and social-welfare maximizing.

Our article should be seen as a contribution to the study of group formation and cooperation in oligopolies. Our approach to strategic collaborations is inspired by recent work on strategic models of network formation (see, e.g., Aumann and Myerson, 1989; Bala and Goyal, 2000; Dutta, van den Nouweland, and Tijs, 1998; Goyal and Joshi, 1999; Jackson and Watts, 1999; Jackson and Wolinsky, 1996; and Kranton and Minehart, 2001). Our article's contribution to this literature is that it provides a model in which the "quality" of the links is endogenously determined—via the choice of R&D efforts—by the players.

The work of Kranton and Minehart (2001) deals with networks between vertically related firms. In contrast, our article studies collaborative ties between horizontally related firms, i.e., firms that compete subsequently in the market. This leads us to incorporate an explicit market-competition element in our collaboration model. Our article should be seen as complementary to their work. The results of our analysis suggest that the nature of market competition has a significant influence on the private as well as the social incentives to form bilateral collaborative links.

We next discuss the relationship with Goyal and Joshi (1999). Their article also studies networks of collaboration between oligopolistic firms. The principal difference concerns the modelling of R&D effort and the cost of link formation. They assume that a collaboration link between two firms involves a fixed cost and leads to an exogenously specified reduction in marginal cost of production. Their analysis focuses on how the costs of forming links affect the architecture of strategically stable networks. By contrast, in the present article the costs of forming links are taken to be negligible, but firms decide independently on a level of R&D, which in turn determines the level of cost reduction endogenously. We thus focus on the impact of collaborative ties on firms' incentives to conduct research, cost reduction, and social welfare.

Issues related to group formation and cooperation have long been a central concern of economic theory, and game theory in particular (see, e.g., Bloch (1995) and Yi (1998b); Bloch (1997) surveys this work). In the literature, group formation is modelled in terms of a *coalition structure,* which is a partition of the set of firms. In our article, we consider two-player relationships. In this sense, our model is somewhat restrictive as compared to the work on coalitions, which allows for groups of arbitrary size. However, the principal distinction concerns the nature of collaboration structures. The coalitions approach requires every player to belong to one group only, while our approach allows for cooperative relationships that are nonexclusive. This generates structures, such as stars and asymmetric architectures, that are very different from those studied in the coalition-formation literature.

Our model is also related to the literature on R&D cooperation in oligopoly. (See, e.g., d'Aspremont and Jacquemin (1988), Kamien, Muller, and Zang (1992), Katz (1986), and Suzumura (1992). For more recent work, see Leahy and Neary (1997) and the references cited therein.) In the terminology of Kamien, Muller, and Zang (1992), our model is a research joint venture competition type of model, where firms forming a collaboration commit to completely share the R&D results arising from research efforts decided unilaterally. This literature typically compares the properties of the grand coalition RJV with the autarchy situation, both in terms of social efficiency as well as from the point of view of industry profits.[6] Perhaps the main contribution of our article to this literature is the finding that social welfare as well as industry profits are maximized at intermediate levels of collaboration. Moreover, our result potentially has implications for policy: When market rivalry is substantial, there is a set of networks of collaboration that dominate both the empty network and the complete network in terms of social efficiency as well as on the basis

[6] An exception to this is Katz (1986). However, he restricts the analysis to collaboration structures in which there is a single group of collaborating firms and the other firms remain isolated. In this article we allow for arbitrary collaboration structures.

of industry profits. The work of Kamien, Muller, and Zang (1992) suggests that collaborative agreements of the RJV competition type are likely to be welfare reducing and should be treated with caution by competition authorities. Our results suggest that this prescription may need to be reconsidered.

The rest of the article is organized as follows. The model is presented in Section 2. In Section 3 we present the results on symmetric networks. Section 4 explores the role of asymmetric networks and knowledge spillovers. Section 5 concludes.

## 2 The model

■ We consider a three-stage game. In the first stage, firms form pair-wise collaboration links. In the second stage, each firm chooses a level of effort in R&D. The R&D efforts, along with the network of collaboration, define the costs of the firms. In stage three, firms operate in the market, taking as given the costs of production.

We are interested in the networks of collaboration that emerge in two different settings. We first study collaborations among firms operating in independent markets. We shall then study collaborations in a homogeneous-product oligopoly with quantity-setting firms.[7] We will also examine the effects of collaboration links on the nature of market outcomes. Finally, we shall compare stable collaboration networks with efficient networks. We now develop the notation and define our notions of stability and efficiency.

□ **Networks.** Let $N = \{1, 2, \ldots, n\}, n \geq 3$ be the set of firms. For any pair of firms $i, j \in N$, the pair-wise relationship between the two firms is represented by a binary variable $g_{ij} \in \{0, 1\}$. When $g_{ij} = 1$, this means that the two firms are linked, while $g_{ij} = 0$ refers to the case of no link. A network $g$ is then a collection of links, i.e., $g = \{g_{ij}\}_{i,j \in N}$. Let $g - g_{ij}$ denote the network obtained by severing an existing link between firms $i$ and $j$ from network $g$, while $g + g_{ij}$ is the network obtained by adding a new link between firms $i$ and $j$ in network $g$. Let $N_i(g)$ be the set of firms with which firm $i$ has a collaboration link in network $g$, and let $\eta_i(g)$ be the cardinality of set $N_i(g)$. We now define the relation " $\geq$ " on the set of networks. We shall say that network $g' \geq g$ if for every pair $i, j \in N$, $g'_{ij} \geq g_{ij}$. Moreover, $g' > g$ if $g'_{ij} \geq g_{ij}$ for all $i, j$, and in addition, there exist $l, m$ such that $g'_{lm} > g_{lm}$. We note that " $\geq$ " defines a partial order on the set of networks.

□ **Effort levels and spillovers.** Given a network $g$, every firm chooses an R&D effort level unilaterally.[8] This effort helps lower its own marginal cost of production. Individual efforts also have positive spillovers on the costs of other firms. We assume that if two firms have a collaboration link, then this spillover is perfect, and if they do not have a collaboration link, then this spillover is imperfect. Let $\beta \in [0, 1)$ be a parameter that reflects the level of spillovers among firms with no collaboration links.[9]

We assume that firms are initially symmetric, with zero fixed costs and identical constant marginal costs $\overline{c}$. Given a network $g$ and the collection of effort levels $\{e_i(g)\}_{i \in N}$, the cost of firm $i$ is given as follows:

$$c_i(\{e_i(g)\}_{i \in N}) = \overline{c} - \sum_{k \in N_i(g)} e_k - \beta \sum_{l \notin N_i(g)} e_l. \tag{1}$$

[7] We have also analyzed a slightly more general model with a horizontal product-differentiation parameter. The analysis of that model yields similar insights. The additional parameter made the computations cumbersome, so we decided to restrict attention to these two limiting cases.

[8] The unilateral choice of R&D effort is consistent with the model of competitive RJV, proposed in Kamien, Muller and Zang (1992).

[9] We note that in our model spillovers across noncollaborating firms are "public" in nature; thus, they are not related to the "distance" between different firms in the collaboration network. In this sense, spillovers are quite different from the indirect benefits that arise in the connections model studied by Bala and Goyal (2000) and Jackson and Wolinsky (1996).

The total cost reduction for firm $i$ stems from its own research, $e_i$, and the research knowledge of other firms, which is either fully absorbed, in the case of existing collaborative links, or partially absorbed, in the case of spillovers. We refer to this total cost reduction as *effective* R&D level. We shall assume that R&D effort is costly. Given a level $e_i \in [0, \bar{c}]$ of effort, the cost of effort is $Z(e_i) = \gamma e_i^2$, $\gamma > 0$. Under this specification, the cost of R&D effort is an increasing function and exhibits decreasing returns. The parameter $\gamma$ measures the curvature of this function. We shall assume that $\gamma$ is sufficiently large so that the second-order conditions hold and equilibria can be characterized in terms of first-order conditions and are interior. For low values of $\gamma$, existence of equilibrium follows from standard results, but equilibrium outcomes are typically corner solutions.

□ **Payoffs.** A network of collaboration $g$ leads to a vector of R&D efforts $\{e_i(g)\}_{i\in N}$, which in turn defines the firms' production costs $\{c_i(g)\}_{i\in N}$. Given these marginal costs, firms operate in the market by choosing quantities $\{q_i(g)\}_{i\in N}$. The demand is assumed to be linear and given by $Q = a - p$, $a > \bar{c}$.[10] In the independent market case, $Q = q_i$ and the profits of firm $i$ in collaboration network $g$ are $\pi_i(g) = [a - q_i - c_i(g)]\, q_i - \gamma e_i^2(g)$. In the homogeneous-good market with quantity-setting firms, $Q = \sum_{i=1}^N q_i$. Thus, the profits of firm $i$ in collaboration network $g$ are given by

$$\pi_i(g) = \left[a - q_i(g) - \sum_{j\neq i} q_j(g) - c_i(g)\right] q_i(g) - \gamma e_i^2(g). \tag{2}$$

□ **Stability and efficiency.** We shall say that a network $g$ is *stable* if and only if for all $i, j \in N$,
(i) if $g_{ij} = 1$, then $\pi_i(g) \geq \pi_i(g - g_{ij})$ and $\pi_j(g) \geq \pi_j(g - g_{ij})$
(ii) if $g_{ij} = 0$ and $\pi_i(g + g_{ij}) > \pi_i(g)$, then $\pi_j(g + g_{ij}) < \pi_j(g)$.

This definition of stability, which is taken from Jackson and Wolinsky (1996), is quite weak and should be seen as a necessary condition for strategic stability.

For any network $g$, social welfare is defined as the sum of consumer surplus and producers' profits. Let $W(g)$ denote aggregate welfare in network $g$. We shall say that a network $g$ is *efficient* if and only if $W(g) \geq W(g')$ for all $g'$. This concept of efficiency is in the spirit of a second best, since efforts and quantities are chosen noncooperatively.[11] When firms operate in independent markets, social welfare is

$$W(g) = \sum_{i=1}^N \left(\frac{q_i^2(g)}{2} + \pi_i(g)\right). \tag{3}$$

Let $Q(g) = \sum_{i\in N} q_i(g)$ be the aggregate output in network $g$ for the homogeneous-product oligopoly. In this case, it is easily seen that

$$W(g) = \frac{Q(g)^2}{2} + \sum_{i=1}^N \pi_i(g). \tag{4}$$

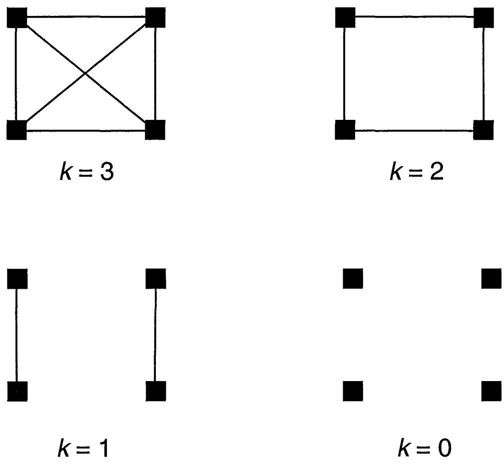
## 3. Symmetric networks

■ A network is said to be *symmetric* if every firm has the same number of collaboration links. In a symmetric network $\eta_i(g) = \eta_j(g) = k$ for any two firms $i$ and $j$. The number $k$ will be

[10] This is admittedly a special setting. We find that even in this simple case, a complete analysis of the interaction between markets and R&D networks is quite complicated, and several interesting questions cannot be completely answered. We intend to take up a more general market specification in future work.

[11] In our model links are free, so the first-best socially optimal network is the complete network along with the competitive output levels and the socially optimal level of R&D effort (which internalizes the spillovers involved in collaboration). We are indebted to the Editor for pointing this out.

FIGURE 1
SYMMETRIC NETWORKS ($n = 4$)



referred to as the *degree* of network $g$ or, equivalently, as the *level of collaborative activity*.[12] We will denote a symmetric network of degree $k$ by $g^k$, $k = 0, 1, \ldots, n-1$. Figure 1 illustrates symmetric networks with four firms. It is worth emphasizing that symmetric networks allow for nonexclusive relationships. For example, a symmetric network of degree 2 involves a firm having links with firms that are not linked to each other.[13] In this section we assume that there are no knowledge spillovers among noncollaborating firms, i.e., $\beta = 0$.

□ **Independent markets.** Collaborations between firms operating in independent markets are commonly observed.[14] In such an environment, individual R&D effort has no implications for the level of market competitiveness of potential collaborators. This setting therefore allows us to isolate the pure effects of collaboration. We find that collaboration between firms increases the level of effort by individual firms. Moreover, every pair of firms has an incentive to form links, and the complete network is the unique stable and efficient network.

*Market outcome.* Given a network $g$, and the R&D efforts levels $\{e_i(g)\}_{i \in N}$, firms choose quantities to maximize their monopoly profits. Standard derivations show that equilibrium quantities are $q_i(g) = [a - c_i(g)]/2$, and profits are

$$\pi_i(g) = \left[\frac{a - c_i(g)}{2}\right]^2 - \gamma e_i^2(g). \tag{5}$$

*R&D efforts.* In the second stage of the game, firms choose their R&D efforts to maximize the reduced-form profits (5). The costs $c_i(g)$ depend on the effort levels undertaken by the firms, which in turn are a function of the existing network. We consider symmetric networks of degree $k$.

[12] If the number of firms is even, then there is always a set of links $l$ such that the resulting network is symmetric of degree $k$, where $k = 0, 1, \ldots, n-1$.

[13] This also allows us to clarify how networks differ from coalition structures. In this four-firm example there are no symmetric coalition structures in which each firm is linked to two other firms, i.e., every firm is a member of a three-firm coalition. More generally, for arbitrary $n$, there do not exist symmetric coalition structures in which every firm is linked to $n/2$ firms or more.

[14] An example of such collaboration is Procordia (food processing) and Biogen (pharmaceuticals); another example is Procordia and Sumitomo (chemicals) (Delapierre and Mytelka, 1998).

Using (1) in (5) and noting that $\beta = 0$, we obtain

$$\pi_i(g^k) = \frac{\left[a - \overline{c} + e_i + \sum_{l \in N_i(g^k)} e_l\right]^2}{4} - \gamma e_i^2.$$

The first-order condition is $(a - \overline{c} + e_i + \sum_{l \in N_i(g^k)} e_l)/4 - \gamma e_i = 0$. We note that profits of firm $i$ are an increasing and convex function of the efforts of collaborating firms; this implies that efforts of linked firms are strategic complements. Invoking symmetry and solving, we obtain[15]

$$e(g^k) = \frac{(a - \overline{c})}{4\gamma - k - 1}; \qquad c(g^k) = \overline{c} - \frac{(a - \overline{c})(k + 1)}{4\gamma - k - 1}. \tag{6}$$

These derivations allow us to state the following:

*Proposition 1.* Suppose firms operate in independent markets. Then individual R&D effort is increasing and costs are decreasing with respect to the level of collaborative activity.

*Strategic stability.* The analysis of stability of networks yields the following result:

*Proposition 2.* Suppose firms operate in independent markets. The complete network is the unique strategically stable network.

*Proof.* See the Appendix.

We note that this result obtains in the class of all networks, symmetric as well as asymmetric. The first step is to show that for any given network $g$, there is a unique pure-strategy equilibrium in effort levels, $e^*(g) = \{e_1^*(g), \ldots, e_n^*(g)\}$. This result follows from the fact that the payoffs in this game satisfy the *contraction property* (Vives, 1999). Let the profit levels corresponding to this equilibrium be given by $\pi^*(g) = \{\pi_1^*(g), \ldots, \pi_n^*(g)\}$. The second step establishes that for any pair of networks $g$ and $g'$ such that $g \leq g'$, $e_l^*(g) \leq e_l^*(g')$ for all $l \in N$ by using Theorem 4.2.2 in Topkis (1999). Finally, we exploit the monotonicity result obtained in step 2 to show $\pi_l^*(g) < \pi_l^*(g + g_{ij})$, for $l = i, j$. This implies that in any network other than the complete network, firms have an incentive to form additional links.

*Welfare.* Our final result looks at the efficiency aspects of collaboration networks.

*Proposition 3.* Suppose firms operate in independent markets. The complete network is uniquely efficient.

*Proof.* From (3) we see that welfare is increasing in the output of the firms and their profits. From the arguments above (in Proposition 2) it follows that for any pair of networks, $g$ and $g' = g + g_{ij}$, $c_l(g') \leq c_l(g)$ for all $l \in N$. As a result $q_l(g) \leq q_l(g')$ and therefore $\pi_l(g) \leq \pi_l(g')$ for all $l \in N$. Finally, we know from above that $\pi_m(g) < \pi_m(g')$, for $m = i, j$; the result follows. *Q.E.D.*

□ **Homogeneous-product oligopoly.** Collaborations between firms operating in the same market are often observed. A firm's R&D effort lowers the costs of its collaborators, which in this setting makes them more competitive. This reduces the marginal returns to investing in R&D as additional links are formed. Our analysis illustrates how this competition effect influences the private and social incentives to collaborate. Our first observation concerns the effects of collaboration on the R&D effort of individual firms.

*Market outcome.* In the market competition stage, we note that given a cost configuration of firms, $\{c_i(g)\}_{i \in N}$, the equilibrium quantity of firm $i$ in a homogeneous-product oligopoly is given

[15] In the independent-markets case, we assume $\gamma > \max\{an/4\overline{c}, n/4\}$. This ensures that second-order conditions are satisfied and that firms' effective costs are positive.

by

$$q_i(g) = \frac{a - nc_i(g) + \sum_{j \neq i} c_j(g)}{n+1}, \tag{7}$$

and the profits of the Cournot competitors are given by

$$\pi_i(g) = \left(\frac{a - nc_i(g) + \sum_{j \neq i} c_j(g)}{n+1}\right)^2 - \gamma e_i^2(g). \tag{8}$$

With this profit expression, we can compute the payoff to the representative firm $i$ in $g^k$. We note that there are three types of firms, namely, (i) the firm $i$, (ii) $k$ firms linked to firm $i$, which we represent with subscript $l$, and (iii) $n - k - 1$ firms not linked to firm $i$, which we represent with subscript $m$.

*R&D efforts.* Let $e_l$ denote the R&D effort undertaken by a firm linked to firm $i$ and $e_m$ be the corresponding effort of firms not linked to firm $i$. Then the cost structure of the different Cournot competitors is as follows:

$$c_i(g^k) = \bar{c} - e_i - ke_l; \qquad c_l(g^k) = \bar{c} - e_l - \sum_{j \in N_l(g)} e_j; \qquad c_m(g^k) = \bar{c} - e_m - \sum_{j \in N_m(g)} e_j. \tag{9}$$

Plugging these costs into the profits function in (8) and using symmetry for the linked and nonlinked firms (with respect to firm $i$), respectively, we obtain

$$\pi_i(g^k) = \frac{[a - \bar{c} + e_i(n-k) + e_l(n-k)k - e_m(k+1)(n-k-1)]^2}{(n+1)^2} - \gamma e_i^2. \tag{10}$$

The first-order condition is[16]

$$(n-k)\,[a - \bar{c} + e_i(n-k) + e_l(n-k)k - e_m(k+1)(n-k-1)] - \gamma(n+1)^2 e_i = 0. \tag{11}$$

Notice that the efforts of linked firms are strategic complements, while the efforts of nonlinked firms are strategic substitutes. This contrasts with the independent markets case where the efforts of nonlinked firms are irrelevant. Invoking symmetry, i.e., $e_i = e_l = e_m = e(g^k)$ and solving for $e(g^k)$, we obtain the equilibrium effort level:

$$e(g^k) = \frac{(a - \bar{c})\,(n-k)}{\gamma(n+1)^2 - (n-k)(k+1)}. \tag{12}$$

Our first observation is that this equilibrium level of R&D effort is declining in the level of collaborative activity $k$. We establish this by showing that

$$e(g^k) - e(g^{k+1}) = \frac{(a - \bar{c})\left[\gamma(n+1)^2 - (n-k)(n-k-1)\right]}{\left[\gamma(n+1)^2 - (n-k)(k+1)\right]\left[\gamma(n+1)^2 - (n-k-1)(k+2)\right]} > 0. \tag{13}$$

We note that the numerator is positive (from the second-order condition), while the denominator is positive because $k \leq n - 1$. This leads us to state the following result:

*Proposition 4.* Suppose firms are competitors in a homogeneous-product market. Then R&D effort of a firm is decreasing in the level of collaborative activity.

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[16] In the homogeneous-product market case we shall assume $\gamma > \max\{n^2/(n+1)^2, a/4\bar{c}\}$. This ensures interiority of the solutions for all the optimization problems we consider except for the proof of Proposition 6, where we use a slightly stronger condition.

We observe that the marginal cost of R&D effort is an increasing function of $e_i$, with slope $2\gamma$ (which is independent of $k$). The marginal revenue of R&D effort is also an increasing function of $e_i$, but with the smaller slope $2(n-k)^2/(n+1)^2$ (this follows from the condition $\gamma > n^2/(n+1)^2$ in footnote 16). Thus, Proposition 4 implies that the marginal return to R&D effort is, at least "locally," declining in $k$. An increase in the degree $k$ leads to an increase in the number of collaborators for every firm. A firm's research effort reduces its own production cost, but it also lowers the costs of collaborators, which makes them tougher competitors. Thus an increase in the number of collaborators has mixed effects. In addition, since every firm has more collaborators, it operates at lower costs, and this lowers the returns to cost reduction for a nonlinked firm. The overall effect of these forces is that R&D effort declines with the degree of the network.

We now examine the nature of cost reduction. By substituting (12) into the cost structure (9), we obtain

$$c(g^k) = \frac{\overline{c}\gamma(n+1)^2 - a(n-k)(k+1)}{\gamma(n+1)^2 - (n-k)(k+1)}.$$

From the difference,

$$c(g^k) - c(g^{k+1}) = \frac{(a-\overline{c})\gamma(n+1)^2(n-2k-2)}{\left[\gamma(n+1)^2 - (n-k)(k+1)\right]\left[\gamma(n+1)^2 - (n-k-1)(k+2)\right]},$$

we can see that an increase in the level of collaborations reduces the cost of the firms if and only if $k < n/2 - 1$.

*Proposition 5.* Suppose firms are competitors in a homogeneous-product market. Then the relationship between cost reduction and the level of collaborative activity is nonmonotonic. Moreover, cost reduction is maximum when each firm is linked with roughly half of the other firms.

This result shows that in relatively sparse networks the cost-reducing benefits of an extra collaboration are substantial as compared to the detrimental effects arising from the induced decrease in the R&D activity of the firms (Proposition 4). On the other hand, if the network is relatively dense, the latter effect dominates and an increase in the level of collaborative activity results in an increase in the firms' operating costs. Thus, cost reduction exhibits a nonmonotonic relationship with respect to the density of the network.

*Strategic stability.* The profits attained by a firm in a symmetric network of degree $k$ can be obtained by substituting the equilibrium level of effort and costs into (8):

$$\pi(g^k) = \frac{(a-\overline{c})^2\gamma\left[\gamma(n+1)^2 - (n-k)^2\right]}{\left[\gamma(n+1)^2 - (n-k)(k+1)\right]^2}. \tag{14}$$

In Proposition 6 below, we assume $\gamma = 1$. This allows us to explicitly compare the relevant expressions for profits. However, we believe that the result is true for larger values of $\gamma$ (as specified in the proof); in these cases, the expressions are complicated and clear comparisons are difficult, so we use a series of plots on the behavior of profits.[17]

*Proposition 6.* Suppose firms are competitors in a homogeneous-product market. Then the empty network is not stable, while the complete network is stable.

*Proof.* See the Appendix.

This result suggests that firms' incentives to form collaborations are quite large, and it leads us to ask: Is the complete network the unique stable symmetric network? For $n = 4$ we have

[17] These graphs are available from the authors upon request. They can also be accessed via the Web page of the Journal (www.rje.org).

shown that this is true. However, we face two problems to answer this question for general $n$. The first problem arises due to the indirect effects of an additional link. To illustrate, consider a six-firm cycle and suppose firms are numbered clockwise. In this network, if firm 1 forms an additional link with firm 3, then there are four types of firms: $\{1, 3\}$, $\{4, 6\}$, $\{2\}$, and $\{5\}$. Firms 1 and 3 have an extra link compared to the rest, while the other groups differ from each other with respect to their "distance" from 1 and 3. Thus, firm 2 is affected directly by firms 1 and 3, while firm 5 is indirectly affected via the efforts of firms 4 and 6. This implies that we must solve a system of four equations. The second problem is that a single additional link can have different effects depending on where it is. To illustrate, consider the same example as above. If firm 1 forms an additional link with firm 4 instead, then we have only two types of firms in the resulting network: $\{1, 4\}$ and $\{2, 3, 4, 6\}$. This implies that we have to solve a very different set of optimization problems, with two equations. These two problems make a complete analysis of stability for arbitrary $n$ quite difficult.

*Aggregate performance.* We first show that in the class of symmetric networks, aggregate profits are highest for intermediate levels of collaboration. To establish this, we first compare the profits a firm obtains under the complete network (the network of degree $k = n - 1$) with the benefits the same firm would obtain in a network of one degree less, i.e., $k = n - 2$. In the Appendix we show that $\pi(g^{n-2}) - \pi(g^{n-1}) > 0$. To complete the argument, it is now sufficient to show that firms obtain higher benefits under the complete network as compared to the empty network ($k = 0$):

$$\pi(g^{n-1}) - \pi(g^0) = \frac{(a - \overline{c})^2 \gamma (n^2 - 1)}{\left[\gamma(n+1)^2 - n\right]^2} > 0.$$

*Proposition 7.* Suppose firms are competitors in a homogeneous-product market. Then there exists an intermediate level of collaborative activity $\overline{k}$ with $0 < \overline{k} < n - 1$ for which firms' profits are maximized.

The intuition behind this result may be seen with the help of an example. Figure 2 depicts firms' profits in an industry with 16 firms for symmetric networks with different degrees of collaboration $k$ (parameters are $a - \overline{c} = 1$, $\gamma = 1$). At the left end of the horizontal axis we have the empty network, while at the right end we have the complete network. It can be seen that profits are lowest under the empty network in this case. As the number of links rises, firm performance improves because the effects of lower research efforts are offset by the saving in costs due to the sharing of R&D. For high values of $k$, however, the impact of lower efforts dominates and profits decline. For further illustration, let us suppose $n$ and $k$ are continuous variables and maximize $\pi(g^k)$ with respect to $k$. We find that the number of links $k_f$ that maximizes aggregate profits satisfies the following equation:

$$\gamma(2n - 3k_f - 1)(n+1)^2 - (n - k_f)^3 = 0. \tag{15}$$

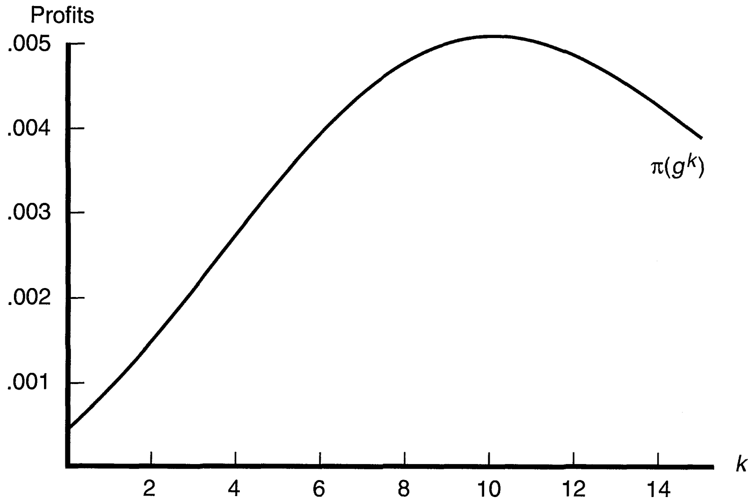
It is easily checked that $dk_f/dn > 0$, which suggests that the number of collaborations that maximizes aggregate profits is greater in larger markets. Likewise, we find that $dk_f/d\gamma > 0$, which implies that firms would collectively wish to collaborate more intensively in environments where R&D activity is more costly.

We now consider the social welfare aspects of collaboration networks. Using the optimal effort $e(g^k)$ in (12) and the cost structure (9) one easily obtains $q(g^k)$. Substituting this value in (4) we obtain welfare in a network of degree $k$:

$$W(g^k) = \frac{(a - \overline{c})^2 n \gamma \left[\gamma\,(n+2)\,(n+1)^2 - 2\,(n-k)^2\right]}{2\left[\gamma(n+1)^2 - (n-k)(k+1)\right]^2}. \tag{16}$$

FIGURE 2

FIRM PROFITS IN A SYMMETRIC NETWORK OF DEGREE $k$ ($n$ = 16)




Our first finding is that the empty network is not efficient. We prove this by showing that social welfare under the complete collaboration agreement is higher than it is in the absence of collaborations. Setting $k = 0$ in (16) yields welfare under the empty network $W(g^e)$. Setting $k = n - 1$ in (16) yields welfare under the complete network $W(g^{n-1})$. It is easily verified that $W(g^{n-1}) - W(g^e) > 0$. This result implies that some degree of collaboration is desirable from the point of view of the society at large.

Secondly, we note that neither industry profits nor cost reductions are maximal under the complete network. This raises questions about the efficiency of the complete network. Indeed, we find that the complete network generally leads to too little effort and is therefore not socially efficient. We prove this by showing that $W(g^{n-2}) - W(g^{n-1}) > 0$. The details of the computations are given in the Appendix.[18]

*Proposition 8.* Suppose firms are competitors in a homogeneous-product market. Then there exists some intermediate level of collaborative activity $\tilde{k}$ with $0 < \tilde{k} < n - 1$ for which social welfare is maximized.

For further illustration, we suppose that $n$ and $k$ are continuous variables and maximize $W(g^k)$ in (16) with respect to $k$. We find that the efficient network (in the class of symmetric networks) has $k_w$ links where $k_w$ satisfies

$$\gamma(n+1)^2\,[(n+3)(n-2k_w)-2] - 2(n-k_w)^3 = 0. \qquad (17)$$

It can be verified that $dk_w/dn > 0$, which suggests that the desirability of collaboration is greater in larger markets. Moreover, $dk_w/d\gamma > 0$, which means that the more costly R&D is, the more collaborations are needed to maximize social welfare. A further implication of equations (15) and (17) is that industry-profit-maximizing symmetric networks exhibit an excessive level of collaborative activity from a welfare viewpoint.[19] This follows from noting that $dW(g^k)/dk|_{k=k_f} < 0$.[20]

[18] We note that the following result obtains in the class of symmetric networks. We also draw attention to the fact that welfare is computed assuming noncooperative firm behavior.

[19] This remark is in line with Yi (1998a) and Yi and Shin (2000).

[20] The details of the derivation are given in the Appendix.

Seen together, the results obtained for independent markets and homogeneous-product markets yield a number of observations. First, we note that firms generally have an incentive to collaborate, irrespective of the product-market setting, so the empty network is never incentive compatible. Second, in the independent-markets case individual R&D effort increases, while in the homogeneous-product case this effort declines with the level of collaborative activity. This is due to the fact that in the latter context, increased collaboration activity generates business-stealing effects that dampen the firms' incentives to invest in cost-reducing activities. Third, this difference in firms' behavior has important consequences for social welfare. In the independent markets case, individual considerations lead firms to form links with all other firms, thereby generating the complete network, which is socially optimal. In contrast, when firms are competitors, the complete network is also consistent with individual incentives but is not optimal.

## 4. Asymmetric networks and knowledge spillovers

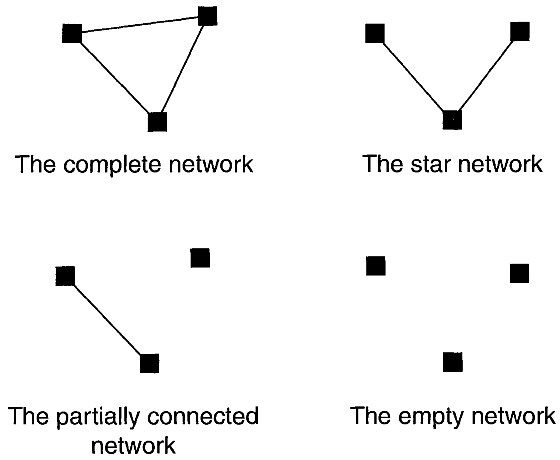
■ In the analysis so far, we have restricted attention to symmetric R&D networks (for the most part) and also assumed that knowledge spillovers across unlinked firms are absent. In this section we model knowledge spillovers and also allow for asymmetric networks. The aim of this section is threefold. First, we show that asymmetric networks play an important role in collaboration because they may be stable and industry-profit maximizing as well as welfare maximizing. Second, we see that asymmetric forms of collaboration may substantially alter the market structure by causing significant disparities between firms and, in extreme cases, even the exit of firms. We shall also find, somewhat surprisingly, that this is not necessarily detrimental from a social perspective. Third, we investigate how substantial knowledge spillovers across firms interact with firms' incentives to form collaborative agreements and their consequences from a social standpoint. Undertaking this analysis in general is quite complicated. So we have chosen to restrict ourselves to the simplest possible case that allows us to illustrate our points.

We consider a three-firm market for a homogeneous good with the following additional features: On the one hand, we shall assume that there is a positive knowledge spillover $\beta \in [0, 1]$ between firms that have no collaboration link. On the other hand, we assume $\gamma = 1$, which suffices to ensure nonnegativity of all variables. There are four possible network architectures in this case: (i) the complete network, $g^c$, in which every pair of firms is linked, (ii) the star network, $g^s$, in which there is one firm that is linked to the other two firms, (iii) the partially connected network, $g^p$, in which two firms have a link and the third firm is isolated, and (iv) the empty network, $g^e$, in which there are no collaboration links. We present these networks in Figure 3. We derive the equilibrium level of R&D effort, the quantity, and the profits of every firm for every network. We then use these values first to observe how a firm R&D effort changes as its links vary, and also as the links of other firms change. Second, we study the incentives of firms to form collaborative relationships. This tells us which networks are stable from a strategic point of view. Finally, we examine the nature of efficient networks, from the firms' perspective as well as from the point of view of society at large.

□ **Market competition.** Given a network $g$ and the R&D efforts $\{e_1(g), e_2(g), e_3(g)\}$, firms choose quantities to maximize profits in (2). Equilibrium quantities are given in (7).

*R&D efforts.* In the second stage of the game, firms choose their R&D efforts to maximize the reduced-form profits given in (8). As before, the costs $c_i(g)$, $i = 1, 2, 3$ depend on the effort levels undertaken by the firms, which in turn are a function of the existing network. We consider the different networks now and derive the equilibrium levels of effort and firm profits. The details of these computations are given in the Appendix. Equilibrium effort levels and profits only depend on $\beta$ (except for a scale parameter) and are exhibited in Figures 4 and 5. In Figure 4, $e_h(g^s)$ refers to the effort of the hub firm, while $e_s(g^s)$ refers to the effort of the spoke firms, respectively, in the star network. Similarly, $e_l(g^p)$ refers to the effort of the linked firms, while $e_i(g^p)$ refers to the effort of the isolated firm, respectively, in the partially connected network. The other effort terms are self-explanatory. In Figure 5, profits curves are interpreted similarly.

FIGURE 3
NETWORKS FOR $n = 3$

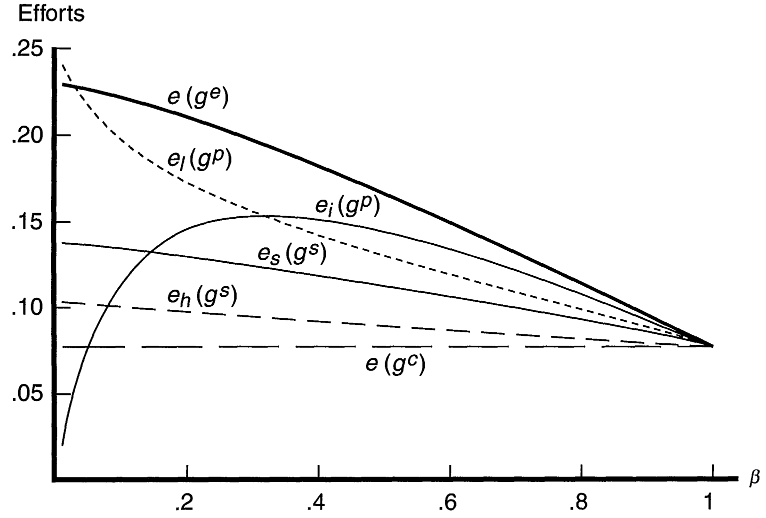



From Figure 4 we observe that typically, as a firm forms additional links, it lowers its effort levels: $e(g^e) > e_l(g^p) > e_h(g^s)$. We also note that a firm decreases effort if the other two firms form an additional link: $e_i(g^p) < e(g^e)$, $e_s(g^s) < e_l(g^p)$, and $e(g^c) < e_h(g^s)$. Using Figure 5, we can study the stability of different networks. Define $\beta_1$ as the solution to equation $\pi_l(g^p) = \pi_h(g^s)$ (the graph shows that $\beta_1$ exists and is unique). Applying our notion of stable network presented above, the following result obtains:[21]

*Proposition 9.* The complete network is stable for all $\beta \in [0, 1]$, while the partially connected network is stable for all $\beta \in [0, \beta_1]$. The star network and the empty network are never stable.

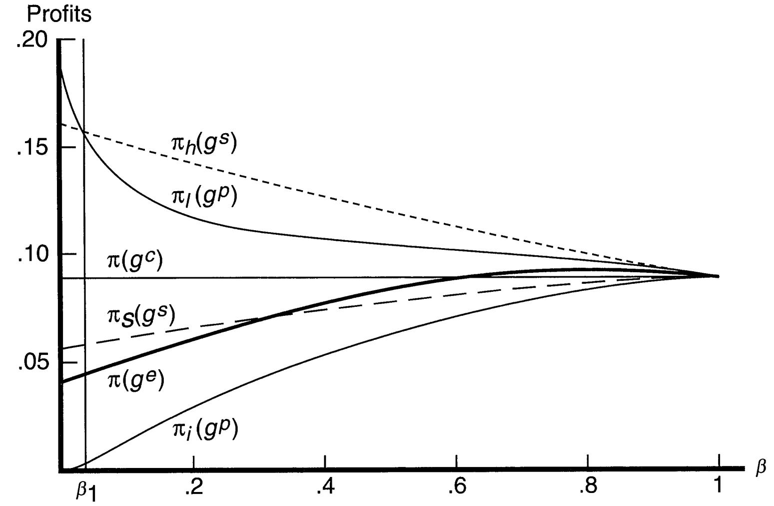
We elaborate on some aspects of this result. Our first observation is that the partially connected network is stable. In this network the structure of the market is very asymmetric, with the isolated firm having a significant cost disadvantage; in some extreme cases, it is driven out of the market altogether. This outcome is in line with the empirically documented observation that firms

FIGURE 4
FIRMS' R&D EFFORTS




[21] The details of the proof are omitted here to economize on space. The proof is available from the authors upon request and can also be accessed via www.rje.org.

FIGURE 5
INDIVIDUAL FIRM PROFITS




often seek collaborative relationships in an attempt to alter market structure to their own benefit (Hagedoorn and Schakenraad, 1990). Seen from another perspective, the fact that firms without collaboration ties can be forced out of the market may be an important reason behind the recently observed proliferation of strategic alliances and interfirm collaboration agreements as businesses seek partners as a survival strategy (Delapierre and Mytelka, 1998).

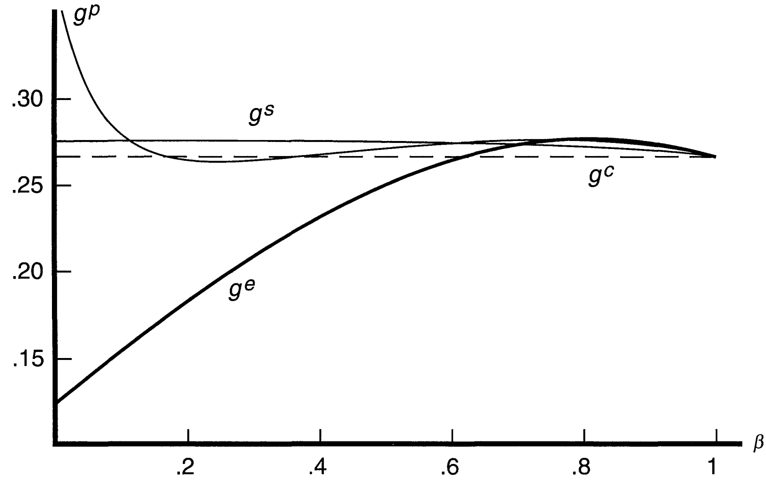
Our second observation pertains to the relationship between public spillovers and networks. As $\beta$ grows, the profits of the firms in the different networks become similar, irrespective of the network structure (in the limiting case $\beta = 1$ they are all equal). Thus network structures are more important when spillovers are modest.

Our third remark concerns the impact of spillovers on the stability of different networks. Greater spillovers destabilize the partially connected network rapidly. The intuition behind this remark is that the stability of the partially connected network relies on the great asymmetry existing between the linked firms and the isolated firm. It is this asymmetry that discourages a linked firm from forming a link with the isolated firm, for low spillovers. As $\beta$ increases, this asymmetry reduces, and that destabilizes the partially connected network. In contrast, the complete network remains stable for all values of $\beta$; we note however that the losses from deleting a link diminish as $\beta$ increases (in this sense the complete network also becomes more vulnerable with increasing $\beta$).

We now explore the aggregate performance of different networks. We plot the aggregate profits of firms in Figure 6. In this figure aggregate profits under the complete, empty, star, and partially connected networks are represented by the curves $g^c$, $g^e$, $g^s$, and $g^p$, respectively. The figure reveals that the complete network is dominated in terms of industry profits for all $\beta$. Moreover, the number of collaborations that maximize aggregate profits varies in a nonmonotonic manner with respect to the spillover parameter.

We now provide some intuition for this pattern. Recall that in our analysis of symmetric networks $\beta = 0$ and an intermediate level of collaboration is desirable in terms of industry profits. Collaborations have two effects that work in opposite directions: on the one hand, denser networks lead to greater interfirm spillovers. On the other hand, well-connected firms undertake very little R&D effort. In the same spirit, here, for low values of $\beta$, the empty and the complete networks are dominated by the partially connected and the star. As $\beta$ increases, the asymmetries in the partially connected network—in efforts as well as profits—become less pronounced, as explained above (in connection with Proposition 9). This leads to an increase in the attractiveness of the star. Finally, when $\beta$ becomes large, public spillovers can be seen as a substitute for collaborative links—they

FIGURE 6
AGGREGATE PROFITS



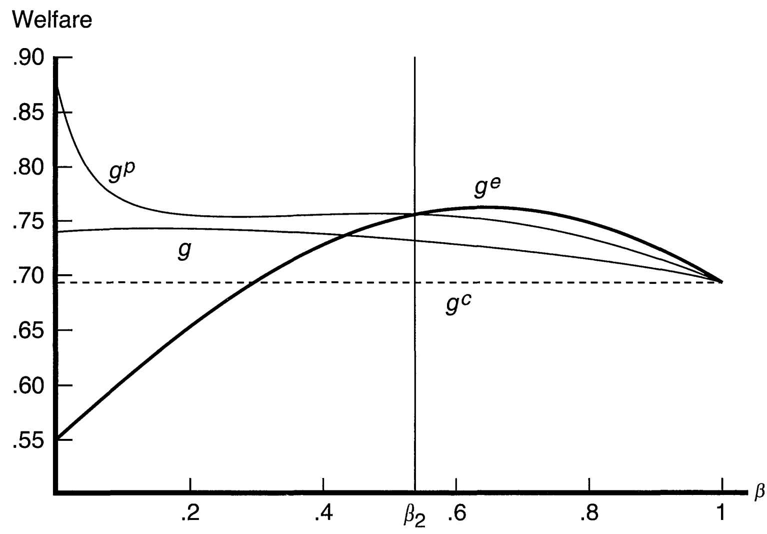

also lead to greater sharing of R&D results and, at the same time, do not lower R&D effort so severely—and thus the partially connected and eventually the empty network is industry-profit maximizing.

We finally examine social welfare under the different networks. To compute social welfare under a network $g$ we substitute equilibrium quantities and profits in the social welfare expression (4). These computations are given in the Appendix. We plot the welfare levels under the different networks in Figure 7. Define $\beta_2$ as the solution to equation $W(g^p) = W(g^e)$. The figure shows that $\beta_2$ exists and is unique. We are ready to state the following proposition:

*Proposition 10.* The partially connected network is efficient for all $\beta \in [0, \beta_2]$, while the empty network is efficient for all $\beta \in [\beta_2, 1]$. Moreover, the partially connected network and the star network always dominate the complete network in terms of social welfare.

The above result shows that the welfare-maximizing number of collaborations declines with respect to the spillover parameter. For low spillover parameters, the partially connected network

FIGURE 7
WELFARE LEVELS



is efficient; the intuition here is the same as in the general model where intermediate levels of collaborations are welfare maximizing. When $\beta$ becomes large, public spillovers can be seen as an efficient substitute for the collaborative link—as mentioned above, they lead to greater sharing of R&D results and do not lower R&D effort severely—and thus the empty network is efficient.

Propositions 9 and 10 show that individual and social incentives to form collaborative agreements need not be aligned. For low-knowledge spillovers, $\beta \in [0, \beta_1]$, the partially connected network is stable, socially efficient, and profit maximizing. However, the complete network is stable for all spillover parameters, while it is never efficient or industry-profit maximizing. Thus, for $\beta > \beta_1$, the private incentives to form collaborations are unambiguously excessive from a social welfare viewpoint.

## 5. Conclusion

■ There is a large body of research that describes the growing use of interfirm collaboration in R&D activity. This literature has emphasized two distinctive features of collaborative relationships: bilateral agreements and nonexclusive relationships. This article develops a strategic model of the formation of collaboration networks that incorporates these two structural features. The model allows us to naturally define the level of collaborative activity in terms of the number of links that a firm maintains in a collaboration network. Our analysis highlights the interaction between market competition, the incentives for R&D, and the architecture of collaboration networks.

We find that if firms are Cournot competitors, then individual R&D effort is declining in the level of collaborative activity. However, due to greater sharing of R&D results under collaboration, cost reduction is initially increasing and then decreasing with respect to the level of collaborative activity. Thus, cost reduction is maximum at intermediate levels of collaboration. We also find that intermediate levels of collaborative activity are better for industry profits as well as for social welfare. An examination of firms' incentives to form collaborative links shows that the complete network is strategically stable. This implies that firms may have excessive incentives to form collaborative links. By contrast, if firms operate in independent markets, then individual R&D effort is increasing in the level of collaborative activity. Cost reduction and social welfare are maximized under the complete network, which is also the unique strategically stable network.

We note that our analysis has been carried out in a simple setting with specific functional forms for demand and cost conditions. Moreover, in the model with an arbitrary number of firms, our analysis has focused on the class of symmetric networks. The analysis of asymmetric networks in a three-firm example suggests that these networks have attractive welfare and stability properties. In future work we hope to explore the relationship between networks, firms' incentives to invest, and markets, in a more general setting.

## Appendix

■ Proofs of Propositions 2, 6, 7, and 8, a proof that $dW(g^k)/dk|_{k=k_f} < 0$, and the derivations for the three-firm example with spillovers follow.

*Proof of Proposition 2.* We first note that "$\leq$" defines a *partial order* on the set of networks $G$. To see this notice that the following properties hold: (i) Reflexive: $g \leq g$ for all $g \in G$. (ii) Antisymmetric: if $g \leq g'$ and $g' \leq g$, then $g = g'$, for all $g, g' \in G$. (iii) Transitive: if $g \leq g'$ and $g' \leq g''$, then $g \leq g''$, for all $g, g', g'' \in G$. We now observe that firm $i$'s strategy set $E_i = [0, \bar{c}]$ is nonempty and compact, and that since $E_i \subset R$, $E_i$ is a lattice. Let $E = E_1 \times E_2 \times \cdots \times E_n \subset R^n$. Firm $i$'s payoff is a function $\pi_i : E \times G \to R$ given by

$$\pi_i\left(\{e_i(g)\}_{i\in N}\right) = \frac{\left[a - \bar{c} + e_i + \sum_{l\in N_i(g)} e_l\right]^2}{4} - \gamma e_i^2. \tag{A1}$$

The first step in the proof is to show that for any given network $g$, there is a *unique* pure-strategy Nash equilibrium in effort levels $\{e_i^*(g)\}_{i\in N}$. To show this, note that the following properties hold in our game: (i) firm $i$'s strategy set is unidimensional; (ii) the profit function is continuous and, for $\gamma$ large (second-order conditions), concave in $e_i$. Then the firms' best replies are unique and given by a function. Notice that in our game the sufficient condition

$$\frac{\partial^2 \pi_i}{\partial e_i^2} = \frac{2}{4} - 2\gamma < \sum_{j \neq i} \left| \frac{\partial^2 \pi_i}{\partial e_i \partial e_j} \right| = \frac{2}{4} - \eta_i(g)$$

is satisfied for $\gamma$ large enough. Therefore, the set of best-reply functions satisfy the contraction property (Vives, 1999), and there is a unique Nash equilibrium in effort levels for any network $g \in G$. Let the profit levels corresponding to this equilibrium be given by $\pi^*(g) = \{\pi_1^*(g), \ldots, \pi_n^*(g)\}$.

The second step establishes that for any pair of networks $g$ and $g'$ in the ordered set $G$ such that $g \leq g'$, $e_l^*(g) \leq e_l^*(g')$ for all $l \in N$. In this step we apply Theorem 4.2.2 in Topkis (1999). We now check that the hypotheses of this theorem are satisfied in our case: (i) $G$ is a partially ordered set. (ii) $\{N, E, \{\pi_i : E \times G \to R\}_{i \in N}\}$ is a collection of strictly supermodular games parametrized by $g \in G$. We have noted that $E_i$ is a compact lattice. The profits function is clearly continuous and supermodular in $e_i$ for fixed $e_{-i}$, and it is easily verified that it exhibits strictly increasing differences in $(e_i, e_{-i})$; the hypothesis follows. (iii) We have noted that the set of strategies $E$ is nonempty and compact. Note also that it is constant with respect to $g$. (iv) Finally, we need to check that the profit function $\pi_i$ exhibits increasing differences in $(e_i, g)$ on $E_i \times G$. Let $e_i' \geq e_i$ and $g' \geq g$. Then, it must be the case that $\pi_i(e_i', e_{-i}; g') - \pi_i(e_i, e_{-i}; g') \geq \pi_i(e_i', e_{-i}; g) - \pi_i(e_i, e_{-i}; g)$. Using (A1) we can rewrite this inequality (after some simple algebraic manipulations) as follows:

$$\left[\sum_{l \in N_i(g)} e_l - \sum_{l \in N_i(g')} e_l\right]\left[2(a - \overline{c}) + 2e_i + \sum_{l \in N_i(g)} e_l + \sum_{l \in N_i(g')} e_l\right]$$
$$\geq \left[\sum_{l \in N_i(g)} e_l - \sum_{l \in N_i(g')} e_l\right]\left[2(a - \overline{c}) + 2e_i' + \sum_{l \in N_i(g)} e_l + \sum_{l \in N_i(g')} e_l\right].$$

Since $\sum_{l \in N_i(g)} e_l - \sum_{l \in N_i(g')} e_l \leq 0$, we have

$$2(a - \overline{c}) + 2e_i + \sum_{l \in N_i(g)} e_l + \sum_{l \in N_i(g')} e_l \leq 2(a - \overline{c}) + 2e_i' + \sum_{l \in N_i(g)} e_l + \sum_{l \in N_i(g')} e_l,$$

which holds since $e_i' \geq e_i$.

In the third step we show that any pair of firms $i$, $j$ that form a new link will strictly increase their profits. Given that $e_l(g) \leq e_l(g + g_{ij})$, for all $l \in N$ (step 2), it follows from an inspection of (A1) that $\pi_i(e_i(g), e_{-i}(g'); g') > \pi_i(e_i(g), e_{-i}(g); g)$. Since $e_i^*(g')$ is optimal, it follows that $\pi_i^*(g) < \pi_i^*(g + g_{ij})$. The same holds for firm $j$. *Q.E.D.*

*Proof of Proposition 6.* We first prove that the empty network is not stable. We can derive the nature of efforts, quantities, and profits in the empty network by setting $k = 0$ in the above expressions. Profits under the empty network are

$$\pi(g^e) = \frac{(a - \overline{c})^2 \gamma \left[\gamma(n + 1)^2 - n^2\right]}{\left[\gamma(n + 1)^2 - n\right]^2}. \tag{A2}$$

Notice that the stability condition (i) is trivially satisfied because the empty network has no links formed at all. Thus, we only need to check whether condition (ii) is satisfied. Consider that some firm $i$ forms a link with some firm $j$. The resulting network is $g^e + g_{ij}$. We next show that firms $i$ and $j$ find such a deviation profitable. In $g^e + g_{ij}$ there are two types of firms: (i) firms $i$ and $j$, which maintain one link; and (ii) the rest $n - 2$ firms, which have no link. We will look for symmetric solutions. Let us denote the research effort of the latter $n - 2$ isolated firms as $e_m$, and the representative firm in this group as firm $l$. The costs of the different types of firms in network $g^e + g_{ij}$ are $c_i = \overline{c} - e_i - e_j$; $c_l = \overline{c} - e_l$. Using these cost expressions we can obtain firm $i$ profits:

$$\pi_i(g^e + g_{ij}) = \frac{\left[a - \overline{c} + (n - 1)(e_i + e_j) - e_l - (n - 3)e_m\right]^2}{(n + 1)^2} - \gamma e_i^2. \tag{A3}$$

The first-order condition is $(n - 1)[a - \overline{c} + (n - 1)(e_i + e_j) - e_l - (n - 3)e_m] - \gamma\,(n + 1)^2 e_i = 0$. Note that the R&D efforts of firms that form a collaboration become strategic complements. The representative firm $l$ in the group of firms with no links maximizes

$$\pi_l(g^e + g_{ij}) = \frac{\left[a - \overline{c} + ne_l - 2(e_i + e_j) - (n - 3)e_m\right]^2}{(n + 1)^2} - \gamma e_l^2.$$

The first-order condition is $n[a - \overline{c} + ne_l - 2(e_i + e_j) - (n-3)e_m] - \gamma(n+1)^2 e_l = 0$. Invoking symmetry, i.e., $e_i = e_j$ and $e_l = e_m$, we obtain the best-response functions of the firms under consideration:

$$e_i = \frac{(n-1)(a - \overline{c} - (n-2)e_l)}{\gamma\,(n+1)^2 - 2(n-1)^2} \qquad e_l = \frac{n(a - \overline{c} - 4e_i)}{\gamma\,(n+1)^2 - 3n}.$$

It can be seen that an interior Nash equilibrium exists only if $\gamma > 2(n-1)/(n+1)$. For $\gamma = 1$, there is a corner solution where $e_l(g^e + g_{ij}) = 0$ and $e_i(g^e + g_{ij}) = (a - \overline{c})(n-1)/(n(6-n) - 1)$ if $n = 4$. Plugging these R&D efforts into (A3) we obtain firm $i$'s deviating profits:

$$\pi_i(g^e + g_{ij}) = \frac{(a - \overline{c})^2\, 4n}{[n(6-n) - 1]^2}. \tag{A4}$$

Setting $\gamma = 1$ in (A2), it is easily seen that $\pi_i(g^e + g_{ij}) - \pi(g^e) > 0$. If $n \geq 6$, then similar computations reveal that deviating profits are greater than equilibrium profits.

We next prove that the complete network is stable. We can derive the nature of efforts, quantities, and profits in the complete network by setting $k = n - 1$ in the expressions above. Note that profits under the complete collaborative agreement are

$$\pi(g^{n-1}) = \frac{(a - \overline{c})^2\,\gamma\left[\gamma(n+1)^2 - 1\right]}{\left[\gamma(n+1)^2 - n\right]^2}. \tag{A5}$$

We only need to check that the stability condition (i) is satisfied because there is no additional link to be formed and thus condition (ii) is trivially satisfied. Consider that some firm $i$ deletes its link with some firm $j$. The resulting network is $g^{n-1} - g_{ij}$. We next show that firm $i$ finds such a deviation unprofitable. In $g^{n-1} - g_{ij}$ there are two types of firms: on the one hand, firms $i$ and $j$, which have $n-2$ links; and on the other hand, the rest of the firms, which have $n-1$ links. We will look for symmetric solutions. Let us denote the effort of the latter $n-2$ firms that maintain $n-1$ collaborations as $e_m$, and the representative firm in this group as firm $l$. The costs of the different types of firms are $c_i = \overline{c} - e_i - (n-2)e_m$; $c_l = \overline{c} - e_l - e_i - e_j - (n-3)e_m$. Using these cost expressions we can write firm $i$ profits as

$$\pi_i(g^{n-1} - g_{ij}) = \frac{\left[a - \overline{c} + 2e_i - (n-1)e_j + (n-2)e_m\right]^2}{(n+1)^2} - \gamma e_i^2.$$

The first-order condition is $2[a - \overline{c} + 2e_i - (n-1)e_j + (n-2)e_m] - \gamma(n+1)^2 e_i = 0$. Note that the R&D efforts of firms breaking a collaboration link become strategic substitutes. The representative firm $l$ in the group of firms with $n-1$ links maximizes

$$\pi_l(g^{n-1} - g_{ij}) = \frac{\left[a - \overline{c} + 2e_i + 2e_j + e_l + (n-3)e_m\right]^2}{(n+1)^2} - \gamma e_l^2.$$

The first-order condition is $a - \overline{c} + 2e_i + 2e_j + e_l + (n-3)e_m - \gamma\,(n+1)^2\,e_l = 0$. Invoking symmetry, i.e., $e_i = e_j$ and $e_l = e_m$, we obtain the R&D efforts of the different types of firms in the deviation network:

$$e_i(g^{n-1} - g_{ij}) = \frac{2\gamma\,(a - \overline{c})\,(n+1)}{\gamma^2(n+1)^3 + \gamma(n+1)(n-4) - 2n + 4}$$
$$e_m(g^{n-1} - g_{ij}) = \frac{(a - \overline{c})\,(\gamma(n+1) + 2)}{\gamma^2(n+1)^3 + \gamma(n+1)(n-4) - 2n + 4}.$$

We can substitute these expressions into (7) and (8) to obtain deviating profits:

$$\pi_i(g^{n-1} - g_{ij}) = \frac{\gamma^2\,(a - \overline{c})^2\,(n+1)^2(\gamma^2(n+1)^2 - 4)}{\left[\gamma^2(n+1)^3 + \gamma(n+1)(n-4) - 2n + 4\right]^2}. \tag{A6}$$

Set $\gamma = 1$ in (A5) and (A6) and establish the comparison

$$\pi(g^{n-1}) - \pi_i(g^{n-1} - g_{ij}) = \frac{(a - \overline{c})^2\,B}{(n^2 + n + 1)^2(n^3 + 4n^2 - 2n + 1)^2},$$

where $B = 4n^7 + 15n^6 - 4n^5 - 20n^4 + 30n^3 + 8n^2 + 12n + 3$. It can be seen that $B > 0$, and the result follows. *Q.E.D.*

*Proof of Proposition 7.* We prove $\pi(g^{n-2}) - \pi(g^{n-1}) > 0$. Setting $k = n-1$ and $k = n-2$, respectively, yields

$$\pi(g^{n-1}) = \frac{(a-\bar{c})^2\gamma\left[\gamma(n+1)^2 - 1\right]}{\left[\gamma(n+1)^2 - n\right]^2}; \qquad \pi(g^{n-2}) = \frac{(a-\bar{c})^2\gamma\left[\gamma(n+1)^2 - 4\right]}{\left[\gamma(n+1)^2 - 2(n-1)\right]^2}.$$

Establishing a comparison between $\pi(g^{n-1})$ and $\pi(g^{n-2})$ yields

$$\pi(g^{n-2}) - \pi(g^{n-1}) = \frac{(a-\bar{c})^2\gamma\left[\gamma^2(2n-7)(n+1)^4 - 3\gamma n(n+1)^2(n-4) - 8n + 4\right]}{\left[\gamma(n+1)^2 - 2(n-1)\right]^2\left[\gamma(n+1)^2 - n\right]^2}.$$

Observe that $\pi(g^{n-2}) - \pi(g^{n-1}) > 0$ if and only if $\gamma^2(2n-7)(n+1)^4 > 3\gamma n(n+1)^2(n-4) + 8n - 4$. For any $n$, both sides of this inequality are increasing in $\gamma$. Note that the left-hand side increases at the rate $2\gamma(2n-7)(n+1)^4$, while the right-hand side rises at the rate $3n(n+1)^2(n-4)$. It is easy to see that, since $\gamma > n^2/(n+1)^2$, the left-hand side increases at a higher rate than the right-hand side. Therefore, if this inequality holds for the lowest possible $\gamma$, then the result follows. Setting $\gamma = n^2/(n+1)^2$ in the inequality above, one obtains $2(n^5 - 5n^4 + 6n^3 - 4n + 2) > 0$ for all $n \geq 4$. *Q.E.D.*

*Proof of Proposition 8.* We prove $W(g^{n-2}) - W(g^{n-1}) > 0$. We observe that $W(g^{n-1})$ and $W(g^{n-2})$ are obtained by setting $k = n-1$ and $k = n-2$ in (16). Establishing a comparison between these welfare levels yields

$$W(g^{n-2}) - W(g^{n-1}) = \frac{(a-\bar{c})^2 n\gamma A}{2\left[\gamma(n+1)^2 - 2n + 2\right]^2\left[\gamma(n+1)^2 - n\right]^2},$$

where $A = 2\gamma^2(n^2-7)(n+1)^4 - \gamma n(n+1)^2(3n^2 - 2n - 20) - 16n + 8$. The sign of $W(g^{n-2}) - W(g^{n-1})$ is positive if and only if $2\gamma^2(n^2-7)(n+1)^4 > \gamma n(n+1)^2(3n^2 - 2n - 20) + 16n - 8$. Observe that both sides of this inequality are increasing in $\gamma$. Note also that the left-hand side rises at the rate $4\gamma(n^2-7)(n+1)^4$, while the right-hand side does so at the rate $n(n+1)^2(3n^2 - 2n - 20)$. It is easy to see that, since $\gamma > n^2/(n+1)^2$, the left-hand side increases faster than the right-hand side. Therefore, if the inequality above is satisfied for the minimum admissible $\gamma$, then the result follows. Set $\gamma = n^2/(n+1)^2$ in the inequality above. It obtains $2n^6 - 3n^5 - 12n^4 + 20n^3 - 16n + 8 > 0$. *Q.E.D.*

*Proof that $dW(g^k)/dk|_{k=k_f} < 0$.* Note that the sign of $dW(g^k)/dk$ is equal to the sign of the left-hand side of (17). Assume $k_f$ solves (15). Then it follows that $\gamma(n+1)^2 = (n-k_f)^3/(2n - 3k_f - 1)$. Plugging this into (17) and rearranging, we have that the sign of $dW(g^k)/dk|_{k=k_f}$ equals the sign of $(n-k_f)^3 n(n - 2k_f - 1)/(2n - 3k_f - 1)$. From (15) it follows that the denominator of this expression is positive. Therefore the sign of $dW(g^k)/dk|_{k=k_f}$ is equal to the sign of $n - 2k_f - 1$. Using (15) again, we have $n - 2k_f - 1 = (n-k_f)\left[(n-k_f)^2 - \gamma(n+1)^2\right]/\gamma(n+1)^2$, which is negative (second-order conditions). As a result, we have $dW(g^k)/dk|_{k=k_f} < 0$. *Q.E.D.*

□ **Equilibrium computations in the three-firm example.** *The complete network* ($g^c$). Standard computations show that the equilibrium effort, quantity, and profits are given by

$$e(g^c) = \frac{a-\bar{c}}{13}; \qquad q(g^c) = \frac{4(a-\bar{c})}{13}; \qquad \pi(g^c) = \frac{15(a-\bar{c})^2}{169}. \tag{A7}$$

*The star network* ($g^s$). In the star network, firms are no longer in a symmetric position. Indeed, two firms have one collaboration agreement each, while the remaining firm has two collaboration agreements. Suppose that firms 1 and 2 are the "spokes" and firm 3 is the "hub" firm, without loss of generality. The cost structure under $g^s$ is then

$$c_1 = \bar{c} - e_1 - \beta e_2 - e_3; \qquad c_2 = \bar{c} - \beta e_1 - e_2 - e_3; \qquad c_3 = \bar{c} - \sum_{i \in N} e_i. \tag{A8}$$

Plugging this cost structure into (2), maximizing profits, and invoking symmetry for the firms at the spokes, i.e., $e_1 = e_2 = e$, we obtain the effort levels of the different firms in the star network:

$$e_s(g^s) = \frac{4(a-\bar{c})(2-\beta)}{7\beta^2 - 13\beta + 58}; \qquad e_h(g^s) = \frac{(a-\bar{c})(\beta^2 - 3\beta + 6)}{7\beta^2 - 13\beta + 58}. \tag{A9}$$

The subscript $s$ applies to the firms at the spokes, and the subscript $h$ refers to the firm at the center of the star network.

Introducing (A8) into (7) and (2) we obtain the equilibrium quantities and profits. For the firms at the spokes we have

$$q_s(g^s) = \frac{16(a-\bar{c})}{58-13\beta+7\beta^2}; \qquad \pi_s(g^s) = \frac{16(a-\bar{c})^2\left(12+4\beta-\beta^2\right)}{\left(58-13\beta+7\beta^2\right)^2}. \tag{A10}$$

For the firm at the hub we obtain

$$q_h(g^s) = \frac{4(a-\bar{c})(\beta^2-3\beta+6)}{58-13\beta+7\beta^2}; \qquad \pi_h(g^s) = \frac{15(a-\bar{c})^2(\beta^2-3\beta+6)^2}{\left(58-13\beta+7\beta^2\right)^2}. \tag{A11}$$

*The partially connected network* ($g^p$). In the partially connected network, $g^p$, two firms are linked while the other firm stays isolated. Suppose without loss of generality that firms 1 and 2 are linked in $g^p$. The cost structure under $g^p$ is then as follows:

$$c_1 = c_2 = \bar{c} - e_1 - e_2 - \beta e_3; \qquad c_3 = \bar{c} - e_3 - \beta(e_1 + e_2). \tag{A12}$$

Proceeding as before and now invoking symmetry for the firms linked to each other, i.e., $e_1 = e_2 = e$, the subgame can be solved to obtain the research efforts of the different firms in the partially connected network $g^p$:

$$e_l(g^p) = \frac{(a-\bar{c})(2+9\beta-9\beta^2+2\beta^3)}{2(4+33\beta-16\beta^2+7\beta^3-2\beta^4)}; \qquad e_i(g^p) = \frac{\beta(a-\bar{c})(9-9\beta+2\beta^2)}{4+33\beta-16\beta^2+7\beta^3-2\beta^4}, \tag{A13}$$

where the subscript $l$ applies to the firms connected in the graph $g^p$ and $i$ refers to the isolated firm in $g^p$. We now obtain the equilibrium quantities and profits of the different firms in $g^p$ by substituting (A13) into (7) and (2). For the firms with collaborative agreements, we have

$$q_l(g^p) = \frac{2(a-\bar{c})(1+5\beta-2\beta^2)}{4+33\beta-16\beta^2+7\beta^3-2\beta^4}; \qquad \pi_l(g^p) = \frac{(a-\bar{c})^2(1+5\beta-2\beta^2)^2(12+4\beta-\beta^2)}{4\left(4+33\beta-16\beta^2+7\beta^3-2\beta^4\right)^2}. \tag{A14}$$

For the firm without research partners, we have

$$q_i(g^p) = \frac{4(a-\bar{c})\beta(3-\beta)}{4+33\beta-16\beta^2+7\beta^3-2\beta^4}; \qquad \pi_i(g^p) = \frac{(a-\bar{c})^2\beta^2(3-\beta)^2\left(7+12\beta-4\beta^2\right)}{\left(4+33\beta-16\beta^2+7\beta^3-2\beta^4\right)^2}. \tag{A15}$$

*The empty network* ($g^e$). The empty network, $g^e$, is the graph representing the case where no firm forms collaborations. In such a situation, the firms' cost structure is $c_i = \bar{c} - e_i - \beta \sum e_j, i, j = 1, 2, 3, j \neq i$. Taking advantage of the symmetry to solve the firms' maximization problems for $e$, we obtain the equilibrium effort, quantity, and profits under the empty network:

$$e(g^e) = \frac{(a-\bar{c})(3-2\beta)}{13-4\beta+4\beta^2}; \qquad q(g^e) = \frac{4(a-\bar{c})}{13-4\beta+4\beta^2}; \qquad \pi(g^e) = \frac{(a-\bar{c})^2(7+12\beta-4\beta^2)}{\left(13-4\beta+4\beta^2\right)^2}. \tag{A16}$$

*Welfare under the different networks.* To compute social welfare under the complete network $W(g^c)$, we plug equilibrium quantities and profits in (A7) into social welfare in (4). It obtains

$$W(g^c) = \frac{9(a-\bar{c})^2}{13}. \tag{A17}$$

Analogously, by substituting equilibrium quantities and profits of the spoke firms, and the quantity and profit of the hub firm into (4), we obtain social welfare under the star network $g^s$:

$$W(g^s) = \frac{(a-\bar{c})^2(2492-1084\beta+579\beta^2-138\beta^3+23\beta^4)}{(58+7\beta^2-13\beta)^2}. \tag{A18}$$

Similarly, we obtain social welfare under the partially connected network,

$$W(g^p) = \frac{(a-\bar{c})^2(28+380\beta+1345\beta^2-802\beta^3-111\beta^4+108\beta^5-12\beta^6)}{2(4+33\beta-16\beta^2+7\beta^3-2\beta^4)^2}, \tag{A19}$$

and, finally, under the empty network,

$$W(g^e) = \frac{3(a-\bar{c})^2(31+12\beta-4\beta^2)}{(13-4\beta+4\beta^2)^2}. \tag{A20}$$

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