Martin Obradovits*

Asymmetric Pricing Caused by Collusion

https://doi.org/10.1515/rne-2024-0023
Received April 29, 2024; accepted May 7, 2024; published online June 3, 2024

Abstract: In many markets, empirical evidence suggests that positive production cost shocks tend to be transmitted more quickly and fully to final prices than negative ones. This article explains asymmetric price adjustment as caused by firms imperfectly colluding on supra-competitive price levels. I consider an equilibrium in which positive cost shocks are transmitted instantaneously, whereas downward price adjustments only occur once aggregate market demand turns out unexpectedly low. This equilibrium exists whenever demand is sufficiently stable and negative cost shocks are not too large.

Keywords: asymmetric pricing; asymmetric price adjustment; rockets and feathers; collusion; cost shocks; stochastic demand

JEL Classification: D21; D43; L13; L41

1 Introduction

A vast body of empirical evidence documents that positive production cost shocks tend to be transmitted more quickly and fully to final prices than negative ones. For example, in a large sample of 77 consumer and 165 producer goods, Peltzman (2000) finds that *asymmetric price adjustment* (or *rockets and feathers*) can be observed in more than two thirds of the markets he examined. Moreover, a multitude of individual empirical studies confirm asymmetric price adjustment in markets related to retail and wholesale gasoline, certain agricultural products, and banking.[1]

1 See e.g. Tappata (2009) for further references.

I am grateful to Daniel Garcia, Maarten Janssen, Paul Pichler, Karl Schlag, Sandro Shelegia, Joshua Sherman, Matan Tsur, Franz Wirl, and various seminar audiences for their invaluable suggestions and discussions.

***Corresponding author: Martin Obradovits**, Department of Economics, University of Innsbruck, Universitätsstraße 15, 6020 Innsbruck, Austria, E-mail: martin.obradovits@uibk.ac.at. https://orcid.org/0000-0002-0813-7058

Ever since the seminal paper of Borenstein, Cameron, and Gilbert (1997), collusion has been mentioned as one possible cause of the phenomenon. However, apart from few specific exceptions which will be discussed below, no rigorous model of collusive asymmetric price adjustment to common (market-wide) production-cost shocks has been provided. The aim of this paper is to fill this gap in the literature.

To this end, I provide a simple model of asymmetric price adjustment caused by firms imperfectly colluding on supra-competitive price levels. The main mechanism, which is inspired by an informal discussion in Borenstein, Cameron, and Gilbert (1997), works as follows. In the considered oligopolistic market, firms would like to coordinate on a high price level, but a prohibition of overt collusion, as well as a large multiplicity of tacitly collusive equilibria, render optimal pricing difficult. Instead, the firms use downward cost shocks as coordinating mechanism. Whenever a negative cost shock hits the market, they use the previous period's price as focal point for collusion, which lets them achieve supra-competitive profits during low-cost periods. On the other hand, whenever a positive cost shock occurs, the firms have no interest in sticking to a low price level, and increase their prices immediately. Asymmetric price transmission results.

However, according to the above mechanism, negative cost shocks would never be transmitted to final prices if the firms' collusive scheme worked perfectly. This would be counterfactual to the rockets-and-*feathers* pattern, which describes slowly falling prices following negative cost shocks. My model accounts for this by incorporating informational frictions. Specifically, I assume that due to spatial distance, opportunity costs, and a frequent possibility of price changes, the firms cannot effectively monitor their rivals' prices.[2] Instead, with a lag of one period, they observe their own demand, which provides a confounded signal of the other firms' past pricing and a random, unobservable aggregate demand level. In the spirit of Green and Porter (1984), this implies that collusion must eventually break down on the equilibrium path, as low demand levels have to be punished in order to make collusion sustainable. It follows that negative price adjustments occur with a delay. This is in contrast to positive cost shocks, which, by the previous argument, are transmitted instantaneously.

My main findings are as follows. First, in order for an equilibrium of this type to be sustainable, it is necessary that each firm's own demand provides a sufficiently precise signal about the other firms' pricing. This is always the case if the variance of the aggregate market demand is sufficiently low. Second, given that the first property is satisfied, a sufficient condition for equilibrium existence is that the size

2 All of the model's main qualitative results prevail if the firms can only sometimes observe their rivals' prices, as long as this happens with sufficiently low probability. Further details can be obtained from the author upon request.

of negative cost shocks is not too large. Third, for any given probability distribution of aggregate demand, asymmetric-pricing equilibria must break down as the number of firms in the market grows large, firms highly discount future profits, or low-cost states become very short-lived in expectation. And finally, a downward price adjustment similar to the one documented in the empirical rockets-and-feathers literature can be generated when one considers the case of multiple independently operating submarkets.

The theoretical literature on asymmetric price transmission caused by collusion is scarce.[3] To the best of my knowledge, the earliest article was given by Damania and Yang (1998), who set up a model of asymmetric price adjustment to firm-specific (idiosyncratic) *demand* (rather than cost) shocks. The intuition is that if a firm is in an implicit collusive agreement and experiences a negative demand shock that is not faced (and observed) by other firms, it might be reluctant to reduce its price, as this may trigger a punishment phase. Because the reverse logic does not hold for positive demand shocks, asymmetric price adjustment to demand shocks may be implied. However, their article cannot explain asymmetric price adjustment to *market-wide cost* shocks, which is the principal focus of the empirical literature.

In an attempt to model the German electricity spot market, Wölfing (2008) considers the case of collusive asymmetric price transmission in supply-function equilibrium. The market structure Wölfing considers is special, as firms have to submit supply functions rather than set prices directly. On top of the limited applicability of this setup to traditional markets, the model cannot endogenously generate negative price adjustment on the equilibrium path. This is because firms have to coordinate on the fraction of a cost shock that is submitted to final prices in each period, and there is no reason why they should not collude perfectly (up to some maximal incentive compatible level). This is in contrast to the present article, which endogenously explains downward price adjustment as coordination failure that must inevitably happen on the equilibrium path.

The most closely related theoretical work is given by Sherman and Weiss (2015). In order to match their empirical setting of a large outdoor market in Jerusalem, they model a specific market structure in which a horizontally differentiated "isolated" firm competes with several homogeneous "rival" firms that compete à la Bertrand, and may engage in implicit collusion. The crucial difference to the present model is a perfect observability of demand and prices, which gives

3 Many other explanations for asymmetric pricing have been proposed. These include consumer search costs (Cabral and Fishman 2012; Lewis 2011; Tappata 2009; Yang and Ye 2008), menu costs (Ball and Mankiw 1994), lags in adjustment of production and finite inventories (Borenstein, Cameron, and Gilbert 1997), habit formation and consumption inertia (Xia and Li 2010), and Edgeworth price cycles that merely resemble asymmetric pricing (Eckert 2002).

rise to contrasting empirical predictions. For example, colluding “rival” firms may instantaneously decrease their prices when costs decrease or aggregate market demand increases, as under some parameter configurations, this implies that the maximally collusive scheme cannot be supported anymore. In contrast, my model predicts downward price adjustment to negative cost shocks as the result of punishment phases on the equilibrium path, which only happen following severe negative demand shocks. Moreover, I impose less structure on the random demand distribution, and do not consider an asymmetric market structure. Due to their different motivation and partly opposing testable predictions, both models may be viewed as complementary to each other.

Out of the ample empirical literature on asymmetric price adjustment, a number of studies report a link between the estimated market power of firms and the degree of asymmetric price adjustment in their market. For the retail gasoline market, these studies include Deltas (2008), Verlinda (2008), and Balmaceda and Soruco (2008). For example, analyzing a wide panel of state-level average retail prices for 48 US-American states, Deltas (2008) finds a significant correlation between average retail markups (as a proxy for market power) and the severity of asymmetric price adjustment. Similar results, based on proximity to rival stations and brand identity as measures for market power, are reported by Verlinda (2008), who uses a disaggregated panel of station-level retail gasoline prices in Southern California.

A comparable price-response asymmetry can also be found in the banking sector. Analyzing the response of consumer deposit interest rates to changes in the market interest rate, Hannan and Berger (1991) and Neumark and Sharpe (1992) document that markets with a more concentrated banking sector are prone to a higher degree of asymmetric pricing. In particular, the researchers find that deposit interest rates rise *slower* following an increase in the market interest rate if the market concentration is high. This is the interest-rate analogue to the more traditional setting where prices rise faster than they fall facing negative cost shocks. As market power typically facilitates collusion, all of the mentioned articles suggest that collusion may play a significant role in explaining the rockets-and-feathers pattern.

The relevance of tacit collusion as a possible cause for asymmetric price transmission is further corroborated in a recent experimental study by Bulutay et al. (2021). In their experimental oligopoly market that abstracts from other possible explanations such as search costs and information asymmetries, the authors document a significant pricing asymmetry in markets with three or more sellers, but not under duopoly (which tends to lead to rather stable pricing patterns that are consistent with persistent collusion). The authors conclude that imperfect tacit collusion is a likely cause for asymmetric price adjustment in their experimental setting.

The remainder of this article is structured as follows. Section 2 introduces the model and solves for the unique symmetric equilibrium of the stage game with arbitrary production cost. In Section 3, a simple asymmetric-pricing strategy combination for an infinitely repeated, dynamic version of the stage game (with fluctuating costs and demand) is constructed. Moreover, necessary and sufficient conditions for equilibrium existence are provided, and a specific example is presented. Section 4 extends the baseline model of Section 3 to the case of multiple separated submarkets, allowing for a more realistic pattern of the pricing asymmetry. Section 5 concludes. Technical proofs are relegated to the Appendix.

2 Model Setup and Equilibrium of the Stage Game

Consider an infinite-horizon game in which N profit-maximizing and risk neutral firms i compete in prices $p_{i,t}$ of some horizontally differentiated good. Importantly, the firms can never directly observe their rivals' prices, both in the current and all bygone periods. Instead, with a lag of one period, firms observe their own demand, which provides an imperfect signal about their competitors' past pricing.

Time is discrete, with $t = 1, 2, \ldots$. In each period, the firms face a common marginal cost c_t, where $c_t \in \{c_H, c_L\}$, with $c_H > c_L \geq 0$. Firms' marginal costs follow a two-state Markov chain, with $\mathbb{P}(c_{t+1} = c_H | c_t = c_H) = \rho_H \in (0,1)$, $\mathbb{P}(c_{t+1} = c_L | c_t = c_H) = 1 - \rho_H$, $\mathbb{P}(c_{t+1} = c_L | c_t = c_L) = \rho_L \in (0,1)$, and $\mathbb{P}(c_{t+1} = c_H | c_t = c_L) = 1 - \rho_L$. Firms discount future profits with a common discount factor $\delta \in (0,1)$.

The demand side is characterized by a continuum of identical consumers with a random total mass $\tilde{\theta}_t$ (henceforth called "aggregate demand") that is drawn from a stationary probability distribution $F(\theta) := \mathbb{P}(\tilde{\theta}_t \leq \theta)$ in each period, with a finite mean that is normalized to 1, $\mathbb{E}(\tilde{\theta}_t) = 1$. F is assumed to be twice continuously differentiable over its support $[0, \overline{\theta}]$, where $\overline{\theta} > 1$ may be infinite.[4] Moreover, for every $\theta \in (0, \overline{\theta})$, $f(\theta) := F'(\theta) > 0$, which means that there are no holes in the aggregate-demand distribution. As with prices, the firms are unable to observe $\tilde{\theta}_t$ directly. Further conditions on F will be discussed later in the analysis.

The consumers always prefer buying over not buying and the total market demand is perfectly inelastic at each point in time. The (random) demand of firm i if it prices at p_i and all other firms price at some vector $\boldsymbol{p_{-i}} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N)$ is given by

$$\tilde{D}_i = \tilde{\theta} * s_i(p_i; \boldsymbol{p_{-i}}), \tag{1}$$

4 Imposing zero as lower bound of the demand distribution implies that in any given period, firms' demand may be arbitrarily low. Later in the analysis, this will guarantee that all negative cost shocks must be transmitted eventually, although prices may be very sticky downward.

where $s_i(p_i;\ \boldsymbol{p}_{-i})$ is a function that maps a vector of prices to a market share $s_i \in [0,1]$ of the aggregate demand $\tilde{\theta}$. Since I will only consider symmetric equilibria in pure strategies, it is sufficient to characterize $s_i(p_i;\ p,\dots,p) := s_i(p_i;\ \boldsymbol{p})$, where p is a price that is commonly chosen by all firms other than i, as well as $s_j(p;\boldsymbol{p}_{(\boldsymbol{p}_i)})$, where $\boldsymbol{p}_{(\boldsymbol{p}_i)}$ denotes the price vector in which the $N-2$ firms other than j and i price at firm j's price p, and firm $i \neq j$ charges p_i. In order to minimize technicalities, I focus on a linear demand specification that can be seen as special case of the well-known "spokes model" of non-localized spatial competition provided by Chen and Riordan (2007).[5] In particular, let firms' market shares be given by

$$s_i(p_i;\boldsymbol{p}) = \begin{cases} 1 & \text{if } p_i < p - \dfrac{N-1}{\alpha N} \\ \dfrac{1}{N} - \alpha(p_i - p) & \text{if } p_i \in \left[p - \dfrac{N-1}{\alpha N}, p + \dfrac{1}{\alpha N}\right] \\ 0 & \text{if } p_i > p + \dfrac{1}{\alpha N}, \end{cases} \tag{2}$$

$$s_j(p;\boldsymbol{p}_{(\boldsymbol{p}_i)}) = \frac{1 - s_i(p_i;\boldsymbol{p})}{N-1}. \tag{3}$$

This specification summarizes the following ideas. First, if all firms price at some common price level p, they split the aggregate market demand evenly. Second, if a firm unilaterally deviates from a common price level, it receives a higher (lower) market share than its rivals if, and only if, it prices lower (higher) than them. In the linear setup, the strength of the marginal market-share response is given by $\alpha > 0$ everywhere. And third, given a unilateral deviation of firm i, all other firms share the residual demand evenly.[6]

Using this setup, it is straightforward to derive firm i's (unique) best response to any price level p that is commonly chosen by all other firms. In a symmetric equilibrium, this best response must be equal to p. A simple calculation then reveals that the unique symmetric stage-game equilibrium price is given by

5 In the relevant variant of the spokes model, N firms are located at the endpoints of different line segments that have a common origin. The consumers are uniformly distributed along these segments, with the disutility of purchasing at any given firm being proportional to the distance to the firm (consumers have to travel along the line segments). Moreover, each consumer may only choose between purchasing at their "preferred" firm (which is closest) and *one* random firm out of the $N-1$ (equally distant) other firms. The considered market-share function follows if $N-1$ firms charge a common price p, and a single firm i charges some arbitrary price p_i (as long as p_i is not too low – in the original spokes model, $s_i(p_i;\ \boldsymbol{p})$ can never exceed $\frac{2}{N}$).

6 All of the model's main results carry over to the case of non-linear demand as long as these properties are preserved, under the additional assumptions that $\left.\frac{\partial s_i(p_i;\boldsymbol{p})}{\partial p_i}\right|_{p_i=p} = -\alpha \quad \forall i,\ p$, and that a stage-game equilibrium exists for $c \in \{c_L, c_H\}$. The first additional assumption means that each firm's market-share response following a marginal deviation from a common price vector $\boldsymbol{p}$ is constant in the price level p. This is consistent with markets in which consumers only care about *absolute* price savings.

$$p^*(c) = c + \frac{1}{\alpha N}, \tag{4}$$

with associated equilibrium profits of

$$\pi^*(c) = \pi^* = \frac{1}{\alpha N^2}. \tag{5}$$

Note that the equilibrium price and profits decrease with the "degree of competition" α and the number of firms N. In the limit as either α or N goes to infinity, each firm prices at marginal cost and makes a profit of zero. Moreover, the equilibrium price shifts one to one with the cost level c, while the equilibrium profits are independent of it.[7] A direct implication is that a cost shock of size Δc is fully transmitted to final prices if and only if the firms' prices also shift by Δc.

Finally, suppose that all firms price at some supra-competitive price level $\hat{p} = p^*(c) + \Delta$, where $\Delta > 0$. Then, firm i's incentive to marginally deviate is given by

$$\frac{\partial}{\partial p_i}\left[(p_i - c)s_i(p_i; \hat{\boldsymbol{p}})\right]\Big|_{p_i=\hat{p}} = -\alpha\Delta < 0.$$

Thus, each firm has an incentive to (marginally) undercut, and this incentive increases in the competition intensity α and the premium over the competitive price level Δ.

3 Equilibrium of the Dynamic Game

In the stage game, each firm has an incentive to lower its price, starting from a collusive price level $\hat{p} > p^*(c)$. The goal of this section is to provide a necessary and sufficient condition for collusion on supra-competitive price levels to be feasible, given the stochastic nature of aggregate market demand and costs, as well as firms' inability to directly observe their rivals' prices.

Unfortunately, due to the various "folk theorems" that have been proven in the literature (see, e.g. Fudenberg and Maskin 1986), it is well-known that any repeated game gives rise to an infinite number of subgame-perfect equilibria, provided that the players' (common) discount factor is sufficiently close to one. Therefore, predicting which specific equilibrium will be played is generally a daunting task. Of course, certain selection criteria, such as Pareto dominance, may pin down equilibria with desirable properties. However, in the context of the present model with

7 Due to the assumption of perfectly price-inelastic aggregate market demand, the latter two features are preserved in the non-linear-demand case as long as $\frac{\partial s_i(p_i;\boldsymbol{p})}{\partial p_i}\Big|_{p_i=p} = -\alpha \quad \forall i, p$ (see also the previous footnote).

imperfect private observability of other players' actions, stochastic aggregate market demand, as well as stochastic costs, it is far from clear whether players would be able to compute the optimal equilibrium strategy, even if this was technically feasible.

The goal of the present paper is *not* to pin down the optimal collusive scheme given the market context and players' informational frictions, but to give an example of a simple and intuitive strategy combination which lets players achieve supra-competitive profits, and gives rise to the familiar rockets-and-feathers pattern. I therefore focus on an equilibrium in which the firms continue to charge a previous period's high price after a negative cost shock has happened, which they keep doing until their demand falls below a certain threshold, or costs rise again. Hence, the firms use past price levels as a natural focal point for collusion.[8]

More precisely, the considered mechanism is based on the described passive pricing behavior of firms combined with a demand tail-test in order to discourage (otherwise profitable) deviations. Once a negative cost shock hits the market such that the marginal production cost drops from c_H to c_L, the firms keep pricing on the supra-competitive price level $p^*(c_H)$ as long as their demand exceeds some critical threshold in each period. Only if firms' demand falls short of this threshold, they enter a punishment phase in which they charge the Nash-equilibrium price $p^*(c_L)$ of the low-cost stage game until the next opportunity for coordination arises.

Formally, I consider symmetric equilibria in which each firm plays the following strategy:

1. Price at $p^*(c_H)$ whenever $c = c_H$. (High-Cost Phase H)
2. If $c = c_L$ and $\tilde{D}_{i,t} \geq k * \frac{(N-1)s_i(p_{i,t};\boldsymbol{p}^*(\boldsymbol{c_H}))}{1-s_i(p_{i,t};\boldsymbol{p}^*(\boldsymbol{c_H}))}$ in every period t since c last switched from c_H to c_L, price at $p^*(c_H)$. (Collusive Phase C)
3. If $c = c_L$ and $\tilde{D}_{i,t} < k * \frac{(N-1)s_i(p_{i,t};\boldsymbol{p}^*(\boldsymbol{c_H}))}{1-s_i(p_{i,t};\boldsymbol{p}^*(\boldsymbol{c_H}))}$ in some period t since c last switched from c_H to c_L, price at $p^*(c_L)$. (Punishment Phase P)

The logic of this construction is as follows. First, in every high-cost period, the firms simply set the equilibrium price of the stage game, $p^*(c_H)$, and do not attempt to coordinate on supra-competitive price levels. Second, whenever costs drop to the low level, the firms try to keep pricing on $p^*(c_H)$, which allows them to collect higher per-period profits than those which would be attained when reducing their price to the new stage-game equilibrium level. However, since rival prices are assumed to be unobservable, each firm can only be deterred from deviating to a lower, even more profitable price (given that its rivals keep pricing on $p^*(c_H)$) if there is a risk

8 See Schelling (1960) for a seminal treatment of focal points.

of triggering a punishment phase. The simplest way to implement this is to punish unusually low demand by reverting to the stage-game equilibrium price of the low-cost state. It can easily be checked that on the equilibrium path (where firm i sticks to pricing at $p^*(c_H)$ after a negative cost shock and $\tilde{D}_{i,t} \geq k * \frac{(N-1)s_i(p_{i,t};\boldsymbol{p}^*(\boldsymbol{c_H}))}{1-s_i(p_{i,t};\boldsymbol{p}^*(\boldsymbol{c_H}))}$ in every period t since c last switched from c_H to c_L), the construction is such that firm i keeps charging the collusive price $p^*(c_H)$ as long as its demand has exceeded the threshold k in each period starting from the last negative cost shock. Off the equilibrium path (where firm i may have chosen prices different from $p^*(c_H)$ in some periods of the collusive phase, although its demand has never fallen below k), the correction factor $\frac{(N-1)s_i(p_{i,t};\boldsymbol{p}^*(\boldsymbol{c_H}))}{1-s_i(p_{i,t};\boldsymbol{p}^*(\boldsymbol{c_H}))}$ ensures that firm i enters the punishment phase if and only if all its rivals do so (or have already done so) as well, assuming that they follow the outlined strategy.[9] Finally, once the punishment phase is reached, the firms keep pricing at the respective stage-game equilibrium price $p^*(c_L)$ until the cycle is reset by a positive cost shock.

If an equilibrium of this structure can be found, it must exhibit asymmetric price transmission. A downward cost shock from c_H to c_L is not transmitted instantaneously to final prices, as the firms keep pricing on $p^*(c_H)$ until demand turns out unexpectedly low. Only in that case (which takes at least one period, as demand is observed with a lag), a punishment phase is entered in which the firms reduce their prices to the equilibrium level $p^*(c_L)$ of the stage game with low costs. On the other hand, upward cost shocks from c_L to c_H are either transmitted immediately (if the firms are currently in the punishment phase and price competitively), or not at all (if the firms are currently in the collusive phase, i.e. a downward cost shock has never been transmitted). In particular, if the low cost state is sufficiently persistent such that an upward cost shock typically happens when the collusive phase has already ended, the well-documented rockets-and-feathers pattern emerges.[10]

I now turn to existence of such equilibria. First, note that each firm's behavior is clearly optimal in both the high-cost phase and punishment phase. Given that all other firms price at $p^*(c_H)$ in the high-cost state ($p^*(c_L)$ in the low-cost state) no matter what happens (and given that an individual firm cannot influence when the high-cost or punishment phase ends), a firm can do no better than to play the stage-game best response $p^*(c_H)$ ($p^*(c_L)$) itself.

9 To see where the correction factor comes from, note that given the outlined strategy combination, any firm can always perfectly deduce the level of aggregate demand. Notably, since $\tilde{D}_{i,t} = \tilde{\theta}_t * s_i(p_{i,t}, \boldsymbol{p}^*(\boldsymbol{c_H}))$ if all other firms price at $p^*(c_H)$, it holds that $\tilde{\theta}_t = \frac{\tilde{D}_{i,t}}{s_i(p_{i,t},\boldsymbol{p}^*(\boldsymbol{c_H}))}$ and hence that $\tilde{D}_{j,t} = \tilde{\theta}_t * \frac{1-s_i(p_{i,t},\boldsymbol{p}^*(\boldsymbol{c_H}))}{N-1} = \tilde{D}_{i,t} * \frac{1-s_i(p_{i,t},\boldsymbol{p}^*(\boldsymbol{c_H}))}{(N-1)s_i(p_{i,t},\boldsymbol{p}^*(\boldsymbol{c_H}))}$.

10 Moreover, from an outside perspective, the high-cost state is always associated with high prices, whereas the low-cost state is only sometimes associated with low prices.

The non-trivial part of the suggested strategy-combination is the collusive phase. As collusion on the supra-competitive price level $p^*(c_H)$ in the low-cost state c_L should be sustainable, each firm has to be deterred from profitably undercutting its rivals (and obtaining a larger market share). In particular, the demand threshold k must necessarily be chosen in such a way that each firm's expected increase in discounted aggregate profits when marginally undercutting $p^*(c_H)$ (as caused by an increase in per-period profits during the collusive phase) is exactly offset by an equally-sized expected loss of discounted aggregate profits due to a higher probability of the collusive phase to end.

Let $r(p_i;\ \boldsymbol{p};\ k)$ denote the probability that any firm j's demand exceeds k, given that all firms $j \neq i$ price at p and firm i prices at p_i. It is then easy to see that[11]

$$r(p_i; \boldsymbol{p}; k) = 1 - F\left(\frac{(N-1)k}{1 - s_i(p_i; \boldsymbol{p})}\right), \tag{6}$$

whereas

$$r(p; \boldsymbol{p}; k) = 1 - F(Nk). \tag{7}$$

As $s_i(p_i;\ \boldsymbol{p})$ decreases in p_i and equals $\frac{1}{N}$ for $p_i = p$, it can be seen that $r(p_i;\ \boldsymbol{p};\ k) < r(p;\ \boldsymbol{p};\ k)$ for $p_i < p$ (as long as $k > 0$, that is, any positive punishment threshold is used). Hence, a firm that deviates downward from the collusive price level $p^*(c_H)$ does in fact decrease the probability of collusion to be continued in each period. The next step is to characterize how k must be chosen in order to ensure that a marginal deviation from the collusive price does not pay.

Denote by Π_i^H, $\Pi_i^C(p_i)$, and Π_i^P firm i's expected discounted profit stream (given the proposed strategy for all other firms) in the high-cost phase, collusive phase, and punishment phase, respectively. Note that firm i's expected discounted profit stream of the collusive phase $\Pi_i^C(p_i)$ has firm i's collusive-phase price p_i as argument.[12] Only if firm i's total expected discounted profit is maximized for $p_i = p^*(c_H)$, the proposed strategy-combination forms an equilibrium.

The following recursive equations then define firm i' expected discounted profit stream in each of the three regimes (where $\pi_i(p_i)$ is a short notation for $(p_i - c_L)s_i(p_i;\ \boldsymbol{p^*(c_H)})$ and $r(p_i)$ is a short notation for $r(p_i;\ \boldsymbol{p^*(c_H)};\ k)$).

$$\Pi_i^H = \pi^* + \rho_H \delta \Pi_i^H + (1 - \rho_H)\delta \Pi_i^C(p_i) \tag{8}$$

$$\Pi_i^C(p_i) = \pi_i(p_i) + \rho_L \left[r(p_i)\delta \Pi_i^C(p_i) + (1 - r(p_i))\delta \Pi_i^P \right] + (1 - \rho_L)\delta \Pi_i^H \tag{9}$$

11 The first equation follows because $r(p_i; \boldsymbol{p}; k) := \mathbb{P}\left(\tilde{\theta} * s_j(p; \boldsymbol{p}_{(p_i)}) > k\right) = \mathbb{P}\left(\tilde{\theta} * \frac{1 - s_i(p_i; \boldsymbol{p})}{N-1} > k\right) = 1 - F\left(\frac{(N-1)k}{1 - s_i(p_i; \boldsymbol{p})}\right)$. This directly implies $r(p;\ \boldsymbol{p};\ k) = 1 - F(Nk)$, as $s_i(p; \boldsymbol{p}) = \frac{1}{N}$.

12 Since the collusive phase is stationary, it suffices to consider one single price p_i that firm i chooses in every period of that phase.

$$\Pi_i^P = \pi^* + \rho_L \delta \Pi_i^P + (1 - \rho_L) \delta \Pi_i^H. \tag{10}$$

The first and third of these equations have a similar structure. The expected discounted profit stream of the high-cost phase (punishment phase) is given by the sum of the phase's expected stage-game profit and the one-time discounted expected continuation profit. With probability ρ_H (ρ_L), costs stay the same in the high-cost state (low-cost state), which gives rise to an expected continuation profit that is equal to the initial expected discounted profit stream. With probability $1 - \rho_H$ ($1 - \rho_L$), costs switch to the other state, which leads to an expected continuation profit that is equal to the expected discounted profit stream of the collusive phase (high-cost phase).

The second equation has the following interpretation. The expected discounted profit stream of the collusive phase, given that firm i prices at p_i in each stage of that phase, can be written as the sum of the expected stage-game profit of pricing at p_i (while all other firms stick to the candidate equilibrium strategy of pricing at $p^*(c_H)$) and the one-time discounted expected continuation profit. This continuation profit has two parts. With probability ρ_L, costs stay low. Then, depending on whether the previous period's demand has exceeded k or not (which happens with probability $r(p_i)$ and $1 - r(p_i)$, respectively), the expected continuation profit is either given by the initial expected discounted profit stream, or the expected discounted profit stream of the punishment phase. With probability $1 - \rho_L$, costs switch to the high state. Then, the expected continuation profit is equal to the expected discounted profit stream of the high-cost phase.[13]

Figure 1 provides a graphical representation of the underlying dynamical system if firm i prices at p_i in every period of the collusive phase, while all other firms follow the candidate equilibrium strategy.

It was already discussed above that the firms only face a non-trivial pricing decision when the game is in the collusive phase. Clearly, continuing to price at $p^*(c_H)$ in the collusive phase is a best response to all other firms' strategies if, and only if, $p^*(c_H)$ is a global maximizer of $\Pi_i^C(p_i)$. Solving the above system of equations, the following lemma can be stated.

Lemma 1. *Firm i's expected discounted profit stream in the collusive phase, given that all other firms price according to the proposed strategy, can be written as*

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13 A different way of writing down Equation (9) is as follows: $\Pi_i^C(p_i) = \int_0^{\frac{(N-1)k}{1-s_i(p_i)}} \left[\pi_i(p_i)\tilde{\theta} + \rho_L \delta \Pi_i^P + (1-\rho_L)\delta \Pi_i^H\right] f(\tilde{\theta})\mathrm{d}\tilde{\theta} + \int_{\frac{(N-1)k}{1-s_i(p_i)}}^{\bar{\theta}} \left[\pi_i(p_i)\tilde{\theta} + \rho_L \delta \Pi_i^C(p_i) + (1-\rho_L)\delta \Pi_i^H\right] f(\tilde{\theta})\mathrm{d}\tilde{\theta}$, where the bound $\frac{(N-1)k}{1-s_i(p_i)}$ is the necessary aggregate demand level that is needed for sustained collusion, given p_i and k. As $r(p_i) = 1 - F\left(\frac{(N-1)k}{1-s_i(p_i;\mathbf{p})}\right)$, it is easy to see that both formulations are equivalent.

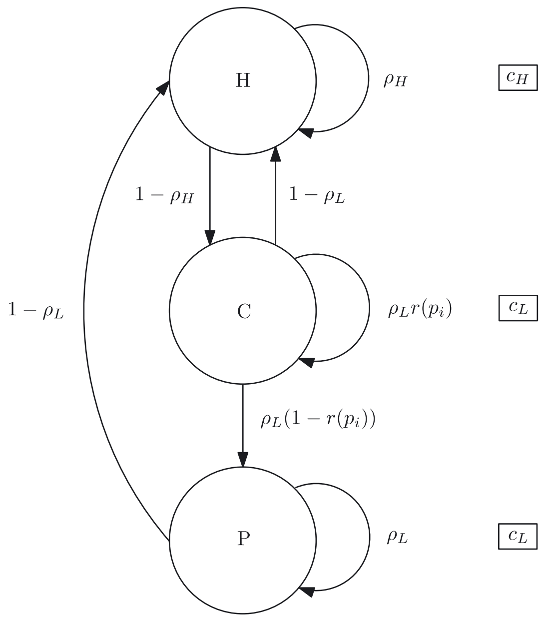

Figure 1: Depiction of the dynamical system that is implied if firm i charges p_i in the collusive phase, given the proposed strategy combination of all other firms. Transition probabilities are found next to the arrows indicating a state change.

$$\Pi_i^C(p_i) = \frac{\pi^*}{1-\delta} + \frac{(1-\delta\rho_H)(1-\delta\rho_L)}{(1-\delta)\left[1+\delta-\delta(\rho_H+\rho_L)\right]} \times \frac{\pi_i(p_i)-\pi^*}{1-\delta\rho_L r(p_i)}. \tag{11}$$

The interpretation to Lemma 1 is straightforward: the expected discounted profit stream of pricing at p_i in every period of the collusive phase is given by the expected discounted profit stream $\frac{\pi^*}{1-\delta}$ of receiving the (competitive) stage-game equilibrium profit in every period, *plus* the expected excess profit over the competitive profit in collusive periods, $\pi_i(p_i)-\pi^*$, appropriately discounted.

Examining Equation (11), it is apparent that $\Pi_i^C(p_i)$ reaches its global maximum at the value of p_i that maximizes $\hat{\Pi}_i(p_i) := \frac{\pi_i(p_i)-\pi^*}{1-\delta\rho_L r(p_i)}$. In order for the proposed strategy-combination to form an equilibrium, this maximum must be reached at $p^*(c_H)$. A necessary condition for this is that the derivative of $\hat{\Pi}_i(p_i)$, evaluated at $p^*(c_H)$, equals zero. Carrying out the corresponding calculation, one arrives at the following proposition.

Proposition 1. *(Necessary condition) In order for the proposed strategy-combination to form an equilibrium, the demand threshold k must be chosen such that $\phi := Nk$ satisfies*

$$h(\phi) := (N-1)\left[1-\delta\rho_L+\delta\rho_L F(\phi)\right] - \delta\rho_L\phi f(\phi) = 0. \tag{12}$$

The intuition to Equation (12) is a simple marginal-cost marginal-benefit trade-off. If a firm marginally deviates downward from the collusive price level $p^*(c_H)$, it makes a higher-stage game profit in expectation (as the best response to all other firms pricing at $p^*(c_H)$ is to price *lower* than $p^*(c_H)$), but this comes at the cost of a higher probability of collusion to end after each period, which decreases the expected length of collusive-phases with supra-competitive profits.

In particular, as the marginal cost of undercutting the collusive price level is proportional to $\phi f(\phi)$, one can see that the above first order condition can only be satisfied for adequately chosen demand thresholds $\phi^* = Nk^*$ if the probability density of aggregate market demand is sufficiently large somewhere in its distribution. Only if that is the case, the probability of sustained collusion following a marginal price decrease can be reduced by so much (when choosing k appropriately) that the firms are successfully discouraged from deviating.

In fact, examining $h'(\phi) = \delta\rho_L\left[(N-2)f(\phi) - \phi f'(\phi)\right]$ and noting that $h(0) > 0$, it is apparent that the necessary condition can never be fulfilled if f is non-increasing (F is weakly concave). This rules out some common cumulative distribution functions, including the uniform, exponential, and Pareto distribution. The interpretation is that these distribution functions provide too weak signals about firms' pricing in order to discourage marginal deviations. No matter how the demand threshold k is chosen, firms can never be deterred from profitably undercutting the collusive price level, as doing so reduces the probability of sustained collusion by too little.

Note moreover that for any given aggregate-demand distribution F, it directly follows from Equation (12) that as N increases without bound or $\delta\rho_L$ decreases towards zero, the first order condition must eventually become unresolvable. Hence, an asymmetric-pricing equilibrium of the analyzed structure can only be supported if there are not too many firms in the market, firms do not highly discount future profits, and the low-cost state is sufficiently persistent.

So far, only a necessary condition in order to allow the collusive price level $p^*(c_H)$ to be a *local* extremum of the expected discounted profit stream of the collusive phase has been provided. However, in order to make pricing at $p^*(c_H)$ a best response to the other firms' strategies, it has to hold that this price is a *global* maximizer of firm *i*'s expected discounted profit stream in the collusive phase. The following proposition provides a sufficient condition for that.

Proposition 2. *(Sufficient Condition) The proposed strategy combination forms an equilibrium ($p^*(c_H)$ is a global maximizer of $\hat{\Pi}_i(p_i)$) if a solution to the necessary* *condition in Equation* (12) *exists, and* $\Delta c := c_H - c_L$ *is sufficiently small.*[14] *In particular, the former is true whenever the variance of aggregate market demand is sufficiently low, that is,* $\mathrm{Var}(\tilde{\theta}) < \left(\frac{3}{6+16\frac{N-1}{\delta\rho L}}\right)^2$.

Thus, asymmetric pricing equilibria of the considered structure exist whenever the variance of aggregate market demand is sufficiently low for given market parameters N, δ and ρ_L, and the size of the negative cost shock is not too large. Moreover, it can be seen that a wide range of plausible distribution functions for modeling stochastic aggregate market demand, e.g. the Log-normal, Gamma, Beta, Log-logistic, and Weibull distribution, give rise to the existence of such asymmetric-pricing equilibria, provided that their variance is sufficiently low. This is because all of these distribution functions can be normalized in such a way that their expectation is set to one, with a free parameter governing their variance.

The intuition to the above proposition is twofold. First, a sufficiently low variance of aggregate market demand guarantees that marginal deviations from the collusive price level are not profitable when the demand threshold k is set properly, as the probability of sustained collusion decreases by too much. And second, also larger deviations from the collusive price level do not pay if the size of the cost shock is sufficiently small. This is because, for a small negative cost shock, the collusive price level lies close to the (new) competitive price level, implying that large deviations from the collusive level cannot pay.

Having established the existence of an asymmetric-pricing equilibrium under suitable model parameters, it is now possible to quantify the *degree* of asymmetry in price adjustment. For this, note that for any solution $\phi^* = Nk^*$ of Equation (12) that does in fact constitute an equilibrium, the probability of the collusive phase to end, conditional on costs remaining low, is given by $F(\phi^*)$ in each period. Thus, following a *persistent* negative cost shock, the number of periods until prices adjust from $p^*(c_H)$ to the lower competitive level of $p^*(c_L)$ is geometrically distributed, with an expectation of[15]

$$L(\phi^*) = \frac{1}{F(\phi^*)} > 1. \tag{13}$$

On the other hand, by construction, positive cost shocks are transmitted instantaneously, given that the firms were pricing at the competitive level $p^*(c_L)$ before.

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14 Explicit sufficient conditions on Δc can be found at the end of the proof of the proposition.

15 To see this, note that the expected number of periods until prices adjust downward is given by $\sum_{i=1}^{\infty} i(1-F(\phi^*))^{i-1}F(\phi^*) = 1/F(\phi^*)$.

Finally, the implicit Equation (12) also allows for comparative statics with respect to the firms' discount factor δ and the persistence of the low-cost state ρ_L. The following proposition is a direct consequence of the implicit function theorem.

Proposition 3. *A marginal increase in δ or ρ_L leads to a more pronounced asymmetry in price transmission if, and only if, $h'(\phi^*)$ is negative.*

Proposition 3 shows that the effect of a marginal increase in δ or ρ_L is directly related to the sign of $h'(\phi^*)$. Clearly, since $h(0) > 0$ (compare with Equation (12)), the lowest solution (if any) to the necessary equilibrium condition $h(\phi) = 0$, say ϕ_1^*, must always satisfy $h'\left(\phi_1^*\right) \leq 0$. While firms would like to coordinate on the lowest possible demand threshold, minimizing the probability of collusion to break down, it might still be the case that non-marginal deviations from $p^*(c_H)$ are profitable for $\phi = \phi_1^*$, but not for a larger solution to the necessary equilibrium condition, say ϕ_2^*, with $h'\left(\phi_2^*\right) > 0$. In principle, it could thus be the case that, as firms become more patient or negative cost shocks become more persistent, negative cost shocks are transmitted more quickly to final prices, even when selecting the Pareto-dominant equilibrium of the considered class. However, in all numerical simulations that I conducted, *equilibria* with positive $h'\left(\phi_2^*\right)$ were accompanied by *equilibria* with negative $h'\left(\phi_1^*\right)$, for some $\phi_1^* < \phi_2^*$. When focusing on the Pareto-dominant equilibrium of the considered type, increases in δ or ρ_L should thus tend to exacerbate the pricing asymmetry.

Finally, it should be noted that comparative statics with respect to N cannot be provided for general demand distributions F, as the implicit function theorem does not work for discrete parameters.

Example. Having established the above general results for arbitrary probability distributions of aggregate market demand, it is instructive to examine a specific example. In what follows, I consider the following family of cumulative distribution functions of aggregate market demand, parametrized by $\beta > 0$:

$$F(\theta;\beta) = \begin{cases} 0 & \text{for } \theta < 0 \\ \left(\dfrac{\theta\beta}{\beta+1}\right)^{\beta} & \text{for } 0 \leq \theta \leq \dfrac{\beta+1}{\beta} \\ 1 & \text{for } \theta > \dfrac{\beta+1}{\beta}. \end{cases} \tag{14}$$

It is easy to check that the corresponding random variable $\tilde{\theta}_\beta$ has a mean of 1 and a variance of

$$\mathrm{Var}(\tilde{\theta}_\beta) = \frac{1}{\beta(\beta+2)},$$

which strictly decreases in β. Moreover, note that $F(\theta;\ \beta)$ is strictly increasing in θ over its support $[0, \frac{\beta+1}{\beta}]$, while it is strictly convex (concave) in θ for $\beta > 1$ ($\beta < 1$). The knife-edge case of $\beta = 1$ corresponds to a uniform distribution over the support $[0, 2]$.

As $F(\theta;\ \beta)$ is weakly concave for $\beta \leq 1$, the discussion after the necessary equilibrium condition provided in Proposition 1 directly implies that no asymmetric-pricing equilibrium of the considered type can exist for $\beta \leq 1$. Indeed, inserting $F(\cdot;\ \beta)$ and solving Equation (12) for ϕ, it turns out that a solution exists if and only if

$$\beta > \frac{N-1}{\delta \rho_L} \quad (> 1), \tag{15}$$

with a unique demand threshold of

$$\phi_\beta^* = Nk_\beta^* = \frac{\beta+1}{\beta}\left[\frac{(1-\delta\rho_L)(N-1)}{\delta\rho_L[\beta-(N-1)]}\right]^{\frac{1}{\beta}}. \tag{16}$$

Note that as $\mathrm{Var}(\tilde{\theta}_\beta)$ strictly decreases in β, inequality (15) implies that the variance of aggregate market demand must be sufficiently low in order to permit a solution to the necessary equilibrium condition.

It should be recalled that the above conditions are not sufficient for the considered equilibrium to exist, as these only rule out marginal deviations from the collusive price level. From Proposition 2, it is however known that equilibrium existence is guaranteed if it further holds that the size of the cost shock, Δc, is sufficiently small.[16]

Now, provided that an equilibrium of the considered type exists, the expected length until a persistent negative cost shock is transmitted to final prices is given by

$$L\left(\phi_\beta^*\right) = \frac{1}{F\left(\phi_\beta^*\right)} = \frac{\delta\rho_L[\beta-(N-1)]}{(1-\delta\rho_L)(N-1)}.$$

Clearly, this expected length strictly increases in δ, ρ_L and β, while it strictly decreases in N. Hence, the showcased demand distribution gives rise to intuitively

16 Interestingly, this condition is not always necessary. With the considered aggregate-demand distribution, it can be shown that the asymmetric-pricing equilibrium exists for arbitrarily large cost shocks, given that $\delta\rho_L$ is sufficiently close to 1. Details are available from the author upon request.

plausible comparative statics: As firms' patience or the persistence of the negative cost shock increases, the expected length of collusive phases, conditional on costs remaining low, increases as well. On the other hand, as the number of firms in the market or the variance of the aggregate demand distribution increases, the expected length of collusive phases, conditional on costs remaining low, decreases.

4 Multiple Submarkets

In the baseline model developed in Sections 2 and 3, I considered the case of a single oligopolistic market in which a small number of firms engages in imperfect collusion on supra-competitive price levels. Under suitable model parameters, the proposed strategy combination forms an equilibrium that entails asymmetric price adjustment to cost shocks. However, the baseline model is counterfactual to the rockets-and-*feathers* pattern in the sense that once collusion breaks down, all prices adapt fully and abruptly to the lower competitive level. In contrast, empirical evidence documents slowly declining prices following negative cost shocks.

The purpose of this section is to reconcile the theoretical model with the patterns that are found in the data. The key argument is that the price series that are typically studied in the literature are not station-specific, but reflect average retail prices in a large market, which may be comprised of several independently operating local submarkets. Hence, due to the stochastic nature of demand, collusion on supra-competitive price levels may persist longer in some submarkets than others. This implies that time series of market-wide average retail prices should be smoothly declining following negative cost shocks.

Consider the baseline model of Sections 2 and 3, and let $M \geq 1$ denote the total number of locally separated submarkets. For simplicity, I focus on the case in which all of these submarkets are identical and characterized by a common probability $\gamma = 1 - F(\phi^*)$ that collusion is continued after each period of the collusive phase, with independent demand realizations.[17] In turn, this implies that the probability that any given submarket will still be in the collusive phase, t periods after a persistent negative cost shock has happened, is equal to γ^t.

Then, the probability $J(m, t)$ that exactly $m \leq M$ of all submarkets will still be in the collusive phase, t periods after a negative cost shock, is binomially distributed, where

$$J(m, t) := \binom{M}{m} (\gamma^t)^m (1 - \gamma^t)^{M-m}. \tag{17}$$

17 It is straightforward to allow for different probabilities across submarkets, reflecting local market circumstances. Moreover, correlated demand can easily be incorporated in numerical simulations.

By a well-known property of binomially distributed random variables, the expected number of submarkets which will still be in the collusive phase after t periods is then given by $M\gamma^t$, which decreases exponentially in t. Hence, the expected average retail price of the whole market, t periods after a persistent negative cost shock has happened, can easily be calculated. The following proposition highlights this finding.

Proposition 4. *Suppose the whole market is comprised of $M \geq 1$ independently operating submarkets, each characterized by a probability $\gamma \in (0,1)$ that collusion is continued after each period of the collusive phase. Then, the expected average retail price of the whole market, t periods after a persistent negative cost shock has occurred, is given by*

$$p^*(c_H)\gamma^t + p^*(c_L)(1-\gamma^t). \tag{18}$$

For a large M, a smooth transition from the collusive price level $p^*(c_H)$ to the new competitive price level $p^*(c_L)$ can be expected.

In the following, I will present a numerical simulation of the extended model. For this, let the parameters of the whole market be given by $M = 50$, $c_H = 20$, $c_L = 16$, $\delta = 0.9999$, $\rho_H = \rho_L = 0.98$. Moreover, for each submarket, let $N = 4$, $\alpha = 0.05$,

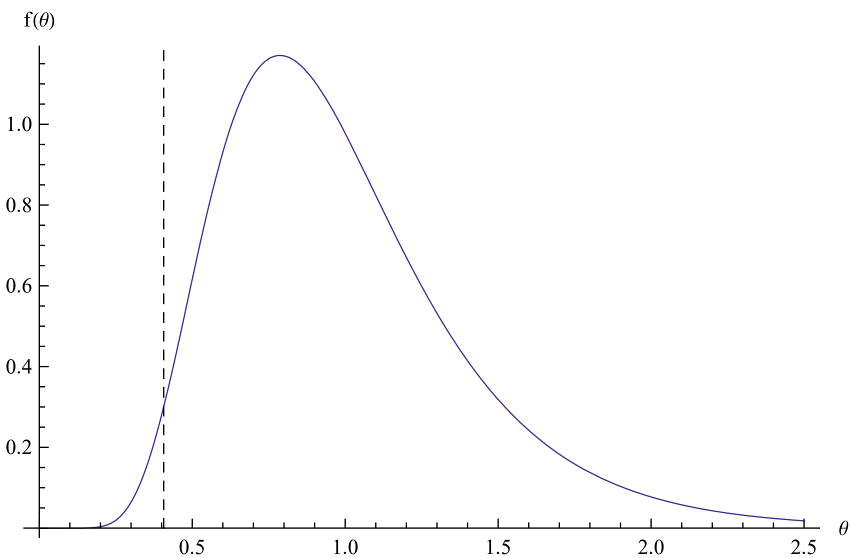

Figure 2: Density function of a log-normal distribution with mean parameter −0.08 and standard-deviation parameter 0.4, implying a mean of 1. Given the selected parameters, the aggregate demand threshold ϕ^* is located at the dashed line.

and $F(\theta)$ log-normal with mean parameter -0.08 and standard-deviation parameter 0.4. The latter implies a mean of the aggregate-market-demand random variable of 1 (as required by the model) and a standard deviation of about 0.417. See Figure 2 for a depiction of the corresponding probability density function $f(\theta)$.

It is now easy to see that $p^*(c_H) = 25$ and $p^*(c_L) = 21$. Also, one can verify numerically that $\phi^* = Nk^* = 0.406724$ is a solution to the first order condition stated in Proposition 1. As $p^*(c_H)$ is also a global maximizer of $\Pi_i^C(p_i)$ for $\phi = \phi^*$,[18] this implies that asymmetric pricing may be observed in equilibrium. In fact, if all firms stick to the punishment threshold ϕ^*, there is a probability of

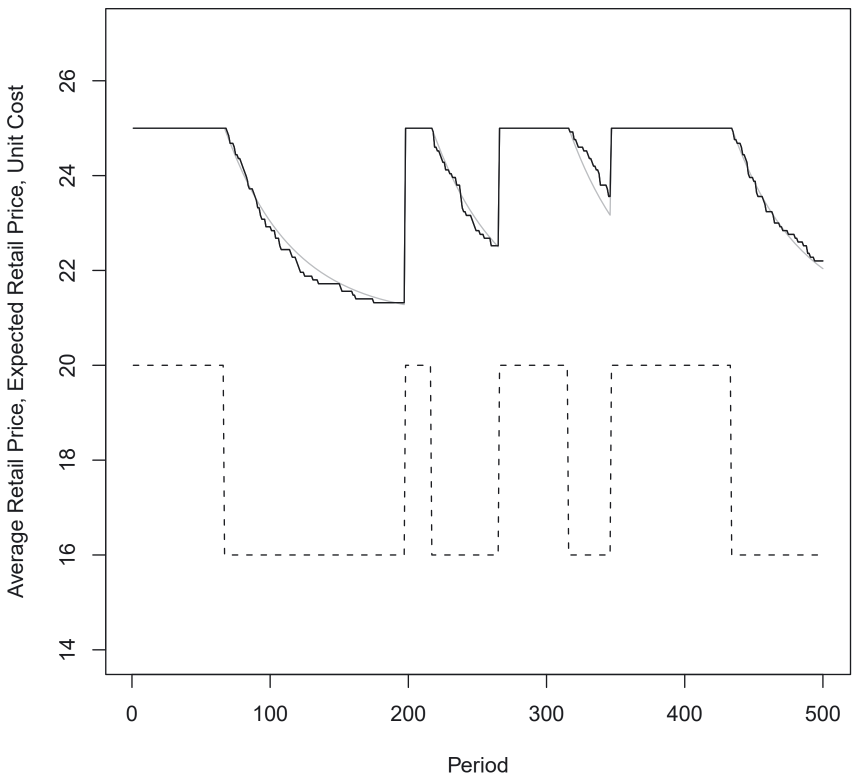

Figure 3: Market simulation for $T = 500$, $M = 50$, $c_H = 20$, $c_L = 16$, $\delta = 0.9999$, $\rho_H = \rho_L = 0.98$, and for each submarket, $N = 4$, $\alpha = 0.05$, $F(\theta)$ log-normal with mean parameter -0.08 and standard-deviation parameter 0.4. The black solid (gray solid) [dashed] line represents the whole market's actual average retail price (expected average retail price) [marginal cost], respectively.

18 A flexible Mathematica code to perform numerical simulations like this can be obtained from the author upon request.

$1 - F(0.406724) \approx 0.98$ that collusion is continued after each period of the collusive phase.

Figure 3 depicts a simulation of the outlined market for a length of $T = 500$ periods (“days”).[19] The well-documented rockets-and-feathers pattern can clearly be discerned.

5 Conclusions

In a wide range of markets, positive production cost shocks tend to be transmitted more quickly and fully to final prices than negative ones. This article provides a simple model of asymmetric price adjustment caused by firms imperfectly colluding on supra-competitive price levels. In the model, negative cost shocks are only transmitted to final prices once collusion breaks down. This happens when an unobservable aggregate-demand variable turns out unexpectedly low, which typically occurs with a delay. On the other hand, positive cost shocks are transmitted instantaneously.

By considering a simple strategy combination under which firms punish unusually low demand, I prove that asymmetric-pricing equilibria exist whenever the variance of aggregate market demand is sufficiently low *and* the size of negative cost shocks is not too large. Conversely, I show that the considered equilibrium can only exist if there are not too many firms in the market, low-cost states are relatively persistent, and the firms do not discount the future by too much. Moreover, in order to discourage marginal deviations, it should not be the case that low aggregate-demand levels are always more probable than high levels (i.e. that the density of aggregate market demand is weakly decreasing). Since all of these features can be examined empirically, a rich array of testable predictions is generated.

Future research might extend the simple model to a more general class of random cost processes, allow firms to endogenously monitor their rivals’ prices, or consider the case of asymmetric market shares. However, already the current model can generate pricing patterns that are similar to the ones observed in the data, given that multiple separated submarkets are considered.

An important agenda is hence to empirically analyze the various predictions of the model and contrast them with those of other theoretical models of asymmetric price adjustment. In particular, if a collusive mechanism similar to the portrayed one causes the phenomenon, it should be observed that prices tend to adjust downward in low-demand periods, and that markets with more stable aggregate demand, more persistent negative cost shocks, and fewer firms, are more likely to exhibit the rockets-and-feathers pattern.

19 The underlying code can be obtained from the author upon request.

Appendix: Technical Proofs

Proof of Lemma 1. First, use Equations (8) and (10) in order to solve for Π_i^H and Π_i^P as functions of $\Pi_i^C(p_i)$. This yields

$$\Pi_i^H = \frac{\pi^* + (1-\rho_H)\delta\Pi_i^C(p_i)}{1-\delta\rho_H}$$

and

$$\Pi_i^P = \frac{\pi^*[1+\delta-\delta(\rho_H+\rho_L)]}{(1-\delta\rho_L)(1-\delta\rho_H)} + \frac{(1-\rho_L)(1-\rho_H)\delta^2}{(1-\delta\rho_L)(1-\delta\rho_H)}\Pi_i^C(p_i).$$

Next, insert the above expressions into Equation (9), isolate $\Pi_i^C(p_i)$ and multiply both sides with $(1-\delta\rho_L)(1-\delta\rho_H)$ in order to get

$$\begin{aligned}
&\Pi_i^C(p_i)\big[(1-\delta\rho_L r(p_i))(1-\delta\rho_L)(1-\delta\rho_H) - \delta^3\rho_L(1-\rho_L)(1-\rho_H)(1-r(p_i)) \\
&\quad - \delta^2(1-\rho_L)(1-\rho_H)(1-\delta\rho_L)\big] \\
&= \pi_i(p_i)(1-\delta\rho_L)(1-\delta\rho_H) + \pi^*\delta\rho_L(1-r(p_i))\big[1+\delta-\delta(\rho_H+\rho_L)\big] \\
&\quad + \pi^*\delta(1-\rho_L)(1-\delta\rho_L).
\end{aligned}$$

Simplify the squared brackets to the right of $\Pi_i^C(p_i)$ and add and subtract $\pi^*(1-\delta\rho_L)(1-\delta\rho_H)$ to the RHS to obtain

$$\begin{aligned}
&\Pi_i^C(p_i)\big\{(1-\delta)\big[1+\delta-\delta(\rho_L+\rho_H)\big](1-\delta\rho_L r(p_i))\big\} \\
&= (1-\delta\rho_L)(1-\delta\rho_H)\big[\pi_i(p_i)-\pi^*\big] + \pi^*\delta\rho_L(1-r(p_i))\big[1+\delta-\delta(\rho_H+\rho_L)\big] \\
&\quad + \pi^*\delta(1-\rho_L)(1-\delta\rho_L) + \pi^*(1-\delta\rho_L)(1-\delta\rho_H).
\end{aligned}$$

Collecting terms with π^* in the RHS and simplifying, this further reduces to

$$\begin{aligned}
&\Pi_i^C(p_i)\big\{(1-\delta)\big[1+\delta-\delta(\rho_L+\rho_H)\big](1-\delta\rho_L r(p_i))\big\} \\
&= (1-\delta\rho_L)(1-\delta\rho_H)\big[\pi_i(p_i)-\pi^*\big] + \pi^*\big[1+\delta-\delta(\rho_L+\rho_H)\big](1-\delta\rho_L r(p_i)),
\end{aligned}$$

which directly implies the equation in the lemma. □

Proof of Proposition 1. Differentiating $\hat{\Pi}_i(p_i) := \frac{\pi_i(p_i)-\pi^*}{1-\delta\rho_L r(p_i)}$ with respect to p_i and eliminating the positive denominator leads to the first order condition

$$\begin{aligned}
&\frac{\partial\pi_i(p_i;\boldsymbol{p}^*(\boldsymbol{c_H}))}{\partial p_i}\big[1-\delta\rho_L r(p_i;\boldsymbol{p}^*(\boldsymbol{c_H});k)\big] + \delta\rho_L\big[\pi_i(p_i;\boldsymbol{p}^*(\boldsymbol{c_H}))-\pi^*\big] \\
&\quad \times \frac{\partial r(p_i;\boldsymbol{p}^*(\boldsymbol{c_H});k)}{\partial p_i} = 0,
\end{aligned}$$

which has to be satisfied for $p_i = p^*(c_H)$.

Inserting the definition of $\pi_i(p_i;\ \boldsymbol{p^*(c_H)}) = (p_i - c_L)s_i(p_i;\ \boldsymbol{p^*(c_H)})$, this can be reformulated to

$$\left[s_i(p_i;\boldsymbol{p}^*(\boldsymbol{c_H})) + (p_i - c_L)\frac{\partial s_i(p_i;\boldsymbol{p}^*(\boldsymbol{c_H}))}{\partial p_i}\right]\left[1 - \delta\rho_L r(p_i;\boldsymbol{p}^*(\boldsymbol{c_H});k)\right]$$
$$+\,\delta\rho_L\left[\pi_i(p_i;\boldsymbol{p}^*(\boldsymbol{c_H})) - \pi^*\right]\frac{\partial r(p_i;\boldsymbol{p}^*(\boldsymbol{c_H});k)}{\partial p_i} = 0, \tag{19}$$

One can now calculate that

$$\frac{\partial r(p_i;\boldsymbol{p}^*(\boldsymbol{c_H});k)}{\partial p_i} = \frac{\partial\left[1 - F\left(\frac{(N-1)k}{1-s_i(p_i;\boldsymbol{p}^*(\boldsymbol{c_H}))}\right)\right]}{\partial p_i}$$
$$= -f\left(\frac{(N-1)k}{1 - s_i(p_i;\boldsymbol{p}^*(\boldsymbol{c_H}))}\right)\frac{(N-1)k}{\left[1 - s_i(p_i;\boldsymbol{p}^*(\boldsymbol{c_H}))\right]^2}\frac{\partial s_i(p_i;\boldsymbol{p}^*(\boldsymbol{c_H}))}{\partial p_i},$$

where the first equality follows from Equation (6). Evaluated at $p^*(c_H)$, this expression simplifies to

$$\frac{\alpha N}{N-1}Nkf(Nk).$$

Moreover, $r(p^*(c_H);\ \boldsymbol{p^*(c_H)};\ k)$ is given by $1 - F(Nk)$, as was already stated in Equation (7).

Evaluating Equation (19) at $p_i = p^*(c_H)$ thus gives

$$\left[\frac{1}{N} + \left(\frac{1}{\alpha N} + c_H - c_L\right)(-\alpha)\right]\left[1 - \delta\rho_L + \delta\rho_L F(Nk)\right]$$
$$+\,\delta\rho_L\frac{c_H - c_L}{N}\frac{\alpha N}{N-1}Nkf(Nk) = 0.$$

Simplifying the squared bracket, canceling out the positive factor $\alpha(c_H - c_L)$, multiplying by $-(N-1)$ and setting $Nk = \phi$ finally yields the expression in the proposition. □

In order to prove Proposition 2, it is convenient to state the subsequent lemma first. In all of what follows, let $u(p_i) := \pi_i(p_i) - \pi^*$ and $v(p_i) := 1 - \delta\rho_L + \delta\rho_L F\left(\frac{(N-1)k^*}{1-s_i(p_i;\,p^*(c_H))}\right)$, where $v(p_i) > 0$.

Lemma 2. *For any set of parameters c_H, c_L, α, δ, ρ_L, N, some price level $p_i \geq 0$ can only be a global maximizer of $\hat{\Pi}_i(p_i)$ if $u(p_i) > 0$ and $u'(p_i) < 0$.*

Proof. As $\hat{\Pi}_i(p^*(c_H)) = \frac{u(p^*(c_H))}{v(p^*(c_H))} = \frac{\Delta c}{Nv(p^*(c_H))}$ is strictly positive for any Δc, it is clear that only positive values of $u(p_i) = \pi_i(p_i) - \pi^*$ are candidates for a global maximizer of $\hat{\Pi}_i(p_i)$. Moreover, note that $\hat{\Pi}_i'(p_i)$ has the same sign has $u'(p_i)v(p_i) -$

$u(p_i)v'(p_i)$. Hence, since $v(p_i)$ is unambiguously positive, $v'(p_i)$ is unambiguously negative, and $u(p_i)$ is unambiguously positive over the relevant range for global maximizers (by the previous observation), it has to hold that $u'(p_i)$ is strictly negative in order for $u'(p_i)v(p_i) - u(p_i)v'(p_i)$ to be non-positive, which must be the case for a global maximizer of $\hat{\Pi}_i(p_i)$. □

Proof of Proposition 2. I will proceed in two steps. First, I will show that whenever $\mathrm{Var}(\tilde{\theta}) < \left(\frac{3}{6+16\frac{N-1}{\delta\rho L}}\right)^2$, at least one solution $\phi^* = Nk^*$ to Equation (12) exists. Second, I will prove that whenever a solution ϕ^* exists and the demand threshold k is set accordingly, $p^*(c_H)$ must be a global maximizer of $\Pi_i^C(p_i)$, provided that $\Delta c := c_H - c_L$ is sufficiently small.

For the first part, note first that by continuity of F and f, a solution ϕ^* to the equation $h(\phi) = (N-1)\left[1 - \delta\rho_L + \delta\rho_L F(\phi)\right] - \delta\rho_L \phi f(\phi) = 0$ must exist whenever there exists some $\hat{\phi}$ such that $h(\hat{\phi}) < 0$, as $h(0) = (N-1)(1-\delta\rho_L) > 0$. Next, observe that the left part of $h(\phi)$, $(N-1)\left[1 - \delta\rho_L + \delta\rho_L F(\phi)\right]$, is bounded above by $N-1$. Hence, it suffices to show that

$$\exists \phi\colon \tilde{h}(\phi) := N - 1 - \delta\rho_L \phi f(\phi) < 0.$$

Now, from Chebyshev's inequality, it is known that for $z \geq 1$, at least $1 - \frac{1}{z^2}$ of the probability mass of any random variable must be not more than z standard deviations away from the mean. Hence, if the aggregate-market-demand random variable $\tilde{\theta}$ has a standard deviation of σ, at least $1 - \frac{1}{z^2}$ of its probability mass must fall in the range $[1 - z\sigma, 1 + z\sigma]$.

As this interval has a length of $2z\sigma$, the average probability density in this interval must at least be given by $\frac{1-\frac{1}{z^2}}{2z\sigma}$. At worst, the maximum density in this interval is then equal to the average probability density (if all values in the interval have the same density), and therefore it must hold that

$$\max_{\phi \in [1-z\sigma, 1+z\sigma]} \phi f(\phi) \geq (1 - z\sigma)\frac{1 - \frac{1}{z^2}}{2z\sigma}.$$

Inserting this minimal maximum of $\phi f(\phi)$ into the condition from above, a solution to Equation (12) is guaranteed whenever

$$N - 1 - \delta\rho_L(1 - z\sigma)\frac{1 - \frac{1}{z^2}}{2z\sigma} < 0.$$

The bound on the variance in the proposition then simply follows by inserting the simple (but generally not tight) value of $z = 2$ and rearranging for σ, which is the square root of the variance. This proves the first part of the statement.

For the second part, note that if $\phi^* = Nk^*$ solves Equation (12) ($p^*(c_H)$ is a local extremum of $\hat{\Pi}_i(p_i) = \frac{u(p_i)}{v(p_i)}$), a sufficient condition for $p^*(c_H)$ to be a global maximizer of $\hat{\Pi}_i(p_i)$ is that this function is strictly concave over the (connected) range of its potential maximizers. Then, $p^*(c_H)$ is a global maximizer of $\hat{\Pi}_i(p_i)$ over the set of its potential maximizers, and hence, $p^*(c_H)$ must indeed maximize the function. Due to Lemma 2, the range of potential maximizers is characterized by values of p_i such that $u(p_i) > 0$ and $u'(p_i) < 0$.[20]

Next, it is easy to calculate that

$$\hat{\Pi}_i''(p_i) = \frac{\left[u''(p_i)v(p_i) - u(p_i)v''(p_i)\right]v(p_i)^2 - 2\left[u'(p_i)v(p_i) - u(p_i)v'(p_i)\right]v(p_i)v'(p_i)}{v(p_i)^4},$$

which has the same sign as

$$\begin{aligned}&\left[u''(p_i)v(p_i) - u(p_i)v''(p_i)\right]v(p_i) - 2\left[u'(p_i)v(p_i) - u(p_i)v'(p_i)\right]v'(p_i)\\ &\quad = -2\alpha v(p_i)^2 - u(p_i)v''(p_i)v(p_i) - 2u'(p_i)v(p_i)v'(p_i) + 2u(p_i)\left(v'(p_i)\right)^2.\end{aligned}$$

For any set of parameters, over the range of potential maximizers of $\hat{\Pi}_i(p_i)$, this expression is smaller than

$$-2\alpha v(p_i)^2 - u(p_i)\left[v''(p_i)v(p_i) - 2\left(v'(p_i)\right)^2\right],$$

which should be negative in order to guarantee strict concavity of $\hat{\Pi}_i(p_i)$ in the relevant region.

Hence, rearranging the last equation from above, a sufficient condition for $p^*(c_H)$ to be a global optimizer of $\hat{\Pi}_i(p_i)$ is that

$$u(p_i)\left[\left(\frac{v'(p_i)}{v(p_i)}\right)^2 - \frac{v''(p_i)}{2v(p_i)}\right] < \alpha \tag{20}$$

over the range of potential maximizers p_i, given the model parameters.

Now, fix any $\overline{s_i} \in \left(\frac{1}{N}, 1\right)$ and note that whenever c_L is sufficiently close to c_H ($\Delta c < \overline{\Delta c}(\overline{s_i}) := \frac{\overline{s_i} - \frac{1}{N}}{2\alpha}$), no firm will ever want to price so low that it obtains a market share larger than $\overline{s_i}$.[21]

20 The fact that this range is connected trivially follows from strict concavity of $u(p_i)$.
21 The inequality in brackets is obtained by solving $s_i(p^D;\, p^*(c_H)) < \overline{s_i}$, where $p^D := \frac{1}{\alpha N} + \frac{c_H + c_L}{2}$ is the solution to the strictly concave program $\max_{p_i}(p_i - c_L)s_i(p_i;\, p^*(c_H))$.

As $F(\theta)$ is twice continuously differentiable, it is easy to see that for any $\overline{s_i} < 1$,

$$\overline{v}(\overline{s_i}) := \max_{p_i \in [s_i^{-1}(\overline{s_i}), s_i^{-1}(0)]} \left[\left(\frac{v'(p_i)}{v(p_i)} \right)^2 - \frac{v''(p_i)}{2v(p_i)} \right]$$

must be finite and independent of c_L. Moreover, $u(p_i)$ is bounded above by $\max_{p_i} u(p_i) = \frac{\Delta c}{N} + \frac{\alpha(\Delta c)^2}{4}$, which can be made arbitrarily small as c_L approaches c_H. Hence, given a fixed $\overline{s_i}$, Equation (20) must be satisfied for all relevant p_i if the following two conditions are met:

$$\Delta c < \overline{\Delta c}(\overline{s_i}) \quad \text{and} \quad \left[\frac{\Delta c}{N} + \frac{\alpha(\Delta c)^2}{4} \right] \overline{v}(\overline{s_i}) \leq \alpha.$$

In particular, this can always be achieved if c_L is sufficiently close to c_H. □

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