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CLUSTERING, COOPERATION, AND SEARCH IN SOCIAL NETWORKS

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Abstract
The paper discusses the important role of clustering in the evolution of social networks, as it affects not only the incentives of players to cooperate but also their ability to search for fresh opportunities. Depending on the volatility of the environment and the social convention in place, we show that network clustering endogenously adapts to achieve and maintain, to the extent possible, a state of high social connectivity in the long run. (JEL: C71, D83, D85)

1. Introduction

There has been a recent burst of interest in the study of networks that springs from a wide diversity of fields: economics, sociology, biology, mathematics, and physics. Despite their common object of concern, the focus and methodology is markedly different in each case.

Thus, in economics and other social sciences, the social network is often taken to be endogenous and the objective is to understand how payoffs and the entailed agents' incentives might explain their pattern of links and the induced outcomes.[1] The problem is frequently formalized as a network formation game, the focus

1. See, for example, Aumann and Myerson (1988), Jackson and Wolinsky (1996), Bala and Goyal (2000) or, for a selected overview, the papers collected in Dutta and Jackson (2003). There is also a very large sociological literature that places less emphasis on individual incentives underlying network formation and more on the performance and payoffs associated to given network architectures and corresponding network positions. Granovetter (1973) and Burt (1982) are good examples of this approach.

then being on how some suitable notion of equilibrium may define the range of (strategically) stable networks. Implicitly, this presumes that the underlying environment is rather stable and/or simple, which allows agents to know (or eventually learn) their relevant range of possibilities. In a sense, therefore, this approach abstracts from the variability and complexity that appears to be a key feature of most interesting setups in the real world.

By contrast, the approach pursued by other disciplines (e.g., the growing literature on so-called complex networks)[2] is of a somewhat polar nature. The main concern has been to unveil large-system regularities that arise from simple mechanisms of network formation, aiming to understand as well their implications for different measures of performance—for example, diffusion, search, or adaptability. Applied to social setups, however, this approach lacks the "microfoundation" on agents' incentives that economists generally insist in essential to truly understand socioeconomics phenomena.

The above considerations suggest that fruitful insights may be gained from integrating the former two approaches. In a nutshell, the basis of this effort should be the recognition that both incentives and complexity are essential ingredients (for us analysts as well as for the agents) in any proper understanding of the process of network formation in most real-world environments. Here, we illustrate the potential of such integration by reflecting on a particular issue: the (endogenous) role of network clustering in supporting the conflicting requirements for cooperation and search in a volatile environment.

The insights obtained in this respect build upon research conducted by Vega-Redondo (2002) and Marsili, Vega-Redondo, and Slanina (2004), hereafter referred to as VR and MVS, respectively. The common setup consists of a given population of agents who continuously search for valuable new links (fresh payoff opportunities), while the links already in place decay stochastically and eventually vanish owing to volatility. The struggle between these two forces, search and volatility, generates a tension reminiscent of what evolutionary biologists label the Red Queen Principle (see Van Valen 1973): "it takes all the running you can do, to keep in the same place" (Carrol 1872).

We find that the resolution of such tension crucially depends on two exogenous features of the situation: (i) the nature of search (i.e., its intensity and depth); and (ii) the prevailing social norm (i.e., whether opportunism is deterred by bilateral or multilateral punishments). We also stress interesting implications for the architecture of the evolving social network as it adapts endogenously to changes in volatility. In particular, our main conclusion is that the impact of increasing volatility on clustering depends crucially on the social convention in place as well as on the characteristics of search. Specifically, we find that clustering grows

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2. See, for example, Watts and Strogatz (1998), Barábasi and Albert (1999), the monograph by Dorogovtsev and Mendes (2003), or the excellent survey paper by Newman (2003).

with volatility if the social convention is multilateral and search is long range, whereas the opposite happens if the convention is purely bilateral and search is short range. This points to an interesting interplay among the social convention in place, the exogenous environmental conditions, and the endogenous architecture of the social network.

2. The Theoretical Framework

Let time be modeled discretely, with $t = 0, 1, 2, \ldots$ indexing the consecutive time periods. The population of players, N, is finite. At every t, each agent $i \in N$ supports a certain number of links that connect her to a set of other agents, $A_i(t) \subset N \setminus \{i.\}$ To simplify matters, let us suppose that the number of links that can be supported by any given agent is at most m, a parameter of the model. Given the set of links $\{A_i(t)\}_{i \in N}$ supported by all agents at t, the prevailing social network $g(t)$ is defined as follows: $g(t) = \{(i, j) \in N \times N : j \in A_i(l) \vee i \in A_j(t)\}$. By construction, therefore, $g(t)$ is undirected so that $(i, j) \in g(t) \Leftrightarrow (j, i) \in g(t)$, and we may simply use the notation ij to refer to the corresponding link. For any agent i, the set $N_i(t) \equiv \{j \in N : ij \in g(t)\}$ is called her *neighborhood* at t.

Every agent i is assumed to play an infinite repetition of a Prisoner's Dilemma (PD) game with each of her neighbors $j \in N_i(t)$. The payoff of the stage PD game are ij-idiosyncratic, as given by the following table:

$$
\begin{array}{c|c|c}
ij & C & D \\
\hline
C & \zeta_{ij}, \zeta_{ij} & \zeta_{ij} - 1, \zeta_{ij} + 1 \\
\hline
D & \zeta_{ij} + 1, \zeta_{ij} - 1 & 0, 0
\end{array}
\tag{1}
$$

where $1 > \zeta_{ij} (= \zeta_{ij}) > 0$. As is customary, C and D are labeled "Cooperate" and "Defect", respectively. Thus, the payoff ζ_{ij} obtained by both players if they jointly cooperate is ij-specific and will change over time in the dynamic ("volatile") environment to be considered later on.

The social dynamics consists of three main components: volatility, search, and link removal. We describe each of them in turn.

1. *Volatility:* At every t, each of the links $ij \in g(t-1)$ prevailing during the preceding period has its corresponding PD cooperation payoffs $\zeta_{ij}(t-1)$ subject to an independent random redraw with probability ε. Thus, with probability ε the new payoff $\zeta_{ij}(t)$ is obtained through a random (say, uniform) draw from some nonnegative real interval $[\underline{\zeta}, \bar{\zeta}]$. With the complementary probability $1 - \varepsilon$, there is payoff persistence; that is, $\zeta_{ij}(t) = \zeta_{ij}(t-1)$; that is, the value of ε is taken to be a measure of the volatility of the environment.
2. *Search:* At every t, every agent i enjoys the possibility of supporting new links, where this opportunity is obtained through either "local" or "global" search.

(a) With probability p, the search is local, which means that it is restricted to lie within the component of i in $g(t-1)$, denoted by $C_i(t-1)$.[3] Two parameters determine the depth and intensity of local search. The depth $\theta\,(\geq 2)$ specifies the geodesic distance from i at which this player can find new potential partners. On the other hand, the intensity is parametrized by $\sigma\,(\geq 1)$, which specifies the maximum number of different players that can be "sampled" by i for new payoff opportunities. Given these parameters, we postulate that any searching player makes σ distinct explorations of new linking opportunities by "moving" along the network at most θ steps away. If the player j thus "met" is not already a neighbor, ($j \notin N_i(t-1)$), then player i obtains an independent payoff draw for each such j, again uniformly on the contemplated interval.

(b) With probability $(1-p)q$ the search is global, which means that player i obtains one new draw for some arbitrarily chosen player $j \in N\backslash N_i$ $(t-1)$.

Once having completed her search (local or global) of new payoff opportunities, player i is entitled to create at most a single new link, possibly dispensing with one of the pre-existing links if she already supports m of them. For simplicity, we assume that the player ends up supporting the admissible collection of links with maximum payoffs, thus ignoring at this stage all strategic considerations.[4]

3. *Link removal*: Let $\tilde{g}(t)$ denote the network resulting from search at t. For every $ij \in \tilde{g}(t)$, the two players involved (i and j) are assumed to verify, prior to actual play, whether their link is worth keeping. Assuming that only links that support cooperation are judged worthwhile, we are interested in contrasting two possibilities. These are associated to different conventions governing agents' strategic interaction.

(i) *Bilateral convention* (BC): In this case, cooperation between i and j must be supported bilaterally in the infinitely repeated PD played by them. This means that the cooperation payoff $\zeta_{ij}(t)$ must satisfy $\zeta_{ij}(t) \geq (1-\delta)/\delta$, where δ denotes as usual the (common) factor at which players discount their stage payoff in the repeated game.

(ii) *Multilateral Convention* (MC): Under this convention, any unilateral deviation from cooperation by a player with one of her neighbors will be punished by any of her other neighbors as soon as the latter learn about it. More specifically, let us suppose that, in every round of the

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3. The component $C_i(t-1)$ consists of all those other nodes j for which there is a network path from i to j in $g(t-1)$.

4. Allowing players to factor strategic considerations into their decisions of link creation has no significant effect in the analysis.

PD games played "in parallel" by each pair of connected players, not only does this interaction take place but also the players share any other strategic information they might have—most importantly, information on whether any other agent has unilaterally deviated from cooperation with some of their partners in a preceding period. Then, one can show (cf. VR) that the relevant condition for cooperation between i and j (under the simplifying assumption that any deviation can affect only one link at a time) is given by the following requirement:

$$\zeta_{ij}(t) \geq \frac{1-\delta}{\delta} - \sum_{k \in N_i \setminus \{j\}} \left(\zeta_{ik}(t) + \frac{1-\delta}{\delta} \right) \delta^{d^i(j,k)-1}, \qquad (2)$$

where $d^i(j, k)$ is the length of the shortest path joining j and k that does not involve player i. The interpretation of this i-excluding distance between j and k is straightforward: it is the number of steps (and thus periods, in the repeated game) that would be required for any information held by j (or k) to reach k (or j) without the concourse of player i. Note, of course, that the requirement of cooperation under MC is never stronger than under BC, and it may be much weaker if the average distance between neighbors (excluding the agent in question) is low. In the language of sociologist James Coleman (1988), this underscores the importance of "social closure" (or network clustering, using the network terminology to be introduced below) in supporting cooperation multilaterally when bilateral incentives may be too weak to do so.[5]

Under whatever convention applies, BC or MC, all links in $\tilde{g}(t)$ that are not anticipated to support cooperation are removed, thus leading to the actual network $g(t)(\subset \tilde{g}(t))$ prevailing at t.

3. Analysis

We want to contrast the implications of BC and MC, the two different social conventions just specified. This comparison has been carried out in VR and MVS under polar search conditions, using numerical simulations as well as mean-field analysis. (Mean-field analysis is an approach commonly use in statistical physics that approximates the dynamics of the system by its expected motion, under the assumption that the forces impinging on each node are stochastically independent.) The main insights delivered by both numerical simulations and

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5. Within the economic literature, the present considerations are reminiscent of the well-known work on multimarket collusion pioneered by Bernheim and Whinston (1990)—see also Spagnolo (1999)—or the historical research conducted by Greif (1993) on early trading institutions. Related work, but with a different perspective, can be found in Haag and Lagunoff (2000) or Annen (2003).
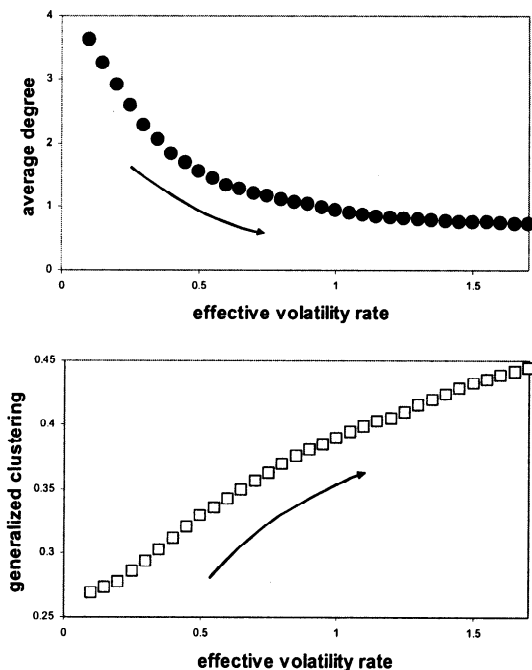
FIGURE 1. Long-run behavior of the process studied in VR operating under MC, as volatility gradually increases and the system stabilizes to corresponding long-run values.

mean-field analysis essentially coincide. Here, owing to space limitations, we restrict attention to the numerical results, and refer the interested reader to VR and MVS for a detailed discussion of the analytical approach.[6]

Our main concern is to understand how a rise in the key environmental parameter, the effective (normalized) rate of volatility ε/p, affects the network architecture and the ability of the society to maintain a high level of interaction. First, Figure 1 depicts the situation under MC in a context with high depth and intensity of search—specifically, we make $\theta = \sigma = n$, where n is the (large but infinite) population size.[7] This diagram traces long-run magnitudes that are obtained as volatility is increased very gradually and, at each step, the system is let to stabilize

around well-defined long-run values. The upper panel shows that, as expected, rising volatility has a detrimental effect on long-run network connectivity. It is also interesting to note that the (negative) effect is of decreasing (absolute) magnitude as volatility rises. But the more interesting observation reported in VR is that, as the volatility rate ε grows, the architecture of the social network endogenously adjusts so that the average distance between neighbors of a randomly selected node,

$$\frac{1}{n} \sum_{i \in N} \left(\frac{1}{|N_i||N_i - 1|} \sum_{j,k \in N_i} d^i(j,k) \right),$$

correspondingly decreases.[8] To cast his conclusion in a way that may be conceptually related to the customary notion of clustering, the lower panel of Figure 1 traces the change in the inverse of the aforementioned average distance—a magnitude that, for the sake of a better name, we simply label the generalized clustering of the network. Its role in supporting cooperation should be clear in view of (2). It is remarkable that such a cooperation-enhancing feature of the social network should emerge in the long run as a cumulative but unintended by-product of agents' independent adjustments.

In contrast, Figure 2 presents the counterpart of the previous results when the underlying social convention is BC. The differences with the scenario governed by MC are quite sharp and interesting. The upper panel shows that the detrimental effect of volatility on network connectivity is already significant at volatility rates that are lower than before by a full order of magnitude. Moreover, these effects become of increasing magnitude as volatility grows, eventually leading to an abrupt transition to a social network with very low average connectivity as the volatility rate ε surpasses a certain threshold. The responsibility for this contrasting state of affairs is to be found in the absence of strategic network effects that, in this scenario, no longer can be relied upon to act as a "buffer" against volatility. This is indeed confirmed by the lower panel of Figure 2, which shows in a clear way the irrelevance of these effects in the present scenario. Since they no longer provide any advantage under BC, generalized clustering remains largely unchanged as volatility increases, provided the social network remains significantly connected. At the point of the transition, however, it falls abruptly as a by-product of the general increase in distances (in particular, between neighbors) brought about by the sharp decrease in overall network connectivity.

The preceding discussion stresses the potentially beneficially role of network clustering in supporting cooperation when the convention governing strategic play is multilateral rather than bilateral. In general, however, it is intuitive that there could be a countervailing negative effect of network (generalized) clustering on

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8. When no path exists between two neighbors of a given node, the entailed "infinite" distance was truncated to n, the size of the population.
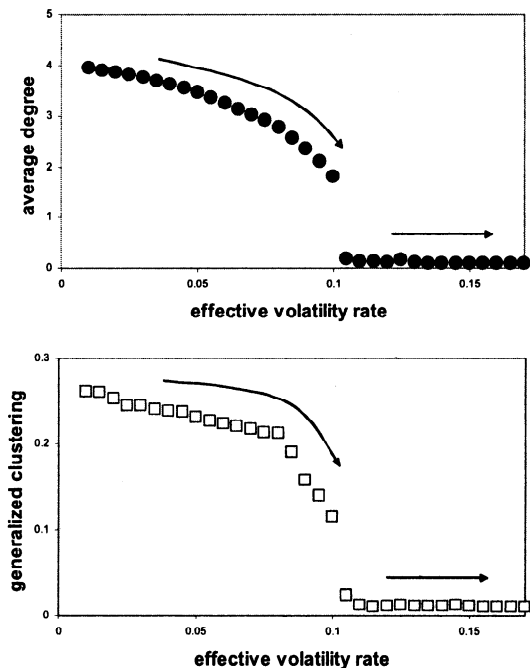
FIGURE 2.   Long-run behavior of the process studied in VR operating under BC, as volatility gradually increases and the system stabilizes to corresponding long-run values.

the effectiveness of search. For example, if many of an agent's neighbors are close to each other, then local search typically induces substantial redundancies: the exploration of new linking options via neighbors will often lead to agents who are already neighbors. These considerations did not have any significant bearing on our former discussion because of the extreme parameter configuration used for the intensity and depth of search. Specifically, we posited $\theta = \sigma = n$, so that search is effectively global; that is, a searching agent samples a fresh opportunity with every other player in the component who is not yet a neighbor. However, if search is substantially less intense and deep (i.e., if the parameters θ and σ display much lower values), then matters should be expected to turn out markedly different.

To shed light on this issue in the starkest fashion, let us contrast the previous search scenario with the polar context in which $\theta = 2$ and $\sigma = 1$ (i.e., only second neighbors may become new fresh neighbors and just one agent is sampled in every search opportunity). Furthermore, suppose for concreteness that the convention in place does not allow for strategic multilateral effects, so that BC applies. This is essentially the context studied by MVS, which can be regarded as a particular case of the foregoing general setup for low-intensity and short-range search. As

volatility increases gradually, the numerical results obtained in the simulatoins
are as depicted in figure 3.[9]

First, its upper panel shows that, as volatility grows, network connectivity
falls, again in an accelerating (and eventually abrupt) fashion, in line with what
was observed in Figure 2 above. The interesting point of contrast, however, is
found in the lower panel of Figure 3. there we depict the effect of increasing
volatility on the usual measure of network clustering, defined as the probability
that two randomly selected neighbors of any given node are themselves neigh-
bors.[10] We observe that an inherent feature of any long-run state with high connec-
tivity is its low clustering. Why is this so? Because, as explained, high clustering
leads to redundancy of search in the present case: the neighbor of a neighbor is
likely to be already a neighbor. This in turn, reduces the effectiveness of search
in offsetting volatility by the creation of new links. Since we have $\theta = 2$, a net-
work with low clustering may be seen as representing a situation where neighbors
are "far apart"—that is, more than the one step away required for local search.
Hence, in this light, we find a sharp contrast between the situation depicted in
Figures 2 and 3. In both, the social norm BC applies so that strategic network
effects are absent and thus clustering has no behavioral implications. Though in
the first case clustering (or neighbor distances) have no effect on search (which
is both extensive and long-range), in the second scenario (where the opposite
applies) this effect is crucial and has a strong bearing on the sustainability of a
high-connectivity state as volatility increases. Only when volatility rises beyond
a certain point and a connected network can no longer be maintained does clus-
tering increase, a manifestation of the fact that the network becomes fragmented
into a large number of small cliques.

We started this paper by putting forward the view that a much richer under-
standing of social networks ought to follow form an integration of the concerns
for incentives and complexity that characterize, respectively, the socioeconomic
and "complex system" literatures. We have briefly illustrated the potential of
such an integration by focusing attention on the important specific issue of how
the independent adjustment of agents endogenously shapes the clustering of the

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9.   Figure 3 is intended to be only suggestive of the main ideas underlying MVS, and it abstracts
form some of the interesting features that arise in the analysis. For example, we found that there is
some hysteresis (dependence of initial conditions) around the point of transition, as is characteristic
of (first-order) phase transitions in models of statistical physics. That is, the threshold on volatility
that marks the transition form a dense to a sparse network as volatility rises is higher than the one
that triggers the opposite transition when volatility falls.

10.   Observe that a network has a maximum clustering of 1 if and only if its generalized clustering (as
defined before) is also 1. However, low levels of clustering may be consistent with much higher levels
of generalized clustering if neighbors tend to be connected by short (albeit not direct) paths. Formally,
note that clustering can be defined by $(1/n) \sum_{i \in N} \{ [|N_i||N_i - 1]^{-1} \sum_{j,k \in N_i} [\tilde{d}^i(j,k)]^{-1} \}$, where
$\tilde{d}^i(j,k) = 1$ if j and k are neighbors and $\tilde{d}^i(j,k) = \infty$ otherwise. This suggests that generalized clus-
tering could have been alternatively defined as $(1/n) \sum_{i \in N} \{ [|N_i||N_i - 1]^{-1} \sum_{j,k \in N_i} [d^i(j,k)]^{-1} \}$,
a modification that should deliver the same qualitative insights as reported in Figures 1 and 2.
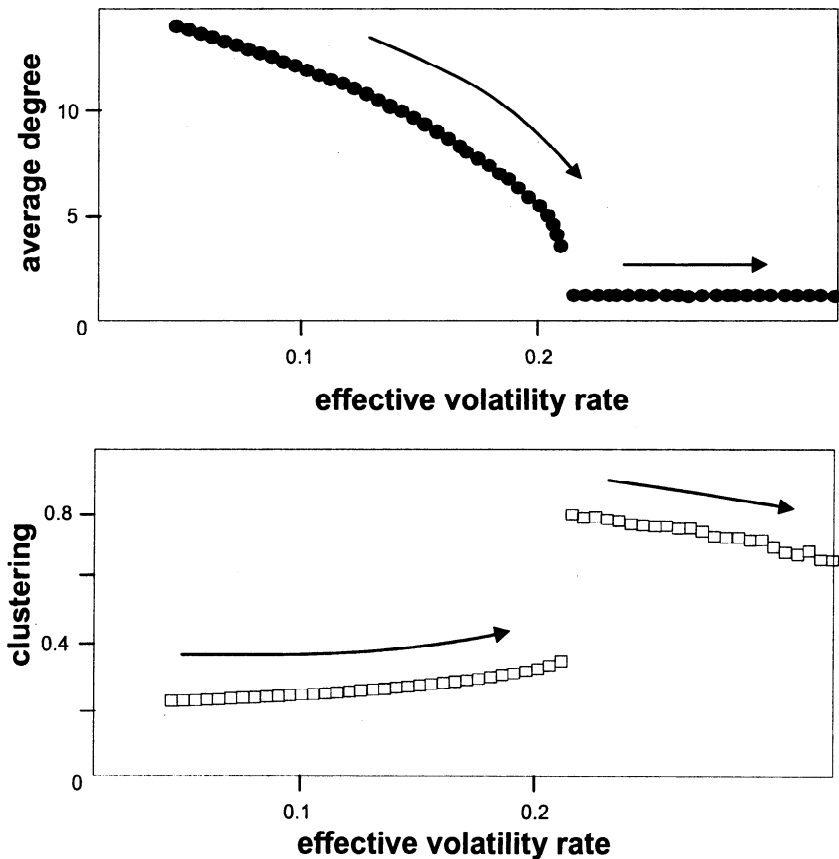
FIGURE 3. Long-run behavior of the process studied in MVS operating under BC, as volatility gradually increases and the system stabilizes to corresponding long-run values.

social network. In essence, our conclusion is that the long-run outcome may be understood as striking a compromise between two conflicting aims: the incentives of agents to behave cooperatively and their ability to search effectively in a complex and volatile environment.

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