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Networks of collaboration in oligopoly

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Abstract

In an oligopoly, prior to competing in the market, firms have an opportunity to form *pair-wise* collaborative links with other firms. These pair-wise links involve a commitment of resources and lead to lower costs of production of the collaborating firms. We study the incentives of firms to form links and the architecture of the resulting collaboration network.

We find that incentives to form links are intimately related to the nature of market competition. Our analysis also suggests that collaborations are used by firms to generate competitive advantage and that strategically stable networks are often asymmetric, with some firms having many links while others have few links or are isolated. Specifically, we show that networks with the dominant group architecture, stars, and inter-linked stars are stable. We also present some results on the architecture of socially efficient networks.

JEL classification: C70; L13; L20

Keywords: Networks; R&D collaboration; Oligopoly

1. Introduction

Empirical work suggests that collaboration between firms is common. This collaboration takes a variety of forms which includes creation and sharing of knowledge about markets and technologies (via joint research activities), setting market standards and sharing facilities (such as distribution channels). Technological partnerships and joint re-

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search activity between firms have become prominent in recent years. In the area of biotechnology, information technology and new materials, for instance, the number of technological collaborations rose from an annual average of 63 in the 1975–1979 period to about 300 per year during 1980–1984, and reached an average of 536 per year in the 1986–1989 period. At the same time, in the area of biotechnology, the number of firms involved in these collaborations also increased sharply, doubling in the 1986–1989 period as compared to the pre-1986 period (Delapierre and Mytelka, 1998; Hagedoorn and Schakenraad, 1990). This empirical work has also highlighted certain structural features of collaborative activity. Typically, collaboration ties are bilateral and are embedded within a broader network of similar ties with other firms.[1] Moreover, considerable asymmetries exist between the level of collaborative activity across firms, with some firms forming several ties while others are poorly linked.[2] These empirical patterns lead us to examine the incentives of firms to collaborate with each other. In particular, we study how the incentives of firms are influenced by the nature of market competition and the costs of forming links.

We consider an oligopoly setting in which firms form *pair-wise* collaborative links with other firms. These pair-wise links involve a commitment of resources on the part of the collaborating firms and yield lower costs of production for firms which form the link.[3] The collection of pair-wise links defines a collaboration network and induces a distribution of costs across the firms in the industry. Given these costs, firms then compete in the market. A distinctive feature of our model is that we allow a firm to form collaboration relations with other firms without seeking prior permission of current collaborators. This has important strategic effects and requires novel methods of analysis.[4]

We start by analyzing the case *where the costs of forming links are small.* To get a first impression of the issues involved we study the textbook example of a market with homogeneous products under both price and quantity competition. We provide a complete characterization of strategically stable and socially efficient networks in this example. We find that *under quantity competition, the complete network is the unique stable and socially efficient network* (Propositions 3.1 and 3.3). The complete network is one in which every pair of firms has a link. We also show that *with price competition the empty network is the unique stable network, while the efficient network is an inter-linked star, with two central firms* (Propositions 3.2 and 3.4). The network with no links is referred to as the empty

[1] For instance, in the late 1980's and 1990's, Bristol-Mayers and Bayer were engaged in several joint projects, but Bayer also had collaborative links with Hoechst, while there were no links between Bristol-Mayers and Hoechst. Thus relations between firms are *non-exclusive/intransitive* (Delapierre and Mytelka, 1998).

[2] During the late 1980's, Daimler Benz had over a dozen collaborations with GEC and Thomson, respectively, which related to cost sharing in the area of information technology, while firms such as Volkswagen and BMW had almost no collaborations in this area. Significant differences in linking activity across firms were also observed in biotechnology (Hagedoorn and Schakenraad, 1990).

[3] We interpret a link as a collaborative R&D project, which involves complementary facilities of the two firms. The project is costly and hence calls forth resources from the collaborating firms; it yields a process innovation which lowers the costs of production of the firms involved.

[4] The number of possible collaboration networks is very large. In a market with n firms, there are $2^{n(n-1)/2}$ possible networks of collaboration. Thus, if $n = 10$, then there are over a billion possible networks of collaboration!

network. An inter-linked star is an asymmetric network in which there are (at least) two groups of firms, a maximally linked group in which every firm is fully linked with all other firms and a group of firms in which every firm has links only with the first group of maximally linked group. Our results thus show that the nature of market competition has an important effect on the type of collaboration networks that arise and this has a bearing on the level of welfare as well.

We then consider network formation under general market conditions, which we classify into *moderate* and *aggressive competition*. In a market with moderate competition, all firms make positive profits but lower cost firms make larger profits. Thus quantity competition under homogeneous or differentiated demand, and price competition under differentiated demand are special cases of this type of competition. We first show that every pair of firms with the same costs must be linked. This implies that in the class of symmetric networks, i.e., networks where every firm has the same number of collaboration links, only the complete network can be stable. *We then develop sufficient conditions for this network to be the unique stable network* (Theorem 3.1). We find that these conditions, though strong, are satisfied by a variety of standard oligopoly models so long as the reduction in marginal cost from the formation of a link is linear. In a market with *aggressive competition* a firm makes profits only if it is has the lowest costs. We focus on the case where all lowest cost firms make positive profits. A patent race with the largest collaborating group winning the race is a special case of this type of competition. We find that under aggressive competition, *stable networks have the dominant group architecture* (Theorem 3.2). In a dominant group architecture, firms divide themselves into two groups, with one group containing at least three firms and having the feature that every pair of firms has a collaboration link, while the second group consists of isolated firms.

We then take up the case where *costs of forming links are significant*. The analysis here focuses on the relationship between these costs and the nature of stable networks. For reasons of tractability we work with the linear demand Cournot model. We first derive a general property of the returns from link formation: firms have increasing returns from links. We then show that, for the interesting class of parameters, *a stable network has the dominant group architecture, with the size of this group being sensitive to the cost of forming links* (Proposition 4.1). An interesting aspect of our analysis is a *non-monotonicity* in the sustainable size of the dominant group as the costs of forming links increase. For small costs of forming links, only a large dominant group is stable, at intermediate costs large as well as small groups can be stable, while for large costs only a medium sized group is stable.

The property of increasing returns from link formation suggests that a firm with many links may have an incentive to induce a firm with few links to form a collaboration relationship by offering to subsidize its costs of link formation. This motivates an examination of stable networks when transfers are allowed between firms. We show that *a stable network has the dominant group architecture or is an inter-linked star* (Propositions 4.2–4.4). We also provide a characterization of the conditions under which the star (which is a special case of inter-linked stars, with just one central firm) is stable. These results are interesting from an empirical point of view, since intransitive network architectures are commonly observed in practice.[5] The stability of the star and inter-linked star architectures also illustrates in a somewhat dramatic fashion how market dominance can arise in a setting with ex-ante identical firms. It also brings out clearly the role of transfers across links, since such structures would not be stable in the absence of transfers.

Our paper is a contribution to the study of group formation and cooperation in oligopolies. The model of collaborative networks we present is inspired by the recent work on strategic models of network formation; see, e.g., Aumann and Myerson (1989), Bala and Goyal (2000), Boorman (1975), Dutta et al. (1995), Goyal (1993), Jackson and Wolinsky (1996), and Kranton and Minehart (2001).[6] To the best of our knowledge, the present paper forms the first attempt to study the relationship between inter-firm networks and market competition. The work of Kranton and Minehart (2001) deals with networks between vertically related firms. By contrast, our paper studies collaborative ties between *horizontally* related firms, i.e., firms which compete in the market subsequently. This leads us to incorporate an explicit market competition element in our collaboration model. The analysis in our paper suggests that market competition has major implications for the nature of collaboration networks.

Issues relating to group formation and cooperation have been a central concern in economics, and game theory in particular. The traditional approach to these issues has been in terms of coalitions. In recent years, there has been considerable work on coalition formation in games; see, e.g., Bloch (1995), Kalai et al. (1979), Ray and Vohra (1997), and Yi (1997). For a survey of this work, refer to Bloch (1997). One application of this theory is to the formation of groups in oligopolies. In this literature, group formation is modeled in terms of a *coalition structure* which is a partition of the set of firms. Each firm therefore can belong to one and only one element of the partition, referred to as a *coalition.* The network approach differs from the coalition approach in two ways. First, the network approach considers two-player or bilateral relationships. In this respect it is somewhat restrictive as compared to the coalition framework, which allows for multilateral relationships among groups of arbitrary size. On the other hand, the network approach allows for a richer class of relationships with regard to the nature of collaboration structures that can emerge. In particular, the network approach allows for collaborative relations which are *intransitive.* Examples of such structures are stars/inter-linked stars, and symmetric networks such as lattices and cycles. These patterns are empirically observed as well as strategically stable but are ruled out in the coalition literature.[7]

Finally, we would like to mention the substantial body of work in sociology, organization theory and business strategy which deals with inter-firm networks (see, e.g.,

[5] See, for instance, Figs. 4.3 and 4.4 in (Delapierre and Mytelka, 1998), which plot the architecture of the collaboration networks in the pharmaceutical and biotechnology industry.

[6] The present paper subsumes our earlier paper (Goyal and Joshi, 1999).

[7] A direct comparison of our results with the coalition literature is difficult in view of the above and there are other substantive differences in the models as well such as the role of spillovers. We therefore discuss the results of Bloch and Yi in greater detail, after presenting our results, in Section 3.

Delapierre and Mytelka, 1998; Nohria and Eccles, 1992; Granovetter, 1994; Podolny and Page, 1998). The contribution of our paper to this literature is a strategic model of network formation.

The paper is organized as follows. In Section 2, we present the basic model. In Section 3, we analyze the formation of networks when the costs of forming links are small, while Section 4 examines the case where costs of forming links are large. Section 5 concludes.

2. The model

We consider a setting in which a set of firms first choose their collaboration links with other firms. These collaboration agreements are pair-wise and costly and help lower marginal costs of production. The firms then compete in the product market. We now develop the required terminology and provide some definitions.

2.1. Networks

Let $N = \{1, 2, \dots, n\}$ denote a finite set of ex-ante identical firms. We shall assume that $n \geqslant 3$. For any $i, j \in N$, the pair-wise relationship between the two firms is captured by a binary variable, $g_{i,j} \in \{0, 1\}$; $g_{i,j} = 1$ means that a link is established between firms i and j while $g_{i,j} = 0$ means that no link is formed. A network $g = \{(g_{i,j})_{i,j \in N}\}$, is a formal description of the pair-wise collaboration relationships that exist between the firms. We let $\mathcal{G}$ denote the set of all networks. Let $g + g_{i,j}$ denote the network obtained by replacing $g_{i,j} = 0$ in network g by $g_{i,j} = 1$. Similarly, let $g - g_{i,j}$ denote the network obtained by replacing $g_{i,j} = 1$ in network g by $g_{i,j} = 0$.

A *path* in g connecting firms i and j is a distinct set of firms $\{i_1, \dots, i_n\}$ such that $g_{i,i_1} = g_{i_1,i_2} = g_{i_2,i_3} = \dots = g_{i_n,j} = 1$. We say that a network is *connected* if there exists a path between any pair $i, j \in N$. A network, $g' \subset g$, is a *component* of g if for all $i, j \in g'$, $i \neq j$, there exists a path in g' connecting i and j, and for all $i \in g'$ and $j \in g$, $g_{i,j} = 1$ implies $g_{i,j} \in g'$. We will say that a component $g' \subset g$ is *complete* if $g_{i,j} = 1$ for all $i, j \in g'$. Finally, let $N_i(g) = \{j \in N \setminus \{i\} \mid g_{i,j} = 1\}$ be the set of firms in $N \setminus \{i\}$ with whom firm i has a link in the network g, and let $\eta_i(g) = |N_i(g)|$.

We now define some networks that play a prominent role in our analysis. The *complete* network, g^c, is a network in which $g_{i,j} = 1$, $\forall i, j \in N$, while the *empty* network, g^e, is a network in which $g_{i,j} = 0$, $\forall i, j \in N$, $i \neq j$. The *dominant group* architecture, g^k, is characterized by one complete non-singleton component with $k \geqslant 2$ firms and $n - k$ singleton firms. Thus, there is a set of firms $N' \subset N$ with the property that $g_{i,j} = 1$ for every pair $i, j \in N'$ while for any $k \in N \setminus N'$, $g_{k,l} = 0$, $\forall l \in N \setminus \{k\}$. An *inter-linked* star is defined as follows. Let $h(g) = \{h_1(g), h_2(g), \dots, h_m(g)\}$ be a partition of the firms with the feature that $\eta_i(g) = \eta_j(g)$, if and only if $i, j \in h_l(g)$ for some $l \in \{1, 2, \dots, m\}$. The groups are numbered in ascending order of links. An inter-linked star is a partition with two features:

(i) $\eta_i(g) = n - 1$ for all firms $i \in h_m(g)$, and
(ii) $N_j(g) = h_m(g)$ for all firms $j \in h_1(g)$.

A firm in the former group is referred to as a *central* firm, while a firm in the latter group is referred to as a *peripheral* firm. The *star* is a special case of this architecture, in which there are only two groups, and $|h_m(g)| = 1$ and $|h_1(g)| = n - 1$.

2.2. Collaboration links and cost reduction

A collaboration link can be interpreted as an agreement to jointly invest in cost-reducing R&D activity. We will suppose that a collaboration link requires a fixed investment, given by $f > 0$, from each firm. In this paper we focus on the role of collaborations as a way to reduce costs of production. This motivates the assumptions that initially firms are symmetric with zero fixed costs and identical constant returns-to-scale cost functions. The constant-returns-to-scale assumption is undoubtedly restrictive. However, this is a first attempt at studying the strategic incentives to form links in a oligopoly setting and it is useful to start with a well understood benchmark model. In this connection we note that the large body of work on collaborative R&D makes similar assumptions. See, for instance, Bloch (1995), d'Aspremont and Jacquemin (1988), Kamien et al. (1992), Leahy and Neary (1997), and Suzumura (1992). This assumption therefore has the advantage of allowing us to place the network approach in perspective with regard to the earlier literature on R&D cooperation between firms.

Collaborations lower marginal costs of production. A network g, therefore, induces a marginal cost vector for the firms which is given by $c(g) = \{c_1(g), c_2(g), \ldots, c_n(g)\}$. We assume that firm i's marginal cost in the network g is a function of the number of collaboration links it has with other firms and is strictly decreasing in the number of these links:[8]

$$c_i(g) = c\big(\eta_i(g)\big), \quad c\big(\eta_i(g) + 1\big) < c\big(\eta_i(g)\big), \quad i \in N. \tag{1}$$

To rule out uninteresting cases, we will always assume that $c_i(g) \geqslant 0$, $\forall i \in N$, $\forall g \in \mathcal{G}$. In some parts of the paper we shall assume that marginal costs are linearly declining in the number of links. Formally,

$$c_i(g) = \gamma_0 - \gamma \eta_i(g), \quad i \in N, \tag{2}$$

where $\gamma_0 > 0$, represents a firm's marginal cost when it has no links, while $\gamma > 0$ is the cost reduction induced by each link formed by a firm.[9]

Given this cost vector, and the specification of the demand functions in the product market, the firms compete in the second stage in the market. For every network g, we assume there is a well-defined Nash equilibrium in the second stage product market game. The profits of firm i, gross of the cost of forming links are given by $\pi_i(g)$.[10]

[8] We are assuming that there are no spillovers across links in this model. We briefly address the issue of spillovers in Section 5.

[9] This is a natural extension to the network framework of the specification used by Bloch (1995) where marginal cost of i decreases linearly in the number of firms belonging to the same coalition as i. Bloch presents a number of examples where a linear reduction in marginal cost is justified. We discuss the implications of non-linear reduction in marginal cost in Section 3.

[10] This implicitly assumes that there are no coordination problems in choosing across different equilibria at this stage.

2.3. Stable networks

A network g is said to be stable if any firm that is linked to another in the network has a strict incentive to maintain the link and any two firms that are not linked have no strict incentive to form a link with each other. This definition is inspired by the notion of stability presented in Jackson and Wolinsky (1996). We need to adapt this general definition slightly to accommodate the different cases of fixed costs we consider. We, therefore, state the formal definitions in Sections 3 and 4.

The requirements above are very weak and should be seen as necessary conditions for a network to be stable. Our analysis illustrates that these weak requirements provide sufficient structure in an interesting class of network formation games.

We have also examined an alternative *non-cooperative* formulation of the network formation game. In this formulation, every firm announces a set of links it intends to form with other firms. A link between two firms i and j is formed if both i and j announce an intention to form such a link. This announcement game of link formation was introduced by Dutta et al. (1995). We assume that there are some positive but small costs to forming links. Given any network, the payoffs to a firm are then defined as in the model presented in Section 3. We examined the structure of networks that arise in Nash equilibrium in undominated strategies of this announcement game. Our analysis yields results analogous to those presented in Section 3 below. We discuss this below after presenting the results.

3. Small costs of link formation

In this section, we provide a fairly general analysis of network formation when the fixed costs of forming links are small. When two firms collaborate, they help lower each other's costs. There are two effects at work here: collaboration lowers a firm's cost but also lowers its competitor's cost. In addition, a collaboration between two firms generates competitive effects on non-participating firms. These effects are in turn related to the nature of market competition. The analysis in this section aims to clarify this relationship by studying the architecture of strategically stable networks under different types of market competition.

We will suppose that there are small but positive costs to forming links. This motivates the following simple definition of strategic stability. A network g is *stable* if the following conditions are satisfied:

(i) For $g_{i,j} = 1$, $\pi_i(g) > \pi_i(g - g_{i,j})$, and $\pi_j(g) > \pi_j(g - g_{i,j})$.
(ii) For $g_{i,j} = 0$, if $\pi_i(g + g_{i,j}) > \pi_i(g)$, then $\pi_j(g + g_{i,j}) \leqslant \pi_j(g)$.

We have adapted this definition from Jackson and Wolinsky (1996). This definition of stability reflects two main ideas. First, while a link can be severed unilaterally, forming a link is a *bilateral* decision, i.e., a link is formed if and only if the two firms involved agree to form the link. Second, there are no transfers possible across links. Taken together with the idea of small but positive costs of link formation, this implies that both firms must make strictly greater profits by forming a link.[11]

3.1. Example: homogeneous product oligopoly

We study the textbook example of a market with homogeneous products and price and quantity competition. We assume the following linear inverse market demand function:

$$p = \alpha - \sum_{i \in N} q_i, \quad \alpha > 0. \tag{3}$$

We assume that marginal cost of firm $i \in N$ is given by (2).

3.1.1. Stable networks

We start with the case of Cournot competition. Given any network g, the Cournot equilibrium output can be written as

$$q_i(g) = \frac{(\alpha - \gamma_0) + n\gamma\eta_i(g) - \gamma\sum_{j \neq i}\eta_j(g)}{(n+1)}, \quad i \in N. \tag{4}$$

In order to ensure that each firm produces a strictly positive quantity in equilibrium, we will assume that $(\alpha - \gamma_0) - (n-1)(n-2)\gamma > 0$. The second stage Cournot profits for firm $i \in N$ are given by $\pi_i(g) = q_i^2(g)$.

We can now characterize the stable collaboration networks under quantity competition.

Proposition 3.1. *Suppose there is quantity competition among the firms. If marginal cost satisfies* (2) *and demand satisfies* (3) *then the complete network, g^c, is the unique stable network.*

Proof. We first show that g^c is stable. There are no links to add so condition (ii) of stability is automatically satisfied. We check condition (i) next. It is easily checked that $\pi_i(g^c) - \pi_i(g^c - g_{i,j}) = \gamma(n-1)(q_i(g^c) + q_i(g^c - g_{i,j}))/(n+1)^2 > 0$. A similar expression obtains for firm j. Thus condition (i) is also satisfied.

We now show that g^c is the unique stable network. Consider a stable network $g \neq g^c$. Then, there exists a pair of firms $i, j \in N$ with $g_{i,j} = 0$. We show that both i and j are strictly better off by forming a link. In the network, $g + g_{i,j}$, the payoff to firm i is given by

$$\pi_i(g + g_{i,j}) - \pi_i(g) = \gamma(n-1)\big(q_i(g + g_{i,j}) + q_i(g)\big)\big/(n+1)^2 > 0. \tag{5}$$

[11] We use positive costs primarily as a way to motivate condition (i) in the definition of stability above. If costs of forming links were zero, then it would be more reasonable to have weak inequalities in condition (i). Proposition 3.1 and Theorem 3.1 will still obtain with this modified definition, but Proposition 3.2 and Theorem 3.2 will have to be modified, since a larger class of networks will be stable in that case.

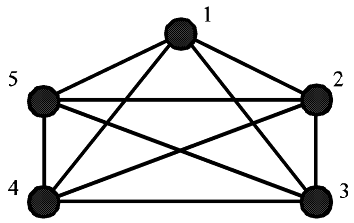

Fig. 1. Complete network ($n = 5$).

Analogous incentives hold for firm j. Thus condition (ii) is violated and g is not stable, a contradiction to our hypothesis.[12] $\square$

Figure 1 gives an example of a complete network. The intuition behind the above result is quite simple. When two firms i and j form a link the positive effects on firm i is of the order of $n\gamma$, while the negative effects are given by γ. Thus the net effect is $(n-1)\gamma$, which is positive and hence link formation is clearly profit enhancing. Thus the only candidate for stability is the complete network. It is straightforward to show that the complete network is indeed stable.

This result is in sharp contrast to the result of Bloch (1995) which shows that there is a unique stable coalition structure and this structure consists of two unequal size coalitions in which the number of firms in the larger coalition is the integer closest to $3(n+1)/4$. To explain this difference we first show why such a coalition structure cannot be stable in our setting. Consider firm i who belongs to a complete component, g', with $k \geqslant 2$ firms. Since intransitive relations are allowed in the networks approach, this firm can initiate a link with some firm $j \notin N(g')$ if it is profitable to do so and Proposition 3.1 shows that i and j will always have an incentive to do so. By contrast, in the coalition model, collaboration links are, by assumption, transitive; therefore, i can form a link with j if and only if all other k firms in the same coalition as i agree to merge with the singleton coalition $\{j\}$. However, it may be no longer profitable for firm i to have a collaboration link with j then. In this case, each of the $k \neq i$ firms in the coalition experiences a reduction in marginal cost of γ. Further, firm j experiences a reduction of $k\gamma$ in its marginal costs. Therefore, the net costs of firm i change by $(n-1)\gamma - 2k\gamma$. Thus if k is large then firm i will not have an incentive to form a link with firm j.

[12] We briefly sketch the argument to show that the complete network is the unique Nash network in undominated strategies. First, observe that in a Nash strategy profile every link announcement must be reciprocated. This follows from the fact that marginal returns of every additional link are positive as shown above. Second, consider a strategy profile with reciprocated links which generates an incomplete network. In this case, it is possible to show that the strategy of a player i who does not have links with all others is weakly dominated by a strategy which announces additional links. Thus the only candidate for equilibrium in undominated strategies is the complete network. Finally, it is easy to see that the complete network is a Nash network and that the strategy of announcing links with everyone is undominated (for small costs of forming links).

It is worth noting that strictness has similar implications in this setting. To see this note that no incomplete network can be sustained as a strict Nash equilibrium. This follows from noting that announcing unreciprocated links is costless. Thus the only candidate for strict Nash network is the complete one. The above argument can now be used to prove that the complete network is the unique strict Nash network as well.

The simultaneous open membership game in (Yi, 1998) obtains the grand coalition as the unique outcome of the game. This approach is similar to one in which the decision to join a coalition is one-sided. In such a game, the member of a smaller group always has an incentive to join a larger group. In our paper, link formation is based on pair-wise incentive compatibility. Thus, our result provides an alternative explanation as to how a grand coalition may endogenously emerge in equilibrium.

We next take up the case of Bertrand competition. Given a network g, what are the payoffs of different firms under Bertrand competition? Standard considerations (exploiting the idea of a finite price grid) allow us to state that there exists an equilibrium, and in this equilibrium a firm will make profits only if it is the unique minimal cost firm in the market. In other words:

$$\pi_i(g) = 0, \quad \text{if } c_i(g) \geqslant c_j(g), \text{ for } i \neq j; \qquad \pi_i(g) > 0, \quad \text{if } c_i(g) < c_j(g), \ \forall j \neq i. \tag{6}$$

Since g is arbitrary, the above expression allows us to specify the payoffs for all possible networks. What are the stable networks of collaboration in this setting of extreme competition? The following result provides a complete answer to this question.

Proposition 3.2. *Suppose there is price competition among the firms. If marginal cost satisfies* (2) *and demand satisfies* (3) *then the empty network, g^e, is the unique stable network.*

The intuition behind this result is simple. Suppose g is a non-empty network and that firm i has a link in this network. It is either the unique minimum cost firm, in which case its collaborators (of whom there must be at least one) have a incentive to delete their links. If, on the other hand, firm i is not the unique minimum cost firm then it has a incentive to delete its links. Thus a network g in which firm i has a link cannot be stable. It is straightforward to show that the empty network is indeed stable. These arguments are very general; in particular, we do not make use of the linear structure of the demand or the cost function. This suggests that the absence of collaborative links is likely to obtain in general settings where competition is extreme.[13]

3.1.2. Efficient networks

For any network g, aggregate welfare, $W(g)$, is defined as the sum of consumer surplus and aggregate profits of the n firms. We shall say that a network g^* is *efficient* if $W(g^*) \geqslant W(g)$, for all $g \in \mathcal{G}$.

We first consider the nature of efficient networks under quantity competition. Let $c(k)$ denote the marginal cost of a firm with k links. To ensure that all firms produce a strictly

[13] It is easy to adopt the above argument to show that no non-empty network can be a Nash network in an simultaneous link announcement game (with small but positive costs of forming links). This leaves the empty network which is clearly Nash. It is possible to show that the strategy of not forming any links is undominated. Thus the empty network is the unique Nash network in undominated strategies. We have already observed that no incomplete network can be a strict Nash. The above argument thus shows that there is no strict Nash network under price competition.

positive output in the Cournot equilibrium corresponding to any network, we will maintain the restriction that $\alpha - nc(0) + (n-1)c(n-2) > 0$. We shall define the social welfare (gross of costs of forming links) from a network g as follows:

$$W(g) = \frac{1}{2}Q^2(g) + \sum_{i \in N} q_i^2(g). \tag{7}$$

Proposition 3.3. *Suppose there is quantity competition among the firms. If marginal cost satisfies* (1) *and demand satisfies* (3) *then the complete network is the unique efficient network.*

Proof.[14] Consider any $g \neq g^c$. Then there exists at least one pair of firms $i, j \in N$ such that $g_{i,j} = 0$ in g. We now proceed in three steps to show that social welfare increases strictly as we move from g to g^c by forming links among all unconnected pairs of firms.

(i) Connect all unconnected pairs of firms (so that all firms have marginal cost $c(n-1)$) while holding the output of each firm i constant at $q_i(g)$. Since aggregate output is unchanged, consumer surplus is unchanged. On the other hand, since $c_i(\eta_i(g)) \geqslant c(n-1)$, and the inequality is strict for at least one pair of firms, aggregate profits (and hence aggregate welfare) strictly increases:

$$\sum_{i \in N}\bigl[p(g) - c(n-1)\bigr]q_i(g) > \sum_{i \in N}\bigl[p(g) - c\bigl(\eta_i(g)\bigr)\bigr]q_i(g). \tag{8}$$

(ii) Equalize output across the n firms so that each firm produces the average output, $Q(g)/n = (\sum_{i \in N} q_i(g))/n$. Once again, since aggregate output is unchanged, consumer surplus is unchanged. Further, aggregate profits are also unchanged:

$$\sum_{i \in N}\bigl[p(g) - c(n-1)\bigr]\frac{Q(g)}{n} = \sum_{i \in N}\bigl[p(g) - c(n-1)\bigr]q_i(g). \tag{9}$$

Therefore, aggregate welfare is unchanged as a consequence of this reallocation of output.

(iii) Note that aggregate output corresponding to a network g is given by $Q(g) = [n\alpha - \sum_{i \in N} c_i(g)]/(n+1)$ and is therefore maximized for g^c. Thus consumers surplus is maximized under g^c. From Eqs. (8) and (9) it follows that aggregate profits are maximized under g^c. From Eq. (7) it then follows that (gross) aggregate welfare is maximized under the complete network. Moreover, note that aggregate welfare is a continuous function of the costs of forming links, since costs enter linearly in the calculation. Thus this also proves that the complete network is the unique efficient network for small costs of forming links. □

We next examine the architecture of efficient networks under price competition. Let $\underline{c}$ be the minimum cost attainable by a firm in any network; this is achieved when a firm has $(n-1)$ links. The following result provides a complete characterization of efficient networks.

[14] The arguments in the proof are analogous to those used in the proof of Proposition 5 in (Yi, 1998). We would like to thank Sang-Seung Yi for drawing attention to this method of proof which greatly simplifies our earlier proof.

Proposition 3.4. *Suppose that there is price competition among the firms. If marginal cost satisfies* (1) *and demand satisfies* (3) *then an efficient network g is an inter-linked star, in which there are two central firms while the others have just two links each.*

Proof. Fix some network g. Let firm i be a minimum cost firm in this network and let its cost be given by $c_i(g) > \underline{c}$. Let equilibrium price be given by $p(g)$. Under price competition, it follows that $p(g) \geqslant c_i(g)$. Hence the consumer surplus is given by $1/2[\alpha - p(g)]^2$, while the profits of firms are bounded above by $[p(g) - c_i(g)][\alpha - p(g)]$. Thus social welfare in a network g is bounded above by the expression

$$\widehat{W}(g) = \frac{[\alpha - p(g)]^2}{2} + \big[p(g) - c_i(g)\big]\big[\alpha - p(g)\big]. \tag{10}$$

It is easily seen that this expression is strictly declining with respect to $p(g)$ as long as $p(g) > c_i(g)$. Thus, for a network g, the potential social welfare is bounded above by the expression, $[\alpha - c_i(g)]^2/2$.

It is easily checked that this maximum potential social welfare is decreasing in $c_i(g)$ and is, therefore, maximized when the price in the market is equal to $\underline{c}$. Thus social welfare is maximized when the product is produced and sold at the minimum marginal cost, $\underline{c}$.

If the minimum cost firm is unique then it will charge a price higher than $\underline{c}$, and earn positive profits in equilibrium. Thus two firms are necessary as well as sufficient for the market price to be equal to the minimum cost level. Observe that aggregate welfare is a continuous function of costs of forming links; thus for positive costs of forming links an efficient network will have exactly two firms which are fully linked. □

In the case of price competition, we observe a conflict between stability and efficiency in networks. The stability result indicates that no firm has any incentive to form a link with another. Efficiency, on the other hand, dictates a connected network.

Will the efficient network also maximize aggregate profits of the n firms? Consider quantity competition first. Yi (1998) demonstrates that if demand is given by (3) and marginal costs by (2) (with $\gamma = 1$), then, relative to the complete network, aggregate profits are higher in a dominant group architecture. This architecture consists of a complete component of k^* firms, with the remaining firms organized as singletons, where k^* is the solution to $4(\alpha - \gamma_0) + 3(n+1)^2(k^* + 1) - 4(n+2)(k^{*2} + (n - k^*)) = 0$.[15]

Next consider price competition with zero costs of link formation. It is clear that aggregate profits are strictly positive in a network g if there is a unique firm, say i, with the lowest marginal cost, $c_i(g)$. Let k be the firm with the next lowest marginal cost, $c_k(g)$. Firm i will operate as a monopoly and charge the monopoly price, $p(g) = (\alpha + c_i(g))/2$, if $(\alpha + c_i(g))/2 < c_k(g)$; otherwise, it will charge $p(g) = c_k(g)$. Since $\pi_i(g)$ is increasing in price if $p < (\alpha + c_i(g))/2$, maximizing profits requires $c_k(g) \geqslant (\alpha + c_i(g))/2$. Therefore, $c_k(g)$ should be at the highest possible level, i.e., the firm k with the second largest number of links should have just one link. Also note that monopoly profits increase as marginal cost falls; therefore, firm i should have $n - 1$ links. It follows that the star architecture will

[15] Yi (1998) shows that the dominant group architecture maximizes aggregate profits in the set of coalition structures. The network architecture that maximizes aggregate profits in the set $\mathcal{G}$ remains an open question.

maximize aggregate profits for zero costs. Since aggregate welfare is a continuous function of the costs of forming links, this result holds for small costs of forming links as well.

3.2. General results

Our analysis of the Cournot and Bertrand models of market competition under homogeneous linear demand and linear reduction in marginal costs suggests that the nature of market competition has a major influence on incentives for collaboration and the architecture of stable networks. We now examine this relationship for a general class of oligopoly models. We shall classify market competition into two types: moderate and aggressive.

Moderate competition: we first consider the case where all firms make positive profits but lower cost firms make higher profits. Such a situation is described as *moderate competition.* Formally, this situation is reflected in the following assumption:

Assumption MC. *Fix some g. $\pi_i(g) > 0$ for all $i \in N$; $\pi_i(g) = \pi_j(g)$ if $c_i(g) = c_j(g)$, while $\pi_i(g) > \pi_j(g)$ if $c_i(g) < c_j(g)$.*

The next assumption concerns the payoffs of firms with the same costs.

Assumption SY1. *Fix some g. Suppose that for a pair of firms i and j, $c_i(g) = c_j(g)$.*

(i) *If $g_{i,j} = 0$ then $\pi_i(g + g_{i,j}) > \pi_i(g) > 0$ and $\pi_j(g + g_{i,j}) > \pi_j(g) > 0$.*
(ii) *If $g_{i,j} = 1$ then $\pi_i(g - g_{i,j}) < \pi_i(g)$ and $\pi_j(g - g_{i,j}) < \pi_j(g)$.*

Yi (1998, Lemma 3) demonstrates that Assumption SY1 holds under a set of reasonable restrictions on general homogeneous demand (downward-sloping, concave) and costs (total cost is convex in own output, total and marginal cost are strictly decreasing with the number of links) along with a joint restriction on demand and costs. These conditions ensure that a favorable cost shock to a pair of symmetric firms will increase their net profits. Yi (1998, Section 5) also shows that Assumption SY1 is valid for symmetrically differentiated demand (where firm i's payoff depends only on the aggregate output of the rival firms). Symmetry in the presence of moderate competition implies the following property of stable networks.

Proposition 3.5. *Suppose that Assumption* SY1 *and* (1) *hold. Consider a stable network g. If $\eta_i(g) = n_j(g)$, then $g_{i,j} = 1$.*

Proof. Let g be stable. If $\eta_i(g) = \eta_j(g) = n$, then by definition $g_{i,j} = 1$. Therefore, consider the case where $\eta_i(g) = \eta_j(g) < n$ and $g_{i,j} = 0$. Under (1) the costs of i and j are identical if $\eta_i(g) = \eta_j(g)$. Under Assumption SY1(i), it follows that $\pi_i(g + g_{i,j}) > \pi_i(g)$ and $\pi_j(g + g_{i,j}) > \pi_j(g)$. This violates requirement (ii) of stability and contradicts the hypothesis that g is stable. □

Proposition 3.5 has several interesting implications for the nature of stable networks. The *first* implication is that a stable network cannot have two or more singleton components. This implies in particular that the empty network cannot be stable. The *second* implication is that an interlinked star is not stable. This is because in all such networks, there are at least two firms i and j who have the same number of links but $g_{i,j} = 0$. By Proposition 3.5, such firms have an incentive to form a link. A *third* implication of this result is that if a stable network contains two or more complete components, then they must be of unequal size. The result above thus implies that if all firms have the same cost, then every pair of firms must be linked; thus, the only (non-empty) symmetric network that can be stable is the complete network. Our next result elaborates on the role of the complete network.

Theorem 3.1. *Suppose that hypotheses in Assumptions* MC *and* SY1 *hold. Then the complete network, g^c, is stable. If in addition, for every network g and any link $g_{i,j} = 0$ it is true that $\pi_i(g + g_{i,j}) > \pi_i(g)$ and $\pi_j(g + g_{i,j}) > \pi_j(g)$ then the complete network, g^c, is the unique stable network.*

Proof. We provide a proof of the first statement. The second statement is immediate and a proof is omitted. In g^c, $\eta_i(g^c) = n - 1$, $\forall i \in N$. Therefore, all firms have the same cost and this is the minimum cost. There are no links to add so requirement (ii) of stability is automatically satisfied. We check requirement (i) next. Suppose we set $g_{i,j} = 0$ for some pair i and j. In the ensuing network, $g^c - g_{i,j}$, Assumption SY1 (ii) implies that both firms i and j lose strictly. This implies that requirement (i) is also satisfied. Thus g^c is stable. □

In view of our comments on Assumption SY1 above, this result shows that the complete network is stable under fairly general conditions. The additional monotonicity condition in Theorem 3.1 for uniqueness is strong. However, it is satisfied by a variety of standard oligopoly models with linear reduction in marginal costs as in expression (2). This includes the standard model of a differentiated oligopoly, with linear demand as well as the case where each of the firms is a monopoly in its own market. This is satisfied by Cournot oligopoly if inverse demand $p(Q)$ *is twice continuously differentiate function with* $p'(Q) < 0$ *and* $p''(Q) \leqslant 0$.[16] The assumption of linear reduction in marginal costs from link formation is critical in yielding the complete network as the unique stable network. We illustrate this with a simple example. Consider the linear homogeneous demand Cournot model and suppose that a firm's costs decline linearly by γ until it forms $(n-2)$ links and then they decline by $\hat{\gamma}$ for the $(n-1)$th link, where $\hat{\gamma} < \gamma/n$. It is then easy to see, using arguments from the proof of Proposition 3.1 that a firm with $(n-2)$ links does not have an incentive to form links with an isolated firm. Thus a dominant group network with a single isolated firm is stable. Note however, that this example satisfies Assumptions MC and SY1 and so from Theorem 3.2 we know that the complete network is also stable.

[16] Due to space constraints, we have omitted proofs of these claims from the paper. The details of these derivations are available from the authors upon request.

Aggressive competition: we next consider the case where only the lowest cost firms make profits. The first case of interest is where the lowest cost firm makes positive profits only if it is the unique such firm. This corresponds to Bertrand competition with general demand and cost reduction functions. The arguments in the proof of Proposition 3.2 generalize in a straightforward way to show that the empty network is the unique strategically stable network. The second case of interest is one in which every lowest cost firm makes positive profits.[17]

Assumption AC. *Fix some* g. *If* $c_i(g) > c_j(g)$, *then* $\pi_i(g) = 0$, *while if* $c_i(g) \leqslant c_j(g)$ *for all* $j \in N \setminus \{i\}$ *then* $\pi_i(g) > 0$.

In our analysis we shall use the following symmetry assumption with respect to the lowest cost firms.

Assumption SY2. *Fix some* g. *Suppose that for a pair of firms* i *and* j, $c_i(g) = c_j(g) = \min_{k \in N} c_k(g)$.

(i) *If* $g_{i,j} = 0$ *then* $\pi_i(g + g_{i,j}) > \pi_i(g) > 0$ *and* $\pi_j(g + g_{i,j}) > \pi_j(g) > 0$.
(ii) *If* $g_{i,j} = 1$ *then* $\pi_i(g - g_{i,j}) < \pi_i(g)$ *and* $\pi_j(g - g_{i,j}) < \pi_j(g)$.

Assumption SY2 is weaker than Assumption SY1 since it applies only to the minimum cost firms. Sufficient conditions on demand and costs under which Assumption SY2 holds are provided in Yi (1998, Lemma 3 and Section 5). Symmetry in the presence of aggressive competition has strong implications for collaboration.

Theorem 3.2. *Let* $n \geqslant 4$. *Suppose that marginal cost is specified by* (1), *and Assumptions* AC *and* SY2 *hold. Then a network is stable if and only if it is a dominant group network* g^k, *with* $k \in \{3, 4, \ldots, n\}$.

Figure 2 shows the dominant group networks for a market with 5 firms. We provide a sketch of the arguments. *First*, we show that any non-singleton component in a stable network must be complete. In proving this property, we also establish that all firms in a non-singleton component must have the same costs and that these costs must be the minimum in the given network. *Second*, we show that there can be at most one non-singleton component in a stable network. These two properties reduce the set of candidates for stable networks

[17] By way of motivation, consider a set of firms who are competing to apply for a patent for a cost-reducing technological process. Each of the firms has some useful complementary knowledge. If they collaborate, then this knowledge can be jointly used to lower costs. Moreover, only the lowest cost technology is patented. Once the patent is available, it is randomly allotted to one of the firms who have the lowest cost technology. Price competition then ensures that only this firm makes profits.

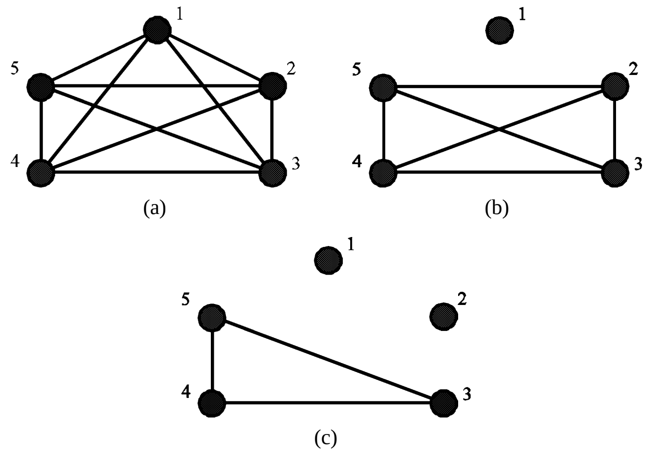

Fig. 2. Dominant group architecture ($n = 5$). (a) Dominant group with no fringe firms. (b) Dominant group with one fringe firm. (c) Dominant group with two fringe firms.

dramatically to a subset of dominant group networks.[18]

We note that the number of stable networks is very small as compared to the number of total networks. For example, when n is 3, 4, 5 or 6, the total number of networks is given by 8, 64, 1024, and 32768, respectively. By contrast, the number of stable networks is given by 3, 5, 16, and 42, respectively. Thus the two simple requirements of stability lead to a strong restriction on the class of networks.

4. Large costs of link formation

In general, R&D collaboration agreements will involve commitment of funds. This leads us to study the model where the costs of forming links are substantial. We suppose that each link imposes a cost of $f > 0$ on each of the two firms forming the link. We start by studying network formation when each firm pays for its own costs of link formation. We then analyze the effects of allowing for firms to make transfers to other firms as a way to subsidize their costs of forming links. Our analysis provides a sharp characterization of stable networks. The results we obtain also illustrate the powerful role of collaborative activity—specially the role of transfers—in generating asymmetries among firms.

We incorporate the fixed costs of forming links in the payoffs as follows. Fix a network g. The *net* profit of each firm $i \in N$ is given by $\Pi_i(g) = \pi_i(g) - \eta_i(g) f$, while the *gross* profit is given by $\pi_i(g) = q_i^2(g)$ Given a network g, let g_{-i} denote the network in which all of firm i's links are deleted. We can now define a stable network as follows.

[18] The proof is omitted due to space constraints; it is available from the authors on request.

The above result is stated for $n \geqslant 4$. It is easily seen that in case of $n = 3$ an analogous result obtains: a stable network is either complete or has two components, one component with two firms and the other component with a singleton firm. We have stated the result for $n \geqslant 4$ as it allows for a simpler statement.

Definition 4.1. Let f be the fixed cost of link formation. A network g is stable, if the following conditions hold.

(1) For $g_{i,j} = 1$, $\pi_i(g) - \pi_i(g - g_{i,j}) \geqslant f$, $\pi_j(g) - \pi_j(g - g_{i,j}) \geqslant f$.
(2) For $g_{i,j} = 0$, $\pi_i(g + g_{i,j}) - \pi_i(g) > f \Rightarrow \pi_j(g + g_{i,j}) - \pi_j(g) < f$.
(3) For every $i \in N$, $\pi_i(g) - \eta_i(g)f \geqslant \pi_i(g_{-i})$.

In words, the first two conditions require respectively that in a stable network, any firm that is linked to another has no incentive to sever the link, and any two firms that are not linked should have no incentive to establish a collaboration link. These two conditions constitute a "marginal" check for stability. The third condition is an "aggregate" or "global" check for stability which requires that a firm should find it profitable to maintain its collaboration links in the network rather than not having any links. This condition can be seen as an individual rationality condition for participation in the network.[19]

In the analysis so far, we have worked with the assumption of negligible costs. This has allowed us to study incentives of link formation simply in terms of the 'sign' of the terms. In the presence of large costs of forming links, an assessment of the incentives to form links requires an explicit measurement of the benefits of links. This complicates the analysis considerably, and to get our main points across easily, we restrict attention to homogeneous demand model with linear demand. We first note that with small costs of forming links the empty network is the unique stable network under price competition. Clearly, the same result will obtain once we assume that there are large costs of forming links.[20] Therefore, in the rest of the analysis in this section, we will focus our attention on quantity competition. Our first result establishes that gross profits of a firm exhibit *increasing returns* with respect to the number of links which the firm has with other firms.

Lemma 4.1. *Consider any network g and distinct firms $i, j, k \in N$ such that $g_{i,j} = g_{i,k} = 0$. Then*:

$$\pi_i(g + g_{i,j} + g_{i,k}) - \pi_i(g + g_{i,j}) > \pi_i(g + g_{i,j}) - \pi_i(g). \tag{11}$$

Proof. First of all note that the Cournot output of firm i is strictly increasing with each additional link: $q_i(g + g_{i,j}) - q_i(g) = \gamma(n-1)/(n+1) > 0$.

Recall that for any network g, the gross profit of i is $\pi_i(g) = q_i^2(g)$. It follows that:

$$\pi_i(g + g_{i,j} + g_{i,k}) - \pi_i(g + g_{i,j}) = \frac{\gamma(n-1)}{(n+1)}\big[q_i(g + g_{i,j} + g_{i,k}) + q_i(g + g_{i,j})\big]$$

[19] A more general version of condition (3) in the stability definition is to require that the current set of links is better than any subset of the links. In our setting, the two requirements are equivalent, due to the increasing returns property that will be derived in Lemma 4.1. Under this property it is optimal to either maintain all links or to delete all of them.

[20] We note that this is only true when no transfers are allowed. Non-empty networks can be stable even under price competition once transfers are allowed; we discuss this issue in the analysis on transfers below.

$$> \frac{\gamma(n-1)}{(n+1)}\big[q_i(g+g_{i,j})+q_i(g)\big]$$
$$= \pi_i(g+g_{i,j})-\pi_i(g). \tag{12}$$

This proves the result. □

We note that by virtue of increasing returns in gross profits, condition (3) implies condition (1) in the definition of stability. Therefore, it suffices to verify conditions (2) and (3) when checking the stability of any network.

We now develop a complete characterization of the architecture of stable networks. We start by noting a 'transitivity' implication of the increasing returns property.

Lemma 4.2. *Let g be a network which is stable under fixed cost f of link formation. Let i and j be two distinct firms. If $\eta_i(g) \geqslant 1$ then $g_{i,j}=1$.*

Proof. The proof is by contradiction. Suppose that g is stable but $g_{i,j}=0$. Since g is stable, it follows that $\pi_i(g)-\pi_i(g-g_{i,k}) \geqslant f$. From the property of increasing returns (Lemma 4.1) it follows that $\pi_i(g+g_{i,j})-\pi_i(g) > \pi_i(g)-\pi_i(g-g_{i,k}) \geqslant f$. Thus firm i has an incentive to form a link with firm j. The only property we have used is that firm i has a link with some other firm. In this respect the situation of firm j is symmetric. Therefore, using an identical argument, we can show that firm j has an incentive to form a link with firm i. This establishes that g is not stable, a contradiction. □

This result has a number of interesting implications. *Firstly*, it implies that any stable network g can have at most one non-singleton component, g'. Furthermore, g' must be complete, i.e., all firms in this component must have links with each other. Thus, every stable network has the dominant group architecture. Recall that g^k denotes the network in which there is one non-singleton complete component of size k and the remaining $n-k$ firms are singletons. *Secondly*, this result implies that there are only two possible *symmetric* stable networks: the empty and the complete. We are now ready to provide a complete characterization of stable networks.

Proposition 4.1. *Suppose that marginal cost satisfies* (2), *demand satisfies* (3) *and that firms compete in quantities. Then there exist numbers F_0, F_1, F_2, and F_3, where $F_0 < F_1 < F_2 < F_3$, with the following properties*:

(1) *For $f < F_0$, g^c is the unique stable network.*
(2) *For $F_0 \leqslant f < F_1$, a network g^k is stable if and only if $k \in \{k(f),\ldots,n\}$, with $k(f) > 1$.*
(3) *For $F_1 \leqslant f < F_3$, a network g^k is stable if and only if $k \in \{\underline{k}(f),\ldots,\bar{k}(f)\}$, with $1 \leqslant \underline{k}(f) < \bar{k}(f) < n$.*
(4) *For $f > F_2$, g^e is a stable network. Moreover, if $f > F_3$ then g^e is the unique stable network.*

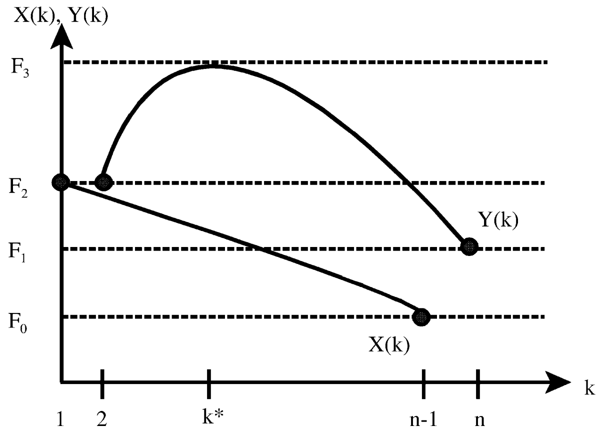

Fig. 3. Pair-wise stability of the dominant group architecture.

Proof. Consider a dominant group network, g^k. A firm in the non-singleton component of size k has no incentive to delete all its links if

$$Y(k) \equiv \frac{(n-1)\gamma}{(n+1)^2}\big[2(\alpha-\gamma_0)+(k-1)(n+3-2k)\gamma\big] \geqslant f. \tag{13}$$

If the above condition is satisfied, then by virtue of the property of increasing returns (Lemma 4.1), a firm in the non-singleton component would always want to form a link with an isolated firm. Therefore, if g is stable, then the isolated firm should have no incentive to form a link with a firm in the non-singleton component. This requires

$$X(k) \equiv \frac{(n-1)\gamma}{(n+1)^2}\big[2(\alpha-\gamma_0)+(n-1)\gamma-2k(k-1)\gamma\big] < f. \tag{14}$$

A network g^k is stable if and only if it satisfies (13) and (14). By inspection, we see that $X(k)$ is declining in k. Further, $X(n-1)=F_0$. Regarding $Y(k)$, it is initially increasing and then decreasing in k. Note that $F_1=Y(n)$, $F_2=Y(2)=X(1)$, and $F_3=Y(k^*)$. Further, $F_0<F_1<F_2<F_3$. The proof now follows from Figs. 3 and 4. □

Figure 4 illustrates the nature of stable architectures, as the cost of forming links f varies. We *first* note that the cost of forming collaboration links has a significant impact on the structure of the collaboration network. In particular, for low costs, the complete network is uniquely stable, for moderate costs only networks with relatively large dominant groups are stable, for high costs, only medium size dominant groups are stable (small and large groups are not sustainable), while for very high costs, the empty network is uniquely stable. Hence, the effect of R&D costs on the size of the dominant group is *non-monotonic*. The intuition for this pattern is as follows: when costs are low, the incentive constraint of the isolated firm to form a link is binding. The marginal payoff to an isolated firm from an additional link is declining in the size of the dominant group. Hence, as the costs of forming R&D collaboration links increase, smaller groups are sufficient to discourage the isolated firm from forming a link. However, beyond a certain cost level, the incentive constraint for a firm in the dominant group to retain its links is binding. The returns from links to a

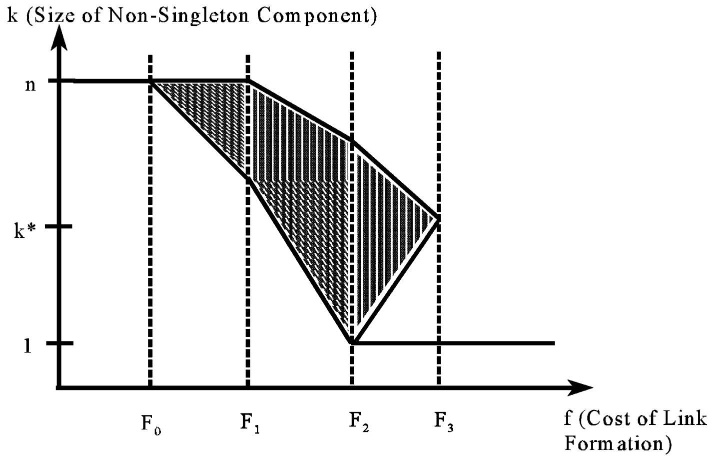

Fig. 4. Non-monotonicity in size of dominant group with respect to cost of link formation.

firm in a dominant group are non-monotonic in the size of the dominant group: they are increasing for group sizes until some critical value k^*, and then declining. This implies that for high cost levels, small and large dominant groups are not stable.

Secondly, we note that for a class of parameters (corresponding to intermediate values of costs of forming links), the asymmetric networks with medium size dominant groups are the only stable outcomes. Since firms are ex-ante identical this result illustrates how collaborative activity can be a means for generating competitive advantage. These advantages are substantive since profits of well connected firms are larger than the profits of isolated firms. To see this note that $\pi_l(g^k) - (k-1)f \geqslant \pi_l(g^k_{-l}) > \pi_i(g^k)$, where l is a linked firm, while i is a isolated firm. The first inequality follows from condition (3) of the stability definition, while the second inequality can be verified by direct computation.

Finally, we note that the key property which drives the characterization results of this section is increasing returns in gross profits. This property ensures that only the empty and complete networks can be stable in the class of symmetric networks, and only the dominant group architecture can be stable in the class of asymmetric networks. Our analysis, therefore, implies that only three architectures—empty, complete and dominant group—will be candidates for stability in any general model of network formation where the reduced form payoff to each player displays increasing returns.

We would also like to characterize efficient networks in the presence of significant costs of forming links. However, we have been unable to obtain a general characterization. We have computed efficient networks for an example with 4 firms (under both price and quantity competition). We briefly report some findings: first, we observe a monotonicity property: the number of links in the efficient network declines as the cost of forming links increases. Second, we observe that social welfare in an asymmetric network—with one dominant firm—exceeds the social welfare from a symmetric network, conditional on the two having the same number of links. Thus, for example, in the set of networks with 2 links, the asymmetric network in which a firm has 2 links generates the higher social welfare. A similar pattern obtains in the set of networks with 3 and 4 links, respectively.[21]

4.1. Transfers

The property of increasing returns suggests that firms with many links may have an incentive to make transfers to firms who are poorly linked to induce them to form links. These considerations motivate an analysis of the nature of stable networks when transfers are allowed across firms. We provide a complete characterization of stable networks when transfers are allowed. This characterization reveals that inter-linked stars and dominant group networks are the only networks which are stable with respect to transfers.

Let $t_i = \{t_i^1, \ldots, t_i^n\}$ be the transfers offered by firm i to other firms. We shall suppose that $t_i^j \geqslant 0$, for all $i, j \in N$, and that $t_i^i = 0$, for all $i \in N$. We modify the concept of stability to accommodate the possibility of transfers. The concept of strategic stability we use is defined as follows.

Definition 4.2. A network g is stable against transfers if:

(1) For all $g_{i,j} = 1$, $[\pi_i(g) - \pi_i(g - g_{i,j})] + [\pi_j(g) - \pi_j(g - g_{i,j})] > 2f$.
(2) For all $g_{i,j} = 0$, $[\pi_i(g + g_{i,j}) - \pi_i(g)] + [\pi_j(g + g_{i,j}) - \pi_j(g)] < 2f$.
(3) There exist transfers $t_i \in R^n$, $i = 1, 2, \ldots, n$, such that

$$\pi_i(g) - \eta_i(g)f + \sum_{j \in N_i(g)} \left(t_j^i - t_i^j\right) \geqslant \pi_i(g_{-i}). \tag{15}$$

We start by noting that an implication of increasing returns in gross profits is that in the class of *symmetric* networks there are only two possible stable networks: the empty and the complete. The proof follows as a corollary of the property of increasing returns (see Lemma 4.1) and is, therefore, omitted. We now turn to the characterization of *asymmetric* networks that are stable with respect to transfers. We first examine connected networks and then unconnected networks.

In this analysis, we make use of the following implication of increasing returns: if firm i has a link with firm j in a network g which is stable against transfers, then it must also have a link with every firm k which has as many links as j in the network $g - g_{i,j}$.

Lemma 4.3. *If g is stable against transfers, then it satisfies the following property: suppose $g_{i,j} = 1$ for distinct $i, j \in N$; then, $g_{i,k} = 1$ for all $k \in N$ satisfying $\eta_k(g - g_{i,j}) \geqslant \eta_j(g - g_{i,j})$.*

The proof of Lemma 4.3 is given in Appendix A. This result provides a simple "marginal" check for stability against transfers by examining the incentive for two firms in

[21] These computations have been omitted from the paper, due to space constraints and are available from the authors on request.

a network g to be jointly better off by forming a link. With this result in hand, we can now provide a characterization of connected networks that are stable with respect to transfers.

Proposition 4.2. *Suppose that marginal cost satisfies* (2), *demand satisfies* (3) *and that firms compete in quantities. Let $g \neq g^c$ be a connected network. If g is stable against transfers then it is an inter-linked star. In addition, it satisfies the following conditions.*

(i) $|\eta_i(g) - \eta_j(g)| \geqslant 2$ *if* $\eta_i(g) \neq \eta_j(g)$.
(ii) *Suppose* $i \in h_{m-s}(g)$, *with* $s \in \{0, 1, 2, \ldots, m-1\}$, *then* $g_{i,j} = 1$ *if and only if* $j \in h_l(g)$, $l > s$.

Proof. We prove the part on inter-linked stars here. The additional properties (i) and (ii) are derived in Appendix A. Suppose g is connected. Then it must be the case that for any $i \in h_1(g)$, $\eta_i(g) \geqslant 1$. We next show that $g_{i,j} = 0$, if $j \notin h_m(g)$. Suppose not. Then it follows from Lemma 4.3 that firm j is linked to all firms. This implies that $j \in h_m(g)$, which contradicts the original hypothesis that $j \notin h_m(g)$. This proves that there is some firm in $h_m(g)$ that has links with firms in $h_1(g)$. It then follows from Lemma 4.3 that this firm has links with all firms in $h_1(g)$ and in fact with all firms in the network. Thus it follows that $\eta_k(g) = n-1$, if $k \in h_m(g)$. The above reasoning also proves that a firm $i \in h_1(g)$ is linked with a firm j if and only if $j \in h_m(g)$; thus $N_i(g) = h_m(g)$, for $i \in h_1(g)$. Thus g must be an inter-linked star. □

Inter-linked stars are illustrated in Fig. 5. For the case $n = 6$, these are the only asymmetric connected networks that are stable against transfers according to Proposition 4.2. Figure 5a presents a star, Fig. 5b presents a network with two inter-linked stars with firms 1 and 2 being central, Fig. 5c presents a network with three inter-linked stars with firms 1, 2, and 3 being central. Finally, Fig. 5d presents a network with firm 1 as central and three highly connected firms 2, 3, and 4, while firms 5 and 6 are only connected with firm 1.

A special case of inter-linked stars is the star network. We now derive the conditions on the parameters under which the star is stable with respect to transfers.

Proposition 4.3. *Let $n \geqslant 4$. Suppose that marginal cost satisfies* (2), *demand satisfies* (3) *and that firms compete in quantities. Then there exist F_H and F_L, where $0 < F_L < F_H$ such that the star architecture is stable against transfers if and only if $F_L < f < F_H$.*

We provide a proof of this result in the text as it illustrates the role of transfers in generating market power and augmenting the profits of well connected firms.

Proof. Suppose that g^s is a star network; denote the central firm by n and typical firms at the periphery by i and j. If firm n deletes all its links then the resulting network is empty, if firm i or firm n deletes a link, the resulting network is $g^s - g_{n,i}$, while if firms i and j form a link then we get the network $g + g_{i,j}$. The gross profits for firms under different networks can be written as follows:

$$\pi_n(g^s) = \frac{[\alpha - \gamma_0 + (n-1)^2\gamma]^2}{(n+1)^2}, \qquad \pi_n(g^e) = \frac{[\alpha - \gamma_0]^2}{(n+1)^2}, \tag{16}$$

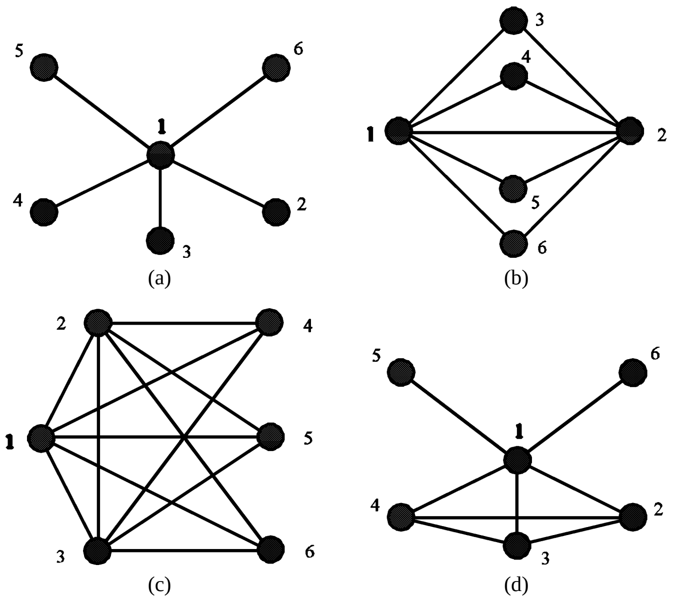

Fig. 5. Asymmetric connected networks that are pair-wise stable against transfers ($n = 6$). (a) Star network with firm 1 as the center. (b) Two inter-linked stars with firms 1 and 2 as the centers. (c) Three inter-linked stars with firms 1, 2, and 3 as the centers. (d) Four asymmetrically-sized inter-linked stars.

$$\pi_n(g^s - g_{n,i}) = \frac{[\alpha - \gamma_0 + (n-2)(n-1)\gamma]^2}{(n+1)^2}, \tag{17}$$

$$\pi_i(g^s) = \frac{[\alpha - \gamma_0 + (3-n)\gamma]^2}{(n+1)^2}, \qquad \pi_i(g^s - g_{n,i}) = \frac{[\alpha - \gamma_0 - 2(n-2)\gamma]^2}{(n+1)^2}, \tag{18}$$

$$\pi_i(g^s + g_{i,j}) = \frac{[\alpha - \gamma_0 + 2\gamma]^2}{(n+1)^2}. \tag{19}$$

We now substitute the above payoff terms in the incentive conditions needed for stability. The requirement that firm n and firm i wish to maintain their link may be written as

$$\frac{\gamma(n-1)[4(\alpha - \gamma_0) + (n-1)\gamma(2n-3) - \gamma(3n-7)]}{(n+1)^2} > 2f. \tag{20}$$

The requirement that firms i and j do not have an incentive to form a link may be written as follows:

$$\frac{2\gamma(n-1)[2(\alpha - \gamma_0) + \gamma(2-n+3)]}{(n+1)^2} < 2f. \tag{21}$$

Define

$$F' = \frac{\gamma(n-1)[4(\alpha - \gamma_0) + (n-1)\gamma(2n-3) - \gamma(3n-7)]}{2(n+1)^2}, \tag{22}$$

$$F_L = \frac{2\gamma(n-1)[2(\alpha - \gamma_0) + \gamma(2-n+3)]}{2(n+1)^2}. \tag{23}$$

Conditions (20) and (21) are satisfied if and only if the fixed costs are such that $F_L < f < F'$. It is easily verified that $F_L < F'$ if $n > 3$.

The requirement that there exists a set of transfers such that firms have no incentives to isolate themselves by deleting all their links is written as follows. There exist transfers t_i, for $i = 1, 2, \ldots, n$, such that

$$\pi_n(g^s) - (n-1)f + \sum_{j \in N_n(g)} \left(t_n^i - t_n^j\right) \geqslant \pi_n(g^e),$$
$$\pi_i(g^s) - f + \left(t_n^i - t_i^n\right) \geqslant \pi_i(g^s - g_{n,i}), \quad \forall i \in N \setminus \{n\}. \tag{24}$$

Finally, we construct the set of transfers. For the star to be stable it must be the case that the peripheral firms do not have an incentive to form a link with each other. Given the symmetry in their situation, it follows that their marginal payoffs from the additional link are the same. This requirement taken along with the increasing returns property implies that if the star is to be stable then it must be the case that none of the peripheral firms has an incentive to form a link with the central firm. Thus transfers have to made by the central firm to each of the peripheral firms. The minimum value of this transfer is given by: $t_n^i = \pi_i(g^s - g_{n,i}) - \pi_i(g^s) + f$. Using earlier computations, we can rewrite this minimum transfer as

$$t_n^i = f - \frac{(n-1)\gamma[2(\alpha - \gamma_0) - \gamma(3n-7)]}{(n+1)^2}. \tag{25}$$

We wish to show that the star satisfies condition (3): $\pi_n(g^s) - (n-1)(f + t_n^i) \geqslant \pi_n(g^e)$. After some rearrangement this requirement can be expressed as

$$\frac{(n-1)\gamma[4(\alpha - \gamma_0) + (n-1)^2\gamma - \gamma(3n-7)]}{(n+1)^2} \geqslant 2f. \tag{26}$$

Define

$$F'' = \frac{(n-1)\gamma[4(\alpha - \gamma_0) + (n-1)^2\gamma - \gamma(3n-7)]}{2(n+1)^2}. \tag{27}$$

It can be checked that $F'' > F_L$, for all $n > 3$. Define $F_H = \min\{F', F''\}$. The proof now follows. □

We note that the transfers in the above construction force the peripheral firms to their outside option, which is to be without a link and therefore isolated. Thus their participation constraint in the collaboration with the central firm is binding. One implication of this is that the profits of the central firm are strictly larger than the profits of the peripheral firms. This follows from noting, first, that the central firm is better off with the star network (and the corresponding transfers) as compared to the empty network, and, second, that $\pi_i(g^s) - f + t_n^i = \pi_i(g^s - g_{n,i}) < \pi_n(g^e)$ (where i is a 'peripheral' firm and n is the central firm).

We now elaborate on different aspects of this result. *First*, in the star, each of the peripheral firms derives a relatively low return from its link, due to the relative cost disadvantage with respect to the center. Hence, the stability of the star architecture is critically dependent on transfers from the center. If transfers were not permitted, each peripheral firm would sever its link with the hub firm. This is also indicative of how market dominance can arise in a setting with ex-ante identical firms.

Second, inspecting the terms F_L and F_H in the proof of Proposition 4.3 shows that the star is stable for all $f \in [0, \infty)$ as $n \to \infty$, i.e., over the entire parameter space. This result is once again a consequence of increasing returns in gross profits: for any fixed cost f of link formation, however high, the center will be able to use transfers to induce peripheral firms to form a link if its marginal profits from the links are high enough; the center's marginal profits in turn will be large enough if there are a sufficient number of peripheral firms (i.e., a large enough value of $n(f)$) for the center to potentially form links with.

Third, Lemma 4.3 highlights an important relationship between a star and the connectedness of a stable network. In particular, it implies that if a network g is stable against transfers, and the non-singleton component is a star, then g must be connected. To see this, let k be some firm which does not belong to the star component. Since there can be at most one non-singleton component in a stable network, $\eta_k(g) = 0$. Now consider a hub, i, and a peripheral firm, j, in the star component. By definition, $\eta_i(g - g_{i,j}) = \eta_k(g - g_{i,j}) = 0$. Then, by Lemma 4.3, i should have had a link with k as well.

Finally, we remark on the possibility of non-empty networks under price competition once transfers are allowed. The construction in the above result suggests that a star can also be sustained under price competition, with the central firm subsidizing each of the peripheral firms exactly to compensate them for the cost of forming links. This argument illustrates once again the role of transfers in generating market power; recall that in the absence of transfers the unique stable network under price competition is the empty network.

We now turn to asymmetric networks which are unconnected. We start by noting an implication of increasing returns from links: there can be at most one non-singleton component in a network which is stable with respect to transfers. What is the architecture of this non-singleton component. A straightforward adoption of the arguments of Proposition 4.2 tells us that if the non-singleton component is incomplete then it must be an inter-linked star. This leaves out only one possibility: the complete component. But an unconnected network with one complete non-singleton component is simply the dominant group network.

The property of increasing returns implies that if a link is formed between a firm in the dominant group and a isolated firm, then the form gains more from the link as compared to the latter. This suggests that the firm in the dominant group has an incentive to subsidize the formation of the link. Thus we should expect that transfers will sometimes eliminate some dominant groups that are stable in the absence of transfers. The following result formalizes this intuition.

Proposition 4.4. *Suppose that cost satisfies* (2), *demand satisfies* (3) *and that firms compete in quantities. If g^k is stable against transfers then it is also stable. The converse, however, is not true.*

Proof. Suppose g^k is stable against transfers. Then, for any i, j such that $g_{i,j} = 0$, net profits must satisfy: $\Pi_i(g) + \Pi_j(g) < \Pi_i(g + g_{i,j}) + \Pi_j(g + g_{i,j})$. This implies that if $\Pi_i(g + g_{i,j}) > \Pi_i(g)$, then $\Pi_j(g + g_{i,j}) < \Pi_j(g)$. Therefore, condition (2) in the definition of stability is also satisfied. Since g^k is stable against transfers, for any i, j such that $g_{i,j} = 1$: $\Pi_i(g) + \Pi_j(g) > \Pi_i(g - g_{i,j}) + \Pi_j(g - g_{i,j})$. However, in g^k, the only firms who are linked are those which belong to the non-singleton complete component. Therefore, these firms have the same net profits, i.e., $\Pi_i(g) = \Pi_j(g)$ and $\Pi_i(g - g_{i,j}) = \Pi_j(g - g_{i,j})$. It follows from the above expressions that $\Pi_i(g) > \Pi_i(g - g_{i,j})$ and $\Pi_j(g) > \Pi_j(g - g_{i,j})$. Therefore, condition (1) in the definition of stability is also satisfied. Since all firms in the non-singleton complete component have identical profits, it follows that any net transfers between these firms must be zero. Therefore, if g^k satisfies condition (3) in the definition of stability against transfers, then it also satisfies condition (3) of stability.

The converse of the above implication is not true: it is possible for g^k to be stable for some range of values of f but not be stable against transfers. Consider the case where $n = 4$ and the network g^2 where firms 1 and 2 are linked while firms 3 and 4 are singletons with no links. It can be verified that g^2 is pairwise stable if

$$f \in \left(\frac{3\gamma}{25}\big[2(\alpha - \gamma_0) - \gamma\big], \frac{3\gamma}{25}\big[2(\alpha - \gamma_0) + 2\gamma\big] \right).$$

However, by virtue of Lemma 4.3, g^2 is not stable against transfers. □

This result shows that in the class of dominant group networks, allowing for transfers refines the set of stable networks.

5. Conclusion

We have developed a simple model of network formation to examine the incentives of firms to form collaboration links with other firms. A collaboration link is interpreted as a technological partnership which is costly but helps lower costs of production of the firms involved. Our interest has been in the interaction between the incentives of firms to collaborate (and the resulting networks of collaboration) and the nature of market competition.

Our analysis has clarified the nature of collaboration structures that are strategically stable under different market conditions. An important finding is that even in settings where firms are (ex-ante) symmetric, strategically stable networks are often asymmetric, with some firms having many collaboration links, while other firms are poorly linked. We characterized such structures, finding that the star, inter-linked stars and the dominant group architecture are strategically stable. These asymmetries translate into different levels of competitiveness for firms and hence have a serious influence on market performance. The model of links between firms which we have used is quite simple and should be seen as a first step in a more systematic analysis of the interaction between firms' collaboration networks and markets. We now briefly discuss some issues that should be explored in future work.

First, we take up the issue of spillovers. Our analysis does not accommodate spillovers across the collaborative links of firms. In the received literature, spillovers from the R&D

activity of firm i is assumed to affect firms $j \neq i$ uniformly.[22] In our framework, one plausible definition of 'distance' between firms in a network is the number of links in the shortest path between the firms. This would allows us to implement the more realistic idea that firms that are 'far apart' receive lower spillovers as compared to firms that are 'close' in the network.

The second issue is social welfare. Our results on the linear homogeneous model and small costs of forming links suggest that under quantity competition there is no conflict between firms' incentives and social incentives: the complete network is uniquely stable as well as efficient. However, we have identified a conflict under price competition. The empty network is uniquely stable while the efficient networks are inter-linked stars with exactly two central firms. We have been unable to characterize efficient networks in the presence of large fixed costs of link formation; this is clearly an important issue which needs to be examined in future research.

Thirdly, we discuss the role of ex-ante asymmetries between firms. In our analysis, we have assumed that all firms are ex-ante symmetric with respect to initial costs and have the same cost reduction function. This seems to us to be the natural starting point, and our results illustrate how significant network asymmetries can emerge even in such a symmetric setting. In some important cases, however, it is natural to start with asymmetric firms. While we expect that asymmetric networks will become more prominent, further work on this subject is needed to clarify the precise structure of such networks.

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Appendix A

Proof of Lemma 4.3. Since g is a stable network, i and j should have no incentive to sever their link. Letting $\Delta\pi_i(g - g_{i,j}) \equiv \pi_i(g) - \pi_i(g - g_{i,j})$ gives

$$\Delta\pi_i(g - g_{i,j}) + \Delta\pi_j(g - g_{i,j}) > 2f. \tag{A.1}$$

Let $T_i(g - g_{i,j}) \equiv (\alpha - \gamma_0) + n\gamma\eta_i(g - g_{i,j}) - \gamma\sum_{l\neq i}\eta_l(g - g_{i,j})$. The above inequality can be written as

$$\frac{(n-1)\gamma}{(n+1)^2}\big[T_i(g - g_{i,j}) + T_j(g - g_{ij}) + (n-1)\gamma\big] > f. \tag{A.2}$$

[22] There is a very large literature on the subject of R&D spillovers. See, e.g., d'Aspremont and Jacquemin (1988), Katz (1986), and Suzumura (1992).

Now consider $k \neq i, j$ such that $\eta_k(g - g_{i,j}) \geqslant \eta_j(g - g_{i,j})$ but $g_{i,k} = 0$. Consider the network $g + g_{i,k}$ and let $\Delta\pi_i(g) = \pi_i(g + g_{i,k}) - \pi_i(g)$. Then

$$\Delta\pi_i(g) + \Delta\pi_k(g) = \frac{2(n-1)\gamma}{(n+1)^2}\bigl[T_i(g) + T_k(g) + (n-1)\gamma\bigr]. \tag{A.3}$$

Note that

$$\begin{aligned}
&\eta_l(g) = \eta_l(g - g_{i,j}) + 1, \quad l = i, j,\\
&\eta_k(g) = \eta_k(g - g_{i,j}) \geqslant \eta_j(g - g_{i,j}),\\
&\eta_l(g) = \eta_l(g - g_{i,j}), \quad l \neq i, j, k.
\end{aligned} \tag{A.4}$$

Therefore, $T_i(g) = T_i(g - g_{i,j}) + (n-1)\gamma$ and $T_k(g) \geqslant T_j(g - g_{i,j}) - 2\gamma$. Substituting in (A.3) and recalling (A.2), it follows that

$$\begin{aligned}
\Delta\pi_i(g) + \Delta\pi_k(g) &\geqslant \frac{2(n-1)\gamma}{(n+1)^2}\bigl[T_i(g - g_{i,j}) + (n-1)\gamma + T_j(g - g_{i,j}) - 2\gamma + (n-1)\gamma\bigr]\\
&= \Delta\pi_i(g - g_{i,j}) + \Delta\pi_j(g - g_{i,j}) + \frac{2(n-1)(n-3)\gamma^2}{(n+1)^2}\\
&> 2f.
\end{aligned} \tag{A.5}$$

The final strict equality holds since $n \geqslant 3$. Therefore, i and k have a profitable deviation from g by forming a link. This contradicts the stability of g against transfers. □

Proof of Proposition 4.2. We prove the properties (i) and (ii) in sequence.

(i) Suppose g is connected and asymmetric. It follows then that g induces a partition with at least two elements. The claim is proved if we show that $\eta_i(g) - \eta_j(g) \geqslant 2$ for any pair $i \in h_{l+1}(g)$ and $j \in h_l(g)$ with $1 \leqslant l \leqslant m-1$. Suppose $\eta_i(g) - \eta_j(g) = 1$. Then there exists some player $k \neq i, j$ such that $g_{i,k} = 1$ but $g_{j,k} = 0$. However, note that $\eta_i(g - g_{i,k}) = \eta_j(g - g_{i,k}) = \eta_j(g)$. Hence, from Lemma 4.3 we infer that g is not stable against transfers, a contradiction.

(ii) We prove this part by induction. Fix $l = 1$. We first note, from the definition of an inter-linked star that if $i \in h_{m-1}(g)$ then $g_{i,j} = 0$, for $j \in h_1(g)$. We next show that if $j \in h_k(g)$, $k > 1$, then $g_{i,j} = 1$. Suppose not and let $g_{i,j} = 0$. Then from Lemma 4.3 it follows that $g_{j,k} = 0$, $\forall k \notin h_m(g)$. Thus $j \in h_1(g)$, which is a contradiction.

Now suppose that the hypothesis is true for $\hat{l} \geqslant 1$, i.e., if $i \in h_{m-\hat{l}}(g)$, then $g_{i,j} = 1$ if and only if $j \in h_k(g)$, $k > \hat{l}$. We now show that the hypothesis is also true for $\hat{l} + 1$. We first prove that if $i \in h_{m-\hat{l}-1}(g)$ and $j \in h_k(g)$, $k \leqslant \hat{l} + 1$, then $g_{i,j} = 0$. Suppose the claim is false. Then $g_{i,j} = 1$ for some $j \in h_l(g)$, $l \leqslant \hat{l} + 1$. From Lemma 4.3 it follows that $g_{i,r} = 1$, $\forall r \in h_l(g)$, $l \geqslant \hat{l} + 1$. Using the induction hypothesis, this means $\eta_i(g) \geqslant \eta_s(g)$ for $s \in h_{m-\hat{l}}(g)$. This contradicts the hypothesis that $i \in h_{m-\hat{l}-1}$.

To prove the converse, we need to show that for any $j \in h_k(g)$, $k > \hat{l} + 1$, implies $g_{i,j} = 1$. Suppose not. Then $g_{i,j} = 0$ for some $j \in h_l(g)$ where $l > \hat{l} + 1$. Then Lemma 4.3 implies that $g_{j,k} = 0$, $\forall k \in h_{l'}(g)$, $l' \leqslant m - \hat{l} - 1$. However, this implies $j \in h_{\hat{l}+1}(g)$, a contradiction. □

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Erratum

Erratum to "Networks of collaboration in oligopoly" [Games Econ. Behav. 43 (1) (2003) 57–85] ☆

Sanjeev Goyal * and Sumit Joshi

The proof of Proposition 3.3 on page 67 contains an error.[1] In particular, the claim that, 'From Eqs. (8) and (9) it follows that aggregate profits are maximized under g^c' is false. The proof can be corrected by replacing the paragraph containing step (iii) with the following steps:

In particular, at this intermediate step aggregate welfare is given by:

$$\begin{aligned}\widehat{W}\big(Q(g)\big) &\equiv \frac{1}{2}Q^2(g) + \big[p(g) - c(n-1)\big]Q(g)\\ &= \big[\alpha - c(n-1)\big]Q(g) - \frac{1}{2}Q^2(g).\end{aligned} \tag{1}$$

(iii) Note that aggregate output corresponding to a network g is given by $Q(g) = [n\alpha - \sum_{i\in N} c_i(g)]/(n+1)$ and is maximized for g^c. Now increase the output of each firm symmetrically to $q_i(g^c) = Q(g^c)/n$. Since $\alpha > c(n-1)$:

$$\begin{aligned}\widehat{W}'\big(Q(g)\big) &= \big[\alpha - c(n-1)\big] - Q(g) > \big[\alpha - c(n-1)\big] - Q(g^c)\\ &= \frac{\alpha - c(n-1)}{(n+1)} > 0.\end{aligned} \tag{2}$$

Therefore, aggregate welfare strictly increases as aggregate output increases. Since aggregate welfare is a continuous function of costs of forming links, this also proves that the complete network is the unique efficient network for small costs of forming links.

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