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Learning from Neighbours

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When payoffs from different actions are unknown, agents use their own past experience as well as the experience of their neighbours to guide their decision making. In this paper, we develop a general framework to study the relationship between the structure of these neighbourhoods and the process of social learning.

We show that, in a connected society, local learning ensures that all agents obtain the same payoffs in the long run. Thus, if actions have different payoffs, then all agents choose the same action, and social conformism obtains. We develop conditions on the distribution of prior beliefs, the structure of neighbourhoods and the informativeness of actions under which this action is optimal. In particular, we identify a property of neighbourhood structures—local independence—which greatly facilitates social learning. Simulations of the model generate spatial and temporal patterns of adoption that are consistent with empirical work.

1. INTRODUCTION

We consider a society with many agents, each of whom faces a similar decision problem: to choose an action at regular intervals without knowing the true payoffs from different actions. The action chosen generates a random reward and also provides information concerning the true payoffs. An agent uses her experience along with the experience of a subset of the society, viz. her *neighbours*, to update her prior beliefs. This experience consists of actions and the corresponding outcomes.[1] Given these beliefs, she chooses an action that maximizes expected utility. We study the evolution of agents' beliefs, actions and utilities.

We wish to understand how the structure of neighbourhoods in a society effects the generation of information (*via* the actions individuals choose) as well as its social dissemination. More specifically, we ask the following questions:

What properties of the neighbourhood structure facilitate/hinder the social adoption of an optimal action?
What are the implications of neighbourhood learning for diversity/conformism?
What are the spatial and temporal patterns of adoption when individuals learn from their neighbours? Are these patterns consistent with empirical observations?

1. Examples of such environments include consumers learning about different brands, farmers learning about the productivity of improved seeds/pesticides/insecticides and doctors learning about the efficacy of new treatments. Empirical work has documented the importance of learning from "others" in several contexts, such as the adoption of new crops (Ryan and Gross (1943)), the diffusion of patent drugs (Coleman (1966)), the choice of new agricultural techniques (Hagerstrand (1969), Rogers (1983)), economic demography (Watkins (1991)) and the purchase of consumer products (Kotler (1986)).

An important aspect of our framework is the generality of the structure of neighbourhoods. We allow for any neighbourhood structure that generates a *connected* society. A society is said to be connected if for every pair of agents i and j, either j is a neighbour of i or there exist agents $i_1, \ldots, i_m$ (depending upon i and j) such that i_1 is a neighbour of i, i_2 is a neighbour of i_1 and so on, until j is a neighbour of i_m.[2]

In our analysis, we make two assumptions on individual behaviour which limit the (Bayesian) rationality of agents. The first assumption says that, in updating her beliefs, an agent does not make inferences concerning the experience of unobserved agents (such as some of the neighbours of her neighbours), from the choice of actions of her neighbours. The second assumption requires that agents are myopic and, given their beliefs, choose an action that maximizes one-period expected utility.

The primary motivation underlying these assumptions is that in our setting a model with fully rational agents would require them to perform highly complex calculations before making their choices in each period. Our model implicitly presumes that agents either do not possess the computational capacity required to undertake these calculations, or do not find the effort required to perform them to be worthwhile. A related reason for our specification of agent behaviour is that it allows us to simplify the formulation of the model and the subsequent analysis. This simplification allows us to focus on the questions raised above. This is worthwhile since we believe that while our results are derived in a model with a specific form of bounded rationality, the insights obtained from our analysis are more general and provide an understanding of the factors relevant for social learning with other decision rules (including a fully rational one) as well. Later in the introduction, we discuss some of the considerations underlying these behavioural assumptions in greater detail, when we relate our work to the literature on Bayesian learning.

We now summarize our results and briefly discuss the intuition underlying them. We begin by establishing that agents' beliefs converge to a limit with probability one (Theorem 3.1). We use this result to demonstrate an important general property of connected societies: *the limiting (expected) utilities of all agents are equal* (Theorem 3.2). In other words, local learning enables an agent to do as well as everyone else in a connected society. The proof of this theorem uses the following arguments. We first establish that if an agent i takes some action x infinitely often then the limiting utility is equal to the true payoffs from action x. Next, we consider two agents i and j and suppose that j is a neighbour of i. If agent j takes some action x' infinitely often then her limiting utility is equal to the true payoffs from action x'. We then establish the following intuitive property: if i observes j then the true payoffs from x must be at least as high as the true payoffs from x'. We note that this property of limiting utilities is transitive. The proof is completed by using the definition of connectedness along with this transitivity of limiting utilities.[3]

Theorem 3.2 implies that, in the long run, different agents cannot choose actions which are payoff ranked. Thus if actions have distinct payoffs then in a connected society everyone takes the same action and conformism obtains with probability one.[4]

2. Familiar examples of connected societies are (a) agents located on points of a d-dimensional lattice in which every agent observes her immediate 2^d neighbours; (b) an organization tree where each person observes their immediate superior and subordinates; (c) agents located around a circle, observing their immediate neighbours and in addition observing a common set of agents who are sampled by a consumer magazine; (d) a group of agents who observe each other.

3. The arguments for this proof are very general and do not rest upon the specific form of bounded rationality assumed. This suggests that a similar result should also obtain in a model with fully rational agents.

4. Diversity refers to a situation in which different groups of agents choose different actions, while conformism describes the outcome with everyone choosing the same action.

The above result naturally leads to the question: do agents learn to choose the optimal action in the long run? It can be shown using standard arguments that learning is generally incomplete in finite agent societies. This motivates the study of learning in large societies, i.e. with a countably infinite number of agents. We begin with an example of incomplete social learning. In this example, every agent has to choose between two actions, one action whose payoffs are known and a second one whose payoffs are unknown; thus agents do not know which action is optimal. We assume that agents are indexed by the set of integers and that each agent observes the agent on either side of her. In addition, there exists a "royal family", i.e. a small set of agents who are observed by everyone.[5] We suppose that the action with unknown payoffs is the optimal action and also that initially everyone's prior beliefs favour the adoption of this action. Thus an infinite number of independent trials of this action are undertaken in the society. Despite this, we show that there is a positive probability that the society will choose the sub-optimal action eventually.

This happens because, in our example, the royal family can generate sufficient negative information that can overwhelm any locally gathered positive information, thereby inducing all agents to switch to the action with known payoffs. This means that no further information is generated and thus the society is locked into an inferior choice.[6] We can also show that, in the absence of the royal family, the society will choose the optimal action in the long run. Thus, this example illustrates an interesting aspect of social learning: *more information links can increase the probability that a society gets locked into a sub-optimal action.*

The example with incomplete learning helps us to identify alternative sets of conditions that ensure complete learning (Propositions 4.1–4.2 and Theorems 4.1–4.2). Our analysis highlights the role of *locally independent* agents. We say that two agents i and i' are locally independent if they have non-overlapping neighbourhoods, i.e. they observe different sets of agents. The general argument proceeds as follows: *First*, for any agent i we construct a set of sample paths A_i having positive probability with the following two properties: one, A_i depends only upon the realizations agent i observes, and two, sample paths in A_i have a uniform upper bound on the amount of negative information concerning optimal actions. *Second*, we observe that if agent i's beliefs are "optimistic" and can overcome this negative information then she will choose an optimal action forever on the set A_i. *Third*, we note that for two locally independent agents, i and i', the corresponding events A_i and $A_{i'}$ are independent. This implies that, given that both i and i' have optimistic prior beliefs, the probability that neither of them tries an optimal action forever is bounded above by the product of the probabilities that neither A_i nor $A_{i'}$ occur. More generally, the probability bound on the event that no one from a set of locally independent agents chooses an optimal action forever is exponentially decreasing in the number of such agents

5. This structure corresponds to situations in which individuals have access to local as well as some common/public source of information. For example, such a structure arises naturally in the context of agriculture where individual farmers observe their neighbouring farmers but all the farmers observe a few large farmers and research laboratories. Another setting with this structure is a consumer goods market; individual consumers discuss purchase decisions with their colleagues and friends and potential customers read one or two consumer magazines which report on some experiments/consumer experiences. A third example pertains to research activity; individual researchers typically keep abreast of developments in their own narrow area of specialization, and also try to keep informed about the work of the pioneers/intellectual leaders in their subject more broadly defined.

6. The reasoning above should apply in any setting where agents choose the informative action only when the posterior belief is above some cut-off value. We therefore conjecture that a similar incomplete learning result can be also be derived in a model with fully rational agents.

and in the limit equals 0.[7] The final step in the argument invokes Theorem 3.2 to show that in a connected society the probability of a society choosing suboptimal actions in the long run is subject to the same upper bound.

Theorem 4.1 makes assumptions on the distribution of prior beliefs while Proposition 4.2 and Theorem 4.2 impose restrictions on the informativeness of actions to ensure that each of the locally independent agents will choose an optimal action in the long run, with strictly positive probability.

We also study the *temporal* and *spatial patterns* of diffusion by simulating the choices of a group of farmers trying to learn the true productivity of a new crop. We find that the temporal pattern (percentage of adopters vs. time) is described quite well by the logistic function, and that the rate of adoption is positively related to the profitability of the new crop. These results are consistent with empirical findings (Griliches (1959), Feder, Just and Zilberman (1985)). We also observe that for different model specifications and parameter values the speed of convergence is fairly rapid. Finally, with regard to the spatial patterns, we find that initially small groups of farmers adopt the new crops and then it slowly spreads as neighbouring agents adopt it as well. Eventually these regions join up and the pace of diffusion accelerates. These findings match empirically observed spatial patterns (see e.g. Hagerstrand (1969)).

Our paper should be seen as a contribution to the theory of Bayesian learning. Blume and Easley (1992) and Vives (1995) survey some of the work in this area; recent work includes papers by Aghion, Paz-Espinosa and Jullien (1993), Bala and Goyal (1994, 1995), Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992) and Bolton and Harris (1992), among others. Research in this tradition has focused on cases where individual agents privately observe a signal and also have access to some *central statistic* which is publicly observed. This central statistic varies, depending on the model. In models of rational expectations learning, for instance, market prices are the central statistic, while in the recent work on herding/information cascades the actions of all previous agents are publicly observable. By contrast, in our framework, agents take actions repeatedly and learn from their own past experience as well as the experience of their neighbours.[8]

In general, the choice of actions by neighbours will reflect their own past experience as well as the past experience of their neighbours. A fully rational agent can therefore try to extract information about the experience of unobserved agents *via* the choice of actions of her neighbours. In the context of our model, this involves several kinds of computations. First, even if the neighbourhood structure is perfectly known, when agents attempt to infer the information in the society as a whole, they must take into account the fact that other agents are simultaneously attempting to make similar types of inferences, and are making choices based upon these inferences. Thus, in order to incorporate the behaviour of other agents, the beliefs of agents would be quite complicated, and the manner in which these beliefs would have to be updated in each period even more so. These difficulties are compounded when we wish to study "large" societies, as the neighbourhood structure is then likely to be imperfectly known. In this case, a fully rational agent would also need

7. The construction of sets such as A_i is central to our argument. This construction is possible in our model because agents do not make inferences from the choices of their neighbours, but only from the realizations of the choices. This is related to our first assumption on bounded rationality. We note, however, that the basic idea behind the concept of locally independent agents is that negative information must spread sufficiently slowly so that individual agents have a chance to generate adequate positive information on the optimal action. We expect that this intuition would also be central to the study of social learning with fully rational agents.

8. Recall that this experience consists of actions and corresponding outcomes. Our formulation is thus quite different from the recent work on observational learning in which agents enter sequentially and choose actions once and learn from others' actions only.

to have beliefs over the set of all neighbourhood structures and update these as well over time. Our assumption about the manner in which agents respond to new information from their neighbours arises from an effort to keep the agents' belief revision process from becoming unmanageable, and our own analysis from becoming intractable. Secondly, if agents were not myopic, their incentives for strategic behavior (such as free riding on the information generated by other agents) would also interact with the imperfect monitoring of the rest of society in very complex ways; the determination of equilibrium strategies of agents would likewise demand great computational effort on their part. For this reason, we suppose in our model that agents are concerned only with current expected utility.[9] However, for the reasons discussed above (in footnotes 3, 6 and 7), we conjecture that our main findings are quite general and would also be pertinent in a model with fully rational agents. Thus our paper contributes to the Bayesian learning literature in two ways: one, by formulating a general model of neighbourhood structures and two, by identifying certain properties of this structure which facilitate information aggregation and adequate learning.

Our paper is also related to the research on social learning with boundedly rational agents. Recent work in this area includes papers by An and Kiefer (1992), Ellison and Fudenberg (1993, 1995) and Smallwood and Conlisk (1983), among others. We briefly discuss the relationship of our paper with the work of Ellison and Fudenberg. In Ellison and Fudenberg's social learning models, agents use only currently available social information such as recent popularity weighting and disregard historical data (including their own past experience) in making decisions. By contrast, in our model agents *do* use historical information. Moreover, the bounded Bayesian decision rule the agents employ precludes the use of popularity weighting. Ellison and Fudenberg study the possibility of obtaining efficient outcomes and social diversity under different levels of popularity weighting and sample sizes. While our paper also studies efficiency and conformism, we focus on the role of prior beliefs and neighbourhood structures. These differences suggest that our paper should be viewed as complementary to their work.

Finally, our paper can also be regarded as studying the dynamics of technology adoption. Our example on incomplete learning, in the presence of a royal family, provides new insights about how the structure of information flows can generate "lock-ins" into inferior technologies.[10] In this connection we would also like to mention the early work of Allen (1982*a*, *b*) which explores the role of neighbourhood influence on the invariant distribution of a process of technology adoption. Our paper extends her work by considering social learning in an explicit model of (Bayesian) individual decision making and learning.

The rest of the paper is organized as follows. Section 2 describes the model. Sections 3–4 present our results while Section 5 discusses simulations of spatial and temporal patterns of social learning. Section 6 concludes.

2. THE MODEL

2.1. *Preliminaries*

Let Θ be a finite set of possible states of the world, X be a finite set of actions and let Y be the space of outcomes. If the state of the world is $\theta \in \Theta$ and an agent chooses action

9. It is possible to assume that agents behave as if they were long lived (i.e. with discount factors greater than zero) but *isolated*. This formulation is somewhat more complicated than the one presented here. Nonetheless, we believe that the results presented here should hold qualitatively in such a setting, since the essential properties of agents' behaviour required for our proofs would be preserved.

10. See e.g. Arthur (1989).

$x \in X$, he observes outcome y with conditional density $\phi(y; x, \theta)$ and obtains reward $r(x, y)$. We make the following assumptions about Y, ϕ and $r(x, y)$.

(A.1) Y is a non-empty, separable metric space. The distribution of outcomes[11] conditional on x and θ can be represented by the density $\phi(\cdot\,; x, \theta)$ with respect to a measure Γ defined on the Borel subsets of Y.

(A.2) For each $x \in X$, $r(x, \cdot)$ is bounded and measurable in Y.

Agents do not know the true state of the world, and they enter with a prior belief in the set $\mathscr{D}(\Theta)$ of beliefs (probability distributions) over the state of nature

$$\mathscr{D}(\Theta) = \{\mu = \{\mu(\theta)\}_{\theta \in \Theta} \mid \text{for all } \theta \in \Theta, \mu(\theta) \geqq 0 \text{ and } \textstyle\sum_{\theta \in \Theta} \mu(\theta) = 1\}. \tag{2.1}$$

Given belief μ an agent's one-period expected utility $u(x, \mu)$ from taking action x is

$$u(x, \mu) = \textstyle\sum_{\theta \in \Theta} \mu(\theta) \displaystyle\int_Y r(x, y)\phi(y; x, \theta) d\Gamma(y). \tag{2.2}$$

Note that $u(x, \cdot)$ is linear on $\mathscr{D}(\Theta)$ for every $x \in X$. We assume that individuals have the same preferences.[12] Let $G: \mathscr{D}(\Theta) \to X$ be the one-period optimality correspondence

$$G(\mu) = \{x \in X \mid u(x, \mu) \geqq u(x', \mu) \text{ for all } x' \in X\}, \qquad \mu \in \mathscr{D}(\Theta). \tag{2.3}$$

Let δ_θ be the point mass belief on the state θ; then $G(\delta_\theta)$ denotes the set of ex post optimal actions if the true state is $\theta \in \Theta$. (In the rest of the paper, we refer to ex post optimal actions as "optimal actions").

We now give two examples which are special cases of the above framework. The first example helps to clarify the basic structure, while the second example is the canonical bandit model (Berry and Fristedt (1985)) and illustrates the generality of our framework.

Example 2.1. There are two actions x_0 and x_1 and two states, θ_0 and θ_1. In state θ_1, action x_1 yields Bernoulli distributed payoffs with parameter $\pi \in (1/2, 1)$; in state θ_0 the payoffs from x_1 are Bernoulli distributed with parameter $1-\pi$. Furthermore, in both states action x_0 yields payoffs which are Bernoulli distributed with parameter $1/2$. Hence, x_1 is the optimal action if the true state is θ_1 while x_0 is the optimal action if θ_0 is the true state. The belief of an agent is a number $\mu \in (0, 1)$, which represents the probability that the true state is θ_1.[13] In this example, the one period optimality correspondence is given by

$$G(\mu) = \begin{cases} x_1 & \text{if } \mu \geqq 1/2; \\ x_0 & \text{if } \mu \leqq 1/2. \end{cases} \tag{2.4}$$

From the point of view of learning, we emphasize that agents do not observe any signals apart from the realized payoffs (1 or 0).

Example 2.2. There is a finite set of actions X; each of the actions can be one of $s \geqq 2$ quality levels or types. We suppose that the s quality types are labelled $\{q_1, \ldots, q_s\}$.

11. In what follows, we shall use the words outcomes/realizations/observations interchangeably.

12. We have also explored the learning process when agents have heterogeneous utilities. Our results on limiting utilities and learning carry over if for each group of agents of a given preference type, taken separately, connectedness obtains.

13. A natural interpretation of this example is to view action x_0 as an established technology, whose payoffs are known, and action x_1 as a new technology, whose payoffs are uncertain.

If an action x is of quality q_m then it generates observations with a density $\phi_m(y)$ and a reward $r(y)$. The expected value of an action of type q_m is

$$V_m \equiv \int_Y r(y)\phi_m(y)d\Gamma(y). \tag{2.5}$$

Let the quality levels be strictly ordered according to ascending expected value, i.e. $V_1 < V_2 < \cdots < V_s$. This induces an ordering $\prec$ among quality levels where $q_j \prec q_k$ if and only if $V_j < V_k$. Any two distinct actions are independent. This implies that a belief μ can be written as

$$\mu = \{\{\mu(x;q)\} | \textstyle\sum_{q\in\{q_1,\dots,q_s\}} \mu(x;q) = 1, \mu(x;q) \geqq 0, \forall x \text{ and } q\}. \tag{2.6}$$

In terms of the model presented earlier, a state $\theta \in \Theta$ is a specification of the quality types of the various actions. Let μ be the initial belief of an agent in the society. We assume as before that the belief is interior, i.e. for each x and each quality type q, $\mu(x;q) > 0$. Recall that $u(x, \mu)$ gives the expected one-period utility of choosing x when the belief is μ. Thus, equation (2.2) can be rewritten as

$$u(x, \mu) = \textstyle\sum_{q_j\in\{q_1,\dots,q_s\}} \mu(x;q_j)V_j. \tag{2.7}$$

Finally, we also allow for an additional kind of action which is completely uninformative i.e. yields the same distribution of outcomes in all states of nature.[14] The set of actions is thus given by $X = X_T \cup \{x_u\}$ where X_T is the set of actions each of which can be one of s types and x_u is the uninformative action.

2.2. *The social structure*

The set of agents is a non-empty set N which can be finite or countably infinite. For each $i \in N$, let $N(i)$ denote the set of *neighbours* of agent i. The statement "j is in $N(i)$" is to be interpreted as saying that agent i has access to the entire past history of agent j's actions and outcomes. By contrast, if j is not a neighbour of i, then i does not observe any of j's actions or outcomes. Throughout this paper we shall suppose $i \in N(i)$ for every agent i. We also assume that the set $N(i)$ is a finite set for all $i \in N$. Let $N^{-1}(i) = \{j \in N | i \in N(j)\}$; the set $N^{-1}(i)$ is the set of all agents who observe agent i.[15] The "royal family" is the set $R = \{j \in N | N^{-1}(j) = N\}$, i.e. those agents who are observed by everyone.

A society comprises of the set of agents and the neighbourhoods of each of the agents. We shall say that a society is *connected* if, for every $i \in N$ and every other agent $j \in N$ there exists a sequence of agents $\{i_1, i_2, \dots, i_m\}$ (depending upon i and j) such that $i_1 \in N(i)$, $i_2 \in N(i_1)$, and so on until $j \in N(i_m)$. The analysis in this paper is restricted to connected societies; we focus on such societies because all other types of societies can be analysed as a collection of connected societies. (See footnote 20 below, for a discussion on this issue.) In what follows, for expositional simplicity, we shall usually omit the term "connected" while referring to societies.

14. In the context of crop choice, this corresponds to a case where the farmer decides not to plant any crop. Likewise, in situations where consumers are making brand choices this action is the "no purchase" option.

15. If the observation relation is symmetric, this set clearly coincides with $N(i)$. However, there are many sources of communication (e.g. radio, television, books, journals and gossip!) which do not possess symmetry. Our framework allows for asymmetric observational links.

2.3. *The dynamics of the model*

Time is discrete and is indexed by $t=1, 2, \ldots$. At the beginning of period 1, each agent i has a prior belief $\mu_{i,1} \in \mathscr{D}(\Theta)$. We assume

$$\text{(A.3)}\quad \text{For all } i \in N,\ \mu_{i,1} \in \text{Int}\,(\mathscr{D}(\Theta)),$$

where Int $(\mathscr{D}(\Theta))$ denotes the interior of the belief space. It is worth noting that we do not restrict the agents to have identical priors.

For each $i \in N$, let $g_i : \mathscr{D}(\Theta) \to X$ be a selection from the one-period optimality correspondence G of equation (2.3) above. In period 1, each agent i plays the action $g_i(\mu_{i,1})$ and observes the outcome. Agent i also observes the actions taken and outcomes obtained by the other agents in $N(i)$. In periods $t=2$ and beyond, each agent i first computes her posterior belief $\mu_{i,t}$ based on the experiences of the agents in $N(i)$. In this regard, we assume that agents employ a "bounded Bayesian" learning algorithm. This algorithm specifies that agents modify their prior beliefs to posterior ones, using Bayes rule in conjunction with the information obtained from their own and their neighbours' experiences. However, they do not attempt to extract any information from the observed choices of their neighbours.[16] After forming her posterior $\mu_{i,t}$ in the manner described above, agent i then chooses the action $g_i(\mu_{i,t})$ which maximizes one-period expected utility, and the process continues in this manner. Thus, agents are being boundedly rational both in choosing their optimal action myopically given their beliefs and also in forming posterior beliefs.

We now briefly sketch the construction of the probability space since the notation is required for the results. Details are provided in the Appendix. For a fixed $\theta \in \Theta$ we define a probability triple $(\Omega, \mathscr{F}, P^\theta)$, where Ω is the space containing sequences of realizations of actions of all agents over time, and P^θ is the probability measure induced over sample paths in Ω by the state $\theta \in \Theta$.

Let Θ be endowed with the discrete topology, and suppose $\mathscr{B}$ is the Borel σ-field on this space. For rectangles of the form $A \times H$ where $A \subset \Theta$ and H is a measurable subset of Ω, let $P_i(A \times H)$ be given by

$$P_i(A \times H) = \textstyle\sum_{\theta \in A} \mu_{i,1}(\theta) P^\theta(H), \tag{2.8}$$

for each agent $i \in N$. Each P_i extends uniquely to all of $\mathscr{B} \times \mathscr{F}$. Since every agent's prior belief lies in the interior of $\mathscr{D}(\Theta)$, the measures $\{P_i\}$ are pairwise mutually absolutely continuous. All stochastic processes are defined on the measurable space $(\Theta \times \Omega, \mathscr{B} \times \mathscr{F})$. A typical sample path is of the form $\omega = (\theta, \omega')$ where θ is the state of nature and ω' is the infinite sequence of sample outcomes denoted by

$$\omega' = ((y^x_{i,1})_{x \in X, i \in N}, (y^x_{i,2})_{x \in X, i \in N}, \ldots), \tag{2.9}$$

where $y^x_{i,t} \in Y^x_{i,t} \equiv Y$. Let $C_{i,t} \equiv g_i(\mu_{i,t})$ denote the action of agent i at time t, $Z_{i,t}$ the outcome of agent i's action at time t (i.e. the signal of her own action from the outcome space Y) and let $(Z_{j,t})_{j \in N(i)}$ be the set of outcomes of the neighbours of i at time t. Also let $U_{i,t}(\omega) = u(C_{i,t}, \mu_{i,t})$ be the expected utility of i with respect to her own action at time t.[17] The

16. This formulation thus rules out the use of popularity weighting and related measures in the learning process. Note, however, that Bayesian updating provides a relatively simple way for each agent to keep track of the information in past history. We also note that while the bounded Bayesian learning rule employed here may not be efficient, it is consistent in our framework.

17. The outcomes of actions are projections of ω onto the respective coordinates. We assume that if agent i has chosen action x' for the t-th time on ω, he observes the coordinate $y^{x'}_{i,t}(\omega)$.

posterior belief of agent i in period $t+1$ is computed as follows

$$\mu_{i,t+1}(\theta)=\frac{\prod_{j\in N(i)}\phi(Z_{j,t};C_{j,t},\theta)\mu_{i,t}(\theta)}{\sum_{\theta'\in\Theta}\prod_{j\in N(i)}\phi(Z_{j,t};C_{j,t},\theta')\mu_{i,t}(\theta')}. \tag{2.10}$$

The σ-field of agent i's information at the beginning of time 1 is $\mathscr{F}_{i,1}\equiv\{\emptyset,\Theta\times\Omega\}$. For every $t\geqq 2$, define $\mathscr{F}_{i,t}$ as the σ-field generated by the past history of agent i's observations of her neighbours' actions and outcomes, i.e. the random variables $(C_{j,1},Z_{j,1})_{j\in N(i)}$, $\ldots,(C_{j,t-1},Z_{j,t-1})_{j\in N(i)}$. Since by the rules of the process, agents only employ the information generated by their neighbours, the set classes $\{\mathscr{F}_{i,t}\}$ are the relevant σ-fields for our purposes. We shall denote by $\mathscr{F}_{i,\infty}$ the smallest σ-field containing all $\mathscr{F}_{i,t}$ for $t\geqq 1$.

3. AGGREGATION OF INFORMATION

In this section we establish that (roughly speaking) in a connected society every agent expects the same utility, in the long run. The first step in the study of the long run distribution of individual utilities establishes convergence of a typical individual's beliefs and utilities. The following result shows that the sequence of posterior beliefs of a typical agent converges almost surely to a limit belief which is measurable with respect to the (direct) limit information of the agent.[18]

Theorem 3.1. *There exists $Q\in\mathscr{B}\times\mathscr{F}$ satisfying $P_i(Q)=1$ for all $i\in N$ and random vectors $\{\mu_{i,\infty}\}_{i\in N}$ such that*

(a) *For each $i\in N$, $\mu_{i,\infty}$ is $\mathscr{F}_{i,\infty}$-measurable.*
(b) *$\omega\in Q\Rightarrow$ for all $i\in N$, $\mu_{i,t}(\omega)\to\mu_{i,\infty}(\omega)$.*

This result is an immediate consequence of the Martingale Convergence Theorem.[19] In what follows, we restrict attention to a specific state of nature which is taken to be the true state. We shall denote this state by θ_1. Clearly, the set

$$Q^{\theta_1}\equiv\{\omega=(\theta,\omega')\in Q\,|\,\theta=\theta_1\},$$

has P^{θ_1} probability 1. (Strictly speaking, the domain of definition of P^{θ_1} is the measurable subsets of Ω, not of $\Theta\times\Omega$. However, we can regard P^{θ_1} as the conditional probability induced by θ_1 on the product space, which is the same for all agents). Without loss of generality we assume that the strong law of large numbers holds on Q^{θ_1}. In what follows statements of the form "which probability one" are with respect to the measure P^{θ_1}.

We next show the convergence of utilities of a typical individual in the society. For each agent i, given $\omega\in Q^{\theta_1}$, let $X^i(\omega)$ be the set of actions which are chosen infinitely often on the sample path. We shall refer to $X^i(\omega)$ as the set of limiting actions (of agent i) on ω. Given that every individual is a myopic optimizer, it seems natural that the set of limiting actions should be optimal with respect to the limiting beliefs. This is true, as part (a) of the following result shows. This result immediately implies that each agent's one period expected payoffs converges as well. Recall that $U_{i,t}(\omega)\equiv u(C_{i,t}(\omega),\mu_{i,t}(\omega))$.

18. It is worth emphasizing that Theorem 3.1 does not preclude the possibility of limit beliefs being different across individual agents.
19. The proofs not given in the text can be found in the appendix.

Lemma 3.1. *Suppose* $\omega \in Q^{\theta_1}$.

(a) *If* $x' \in X^i(\omega)$ *then* $x' \in \operatorname{argmax}_{x \in X} u(x, \mu_{i,\infty}(\omega))$.
(b) *There exists a real number* $U_{i,\infty}(\omega)$ *such that* $\{U_{i,t}(\omega)\} \to U_{i,\infty}(\omega)$. *Furthermore,* $U_{i,\infty}(\omega) = u(x', \mu_{i,\infty}(\omega))$ *where* x' *is any member of* $X^i(\omega)$.

We now examine the distribution of these limiting utilities and actions in the society. Our analysis is summarized in the following result.

Theorem 3.2. *Suppose that the society is connected. Then* $U_{i,\infty}(\omega) = U_{j,\infty}(\omega)$ *for all agents* i *and* j *in* N, *with probability* 1.

The proof of this result employs the following arguments. On a fixed sample path, consider two agents i and j and suppose $i \in N(j)$. We show that if x' is an action taken infinitely often by j then j's long run expected utility $U_{j,\infty}$ will be $u(x', \delta_{\theta_1})$. Likewise, if i chooses x infinitely often, then $U_{i,\infty} = u(x, \delta_{\theta_1})$. Furthermore, the assumption that j observes i is shown to imply that $u(x', \delta_{\theta_1}) \geqq u(x, \delta_{\theta_1})$.[20] Thus, $U_{j,\infty} \geqq U_{i,\infty}$. Connectedness of the society now yields the result.

One interesting implication of the result is that if actions have different payoffs then, on a set of probability one, all agents will choose the same action in the long run and social conformism obtains.[21]

4. LONG RUN SOCIAL LEARNING

In this section we study the optimality of long run actions. An important implication of Theorem 3.2 is that, on a given sample path, if even one agent eventually learns to choose ex post optimal actions, then so will the rest of society. We exploit this observation and develop conditions on the distribution of prior beliefs, the structure of neighbourhoods and the informativeness of actions that ensure that everyone in a society chooses an optimal action in the long run. We start with some definitions. Assume as before that θ_1 is the true state of nature.

Definition 4.1. Given a sample path ω, the long run actions of agent i are said to be optimal on ω if $X^i(\omega) \subset G(\delta_{\theta_1})$. Social learning is said to occur if

$$P^{\theta_1}\left(\textstyle\bigcap_{i \in N} \{X^i(\omega) \subset G(\delta_{\theta_1})\}\right) > 0. \tag{4.1}$$

Social learning is said to be complete if the probability on the left-hand side above is equal to 1; it is said to be incomplete if this probability is less than 1.

Definition 4.2. An action $x \in X$ is said to be fully informative if, for all θ, θ' in Θ such that $\theta \neq \theta'$ we have:

$$\int_Y |\phi(y; x, \theta) - \phi(y; x, \theta')| d\Gamma(y) > 0. \tag{4.2}$$

20. This intermediate result is also useful for addressing questions concerning limit utilities in societies that are not connected. Any such society can be partitioned into a collection of (internally) connected sub-societies. Consider two such sub-societies, N^1 and N^2. The above result says that if an agent $i \in N^1$ observes some agent $j \in N^2$ then $U_{k,\infty} \geqq U_{k,\infty}$, for all $l \in N^1$ and all $k \in N^2$.

21. In an earlier version of the paper, we also studied the likelihood of conformism and diversity in an example where different actions have the same payoff. We showed that the long run outcome is related to the structure of neighbourhoods, with conformism being more likely in more "integrated" societies. Details of these results are available from the authors upon request.

We shall say that an action x_u is uninformative if $\phi(\cdot\,; x_u, \theta)$ is independent of θ.[22]

Thus a fully informative action x is statistically capable of distinguishing between any two distinct states in the long run.

We start by noting that social learning will typically be incomplete in finite societies.[23] This motivates the study of learning in societies with *infinitely* many agents. We begin our analysis with two observations. The first observation concerns the importance of the initial distribution of priors. It is easy to see (with the help of Example 2.1) that even in a large (infinite agent) society, learning will not occur if all agents start out with prior beliefs that lead them to choose the uninformative action. *Thus for social learning to occur some restrictions on the distribution of prior beliefs are necessary.* Our second observation is that even when beliefs are favourable, the social structure of information flows may preclude learning. The following example illustrates this point and also helps us derive sufficient conditions for complete social learning subsequently.

Example 4.1. Consider the setting of Example 2.1. Suppose that the true state is θ_1 and that the society has an infinite number of agents. Assume that the prior beliefs of agents satisfy the following condition

$$\inf_{i\in N} \mu_{i,1} > \frac{1}{2}, \qquad \sup_{i\in N} \mu_{i,1} < \frac{1}{1+p^2}, \tag{4.3}$$

where $p=(1-\pi)/\pi\in(0, 1)$. The above assumption implies that in period 1 all agents will choose the optimal (and informative) action x_1. We suppose that society N is given by the one dimensional integer lattice. For $i\in N$, the set of neighbours is assumed to be $N(i)=\{i-1, i, i+1\}\cup R$, where $R=\{1, 2, 3, 4, 5\}$ constitute the royal family. We now note the possibility of incomplete learning: *there is a strictly positive probability that every agent will choose the suboptimal action x_0 for all $t\geqq 2$.*

The argument underlying this claim is as follows: Define $\bar{Q}=\{Z_{j,1}=0, \text{ for all } j\in R\}$; by construction, $P^{\theta_1}(\bar{Q})=(1-\pi)^5>0$. We show that if $\omega\in\bar{Q}$, then $C_{i,t}(\omega)=x_0$ for all $t\geqq 2$, for $i\in N$. Note that on $\omega\in\bar{Q}$, an agent $i\in N$ observes 5 "negative" realizations from the royal family, while the maximum number of "positive" realizations that can be observed locally is 3. Thus there is a minimum amount of residual negative information. Since $\mu_{i,1}<1/(1+p^2)$ this negative residual information is sufficient to push the posterior belief $\mu_{i,2}$ below $1/2$, making agent i choose the uninformative action. The argument is completed by noting that i has been chosen arbitrarily.[24]

22. It is worth remarking that if the set of uninformative actions X_U is non-empty, then there is no essential loss of generality in assuming that it consists of a single element x_u.

23. To see why this is true consider the set up of Example 2.1. Suppose that the true state is θ_1 and that the society is finite. Prior beliefs of agents are then represented by a number $\mu_{i,1}$ which is the probability that true state is θ_1. Let $\inf_{j\in N}\mu_{j,1}>1/2$ and focus on the agent with the highest value of $\mu_{i,1}$. Standard arguments imply that there exists a finite sequence of T realizations of 0, such that this agent would switch to action x_0. Now consider the set of sample paths on which all agents get realizations of 0 for the first T periods. The probability of this set is positive given that realizations are independent and the number of agents finite. The argument is completed by observing that on any sample path in this set every agent will choose the sub-optimal (and uninformative) action x_0 after time period T.

24. The above example illustrates the possibility of incomplete social learning in perhaps the simplest setting. The phenomenon itself is more general and arises in a larger class of societies. Consider, for example, a society with agents located on the one-dimensional lattice, and observing their two immediate neighbours. Suppose in addition that there are S disjoint, finite groups of agents $\{R_s\}_{s=1}^{S}$, and for each $i\in N$ outside of a finite set $\bar{N}$ of agents, $R_s\subset N(i)$ for at least one s. Thus, the groups $\{R_s\}$ are *nearly royal* families. It is not difficult to show, using the arguments above, that if each group R_s is sufficiently large, then learning will be incomplete in such societies.

In the example above, one reason for incomplete social learning is that the prior beliefs of agents are not very dispersed. This allows a "little" bad experience of a few people to convince everyone to switch to the uninformative action. This aspect of the example motivates a study of connected societies with dispersed prior beliefs. We formalize the idea of dispersed beliefs in the following definition.

(H) The distribution of prior beliefs is *heterogeneous* if for every $\theta \in \Theta$, and for any open neighbourhood around δ_θ, there exists an agent whose prior belief lies in that neighbourhood.

Heterogeneity of beliefs may be interpreted as saying that the truth must lie in the support of the distribution of prior beliefs across agents. Since the true state is unknown, this requirement leads naturally to the formulation above, where for any θ, δ_θ lies in the support of the distribution of prior beliefs.

We can now specify circumstances under which (H) ensures "almost" complete social learning. The essential idea is to find an agent i who puts a large prior probability on the true state, and a set of sample paths A_i on which the agent's favourable prior "overcomes" any negative information generated by her neighbours concerning the true state. This will ensure that she will choose only optimal actions in the long run. Theorem 3.2 can then be used to show that the same holds for all agents in the society. In order to apply this idea, we need to ensure that the maximum amount of negative information that an agent receives from her neighbours is bounded. This motivates the following restriction on the size of individual neighbourhoods.

(B) There exists a number $K>0$ such that $\sup_{i \in N} |N(i)| \leqq K$.

We are now in a position to state our first learning result.

Proposition 4.1. *Consider a connected society. Suppose that conditions* (H) *and* (B) *are satisfied. Then for any* $\lambda \in (0, 1)$, *we have*

$$P^{\theta_1}\left(\bigcap_{i \in N} \{X^i(\omega) \subset G(\delta_{\theta_1})\}\right) \geqq \lambda. \tag{4.4}$$

This Proposition follows as a corollary of Lemmas 4.1 and 4.2. These lemmas are also central to an understanding of our subsequent results on learning and so we present their proofs in the text.

Lemma 4.1 makes use of the following property of the one-period optimality correspondence G.

Remark 4.1. Let $\mu \in \mathscr{D}(\Theta)$. There exists a number $\hat{d} \in (0, 1)$ such that if $\mu(\theta_1) \geqq \hat{d}$ then $G(\mu) \subset G(\delta_{\theta_1})$.

The existence of $\hat{d}$ follows directly from the finiteness of the action space X and the continuity of the utility function with respect to beliefs.

Lemma 4.1. *Fix some agent* $i \in N$. *For any* $\lambda \in (0, 1)$ *there exists a set of sample paths* A_i *satisfying* $P^{\theta_1}(A_i) \geqq \lambda$ *and* $d(\lambda) \in (0, 1)$ *such that if* $\mu_{i,1}(\theta_1) \geqq d(\lambda)$ *then*

$$\omega \in A_i \Rightarrow X^i(\omega) \subset G(\delta_{\theta_1}). \tag{4.5}$$

Proof. Consider agent i in isolation, and suppose she only chooses action x for $t-1$ periods, and observes a sequence $\{y^x_{i,\tau}\}^{t-1}_{\tau=1}$, where each $y^x_{i,\tau} \in Y^x_{i,t} \equiv Y$. The agent's information about state $\theta \neq \theta_1$ based upon the observations generated by her choices can be summarized by the *product likelihood ratio* $r^{x,\theta}_{i,t}$, defined as

$$r^{x,\theta}_{i,t}(\omega) = \frac{\prod^{t-1}_{\tau=1} \phi(y^x_{i,\tau}(\omega); x, \theta)}{\prod^{t-1}_{\tau=1} \phi(y^x_{i,\tau}(\omega); x, \theta_1)}. \tag{4.6}$$

(If $t=1$, we follow the convention that $r^{x,\theta}_{i,t}=1$). It follows from an application of the law of large numbers that $r^{x,\theta}_{i,t} \to \bar{r}^{x,\theta}_i$ where $\bar{r}^{x,\theta}_i < \infty$, almost surely (see e.g. DeGroot (1970) pp. 201–204). Since this is true for all $\theta \neq \theta_1$ and all $x \in X$, there exists σ and a set A^σ_i of sample paths defined as

$$A^\sigma_i = \prod_{x\in X} \left\{ \max_{\theta \in \Theta \backslash \theta_1} \sup_{t \geqq 1} r^{x,\theta}_{i,t} \leqq \sigma \right\} \times \prod^\infty_{t=1} \prod_{j' \in N \backslash i} \prod_{x \in X} Y^x_{j',t}, \tag{4.7}$$

such that $P^{\theta_1}(A^\sigma_i) \geqq \delta$, where $\delta \equiv \lambda^{1/K} > 0$. It follows from our convention that $\sigma \geqq 1$. Intuitively, on a sample path $\omega \in A^\sigma_i$, the maximum amount of "negative information" about state θ_1 *vis-a-vis* state θ that i can obtain from her own actions is bounded above by $\sigma^{|X|}$. We now consider each agent $j \in N(i)$ other than i. Since the realizations of individual agents from their own actions are identically distributed (conditional on θ_1), it follows that for each neighbour $j \in N(i) \backslash i$, there exists a similarly defined set A^σ_j with $P^{\theta_1}(A^\sigma_j) = P^{\theta_1}(A^\sigma_i) = \delta$. (This is done by just replacing i by j everywhere in equation (4.7)). Define the set $A_i = \bigcup_{j \in N(i)} A^\sigma_j$. Using the independence of observations obtained from different individuals, it follows that

$$P^{\theta_1}(A_i) \geqq \delta^{|N(i)|} \geqq \delta^K = \lambda, \tag{4.8}$$

where we use the inequality $|N(i)| \leqq K$, from assumption (B). Note that individual i's posterior belief about state θ_1 at time t can be written as

$$\mu_{i,t}(\theta_1)(\omega) = \frac{\mu_{i,1}(\theta_1)(\omega)}{\mu_{i,1}(\theta_1)(\omega) + \sum_{\theta \neq \theta_1} \prod_{j \in N(i)} \prod_{x \in X} r^{x,\theta}_{j,t}(\omega) \mu_{i,1}(\theta)(\omega)}, \tag{4.9}$$

where $r^{x,\theta}_{j,t}(\omega)$ now refers to the product likelihood ratio along the sample path when the actions $\{C_{j,\tau}\}$ are chosen. For $\omega \in A_i$ we have

$$\mu_{i,t}(\theta_1)(\omega) \geqq \frac{\mu_{i,1}(\theta_1)(\omega)}{\mu_{i,1}(\theta_1)(\omega) + \sum_{\theta \neq \theta_1} \sigma^{K|X|} \mu_{i,1}(\theta)(\omega)}, \tag{4.10}$$

by construction of the set A_i.[25]

Let $\hat{d}$ be the number defined in Remark 4.1 above. Since the expression on the right side of (4.10) is independent of t, it is evident that there will exist a number $d(\lambda) \in (0, 1)$ such that if $\mu_{i,1}(\theta_1) \geqq d(\lambda)$ and $\omega \in A_i$, then $\mu_{i,t}(\theta_1)(\omega) \geqq \hat{d}$ for all $t \geqq 1$. By the definition of $\hat{d}$ this means $G(\mu_{i,t}(\omega)) \subset G(\delta_{\theta_1})$ for each t. Since $C_{i,t}(\omega) \in G(\mu_{i,t}(\omega))$ for each t, we have $\omega \in \bigcap_{t \geqq 1} \{C_{i,t} \in G(\delta_{\theta_1})\}$. It follows easily that $X^i(\omega) \subset G(\delta_{\theta_1})$ as well. ||

Thus, if an agent i can be found whose prior belief $\mu_{i,1}(\theta_1) \geqq d(\lambda)$, then by construction, for each sample path $\omega \in A_i$ the agent will only choose optimal actions in the long

25. On this set of sample paths, irrespective of the choice of actions by $j \in N(i)$ up to time $t-1$ the corresponding $r^{x,\theta}_{j,t}$ will be bounded above by σ. See the discussion following equation (2.9) in Section 2.

run.[26] Theorem 3.2 can then be used to show that the long run behaviour of the society is the same as agent i. The following lemma formalizes the argument.

Lemma 4.2. *Suppose that the society is connected. If for some $\omega \in Q^{\theta_1}$ there exists an agent $i(\omega)$ such that $X^{i(\omega)}(\omega) \subset G(\delta_{\theta_1})$ then for every $j \in N$, $X^j(\omega) \subset G(\delta_{\theta_1})$ as well.*

Proof. Let $x' \in X^{i(\omega)}(\omega)$ so that $x' \in G(\delta_{\theta_1})$ as well. By Remark 1 in the Appendix, if $x' \in X^{i(\omega)}(\omega)$ then $U_{i(\omega),\infty} = u(x', \delta_{\theta_1})$. Fix $j \in N$. Connectedness (and hence Theorem 3.2) implies that $U_{j,\infty}(\omega) = U_{i(\omega),\infty}(\omega)$. Hence $U_{j,\infty}(\omega) = u(x', \delta_{\theta_1})$. Let $\bar{x} \in X^j(\omega)$. Using Remark 1 again, $U_{j,\infty}(\omega) = u(\bar{x}, \delta_{\theta_1})$ so that $u(\bar{x}, \delta_{\theta_1}) = u(x', \delta_{\theta_1})$. Since $x' \in G(\delta_{\theta_1})$ we have $\bar{x} \in G(\delta_{\theta_1})$ as well. As $\bar{x}$ has been arbitrarily chosen from $X^j(\omega)$ we obtain $X^j(\omega) \subset G(\delta_{\theta_1})$. Since $j \in N$ is also arbitrary, the result follows. ||

The proof of Proposition 4.1 is now straightforward. From the first lemma, if i is an agent whose prior belief puts probability weight of at least $d(\lambda)$ on θ_1, then for each $\omega \in A_i$ he will choose only optimal actions in the long run. By (H), there will exist an agent i for whom $\mu_{i,1}(\theta_1) \geqq d(\lambda)$. Hence

$$A_i \subset \{X^i(\omega) \subset G(\delta_{\theta_1})\} \subset \bigcap_{j \in N} \{X^j(\omega) \subset G(\delta_{\theta_1})\}, \tag{4.11}$$

where the first relation derives from Lemma 4.1 and the second from Lemma 4.2. The lower bound on the probability of the event on the right-hand side in (4.4) follows, since by construction, $P^{\theta_1}(A_i) \geqq \lambda$.

Proposition 4.1 applies to a large class of societies, including those with a royal family, or with "nearly-royal" families (see footnote 24). The assumption of heterogeneity, as expressed in condition (H), is, however, very strong since a single agent with a belief suitably close to the truth is responsible for all the learning that occurs.[27] This motivates an alternative approach to the question of complete social learning: we try to find an infinite number of agents who each play an optimal action forever with probability bounded away from zero, ensuring at the same time that these events are independent.

In this context, we introduce the notion of *locally independent agents.* Two agents i and i' are locally independent if they have non-overlapping neighbourhoods, i.e. satisfying $N(i) \cap N(i') = \varnothing$. A *pairwise locally independent group of agents* is a subset of N such that any two agents in the group are locally independent. Fix a number $K > 0$ and a $\bar{\lambda} \in (0, 1)$. Let $\bar{d} \equiv d(\bar{\lambda})$ be the corresponding value whose existence is ensured by Lemma 4.1. Consider the collection of agents $i \in N$ such that $|N(i)| \leqq K$ and satisfying $\mu_{i,1}(\theta_1) \geqq \bar{d}$. Let $N_{K,\bar{d}}$ be a maximal group of pairwise locally independent agents chosen from this collection, i.e. a subset of the above collection which has the highest cardinality.[28] We are now ready to state and prove the following general complete learning result.

26. As can be seen, the above proof shows that agent i will choose an optimal action forever, which is stronger than the implication $X^i(\omega) \subset G(\delta_{\theta_1})$ found in the statement of the lemma. We retain this formulation since the weaker implication is the one needed for subsequent proofs.

27. In particular, the proof of Lemma 4.1 requires in most cases of interest that $d(\lambda) \to 1$ as $\lambda \to 1$. Technically, this is because the supremum (over all $t \geqq 1$) of the product likelihood ratio defined in (4.6) can be arbitrarily large with positive probability. In other words, for social learning to occur with probability close to 1, a prior belief close to 1 in favour of the true state is needed.

28. It is worth noting that there may be many such maximal groups of agents. For instance let N be the set of integers, with $N(i) = \{i-1, i, i+1\}$ for all $i \in N$. Here, we can fix $K = 3$. Suppose all agents $i \in N$ satisfy $\mu_{i,1}(\theta_1) \geqq \bar{d}$. Then the sets of agents $\{0, 3, 6, 9, 12, \dots\}$, and $\{\dots, -6, -2, 2, 6, 10, \dots\}$ are just two of infinitely many possible candidates for $N_{K,\bar{d}}$.

Theorem 4.1. *Consider a connected society. Let* $\bar{\lambda}>0$, $\bar{d}=d(\bar{\lambda})$ *and* $N_{K,\bar{d}}$ *be as defined above. Then*

$$P^{\theta_1}(\bigcup_{i\in N}\{X^i(\omega)\not\subset G(\delta_{\theta_1})\})\leqq(1-\bar{\lambda})^{|N_{K,\bar{d}}|}. \tag{4.12}$$

In particular, if for some $\bar{\lambda}>0$ *and* $\bar{d}=d(\bar{\lambda})$ *we have* $|N_{K,\bar{d}}|=\infty$, *then complete social learning obtains.*

Proof. Let $i\in N_{K,\bar{d}}$. Let A_i be the set of sample paths specified in Lemma 4.1 corresponding to $\bar{\lambda}$, so that $P^{\theta_1}(A_i)\geqq\bar{\lambda}$. By definition, $i\in N_{K,\bar{d}}$ implies $\mu_{i,1}(\theta_1)\geqq\bar{d}$. Applying Lemma 4.1 and Lemma 4.2, we get $A_i\subset\bigcap_{j\in N}\{X^j(\omega)\subset G(\delta_{\theta_1})\}$. Since $i\in N_{K,\bar{d}}$ is arbitrary, we have

$$\bigcup_{i\in N_{K,\bar{d}}}A_i\subset\bigcap_{j\in N}\{X^j(\omega)\subset G(\delta_{\theta_1})\} \tag{4.13}$$

as well. Hence

$$\bigcup_{j\in N}\{X^j(\omega)\not\subset G(\delta_{\theta_1})\}\subset\bigcap_{i\in N_{K,\bar{d}}}A_i^c. \tag{4.14}$$

However, as the agents in the set $N_{K,\bar{d}}$ are pairwise locally independent, the events $\{A_i^c\}_{i\in N_{K,\bar{d}}}$ are independent. Thus

$$P^{\theta_1}(\bigcup_{j\in N}\{X^j(\omega)\not\subset G(\delta_{\theta_1})\})\leqq P^{\theta_1}(\bigcap_{i\in N_{K,\bar{d}}}A_i^c)\leqq(1-\bar{\lambda})^{|N_{K,\bar{d}}|} \tag{4.15}$$

where we have used the fact that $P^{\theta_1}(A_i)\geqq\bar{\lambda}$ for each i. The result follows. ||

We make a number of remarks concerning the above result. First, it is useful to compare it with the proposition established earlier. In Theorem 4.1, $\bar{\lambda}$ can be an arbitrarily small positive number. The construction in Lemma 4.1 then suggests (informally) that $\bar{d}=d(\bar{\lambda})$ is also relatively "small". Thus, a large number of agents with priors "slightly" in favour of the true state can play the role of the agent in Proposition 4.1 whose prior is strongly in favour of the true state. This can be seen most clearly in the context of Example 4.1. Suppose that everything were the same as in that example, except that there was no royal family, i.e. $N(i)=\{i-1, i, i+1\}$ and hence $K=3$. The theory of random walks can be applied to show that $\bar{d}$ can be chosen to be any number greater than $1/2$.[29] The assumption made in the example that $\inf_{i\in N}\mu_{i,1}>1/2$ implies that $|N_{K,\bar{d}}|=\infty$. We can then apply Theorem 4.1 to conclude that complete social learning obtains. Thus, the condition that infinitely many agents have priors which make them choose the optimal action in the first period is essentially sufficient for complete learning.[30]

Second, we note that unlike Proposition 4.1, the above theorem has virtually no implications for societies where a royal family is present, since in this case, $R\subset N(i)\cap N(i')$ and no two agents can be pairwise locally independent. It then follows that $|N_{K,\bar{d}}|\leqq 1$ so that (4.12) does not ensure complete social learning. Indeed, we know from Example 4.1 that if we re-admit the information links of the royal family (by having $N(i)=\{i-1, i, i+1\}\cup R$ for all $i\in N$), then social learning is incomplete. Thus, more information links can increase the chances of a society getting locked into a sub-optimal action!

29. In the terminology of Lemma 4.1, there exists a $\bar{\lambda}>0$ such that σ may be chosen to equal 1. Clearly, the value $\hat{d}$ can be any number larger than $1/2$. Then $\bar{d}=\hat{d}$ suffices.

30. Note also that Theorem 4.1 requires only that the agents in $N_{K,\bar{d}}$ have at most K neighbours each. For the remaining agents, the set of neighbours can be any finite set. This is weaker than condition (**B**) used in Proposition 4.1.

Thirdly, we elaborate on the term 'local independence'. Since the society is connected, it is clear that no agent is truly independent of any other agent. Our terminology refers to the following property of the model: if i and j are distinct agents in $N_{K,\bar{d}}$, then there exist independent sets of sample paths A_i and A_j where i's and j's choices respectively are determined only by their locality, i.e. by the information derived from $N(i)$ and $N(j)$ respectively. In particular, on these sets of sample paths, each agent's choices (but not necessarily beliefs) are independent of the information—either positive or negative—generated by the agents outside their own neighbourhoods.

Finally, to obtain a better idea of the rate at which the probability of incomplete social learning decreases with the number of locally independent agents, we briefly discuss some simulations of Example 2.1. We suppose the agents in N are arranged in a circle with $N(i)=\{i-1, i, i+1\}$ (no royal family). Figure 1 displays the probability of incomplete learning as a function of societal size $|N|$ assuming the payoffs in Example 2.1 are Bernoulli distributed, while Figure 2 concerns the Normal case.

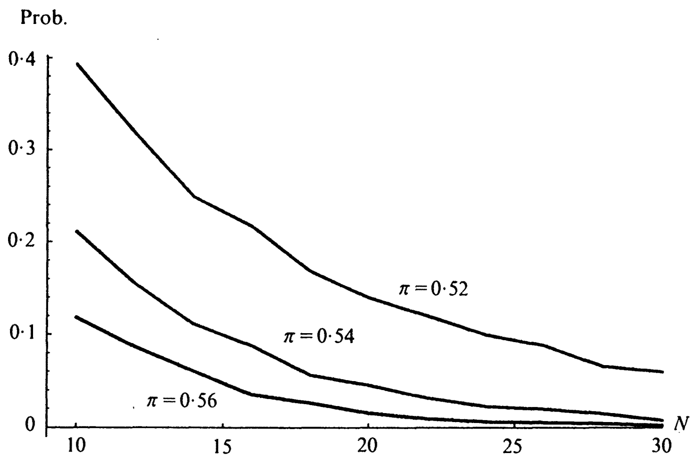

FIGURE 1
Incomplete social learning

We note that the probability decays quite rapidly with the size of the society; furthermore, a regression of the log incomplete learning probability on $|N|$ yields a very good fit (the R^2 values are all above 0·99 and between 0·94 and 0·98 in the Bernoulli and Normal cases respectively), and suggests that the bound established in (4.12) is tight.

While Theorem 4.1 demonstrates the role of locally independent agents in generating socially optimal long run behaviour, as the above discussion shows, it does restrict the class of societies. In particular, it effectively excludes societies with a royal family. This motivates an examination of conditions under which we can obtain complete learning in more general societies, where prior beliefs may not satisfy (H). Example 4.1 is again a good starting point: one reason why the example "works" is because the negative information generated by the royal family exceeds any positive information that a local neighbourhood can produce. This suggests that if an agent (or a group of them) is able to generate an arbitrarily "large" amount of positive information with non-zero probability, then complete social learning may obtain. This motivates the concept of Unbounded Positive

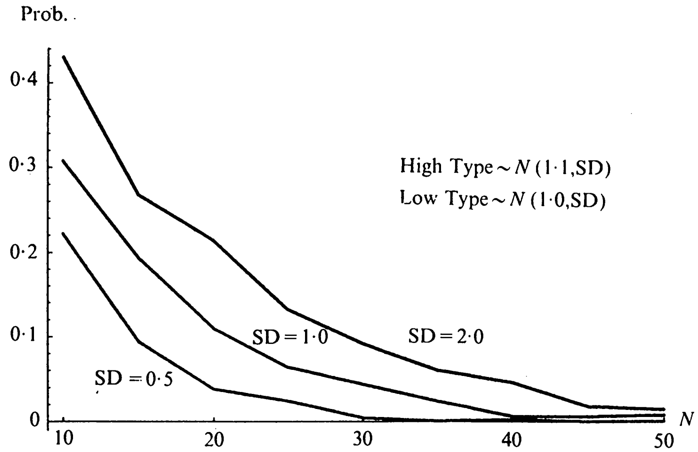

FIGURE 2
Incomplete social learning

Information

(UPI) An action $x \in X$ generates unbounded positive information concerning the true state θ_1, if for every $\alpha \in (0, 1)$, there is a set $B^{x,\alpha} \subset Y$ with $\int_{B^{x,\alpha}} \phi(y; x, \theta_1) d\Gamma > 0$ such that

$$y \in B^{x,\alpha} \Rightarrow \max_{\theta \in \Theta \setminus \theta_1} \frac{\phi(y; x, \theta)}{\phi(y; x, \theta_1)} \leqq \alpha. \tag{4.16}$$

Assumption (UPI) may be used to prove complete social learning in a large class of societies, including those with a royal family. Let R be an arbitrary finite set of agents. Consider the following generalization of local independence: two individuals $i \notin R$ and $i' \notin R$ are called *quasi-locally independent* if $N(i) \cap N(i') \subset R$. For more than two agents the corresponding condition is that of *pairwise quasi-local independence*. For $\mu \in \mathscr{D}(\Theta)$, recall from Remark 4.1 above that $\hat{d} \in (0, 1)$ is such that if $\mu(\theta_1) \geqq \hat{d}$ then $G(\mu) \subset G(\delta_{\theta_1})$. Fix $K > 0$ and let $N_{K,\hat{d}}$ be a maximal group of pairwise quasi-locally independent agents having at most K neighbours each and whose prior beliefs satisfy $\mu_{i,1}(\theta_1) \geqq \hat{d}$. We now have:

Proposition 4.2. *Consider a connected society with* $|N_{K,\hat{d}}| = \infty$. *If each* $x \in G(\delta_{\theta_1})$ *satisfies condition* (UPI), *then social learning is complete.*

The proof is (roughly) along the lines of Theorem 4.1 and may be found in the Appendix. The main difference arises in the case where $|R| > 0$. The argument for this case proceeds by contradiction. Suppose social learning is incomplete: then there must exist a set which has positive probability, on which the negative information generated by the agents in R concerning the true state is bounded above by some number and yet learning is incomplete. We use a construction similar to the one used in Theorem 4.1 to establish that if there are an infinite number of quasi-locally independent agents then at least one of them will get sufficiently positive information in their first trial with an optimal action to offset this negative information. Thus at least one agent will try an optimal action forever on any sample path of this set. This observation taken along with the connectedness of society yields a contradiction and completes the proof.[31]

Assumption (UPI) allows us to demonstrate complete social learning in a number of societies not covered by Theorem 4.1. For instance, the case of a royal family is one where $R \subset N(i)$ for each $i \in N$; hence, the quasi-local independence of i' and i'' is equivalent to requiring $N(i') \cap N(i'') = R$. More generally, Proposition 4.2 will also apply if there are S "nearly-royal" families (see footnote 24).

In the results described so far, social learning relies on the set of locally independent agents who each try optimal actions with positive probability from the first period onwards. We now examine the possibilities of complete learning when agents do not necessarily start with prior beliefs that favour optimal actions. In this setting, the likelihood of social learning is sensitive to the nature of information generated by non-optimal actions across agents, both regarding the payoffs of these actions themselves as well as the payoffs of optimal actions.[32]

We provide two alternative sets of sufficient conditions on the informativeness of actions. One set of conditions applies when realizations from an action convey no payoff relevant information concerning any other action. The second set of conditions deal with the complementary situation when realizations on an action can reveal information about other actions. The conditions are used in Theorem 4.2.

To state the result we need to introduce additional concepts. First note that $x \in X$ induces an ordered partition of the states denoted by $\Theta_1(x) \prec_x \Theta_2(x) \prec_x \cdots \prec_x \Theta_{s(x)}(x)$ such that

(a) for each $k = 1, \ldots, s(x)$, the expected payoff $u(x, \delta_{\theta_k})$ is constant for all $\theta_k \in \Theta_k(x)$.

(b) if $\Theta_m(x) \prec_x \Theta_k(x)$ then $u(x, \delta_{\theta_m}) < u(x, \delta_{\theta_k})$ for $\theta_m \in \Theta_m(x)$ and $\theta_k \in \Theta_k(x)$.

For $x \in X$, let $k(x)$ denote the payoff equivalent set of states of nature which contains θ_1, i.e. $\theta_1 \in \Theta_{k(x)}(x)$. Also let $\Theta(x)^{++} \equiv \bigcup_{m > k(x)} \Theta_m(x)$, $\Theta(x)^{+} \equiv \bigcup_{m \geqq k(x)} \Theta_m(x)$ and $\Theta(x)^{-} \equiv \bigcup_{m < k(x)} \Theta_m(x)$. The first set of assumptions on informativeness of actions are given by condition (I) stated below:

(Ia) For $x, x' \in X$, where $x' \neq x$, if action x' is chosen and $y \in Y$ is observed, then for any $\mu \in \mathscr{D}(\Theta)$ the posterior belief $\mu(\Theta_m(x))' = \mu(\Theta_m(x))$ for each $m = 1, \ldots, s(x)$.

(Ib) There exists $x_1 \in G(\delta_{\theta_1})$ such that if x_1 is chosen and $y \in Y$ is observed, then $\phi(y; x_1, \theta)/\phi(y; x_1, \theta_1) = 1$ for all $\theta \in \Theta_{k(x_1)}(x_1)$.

(Ic) For x_1 as above, there exists a set $B^{x_1} \subset Y$ satisfying $\int_{B^{x_1}} \phi(y; x_1, \theta_1) d\Gamma(y) > 0$ and $\alpha \in (0, 1)$ such that

$$y \in B^{x_1} \Rightarrow \frac{\phi(y; x, \theta)}{\phi(y; x, \theta_1)} \leqq \alpha < 1, \tag{4.17}$$

for all $\theta \in \Theta(x_1)^{-}$.

Condition (I) can be best understood in terms of the canonical bandit model of Example 2.2. Condition (Ia) requires that there be no essential information flows across

31. To continue Example 4.1 further, suppose that when x_1 is chosen the outcome is distributed according to the normal, exponential, Poisson or geometric distributions. Then (UPI) is satisfied and the above result applies. It also holds more generally: for example if the density functions take one of the above forms and $x \in G(\delta_{\theta_1}) \Rightarrow u(x, \delta_{\theta_1}) > u(x, \delta_\theta)$ for all $\theta \neq \theta_1$ then complete learning obtains.

32. In this context it is also worth noting that Proposition 4.2 also holds if an infinite number of quasi-locally independent agents have priors that lead them to try sub-optimal actions provided that these actions satisfy the condition (UPI).

actions, i.e. actions are independent of each other. Condition (Ib) says that the action x_1 is incapable of distinguishing between states which are payoff equivalent for it: in the bandit model, payoff equivalent states for x_1 correspond to states where the quality types of actions other than x_1 vary. As actions are independent, x_1 will not be able to distinguish between these states. Condition (Ic) requires that x_1 should be capable of generating a minimum amount of negative information concerning payoff inferior states. In the bandit model if the conditional density functions $\{\phi(\cdot)\}$ have the standard monotone likelihood ratio property (MLRP), then (Ic) holds.

We now impose some restrictions on beliefs. Let $x_1 \in G(\delta_{\theta_1})$ be as above. By definition, it must be the case that $u(x_1, \delta_{\theta_1}) > \max_{x \in X \setminus G(\delta_{\theta_1})} u(x, \delta_{\theta_1})$. Hence we can find $\xi \in (0, 1)$ and $\varepsilon > 0$ such that

$$\xi u(x_1, \delta_{\theta_1}) + (1 - \xi) u(x_1, \delta_{\theta_L}) \geqq \max_{x \in X \setminus G(\delta_{\theta_1})} u(x, \delta_{\theta_1}) + \varepsilon \equiv u_{\min}, \tag{4.18}$$

where $\theta_L \in \Theta_1(x_1)$. Recall that $\Theta_{k(x_1)}(x_1)$ is the set of states payoff equivalent to state θ_1 for action x_1. Consider the collection of agents $i \in N$, who have at most K neighbours each and such that $\mu_{i,1}(\Theta_{k(x_1)}(x_1)) \geqq \xi$ for each i. Let $N_{K,\xi}$ be a maximal group of pairwise locally independent agents chosen from this collection. The restriction on the belief of an agent $i \in N_{K,\xi}$ ensures that i will choose x_1 at least once; however, it does not preclude suboptimal actions from being chosen at the outset.

We next consider the class of situations where actions potentially provide information on states which are payoff relevant for other actions. Recall that X_I is the set of fully informative actions. Assume that $X = X_I \cup \{x_u\}$ and let $x_1 \in G(\delta_{\theta_1})$ be given. The case where $x_u \in G(\delta_{\theta_1})$ is trivial and thus there is no loss of generality in assuming that $x_1 \in X_I$. We now state the alternative conditions on the informativeness of actions.

- (Ia*) For each $x \in X \setminus x_u$, there exists a set $B^x \subset Y$ satisfying $\int_{B^x} \phi(y; x, \theta_1) d\Gamma(y) > 0$ such that if action x is chosen and $y \in B^x$ is observed, then for any $\mu \in \mathscr{D}(\Theta)$ the posterior belief $\mu(\Theta(x_1)^+)' \geqq \mu(\Theta(x_1)^+)$.
- (Ib*) For each $x \in X \setminus x_u$ and for B^x as in (Ia*) above there exists $\alpha(x) \in (0, 1)$ such that $y \in B^x$ implies $\max_{\theta \in \Theta \setminus \theta_1} \phi(y; x, \theta)/\phi(y; x, \theta_1) \leqq \alpha(x)$.

Condition (Ia*) requires that for each informative action x, if $y \in B^x$ is observed then this does not yield negative information concerning payoffs of the optimal action x_1. Condition (Ib*) requires that all informative actions should be capable of generating a certain minimum amount of positive information concerning the true state. We note that condition (I*) is always satisfied when $|\Theta| = 2$.

As before, fix $\varepsilon > 0$ and $\xi^* \in (0, 1)$ such that $\xi^* u(x_1, \delta_{\theta_1}) + (1 - \xi^*) u(x_1, \delta_{\theta_L}) \geqq u(x_u) + \varepsilon$ where $\theta_L \in \Theta_1(x_1)$. Let N_{K,ξ^*} be a maximal collection of locally independent agents having at most K neighbours each and whose beliefs satisfy $\mu_{i,1} \geqq \xi^*$. We can now state the following theorem.

Theorem 4.2. *Consider a society which is connected.* (a) *Suppose that actions satisfy condition* (I). *Then there exists* $\bar{\lambda} \in (0, 1]$ *such that*

$$P^{\theta_1}\left(\bigcup_{i \in N} \{X^i(\omega) \not\subset G(\delta_{\theta_1})\}\right) \leqq (1 - \bar{\lambda})^{|N_{K,\xi}|}. \tag{4.19}$$

In particular if $|N_{K,\xi}| = \infty$ *then complete learning obtains.* (b) *The above conclusions continue to hold if condition* (I) *is replaced by condition* (I*) *and* $N_{K,\xi}$ *by* N_{K,ξ^*} *everywhere.*

We now summarize the intuition underlying Theorem 4.2.[33] The basic difference from the earlier results lies in the construction of the set A_i. In part (a) we show that for a sample path in A_i, agent i will observe a critical number of trials T with the optimal action x_1. By virtue of (Ic) this is sufficient to ensure that the agent will choose only the optimal action from some finite time onwards.

We briefly discuss how condition (I*) is used in part (b). As in Lemma 4.1, we can isolate a set of sample paths A_i, on which the amount of negative information obtained by agent $j \in N(i)$ concerning θ_1 is uniformly bounded above by a number σ. Recall that $\hat{d}$ is a number such that $\mu(\theta_1) \geqq \hat{d}$ implies $G(\mu) \subset G(\delta_{\theta_1})$. Let $\alpha = \max_{x \in X \backslash x_u} \alpha(x)$; since $\alpha < 1$, we can choose T to satisfy $\xi^*/(\xi^* + \alpha^T \sigma^{K|X|}(1-\xi^*)) \geqq \hat{d}$. Define A_j^σ as follows

$$A_j^\sigma = \prod_{x \in X \backslash x_u} \left\{ \prod_{t=1}^{T} B_{j,t}^x \times \left\{ \max_{\theta \in \Theta \backslash \theta_1} \sup_{t \geqq T+1} r_i^{x,\theta}(T+1, t) \leqq \sigma \right\} \right\}$$
$$\times \prod_{t=1}^{\infty} \prod_{j' \in N \backslash j} \prod_{x \in X} Y_{j',t}^x, \tag{4.20}$$

where $B_{j,t}^x = B^x$ for all $t \geqq 1$ and $r_i^{x,\theta}(T+1, t)$ is the product likelihood ratio from choosing x between periods $T+1$ and $t-1$. Let $A_i = \bigcap_{j \in N(i)} A_j^\sigma$; familiar arguments can be used to establish that $P^{\theta_1}(A_i) = \bar{\lambda} > 0$. Using condition (Ia*) we next show that along sample paths in A_i, the choices $C_{i,t} \neq x_u$, for all $t \leqq T$. This guarantees that agent i tries an informative action long enough and generates positive information that is sufficient to offset any subsequent negative information concerning state θ_1. The rest of the proof is standard.

The discussion so far has focused on the optimality of long run actions: we now summarize our findings on the distribution of limit beliefs. Recall that the beliefs of every agent converge almost surely (Theorem 3.1). An issue of importance is whether agents learn the truth, i.e. if limit beliefs place point mass on the true state. In general, even in cases where long run actions are optimal, there is no guarantee that beliefs will converge to the truth. This is because the support of the limiting beliefs distribution depends crucially on the informativeness of the optimal actions.[34] However, if an agent chooses optimal actions in the long run and these actions are fully informative about the true state then the agent will learn the truth.

5. TEMPORAL AND SPATIAL PATTERNS OF LEARNING

While the results of Sections 3 and 4 characterize the long run outcomes in our framework, they do not tell us much about the temporal and spatial evolution of social learning. In this section we discuss simulations of our framework to get some idea about these issues. In particular, we wish to compare the results of our simulations with the findings of the extensive empirical literature on diffusion as a means of validating our theoretical paradigm.

We assume the following social structure: The set of farmers N is arranged in a $k \times k$ grid, with each farmer owning a single plot of land. In our simulations we take $k = 20$, so that we have a total of 400 agents. Each farmer i observes the actions and payoffs (observations) of her surrounding 8 neighbours, in addition to her observations corresponding to

33. The proof of part (a) is given in the Appendix. The proof of part (b) is similar, and is omitted.
34. It is not difficult to construct instances of Example 2.2 (the canonical bandit model) where beliefs fail almost surely to place point mass on the truth despite long run actions being optimal.

her own actions. We perform simulations under different specifications, which are special cases of Example 2.2. We now summarize our findings.[35]

Termporal patterns In the first simulation, we assume that there are two crops, one of which (Crop 0) has known payoffs of 1/2, while the other (Crop 1) represents a new, unknown technology. Crop 1 can be of quality level $q_1 = 0{\cdot}45$ or $q_2 = 0{\cdot}55$; if the crop is of quality q_k for $k = 1, 2$ then its payoffs are Bernoulli-distributed with parameter q_k. We suppose that the true quality of Crop 1 is q_2, and so it is better than Crop 0. We also assume that the farmers' beliefs at the beginning of period 1 are heterogeneous, with about 1% of the farmers having a prior above 1/2 and therefore experimenting with the new crop.

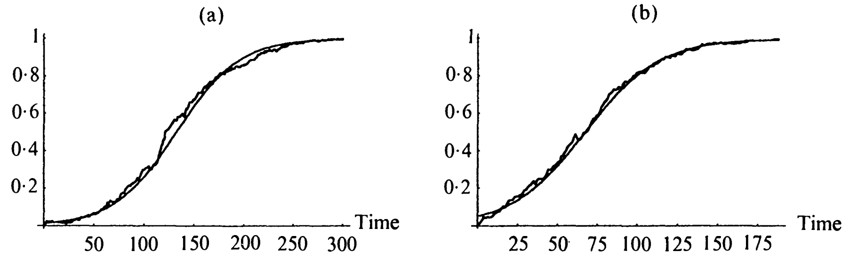

FIGURE 3
Diffusion curve

The diffusion curve of a typical simulation is given in Figure 3(a). As can be seen, the logistic curve fitted from the data matches the adoption curve quite well. The R^2 is $0{\cdot}987$, which is in the same range as obtained by Griliches (1957) in his study of the diffusion of hybrid corn. We also report a simulation where the new technology is more profitable than in the earlier case (we chose $q_1 = 0{\cdot}43$ and $q_2 = 0{\cdot}57$ as the quality levels). The adoption curve for this simulation is given in the Figure 3(b).

As can be seen, the logistic still provides a good fit ($R^2 = 0{\cdot}99$); however, the adoption rate is far higher, as it takes approximately half the time for the population to adopt compared to the earlier case. This is consistent with the result of Griliches, who found that the adoption rate was strongly positively linked to profitability. Finally, we also note that both adoption curves exhibit small downward fluctuations, an empirical phenomenon which has been discussed by Rogers (1983). As a check on the robustness of these patterns, we also ran simulations of a two crop model in which the returns of the new crop were normally distributed, with unknown means. Two typical simulations are plotted in Figures 4(a) and (b). The R^2 values are $0{\cdot}988$ and $0{\cdot}982$ respectively. These figures corroborate the findings that emerged from the Bernoulli case.[36]

Spatial patterns. We consider a simulation of the two crop model discussed above when $q_1 = 0{\cdot}45$ and $q_2 = 0{\cdot}55$ to obtain an idea of the spatial evolution of the process. Figure 5 depicts the results at different points in time.

35. In our simulations, the opposite edges of the rectangular grid are identified with each other to ensure that all farmers have 8 neighbours apart from themselves, including those living along an edge.

36. All four figures also reveal high positive serial correlation of the residuals from the logistic fit. Given the local learning structure of our model, this is intuitive, and suggests a (reduced-form) test of the hypothesis of neighbourhood learning.

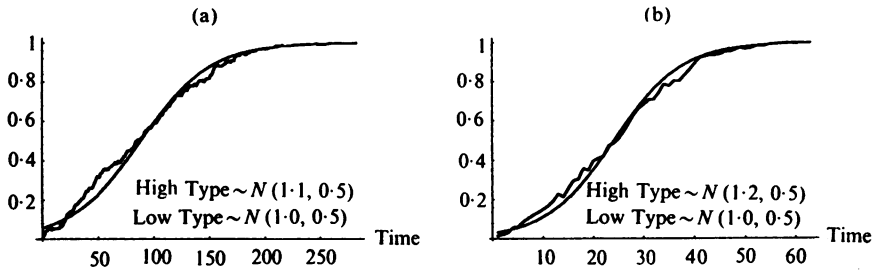

FIGURE 4
Diffusion curve

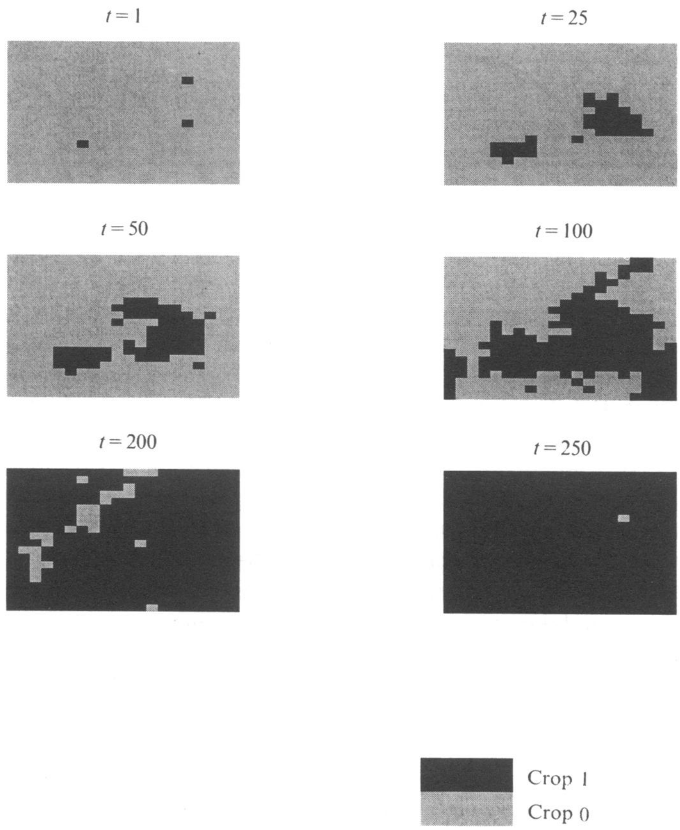

FIGURE 5
Spatial diffusion

Initially, there are only 3 farmers who experiment with the new crop. By $t = 25$, one farmer has dropped out due to bad experiences with the new crop. However, a group of agents around the other two farmers have chosen the new crop as well. By $t = 50$ the two clusters are almost in contact with each other, after which the adoption rate increases rapidly. (At $t = 50$, the proportion of adopters is about 0·15, while at $t = 100$ it has almost tripled to 0·41). By $t = 200$ adoption is nearly complete. We note that this pattern of spatial diffusion is consistent with empirical evidence (Hagerstrand (1969), Rogers (1983)).

6. CONCLUDING REMARKS

When payoffs from different actions are unknown, agents use their own past experience as well as the experience of their colleagues, friends and acquaintances as a guide for current decisions. We model these information flows across agents in terms of neighbourhoods of individual observation. Our analysis suggests that the structure of these neighbourhoods has important implications for the likelihood of adoption of new technologies, the coexistence of different practices, and for the temporal and spatial patterns of diffusion in a society. These findings raise an important question: what types of neighbourhood structures are likely to emerge in a society?

APPENDIX

We begin with a construction of the probability space, $(\Omega, \mathscr{F}, P^\theta)$. Fix $\theta \in \Theta$. For each $i \in N$, $x \in X$ and $t = 1, 2, \ldots$ let $Y^x_{i,t} \equiv Y$. For each $t = 1, 2, \ldots$ let $\Omega_t = \prod_{i\in N}\prod_{x\in X} Y^x_{i,t}$ be the space of the t-th outcomes across all agents and all actions. Ω_t is endowed with the product topology. Let $H_t \subset \Omega_t$ be of the form

$$H_t = \prod_{i\in N}\prod_{x\in X} H^x_{i,t}, \tag{A.1}$$

where $H^x_{i,t}$ is a Borel subset of $Y^x_{i,t}$ for each $i \in N$ and $x \in X$. (If the number of agents n is countably infinite, $H^x_{i,t} \equiv Y^x_{i,t}$ for all but a finite set of i's). Define the probability P^θ_t of the set H_t as

$$P^\theta_t(H_t) = \prod_{i\in N}\prod_{x\in X}\int_{H^x_{i,t}} \phi(y; x, \theta)d\Gamma(y). \tag{A.2}$$

P^θ_t extends uniquely to the σ-field on Ω_t generated by sets of the form H_t. Let $\Omega = \prod_{t=1}^\infty \Omega_t$. For cylinder sets $H \subset \Omega$ of the form

$$H = \prod_{t=1}^T H_t \times \prod_{t=T+1}^\infty \Omega_t, \tag{A.3}$$

let $P^\theta(H)$ be defined as $P^\theta(H) = \prod_{t=1}^T P^\theta_t(H_t)$. Let $\mathscr{F}$ be the σ-field on Ω generated by sets of the type given by (A.3). P^θ extends uniquely to the sets in $\mathscr{F}$. This completes the construction of the probability space $(\Omega, \mathscr{F}, P^\theta)$.

Let Θ be endowed with the discrete topology, and suppose $\mathscr{B}$ is the Borel σ-field on this space. For rectangles of the form $A \times H$ where $A \subset \Theta$ and H is a measurable subset of Ω, let $P_i(A \times H)$ be given by

$$P_i(A \times H) = \sum_{\theta\in A} \mu_{i,1}(\theta)P^\theta(H), \tag{A.4}$$

for each agent $i \in N$. Each P_i extends uniquely to all of $\mathscr{B} \times \mathscr{F}$. Since every agent's prior belief lies in the interior of $\mathscr{D}(\Theta)$, the measures $\{P_i\}$ are pairwise mutually absolutely continuous.

Proof of Theorem 3.1

For each $\theta \in \Theta$, the belief $\mu_{i,t}(\theta)$ of agent i at the beginning of time t can be regarded as a version of the conditional expectation $E[\mathbf{1}_{\{\theta\}\times\Omega} \| \mathscr{F}_{i,t}]$ where the expectation is with respect to the measure P_i. Since this sequence of random variables is a uniformly bounded martingale (see Easley and Kiefer (1988)) with respect to the increasing sequence of σ-fields $\{\mathscr{F}_{i,t}\}$ the Martingale Convergence Theorem applies, so that $\mu_{i,t}$ converges almost surely to the $\mathscr{F}_{i,\infty}$-measurable limit belief $\mu_{i,\infty}$. Let Q_i be the set of sample paths on which agent i's beliefs converge, where $P_i(Q_i) = 1$. Since the measures are pairwise mutually absolutely continuous and the set of agents N is at most countable, the set $Q = \bigcap_{i\in N} Q_i$ also has P_i measure 1 for each i. ||

Proof of Lemma 3.1

Let $x \in X$. Since $x' \in X^i(\omega)$ there exists a subsequence $\{t_k\}$ such that $u(x', \mu_{i,t_k}(\omega)) \geqq u(x, \mu_{i,t_k}(\omega))$. Taking limits and using the continuity of u on the set $\mathscr{D}(\Theta)$, we get $u(x', \mu_{i,\infty}(\omega)) \geqq u(x, \mu_{i,\infty}(\omega))$. Since x is arbitrary, this proves statement (a). Statement (b) follows from the maximum theorem and part (a). ||

Let supp (μ) denote the support of a probability distribution μ. We have:

Lemma 3.2. *Suppose $i \in N(j)$ and $\omega \in Q^{\theta_1}$. If, for some $\theta \neq \theta_1$, $\theta \in \text{supp}\,(\mu_{j,\infty}(\omega))$ then $u(x, \delta_\theta) = u(x, \delta_{\theta_1})$ for all $x \in X^i(\omega) \cup X^j(\omega)$.*

Proof. Suppose the conditions of Lemma 3.2 hold but $u(x, \delta_{\theta_1}) \neq u(x, \delta_\theta)$ for some $x \in X^i(\omega) \cup X^j(\omega)$. Then, by definition, we have

$$\int_Y |\phi(y; x, \theta_1) - \phi(y; x, \theta)| d\Gamma(y) > 0. \tag{B.1}$$

Since x is chosen infinitely often either by agent i or by j (or both), and agent j observes agent i, the law of large numbers ensures that $\mu_{j,\infty}(\theta)(\omega) = 0$, so that θ is not in the support of $\mu_{j,\infty}(\omega)$. This contradiction establishes the result. ||

Remark 1. Since $i \in N(i)$ for every $i \in N$, the above lemma implies that for every $x \in X^i(\omega)$, $u(x, \cdot)$ is constant on the set

$$\{\mu | \operatorname{supp}(\mu) \subset \operatorname{supp}(\mu_{i,\infty}(\omega))\}.$$

In particular, $U_{i,\infty}(\omega) \equiv u(x, \mu_{i,\infty}(\omega)) = u(x, \delta_{\theta_1})$ for each $x \in X^i(\omega)$.

Lemma 3.3. *Suppose* $\omega \in Q^{\theta_1}$. *If* $i \in N(j)$, *then* $U_{j,\infty}(\omega) \geqq U_{i,\infty}(\omega)$.

Proof. We shall show that if $x' \in X^j(\omega)$, then $u(x', \delta_{\theta_1}) \geqq u(x, \delta_{\theta_1})$, for all $x \in X^i(\omega)$. This will suffice for the proof since from Lemma 3.2 and Remark 1 we have

$$U_{j,\infty}(\omega) \equiv u(x', \mu_{j,\infty}(\omega)) = u(x', \delta_{\theta_1}) = u(x', \delta_\theta) \text{ for all } \theta \in \operatorname{supp}(\mu_{j,\infty}), \tag{B.2}$$

and

$$U_{i,\infty}(\omega) \equiv u(x, \mu_{i,\infty}(\omega)) = u(x, \delta_{\theta_1}) = u(x, \delta_\theta) \text{ for all } \theta \in \operatorname{supp}(\mu_{j,\infty}). \tag{B.3}$$

There are two cases: if $\mu_{j,\infty}(\omega) = \delta_{\theta_1}$ the result follows trivially from Lemma 3.1. In the second case, suppose that $\theta \neq \theta_1$ also lies in the support of $\mu_{j,\infty}(\omega)$. We now proceed by contradiction. Assume that $u(x', \delta_{\theta_1}) < u(x, \delta_{\theta_1})$. Since $\theta \neq \theta_1$ lies in the support of $\mu_{j,\infty}(\omega)$, Lemma 3.2 above together with the facts that $x' \in X^j(\omega)$ and $x \in X^i(\omega)$ implies that $u(x', \mu_{j,\infty}(\omega)) < u(x, \mu_{j,\infty}(\omega))$. However this contradicts Lemma 3.1 above and hence $u(x', \delta_{\theta_1}) \geqq u(x, \delta_{\theta_1})$. ||

Proof of Theorem 3.2 If i and j are two agents in N, then either $i \in N(j)$ or there exist agents $j_1, \ldots, j_m$ such that $j_1 \in N(j)$, $j_2 \in N(j_1)$ and so on until $i \in N(j_m)$. In the first case, Lemma 3.3 applies directly to show that $U_{j,\infty}(\omega) \geqq U_{i,\infty}(\omega)$ while in the latter case the same is true by transitivity. The result follows by interchanging the roles of i and j. ||

Let $i \in N$. If agent i were to choose $x \in X$ between period t and $t' - 1$ and observe the corresponding outcomes $\{y^x_{i,n}\}_{n=t}^{t'-1}$, the product likelihood ratio of state θ with respect to θ_1 at the beginning of time t' would be

$$r_i^{x,\theta}(t, t') = \prod_{n=t}^{t'-1} \frac{\phi(y^x_{i,n}; x, \theta)}{\phi(y^x_{i,n}; x, \theta_1)}. \tag{C.1}$$

By convention we assume that $r_i^{x,\theta}(t, t') = 1$ if $t = t'$. Moreover, if $t = 1$ we write $r_i^{x,\theta}(1, t')$ simply as $r_{i,t'}^{x,\theta}$.

Proof of Proposition 4.2 (Sketch)

Let $j \in N$. For $\alpha \in (0, 1)$ and $x \in G(\delta_{\theta_1})$ let $B_{j,1}^{x,\alpha}$ be the set $B^{x,\alpha}$ whose existence is assumed in condition (UPI). Using arguments analogous to Lemma 4.1, we can establish that there exists a $\sigma \geqq 1$, an $\alpha \in (0, 1)$ such that $\alpha\sigma^{K|X|} < 1$, a set A_j^σ defined as

$$A_j^\sigma = \prod_{x \in G(\delta_{\theta_1})} B_{j,1}^{x,\alpha} \times \left\{ \max_{x \in G(\delta_{\theta_1})} \sup_{\tau \geqq 2} r_j^{x,\theta}(2, \tau) \leqq \sigma \right\}$$

$$\times \left\{ \max_{x \in X \backslash G(\delta_{\theta_1})} \sup_{t \geqq 1} r_{j,t}^{x,\theta} \leqq \sigma \right\} \times \prod_{t=1}^{\infty} \prod_{j' \in N \backslash j} \prod_{x \in X} Y_{j',t}^x, \tag{C.2}$$

and $\bar{\delta} > 0$ such that $P^{\theta_1}(A_j^\sigma) = \bar{\delta} > 0$ (by using the assumption that each $x \in G(\delta_{\theta_1})$ satisfies the (UPI) property). Fix $i \in N_{K,\hat{a}}$. Define $A_i = \bigcap_{j \in N(i)} A_j^\sigma$. Clearly $P^{\theta_1}(A_i) \geqq \bar{\delta}^K > 0$. Note that since agent i is assumed to have a belief

$\mu_{i,1}(\theta_1)\geqq\hat{d}$, she will choose an action $x\in G(\delta_{\theta_1})$; by construction of the set A_i, she will observe an outcome $y\in B^{x,\alpha}$. For $\omega\in A_i$ we have

$$\mu_{i,t}(\theta_1)(\omega)\geqq\frac{\mu_{i,1}(\theta_1)(\omega)}{\mu_{i,1}(\theta_1)(\omega)+\sum_{\theta\neq\theta_1}\alpha\sigma^{K|X|}\mu_{i,1}(\theta)(\omega)}. \tag{C.3}$$

Since $\alpha\sigma^{K|X|}<1$ by construction, for all $t\geqq 1$ we have $\mu_{i,t}(\theta_1)\geqq\mu_{i,1}(\theta_1)\geqq\hat{d}$, so that $X^i(\omega)\subset G(\delta_{\theta_1})$. The proof for the case of $|R|=0$ now follows along the lines of Lemma 4.2 and Theorem 4.1, and is omitted.

The case $|R|>0$. Let $\hat{Q}=\bigcup_{i\in N}\{X^i(\omega)\not\subset G(\delta_{\theta_1})\}$. We shall assume $P^{\theta_1}(\hat{Q})>0$ initially. Clearly, there exists $\sigma\geqq 1$ (without loss of generality having the same value as above) such that $P^{\theta_1}(\hat{Q}\cap A_R^\sigma)>0$, where A_R^σ is the set

$$A_R^\sigma=\bigcap_{j\in R}\left\{\max_{\theta\in\Theta\backslash\theta_1}\max_{x\in X}\sup_{t\geqq 1} r_{j,t}^{x,\theta}\leqq\sigma\right\}\times\prod_{t=1}^{\infty}\prod_{j\in N\backslash R}\prod_{x\in X}Y_{j,t}^x. \tag{C.4}$$

For $i\in N_{K,\hat{d}}$ consider the set A_i constructed as above, but excluding all $j\in N(i)$ who are members of R. The probability of A_i conditional on A_R^σ satisfies

$$P^{\theta_1}(A_i|A_R^\sigma)=\frac{\prod_{j\in N(i)\backslash R}P^{\theta_1}(A_j^\sigma)\times P^{\theta_1}(A_R^\sigma)}{P^{\theta_1}(A_R^\sigma)}\geqq\bar{\delta}^K>0. \tag{C.5}$$

Using (C.5) we can establish the analogue of the argument used in Theorem 4.1, i.e.

$$P^{\theta_1}\left(\bigcap_{i\in N_{K,\hat{d}}}A_i^c\,|\,A_R^\sigma\right)\leqq\lim_{|N_{K,\hat{d}}|\to\infty}(1-\bar{\delta}^K)^{|N_{K,\hat{d}}|}=0. \tag{C.6}$$

Note that for $\omega\in A_i\cap A_R^\sigma$, as $\mu_{i,1}\geqq\hat{d}$, our construction ensures that $C_{i,t}\in G(\mu_{i,t})\subset G(\delta_{\theta_1})$ for all $t\geqq 1$. Thus on the set $A_i\cap A_R^\sigma$, agent i will always choose an action in $G(\delta_{\theta_1})$. As $i\in N_{K,\hat{d}}$ is arbitrary, we get $(\bigcup_{i\in N_{K,\hat{d}}}A_i)\cap A_R^\sigma\subset\bigcap_{j\in N}\{X^j(\omega)\subset G(\delta_{\theta_1})\}$, using the argument of Lemma 4.2. However, using (C.6) this implies $P^{\theta_1}(\hat{Q}\cap A_R^\sigma)=P^{\theta_1}(\bigcup_{i\in N}\{X^i(\omega)\not\subset G(\delta_{\theta_1})\}\cap A_R^\sigma)=0$, which contradicts our earlier supposition that $P^{\theta_1}(\hat{Q}\cap A_R^\sigma)>0$. The result follows. ‖

Proof of Theorem 4.2

We suppose for simplicity that $G(\delta_{\theta_1})$ is a singleton. The steps presented below extend easily to cover the case where there are multiple optimal actions. We first establish the following lemma:

Lemma 4.3. *Suppose* (Ia)–(Ic) *hold. Let* $\mu\in\mathscr{D}(\Theta)$ *satisfy* $\mu(\Theta_{k(x_1)}(x_1))\geqq\xi$. (a) *If action* x_1 *is chosen* t *times, and outcomes* $y_1\in B^{x_1},\ldots,y_t\in B^{x_1}$ *are observed, then the posterior belief* $\mu(\Theta(x_1)^+)'\geqq\xi$. (b) *The conclusion in* (a) *is unaffected if an action* $x\in X\backslash x_1$ *has also been chosen and* $y\in Y$ *is observed.*

The proof exploits condition (Ib) and involves some straightforward calculations. We omit it due to space constraints. Lemma 4.3 is useful since if $\mu\in\mathscr{D}(\Theta)$ satisfies $\mu(\Theta(x_1)^+)\geqq\xi$ then $u(x_1,\mu)\geqq u_{\min}$.

Proof. (Theorem 4.2). Let $j\in N$. Arguments analogous to those used in Lemma 4.1 establish that there exists a real number $\sigma\geqq 1$ and $\bar{\delta}>0$ such that

$$P^{\theta_1}\left(\sup_{t'>t}\max_{\theta\in\Theta(x_1)^-}r_j^{x_1,\theta}(t,t')\leqq\sigma\right)=\bar{\delta}>0. \tag{C.7}$$

Choose T to satisfy $\alpha^T\sigma^K<1$, where $\alpha\in(0,1)$ is the number assumed in condition (Ic). Let A_j^σ be defined as

$$\begin{aligned}A_j^\sigma=\prod_{t=1}^{T}B_{j,t}^{x_1}\times&\left\{\sup_{t'>T}\max_{\theta\in\Theta(x_1)^-}r_j^{x_1,\theta}(T+1,t')\leqq\sigma\right\}\\&\times\prod_{x\in X\backslash x_1}\prod_{t=1}^{\infty}Y_{j,t}^x\times\prod_{j'\in N\backslash j}\prod_{x\in X}\prod_{t=1}^{\infty}Y_{j',t}^x,\end{aligned} \tag{C.8}$$

where we have written B^{x_1} as $B_{j,t}^{x_1}$ to avoid confusion. Fix $i\in N_{K,\xi}$. Let $A_i=\bigcap_{j\in N(i)}A_j^\sigma$. By construction $P^{\theta_1}(A_i)=\bar{\delta}^{|N(i)|}\geqq\bar{\delta}^K>0$.

We claim that if $\omega \in A_i$ then agent i will choose the optimal action x_1, for all time periods after some finite point. The first step is to show that agent i will observe at least T trials of action x_1. We begin by showing it is tried at least once by some agent $j \in N(i)$. The proof is by contradiction. Suppose not. This implies, in particular, agent i observes infinitely many trials of some action $x \in X \backslash x_1$. Since x is suboptimal, the strong law of large numbers will ensure that $\lim_{t\to\infty} \mu_{i,t}(\theta)=0$ for all states θ where $u(x, \delta_\theta) > u(x, \delta_{\theta_1})$. Choose $\bar{\varepsilon}>0$ such that $u_{\min}-\bar{\varepsilon} > \max_{x\in X\backslash x_1} u(x, \delta_{\theta_1})$. The above argument implies that at a finite time t', agent i's expected utility $u(x, \mu_{i,t'}) \leqq u_{\min}-\bar{\varepsilon}$. Since x_1 has not been chosen and the choice of other actions does not affect i's beliefs concerning $\Theta_{k(x_1)}(x_1)$, we have $\mu_{i,t'}(\Theta_{k(x_1)}(x_1)) \geqq \xi$. By the observation following Lemma 4.3 this implies $u(x_1, \mu_{i,t'}) \geqq u_{\min}$, which implies that x_1 would be preferable to x at the time of the next choice of x by agent i. Thus action x_1 must be tried by agent i at some time $\hat{t}$, and this contradicts our original supposition.

We now make the following observation. Suppose that at time t each agent $j \in N(i)$ has chosen action x_1 for $0 \leqq t_j \leqq T$ periods. Hence up to time t, for each $j\in N(i)$ agent i observes the outcomes $y^{x_1}_{j,1} \in B^{x_1}_{j,1}, \ldots, y^{x_1}_{j,t_j} \in B^{x_1}_{j,t_j}$ for $t=1,\ldots,t_j$. It follows from Lemma 4.3 that agent i's posterior belief $\mu_{i,t}(\Theta(x_1)^+) \geqq \xi$. Note by Lemma 4.3(b) that the possibility that agents $j\in N(i)$ may have also chosen actions in $X\backslash x_1$ does not alter the conclusion. The same argument can be repeated in conjunction with this observation to show that agent i must observe at least T choices of action x_1 by agents $j\in N(i)$.

Let $t(T)$ be the time when agent i has observed a trial of x_1 for the T-th time. Let $\hat{\mu}_{i,t'} \in \mathscr{D}(\Theta)$ be agent i's belief after incorporating all information about actions $x\in X\backslash x_1$ up to time $t' \geqq t(T)$. We get

$$\mu_{i,t'}(\Theta(x_1)^-) = \frac{\sum_{\theta\in\Theta(x_1)^-} \hat{\mu}_{i,t'}(\theta) \prod_{j\in N(i)} r_j^{x_1,\theta}(1,t_j)}{\hat{\mu}_{i,t'}(\Theta_{k(x_1)}(x_1)) + \sum_{\theta\in\Theta(x_1)^{++}} \hat{\mu}_{i,t'}(\theta) \prod_{j\in N(i)} r_j^{x_1,\theta}(1,t_j) + \sum_{\theta\in\Theta(x_1)^-} \hat{\mu}_{i,t'}(\theta) \prod_{j\in N(i)} r_j^{x_1,\theta}(1,t_j)}. \tag{C.9}$$

Since $\omega\in A_i$ by assumption we have $\prod_{j\in N(i)} r_j^{x_1,\theta}(1,t_j) \leqq \alpha^T\sigma^K < 1$ for all $\theta\in\Theta(x_1)^-$. This is because, by construction of the set A_i, for the first T observation of x_1 by agent i, the product likelihood ratio $r^{x_1,\theta}$ for any $\theta\in\Theta(x_1)^-$ is at most α^T, and in all subsequent trials for each agent $j\in N(i)$ the product likelihood ratio is at most σ. However, by (Ia) we have $\hat{\mu}_{i,t'}(\Theta_{k(x_1)}(x_1)) = \mu_{i,1}(\Theta_{k(x_1)}(x_1)) \geqq \xi$ and $\hat{\mu}_{i,t'}(\Theta(x_1)^-) \leqq 1-\xi$. Thus $\sum_{\theta\in\Theta(x_1)^-} \mu_{i,t'}(\theta) \leqq \alpha^T\sigma^K(1-\xi) < (1-\xi)$. It follows from (C.9) that $\mu_{i,t'}(\Theta(x_1)^-) < (1-\xi)/(\xi+(1-\xi)) = 1-\xi$. Thus $\mu_{i,t'}(\Theta(x_1)^+) \geqq \xi$ and hence $u(x_1, \mu_{i,t'}) \geqq u_{\min}$. As t' is arbitrary, this means that agent i's belief on ω will henceforth never fall below $u_{\min}$. As all suboptimal actions will fall below $u_{\min}-\bar{\varepsilon}$ in finite time, agent i must choose action x_1 from some finite time onwards. The rest of the proof now proceeds as in Lemma 4.2 and Theorem 4.1. ||

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