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Structural holes in social networks

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Abstract

We consider a setting where every pair of players that undertake a transaction (e.g. exchange goods or information) creates a unit surplus. A transaction can take place only if the players involved have a connection. If the connection is direct the two players split the surplus equally, while if it is indirect then intermediate players also get an equal share of the surplus. Thus, individuals form links with others to create surplus, to gain intermediation rents, and to circumvent others who are trying to become intermediary.

Our analysis clarifies the interplay between these forces in the process of strategic network formation. First, we show that, in the absence of capacity constraints on links, it leads to the emergence of a star network where a single agent acts as an intermediary for all transactions and enjoys significantly higher payoffs. Second, we study the implications of capacity constraints in the ability of agents to form links. In this case, distances between players must be long, which induces players who are "far off" to connect in order to avoid paying large intermediation rents. A cycle network then emerges, payoffs being equal across all players.

JEL classification: C72; C78; D85

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1. Introduction

It is now widely agreed that knowledge of the structure of interaction among individuals is important for a proper understanding of a number of important questions in economics, such

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as the spread of new ideas and technologies, the patterns of employment and wage inequality, competitive strategies in dynamic markets, and career profiles of managers. [1]

Connections facilitate timely access to important information—on trade opportunities, job vacancies, project deadlines, and novel ideas for research. In some important instances—e.g., trade opportunities—the payoffs an individual entity gets in a network will clearly depend on his relative importance in bridging gaps in the network between others. The potential benefits from bridging different parts of a network were important in the early work of Granovetter [23] and are central to the notion of *structural holes* developed by Burt [7]. [2] In recent years, a number of empirical studies have also shown that individuals or organizations who bridge "structural holes" in networks gain significant payoff advantages. [3] For instance, the work on promotions and performance evaluation argues that the differences in structural location of individuals—in particular whether they bridge structural holes in the social network—explain a significant part of the variation in promotion timing of otherwise similar people. Given these significant payoffs effects, it seems natural that an individual will make investments in connections so as to become structurally important, while other individuals will likewise form connections to circumvent such attempts. Are special structural positions and the corresponding large payoff differences sustainable when individual entities form connections strategically?

We develop a simple model of network formation to address this question. We consider a setting where interaction between every pair of individuals generates a surplus. If two individuals are directly linked then they split this surplus equally, while if they are indirectly connected—there are other players in the "path" between them—then the division of surplus depends on the competition between these intermediaries. In this setting, there are three types of incentives for individuals to form links with others. The first incentive is the desire to create surpluses: individuals would like to join the network so as to create exchange possibilities which in turn create surpluses. The second incentive is related to the rewards from intermediation: players would like to place themselves between others in order to extract rents from intermediation. The third incentive arises out of the desire to avoid sharing surpluses with intermediaries; in other words, individuals will try to circumvent intermediate players to retain more of the surplus for themselves.

Our analysis brings out the interplay between these incentives pressures. The first pressure rules out partially connected networks: equilibrium networks are either connected or empty. The second incentive pressure pushes towards a star structure, while the third pressure pushes toward a cycle in which no one earns any intermediation rents. In our benchmark model the main result is that the second effect dominates and the star emerges as the unique (non-empty) equilibrium network. This result is obtained in a context where *all bilateral deviations are allowed* and there are *no capacity constraints*.

[1] We mention a small sample of this work. See Bala and Goyal [3], Coleman et al. [11], and Ellison and Fudenberg [15] for the role of communication networks in technological diffusion; Rees [36], Granovetter [23], Montgomery [32], and Calvo and Jackson [10] for effects of social networks on employment and wage inequality; Ahuja [1], Goyal and Moraga [21], and Gulati et al. [25] on the role of strategic partnering in generating competitive advantage in markets; Burt [7] for the effects of organizational networks on individual promotions and salaries; and Rauch [35] on the role of social and economic networks in international trade.

[2] See also Burt [8] for an exploration of the influence of individual position in social networks in shaping the generation of creative ideas.

[3] See Burt [7] and Mehra et al. [31] for influence of structural positions on promotions and performance evaluation, Podolny and Baron [34] for work on network positions and mobility, and Ahuja [1] for the influence of a firm's position in interorganizational networks on its innovativeness and overall performance.

We next examine the effects of requiring that bilateral deviations be subject to an internal consistency requirement: any two players who agree on a deviation should have an individual incentive to implement their part of the deal. Now we find that the set of equilibrium architectures expand: in fact both the star as well as the cycle can be sustained in equilibrium. This is because the deviations which destroyed the cycle earlier are shown to violate the individual compatibility requirement. This multiplicity leads us to examine the relative dynamic stability of the two networks, and the main result of this part of the paper is that the star is the unique stochastically stable network.

We turn next to the issue of capacity constraints. If these constraints are stringent, then the star is no longer feasible. In this case, the key insight is that, in any feasible network, distances between players will be long. This creates incentives for players to form links to avoid paying large intermediation rents, which in turn leads to the emergence of the cycle—a network in which payoffs across players are equal. However, the cycle is itself not robust to arbitrary bilateral deviations, as was noted in the discussion above. Thus, we show that the static model of network formation has no (non-empty) equilibrium network.

Given this state of affairs, we finally turn to an examination of a setup where, under capacity constraints, bilateral deviations are required to be internally consistent. Under these conditions, we show that existence of equilibrium is restored and the unique (non-empty) equilibrium architecture is the cycle. This shows that the third incentive pressure we mentioned above then dominates the process.

We conclude this introductory section with a review of related literature.

1.1. Relation to the literature

Our paper is a contribution to the theory of network formation. In recent years, this has been an active area of research, see e.g., Aumann and Myerson [2] and Boorman [6], Bala and Goyal [3], Dutta et al. [14], Jackson and Wolinsky [28], and Kranton and Minehart [30]. In our model, individuals form links with each other which involve a trade-off between the benefits of accessing others and the costs involved in forming links. In particular, an important idea here is that when a player i forms a link with another player j, she also gains access to players whom player j is accessing via her own links. In previous work it has been assumed that the benefits that i and j enjoy from the connections of j are non-rival. By contrast, we study a setting in which the benefits are rival. In such a context, a key issue that arises is how the benefits are shared between different players. In other words, we need a theory of how intermediation rents are determined via the structure of the network.

The allocation of intermediation rents in turn requires a model of competition between intermediaries and a related contribution of our paper is a method to assign these benefits as a function of the network structure. We introduce the idea of *essential* players—players without whom an interaction cannot take place—to determine which intermediaries will earn rents. The incorporation of intermediary rents and payments has powerful implications for equilibrium networks which are quite different from earlier work. To see this clearly, it is useful to relate our work to the results that are obtained in the non-rival benefits version of our model, the well-known connections model due to Jackson and Wolinsky [28]. They show that a star can only be sustained over a *relatively small range* of parameter values, and that the central player who is bridging the structural holes actually earns a *lower payoff* than the peripheral players (see Proposition 2 in their paper). By contrast, in the present model, the star is sustainable for practically the *entire range of parameters* and the central player earns a much *higher payoff* as compared to the peripheral players.

We next clarify the relation between our paper and some other recent papers who also explore the emergence of star networks. Bala and Goyal [3], Feri [16], Galeotti et al. [18], and Hojman and Szeidl [26] study a model of network formation in which the benefits are *non-rival* and link formation is *unilateral*. It is precisely this second feature that, taken together with the presence of decay, allows for the establishment of "periphery-sponsored" stars in these models: all links are initiated and paid for by peripheral players. In contrast, we consider a model of link formation in which both players have to agree on the link, as in the aforementioned model of Jackson and Wolinsky [28]. Arguably, two sided link formation is more natural in economic applications. However, a *characterization* of equilibrium in such models has proved elusive.

Two other recent papers on network formation which also discuss stars and middlemen are Gilles et al. [20] and Galeotti and Meléndez [19]. Gilles et al. [20] derive sufficient conditions on payoffs which ensure that efficient networks are sustained in a strong equilibrium. This sufficient condition is interpreted by the authors in terms of a role for intermediary players. Galeotti and Meléndez [19] also study the formation of star networks; however, in their model players are involved in an infinitely repeated prisoner's dilemma that determines how linking costs are shared. [4]

The ideas of access advantages and strategic positioning are important building blocks for the notion of structural holes, introduced by Burt [7], and have been an important part of the tradition in sociology since the work of Granovetter [23]. As we mentioned earlier, empirical research by sociologists shows that differences in structural location of otherwise similar individuals—in particular whether they bridge structural holes in the social network—explain a significant part of this variance. Our paper shows that the presence of structural holes and corresponding payoff differences is consistent with a model of rational players who strategically seek to create positional advantages for themselves and also have an incentive in preventing others from becoming central. To the best of our knowledge our paper is the first to provide a formal model which directly addresses these concerns. [5]

The paper is organized into five sections. Section 2 lays out the basic model. Section 3 presents the main results on equilibrium and efficient networks. Section 4 discusses the extensions of the basic model, while Section 5 concludes. All the proofs are given in an Appendix at the end of the paper.

2. The model

We consider a population composed of finite set of ex ante identical agents, $N = \{1, 2, \ldots, n\}$ where $n \geqslant 3$. These agents play a network-formation game where every one of them makes a simultaneous announcement of *intended* links. An intended link $s_{i,j} \in \{0, 1\}$, where $s_{ij} \equiv s_{ij} = 1$ means that player i intends to form a link with player j, while $s_{ij} = 0$ means that player i does

[4] After we finished writing the paper, we became aware of a paper by Buskens and van de Rijt [9] which also studies structural holes using network-formation games. Their payoff function is different, being based on Burt's [7] "constraint" measure. They find that stable networks are complete and balanced *bipartite networks*—in particular, star networks are not stable.

[5] The modern theory of complex networks also explores a variety of different network-formation models where some nodes display significantly higher connectivity than others (see e.g., [40] for a survey). A prominent example is the dynamic setup proposed by Barabasi and Albert [4] where, at every point in time, a new node arrives and forms new links with existing nodes. The probability that these links connect to any given existing node is taken to be increasing in the number of links the node has; this is referred to as *preferential attachment*. This literature, however, does not provide us with any reasons for why a highly linked older player and a newly arriving player should want to connect with each other. We show that large intermediation rents for the older player and the desire of the new player to minimize the number of intermediaries (and thereby reduce payments) jointly provide a simple explanation for preferential attachment.

not intend to form such a link. Thus, a strategy of player i is given by $s_i = \left[s_{ij}\right]_{j \in N \setminus \{i\}}$, with S_i denoting the strategy set of player i.

A link between two players i and j is formed if and only if $s_{ij} = s_{ji} = 1$. We denote the formed (undirected) link by $g_{ij} \equiv g_{ji} = 1$ and the absence of a link by $g_{ij} \equiv g_{ji} = 0$. Any given strategy profile $s = (s_1, s_2, \ldots, s_n)$ therefore induces a network $g(s)$. The *network* $g(s) = \{(g_{ij})\}_{i,j \in N}$ is a formal description of the pairwise links that exist between the players. There exists a *path* between i and j in a network g if either $g_{ij} = 1$ or if there is a distinct set of players $\{i_1, \ldots, i_n\}$ such that $g_{i,i_1} = g_{i_1,i_2} = g_{i_2,i_3} = \cdots = g_{i_n,j} = 1$. All players with whom i has a path defines the component of i in g, which is denoted by $C_i(g)$.

Suppose that players are traders who can exchange goods and that this exchange creates a surplus of 1. We assume that exchange can be carried out only if these traders know each other personally or there is a sequence of personal connections, i.e., there is a path which indirectly links the two traders. [6] The central issue here is how are potential surpluses allocated between the different parties to the trade. In the case where traders know each other, we assume that they each get one-half of the surplus. If they are linked indirectly then the allocation of the surplus depends on the competition between the intermediary agents. One way to proceed is to think of each "path" of intermediaries between these two players as providing a service: the service of intermediation. Thus, the allocation of surpluses arises out of player demands, counter-demands, etc.

There are many ways in which this demand game can be modelled. Here, we shall use the idea of essential players in constructing the payoffs: a player i is said to be *essential* for j and k if i lies on *every* path that joins j and k in the network. Denote by $E(j, k; g)$ the set of players who are essential to connect j and k in network g and let $e(j, k; g) = |E(j, k; g)|$. [7] We will assume that *non-essential players between j and k get a zero share of the surplus*, *while the essential players and j and k divide the surplus equally*.

We now discuss a simple non-cooperative game that supports this division rule. Fix a network g, and consider the pairs of players who have a path in this network. Let $I^i_{j,k}(g)$ be the indicator function that takes value 1 if and only if player i lies on a path between j and k in g or if $i = j$ or k. Every player i sets a demand $x^i_{j,k}$ for every transaction $\{j, k\}$ such that $I^i_{j,k} = 1$. If demands for a transaction $\{j, k\}$ in network g are feasible, i.e., $\sum_{i: I^i_{j,k}=1} x^i_{j,k} \leqslant 1$, then they are implemented. If the demands add up to more than 1, then the transaction is cancelled and all players get a zero payoff from that transaction. In this demand game, the above allocation rule with essential players getting equal surpluses and non-essential players getting no surpluses can be supported as an equilibrium as follows: for a transaction $\{j, k\}$ every essential player to that transaction and players j and k demand $1/(e(j, k : g) + 2)$, while all non-essential players demand 0. [8]

[6] One possible interpretation of a path is that it reflects trust between members of a group; see Dixit [13] for a model of trade in which enforcement problems lead players to trade with those near by.

[7] For example, the number of essential players between j and k is zero if the players have a direct link, or if players are located around a circle—in the latter case, for every pair of players j and k and every other player i, there is always a path joining j and k that does *not* include i. On the other hand, note that in a star every pair of peripheral players has a single and common essential player, namely the center of the star.

[8] However, this non-cooperative demand game also has other equilibria—e.g., consider the case in which every player asks for the entire surplus. This has led us to investigate the cooperative foundations for the surplus allocation rule. In this approach, the unit surplus may be regarded as divided among all agents in N according to some imputation $z^{jk} = (z^{jk}_i)_{i \in N}$ of non-negative shares. The Appendix shows that the payoff division we postulate is the unique allocation in the *kernel* of the corresponding cooperative game (cf. [12]).

We suppose that agents have to pay a fixed (marginal) cost c for each link they establish. Then, for every strategy profile $\mathbf{s} = (s_1, s_2, \ldots, s_n)$, the (net) payoffs to player i are given by

$$\Pi_i(s_i, \mathbf{s}_{-i}) = \sum_{j \in C_i(g)} \frac{1}{e(i, j; g) + 2} + \sum_{j,k \in N} \frac{I_{\{i \in E(j,k)\}}}{e(j, k; g) + 2} - \eta_i(g)c, \tag{1}$$

where $I_{\{i \in E(j,k)\}} \in \{0, 1\}$ stands for the indicator function specifying whether i is essential for j and k, and $\eta_i(g) \equiv \left|\{j \in N : j \neq i,\ g_{ij} = 1\}\right|$ denotes the number of players with whom player i has a link.

We note that even if a player lies on several paths between two players, she may still get no intermediation payoffs if she is not essential. In an earlier working-paper version [22] we examined in detail the case where a player i's intermediation rents from connecting j and k are an increasing function of the number of shortest paths (between j and k that) she lies on. Finally, we also note that the payoff function above assumes that the costs of link formation are constant. In Section 4 we study the implications of increasing costs by considering the case of capacity constraints on the number of links an individual can form.

The main objective of the paper is to study the architecture of networks that are strategically stable and assess their efficiency. Our notion of strategic stability is a refinement of Nash equilibrium that allows for coordinated two-person deviations.

Definition 1. A strategy profile $\mathbf{s}^*$ is a *bilateral equilibrium* (*BE*) if the following conditions hold:

- for any $i \in N$ and every $s_i \in S_i$, $\Pi_i(\mathbf{s}^*) \geqslant \Pi_i(s_i, \mathbf{s}^*_{-i})$;
- for any pair of players $i, j \in N$ and every strategy pair (s_i, s_j),

$$\Pi_i(s_i, s_j, \mathbf{s}^*_{-i-j}) > \Pi_i(s_i^*, s_j^*, \mathbf{s}^*_{-i-j}) \Rightarrow \Pi_j(s_i, s_j, \mathbf{s}^*_{-i-j}) < \Pi_j(s_j^*, s_j^*, \mathbf{s}^*_{-i-j}).$$

Thus, a given strategy profile is a 'BE' if no player or pair of players can deviate (unilaterally or bilaterally, respectively) and benefit from the deviation (at least one of them strictly, for bilateral deviations). This notion *refines* the original formulation of pairwise stability due to Jackson and Wolinsky [28] by allowing pairs of players to form and delete links simultaneously. As it will be apparent from the analysis, this possibility of joint creation and deletion of links is crucial to our results.

In fact, our analysis will focus on a refinement of BE that we call *strict*. It rules out the existence of deviations (unilateral or bilateral) that have some consequence (i.e., affect the network) but nevertheless are payoff indifferent for the agents involved. In part, the motivation of this equilibrium concept is dynamic: a gradual adjustment process that allows pairs of players to revise the network in sequence will (only) be absorbed by equilibria of this sort—see below for an elaboration.

Definition 2. A strategy profile $\mathbf{s}^*$ is a *strict bilateral equilibrium* (*SBE*) if the following conditions hold:

- for any $i \in N$ and every $s_i \in S_i$ such that $g(s_i, \mathbf{s}^*_{-i}) \neq g(\mathbf{s}^*)$, $\Pi_i(\mathbf{s}^*) > \Pi_i(s_i, \mathbf{s}^*_{-i})$;
- for any pair of players, $i, j \in N$ and every strategy pair (s_i, s_j) with $g(s_i, s_j, \mathbf{s}^*_{-i-j}) \neq g(\mathbf{s}^*)$,

$$\Pi_i(s_i, s_j, \mathbf{s}^*_{-i-j}) \geqslant \Pi_i(s_i^*, s_j^*, \mathbf{s}^*_{-i-j}) \Rightarrow \Pi_j(s_i, s_j, \mathbf{s}^*_{-i-j}) < \Pi_j(s_j^*, s_j^*, \mathbf{s}^*_{-i-j}).$$

We note that an SBE is a BE.

Finally, we introduce the notion of efficiency. In line with the assumption that the bargaining setup involves transferable utility, different networks g are assessed in terms of the total surplus generated, $W(g) \equiv \sum_{i \in N} \Pi_i(g)$. Let $\mathcal{G}$ denote the set of all possible networks (i.e., all undirected graphs with n vertices).

Definition 3. A network $\tilde{g}$ is efficient if $W(\tilde{g}) \geqslant W(g)$ for all $g \in \mathcal{G}$.

Before undertaking the analysis of the model, it might be useful to review some standard graph-theoretic notions that will be used repeatedly. A network is said to be *connected* if there exists a path between any pair $i, j \in N$. Given any $g' \subset g$, let $N(g') \equiv \{i \in N : g'_{ij} = g'_{ji} = 1 \text{ for some } j\}$ be the subset of nodes which display some link in g'. Then, a network, $g' \subset g$, is a *component* of g if for all $i, j \in N(g')$, $i \neq j$, there exists a path in g' connecting i and j, and for all $i \in N(g')$ and $k \in N$, $g_{ik} = 1$ implies $k \in N(g')$. A component $g' \subset g$ is *complete* if $g'_{ij} = 1$ for all $i, j \in N(g')$.

Two networks g and g' are said to have the same architecture if one network can be obtained from the other by a permutation of the players' labels. A network is said to be symmetric [9] if all players have the same number of links, say η. The *complete* network, g^{c}, is a symmetric network in which $\eta = n - 1$, $\forall i \in N$, while the *empty* network, g^{e}, is a symmetric network in which $\eta = 0$. Another example of a symmetric network is a *cycle* where $\eta = 2$ and the whole set of nodes can be ordered in a list $i_1, i_2, \ldots i_n$ with $g_{i_1,i_2} = g_{i_2,i_3} = \cdots = g_{i_n,i_1} = 1$ and no other links exist.

Finally, we shall say that a network is asymmetric if there is at least one pair of players who have a different number of links. One important example is the *star* where there is a single node, i_{c}, with $\eta_{i_{\mathrm{c}}} = n - 1$ while $\eta_i = 1$ for all other $i \neq i_{\mathrm{c}}$. An interesting mixture is what we shall call the *hybrid cycle–star* network where there is a subset of k nodes $C = \{i_1, i_2, \ldots i_k\}$ arranged in a cycle and some particular player $i_x \in C$ such that $g_{j,i_x} = 1$ for all $j \in N \backslash C$. Fig. 1 illustrates these different types of networks.

3. Analysis

Our first main result is that, in the absence of other considerations (e.g., capacity constraints or restricted bilateral deviations), equilibrium networks must be stars. This result shows that structural holes can arise endogenously in the starkest possible manner—a single individual (the center of the star) is *essential* to the value generated in the *whole* network. This in turn implies that the center also appropriates a correspondingly large part of the total surplus generated by the network.

We start by noting a property of equilibrium networks.

Proposition 1. *A BE network is either empty or connected.*

Proof. See the Appendix.

Suppose that, contrary to what is asserted, an equilibrium network is split into two or more components. Consider two individuals, i and j, in different components. First, we observe that

[9] The notion of symmetric networks used here only presumes internode symmetry concerning their degree. It corresponds to what in graph theory is often labelled as regular networks.

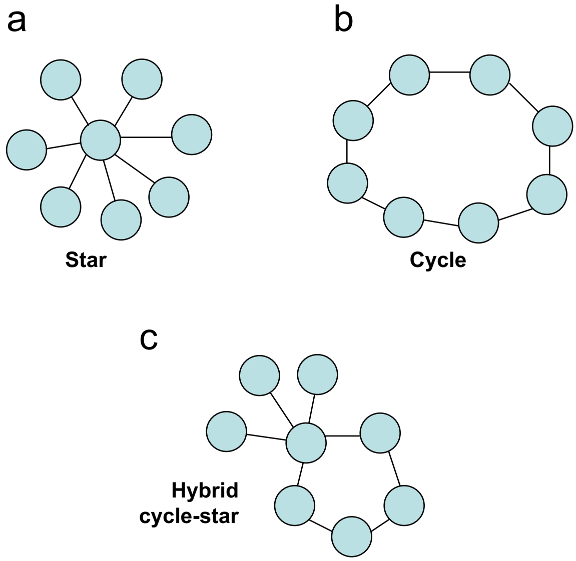

Fig. 1. Main architectures. (a) Star, (b) cycle, and (c) hybrid cycle–star.

their marginal payoffs of establishing a link between them (thereby merging the two components) are exactly the same for both players. This happens because the additional "access payoffs" of i from connecting to the individuals in the other component are identical to the corresponding intermediation payoffs earned by j. Therefore, since gross payoffs are the sum of access and intermediation payoffs, both players enjoy the same marginal gross benefit from linking to each other. Now, we argue that it cannot be optimal for these two players to remain in separate components. The reason is as follows. Suppose that i's component contains more than one player. Then we show that the component must have some agent (let us suppose that it is i herself) who enjoys no intermediation payoffs, either because she is extremal or an "inessential" part of a cycle. Let us consider each of these two possibilities in turn. In the first case (when i is extremal), since some player ℓ finds it optimal to connect to i, she will also find it optimal to connect to j, hence ℓ and j cannot lie at equilibrium in different components. In the second case, the payoffs of j from linking with i are, in addition to the payoff from the direct link, a "scaled down replica" of the access payoffs of i. It can then be shown that the gross payoffs to j must exceed the linking cost if, as assumed, it is optimal for player i to incur the cost of *two* links to have her (pure access) payoffs.

What type of equilibrium networks are then possible? We start by noting that two types of architectures are always possible, unless the cost is very low or the population is quite small. On the one hand, the empty network is an SBE if $c > \frac{1}{2}$, even if the population is arbitrarily large. This reflects a simple instance of "coordination failure": if no links are formed, then the creation of any link has to be judged on a stand-alone basis, which is unprofitable if the linking cost exceeds half of the unit surplus earned by an isolated pair of players.

On the other hand, it is also clear that the star is an SBE if the population is large enough and the linking cost is no so low as to justify a direct connection between peripheral players. Specifically, suppose that $\frac{1}{6} < c < \frac{1}{2} + (n-2)/6$. Then, in a star, the payoffs to the center are positive and equal to

$$\frac{n-1}{2} + \frac{1}{3}\frac{(n-1)(n-2)}{2} - (n-1)c,$$

whereas if she were to keep only k of the $n-1$ links she would earn the lower payoffs:

$$\frac{k}{2} + \frac{1}{3}\frac{k(k-1)}{2} - kc.$$

As for the incentives of the peripheral players, in the star they earn $\frac{1}{2} + (n-2)/3 - c$, while if they created an additional link the entailed payoffs would be $\frac{1}{2} + \frac{1}{2} + (n-3)/3 - 2c$. The latter is smaller than the former if $c > \frac{1}{6}$.

The star is a (strict) BE (SBE) network for a wide range of parameters and this is due to the fact that centrality generates large payoffs from essentialness. However, a star also exhibits an extreme form of such essentialness, with only one player being essential for all pairs of players. This raises the question: are there other—perhaps more egalitarian—network architectures that can arise in equilibrium? The following result provides a complete (negative) answer to this question for large societies.

Theorem 1. *Given $c > \frac{1}{6}$, if n is sufficiently large*[10] *a star is the unique non-empty SBE network. The empty network is an SBE for $c > 1/2$.*

Proof. See the Appendix.

The proof presented in the Appendix proceeds by showing that all networks other than the empty network and the star are not sustainable in a strict equilibrium. The arguments rely on the three kinds of incentives that arise in our model: accessing others, gaining intermediation rents, and avoiding intermediation payments. The intuition underlying the main steps can be outlined as follows:

1. *If an SBE network is minimally connected* (*i.e.*, *a tree*), *it must be a star*: This follows from two observations. First, the number of essential players between any two players is bounded above, independent of the population size (if it were too large, the incentives of these players to establish a link and thus gain shorter access to each of them as well as to many others would certainly exceed the cost of the link). Secondly, "extremal" and "central" players always have much to gain by connecting directly since by doing so the gains are proportional to population size but (opportunity) costs are bounded. A simple illustration of the latter point is depicted in Fig. 2.

[10] More precisely, there is a function $F(c)$ such that, if $n > F(c)$, the stated conclusion is obtained. This is the interpretation of all our results in the paper that require n being sufficiently large. For the present result, the particular form of F can be obtained from the bounds contemplated in the proofs of Lemmas 3, 6, and 7. Specifically, for some $\hat{e}(c)$ and $y(c)$ that can be chosen as explained in those proofs, independent of n, we have

$$F(c) = \max \left\{ \begin{array}{cc} \multicolumn{2}{c}{4 + 2c\hat{e}(c)(\hat{e}(c)-1);} \\ y(c)[c\hat{e}(c)(\hat{e}(c)-1)+1]; & 4 + y(c) + 2c\hat{e}(c)(\hat{e}(c)-1); \\ \frac{(y(c)-1)(y(c)+1)}{2(y(c)-2)} + y(c); & \frac{6(y(c)-1)(y(c)+1)}{11y(c)-37} + y(c), \end{array} \right\}$$

where the first term reflects the bound used in the proof of Lemma 3, the next two in that of Lemma 6, and the final two in that of Lemma 7.

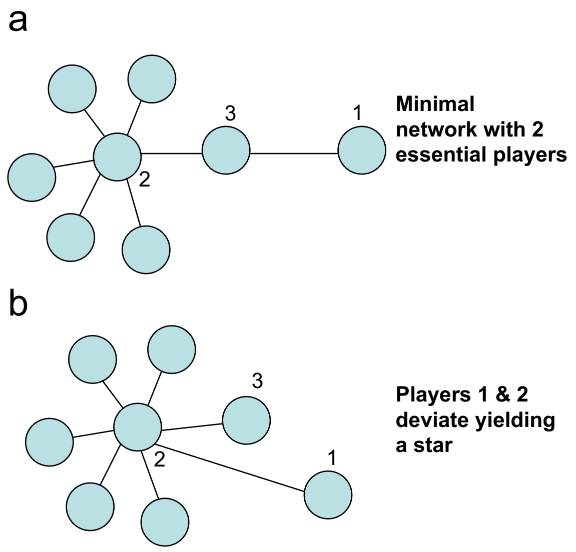

Fig. 2. Incentives in minimal networks. (a) Minimal network with two essential players and (b) players 1 and 2 deviate yielding a star.

2. *There can be at most one cycle in an SBE network*: Here, the main point is that, if several cycles exist in a network, it is always possible for two players to establish a new link and at least remain equally welloff. Two possible cases need to be distinguished in this respect, illustrated in turn by Figs. 3a and b. In the first case, the two cycles have no players in common and players such as 3 and 4 in Fig. 3(I) have a strict incentive to connect and destroy their links to essential players 1 and 2. This change leads to the network in Fig. 3a(II), where players 3 and 4 access the same number of other players, avoid the need of sharing some payoffs with players 1 and 2, and incur the same linking costs. In the second case, there are common players in both cycles (i.e., player 1 in Fig. 3b(I)), so that a deviation by two players (1 and 2) can transform those two cycles into just one (Fig. 3b(II)) where the payoffs to both of them are exactly the same as before.[11] This means that the original network fails to be an SBE.[12]
3. *A single cycle cannot be an SBE network*: A cycle provides incentives for players lying on opposite sides of it to connect and, by also deleting two of their links, become markedly central and thus earn large intermediation rents (cf. Fig. 4). It is true that by doing so the two players in question have to make intermediation payments to others where none of these existed under the cycle. We find, however, that the intermediation rents dominate the intermediation payments and as a result the cycle is not sustainable.
4. *If an SBE network includes a cycle*, *all other players not belonging to it are connected to the same player in the cycle*, *i.e.*, *it is a hybrid cycle–star network*: The starting point of the argument is the observation that, given the linking cost c, the cycle cannot include more than

[11] This change in link pattern, however, strictly increases the payoffs of player 3 since player 3 has the same access benefits, no essential players, but forms one link less in the new network.

[12] Note that this is the only point in the proof of Theorem 1 where we use the strictness of equilibrium condition. It can be checked that a network with two cycles is sustainable in a non-strict equilibrium.

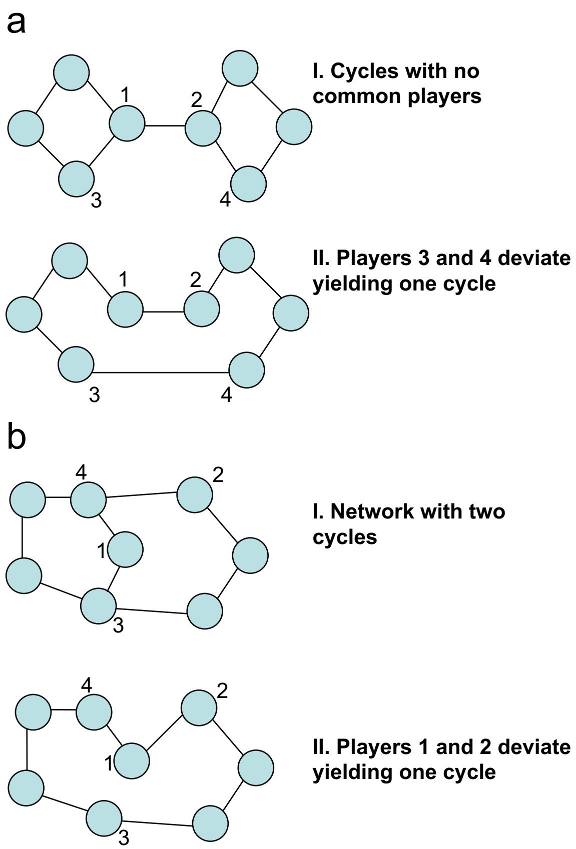

Fig. 3. (a) Instability of networks with two cycles: (I) cycles with no common players and (II) players 3 and 4 deviate yielding one cycle and (b) Instability of networks with two cycles: (I) network with two cycles and (II) players 1 and 2 deviate yielding one cycle.

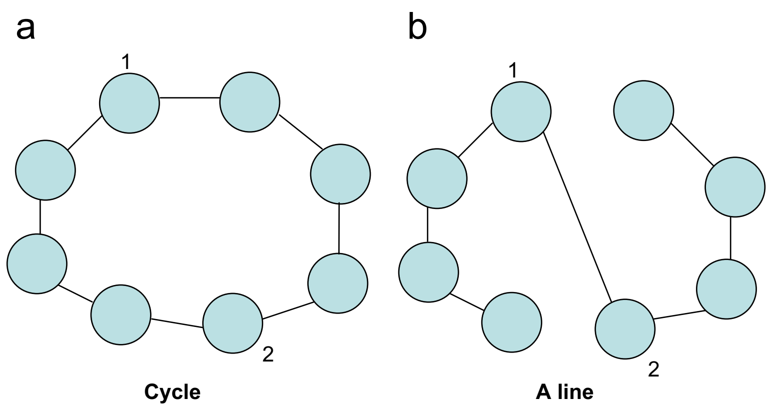

Fig. 4. Bilateral deviations from cycle (a) Cycle and (b) a line.

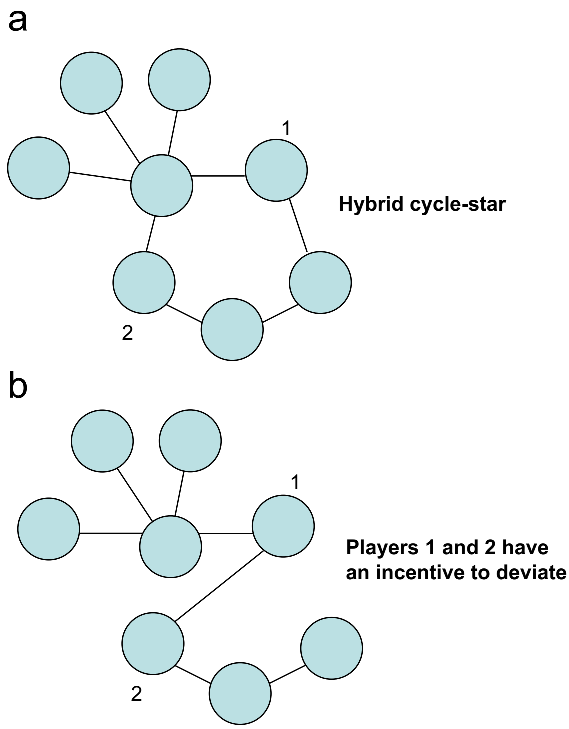

Fig. 5. Instability of hybrid cycle–star. (a) Hybrid cycle–star and (b) players 1 and 2 have an incentive to deviate.

a certain number of players—otherwise, creating a link that bypasses the " gate player" that is essential to accessing the cycle would become profitable. Then, since almost all players (if the population is large) must lie outside the cycle, we can rely on an adaptation of the ideas used in Step 1 above to argue that, in the minimally connected part of the network that lies outside the cycle, players must be arranged in a star configuration.

5. *The only hybrid cycle–star network that defines an SBE is the "degenerate" star network*: As explained above, in an SBE network that involves a unique cycle, most of the players in the (large) population must lie outside the cycle and connect to it through a single player. Therefore, much as it was argued for Step 3 that configuration opens the possibility that the cycle be broken by the concerted deviation of two players in it, who can then obtain high intermediation payoffs. The nature of such a deviation is illustrated in Fig. 5. It rules out that a hybrid cycle–star SBE network may include any genuine cycle, thus leaving us with the strict star network as the only possibility for a non-empty SBE. Since it is clear that a star network is an SBE (see the discussion preceding the statement of the theorem), the conclusion follows.

The above result provides us with a complete characterization of SBE networks for large societies if the linking cost is not very small.[13] We now elaborate on two aspects of this result:

[13] If $c < \frac{1}{6}$, it is easy to see that an SBE (in pure strategies) does not exist. The reason is that, under those conditions, no player can earn intermediation rents at equilibrium. (The linking cost is so low that it would always pay off to bypass such an agent.) Then, by point 2 above, the only candidate SBE network is a single cycle, but this cannot be the result, an equilibrium, for the reasons explained in point 3.

(i) the role of population size and (ii) the relationship between equilibrium, efficiency, and payoff heterogeneity.

(i) *Population size*: There are two points we want to bring out here. First, we discuss the minimum value of n which is needed in the above result. Second, we consider the possibilities available in small societies and illustrate matters through the complete characterization of equilibrium networks for a society with four players.

There are essentially two constraints on the value of n in Theorem 1. The first constraint arises from the argument in step 1 above: the incentives of the central and extremal players to connect increase with the number of players. The second constraint arises from the argument in step 5 above: the incentives of players in a hybrid–cycle network to destroy the cycle are also related to number of players in the population. In both instances the players will trade off the costs of forming an additional link with the potential (gross) gains, and so the lower bound on n is a function of the costs of forming links c.

However, it is important to understand that, even in small societies, equilibrium networks will generate intermediation rents induced by the existence of structural holes. To see this, note that steps 2 and 3 above do not rely on population size, and so a single cycle—or a network with several cycles—cannot arise even in small societies. (Also note that a star is an equilibrium network for small societies so long as $\frac{1}{6} < c < \frac{1}{2} + (n-2)/6$.) On the other hand, observe that step 4 (which again does not depend on n) shows that, in a network with one cycle and some players outside it, there is an upper bound to the number of players in the cycle. In other words, all but a given number of players must be peripheral to a "local" star.

To make the aforementioned considerations concrete, consider a society with four players. In this context, it is easy to see that, by an application of the above arguments along with some simple computations, the following characterization arises. An SBE network is either a star, a line, or empty. In particular, a star is an SBE if $\frac{1}{6} < c < \frac{10}{12}$, a line is an BE if $\frac{7}{12} < c < \frac{23}{24}$, while the empty network is an SBE if $\frac{6}{12} < c$.

(ii) *Equilibrium*, *efficiency*, *and payoff heterogeneity*: Concerning efficiency, the first point to note is that, from a social point of view, there is no gain in adding links between nodes in the same component, while there is a cost to adding those links since $c > 0$. So all components of efficient networks must be minimally connected (possibly singletons). On the other hand, in our setting where positive surpluses are earned from every direct and indirect connection, it is easy to see that networks with distinct (non-singleton) components cannot be efficient. Thus, an efficient network must either be empty or minimally connected. Finally, we note that the aggregate net payoff in the latter case is $\frac{1}{2}n(n-1) - 2c(n-1)$, which is positive if, and only if, $c < n/4$. Based on these observations, the following result readily follows.

Proposition 2. *If $c < n/4$ then an efficient network is minimally connected, while if $c > n/4$ then an efficient network is empty.*

Clearly, the star is a minimally connected network and is therefore efficient for large n. Therefore, combining Proposition 2 and Theorem 1, we conclude that efficiency is guaranteed at a non-empty SBE network, provided the population is large.

The fact that all efficient networks are minimally connected implies that, as it is the case for non-trivial equilibrium networks, efficient ones always generate intermediation rents. This in turn gives rise to payoff asymmetries, with some players (in particular, those in extremal positions) yielding some of the surplus they produce to other players who are essential for them. It is interesting to observe that this asymmetry reaches its highest level for those efficient networks that are also equilibrium ones, i.e., for star networks. In these networks, the central player earns $(n-1)[1/2+(n-2)/6-c]$, while each peripheral player earns $1/2+(n-2)/3-c$. Clearly, the entailed payoff difference increases unboundedly as n grows. [14]

4. Discussion and extensions

Our former analysis shows that in a setting with access benefits and intermediation rents, strategic network formation can generate sharp predictions: for large societies, all non-empty SBE networks are stars. In this section we will examine how sensitive this result is to some of the specific assumptions we made in the analysis. First, in Section 4.1, we check whether our analysis is robust to the requirement of "credibility" (or internal consistency) of bilateral deviations. Second, we examine the implications of capacity constraints with regard to the number of links that any individual can simultaneously hold.

4.1. Two-person coalition proof networks

In our analysis so far we have assumed that a deviation by two players is credible so long as it yields higher payoffs to both players. We have not looked at profitable deviations from the agreed upon deviation, thus ignoring considerations that are in the spirit of coalition proofness. In this section we will examine the consequences of this further requirement, which of course can only enlarge the set of equilibria—in general, only a subset of bilateral deviations may qualify as valid. We start with a definition of *bilateral proofness* in our context, focusing directly on the "strict" version of this concept.

Definition 4. A strategy profile $\mathbf{s}^*$ is a *strict bilateral-proof equilibrium* (*SBPE*) if the following conditions hold:

1. for any $i \in N$ and every $s_i \in S_i$ such that $g(s_i, \mathbf{s}^*_{-i}) \neq g(\mathbf{s}^*)$, $\Pi_i(\mathbf{s}^*) > \Pi_i(s_i, \mathbf{s}^*_{-i})$;
2. for any pair of players $i, j \in N$ and every strategy pair (s_i, s_j) with $g(s_i, s_j, \mathbf{s}^*_{-i-j}) \neq g(\mathbf{s}^*)$ and $\Pi_i(s_i, s_j, \mathbf{s}^*_{-i-j}) \geqslant \Pi_i(s_i^*, s_j^*, \mathbf{s}^*_{-i-j})$, one of the following two conditions hold:
   (a) $\Pi_j(s_i, s_j, \mathbf{s}^*_{-i-j}) < \Pi_j(s_i^*, s_j^*, \mathbf{s}^*_{-i-j})$.
   (b) $\exists k, \ell \in \{i, j\}, k \neq \ell$, and some $\tilde{s}_k \in S_k$, such that $\Pi_k(\tilde{s}_k, s_\ell, \mathbf{s}^*_{-k-\ell}) > \Pi_k(s_k, s_\ell, \mathbf{s}^*_{-k-\ell})$.

Most of the insights obtained in Section 3 through the SBE concept are maintained if we consider the less demanding SBPE concept. Indeed, reviewing the propositions and lemmas that underlie the proof of Theorem 1, it is straight forward to check that all of them continue to apply for the SBPE concept, except for Lemmas 5 and 7 (corresponding to steps 3 and 5 in the discussion following Theorem 1). Thus, the only candidates for SBPE are the empty network and the hybrid cycle–star networks (with the star and the cycle as degenerate special cases). The following result shows that the requirement of "proofness" has powerful implications: the cycle network—which is perfectly egalitarian—can be now sustained in equilibrium.

[14] This payoff advantage for the central player, however, suggests that players may want to compete to become the central player. In our analysis we allow players to form an additional link and/or delete links but it is possible to envisage richer strategies. For instance, a player can propose new networks involving subsets of players or may propose the formation of links along with a set of transfers. An analysis of these richer strategic possibilities is clearly important but lies outside the scope of the present paper.

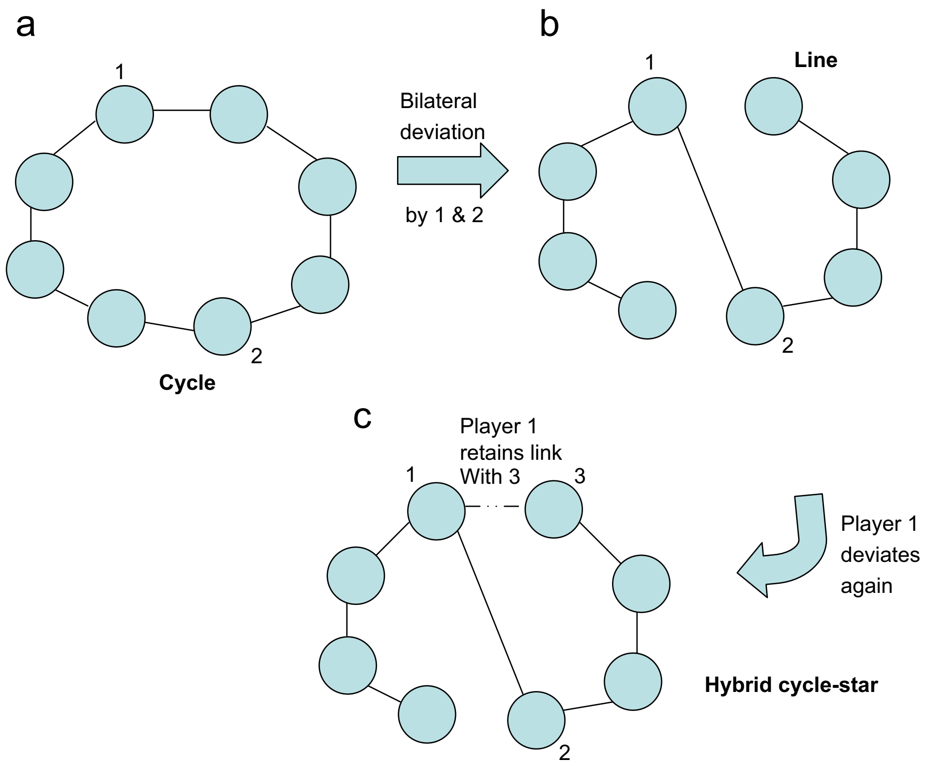

Fig. 6. Bilateral proof deviations. (a) Cycle, (b) line, and (c) hybrid cycle–star.

Proposition 3. *Given any $c > 0$, suppose that n is large enough. Then, any cycle containing all players can be supported by an SBPE.*

Proof. See the Appendix.

The proof of this result is presented in the Appendix. It shows that the deviation by two distant players in a cycle which takes the cycle to a line is not credible since each of the players has an incentive to renege on the deviation and retain both their erstwhile links, under the assumption that the other player will delete one of her links. This incentive is clarified in Fig. 6. Recall that, in Theorem 1 (cf. point 4 following its statement), a cycle was broken by a deviation in which, say, players 1 and 2 in Figs. 6a and b move and create a line. The proof of Proposition 3 shows that this deviation is vulnerable to a further deviation in which player 1 retains both the links that she had. This deviation yields network 6c. By moving to this network, player 1 is able to circumvent a large number of essential players in the line. This deviation is worthwhile if there are enough players on the line going from 1 to 3. Indeed, analogous reasoning can be used to rule out bilateral deviations from a hybrid cycle–star network: a joint deviation by two players in the cycle is not in the interest of any one of them, under the assumption that the other abides by it.

The above discussion indicates that, under the requirement of bilateral proofness, a richer set of network architectures can be supported at equilibrium—specifically, the cycle and a hybrid cycle–star network are also possible, in addition to full stars. Thus, in this sense, the pressure towards polarization that proved so acute under unqualified bilateral stability is mitigated when we insist that bilateral deviations be internally consistent. The multiplicity of equilibrium networks however raises questions about their relative robustness.

We will argue now that egalitarian structures such as the cycle and hybrid cycle–star networks do not survive under the pressure of occasional perturbations that affect some of the links. A natural way of formulating precisely this idea is to check the performance of some suitably modelled dynamics of adjustment under the pressure of occasional random decay of links. In what follows, we describe and analyze such a dynamic approach.

Consider a dynamic process modelled in discrete time where, at each period $t = 1, 2, \ldots,$ the state of the system is given by the prevailing network $g(t)$. At every t, two players are selected at random and given the option to create a link between them (if this link is not already in place) and, simultaneously, destroy any subset of the existing links that involve either of them. In line with the considerations that underlie the SBPE concept, we suppose that, in evaluating any such bilateral move, players view as credible only those adjustments that are internally consistent (i.e., immune to a further unilateral deviation).

More precisely, consider two players, say i and j, who are given the opportunity to revise the network at t. On the one hand, if they were not previously linked at $t-1$ (i.e., $g_{ij}(t-1) = 0$) they can create the corresponding link and have $g_{ij}(t) = 1$. On the other hand, in conjunction with this change, they can also consider the possibility of removing any of the preexisting links, i.e., they can make $g_{ik}(t) = 0$ or/and $g_{jk}(t) = 0$ for any $k \in N$. The requirements demanded from any such combined adjustment are two-fold:

(i) For each $\ell = i, j$, $\Pi_\ell(g(t)) \geqslant \Pi_\ell(g(t-1))$.
(ii) There exists *no* $\ell \in \{i, j\}$ such that $\Pi_\ell(g'(t)) > \Pi_\ell(g(t))$ for some $g'(t)$ that differs from $g(t)$ only in components involving index ℓ and if $g'_{\ell k}(t) = 1 \neq g_{\ell k}(t)$, then $g_{\ell k}(t-1) = 1$.

The requirements on bilateral adjustment embodied by (i) and (ii) reflect the notion of bilateral proofness underlying Definition 4. Condition (i) simply states that a bilateral adjustment cannot be detrimental to either of the players involved. Condition (ii), on the other hand, rules out that there could exist an individual deviation from any such bilateral adjustment that strictly benefits the deviating agent. We shall assume that any bilateral adjustment consistent with (i) and (ii) has positive (say, uniform) probability of being implemented. It is clear, therefore, that every stationary point of the induced dynamics defines an SBPE network.

The adjustment rules (i) and (ii) are to be regarded as the "core" part of the network dynamics. In addition to it, we posit that, at the end of every t (i.e., "right before" entering the subsequent period $t+1$), there is some "small" probability $\varepsilon > 0$ so that, in a *stochastically independent* way for *every* possible pair of players, $k, k' \in N$ $(k \neq k')$, a link between them is created (if currently absent) or destroyed (if currently in place). This part of the dynamics is to be interpreted in the same vein as modern evolutionary theory does, i.e., as a small perturbation/mutation of the core dynamics.[15] In essence, its role is to render the overall process ergodic, thus allowing an assessment of the relative robustness of each of the limit states of the core dynamics, independent of initial conditions.

Formally, our analysis centers around the standard notion of stochastic stability. Adapting it to the present framework, we shall say that a network g^* is *stochastically stable* if it is attributed

[15] See the early work of Kandori et al. [29] and Young [41] or, for a more extensive discussion of the approach and the methodology, the monographs by Samuelson [37], Vega-Redondo [39], and Young [42].

positive probability by the (unique) long-run invariant distribution of the process when the mutation probability $\varepsilon \to 0$. The following result, which is proven in the Appendix, establishes that, if the linking cost is not too low, every stochastically stable network has a star architecture.

Proposition 4. *Suppose $c > \frac{5}{12}$ and that n is large enough.*[16] *Then, every stochastically stable network is a star.*

Proof. See the Appendix.

Intuitively, this result hinges upon a comparison of the relative robustness displayed by the different networks arising in the long run from the operation of the unperturbed adjustment dynamics. Since this dynamics turns out to converge to some SBPE network, the only candidates for stochastic stability are precisely those type of networks, i.e., cycles, cycle–star hybrids, stars, or the empty network. Not all of them, however, are comparably resilient in the face of perturbations. As we explain next, the dynamic forces that bring the system towards star networks are comparably stronger than for the remaining cases. This leads, in the end, to the long-run selection result stated in Proposition 4.

First, note that cycles and cycle–star hybrids are very fragile configurations since both of them are "easily" abandoned through the concourse of just *one* mutation. Once there is a single mutation that breaks the cycle, either a "central agent" appears afresh or, if one already existed (in the cycle–star hybrid), she has her intermediation role reinforced. This allows one to construct a subsequent path of unperturbed adjustment that eventually leads to an all-encompassing star network. Along this path, one simply needs to have the central player matched in turn with a suitably selected "peripheral player." A mutually beneficial link between them is then formed that reaps the substantial gains (intermediation rents and access benefits) that are made possible by the central player in a large population.

In contrast, we observe that a similar fragility does not affect a star network. Starting from any such configuration, if just one link is created or destroyed, any adjustment that can follow thereafter brings the system back to the original star.

Finally, concerning the empty network, it is certainly true that, in general, *several* simultaneous mutations will be needed to escape the situation if the linking cost is high. But, given this cost, such a number is given independent of the population size n. This contrasts with the situation that must arise for a reversion to the empty network from any of the other (connected) SBPE networks. For these latter transitions, the number of simultaneous mutations required grows unboundedly with n, thus making them so much less likely if the population is large.

The aforementioned considerations indicate that the star architecture continues to be singled out by the model, even when the agents have their possible adjustments/deviations restricted by the demand of bilateral proofness. In this case, however, its selection reflects dynamic considerations: when agents' adjustment is subject to small (infinitesimal) noise, only star networks will be observed at any significant fraction of time in the long run.

4.2. Capacity constraints

In the basic model we assumed that the marginal costs of linking between players are constant and in particular do not depend on the number of links a player forms. In some settings it seems

[16] If $c \leqslant \frac{5}{12}$, the transition from a star network to a cycle can be implemented through the unperturbed dynamics after just one mutation. As explained, in the Appendix, this renders both the cycle and the star stochastically stable networks.

more natural to suppose that the costs per link are increasing in the number of links. Or, analogously, we should expect that an individual player will be subject to some capacity constraints. In this section we discuss the implications of such constraints on equilibrium networks. To bring out the role of capacity constraints clearly we will focus on the case where each player is constrained to form at most K links and K is small relative to the number of players, i.e., $K << n$. We start with a discussion of BE networks and then briefly discuss the nature of bilateral-proof networks.

First, consider the nature of BE networks in the presence of capacity constraints. The simplest way to proceed is to examine the effects of capacity constraints on arguments developed in Section 3. We first note that a non-empty (strict) equilibrium network must be connected. Suppose not. Then consider a network g and any non-singleton component in it. Suppose that the component is non-minimal. We can then use the arguments in steps 2 and 3 of Theorem 1 (in Section 3) to conclude that this component must be a hybrid cycle–star network (with one or more players outside the cycle). The other possibility is that this component is minimal. We therefore conclude that in any non-singleton component there exists a player i who has only one link. Moreover, there is a player j such that $g_{i,j} = 1$. It follows that j can earn weakly higher payoffs by deleting the link with i and instead linking with a player k in some other component. We now look at the incentives of players in other components. A player in a singleton component will want to link with j (since i finds it profitable to link with j). Finally, in any other non-singleton component, by our argument, there always exists a player with a single link and clearly this player will find it profitable to link with j (given that i finds it profitable to link with j). Thus, g is not an SBE network, a contradiction to our hypothesis.

We now turn to connected networks and reexamine steps 1–5 in the proof of Theorem 1. We first note that minimal networks cannot be sustained in equilibrium. To see why this is the case, fix some K and allow n to grow: the diameter of a minimal network will grow without bound. This in turn means that the intermediation rents that any two end players have to pay to access the path between them will grow without bound, creating incentives for them to form a link. This proves that no connected minimal network can be sustained in equilibrium. Next consider non-minimal networks. The arguments in steps 2 and 3 discussed above do not rely on capacity constraints, so they also apply in the present setting. This means that any equilibrium network must have a cycle and some trees rooted in the cycle. Next, note that for $K > 2$, the arguments in step 4 can be applied to ensure that the number of players in the cycle in such a network is bounded above by a number that is independent of n. This in turn means that the number of players in any of those trees can be made arbitrarily large by increasing n. Given the assumption on capacity constraints, however, players in these trees who are located far from their respective roots will then have an incentive to form a link with a player in the cycle, who would be willing to switch a link from someone in the cycle to such an end player. We have thus established:

Proposition 5. *Given $K > 2$, suppose that n is sufficiently large. Then, if $c < \frac{1}{2}$, there exists no* (*strict*) *BE. On the other hand, if $c > \frac{1}{2}$, the unique* (*strict*) *BE is the empty network.*

This is a striking result and it is important to discuss the incentive pressures that lead to it. Recall that in our model there are three types of incentives driving individuals to form links. The first incentive arises from access advantages, the second incentive is the rewards from intermediation, while the third incentive arises out of the desire to circumvent intermediate players in order to retain more of the surplus for themselves. In the basic model, we showed these pressures lead to a star network. The star is immune to the pressures from the third incentive since the central player will remain critical to all but one exchange, *even after the formation of an additional link*.

In other words, it is the extreme centrality of the hub player which discourages the formation of additional links. In a setting with significant capacity constraints, no player can occupy such a central position, implying that there will be several intermediaries, and this creates incentives for players to form additional links.

This negative result motivates an examination of circumstances under which equilibrium can be established. A natural possibility is to examine solution concepts which restrict the set of deviations. The following result summarizes the analysis of bilateral-proof equilibrium networks.

Proposition 6. *Given any $c > 0$ and $K > 2$, suppose that n is sufficiently large. Then an SBPE network is either empty or a cycle.*

Proof. See the Appendix.

The proof is given in the Appendix. The intuition underlying the result is as follows. When capacity constraints are binding (relative to population size), we have already argued that the star is no longer feasible and hence cannot be supported in equilibrium. On the other hand, no more than one cycle can prevail at an SBPE (this follows from earlier arguments developed in Section 3). So there are just three candidates to consider: the empty network, the full cycle, or a small cycle to which some (one or several) long trees are appended. But the latter cannot possibly be an equilibrium since agents have deviations available that are feasible (if $K > 2$) and would save them substantial intermediation costs. In the end, this leaves the cycle and the empty network as the only network architectures that are possible in SBPE.

In line with Proposition 4, we may raise the question of whether a dynamic approach can be selected between these two networks. A study of perturbed dynamics along the lines of Section 4.1 yields the following clearcut result.

Proposition 7. *Given any $c > 0$ and $K > 2$, suppose that n is sufficiently large. Then every stochastically stable network is a cycle.*

The proof of this result follows from a direct application of the methods used for Proposition 4, so we omit it. The result highlights the implications of capacity constraints: if these constraints are stringent enough (relative to population size) the struggle between the different forces at work in the model is resolved quite differently as compared to the basic model. The key observation here is that a relatively small capacity for linking implies that in a non-cycle network there will exist players who have to pay rents to a *large number of intermediaries*. This, however, creates incentives for those player to form a link to circumvent such payments. In contrast, notice that when all players are arranged in an all-encompassing star, every peripheral player has only one intermediary—therefore, any new link only affords modest savings in intermediation rents.

5. Conclusion

This paper has studied a simple model of network formation where agents may exploit positional advantages if these provide them with the ability to block profitable bilateral interaction between two players who are not direct neighbors. We obtain two principal insights.

First, we show that the strategic struggle for the aforementioned advantages leads to a polarized star architecture where a single player becomes essential to connecting every other pair of players. This represents a clearcut formalization of the notion found in the sociological literature that structural holes open the potential for large benefits to those individuals who succeed in bridging them. This conclusion is essentially unchanged whether we suppose that players may commit to bilateral deviations or they cannot (and thus any such deviation must be internally consistent).

Our second main insight concerns a setting in which players have a small capacity to form links relative to the number of players. In this context, one key observation is that, in any feasible network, distances between players must be long. This creates incentives for "faroff" players to form an additional link in order to avoid paying large intermediation rents. This pressure leads to problems of existence (in the class of non-empty networks) if bilateral deviations are unrestricted. However, if they are required to be internally consistent, then a cycle emerges in which payoffs are equal across players.

The above results raise the question of how would the results be affected if we relaxed the important assumption that there is no decay in value as exchange takes place through long paths, as in a cycle. The analysis of decay in the presence of capacity constraints is an interesting topic which is left for further research.

Acknowledgment

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Appendix A.

A.1. The surplus bargaining game

Consider any pair of players, i and j, which may generate a unit of surplus (i.e., have at least a network path joining them). Given the prevailing network g, the considerations explained in the text induce a coalitional form with transferable utility given by the characteristic function $v : 2^N \to \mathbb{R}^n$ defined, for each $S \subset N$, as follows:

$$v(S) = \begin{cases} 1 & \text{if } \exists\{i_1, \ldots, i_n\} \subset S \quad \text{s.t. } g_{i,i_1} \cdot g_{i_1,i_2} \cdot g_{i_2,i_3} \cdots g_{i_n,j} = 1, \\ 0 & \text{otherwise.} \end{cases} \tag{A.1}$$

Given any imputation $z \in \mathbb{R}^n$ define the *excess payoff* that can be earned by any *coalition S* by

$$U(S, z) = v(S) - \sum_{i \in S} z_i$$

and the *excess payoff* that can be earned by some player k *against* other player l by

$$u_{k\ell}(z) = \max\{U(S, z) : S \subset N,\ k \in S, \ell \notin S\}.$$

Then the kernel of the game induced by the network g which, in this case, also happens to be in the core [17] is defined as the set of imputations $\hat{z}$ that satisfy, $\forall k, \ell \in N$, one of the following two

[17] It is easy to check that the core of the game associated to the surplus generated by i and j consists of *all* those imputations where inessential players receive a null share. For another application of the concept of kernel to network games, see Binenstock and Bonacich [5].

conditions:

$$u_{k\ell}(\hat{z}) \geqslant u_{\ell k}(\hat{z}), \tag{A.2}$$

$$u_{k\ell}(\hat{z}) < u_{\ell k}(\hat{z}) \Rightarrow \hat{z}_k = v(\{k\}) = 0. \tag{A.3}$$

The intuitive basis for (A.2)–(A.3) is the idea that each player evaluates any imputation $\hat{z}$ contemplated throughout the bargaining process on the basis of the induced excess payoffs $u_{k\ell}(\hat{z})$. Thus, if there is a bilateral "imbalance" in these magnitudes for some pair of players, this is sure to trigger a "reasonable objection" from the unfavored party (i.e., the agent with the higher excess) unless the other is already at her minimum (individually rational) payoff.

We now argue that, given the characteristic function (A.1) associated to a particular pair of players i and j, the induced kernel consists of the *unique* imputation vector $\hat{z}$ satisfying

$$\hat{z}_k = \begin{cases} \frac{1}{e(i,j)+2} & \text{if } k \in E(i,j) \cup \{i,j\}, \\ 0 & \text{otherwise,} \end{cases}$$

thus inducing the payoff function specified in (1). This conclusion follows from the following two claims.

Claim 1. *Consider any pair of players $k, \ell \in E(i,j) \cup \{i,j\}$. Then, any kernel imputation $\hat{z}$ satisfies $\hat{z}_k = \hat{z}_\ell$.*

Claim 2. *Consider any player $k \notin E(i,j) \cup \{i,j\}$. Then, any kernel imputation $\hat{z}$ satisfies $\hat{z}_k = 0$.*

To show Claim 1, suppose that $k, \ell \in E(i,j) \cup \{i,j\}$ but $\hat{z}_k > \hat{z}_\ell$. Given that for any $S \subset N \backslash \{k\}$ and for any $S' \subset N \backslash \{\ell\}$, we have $v(S) = v(S') = 0$. Therefore,

$$\begin{aligned} u_{k\ell}(\hat{z}) &= \max\{U(S,\hat{z}) : S \subset N, k \in S, \ell \notin S\} = U(\{k\},\hat{z}) = -\hat{z}_k \\ &< -\hat{z}_\ell = U(\{\ell\},\hat{z}) \leqslant \max\{U(S,\hat{z}) : S \subset N, k\ell \in S, k \notin S\} = U_{\ell k}(\hat{z}). \end{aligned}$$

By virtue of (A.3), this requires that $\hat{z}_k = 0$, which is a contradiction with the fact that $\hat{z}_\ell \geqslant 0$.

Next, to establish Claim 2, suppose that $k \notin E(i,j) \cup \{i,j\}$ but $\hat{z}_k > 0$. Consider then some other $\ell \in E(i,j) \cup \{i,j\}$. Again, since $v(S) = 0$ for any $S \subset N \backslash \{\ell\}$, we have that

$$u_{k\ell}(\hat{z}) = -\hat{z}_k < 0.$$

Considering now the reciprocal excess payoff $u_{\ell k}(\hat{z})$, note that $S_0 \equiv E(i,j) \cup \{i,j\} \subset N \backslash \{k\}$ so that $v(S_0) = 1$ and, therefore,

$$\begin{aligned} u_{\ell k}(\hat{z}) &= \max\{u(S,z) : S \subset N,\ k \in S,\ \ell \notin S\} \\ &\geqslant u(S_0, z) \geqslant 1 - \sum_{u \neq k} \hat{z}_u = \hat{z}_k > 0. \end{aligned}$$

Therefore, (A.3) requires that $\hat{z}_k = 0$, a contradiction.

Proof of Proposition 1. The proof of the result requires two preliminary lemmas. The first one concerns critical links, i.e., links that define the only path between the two end players (and whose deletion, therefore, would increase the number of components). It establishes that the marginal payoff of any such critical link is equal for the two players involved. The second lemma shows that in any component of a network, there are at least two non-essential players, i.e., players who are not essential for any interaction (and therefore enjoy only access payoffs).

Lemma 1. *Consider any network g. If $g_{ij} = 1$ and the link is critical then $M_i(g_{ij}; g) = M_j(g_{ij}; g)$.*

Proof. By hypothesis g_{ij} is critical, and so it follows that i and j lie in different components in the network $g - g_{ij}$. Let $C_i(g)$, $C_j(g)$ be the components that contain i and j, respectively, where we shall usually dispense with an explicit account of the dependence and simply write C_i and C_j. The marginal payoff of the link ij for player i is given by

$$\begin{aligned} &M_i(g_{ij}; g) \\ &\quad = \frac{1}{2} + \sum_{k \in C_j \setminus \{j\}} \frac{1}{e(i,k)+2} + \sum_{l \in C_i \setminus \{i\}} \sum_{k \in C_j \setminus \{j\}} \frac{1}{e(l,k)+2} + \sum_{l \in C_i \setminus \{i\}} \frac{1}{e(l,j)+2} - c, \end{aligned}$$

where the first two terms refer to access benefits while the latter two terms refer to essentiality benefits. Similarly, we can write the marginal payoffs of player j form link g_{ij} as

$$\begin{aligned} &M_j(g_{ij}; g) \\ &\quad = \frac{1}{2} + \sum_{l \in C_i \setminus \{i\}} \frac{1}{e(j,l)+2} + \sum_{l \in C_i \setminus \{i\}} \sum_{k \in C_j \setminus \{j\}} \frac{1}{e(l,k)+2} + \sum_{k \in C_j \setminus \{j\}} \frac{1}{e(i,k)+2} - c. \end{aligned}$$

It follows then that $M_i(g_{ij}; g) = M_j(g_{ij}; g)$ and the proof of the lemma is complete. □

Lemma 2. *In a network g, a component with m players has at least two non-essential players.*

Proof. Consider any arbitrary component of the network with m nodes. First, we note that from any arbitrary connected network one can reach a line network by a series of steps that involve only two operations: (i) removal of links until a tree is obtained, clearly this increases the number of essential players. (ii) Now fix two players who are extremal and look at players who do not lie on the path between them. Pick any extremal player from this set and delete her single link. Relink this player with one of the two extremal players identified above. It is easy to see that any such operation will (weakly) increase the number of essential players. Since in the line network the two extremal players are non-essential, it follows that the maximum number of essential players in the original component can be no higher than $m - 2$. Thus, any component in a network with m players must have at least two non-essential players. □

Equipped with Lemmas 1 and 2, we proceed with the proof of the proposition. Let g be a non-empty BE network, and suppose it is not connected. Let $\hat{C}$ be the largest component in g, which must therefore contain at least two players. We claim that there is a player $j \notin \hat{C}$ that can establish a mutually profitable link with some player in $\hat{C}$. For simplicity, we shall consider the extreme (and less favorable) case where j has no connections, i.e., defines a singleton component.

By Lemma 2, we know that $\hat{C}$ has some non-essential player $i \in \hat{C}$. Two possibilities need to be considered separately. One is that i is extremal, i.e., she has only one link that connects her to some other player ℓ in the component. Then, it is clear that, since i and ℓ both find it profitable to keep their link, player ℓ would find it optimal to create a link with j, and so would player j. This contradicts the hypothesis that g is a BE network.

The second possibility is that i is non-extremal and therefore $\eta_i \geqslant 2$. Let $\mathcal{N}_i^m(g) = |\{j \in C_i : e(i, j) = m\}|$ be the players whom i accesses via m essential players and let $\eta_i^m(g) = |\mathcal{N}_i^m(g)|$. The payoffs of this player i in network g are then given by

$$\frac{\eta_i^0(g)}{2} + \frac{\eta_i^1(g)}{3} + \frac{\eta_i^2(g)}{4} + \cdots + \frac{\eta_i^r(g)}{r+2} - \eta_i(g)c \tag{A.4}$$

for some $r \leqslant n-2$.

Since g is an equilibrium, it follows then that

$$\frac{1}{\eta_i(g)}\left[\frac{\eta_i^0(g)}{2} + \frac{\eta_i^1(g)}{3} + \frac{\eta_i^2(g)}{4} + \cdots + \frac{\eta_i^r(g)}{r+2}\right] \geqslant c. \tag{A.5}$$

Now let us examine marginal returns for player $j \notin \hat{C}$ from a link with player i. Suppose, for simplicity, that player j is a singleton component. Then the *marginal returns* to j are given as follows:

$$\frac{\eta_j^0(g+g_{ij})}{2} + \frac{\eta_j^1(g+g_{ij})}{3} + \frac{\eta_i^2(g+g_{ij})}{4} + \cdots + \frac{\eta_i^r(g+g_{ij})}{r+2} - c, \tag{A.6}$$

where, by analogy with previous notation, $g+g_{ij}$ simply denotes the network obtained by replacing g_{ij} in network g by a new $g_{ij} = 1$.

Note now that for every $k \in \hat{C}\backslash\{i\}, e(j, k; g+g_{ij}) = e(i, k; g)+1$. Thus $\eta_j^m(g+g_{ij}) = \eta_i^{m-1}(g)$ for every $m \geqslant 1$ and $\eta_j^0(g+g_{ij}) = 1$. Using these facts we can write the marginal returns of player j from the link with i as follows:

$$\frac{1}{2} + \frac{\eta_i^0(g)}{3} + \frac{\eta_i^1(g)}{4} + \cdots + \frac{\eta_i^r(g)}{r+3} - c. \tag{A.7}$$

Now we argue that

$$\begin{aligned}
&\frac{1}{2} + \frac{\eta_i^0(g)}{3} + \frac{\eta_i^1(g)}{4} + \cdots + \frac{\eta_i^r(g)}{r+3} \\
&\quad > \frac{1}{2}\left[\frac{\eta_i^0(g)}{2} + \frac{\eta_i^1(g)}{3} + \frac{\eta_i^2(g)}{4} + \cdots + \frac{\eta_i^r(g)}{r+2}\right] \\
&\quad \geqslant \frac{1}{\eta_i(g)}\left[\frac{\eta_i^0(g)}{2} + \frac{\eta_i^1(g)}{3} + \frac{\eta_i^2(g)}{4} + \cdots + \frac{\eta_i^r(g)}{r+2}\right] \\
&\quad \geqslant c.
\end{aligned}$$

The first inequality is immediate, while we use $\eta_i(g) \geqslant 2$ in deriving the second inequality and Eq. (A.5) in deriving the final inequality. We now apply Lemma 1 to conclude that player i also has a strict incentive to form a link with j, given that all existing links are retained. But note that given that link g_{ij} is formed, player i has no incentive to delete any of his erstwhile links since (roughly speaking) the marginal returns from each of these links has actually increased. Thus players i and j have a strict incentive to form an additional link. These arguments extend directly to cover the case where j belongs to a non-singleton component. Thus g is not a BE network, a contradiction that completes the proof. □

Proof of Theorem 1. As indicated in the text, the proof can be decomposed into five steps. In what follows, each of these steps is formally embodied by a corresponding lemma. All of them assume that $c > \frac{1}{6}$ and $n \geqslant \hat{n}(c)$ for some suitable $\hat{n}(c)$.

Lemma 3. *The star is the only minimal network which can be sustained in an SBE.*

Proof. Consider a g that is minimally connected but not a star. Then, there are two players, say i and j, such that (a) $e(i, j; g) \geqslant 2$; (b) they are "end players", i.e., $\eta_i(g) = \eta_j(g) = 1$. Then, it readily follows that for at least one of them, say i, the following holds: there is a player x with $e(i, x; g) = 1$ (thus, x is two steps away from i in g) and the set of players k for whom x is essential in connecting i and k has a cardinality that is at least $(n-4)/2$, i.e., $|\{k \in N : x \in E(i, k; g)y\}| \geqslant (n-4)/2$. (This is true because, roughly speaking, in a tree every link is a bridge between two components, one of which has at least half the players.)

We argue that such a network g cannot be induced by an equilibrium. First, note that, given the linking cost c, the number of essential players that can be supported in equilibrium between any two players has some (finite) upper bound $\hat{e}(c)$, independent of n. This is because the benefits only depend on the length of the path and do not depend on the total number of players. Consider then the possibility that i and x were to form a link. The gross gains, $\Delta\pi_i$ and $\Delta\pi_x$, induced by that change for i and x (if all other links were to remain in place) are bounded below as follows:

$$\min\{\Delta\pi_i, \Delta\pi_x\} \geqslant \frac{(n-4)/2}{\hat{e}(c)-1} - \frac{(n-4)/2}{\hat{e}(c)} = \frac{n-4}{2\hat{e}(c)(\hat{e}(c)-1)}.$$

This expression is larger than c if n is large enough, which implies that both i and x benefit from a deviation that creates a link between them and keeps all other links. □

Lemma 4. *There can be at most one cycle in an SBE network.*

Proof. Suppose g is an equilibrium network and there are two or more cycles in it. Let $\chi_1 = (i_1, i_2, \ldots, i_n)$ be ordered set of players in one cycle, and let $\chi_2 = (j_1, j_2, \ldots, j_m)$ be those in the other cycle. Since g is connected it follows that there are two possibilities: (1) cycles have common players and (2) cycles have no common players. We take these up in turn.

(1) Cycles have players in common: If there is a single common player i_1 in the two cycles then it is easy to see that the partners of i_1 (say) $i_2 \in \chi_1$ and $j_2 \in \chi_2$ have a strict incentive to delete their links with i_1 and instead form a link with each other. (Throughout, we shall abuse notation and write $i \in \chi$ if i is one of the nodes in the ordered collection of nodes specifying χ) Consider next the case with two or more players in common. Let $(i_1, i_2, \ldots, i_k)$ be the players in common. Suppose that $k \geqslant 3$; the case of $k = 2$ is simple and omitted. Then there exist players i_1, i_x, and j_y with the following properties: $i_x \in \chi_1$ but $i_x \notin \chi_2$, while $j_y \in \chi_2$, and $j_y \notin \chi_1$, and $g_{i_1,i_x} = g_{i_1,j_y} = g_{i_1,i_2} = \cdots = g_{i_{k-1},i_k} = 1$. Note also that like player i_1, i_k must again have links with a player who belongs to one of χ_1 and χ_2 only. It then follows that i_{k-1} and j_y have at least a weak incentive to delete their current link with i_k and i_1, respectively, and instead form a link with each other. It then follows that g cannot be sustained by an SBE.

(2) Cycles have no common players: Since g is an SBE network it is connected and so there exists a path between the two cycles. Let $(i_1, i_2, \ldots, i_k)$ be members of such a path with $i_1 \in \chi_1$ while $i_k \in \chi_2$. Suppose $g_{i_1,i_x} = 1$ and $g_{i_k,j_y} = 1$, where $i_x \in \chi_1$ and $j_y \in \chi_2$. Now it is easy to use a variant of the earlier argument for case 1 above to show that players i_x and j_y have a

strict incentive to delete their link with i_1 and i_k and instead form a link with each other. The proof is complete. □

Lemma 5. *A cycle containing all players cannot be sustained in a BE.*[18]

Proof. Consider two players i and j who are furthest apart in terms of geodesic distance in the cycle. Now consider the deviation in which each of the players deletes one link and they form a link with each other in such a way that they create a line. Assume, for simplicity, that n is even, so that there are $(n-2)/2$ players to one side of player i and $(n-2)/2$ players to the other side of player j in the line created. We now show that players i and j will strictly increase their payoff with this coordinated deviation.

We proceed in two steps: the first step is to show that individual payoffs are strictly increasing as we move toward the center of the line. The payoffs of an individual player consist of two components, the returns from accessing others and the returns from being essential on paths between pairs of other players. Number the players on a line as $1, 2, \ldots, n$. The access returns to player l are given by

$$\frac{1}{l} + \frac{1}{l-1} + \cdots + \frac{1}{2} + \frac{1}{2} + \cdots + \frac{1}{n-l+1}, \tag{A.8}$$

while the access returns to player $l+1$ are given by

$$\frac{1}{l+1} + \frac{1}{l} + \cdots \frac{1}{2} + \frac{1}{2} + \cdots + \frac{1}{n-l}. \tag{A.9}$$

It now follows that the access returns for player $l+1$ are larger than the access returns for player l if $l < n/2$.

We now turn to the returns from being essential. The essentialness payoff to player l can be written as follows:

$$\sum_{i=1}^{l-1} \sum_{j=l+2}^{n} \frac{1}{e(i,j)+2} + \sum_{i=1}^{l-1} \frac{1}{e(i,l+1)+2}. \tag{A.10}$$

Similarly, the essentialness payoffs to player $l+1$ can be written as follows:

$$\sum_{i=1}^{l-1} \sum_{j=l+2}^{n} \frac{1}{e(i,j)+2} + \sum_{j=l+2}^{n} \frac{1}{e(l,j)+2}. \tag{A.11}$$

The first part of the essentialness payoffs to the two players are equal, while the second part of the payoffs are greater for player $l+1$ if $l < n/2$.

Let g^{C} and g^{L} denote the networks prevailing before the contemplated deviation and after it (i.e., the cycle and the line with i and j at the center, respectively). To show that i and j indeed obtain higher payoffs under g^{L}, note that the aggregate gross payoffs obtained in both cases are the same. The above argument implies that i and j enjoy a higher share of total gross value in the line as compared to the other players. This implies that players i and j earn a higher gross payoff in the line. Since their linking cost is the same in both cases (i.e., $2c$,) it follows that they obtain higher net payoffs as well, which completes the proof. □

[18] Here, we need that $\hat{n}(c) \geqslant 4$, since for $n = 3$ a complete network can be sustained in equilibrium for $c < \frac{1}{6}$.

Lemma 6. *An SBE network with a cycle has the hybrid star–cycle architecture, for large enough n.*

Proof. Suppose that g is a non-empty SBE network with a cycle. Let X be the set of players and x the number of players who are outside the cycle, while Y is the set of players in cycle and define $y = n - x$ to be the number of players in cycle. Note that $y \geqslant 4$ since $y = 3$ cannot be sustained in equilibrium given that $c > \frac{1}{6}$. Next we argue that, for any such c, there is a $y(c)$ such that $y \leqslant y(c)$ in equilibrium. Given step 2 above, clearly $x \geqslant 1$. Suppose $g_{ij} = 1$ for some $i \in X$ and $j \in Y$. Since $j \in Y$, there is some $k \in Y$ such that $g_{jk} = 1$. Clearly, player k prefers strictly to switch link from j to i. This reduces the essentialness payoffs he has to pay out to j and keeps his costs constant. On the other hand, i benefits from such an adjustment if $(y - 1)/6 > c$. So given a c, in order for the cycle to be stable, we must have $y \leqslant y(c) \equiv 6c + 1$.

Next we show that all nodes that do not belong to the cycle must be connected to a *single* node in the cycle. The initial observation is that, if n is large enough, there cannot be two distinct trees with different roots lying in the cycle. To see this, consider one of those subtrees that has a number of nodes at least equal to $[n - y(c)]/y(c) = (n/y(c)) - 1$. At least one such tree must exist since there are at most $y(c)$ nodes in the cycle. Let i_1 be the root of this tree and i_2 be the root of any other tree. Now consider any node j in the second tree different from its root, i_2. A straightforward adaptation of the arguments used in the proof of Lemma 3 lead to the conclusion that, if n is large, j and i_1 can both profit from forming a link, whether or not j maintains its link with i_2. The reason is that, since the number of essential players $e(\ell, \ell')$ for any pair of players, ℓ and ℓ', is bounded by some $\hat{e}(c)$, independent of n, the gross gains from the link between j and i_1 grow linearly with n for both players, independent of whether player j maintains the link to i_2. Specifically, it is easy to compute that these gains are bounded below by $(\frac{n}{y(c)} - 1)(\frac{1}{\hat{e}(c)-1} - \frac{1}{\hat{e}(c)})$, where recall that $y(c)$ is the maximum number of agents who can lie in a cycle at equilibrium.[19]

Once established that there can be at most one tree connected to the cycle through its root, again relying on the arguments introduced in Lemma 3 we arrive at the conclusion that, for any two nodes in the tree, u and v, the number of essential players $e(u, v) \leqslant 1$. This still leaves open the possibility that the tree consists of a star with a center at some node i_c that is not part of the cycle but has a link to a node in the cycle. But, in that case, the node i_c and any of the neighbors of k in the cycle, say k', both profit from establishing a direct link, for large enough n. □

Lemma 7. *The star is the unique hybrid–cycle equilibrium.*

Proof. As argued in the previous step, $y \leqslant y(c)$ for a function $y(c)$ that is independent of n. Thus, fixing some c, consider the class of hybrid networks g in which $y \leqslant y(c)$. Let i be the player in the center of the star and suppose that $j, k \in Y$ with $g_{ij} = g_{ik} = 1$. We now show that j and k have a strict incentive to form a link if number of peripheral players $x \geqslant n - y(c)$ is sufficiently large. The payoffs of players j and k in the hybrid network g are given by

$$\pi_j(g^{\mathrm{H}}) = \pi_k(g^{\mathrm{H}}) = \frac{x}{3} + \frac{y-1}{2} - 2c. \tag{A.12}$$

[19] Note that player j and each of those in the tree rooted at i_1 have to share their respective surplus with at most $\hat{e}(c)$ players at equilibrium. Thus, by circumventing i_2, j, and i_1 benefit by $(\frac{1}{\hat{e}(c)-1} - \frac{1}{\hat{e}(c)})$ per each of the transactions (at least $\frac{n-y(c)}{y(c)}$ of them) between j and every player in that tree.

Now consider a deviation by players j and k in which player k deletes his link with player i and player j deletes his link with player m in the cycle and instead players j and k form a link with each other. The resulting network is a minimal network g' in which there are x peripheral players and a line starting with player i which consists of y players. The payoffs of player j in g' are given by

$$\pi_j(g') = \frac{x}{3} + \frac{1}{2} + \sum_{k=2}^{y-1} \frac{1}{k} + x \sum_{k=4}^{y+1} \frac{1}{k} + \sum_{k=3}^{y} \frac{1}{k} - 2c. \tag{A.13}$$

These payoffs are bounded below by

$$\frac{x}{3} + x\frac{y-2}{y+1} - 2c = M. \tag{A.14}$$

Next note that $M > x/3 + (y-1)/2 - 2c$ if

$$x > \frac{y-1}{2}\frac{y+1}{y-2}. \tag{A.15}$$

Since $y \geqslant 4$, the right-hand side is increasing in y and bounded above by $[y(c)-1][y(c)+1]/2[y(c)-2]$. The final step is to note that $x = n - y \geqslant n - y(c)$ and so (A.15) applies for sufficiently large n. Thus player j has a strict incentive to switch links to player k for large n We now turn to the incentives of player k.

The payoffs of player k in g' are given by

$$\pi_k(g') = \frac{x}{4} + \frac{1}{3} + \frac{1}{2} + \sum_{k=2}^{y-2} \frac{1}{k} + x \sum_{k=5}^{y+1} \frac{1}{k} + \sum_{k=4}^{y} \frac{1}{k} - 2c. \tag{A.16}$$

These payoffs are bounded below by

$$\frac{x}{4} + x\frac{y-3}{y+1} - 2c = M'. \tag{A.17}$$

Note that $M' > x/3 + (y-1)/2 - 2c$ if

$$x > \frac{6(y-1)(y+1)}{11y-37}. \tag{A.18}$$

Since $y \geqslant 4$, the right-hand side is positive and increasing in y and so is bounded above by $6[y(c)-1][y(c)+1]/[11y(c)-37]$. Note that $x = n - y \geqslant n - y(c)$ is larger than this term for sufficiently large n. Thus, player k has a strict incentive to switch links to player j, for sufficiently large n. □

Combining Lemmas 3–7 the proof of Theorem 1 is complete. □

Proof of Proposition 3. In a cycle, all players are symmetrically located, so we need only consider the incentives for a typical player, say some player i. First note that the payoffs of player i in a cycle are $(n-1)/2 - 2c$. The payoffs from deleting both links are 0 and clearly for a given c, there is always a large enough n such that deleting both links is not optimal. Consider next the deviation of deleting one link. If player i deletes one link and keeps the other link as in the cycle then he becomes an end player of the line network. In this network his payoffs are given by

$$\sum_{k=2}^{n} \frac{1}{k} - c. \tag{A.19}$$

The payoff from both links in cycle are higher if

$$\frac{n-1}{2} - \sum_{k=2}^{n} \frac{1}{k} > c. \tag{A.20}$$

Clearly this inequality holds for large enough n. We next consider the deviation in which player i deletes both links but forms a new link (say) with the center of the line network that arises. Again, a variation of the above argument shows that for large n this reduces payoffs.

We finally consider deviations by player i which involve coordinated deviation with one other player. Suppose that in the cycle he has a link with $i-1$ and $i+1$. In the deviation, he maintains the link with $i-1$ but deletes the link with $i+1$ and instead forms a link with some player k. If player k retains all his links as in the cycle, it is easy to see that the payoffs of the player go down strictly since the costs remain the same (he maintains two links) while the gross payoffs decline since player k is essential for accessing at least one player, namely $i+1$. So this deviation is not profitable. If player k deletes both his links then the payoffs from the deviation are still lower and so it is clearly not profitable. We turn to the final case, where players i and k coordinate and link up with each other but delete a link each so that the new network is a line. From Lemma 5 it follows that for players such a deviation is profitable. However, now we need to check whether this deviation is credible: do the players have an incentive to actually delete one of their links? We show that for large n, at least one of the players has a strict incentive to retain their links in the cycle. Number the players in the line as $1, 2, \ldots, n$ from left to right. So there are $i-1$ players to the left of player i while there are $n-k$ players to the right of player k in the line. Suppose without loss of generality that $i-1 \geqslant n-k$. Player k gets a payoff $\sum_{l=2}^{i+1} 1/l$ in the line network from these players $1, 2, \ldots, i$. The payoffs can be increased to $i/2$ if player k forms a link with player 1. Clearly, it is profitable for player k to deviate from the deviation and form a link with player 1, if n is large. It is similarly possible to show that player 1 would have an incentive to form a link with player k, if n is large. Thus, the deviation in which i and k deviate to create a line is not two-person coalition proof. Finally, we note that starting from a cycle it is not profitable for player i to form any additional links. The proof for cycle being an SBPE network is complete. □

Proof of Proposition 4. First, we note that an application of the arguments used in the proofs of Proposition 1 and Lemmas 3, 4, and 6 establishes that, starting from any arbitrary network, there is a feasible path of unperturbed adjustment that leads, in an finite number of steps, to an SBPE network.[20] Then, since by construction an SBPE network is stationary under the adjustment rules (i) and (ii) contemplated in Section 4.1, it follows that the unperturbed dynamics (with $\varepsilon = 0$)

[20] Thus, for example, if the original network is disconnected into several components, there is positive probability that it becomes connected or empty; or, if it includes several cycles, there is positive probability that these cycles eventually merge them into a single one; or if connected to a singe cycle where there are several trees, there is positive probability that these trees merge into a single one; or if a single cycle has a single tree attached to it, there is positive probability that the latter becomes a star attached to the cycle.

converges almost surely to a network that is either empty, or displays a cycle, a hybrid cycle–star, or a star architecture.

Consider now the perturbed dynamics induced by some $\varepsilon > 0$. Clearly, this dynamics is ergodic and thus a transition across all stationary states of the unperturbed dynamics is possible. Then, in essence, the proof involves showing that some of the transitions across those stationary states are infinitely more unlikely than others when ε is infinitesimally small. The line of argument can be decomposed into the following steps.

Step 1: Starting from a cycle or a hybrid cycle–star network, just one mutation can trigger an unperturbed adjustment path leading to a star network.

For concreteness, let us focus on the case where the original network is a strict cycle, since it should be clear how the argument can be adapted to the case where it is a hybrid cycle–star. Suppose that the mutation deletes one of the existing links. Then, the resulting network is a line, so let players be indexed consecutively from one end to the other, $i = 1, 2, \ldots, n$, with n being odd for simplicity and player $\frac{n+1}{2}$ representing the central player. Suppose that player $\frac{n-3}{2}$ is given the option of linking to that central player and, simultaneously, considers deleting her link to player $\frac{n-1}{2}$. Both players $\frac{n-3}{2}$ and $\frac{n+1}{2}$ strictly benefit from this adjustment if the population is large enough. Concerning player $\frac{n-3}{2}$, this happens because the payoffs she obtains from players $\frac{n-1}{2}$ and $\frac{n+1}{2}$ are simply permuted, whereas in the modified network the number of essential players that $\frac{n-3}{2}$ must count on in order to access each $i = \frac{n+3}{2}, \frac{n+5}{2}, \ldots, n$ is one less than before. On the other hand, to see that player $\frac{n+1}{2}$ also benefits strictly from the adjustment note that her gain in *gross* payoffs $\Delta\Pi$ is given by

$$\begin{aligned}
\Delta\Pi = & \left[\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{\frac{n-1}{2}-1}-\frac{1}{\frac{n-1}{2}}\right)\right] \\
& +\left[\left(\frac{1}{2+1}-\frac{1}{3+1}\right)+\left(\frac{1}{3+1}-\frac{1}{4+1}\right)+\cdots+\left(\frac{1}{\frac{n-1}{2}-1+1}-\frac{1}{\frac{n-1}{2}+1}\right)\right] \\
& +\cdots+\left[\left(\frac{1}{2+\frac{n-1}{2}}-\frac{1}{3+\frac{n-1}{2}}\right)+\left(\frac{1}{3+\frac{n-1}{2}}-\frac{1}{4}\right)\right. \\
& \left.+\cdots+\left(\frac{1}{\frac{n-1}{2}-1+\frac{n-1}{2}}-\frac{1}{\frac{n-1}{2}+\frac{n-1}{2}}\right)\right] \\
= & \left[\frac{1}{2}-\frac{1}{\frac{n-1}{2}}\right]+\left[\frac{1}{2+1}-\frac{1}{\frac{n-1}{2}+1}\right]+\left[\frac{1}{2+\frac{n-1}{2}}-\frac{1}{\frac{n-1}{2}+\frac{n-1}{2}}\right] \\
= & \sum_{r=0}^{\frac{n}{2}}\left[\frac{1}{2+r}-\frac{1}{\frac{n-1}{2}+r}\right].
\end{aligned}$$

As n grows, the asymptotic behavior of the above expression can be approximated by

$$\int_0^{\frac{n}{2}}\left[\frac{1}{2+r}-\frac{1}{\frac{n-1}{2}+r}\right]dr = \ln\left(2+\frac{n-1}{2}\right)$$

which increases unboundedly as $n \uparrow \infty$. The *net* gains of the contemplated adjustment for player $\frac{n+1}{2}$ consist of the gross gain $\Delta\Pi$ net of the cost c for the additional link she supports with $\frac{n-3}{2}$. It follows, therefore, that, given any c, the net gains $\Delta\Pi - c > 0$ as long as n is large enough.

However, in connection to the previous bilateral adjustment by players $\frac{n-3}{2}$ and $\frac{n+1}{2}$, the question remains as to whether it is *bilateral proof*. Clearly, no deviation from such an adjustment can be optimal for player $\frac{n+1}{2}$. On the other hand, for player $\frac{n-3}{2}$, it is still conceivable that she might want to deviate and maintain the link to $\frac{n-1}{2}$, since this saves the need to pay intermediation costs to player $\frac{n+1}{2}$ when player $\frac{n-1}{2}$ accesses $\frac{n-3}{2}$ and all players $i < \frac{n-3}{2}$. The gross gain derived from such a deviation would be

$$\begin{aligned}\Delta\Pi &= \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{\frac{n-1}{2}}-\frac{1}{\frac{1}{n+1}}\right)\\ &= \frac{1}{2}-\frac{2}{n+1}\end{aligned}$$

which is always lower than c (assumed no lower than $\frac{1}{2}$). Thus, such a deviation is not profitable and, consequently, the contemplated adjustment is bilateral proof. Now we may proceed iteratively with players that are two steps apart from the central player in the current network (e.g., either player $\frac{n-5}{2}$ or $\frac{n+3}{2}$ at the next iteration). It is immediate to see that the former considerations concerning the profitability and bilateral proofness of the adjustment continue to apply until a star centered at player $\frac{n+1}{2}$ is finally reached. At this point, no further adjustment possibilities will change the network.

Step 2: An unperturbed adjustment path can be constructed from the empty network to a star network after a suitable set of mutations whose number is bounded below depending on c but independent of n.

Simply consider a set of simultaneous mutations that have m agents forming a star centered on, say, agent 1. It is immediate to check that as long as $m-2 > 6c$, all m agents involved are interested in maintaining their links. Thereafter, consider a path along which each of the $n-m$ agents not yet part of the star are given the option of forming a link with player 1. Each of these players as well as player 1 finds it profitable to create the link. This eventually leads to a full star centered on agent 1 involving all n agents in the population.

Step 3: Starting from a cycle, a star, or a hybrid cycle–star network the number of mutations required to have a subsequent adjustment path leading to the empty network grows unboundedly with the population size n.

To see this, suppose that the number of mutations were uniformly bounded. Then, after these mutations, the resulting network would display some component whose size grows unboundedly with n. Consider the set of agents lying in one such component. Given the cost c, if n is large enough, every path of unperturbed adjustment should have those agents continue being in a common component. This rules out convergence to the empty network.

Step 4: If any star network is perturbed by just one mutation, the unperturbed dynamics must return to the original star network, almost surely, if $c > \frac{5}{12}$.

Starting from a star network, let a single mutation arise, introducing a new link or destroying an existing one. Consider first the possibility that a new link is created, which must be between two "spoke" players, say i and j. Since these are connected indirectly through the hub of the star network, they will remove their link if given the opportunity because $c > \frac{1}{6}$. Once this happens, the original star network is reached again and the system remains stationary thereafter in the absence of any further mutations. Let us now suppose that the mutation in question destroys one of the existing links between the hub player and some spoke player i. If i meets the hub again, their link is restored and the system returns to the original star network. Instead, if i meets some other spoke player j, a link between them is profitable to both (if n is large enough) and then will be established with positive probability. If this occurs, however, we can be sure that no additional link between i and some other spoke agent $k \neq j$ will be created, as long as her link with j remains in place. This is a consequence of the assumption that $c > \frac{5}{12}$.[21] Thus, when i comes to be matched again with the hub agent, there is positive probability that a link is established between them alongside the removal of the link between i and j. This, which will eventually happen almost surely, restores the original star network.

Combining the aforementioned conclusions, we can rely on the graph-theoretic techniques developed by Freidlin and Wentzell [17] to characterize the stochastically stable networks of the process. Since this approach is by now standard in modern evolutionary theory, we simply outline the argument, dispensing with formal notation and specific details.[22]

Let g^* be a star network and denote by $\tilde{g}$ some other network that also defines an SBPE but is *not* a star. Let us first suppose that $\tilde{g}$ is a cycle network. In essence, what we need to show is that the minimum *aggregate* number of mutations required to implement transitions from *every* other stationary state g to the network g^* is lower than that for $\tilde{g}$. In the language used by modern evolutionary literature, a pattern of possible paths of transition from all other states towards the particular states g^* and $\tilde{g}$ is formalized by so-called g^*- and $\tilde{g}$-trees, i.e., directed trees in which there is a unique path linking any other state g to the state (root) g^* or $\tilde{g}$, respectively. Each mutation involved in these transitions is regarded as a (mutation) *cost* and the overall number of mutations contemplated in any such tree is called its *resistance*. Thus, in these terms, our aim is to show that the lowest resistance among all g^*-trees is lower than that achievable among all $\tilde{g}$-trees.

Suppose, for the sake of contradiction, that the lowest-resistance g^*-tree has a resistance no smaller than some $\tilde{g}$-tree. In the latter tree, there is of course an arrow joining state g^* to some other stationary state of the unperturbed dynamics, say g'.[23] Thus, departing from that $\tilde{g}$-tree, a g^*-tree can be constructed as follows. First, delete the arrow originating from state g^* in the $\tilde{g}$-tree. Second, add an arrow connecting $\tilde{g}$ to g^*. Clearly, the former two "tree-pruning" operations give rise to a well-defined g^*-tree. Moreover, the first operation removed a link with a (mutation) cost no lower than 2, in view of step 4 above, while the second operation included a link whose cost is just 1, by virtue of step 1. Thus, in total, we find that the resulting g^*-tree has a cost strictly lower than the original $\tilde{g}$-tree, a contradiction that establishes the desired conclusion.

It should be clear that a similar logic applies if the alternative SBPE network $\tilde{g}$ is a hybrid cycle–star. Thus consider the remaining case where $\tilde{g}$ is the empty network. We now discard the possibility that $\tilde{g}$ is stochastically stable. Consider any $\tilde{g}$-tree. By step 3, this tree has an arrow originating in some other (non-empty) SBPE network $\hat{g}$ whose cost is arbitrarily high for large enough n. Suppose now that this link is removed and instead an arrow is added from $\tilde{g}$ (the empty network) to another SBPE network (possibly $\hat{g}$ itself) with a cost that is bounded above uniformly, independent of n. The establishment of this arrow is possible, in view of step 2. The resulting

[21] If $c \leqslant \frac{5}{12}$, it is profitable for agent i to sustain both the link to j and the link to some other k, thus giving rise to a hybrid cycle–star. In fact, under these conditions, such a low linking cost allows for a subsequent transition—implemented through unperturbed adjustment alone—from such resulting hybrid cycle–star to a full cycle. (Here, we can rely on the same considerations that underlie the argument for step 4 in our main theorem.) The fact that an unperturbed transition can be triggered both ways by a single mutation and subsequent unperturbed adjustment indicates (following the standard logic in evolutionary theory) that both the cycle and the star architectures are stochastically stable in this case.

[22] We again refer to the bibliography listed in footnote 15 for a detailed explanation of the approach and methodology used.

[23] It is well known (see e.g., [41]) that it is enough to restrict considerations to directed g-trees that include only stationary states of the unperturbed dynamics.

$\hat{g}$-tree has a cost lower than the original $\tilde{g}$-tree if n is large enough, which confirms that $\tilde{g}$ cannot be stochastically stable and completes the proof. □

Proof of Proposition 6. The proof is presented in terms of a number of claims.

Claim 1. *Any SBPE network is either connected or empty*.

The stability of the empty network for $c > \frac{1}{2}$ is immediate, so we focus on the connectedness part. Suppose there is a non-empty component which does not include all players. First consider the case that this component is a tree. There exists a player x who is linked to some end player. Suppose for the sake of argument that x is capacity constrained. This player x is weakly betteroff by deleting the link with the end player and forming a link with another player outside the component. On the other hand, the latter player must be weakly betteroff by such a link if she is either isolated or an end player. If no such player (isolated or end player) exists outside the component under consideration, then we can simply use arguments in Proposition 1 above to infer that there is some player in a cycle who is strictly betteroff by linking with x. This rules out that a tree component not including all players could be part of an equilibrium. Second, consider the case where the component in question contains a cycle. Then, simple variations on arguments in Proposition 1 along with the above argument (which takes into account capacity constraints) rule out partially connected networks.

Claim 2. *Any connected SBPE network contains at most one cycle*.

Note that Lemma 4 does not involve the formation of additional links, and so it can be applied to prove this claim.

Claim 3. *In an SBPE network with a cycle all players must belong to the cycle*.

Consider an SBPE network with an m-player cycle, $m \geqslant 3$. A variation of arguments in Lemma 6 can be used to show that given c, if m exceeds some suitably specified upper bound $y(c)$, there is a profitable (and consistent) bilateral deviation by two players—one in the cycle and another outside it—that expands the cycle by bringing the latter player into it. Thus if there are players outside a cycle then the number of players in the cycle are bounded above by $y(c)$, which is independent of n.

Suppose, for the sake of contradiction, that there is an SBPE network where some player i does not belong to the cycle. Given any K and c, if n is large enough, we can choose i so that its distance to some player in the cycle is arbitrarily long, say no shorter than any d. Consider now a possible bilateral deviation by player i and some player j in the cycle. Clearly, if d is high enough, both player i and j benefit from establishing a new link and also keeping all their other links. This contradicts the hypothesis that the original network was induced by an equilibrium.

Claim 4. *Every SBPE network includes a cycle*.

Consider SBPE network that has no cycles. Let i and j be two extremal players (who have one link each) which are maximally apart, denoted by d. For fixed K, d can be made arbitrarily long if n is large enough. Thus, i and j have a profitable and consistent bilateral deviation, which involves keeping all their previous links and establishing a new one between them.

The proof follows from Claims 1–4. □

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