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Clustering in network games

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ABSTRACT

In many circumstances, behavior and well-being of people depend on the actions taken by their social contacts. Previous research has mainly studied how individuals' choices are shaped by their number of connections, while the incidence that other features of their social networks have on their behavior has been understudied. This paper analyzes the role of network clustering in Bayesian games of strategic substitutes and strategic complements played on networks, which reflect for instance public good provision and technology adoption, respectively. In our framework, players have incomplete information about the interaction network that includes the number of triads in the network — information labeled as *perceived clustering* throughout the paper. We show that equilibrium actions are non-decreasing (non-increasing) in perceived clustering under strategic substitutes (complements). Greater perceived clustering thus increases public good provision and reduces the adoption of complementary technologies in the Bayesian equilibria.

1. Introduction

How does network clustering—a prevalent feature of real-life social networks (Jackson, 2010)—shape human behavior? The literature has identified different mechanisms of influence. One is information-based: news flow faster and more reliably through clustered networks, what may discourage misbehavior because of reputational effects (Burt, 2005). As a second mechanism, clustering promotes collective sanctioning systems: people may coordinate efforts to punish an individual more easily if their networks are tightly-clustered (Coleman, 1988a,b). These mechanisms constitute the basis of theories of social capital that posit that clustering might prevent the emergence of free-riding attitudes. Yet, there is a paucity of work that provides foundations for how network clustering relates to behavior (see e.g. Jackson et al., 2012, for an exception).

There are two main challenges when studying network games. First, it is problematic to isolate the incidence that each network feature has on behavior because changing one network property affects the whole network architecture. Second, even if one focuses on a particular network, a bewildering range of equilibria arises when players have complete information about the network they belong to (e.g. Bramoullé and Kranton, 2007). One approach to solve these problems is the introduction of incomplete network information. Under incomplete information, behavior may only depend on anticipated interaction patterns, simplifying the analysis. Moreover, in many real-life situations, people indeed have incomplete information about their network and, even if complete information is available, people exhibit cognitive limitations while recalling information about the network architecture (Dessi et al., 2016).

This paper explores the impact of clustering in network games of strategic substitutes and strategic complements, reflecting decisions such as public good provision and technology adoption, respectively. We extend the incomplete-information framework of Galeotti et al. (2010) by incorporating players' information about network clustering. In particular, we assume that players privately learn their own degree, while both the degree distribution and the (positive) number of triangles in the network – information that we label *perceived clustering* – are common knowledge.[1] The ignorance of players about other network aspects induces a Bayesian game where players' degrees are interpreted as their types.

Our information structure applies to many real life situations. Newcomers to a small village may expect that their potential partners are likely to know each other. On the contrary, first-year students may believe that their potential colleagues come

[1] Galeotti et al. (2010) abstract from clustering. Their framework can be considered as a special case of ours when players know that there are no triangles in the network.

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from distant places and are not linked. In most real-life situations, people's expectations about clustering lay in between these two extremes. We explore how behavior changes as perceived clustering varies and why.

We show that greater perceived clustering discourages people from free-riding. Although this direction is in line with the effect of clustering from Coleman (1988a), our result is not driven by the emergence of collective sanctions. Rather, we uncover a novel mechanism through which clustering may influence behavior: if the connectivity of one's neighbors is unknown, more clustering induces players to believe that their neighbors' degrees are more likely to be correlated and thus similar. Such increased correlation in the perceived distribution of neighbors' degrees affects behavior differently in the two games, leading to greater activity levels under strategic subsitutes and to lower ones under strategic complements.

2. Network games

Network. Let $g = (N, E)$ be a finite network composed of set of players $N = \{1, \ldots, n\}$ and a set of links E between them; $n = |N|$. We write $g_{ij} = 1$ if agents $i \in N$ and $j \in N$ are directly linked in g and $g_{ij} = 0$ otherwise, with $g_{ij} = g_{ji}$, $\forall i, j \in$ N. We sometimes denote a link between i and j by ij. Define $N_i(g) = \{j \in N : g_{ij} = 1\}$ as i's neighborhood. The degree of i is $k_i(g) = |N_i(g)|$.

A triangle $Z^3(g)$ is a set of nodes $\{i, j, k\}$ such that $g_{ij}g_{jk}g_{ki} = 1$. The set of all triangles in g is $S^3(g)$, and $S_i^3(g) = \{Z^3(g) : i \in Z^3(g)\}$ is the set of triangles to which i belongs. The clustering coefficient of i is $C_i(g) = \frac{2\left|S_i^3(g)\right|}{k_i(g)\left(k_i(g)-1\right)}$.

Games. We consider the games of strategic substitutes and strategic complements in Bramoullé and Kranton (2007)[2] and Jackson (2010), respectively. In both games, each agent chooses simultaneously an action in $X = \{0, 1\}$. Playing 1 bears a cost c, with $0 < c < 1$; playing 0 bears no cost. Let x_i be the action of player $i \in N$. The utility of each i is $u_i(x_i, \bar{x}_{N_i(g)})$, where $\bar{x}_{N_i(g)} = \sum_{j \in N_i(g)} x_j$.

Under strategic substitutes:

$$u_i(0, \bar{x}_{N_i(g)}) = 0 \quad \text{if} \quad \bar{x}_{N_i(g)} = 0$$

$$u_i(0, \bar{x}_{N_i(g)}) = 1 \quad \text{if} \quad \bar{x}_{N_i(g)} \geq 1$$

$$u_i(1, \bar{x}_{N_i(g)}) = 1 - c \quad \text{for any} \quad \bar{x}_{N_i(g)}$$

Under strategic complements:

$$u_i(1, \bar{x}_{N_i(g)}) = 1 - c \quad \text{if} \quad \bar{x}_{N_i(g)} \geq 1$$

$$u_i(1, \bar{x}_{N_i(g)}) = -c \quad \text{if} \quad \bar{x}_{N_i(g)} = 0$$

$$u_i(0, \bar{x}_{N_i(g)}) = 0 \quad \text{for any} \quad \bar{x}_{N_i(g)}$$

Information. Players know that each link in the network was formed randomly with probability p as in the Erdös–Renyi model. They also know their own degree and the set of feasible degrees in the network, denoted $F(g) = \{1, 2, \ldots, \bar{k}\}$, but not the degree of other players. Besides, players have information about the number of triangles in g, denoted $\tau(g) = \left|S^3(g)\right|$. We refer to $\tau(g)$ as *perceived clustering*. The information set of each $i \in N$ is then $I_i(g) = \{k_i(g), p, F(g), \tau(g)\}$.

Players learn the degree distribution from the joint knowledge of p and $F(g)$. Given $I_i(g)$, the probability that $j \in N_i(g)$ has degree k is:

$$P[K_j(g) = k \mid I_i(g)] = \binom{\bar{k} - 1}{k - 1} p^{k-1}(1 - p)^{\bar{k}-k} \tag{1}$$

where $K_j(g)$ is the random variable of the degree of $j \in N_i(g)$.

We intentionally assume that players do not know the network size. The reason is that, if they did, any variation in perceived clustering may be accompanied by a variation in the perceived connectivity of players. Suppose for instance that $I_i(g) = \{k_i(g), p, n, \tau(g)\}$. Then, the maximal degree that neighbor $j \in N_i(g)$ can have is $\bar{k} = n - 1 - k_i(g_1) + \tau(g)$ in i's beliefs, since j may form a maximum of $\tau(g)$ links with agents in $N_i(g)$, and a maximum of $|N \setminus \{j\}| - k_i(g) = n - 1 - k_i(g)$ links with agents outside $N_i(g)$. If rather $I_i'(g) = \{k_i(g), p, n, \tau(g) + 1\}$, the maximal degree of $j \in N_i(g)$ is $\bar{k}' = n - 1 - k_i + \tau(g) + 1$, implying:

$$P[K_j(g) = k \mid I_i(g)] = \binom{\bar{k} - 1}{k - 1} p^{k-1}(1 - p)^{\bar{k}-k}$$

$$\neq \binom{\bar{k}' - 1}{k - 1} p^{k-1}(1 - p)^{\bar{k}'-k} = P[K_j(g) = k \mid I_i'(g)] \tag{2}$$

Hence, to isolate the effect of $\tau(g)$ from that driven by a different perceived connectivity of neighbors, we assume that players do not know n

Players' types are given by their degrees. A symmetric strategy σ is a mapping that specifies a player's action as a function of his or her type, $\sigma(k_i(g)) \in [0, 1]$. Following Galeotti et al. (2010), we focus on the symmetric equilibria of the games, which are the only possible equilibria since:

(i) All degree-k players have the same probability of having a particular neighbor;

(ii) The maximal number of links that can exist among each player's neighbors is $\tau(g)$ and the probability of these links is the same for all players;

(iii) The utility function is identical for all agents.

Under these conditions, all degree-k players face the same probability over neighbors' actions, and thereby, the same decision problem.

3. Results

Triangle-free networks. In Galeotti et al. (2010), the information set of each $i \in N$ is $I_i^G(g) = \{k_i(g), p, F(g), \tau(g) = 0\}$ (or, equivalently $I_i^{G'}(g) = \{k_i(g), p, n, \tau(g) = 0\}$).[3] Players know that their neighbors are not linked ($\tau(g) = 0$), and thus have degrees with independent probabilities. Since the probability of having at least one neighbor playing 1 increases with degree, players with higher degrees have greater incentives to play action 0(1) under strategic substitutes (complements). Hence, under strategic substitutes (complements) the equilibrium is characterized by a threshold and that is non-increasing (non-decreasing).

Let t_{ss} the value for which:

$$1 - \left[P[K_j(g) > t_{ss} \mid I_i^G(g)]\right]^{t_{ss}} = 1 - c$$

holds, with $P[K_j(g) > t_{ss} \mid I_i^G(g)] = 1 - \sum_{k=1}^{t_{ss}} \binom{\bar{k}-1}{k-1} p^{k-1}(1 - p)^{\bar{k}-k}$.

Under strategic substitutes, any equilibrium strategy σ satisfies: $\sigma(k) = 1$ for $k < t_{ss}$, $\sigma(k) = 0$ for $k > t_{ss}$ and $\sigma(k) \in [0, 1]$ for $k = t_{ss}$.

[2] We consider a simplified version of their model.

[3] See Remark 1 in Appendix.

Reasoning is analogous for strategic complements. Here, any (non-trivial)[4] equilibrium strategy σ satisfies: $\sigma(k) = 0$ for $k < t_{sc}$, $\sigma(k) = 1$ for $k > t_{sc}$ and $\sigma(k) \in [0, 1]$ for $k = t_{sc}$, where t_{sc} is the value for which:

$$1 - \left[P[K_j(g) < t_{sc} \mid I_i^G(g)] \right]^{t_{sc}} = c$$

satisfies, $P[K_j(g) < t_{sc} \mid I_i^G(g)] = 1 - \sum_{k=t_{sc}}^{\bar{k}} \binom{\bar{k}-1}{k-1} p^{k-1}(1-p)^{\bar{k}-k}$.

Clustered networks. We contribute to the literature by analyzing the effects of varying perceived clustering on equilibrium behavior. To that aim, we compare two networks g_1 and g_2. In both networks, each i expects to have a degree-k neighbor j with the probability in . However, perceived clustering is $\tau(g_1)$ in g_1 and $\tau(g_2) = \tau(g_1) + 1$ in g_2. Thus, players' have the same beliefs regarding the popularity of their neighbors in both networks, but different beliefs about the extent to which their neighbors can be connected.[5]

Proposition 1. *Let $g_1 = (N, E)$ and $g_2 = (N', E')$, with N=N'. Assume for all $i \in N$: $I_i(g_1) = \{k_i(g_1), p, F(g_1), \tau(g_1)\}$, $I_i(g_2) = \{k_i(g_2), p, F(g_2), \tau(g_2)\}$, $F(g_1) = F(g_2)$, $k_i(g_1) = k_i(g_2) = k_i$, $p \in (0, 1)$ and $\tau(g_2) = \tau(g_1) + 1$. Every equilibrium strategy $\sigma_x(k)$ in network g_x, $x \in \{1, 2\}$, satisfies:*

1. Under strategic substitutes: $\sigma_x(k) = 1$ for $k < t_x$, $\sigma(k) = 0$ for $k > t_x$ and $\sigma_x(k) \in [0, 1]$ for $k = t_x$.
2. Under strategic complements: $\sigma_x(k) = 0$ for $k < t_x$, $\sigma(k) = 1$ for $k > t_x$ and $\sigma_x(k) \in [0, 1]$ for $k = t_x$, where $\sigma_x(k) \neq \sigma_x(k')$ for some k and k'.
3. In both games, $t_2 > t_1$.

In words, the threshold above which a player best-replies with action 0(1) under strategic substitutes (complements) increases with perceived clustering. The intuition is the following. As in Galeotti et al. (2010), players' degrees determine their equilibrium actions, and therefore players' expectations about their neighbors' actions are given by the distribution of their neighbors' degrees. Greater perceived clustering leads to a higher expected correlation in neighbors' degrees.[6] This increased correlation reduces players' incentives to play 0(1) under strategic substitutes (complements), as it reduces the probability that at least one of their neighbors plays 1. Since the expected payoffs decrease with perceived clustering in both games, lower perceived clustering benefits players as much as social connections do irrespective of whether the game exhibits strategic complements or substitutes.

Galeotti et al. (2010) show that inducing a shift in the degree distribution in the sense of first order stochastic dominance (i.e. moving from p to p', $p' > p$) neither reduces overall contribution to public good provision nor technology adoption in equilibrium. We show that, holding constant the degree distribution, an increase in perceived clustering has the same (the opposite) effect as an increase in network connectivity in the sense of first order stochastic dominance under strategic substitutes (complements). Hence, if greater expectations about network clustering are accompanied by greater expectations about neighbors' connectivity as it occurs in random networks[7] the effects would reinforce each other under strategic substitutes, while there would be a conflict in the effects of clustering and connectivity under strategic complements.

[4] Under strategic complements, there is always an equilibrium where all players play the same action.

[5] Proposition 1 analyzes the effects of varying perceived clustering in the equilibria that are non-trivial. The proof is relegated to Appendix.

[6] Note that correlation in degrees comes from the fact that agents can be linked. For example, if $n = 2$, either both nodes have degree 1 (with probability p) or 0 (with probability $1-p$). Hence, the greater the clustering, the greater the number of neighbors that can be connected and thus have degrees with non-independent probabilities.

4. Conclusions

We focus on games in which players only require that one neighbor plays a certain action to be best responding with the same action (under strategic complements) or with the opposite one (under strategic substitutes). However, in certain contexts people may require social reinforcement from multiple neighbors to adopt particular behaviors. Empirical evidence shows that clustering fosters the spread of behaviors in these contexts (Centola, 2010). When such a social reinforcement is required, do our results maintain? Could the correlation in degrees induced by clustering explain the advantage of clustered networks to spread behaviors that require social reinforcement to be adopted? We leave these questions for future research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Remark 1. Assuming $I_i^G(g) = \{k_i(g), p, F(g), \tau(g) = 0\}$ or $I_i^{G'}(g) = \{k_i(g), p, n, \tau(g) = 0\}$ is equivalent: whenever $F(g) = \{1, 2, \ldots, \bar{k}\}$ and $\bar{k} = n - 1$, players' beliefs about their neighbors are equal under both information setups,

$$P[K_j(g) = k \mid I_i^G(g)] = \binom{\bar{k}-1}{k-1} p^{k-1}(1-p)^{\bar{k}-k}$$

$$= \binom{n-2}{k-1} p^{k-1}(1-p)^{n-k-1} = P[K_j(g) = k \mid I_i^{G'}(g)]$$

for all $i \in N, j \in N_i(g)$. ∎

[7] The average clustering of a random network is p. Hence, an increment in p increases both players' expectations about their neighbors' degrees and about network clustering.

Claim 1 (*Conditional Independence*). *Let $g = (N, E)$ be an Erdös–Renyi network, where $p \in (0, 1)$ is the link probability. For all $j, l \in N$:*

$$p[K_j(g)=k_j, K_l(g)=k_l \mid g_{jl}] = p[K_j(g) = k_j \mid g_{jl}] * p[K_l(g) = k_l \mid g_{jl}]$$

By construction, each link between $j(l) \in N$ and any agent in $N \setminus \{j, l\}$ occurs with probability p, independently of any other edge. Then,

$$p[K_j(g) = k_j \mid K_l(g) = k_l] = p[K_j(g) = k_j \mid g_{jl} + \sum_{m \neq j} g_{ml} = k_l]$$

$$= \binom{n-2}{k_j - g_{jl}} p^{k_j - g_{jl}}(1-p)^{n-2-k_j+g_{jl}} = p[K_j(g) = k_j \mid g_{jl}]$$

which means that $K_j(g)$ and $K_l(g)$ are conditionally independent given g_{jl}. Likewise,

$$p[K_j(g) = k_j \mid K_l(g) = k_l, \dots, K_m(g) = k_m] = p[K_j(g) = k_j \mid g_{jl}, \dots, g_{jm}]$$

i.e. $p[K_j(g) = k \mid K_l(g) = k_l, \dots, K_m(g) = k_m]$ is independent of any link that does not involve j. ∎

Lemma 1. *Consider two Erdös–Renyi networks $g_1 = (N, E)$ and $g_2 = (N', E')$, where $p \in (0, 1)$ is the link probability. Let $I_i(g_1) = \{k_i(g_1), p, F(g_1), g_{jl} = 0\}$ be the information set of each $i \in N$, and $I_i(g_2) = \{k_i(g_2), p, F(g_2)\}$ the information set of each $i \in N'$, with $F(g_1) = F(g_2)$. If $D \subseteq F(g_1)$ is a subset of degree values,*

$$p[K_j(g_2), K_l(g_2) \in D \mid I_i(g_2)] > p[K_j(g_1), K_l(g_1) \in D \mid I_i(g_1)]$$
$$= \Big[p[K_j(g_1) \in D \mid I_i(g_1)]\Big]^2 \qquad \forall j, l \in N_i(g_1) \cap N_i(g_2)$$

Proof.

1. **Network** g_2. Recall that $F(g_1) = \{1, 2, \dots, \bar{k}\}$ is the set of feasible degrees in network g_1, and $\bar{k}$ is the maximal element in this set. Each i does not know whether $j, l \in N_i(g_2)$ are linked in g_2. Since $j \in N_i(g_2)$ can be linked to a maximum of $\bar{k}$ agents (including i and l), the probability that j has a degree in set D, conditioned on $g_{jl} = 1$ and on $I_i(g_2)$, is the probability that j forms $k - g_{ij} - g_{jl} = k - 2$ additional links with some of the $\bar{k} - |\{i, l\}| = \bar{k} - 2$ remaining agents:

$$p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 1] = \sum_{k \in D} \binom{\bar{k}-2}{k-2} p^{k-2}(1-p)^{\bar{k}-k}$$

and

$$p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 1] = p[K_l(g_2) \in D \mid I_i(g_2), g_{jl} = 1].$$

Conditioned on $g_{jl} = 0$, the probability that the realization of $K_j(g_2)$ falls inside D is, given $I_i(g_2)$:

$$p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 0] = \sum_{k \in D} \binom{\bar{k}-2}{k-1} p^{k-1}(1-p)^{\bar{k}-k-1}$$

with

$$p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 0] = p[K_l(g_2) \in D \mid I_i(g_2), g_{jl} = 0].$$

Agent i knows that each link is formed randomly and independently with probability $p \in (0, 1)$. Then, $K_j(g_2)$ and $K_l(g_2)$ are conditionally independent given g_{jl} (see Claim 1), and $P[K_j(g_2), K_l(g_2) \in D \mid I_i(g_2)]$ can be expressed as a sum of two conditional probabilities: the probability that both j and l have a degree in set D conditioned on $g_{jl} = 1$, and the probability that they both have a degree in D conditioned on $g_{jl} = 0$.

$$\begin{aligned}P[K_j(g_2), K_l(g_2) \in D \mid I_i(g_2)] = p* \\ \Big[p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 1] * p[K_l(g_2) \in D \mid I_i(g_2), g_{jl} = 1]\Big] \\ + (1-p)* \\ \Big[p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 0] * p[K_l(g_2) \in D \mid I_i(g_2), g_{jl} = 0]\Big]\end{aligned} \tag{3}$$

Since $p[K_j(g_2) \in D \mid I_i(g_2), g_{jl}] = p[K_l(g_2) \in D \mid I_i(g_2), g_{jl}]$, (3) equals:

$$p * \Big[p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 1]\Big]^2 + (1-p) * \Big[p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 0]\Big]^2 \tag{4}$$

2. **Network** g_1. As in network g_2, i knows that $j \in N_i(g_1)$ can be linked to a maximum of $\bar{k}$ agents (included i). However, i knows that none of these agents is l. Hence, the probability that each $j \in N_i(g_1)$ has a degree in set D given $I_i(g_1)$ is the probability that j forms $k - g_{ij} = k - 1$ additional links with some of the $\bar{k} - |\{i\}| = \bar{k} - 1$ remaining agents to which j can be linked:

$$p[K_j(g_1) \in D \mid I_i(g_1)] = \sum_{k \in D} \binom{\bar{k}-1}{k-1} p^{k-1}(1-p)^{\bar{k}-k}$$

In network g_1, each $i \in N$ knows that $g_{jl} = 0$, what implies that j and $l \in N_i(g_1)$ have degrees with independent probabilities (see Claim 1). Hence, player i believes that both j and l have a degree in set D with probability:

$$\begin{aligned}&p[K_j(g_1), K_l(g_1) \in D \mid I_i(g_1)] \\ &\quad = p[K_j(g_1) \in D \mid I_i(g_1)] * p[K_l(g_1) \in D \mid I_i(g_1)] \\ &\quad = p[K_j(g_1) \in D \mid I_i(g_1)]^2\end{aligned}$$

since $p[K_j(g_1) \in D \mid I_i(g_1)] = p[K_l(g_1) \in D \mid I_i(g_1)]$. Given that $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$:

$$\begin{aligned}&p[K_j(g_1) \in D \mid I_i(g_1)] \\ &\quad = \sum_{k \in D} \binom{\bar{k}-1}{k-1} p^{k-1}(1-p)^{\bar{k}-k} \\ &\quad = \sum_{k \in D} \Big[\binom{\bar{k}-2}{k-2} + \binom{\bar{k}-2}{k-1}\Big] p^{k-1}(1-p)^{\bar{k}-k} \\ &\quad = \Big[\sum_{k \in D} \binom{\bar{k}-2}{k-2} p^{k-1}(1-p)^{\bar{k}-k}\Big] \\ &\qquad + \Big[\sum_{k \in D} \binom{\bar{k}-2}{k-1} p^{k-1}(1-p)^{\bar{k}-k}\Big] \\ &\quad = p * \Big[\sum_{k \in D} \binom{\bar{k}-2}{k-2} p^{k-2}(1-p)^{\bar{k}-k}\Big] \\ &\qquad + (1-p) * \Big[\sum_{k \in D} \binom{\bar{k}-2}{k-1} p^{k-1}(1-p)^{\bar{k}-k-1}\Big] \\ &\quad = p * p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 1] \\ &\qquad + (1-p) * p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 0]\end{aligned}$$

Then, the probability that both j and l have a degree in set $k \in D$ given $I_i(g_1)$ is:

$$\begin{aligned}&p[K_j(g_1), K_l(g_1) \in D \mid I_i(g_1)] = p[K_j(g_1) \in D \mid I_i(g_1)]^2\\ &\quad= \Big[p * p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 1]\\ &\quad+(1-p) * p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 0]\Big]^2\\ &\quad= p^2 * \Big[p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 1]\Big]^2 + 2p(1-p)\\ &* \Big[p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 1] * p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 0]\Big]\\ &\quad+ (1-p)^2 * \Big[p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 0]\Big]^2\end{aligned} \tag{5}$$

Subtracting (4)–(5): =

$$\begin{aligned}&p(1-p) * \Big[p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 1]\\ &\quad- p[K_j(g_2) \in D \mid I_i(g_2), g_{jl} = 0]\Big]^2 > 0\end{aligned}$$

. ■

Proof of Proposition 2. Let g_x denote network g_x, with $x \in \{1, 2\}$. Define $D = \{k \in F : \sigma(k) = 0\}$ as the set of degree values for which a symmetric strategy σ specifies action 0.

A. Strategic substitutes. The expected utility of i in network g_x is the probability that at least one of agent in $N_i(g_x)$ plays 1. When all agents play the symmetric strategy σ, i's expected utility of playing 0 is the probability that at least one of agent in $N_i(g_x)$ does not have a degree in set D:

$$E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big) = 1 - \Big[p\Big[K_1(g_x), K_2(g_x), \dots, K_{k_i(g_x)}(g_x) \in D\Big]\Big]$$

where $K_1(g_x), K_2(g_x), \dots, K_{k_i(g_x)}(g_x)$ are the random variables of the degrees of the agents in $N_i(g_x) = \{1, 2, \dots, k_i(g_x)\}$, and $p[K_1(g_x), K_2(g_x), \dots, K_{k_i(g_x)} \in D]$ is the joint probability degree distribution of i's neighbors, conditioned on $I_i(g_x)$. Applying the probability chain rule, we can rewrite $E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big)$ as:

$$\begin{aligned}&E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big)\\ &\quad= 1 - \Big[p\Big[K_1(g_x) \in D\Big] * p\Big[K_2(g_x) \in D \mid K_1(g_x) \in D\Big] * \dots *\\ &\quad p\Big[K_{k_i(g_x)}(g_x) \in D \mid K_1(g_x), K_2(g_x), \dots, K_{k_i(g_x)-1}(g_x) \in D\Big]\Big]\end{aligned} \tag{6}$$

Player i is best responding with action 0 if $E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big) \geq E_{U_i(g_x)}\big(1, \sigma, I_i(g_x)\big) = 1 - c$ and with action 1 otherwise.

Let i and j two players of degree $k_i(g_x)$ and $k_j(g_x) = k_i(g_x) + 1$, respectively. $E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big)$ is increasing in $k_i(g_x)$, since:

$$\begin{aligned}&E_{U_j(g_x)}\big(0, \sigma, I_j(g_x)\big) - E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big)\\ &\quad= p\Big[K_1(g_x) \in D\Big] * p\Big[K_2(g_x) \in D \mid K_1(g_x) \in D\Big]\\ &\quad* \dots * p\Big[K_{k_i(g_x)}(g_x) \in D \mid K_1(g_x), K_2(g_x), \dots, K_{k_i(g_x)-1}(g_x) \in D\Big]\\ &\quad* \Big[1 - p\Big[K_{k_i(g_x)+1}(g_x) \in D \mid K_1(g_x), K_2(g_x), \dots, K_{k_i(g_x)}(g_x) \in D\Big]\Big]\\ &\quad> 0\end{aligned}$$

Hence, if i is best-responding with action 0, any player with degree above $k_i(g_x)$ must be best-responding with this action as well. It follows then that any equilibrium strategy σ satisfies: $\sigma(k) = 1$ for $k < t_x$, $\sigma(k) = 0$ for $k > t_x$ and $\sigma(k) \in [0, 1]$ for $k = t_x$, where t_x it the value of $k_i(g_x)$ for which (6) equals $1 - c$, i.e. the value of $k_i(g_x)$ for which:

$$\begin{aligned}&E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big)\\ &\quad= 1 - \Big[p\Big[K_1(g_x) > t_x\Big] * p\Big[K_2(g_x) > t_x \mid K_1(g_x) > t_x\Big]\\ &\quad* \dots * p\Big[K_{k_i(g_x)}(g_x) > t_x \mid K_1(g_x), K_2(g_x), \dots, K_{k_i(g_x)-1}(g_x) > t_x\Big]\Big]\\ &\quad= 1 - c\end{aligned}$$

holds.

Comparison between g_1 and g_2. We can rewrite (6) as:

$$\begin{aligned}&E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big) = 1 - \Big[p\Big[K_1(g_x)K_2(g_x) \in D\Big]\\ &\quad* p\Big[K_3(g_x) \in D \mid K_1(g_x), K_2(g_x) \in D\Big]\\ &\quad* p\Big[K_4(g_x) \in D \mid K_1(g_x), K_2(g_x), K_3(g_x) \in D\Big]\\ &\quad* \dots * p\Big[K_{k_i(g_x)}(g_x) \in D \mid K_1(g_x), K_2(g_x), \dots, K_{k_i(g_x)-1}(g_x) \in D\Big]\Big]\end{aligned} \tag{7}$$

where all the probabilities in (7) are conditional on $I_i(g_x)$.

Define $L(g_x)$ as a set of links among the agents in $N_i(g_x)$ that are feasible given $I_i(g_x)$. If $\frac{k_i(k_i-1)}{2} \leq \tau(g_1)$, $\big|L(g_2)\big| = \big|L(g_1)\big| = \frac{k_i(k_i-1)}{2}$, since $\tau(g_x)$ may be equal to $\big|S_i^3(g_x)\big|$ given $I_i(g_x)$. Suppose on the contrary that $\frac{k_i(k_i-1)}{2} > \tau(g_1)$. Given $I_i(g_1)$, there are $\big|L(g_1)\big| = \tau(g_1)$ links among the agents in $N_i(g_1)$ that are feasible and at least one link, say, $12 \notin L(g_1)$, that is not. Then, the probability that $1 \in N_i(g_1)$ has a degree in set D given $I_i(g_1)$ is independent of the probability that $2 \in N_i(g_1)$ does (see Claim 1):

$$\begin{aligned}p[K_1(g_1), K_2(g_1) \in D] &= p[K_1(g_1) \in D] * p[K_2(g_1) \in D]\\ &= p[K_1(g_1) \in D]^2\end{aligned} \tag{8}$$

Given $I_i(g_2)$, on the contrary, there are $\tau(g_2) = \big|L(g_2)\big| = \big|L(g_1)\big| + 1$ links that are feasible, say, $L(g_2) = L(g_1) \cup 12$. Since $\{1, 2\} \subseteq N_i(g_2)$ may be connected given $I_i(g_2)$:

$$p[K_1(g_2), K_2(g_2) \in D] \neq p[K_1(g_2) \in D]^2 \tag{9}$$

Observe in (7) that this is the only difference between $E_{U_i(g_1)}\big(0, \sigma, I_i(g_1)\big)$ and $E_{U_i(g_2)}\big(0, \sigma, I_i(g_2)\big)$. That is,

$$\begin{aligned}&p\Big[K_3(g_1) \in D \mid K_1(g_1), K_2(g_1) \in D\Big] =\\ &p\Big[K_3(g_2) \in D \mid K_1(g_2), K_2(g_2) \in D\Big],\end{aligned}$$

$$\begin{aligned}&p\Big[K_4(g_1) \in D \mid K_1(g_1), K_2(g_1), K_3(g_1) \in D\Big] =\\ &p\Big[K_4(g_2) \in D \mid K_1(g_2), K_2(g_2), K_3(g_2) \in D\Big]\end{aligned}$$

and so on, since none of these probabilities depends on whether $g_{12} = 1$ or $g_{12} = 0$ (see Claim 1). Hence, $E_{U_i(g_1)}\big(0, \sigma, I_i(g_1)\big) - E_{U_i(g_2)}\big(0, \sigma, I_i(g_2)\big)$ corresponds to the difference between (8) and (9), which is positive by Lemma 1. As a result, $t_2 > t_1$.

B. Strategic complements. In this case:

$$
\begin{aligned}
&E_{U_i(g_x)}\big(1, \sigma, I_i(g_x)\big) \\
&\quad = -c + 1 - \Bigg[p\Big[K_1(g_x) \in D\Big] * p\Big[K_2(g_x) \in D \mid K_1(g_x) \in D\Big] \\
&\quad * \ldots * p\Big[K_{k_i(g_x)}(g_x) \in D \mid K_1(g_x), K_2(g_x), \ldots, K_{k_i(g_x)-1}(g_x) \in D\Big]\Bigg]
\end{aligned}
\tag{10}
$$

which is increasing in $k_i(g_x)$. Then, each non-trivial equilibrium strategy σ satisfies: $\sigma(k) = 0$ for $k < t_x$, $\sigma(k) = 1$ for $k > t_x$ and $\sigma(k) \in [0, 1]$ for $k = t_x$, where t_x it the degree value for which (10) equals $E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big) = 0$ in g_x. That is, the value for which:

$$
\begin{aligned}
&E_{U_i(g_x)}\big(1, \sigma, I_i(g_x)\big) \\
&\quad = -c + 1 - \Bigg[p\Big[K_1(g_x) < t_x\Big] * p\Big[K_2(g_x) < t_x \mid K_1(g_x) < t_x\Big] \\
&\quad * \ldots * p\Big[K_{k_i(g_x)}(g_x) < t_x \mid K_1(g_x), K_2(g_x), \ldots, K_{k_i(g_x)-1}(g_x) < t_x\Big]\Bigg] \\
&\quad = 0
\end{aligned}
$$

satisfies.

Note that (10) can be rewritten as:

$$
\begin{aligned}
&E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big) = -c + 1 - \Bigg[p\Big[K_1(g_x)K_2(g_x) \in D\Big] \\
&\quad * p\Big[K_3(g_x) \in D \mid K_1(g_x), K_2(g_x) \in D\Big] \\
&\qquad * p\Big[K_4(g_x) \in D \mid K_1(g_x), K_2(g_x), K_3(g_x) \in D\Big] \\
&\quad * \ldots * p\Big[K_{k_i(g_x)}(g_x) \in D \mid K_1(g_x), K_2(g_x), \ldots, K_{k_i(g_x)-1}(g_x) \in D\Big]\Bigg]
\end{aligned}
$$

Applying the same reasoning as for strategic substitutes, $E_{U_i(g_1)}\big(1, \sigma, I_i(g_1)\big) > E_{U_i(g_2)}\big(1, \sigma, I_i(g_2)\big)$, and the result follows. ■

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