

(Downloaded from <https://cs.stanford.edu/~knuth/programs.html> and typeset on September 17, 2017)

**1. Intro.** This is a transcription of my “random matroid” program in #P72.

Standard input contains a sequence of integers. The first of these is the universe size,  $n$ , which should be at most 16. Then comes, for  $r = 1, 2, \dots$ , a list of sets that are stipulated to have rank  $\leq r$ . Sets are specified in hexadecimal notation, and each list is terminated by 0. Thus, the  $\pi$ -based example in my paper corresponds to the standard input

10 1a 222 64 128 288 10c

because #1a =  $2^4 + 2^3 + 2^1$  represents the set  $\{1, 3, 4\}$ , and #222 represents  $\{1, 5, 9\}$ , etc. The program appends zeros to the data on standard input if necessary, so trailing zeros can be omitted. Similarly, the standard input

5 7 0 1e

specifies a five-point matroid in which  $\{0, 1, 2\}$  has rank  $\leq 2$  and  $\{1, 2, 3, 4\}$  has rank  $\leq 3$ .

#define nmax 16 /\* to go higher, extend *print\_set* to larger-than-hex digits \*/

#define lmax 25742 /\*  $2(\binom{16}{8} + 1)$ , a safe upper bound on list size \*/

#include <stdio.h>

int n; /\* number of elements in the universe \*/

int mask; /\*  $2^n - 1$  \*/

int S[lmax + 1], L[lmax + 1]; /\* list memory \*/

int r; /\* the current rank \*/

int h; /\* head of circular list of closed sets for rank  $r$  \*/

int nh; /\* head of circular list being formed for rank  $r + 1$  \*/

int avail; /\* beginning the list of available space \*/

int unused; /\* the first unused slot in  $S$  and  $L$  arrays \*/

int x; /\* a set used to communicate with the *insert* routine \*/

int rank[1 << nmax]; /\* 100 + cardinality, or assigned rank \*/

<Subroutines 8>

main()

{

register int i, j, k;

if (scanf("%d", &n)  $\neq$  1  $\vee$   $n > 16 \vee n < 0$ ) {  
fprintf(stderr, "Sorry, I can't deal with a universe of size %d.\n", n);  
exit(-1);  
}

mask = (1 << n) - 1;

<Set initial contents of *rank* table 2>;

<Initialize list memory to available 3>;

rank[0] = 0, r = 0;

while (rank[mask] > r) <Pass from rank  $r$  to  $r + 1$  4>;

print\_circuits();

}

**2.** <Set initial contents of *rank* table 2>  $\equiv$

$k = 1$ ;

rank[0] = 100;

while ( $k \leq \text{mask}$ ) {

for ( $i = 0$ ;  $i < k$ ;  $i++$ ) rank[ $k + i$ ] = rank[ $i$ ] + 1;

$k = k + k$ ;

}

This code is used in section 1.

**3.** The published paper had a comparatively inefficient algorithm here; it initialized thousands of links that usually remained unused.

```

⟨ Initialize list memory to available 3 ⟩ ≡
    L[1] = 2;
    L[2] = 1;
    S[2] = 0;
    h = 1; /* list containing the empty set */
    unused = 3;

```

This code is used in section 1.

```

4. ⟨ Pass from rank  $r$  to  $r + 1$  4 ⟩ ≡
{
    ⟨ Create empty list 5 ⟩;
    generate();
    if (r) enlarge();
    ⟨ Return list  $h$  to available storage 6 ⟩;
    r++;
    h = nh;
    sort(); /* optional */
    print_list(h);
    ⟨ Assign rank to sets and print independent ones 7 ⟩;
}

```

This code is used in section 1.

```

5. ⟨ Create empty list 5 ⟩ ≡
    nh = avail;
    if (nh) avail = L[nh];
    else nh = unused++;
    L[nh] = nh;

```

This code is used in section 4.

```

6. ⟨ Return list  $h$  to available storage 6 ⟩ ≡
    for (j = h; L[j] ≠ h; j = L[j]) ;
    L[j] = avail;
    avail = h;

```

This code is used in section 4.

```

7. ⟨ Assign rank to sets and print independent ones 7 ⟩ ≡
    printf("Independent sets for rank %d:", r);
    for (j = L[h]; j ≠ h; j = L[j]) mark(S[j]);
    printf("\n");

```

This code is used in section 4.

8. The *generate* procedure inserts minimal closed sets for rank  $r + 1$  into a circular list headed by  $nh$ . (It corresponds to “Step 2” in the published algorithm.)

⟨Subroutines 8⟩ ≡

```

void insert(void);    /* details coming soon */
void generate(void)
{
    register int  $t, v, y, j, k$ ;
    for ( $j = L[h]; j \neq h; j = L[j]$ ) {
         $y = S[j]$ ;    /* a closed set of rank  $r$  */
         $t = mask - y$ ;
        ⟨Find all sets in list  $nh$  that already contain  $y$  and remove excess elements from  $t$  9⟩;
        ⟨Insert  $y \cup a$  for each  $a \in t$  10⟩;
    }
}

```

See also sections 11, 12, 13, 14, 15, 16, 17, and 18.

This code is used in section 1.

9. ⟨Find all sets in list  $nh$  that already contain  $y$  and remove excess elements from  $t$  9⟩ ≡

```

for ( $k = L[nh]; k \neq nh; k = L[k]$ )
    if  $((S[k] \& y) \equiv y) \ t \&= \sim S[k]$ ;

```

This code is used in section 8.

10. ⟨Insert  $y \cup a$  for each  $a \in t$  10⟩ ≡

```

while ( $t$ ) {
     $x = y \mid (t \& -t)$ ;
    insert();    /* insert  $x$  into  $nh$ , possibly enlarging  $x$  */
     $t \&= \sim x$ ;
}

```

This code is used in section 8.

11. The following key procedure basically inserts the set  $x$  into list  $nh$ . But it augments  $x$  if necessary (and deletes existing entries of the list) so that no two entries have an intersection of rank greater than  $r$ . Thus it incorporates the idea of “Step 4,” but it is more efficient than a brute force implementation of that step.

```

⟨Subroutines 8⟩ +=
void insert(void)
{
    register int  $j, k$ ;
     $j = nh$ ;
    store:  $S[nh] = x$ ;
    loop:  $k = j$ ;
    continu:  $j = L[k]$ ;
    if ( $rank[S[j] \& x] \leq r$ ) goto loop;
    if ( $j \neq nh$ ) {
        if ( $x \equiv (x \mid S[j])$ ) { /* remove from list and continue */
             $L[k] = L[j], L[j] = avail, avail = j$ ;
            goto continu;
        } else { /* augment  $x$  and go around again */
             $x \mid= S[j], nh = j$ ;
            goto store;
        }
    }
     $j = avail$ ;
    if ( $j$ )  $avail = L[j]$ ;
    else  $j = unused++$ ;
     $L[j] = L[nh]$ ;
     $L[nh] = j$ ;
     $S[j] = x$ ;
}

```

12. The *enlarge* procedure inserts sets that are read from standard input until encountering an empty set. (It corresponds to “Step 3.”)

```

⟨Subroutines 8⟩ +=
void enlarge(void)
{
    while (1) {
         $x = 0$ ;
        scanf("%x", & $x$ );
        if ( $\neg x$ ) return;
        if ( $rank[x] > r$ ) insert();
    }
}

```

**13.** We don't output a set as a hexadecimal number according to the convention used on standard input; instead, we print an increasing sequence of hexadecimal digits that name the actual set elements. For example, the set that was input as **1a** would be output as **134**.

⟨Subroutines 8⟩ +≡

```
void print_set(int t)
{
    register int j, k;
    printf("_");
    for (j = 1, k = 0; j ≤ t; j <<= 1, k++)
        if (t & j) printf("%x", k);
}
```

**14.** ⟨Subroutines 8⟩ +≡

```
void print_list(int h)
{
    register int j;
    printf("Closed_sets_for_rank%d:", r);
    for (j = L[h]; j ≠ h; j = L[j]) print_set(S[j]);
    printf("\n");
}
```

**15.** The subroutine *mark*(*m*) sets  $\text{rank}[m'] = r$  for all subsets  $m' \subseteq m$  whose rank is not already  $\leq r$ , and outputs  $m'$  if it is independent (that is, if its rank equals its cardinality).

⟨Subroutines 8⟩ +≡

```
void mark(int m)
{
    register int t, v;
    if ( $\text{rank}[m] > r$ ) {
        if ( $\text{rank}[m] \equiv 100 + r$ ) print_set(m);
         $\text{rank}[m] = r$ ;
        for (t = m; t; t = v) {
            v = t & (t - 1);
            mark(m - t + v);
        }
    }
}
```

16. I've added a *tl* array to the data structure, to speed up and shorten this routine.

⟨Subroutines 8⟩ +≡

```

void sort()
{
    register int i, j, k;
    int hd[101 + nmax], tl[101 + nmax];
    for (i = 100; i ≤ 100 + n; i++) hd[i] = -1;
    j = L[h];
    L[h] = h;
    while (j ≠ h) {
        i = rank[S[j]];
        k = L[j];
        L[j] = hd[i];
        if (L[j] < 0) tl[i] = j;
        hd[i] = j;
        j = k;
    }
    for (i = 100; i ≤ 100 + n; i++)
        if (hd[i] ≥ 0) L[tl[i]] = L[h], L[h] = hd[i];
}

```

17. The parameter *card* is 100 plus the cardinality of *m* in the following subroutine.

⟨Subroutines 8⟩ +≡

```

void unmark(int m, int card)
{
    register t, v;
    if (rank[m] < 100) {
        rank[m] = card;
        for (t = mask - m; t; t = v) {
            v = t & (t - 1);
            unmark(m + t - v, card + 1);
        }
    }
}

```

18. ⟨Subroutines 8⟩ +≡

```

void print_circuits(void)
{
    register int i, k;
    printf("The_circuits_are:");
    for (k = 1; k ≤ mask; k += k)
        for (i = 0; i < k; i++)
            if (rank[k + i] ≡ rank[i]) {
                print_set(k + i);
                unmark(k + i, rank[i] + 101);
            }
    printf("\n");
}

```

**19. Index.**

*avail*: [1](#), [5](#), [6](#), [11](#).

*card*: [17](#).

*continuu*: [11](#).

*enlarge*: [4](#), [12](#).

*exit*: [1](#).

*fprintf*: [1](#).

*generate*: [4](#), [8](#).

*h*: [1](#), [14](#).

*hd*: [16](#).

*i*: [1](#), [16](#), [18](#).

*insert*: [1](#), [8](#), [10](#), [11](#), [12](#).

*j*: [1](#), [8](#), [11](#), [13](#), [14](#), [16](#).

*k*: [1](#), [8](#), [11](#), [13](#), [16](#), [18](#).

*L*: [1](#).

*lmax*: [1](#).

*loop*: [11](#).

*m*: [15](#), [17](#).

*main*: [1](#).

*mark*: [7](#), [15](#).

*mask*: [1](#), [2](#), [8](#), [17](#), [18](#).

*n*: [1](#).

*nh*: [1](#), [4](#), [5](#), [8](#), [9](#), [10](#), [11](#).

*nmax*: [1](#), [16](#).

*print\_circuits*: [1](#), [18](#).

*print\_list*: [4](#), [14](#).

*print\_set*: [1](#), [13](#), [14](#), [15](#), [18](#).

*printf*: [7](#), [13](#), [14](#), [18](#).

*r*: [1](#).

*rank*: [1](#), [2](#), [11](#), [12](#), [15](#), [16](#), [17](#), [18](#).

*S*: [1](#).

*scanf*: [1](#), [12](#).

*sort*: [4](#), [16](#).

*stderr*: [1](#).

*store*: [11](#).

*t*: [8](#), [13](#), [15](#), [17](#).

*tl*: [16](#).

*unmark*: [17](#), [18](#).

*unused*: [1](#), [3](#), [5](#), [11](#).

*v*: [8](#), [15](#), [17](#).

*x*: [1](#).

*y*: [8](#).

- ⟨ Assign rank to sets and print independent ones 7 ⟩ Used in section 4.
- ⟨ Create empty list 5 ⟩ Used in section 4.
- ⟨ Find all sets in list  $nh$  that already contain  $y$  and remove excess elements from  $t$  9 ⟩ Used in section 8.
- ⟨ Initialize list memory to available 3 ⟩ Used in section 1.
- ⟨ Insert  $y \cup a$  for each  $a \in t$  10 ⟩ Used in section 8.
- ⟨ Pass from rank  $r$  to  $r + 1$  4 ⟩ Used in section 1.
- ⟨ Return list  $h$  to available storage 6 ⟩ Used in section 4.
- ⟨ Set initial contents of *rank* table 2 ⟩ Used in section 1.
- ⟨ Subroutines 8, 11, 12, 13, 14, 15, 16, 17, 18 ⟩ Used in section 1.



ERECTION

	Section	Page
Intro .....	<a href="#">1</a>	1
Index .....	<a href="#">19</a>	7