(Downloaded from https://cs.stanford.edu/~knuth/programs.html and typeset on September 17, 2017)

1. Intro. This program is part of a series of "SAT-solvers" that I'm putting together for my own education as I prepare to write Section 7.2.2.2 of *The Art of Computer Programming*. My intent is to have a variety of compatible programs on which I can run experiments to learn how different approaches work in practice.

After experience with ten previous approaches, I finally feel ready to write the program that I plan to describe first: a very simple "no-frills" algorithm that does pretty well on not-too-large problems in spite of being rather short and sweet. The model for this program is the "fast one-level algorithm" of Cynthia A. Brown and Paul W. Purdom, Jr., found in their paper "An empirical comparison of backtracking algorithms," *IEEE Transactions on Pattern Analysis and Machine Intelligence* **PAMI-4** (1982), 309–316. This almost-forgotten paper introduced the idea of watched literals, a concept that became famous when it was rediscovered and generalized almost two decades later. Brown and Purdom noticed that the operations of backtracking became quite simple when there is one watched literal in each clause; later researchers, unaware of this previous work, discovered how to speed up the process of so-called unit propagation by having two watched literals per clause. By presenting the Brown–Purdom algorithm first, I hope to introduce my readers to this concept in a natural and gradual way.

[Note: This program, SAT10, is essentially the prototype for Algorithm 7.2.2.2D.]

If you have already read SAT0 (or some other program of this series), you might as well skip now past all the code for the "I/O wrapper," because you have seen it before.

The input on stdin is a series of lines with one clause per line. Each clause is a sequence of literals separated by spaces. Each literal is a sequence of one to eight ASCII characters between ! and }, inclusive, not beginning with  $\tilde{\ }$ , optionally preceded by  $\tilde{\ }$  (which makes the literal "negative"). For example, Rivest's famous clauses on four variables, found in 6.5–(13) and 7.1.1–(32) of TAOCP, can be represented by the following eight lines of input:

Input lines that begin with  $\tilde{\ }_{\sqcup}$  are ignored (treated as comments). The output will be ' $\tilde{\ }$ ' if the input clauses are unsatisfiable. Otherwise it will be a list of noncontradictory literals that cover each clause, separated by spaces. ("Noncontradictory" means that we don't have both a literal and its negation.) The input above would, for example, yield ' $\tilde{\ }$ '; but if the final clause were omitted, the output would be ' $\tilde{\ }$ x1  $\tilde{\ }$ x2 x3', in some order, possibly together with either x4 or  $\tilde{\ }$ x4 (but not both). No attempt is made to find all solutions; at most one solution is given.

The running time in "mems" is also reported, together with the approximate number of bytes needed for data storage. One "mem" essentially means a memory access to a 64-bit word. (These totals don't include the time or space needed to parse the input or to format the output.)

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```
2. So here's the structure of the program. (Skip ahead if you are impatient to see the interesting stuff.)
#define o mems ++
                           /* count one mem */
#define oo mems += 2 /* count two mems */
                                /* count three mems */
#define ooo mems += 3
#define O "%"
                        /* used for percent signs in format strings */
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include "gb_flip.h"
  typedef unsigned int uint;
                                     /* a convenient abbreviation */
  typedef unsigned long long ullng; /* ditto */
  \langle \text{Type definitions 5} \rangle;
  \langle \text{Global variables 3} \rangle;
  \langle \text{Subroutines } 28 \rangle;
  main(int argc, char *argv[])
    register uint h, i, j, l, p, q, r, level, kk, pp, qq, ll, force, nextmove;
    register int c, cc, k, v\theta, v, vv, vvv;
    \langle \text{Process the command line 4} \rangle;
    ⟨Initialize everything 8⟩;
    \langle \text{Input the clauses 9} \rangle;
    if (verbose & show_basics) \langle Report the successful completion of the input phase 22 \rangle;
    \langle Set up the main data structures 33\rangle;
    imems = mems, mems = 0;
    \langle Solve the problem 40 \rangle;
  done: if (verbose & show_basics)
       fprintf(stderr, "Altogether_{\square}"O"llu+"O"llu_mems,_{\square}"O"llu_bytes,_{\square}"O"llu_nodes.\n", imems,
            mems, bytes, nodes);
  }
```

 $\S3$  SAT10 INTRO 3

```
#define show_basics 1 /* verbose code for basic stats */
#define show_choices 2
                           /* verbose code for backtrack logging */
                            /*\ verbose\ {\it code}\ {\it for}\ {\it further}\ {\it commentary}\ */
#define show_details 4
                                 /* verbose code to list variables not in solution */
#define show_unused_vars 8
\langle \text{Global variables 3} \rangle \equiv
  int random\_seed = 0;
                          /* seed for the random words of gb\_rand */
  int \ verbose = show\_basics + show\_unused\_vars;
                                                 /* level of verbosity */
  int show\_choices\_max = 1000000;
                                     /* above this level, show_choices is ignored */
  int hbits = 8;
                   /* logarithm of the number of the hash lists */
  int buf\_size = 1024;
                         /* must exceed the length of the longest input line */
                     /* file for optional output */
  FILE *out_file;
  \mathbf{char} * out\_name;
                     /* its name */
  FILE *primary_file;
                         /* file for optional input */
                          /* its name */
  char *primary\_name;
                      /* the number of primary variables */
  int primary_vars;
  ullng imems, mems; /* mem counts */
                 /* memory used by main data structures */
  ullng bytes;
  ullng nodes;
                  /* total number of branch nodes initiated */
  ullng thresh = 0; /* report when mems exceeds this, if delta \neq 0 */
                   /* report every delta or so mems */
  ullng delta = 0;
  float eps = 0.1;
                    /* parameter for the minimum score of a watch list */
See also sections 7 and 27.
```

This code is used in section 2.

4 INTRO SAT10 §4

- **4.** On the command line one can specify any or all of the following options:
- 'v (integer)' to enable various levels of verbose output on *stderr*.
- 'c positive integer' to limit the levels on which clauses are shown.
- 'h' positive integer' to adjust the hash table size.
- 'b' positive integer' to adjust the size of the input buffer.
- 's (integer)' to define the seed for any random numbers that are used.
- 'd\langle integer \rangle' to set delta for periodic state reports.
- 'e' float' to change the eps parameter in rankings of variables for branching.
- 'x\( filename \)' to copy the input plus a solution-eliminating clause to the specified file. If the given problem is satisfiable in more than one way, a different solution can be obtained by inputting that file.
- ' $V\langle$  filename $\rangle$ ' to input a file that lists the names of all "primary" variables. A nonprimary variable will not be used for branching unless its value is forced, or unless all of the primary variables have already been assigned a value.
- 'T\' integer \'' to set timeout: This program will abruptly terminate, when it discovers that mems > timeout.

```
\langle \text{ Process the command line 4} \rangle \equiv
  for (j = argc - 1, k = 0; j; j - -)
    switch (argv[j][0]) {
    case 'v': k = (sscanf(argv[j] + 1, ""O"d", \&verbose) - 1); break;
    case 'c': k = (sscanf(argv[j] + 1, ""O"d", \&show\_choices\_max) - 1); break;
    case 'h': k = (sscanf(argv[j] + 1, ""O"d", \&hbits) - 1); break;
    case 'b': k = (sscanf(arqv[j] + 1, ""O"d", \&buf\_size) - 1); break;
    case 's': k = (sscanf(argv[j] + 1, ""O"d", \&random\_seed) - 1); break;
    case 'd': k = (sscanf(argv[j] + 1, ""O"11d", \&delta) - 1); thresh = delta; break;
    case 'e': k = (sscanf(argv[j] + 1, ""O"f", \&eps) - 1); break;
    case 'x': out\_name = argv[j] + 1, out\_file = fopen(out\_name, "w");
       if (\neg out\_file) fprintf(stderr, "I_{\sqcup}can't_{\sqcup}open_{\sqcup}file_{\sqcup}'"O"s'_{\sqcup}for_{\sqcup}output! \n", out\_name);
    case 'V': primary\_name = argv[j] + 1, primary\_file = fopen(primary\_name, "r");
       if (¬primary_file) fprintf(stderr, "I_can't_open_file_'"O"s'_for_input!\n", primary_name);
    case 'T': k = (sscanf(argv[j] + 1, "\%11d", \&timeout) - 1); break;
    default: k = 1;
                         /* unrecognized command-line option */
  if (k \lor hbits < 0 \lor hbits > 30 \lor buf\_size \le 0) {
    fprintf(stderr, "Usage: \_"O"s\_[v<n>]\_[c<n>]\_[h<n>]\_[b<n>]\_[s<n>]\_[d<n>]\_[e<f>]", <math>argv[0]);
    fprintf(stderr, "| [x<foo>]| [V<foo>]| [T<n>]| < | foo.sat\n");
    exit(-1);
```

This code is used in section 2.

 $\S5$  SAT10 THE I/O WRAPPER 5

5. The I/O wrapper. The following routines read the input and absorb it into temporary data areas from which all of the "real" data structures can readily be initialized. My intent is to incorporate these routines into all of the SAT-solvers in this series. Therefore I've tried to make the code short and simple, yet versatile enough so that almost no restrictions are placed on the sizes of problems that can be handled. These routines are supposed to work properly unless there are more than  $2^{32}-1=4,294,967,295$  occurrences of literals in clauses, or more than  $2^{31}-1=2,147,483,647$  variables or clauses.

In these temporary tables, each variable is represented by four things: its unique name; its serial number; the clause number (if any) in which it has most recently appeared; and a pointer to the previous variable (if any) with the same hash address. Several variables at a time are represented sequentially in small chunks of memory called "vchunks," which are allocated as needed (and freed later).

```
/* preferably (2^k - 1)/3 for some k */
#define vars_per_vchunk 341
\langle \text{ Type definitions 5} \rangle \equiv
  typedef union {
    char ch8[8];
    uint u2[2];
    long long lng;
  } octa;
  typedef struct tmp_var_struct {
    octa name:
                     /* the name (one to eight ASCII characters) */
                     /* 0 for the first variable, 1 for the second, etc. */
    uint serial;
                    /* m if positively in clause m; -m if negatively there */
    int stamp;
                                         /* pointer for hash list */
    struct tmp_var_struct *next;
  } tmp_var;
  typedef struct vchunk_struct {
                                         /* previous chunk allocated (if any) */
    struct vchunk_struct *prev;
    tmp_var var[vars_per_vchunk];
  } vchunk:
See also sections 6, 24, 25, and 26.
This code is used in section 2.
```

**6.** Each clause in the temporary tables is represented by a sequence of one or more pointers to the **tmp\_var** nodes of the literals involved. A negated literal is indicated by adding 1 to such a pointer. The first literal of a clause is indicated by adding 2. Several of these pointers are represented sequentially in chunks of memory, which are allocated as needed and freed later.

```
#define cells_per_chunk 511 /* preferably 2<sup>k</sup> − 1 for some k */

⟨Type definitions 5⟩ +≡

typedef struct chunk_struct {

struct chunk_struct *prev; /* previous chunk allocated (if any) */

tmp_var *cell[cells_per_chunk];

} chunk;
```

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```
7. \langle \text{Global variables } 3 \rangle + \equiv
                   /* buffer for reading the lines (clauses) of stdin */
  \mathbf{char} * buf;
                          /* heads of the hash lists */
  tmp_var **hash;
  uint hash\_bits[93][8];
                              /* random bits for universal hash function */
                               /* the vchunk currently being filled */
  vchunk *cur\_vchunk;
  tmp\_var * cur\_tmp\_var;
                                  /* current place to create new tmp_var entries */
  tmp\_var *bad\_tmp\_var;
                                  /* the cur_tmp_var when we need a new vchunk */
  chunk *cur\_chunk;
                             /* the chunk currently being filled */
  tmp\_var **cur\_cell;
                              /* current place to create new elements of a clause */
  tmp_var **bad_cell;
                              /* the cur_cell when we need a new chunk */
                    /* how many distinct variables have we seen? */
  ullng vars;
  ullng clauses;
                       /* how many clauses have we seen? */
                           /* how many of them were null? */
  ullng nullclauses;
  ullng cells:
                    /* how many occurrences of literals in clauses? */
                        /* is the current clause ignorable? */
  int non_clause;
8. \langle \text{Initialize everything } 8 \rangle \equiv
  qb\_init\_rand(random\_seed);
  buf = (\mathbf{char} *) \ malloc(buf\_size * \mathbf{sizeof}(\mathbf{char}));
  if (\neg buf) {
    fprintf(stderr, "Couldn't_{la}llocate_{l}the_{l}input_{l}buffer_{l}(buf_size="O"d)!\n", buf_size);
     exit(-2);
  hash = (\mathbf{tmp\_var} **) \ malloc(\mathbf{sizeof}(\mathbf{tmp\_var}) \ll hbits);
    fprintf(stderr, "Couldn't_allocate_"O"d_hash_list_heads_(hbits="O"d)!\n", 1 \ll hbits, hbits);
  for (h = 0; h < 1 \ll hbits; h \leftrightarrow) hash[h] = \Lambda;
See also section 15.
This code is used in section 2.
```

 $\S9$  SAT10 THE I/O WRAPPER 7

9. The hash address of each variable name has h bits, where h is the value of the adjustable parameter hbits. Thus the average number of variables per hash list is  $n/2^h$  when there are n different variables. A warning is printed if this average number exceeds 10. (For example, if h has its default value, 8, the program will suggest that you might want to increase h if your input has 2560 different variables or more.)

All the hashing takes place at the very beginning, and the hash tables are actually recycled before any SAT-solving takes place; therefore the setting of this parameter is by no means crucial. But I didn't want to bother with fancy coding that would determine h automatically.

```
\langle \text{Input the clauses 9} \rangle \equiv
      if (primary_file) \langle Input the primary variables 10 \rangle;
      while (1) {
            if (\neg fgets(buf, buf\_size, stdin)) break;
            clauses ++;
            if (buf[strlen(buf) - 1] \neq '\n') {
                  fprintf(stderr, "The \clause \cupon \cupul ine \cupu" O" \cupu" \cupu" O" \cupu" \cupu on \cupu ine \cupu" on \cupu ine \cupu" on \cupu on \cupu on \cupu on \cupu on \cupu on \cupu" on \cupu on \cupu
                  fprintf(stderr, "limylibuf_size_lis_lonly_l"O"d!\n", buf_size);
                  fprintf(stderr, "Please_use_the_command-line_option_b<newsize>. \n");
                  exit(-4);
            \langle \text{ Input the clause in } buf \ 11 \rangle;
      if (\neg primary\_file) primary\_vars = vars;
      if ((vars \gg hbits) > 10) {
            fprintf(stderr, "There\_are\_"O"lld\_variables\_but\_only\_"O"d\_hash\_tables; \n", vars, 1 \ll hbits);
            while ((vars \gg hbits) \ge 10) \ hbits ++;
            fprintf(stderr, "\_maybe\_you\_should\_use\_command-line\_option\_h"O"d?\n", hbits);
      clauses -= nullclauses;
      if (clauses \equiv 0) {
            fprintf(stderr, "No_{\square}clauses_{\square}were_{\square}input! \n");
            exit(-77);
      if (vars > #8000000) {
            fprintf(stderr, "Whoa, \_the \_input\_had \_"O"llu\_variables! \n", vars);
            exit(-664);
      if (clauses \ge *80000000) {
            fprintf(stderr, "Whoa, | the | input| had | "O" llu| clauses! \n", clauses);
            exit(-665);
      if (cells \ge #10000000) {
            fprintf(stderr, "Whoa, \_the \_input \_had \_"O"llu \_occurrences \_of \_literals! \n", cells);
            exit(-666);
This code is used in section 2.
```

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We input from primary\_file just as if it were the standard input file, except that all "clauses" are discarded. (Line numbers in error messages are zero.) The effect is to place the primary variables first in the list of all variables: A variable is primary if and only if its index is  $\leq primary\_vars$ .

```
\langle \text{Input the primary variables } 10 \rangle \equiv
     while (1) {
       if (¬fgets(buf, buf_size, primary_file)) break;
        if (buf[strlen(buf) - 1] \neq '\n') {
          fprintf(stderr, "The_{\sqcup}clause_{\sqcup}on_{\sqcup}line_{\sqcup}"O"lld_{\sqcup}("O".20s...)_{\sqcup}is_{\sqcup}too_{\sqcup}long_{\sqcup}for_{\sqcup}me; \n",
                clauses, buf);
          fprintf(stderr, \verb"\my\n"buf_size\_is\_only\_"O"d!\n", buf\_size);
          fprintf(stderr, "Please_use_the_command-line_option_b<newsize>.\n");
          exit(-4);
        \langle \text{ Input the clause in } buf 11 \rangle;
        (Remove all variables of the current clause 19);
     cells = null clauses = 0;
     primary\_vars = vars;
     if (verbose & show_basics)
        fprintf(stderr, "("O"d_iprimary_ivariables_iread_ifrom_i"O"s)\n", primary_vars, primary_name);
This code is used in section 9.
11. (Input the clause in buf 11) \equiv
  for (j = k = non\_clause = 0; \neg non\_clause;) {
     while (buf[j] \equiv ' \sqcup ') j++;
                                          /* scan to nonblank */
     if (buf[j] \equiv '\n') break;
     if (buf[j] < , , \vee buf[j] > , ,) {
       fprintf(stderr, "Illegal \ character \ (code \ \#"O"x) \ in \ the \ clause \ on \ line \ "O"lld! \ ",
             buf[j], clauses);
        exit(-5);
     if (buf[j] \equiv , \sim, ) \ i = 1, j ++;
     else i=0;
     \langle Scan \text{ and record a variable}; \text{ negate it if } i \equiv 1 \text{ 12} \rangle;
  if (k \equiv 0 \land \neg non\_clause) {
     fprintf(stderr, "(Empty_line_l"O"lld_lis_being_lignored)\n", clauses);
     nullclauses ++;
                           /* strictly speaking it would be unsatisfiable */
  if (non_clause) (Remove all variables of the current clause 19);
  cells += k;
```

This code is used in sections 9 and 10.

 $\S12$  SAT10 THE I/O WRAPPER S

```
We need a hack to insert the bit codes 1 and/or 2 into a pointer value.
#define hack_in(q,t) (tmp_var *)(t | (ullng) q)
\langle Scan and record a variable; negate it if i \equiv 1 12\rangle \equiv
     register tmp_var *p;
     if (cur\_tmp\_var \equiv bad\_tmp\_var) (Install a new vchunk 13);
     \langle \text{Put the variable name beginning at } buf[j] \text{ in } cur\_tmp\_var \neg name \text{ and compute its hash code } h \text{ 16} \rangle;
     if (\neg non\_clause) {
        \langle \text{Find } cur\_tmp\_var \neg name \text{ in the hash table at } p \mid 17 \rangle;
        if (clauses \land (p \neg stamp \equiv clauses \lor p \neg stamp \equiv -clauses)) \land Handle a duplicate literal 18 \rangle
           p \rightarrow stamp = (i ? -clauses : clauses);
           if (cur\_cell \equiv bad\_cell) (Install a new chunk 14);
           *cur\_cell = p;
           if (i \equiv 1) *cur\_cell = hack\_in(*cur\_cell, 1);
           if (k \equiv 0) *cur\_cell = hack\_in(*cur\_cell, 2);
           cur\_cell++, k++;
  }
This code is used in section 11.
      \langle \text{Install a new vchunk } 13 \rangle \equiv
13.
  {
     register vchunk *new_vchunk;
     new\_vchunk = (\mathbf{vchunk} *) \ malloc(\mathbf{sizeof}(\mathbf{vchunk}));
     if (\neg new\_vchunk) {
        fprintf(stderr, "Can't_allocate_a_new_vchunk!\n");
        exit(-6);
     new\_vchunk \neg prev = cur\_vchunk, cur\_vchunk = new\_vchunk;
     cur\_tmp\_var = \&new\_vchunk \neg var[0];
     bad\_tmp\_var = \&new\_vchunk \rightarrow var[vars\_per\_vchunk];
This code is used in section 12.
14. \langle \text{Install a new chunk } 14 \rangle \equiv
     register chunk *new_chunk;
     new\_chunk = (\mathbf{chunk} *) \ malloc(\mathbf{sizeof}(\mathbf{chunk}));
     if (\neg new\_chunk) {
        fprintf(stderr, "Can't_allocate_a_new_chunk!\n");
        exit(-7);
     new\_chunk \neg prev = cur\_chunk, cur\_chunk = new\_chunk;
     cur\_cell = \&new\_chunk \neg cell[0];
     bad\_cell = \&new\_chunk \neg cell[cells\_per\_chunk];
This code is used in section 12.
```

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15. The hash code is computed via "universal hashing," using the following precomputed tables of random bits.

```
\langle \text{ Initialize everything } 8 \rangle + \equiv
  for (j = 92; j; j--)
     for (k = 0; k < 8; k++) hash\_bits[j][k] = gb\_next\_rand();
16. \(\rightarrow\) Put the variable name beginning at buf[j] in cur\_tmp\_var \neg name and compute its hash code h 16\) \(\equiv \)
   cur\_tmp\_var \rightarrow name.lng = 0;
   for (h = l = 0; buf[j + l] > ' ' \land buf[j + l] \leq ' " ; l \leftrightarrow )
     if (l > 7) {
        fprintf(stderr, "Variable \_ name \_ "O".9s... \_ in \_ the \_ clause \_ on \_ line \_ "O" lld \_ is \_ too \_ long! \n",
              buf + j, clauses);
         exit(-8);
     h \oplus = hash\_bits[buf[j+l] - '!'][l];
     cur\_tmp\_var \rightarrow name.ch8[l] = buf[j+l];
  if (l \equiv 0) non_clause = 1; /* '~' by itself is like 'true' */
   else j += l, h \&= (1 \ll hbits) - 1;
This code is used in section 12.
17. \langle \text{Find } cur\_tmp\_var \neg name \text{ in the hash table at } p \mid 17 \rangle \equiv
   for (p = hash[h]; p; p = p \rightarrow next)
     if (p \rightarrow name.lng \equiv cur\_tmp\_var \rightarrow name.lng) break;
                  /* new variable found */
  if (\neg p) {
     p = cur\_tmp\_var ++;
     p \rightarrow next = hash[h], hash[h] = p;
     p \rightarrow serial = vars ++;
     p \rightarrow stamp = 0;
This code is used in section 12.
```

18. The most interesting aspect of the input phase is probably the "unwinding" that we might need to do when encountering a literal more than once in the same clause.

This code is used in section 12.

 $\S19$  SAT10 THE I/O WRAPPER 11

19. An input line that begins with ' $\sim$ <sub> $\square$ </sub>' is silently treated as a comment. Otherwise redundant clauses are logged, in case they were unintentional. (One can, however, intentionally use redundant clauses to force the order of the variables.)

```
\langle Remove all variables of the current clause 19\rangle \equiv
     while (k) {
        \langle \text{Move } cur\_cell \text{ backward to the previous cell } 20 \rangle;
     if (non\_clause \land ((buf[0] \neq `, `, `) \lor (buf[1] \neq `, \bot, `)))
        fprintf(stderr, "(The_{\sqcup}clause_{\sqcup}on_{\sqcup}line_{\sqcup}"O"lld_{\sqcup}is_{\sqcup}always_{\sqcup}satisfied) \n", clauses);
     null clauses ++;
This code is used in sections 10 and 11.
20. \langle \text{Move } cur\_cell \text{ backward to the previous cell } 20 \rangle \equiv
  if (cur\_cell > \& cur\_chunk \neg cell[0]) \ cur\_cell --;
  else {
     register chunk *old\_chunk = cur\_chunk;
     cur\_chunk = old\_chunk \rightarrow prev; free(old\_chunk);
     bad\_cell = \& cur\_chunk \neg cell[cells\_per\_chunk];
     cur\_cell = bad\_cell - 1;
This code is used in sections 19 and 36.
21. (Move cur\_tmp\_var backward to the previous temporary variable 21) \equiv
  if (cur\_tmp\_var > \& cur\_vchunk \neg var[0]) cur\_tmp\_var ---;
  else {
     register vchunk *old\_vchunk = cur\_vchunk;
     cur\_vchunk = old\_vchunk \neg prev; free(old\_vchunk);
     bad\_tmp\_var = \& cur\_vchunk \neg var[vars\_per\_vchunk];
     cur\_tmp\_var = bad\_tmp\_var - 1;
  }
This code is used in section 37.
     \langle Report the successful completion of the input phase 22 \rangle \equiv
  fprintf(stderr, "("O"lld_variables, "O"lld_clauses, "O"llu_literals_successfully_read)\n",
        vars, clauses, cells);
This code is used in section 2.
```

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**23. SAT solving, version 10.** Okay, now comes my hypothetical recreation of the Brown–Purdom SAT solver. (It's unfortunate that no copy of their original program survives.)

The algorithm below essentially solves a satisfiability problem by backtracking. At each level it tries two possibilities for some unset variable, unless it finds an unset variable for which there's only one viable possibility based on previously set variables (thus making a forced move), or unless it finds an unset variable with *no* viable possibilities (in which case it backs up to the previous level of branching).

The key idea is that the first literal in every clause is considered to be "watched," and the watched literal has not been set false. If the algorithm does want to make that literal false, it must first swap another literal of the clause into the first position.

This method can be implemented with extremely simple data structures:

- For each clause c, there's a sequential list of the literals in c.
- For each variable v, there are linked lists of the clauses that are watching v and  $\bar{v}$ .
- Each variable is either set to true, set to false, or unknown.
- There's a circular list containing all the unset variables whose literals are watched by at least one clause. (This list is called the active ring; we're done when it becomes empty at decision time.)

And of course we remember the current trail of decisions made at each level of the implicit backtrack tree.

24. Each link is a 32-bit integer. (I don't use C pointers in the main data structures, because they occupy 64 bits and clutter up the caches.) Links in the watch lists are indexes of clauses; links in the active ring are indexes of variables. A zero link indicates the end of a list.

The literals within a clause, called "cells," are 32-bit unsigned integers kept in a big array called mem. Variable number k, for  $1 \le k \le vars$ , corresponds to the literals numbered 2k and 2k + 1.

Each clause is represented by a pointer to its first cell and by a link to the successor clause (if any) with the same watched literal.

```
⟨Type definitions 5⟩ +≡
typedef struct {
  uint start; /* the address in mem where the cells for this clause start */
  uint wlink; /* link to another clause in the same watch list */
} clause;
```

25. Several items are stored for each variable: The heads of its two watch lists; the link to the next active variable; a spare field for miscellaneous use; and the 8-byte symbolic name.

(We also keep the current values of variables in a separate array val, with one byte for each variable.)

```
#define false 0
                       /* val code for a false literal */
                       /* val code for a true literal */
#define true 1
#define unknown
                     -1
                             /* val code for an unset literal */
\langle \text{Type definitions 5} \rangle + \equiv
  typedef struct {
    uint wlist0, wlist1;
                              /* heads of the watch lists */
    int next;
                  /* next item in the ring of active variables */
                    /* extra field used only by sanity at the moment */
    uint spare;
                     /* the variable's symbolic name */
    octa name;
  } variable;
```

**26.** The backtracking process maintains a sequential stack of state information.

```
⟨Type definitions 5⟩ +≡
typedef struct {
  int var; /* variable whose value is being set */
  int move; /* code for what we're setting it */
} state;
```

printf("Ring:"); **if** (active) {

 $printf("\n");$ 

if  $(p \equiv active)$  break;

for (p = vmem[active].next; ; p = vmem[p].next) {  $printf("_{\sqcup}"O".8s", vmem[p].name.ch8);$ 

§27

```
27. \langle \text{Global variables } 3 \rangle + \equiv
                    /* the master array of cells */
  uint *mem;
                       /* the master array of clauses */
  clause *cmem;
  variable *vmem;
                        /* the master array of variables */
  \mathbf{char} *val;
                  /* the master array of variable values */
  state *smem;
                      /* the stack of choices made so far */
  uint active:
                    /* an item in the active ring, or zero if that ring is empty */
```

28. Here is a subroutine that prints a clause symbolically. It illustrates some of the conventions of the data structures that have been explained above. I use it only for debugging.

Incidentally, the clause numbers reported to the user after the input phase may differ from the line numbers reported during the input phase, when null clauses > 0.

```
\langle Subroutines 28\rangle \equiv
  void print_clause(int c)
    register uint k, l;
     printf(""O"d:", c);
                               /* show the clause number */
    for (k = cmem[c].start; k < cmem[c-1].start; k++) {
       l = mem[k];
       printf("_{\sqcup}"O"s"O".8s", l \& 1?""":"", vmem[l \gg 1].name.ch8);
                                                                                     /* kth literal */
     printf("\n");
See also sections 29, 30, 31, and 32.
This code is used in section 2.
29. Similarly we can print out all of the clauses that currently watch a particular literal.
\langle \text{Subroutines } 28 \rangle + \equiv
  void print_watches_for(int l)
    register int c;
    if (l \& 1) \ c = vmem[l \gg 1].wlist1;
     else c = vmem[l \gg 1].wlist0;
     for (; c; c = cmem[c].wlink) print_clause(c);
30. \langle Subroutines 28 \rangle + \equiv
  void print_ring(void)
     register int p;
```

31. Speaking of debugging, here's a routine to check if the redundant parts of our data structure have gone awry.

```
#define sanity_checking 0
                                     /* set this to 1 if you suspect a bug */
\langle \text{Subroutines } 28 \rangle + \equiv
  void sanity(void)
     register int k, l, c, v;
     if (active) {
       for (v = vmem[active].next; ; v = vmem[v].next) {
                                     /* all spare fields assumed zero otherwise */
          vmem[v].spare = 1;
          if (v \equiv active) break;
       }
     k = 0;
     for (v = 1; v < vars; v ++) {
       for (c = vmem[v].wlist0; c; c = cmem[c].wlink) {
          k++;
          if (mem[cmem[c].start] \neq v + v)
            fprintf(stderr, "Clause_{\square}"O"d_{\square}watches_{\square}"O"u,_{\square}not_{\square}"O"u!\\n", c, mem[cmem[c].start], v+v);
          else if (val[v] \equiv false)
            fprintf(stderr, "Clause_{||}"O"d_{||}watches_{||}the_{||}false_{||}literal_{||}"O"u!\n", c, (v+v));
       for (c = vmem[v].wlist1; c; c = cmem[c].wlink) {
          k++;
          if (mem[cmem[c].start] \neq v + v + 1)
            else if (val[v] \equiv true)
            fprintf(stderr, "Clause_{\sqcup}"O"d_{\sqcup}watches_{\sqcup}the_{\sqcup}false_{\sqcup}literal_{\sqcup}"O"u! \n", c, (v+v+1));
       if (vmem[v].spare \equiv 0 \land val[v] \equiv unknown \land (vmem[v].wlist0 \lor vmem[v].wlist1))
          fprintf(stderr, "Variable_{\square}"O".8s_{\square}should_{\square}be_{\square}in_{\square}the_{\square}active_{\square}ring! n", vmem[v].name.ch8);
       if (vmem[v].spare \equiv 1 \land (val[v] \neq unknown \lor ((vmem[v].wlist0 \mid vmem[v].wlist1) \equiv 0)))
          fprintf(stderr, "Variable_{\square}"O".8s_{\square}should_{\square}not_{\square}be_{\square}in_{\square}the_{\square}active_{\square}ring!\\",
               vmem[v].name.ch8);
       vmem[v].spare = 0;
     if (k \neq clauses)
       fprintf(stderr, "Oops: \_"O"d_Uof_U"O"lld_Uclauses_Uare_Ubeing_Uwatched! \n", k, clauses);
  }
32. In long runs it's helpful to know how far we've gotten.
\langle \text{Subroutines } 28 \rangle + \equiv
  void print_state(int l)
     register int k;
     fprintf(stderr, "\_after\_"O"lld\_mems: ", mems);
     for (k = 1; k \le l; k++) fprintf (stderr, ""O"c", smem[k].move + '0');
     fprintf(stderr, "\n");
     fflush(stderr);
```

**33. Initializing the real data structures.** We're ready now to convert the temporary chunks of data into the form we want, and to recycle those chunks. The code below is, of course, similar to what has worked in previous programs of this series.

```
\langle \text{ Set up the main data structures } 33 \rangle \equiv
  \langle Allocate the main arrays 34 \rangle;
   Copy all the temporary cells to the mem and cmem arrays in proper format 35;
   (Copy all the temporary variable nodes to the vmem array in proper format 37);
  \langle \text{ Check consistency } 38 \rangle;
  if (out\_file) \langle Copy all the clauses to <math>out\_file 39 \rangle;
This code is used in section 2.
34. \langle Allocate the main arrays 34 \rangle \equiv
                                /* a tiny gesture to make a little room */
  free(buf); free(hash);
  mem = (\mathbf{uint} *) \ malloc(cells * \mathbf{sizeof}(\mathbf{uint}));
  if (\neg mem) {
     fprintf(stderr, "Oops, \sqcup I_{\sqcup}can't_{\sqcup}allocate_{\sqcup}the_{\sqcup}big_{\sqcup}mem_{\sqcup}array! \n");
     exit(-10);
  bytes = cells * sizeof(uint);
  cmem = (clause *) malloc((clauses + 1) * sizeof(clause));
  if (\neg cmem) {
     fprintf(stderr, "Oops, \sqcup I_{\sqcup} can't_{\sqcup} allocate_{\sqcup} the_{\sqcup} cmem_{\sqcup} array! \n");
     exit(-11);
  bytes += (clauses + 1) * sizeof(clause);
  vmem = (variable *) malloc((vars + 1) * sizeof(variable));
  if (\neg vmem) {
     exit(-12);
  bytes += (vars + 1) * sizeof(variable);
  smem = (state *) malloc((vars + 1) * sizeof(state));
  if (\neg smem) {
     exit(-13);
  bytes += (vars + 1) * sizeof(state);
  val = (\mathbf{char} *) \ malloc((vars + 1) * \mathbf{sizeof}(\mathbf{char}));
  if (\neg val) {
     fprintf(stderr, "Oops, \sqcup I_{\sqcup}can't_{\sqcup}allocate_{\sqcup}the_{\sqcup}val_{\sqcup}array! \n");
  bytes += (vars + 1) * sizeof(char);
This code is used in section 33.
```

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```
35.
      \langle \text{Copy all the temporary cells to the } mem \text{ and } cmem \text{ arrays in proper format } 35 \rangle \equiv
  for (j = 1; j \le vars; j ++)  {
     o, vmem[j].wlist0 = vmem[j].wlist1 = 0;
     o, val[j] = unknown;
  for (c = clauses, j = 0; c; c--) {
     o, cmem[c].start = k = j;
     (Insert the cells for the literals of clause c 36);
     l = mem[k];
     if (l \& 1) ooo, p = vmem[l \gg 1].wlist1, cmem[c].wlink = p, vmem[l \gg 1].wlist1 = c;
     else ooo, p = vmem[l \gg 1].wlist0, cmem[c].wlink = p, vmem[l \gg 1].wlist0 = c;
  if (j \neq cells) {
     fprintf(stderr, "Oh_loh,_lsomething_lhappened_lto_l"O"d_lcells!\n", (int) cells = j);
     exit(-15);
  o, cmem[c].start = j;
This code is used in section 33.
      The basic idea is to "unwind" the steps that we went through while building up the chunks.
#define hack_out(q) (((ullng) q) & #3)
#define hack\_clean(q) ((tmp_var *)((ullng) q \& -4))
(Insert the cells for the literals of clause c 36) \equiv
  for (i = 0; i < 2;)
     \langle \text{Move } cur\_cell \text{ backward to the previous cell } 20 \rangle;
     i = hack\_out(*cur\_cell);
     p = hack\_clean(*cur\_cell) \rightarrow serial;
    p += p + (i \& 1) + 2;
     o, mem[j++] = p;
This code is used in section 35.
37. (Copy all the temporary variable nodes to the vmem array in proper format 37) \equiv
  for (c = vars; c; c--) {
     \langle \text{Move } cur\_tmp\_var \text{ backward to the previous temporary variable } 21 \rangle;
     o, vmem[c].name.lng = cur\_tmp\_var \rightarrow name.lng;
This code is used in section 33.
38. We should now have unwound all the temporary data chunks back to their beginnings.
\langle \text{ Check consistency } 38 \rangle \equiv
  if (cur\_cell \neq \&cur\_chunk \neg cell[0] \lor cur\_chunk \neg prev \neq \Lambda \lor cur\_tmp\_var \neq
          & cur\_vchunk \rightarrow var[0] \lor cur\_vchunk \rightarrow prev \neq \Lambda) {
     fprintf(stderr, "This_can't_happen_(consistency_check_failure)! \n");
     exit(-14);
  free(cur_chunk); free(cur_vchunk);
This code is used in section 33.
```

```
39. \langle Copy all the clauses to out\_file 39\rangle \equiv {
    for (k=0,c=clauses;\ c;\ c--) {
        for (;\ k< cmem[c-1].start;\ k++) {
            l=mem[k];
            fprintf(out\_file,"\"O"s"O".8s",l&1?"~":"",vmem[l\gg 1].name.ch8); /* kth literal */ }
        }
        fprintf(out\_file,"\n");
    }
    fflush(out\_file); /* complete the copy of input clauses */
}
```

This code is used in section 33.

18 Doing it satio \$40

**40. Doing it.** Now comes ye olde basic backtrack, but simplified because updates to the watch lists don't have to be undone.

At level l of the backtrack process we record the variable, smem[l].var, whose value is being specified, and its chosen value, smem[l].move. The latter value is 0 or 1 if we're making a binary branch and we're trying first to make the variable true or false, respectively; it is 3 or 2 if that move failed and we're trying the other alternative. It is 4 or 5 if the move was forced and the variable had to be set respectively to true or false.

```
\langle Solve the problem 40 \rangle \equiv
  o, level = 0, smem[0].move = 0;
  \langle Initialize the active ring 41\rangle;
choose: if (sanity_checking) sanity();
  if (delta \land (mems \ge thresh)) thresh += delta, print\_state(level);
  if (mems > timeout) {
     fprintf(stderr, "TIMEOUT!\n");
     goto done;
  (Decide what to do next, going either to branch or forcedmove or backup or satisfied 42);
branch: o, nextmove = (vmem[v].wlist0 \equiv 0 \lor vmem[v].wlist1 \neq 0);
  nodes ++;
forcedmove: level ++;
                           /* at this point vmem[active].next = v is the branch variable */
  o, smem[level].var = v, smem[level].move = nextmove;
                                    /* the ring becomes empty */
  if (active \equiv v) active = 0;
  else oo, h = vmem[v].next, vmem[active].next = h;
                                                              /* delete v from the ring */
makemove: \langle \text{Set } v \text{ and update the watch lists for its new value } 45 \rangle;
  goto choose;
backup: (Backtrack to the most recent unforced move 48);
This code is used in section 2.
41. \langle Initialize the active ring \langle 41\rangle \equiv
  for (active = j = 0, k = vars; k; k--)
     if ((o, vmem[k].wlist0) \lor (vmem[k].wlist1)) {
       if (active \equiv 0) active = k;
       o, vmem[k].next = j, j = k;
  if (active) o, vmem[active].next = j; /* complete the circle */
This code is used in section 40.
```

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**42.** The basic operation we need to do at each level is to decide which variable in the active ring should be set next. And experience with SAT problems shows that, once we get going, there's usually at least one unset variable whose value is forced by previous clauses.

A literal is forced to be true if and only if there's a clause in its watch list such that all other literals of that clause are already set to false. Therefore we go through the watch list of every active variable until we either find such a literal or discover that there is no forcing at the present time.

When a forced literal is found, we'll want to resume searching for another one at the same place where we left off, thus going cyclically through the active ring. (For if we were to start searching again at the beginning of that list, we'd be covering more or less the same ground as before.)

```
\langle Decide what to do next, going either to branch or forcedmove or backup or satisfied 42 \rangle \equiv
  if (active \equiv 0) goto satisfied;
  if (verbose & show_details) {
     fprintf(stderr, "_active_ring:");
     for (v = vmem[active].next; ; v = vmem[v].next) {
       fprintf(stderr, ""O".8s", vmem[v].name.ch8);
       if (v \equiv active) break;
     fprintf(stderr, "\n");
  vv = active, vvv = 0;
newv: o, v = vmem[vv].next;
                                      /* during the search, v is one step ahead of vv */
  force = 0:
  \langle \text{Set } force = 1 \text{ if variable } v \text{ must be true } 43 \rangle;
  \langle \text{ Set } force \neq 2 \text{ if variable } v \text{ must be false } 44 \rangle;
  if (force \equiv 3) goto backup;
  if (force) {
     nextmove = force + 3;
     active = vv;
     goto forcedmove;
  if (vvv \equiv 0 \land v \le primary\_vars) \ vvv = vv; /* vvv precedes the first active primary variable */
  if (v \equiv active) {
     if (vvv) vv = active = vvv;
     v = vmem[active].next;
     goto branch;
  vv = v; goto newv;
This code is used in section 40.
```

20 Doing it satisfy \$43

**43.** When literal l is watched in clause c, we know that l is the first literal of c. We scan through the other literals until either reaching a literal that's currently unknown or true (whence nothing is forced), or reaching the end (whence l is forced).

If we encounter a true literal l', we could swap it into first position, thereby moving clause c from the watch list of l to the watch list of l', where it probably won't need to be examined as often. But that's a complication that I will postpone for future study, to be explored in variants of this program.

```
\langle \text{ Set } force = 1 \text{ if variable } v \text{ must be true } 43 \rangle \equiv
  for (o, c = vmem[v].wlist0; c; o, c = cmem[c].wlink) {
     for (oo, k = cmem[c].start + 1; k < cmem[c - 1].start; k++)
        if (oo, val[mem[k] \gg 1] \neq (mem[k] \& 1)) goto unforced0;
     if (verbose & show_details)
       fprintf(stderr, "(Clause_{\square}"O"d_{\square}reduced_{\square}to_{\square}"O".8s)\n", c, vmem[v].name.ch8);
     force = 1;
     goto forced0;
  unforced\theta: continue;
  forced0:
This code is used in section 42.
44. \langle Set force += 2 if variable v must be false 44 \rangle \equiv
  for (o, c = vmem[v].wlist1; c; o, c = cmem[c].wlink) {
     for (oo, k = cmem[c].start + 1; k < cmem[c - 1].start; k++)
        if (oo, val[mem[k] \gg 1] \neq (mem[k] \& 1)) goto unforced1;
     if (verbose & show_details)
       fprintf(stderr, "(Clause_{\sqcup}"O"d_{\sqcup}reduced_{\sqcup}to_{\sqcup}^{\sim}"O".8s)\n", c, vmem[v].name.ch8);
     force += 2;
     goto forced1;
  unforced1: continue;
  forced1:
This code is used in section 42.
```

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```
45.
      \langle Set v and update the watch lists for its new value 45\rangle \equiv
  if ((verbose \& show\_choices) \land level \le show\_choices\_max) {
     fprintf(stderr, "Level," O"d,,,", level);
     switch (nextmove) {
     case 0: fprintf(stderr, "trying_{\sqcup}"O".8s", vmem[v].name.ch8); break;
     case 1: fprintf(stderr, "trying_""O".8s", vmem[v].name.ch8); break;
     case 2: fprintf(stderr, "retrying_"O".8s", vmem[v].name.ch8); break;
     \mathbf{case}\ 3:\ \mathit{fprintf}(\mathit{stderr}, \texttt{"retrying}\_\texttt{``}"O".8s", \mathit{vmem}[v].\mathit{name.ch8});\ \mathbf{break};
     case 4: fprintf(stderr, "forcing_"O".8s", vmem[v].name.ch8); break;
     case 5: fprintf(stderr, "forcing_""O".8s", vmem[v].name.ch8); break;
     fprintf(stderr, ", "O"lld_mems \n", mems);
  if (nextmove & 1) {
     o, val[v] = false;
     oo, c = vmem[v].wlist0, vmem[v].wlist0 = 0, ll = v + v;
  } else {
     o, val[v] = true;
     oo, c = vmem[v].wlist1, vmem[v].wlist1 = 0, ll = v + v + 1;
  \langle Clear the watch list for ll that starts at c 46\rangle;
This code is used in section 40.
46. Clear the watch list for ll that starts at c 46 \equiv
  for ( ; c; c = cc)  {
     o, cc = cmem[c].wlink;
     for (oo, j = cmem[c].start, k = j + 1; k < cmem[c - 1].start; k++)  {
       o, l = mem[k];
       if (o, val[l \gg 1] \neq (l \& 1)) break;
     if (k \equiv cmem[c-1].start) {
       fprintf(stderr, "Clause_{\square}"O"d_{\square}can't_{\square}be_{\square}watched! \n", c);
                                                                               /* "can't happen" */
       exit(-18);
     oo, mem[k] = ll, mem[j] = l;
     \langle \text{ Put } c \text{ into the watch list of } l \text{ 47} \rangle;
This code is used in section 45.
```

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47. The variable corresponding to l might become active at this point, because it might not be watched anywhere else. In such a case we insert it at the "beginning" of the active ring (that is, just after active). We always have vmem[active].next = h at this point, unless active = 0.

```
 \begin{array}{l} \langle \operatorname{Put}\ c \ \operatorname{into}\ \operatorname{the}\ \operatorname{watch}\ \operatorname{list}\ \operatorname{of}\ l\ 47 \rangle \equiv \\ & \mathbf{if}\ (\operatorname{verbose}\ \&\ \operatorname{show\_details})\ \operatorname{fprintf}\ (\operatorname{stderr}, "(\operatorname{Clause}_{\square}"O"\operatorname{d}_{\square}\operatorname{now}_{\square}\operatorname{watches}_{\square}"O"\operatorname{s}"O".8\operatorname{s}) \\ & l\ \&\ 1\ ?\ "^{"}:\ "",\ \operatorname{vmem}\ [l\gg 1].\operatorname{name.ch8}); \\ & o,p = \operatorname{vmem}\ [l\gg 1].\operatorname{wlist0},\ q = \operatorname{vmem}\ [l\gg 1].\operatorname{wlist1}; \\ & \mathbf{if}\ (\operatorname{val}\ [l\gg 1]\equiv \operatorname{unknown}\land p\equiv 0\land q\equiv 0)\ \{ \\ & \mathbf{if}\ (\operatorname{active}\equiv 0)\ o,\operatorname{active}=h=l\gg 1,\operatorname{vmem}\ [\operatorname{active}].\operatorname{next}=h; \\ & \mathbf{else}\ oo,\operatorname{vmem}\ [l\gg 1].\operatorname{next}=h,h=l\gg 1,\operatorname{vmem}\ [\operatorname{active}].\operatorname{next}=h; \\ & \mathbf{if}\ (l\ \&\ 1)\ oo,\operatorname{cmem}\ [c].\operatorname{wlink}=q,\operatorname{vmem}\ [l\gg 1].\operatorname{wlist1}=c; \\ & \mathbf{else}\ oo,\operatorname{cmem}\ [c].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1].\operatorname{wlist0}=c; \\ & \mathbf{clse}\ oo,\operatorname{cmem}\ [c].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1]. \\ & \mathbf{clse}\ oo,\operatorname{cmem}\ [c].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1]. \\ & \mathbf{clse}\ oo,\operatorname{vmem}\ [l\gg 1].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1]. \\ & \mathbf{clse}\ oo,\operatorname{vmem}\ [l\gg 1].\operatorname{wlink}=p,\operatorname{vmem}\ [l\gg 1].
```

48. If variables need to be reactivated here, we put them just before the place where a conflict was found.

```
\langle \text{Backtrack to the most recent unforced move 48} \rangle \equiv
  active = vv, h = v;
  while (o, smem[level].move \ge 2) {
    v = smem[level].var;
    o, val[v] = unknown;
    if ((o, vmem[v].wlist0 \neq 0) \lor (vmem[v].wlist1 \neq 0))
       oo, vmem[v].next = h, h = v, vmem[active].next = h;
    level ---;
  if (level) {
    nextmove = 3 - smem[level].move;
    oo, v = smem[level].var, smem[level].move = nextmove;
    goto makemove;
  if (1) {
    printf(" \sim n");
                         /* the formula was unsatisfiable */
    if (verbose & show_basics) fprintf(stderr, "UNSAT\n");
  } else {
  satisfied: if (verbose & show_basics) fprintf(stderr, "!SAT!\n");
    \langle Print the solution found 49\rangle;
```

This code is used in section 40.

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```
49. \langle Print the solution found 49\rangle \equiv for (k=1;\ k\leq level;\ k++) { l=(smem[k].var\ll 1)+(smem[k].move\ \&\ 1); printf("\upsilon"0"s"O".8s",l\ \&\ 1?"":"",vmem[l\gg 1].name.ch8); if (out\_file)\ fprintf(out\_file,"\upsilon"0"s"O".8s",l\ \&\ 1?"":"\upsilon"vmem[l\gg 1].name.ch8); } printf("\upsilon"); if (level<vars) { if (verbose\ \&\ show\_unused\_vars)\ printf("(Unused:"); for (v=0;\ v<vars;\ v++) if (val[v]\equiv unknown) { if (verbose\ \&\ show\_unused\_vars)\ printf("\upsilon"0".8s",vmem[v].name.ch8); if (out\_file)\ fprintf(out\_file,"\upsilon"0".8s",vmem[v].name.ch8); } if (verbose\ \&\ show\_unused\_vars)\ printf(")\n"); } if (out\_file)\ fprintf(out\_file,"\upsilon"); This code is used in section 48.
```

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## 50. Index.

active: <u>27,</u> 30, 31, 40, 41, 42, 47, 48. i:  $\underline{2}$ . imems:  $2, \underline{3}$ . argc: 2, 4. $argv: \underline{2}, \underline{4}.$ j:  $\underline{2}$ .  $k: \ \underline{2}, \ \underline{28}, \ \underline{31}, \ \underline{32}.$ backup:  $\underline{40}$ ,  $\underline{42}$ .  $kk: \underline{2}.$  $bad\_tmp\_var$ : 7, 12, 13, 21.  $l: \ \underline{2}, \ \underline{28}, \ \underline{29}, \ \underline{31}, \ \underline{32}.$ level: 2, 40, 45, 48, 49. branch:  $\underline{40}$ ,  $\underline{42}$ . buf: 7, 8, 9, 10, 11, 16, 19, 34. ll: 2, 45, 46. $buf\_size: \ \ \underline{3}, \ 4, \ 8, \ 9, \ 10.$  $lng: \underline{5}, 16, 17, 37.$ bytes:  $2, \underline{3}, 34.$  $main: \underline{2}.$  $c: \ \underline{2}, \ \underline{28}, \ \underline{29}, \ \underline{31}.$  $makemove: \underline{40}, 48.$  $cc: \ \underline{2}, \ 46.$ malloc: 8, 13, 14, 34. cell: 6, 14, 20, 38. mem: 24, 27, 28, 31, 34, 35, 36, 39, 43, 44, 46. cells: 7, 9, 10, 11, 22, 34, 35. mems:  $2, \underline{3}, 4, 32, 40, 45.$  $cells\_per\_chunk: \underline{6}, 14, 20.$ move: <u>26</u>, 32, 40, 48, 49. choose:  $\underline{40}$ . name:  $\underline{5}$ , 16, 17,  $\underline{25}$ , 28, 30, 31, 37, 39, 42, 43, **chunk**:  $\underline{6}$ , 7, 14, 20. 44, 45, 47, 49. chunk\_struct: 6.  $new\_chunk$ : 14. *ch8*: <u>5, 16, 28, 30, 31, 39, 42, 43, 44, 45, 47, 49.</u>  $new\_vchunk$ : 13. clause: 24, 27, 34.newv: 42.clauses: 7, 9, 10, 11, 12, 16, 19, 22, 31, 34, 35, 39. next: 5, 17, 25, 30, 31, 40, 41, 42, 47, 48. *cmem*: <u>27,</u> 28, 29, 31, 34, 35, 39, 43, 44, 46, 47. nextmove:  $\underline{2}$ , 40, 42, 45, 48. cur\_cell: 7, 12, 14, 20, 36, 38. nodes:  $2, \underline{3}, 40.$ cur\_chunk: 7, 14, 20, 38. non\_clause: 7, 11, 12, 16, 18, 19. cur\_tmp\_var: 7, 12, 13, 16, 17, 21, 37, 38. nullclauses: 7, 9, 10, 11, 19, 28.  $cur\_vchunk$ :  $\underline{7}$ , 13, 21, 38.  $O: \quad \underline{2}.$ delta:  $\underline{3}$ , 4, 40.  $o: \underline{2}$ . done: 2, 40. octa:  $\underline{5}$ ,  $\underline{25}$ . eps: 3, 4. $old\_chunk$ : 20. exit: 4, 8, 9, 10, 11, 13, 14, 16, 34, 35, 38, 46.  $old\_vchunk$ : 21. oo: 2, 40, 43, 44, 45, 46, 47, 48. false: 25, 31, 45.fflush: 32, 39.*ooo*:  $\underline{2}$ , 35. *fgets*: 9, 10. out\_file: 3, 4, 33, 39, 49. fopen: 4. $out\_name: 3, 4.$ force:  $\underline{2}$ , 42, 43, 44.  $p: \ \underline{2}, \ \underline{12}, \ \underline{30}.$ forcedmove:  $\underline{40}$ ,  $\underline{42}$ .  $pp: \underline{2}.$  $forced0: \underline{43}.$ prev:  $\underline{5}$ ,  $\underline{6}$ , 13, 14, 20, 21, 38.  $forced1: \underline{44}.$  $primary\_file: \underline{3}, 4, 9, 10.$ fprintf: 2, 4, 8, 9, 10, 11, 13, 14, 16, 19, 22, 31, 32,  $primary\_name: \underline{3}, 4, 10.$ primary\_vars:  $\underline{3}$ , 9, 10, 42. 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49. free: 20, 21, 34, 38.  $print\_clause$ : 28, 29.  $gb\_init\_rand$ : 8.  $print\_ring: \underline{30}.$  $gb\_next\_rand$ : 15.  $print\_state: 32, 40.$  $gb\_rand$ : 3.  $print\_watches\_for$ : 29.  $h: \underline{2}.$ printf: 28, 30, 48, 49.  $hack\_clean: \underline{36}.$  $q: \underline{2}$ .  $hack_in: \underline{12}.$  $qq: \underline{2}.$  $hack\_out$ : 36.  $r: \underline{2}.$ hash: <u>7</u>, 8, 17, 34.  $random\_seed: \underline{3}, 4, 8.$ sanity: 25, 31, 40. $hash\_bits: \frac{7}{2}, 15, 16.$ hbits: 3, 4, 8, 9, 16.  $sanity\_checking: 31, 40.$ 

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26 NAMES OF THE SECTIONS SAT10

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(Allocate the main arrays 34) Used in section 33.
(Backtrack to the most recent unforced move 48) Used in section 40.
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\langle \text{ Input the clauses 9} \rangle Used in section 2.
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(Put the variable name beginning at buf[j] in cur\_tmp\_var \neg name and compute its hash code h 16) Used
     in section 12.
\langle \text{ Put } c \text{ into the watch list of } l \text{ 47} \rangle Used in section 46.
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