Solve the following :

$$\int_{x=0}^{\pi/4} cos2x\,\sqrt{4-sin2x}\,dx$$

Let $u = sin2x$, then

$$du = 2cos2x\,dx$$

$$\frac{du}{2cos2x} = dx$$

Substituting the vaule back into the original integral, we get

$$\int_{x=0}^{\pi/4} cos2x\,\sqrt{4-u}\,\frac{du}{2cos2x}$$

$$\frac{1}{2}\int_{x=0}^{\pi/4} \sqrt{4-u}\,du$$

$$\frac{1}{2}\int_{x=0}^{\pi/4} (4-u)^{\frac{1}{2}}\,du$$

$$\left[-\frac{1}{2} * \frac{(4-u)^{\frac{1}{2}+1}}{(\frac{1}{2}+1)} + C\right]_0^{\frac{\pi}{4}}$$

$$\left[-\frac{1}{2} * \frac{2}{3}(4-u)^{\frac{3}{2}} + C\right]_0^{\frac{\pi}{4}}$$

$$\left[-\frac{1}{3}(4-u)^{\frac{3}{2}} + C\right]_0^{\frac{\pi}{4}}$$

Substituting the value for u back into the solution we get

$$\left[-\frac{1}{3}(4-sin2x)^{\frac{3}{2}} + C\right]_0^{\frac{\pi}{4}}$$

$$(-\frac{1}{3}(4-sin\frac{\pi}{2})^{\frac{3}{2}} + C) - (-\frac{1}{3}(4-sin0)^{\frac{3}{2}} + C)$$

$$(-\frac{1}{3}(4-1)^{\frac{3}{2}} + C) - (-\frac{1}{3}(4-0)^{\frac{3}{2}} + C)$$

$$(-\frac{1}{3}(3)^{\frac{3}{2}} + C) - (-\frac{1}{3}(4)^{\frac{3}{2}}x + C)$$

$$(-3^{\frac{3}{2}-1} + C) - (-\frac{8}{3} + C)$$

$$-3^{\frac{1}{2}} + C + \frac{8}{3} - C$$

$$\frac{8}{3} - \sqrt{3}$$