ME 541 Project

Decision Making using Expected Utility Theory for Vehicle Maintenance

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ME 541 – Engineering Design: A Decision-based Perspective
Fall 2023

Project: Decision Making Using Expected Utility Theory
Due: 11:59 PM (Eastern timezone) on November 21, 2023

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Maximum Points: 100

Submission: The complete project report must be submitted online as a single PDF file on Gradescope. No intermediate deliverables are required.

Objective: To gain an understanding of multi-attribute utility theory through application to an engineering design/manufacturing problem of your interest.

Tasks: (Use this structure for your report also. Target completion dates have been suggested for each task. Tasks 3 and 5 can be carried out in parallel.)

1. **Decision Situation:** (10 points) Identify a decision situation related to your area of expertise/interest. In your report, describe the problem context. Discuss what kind of decision you need to make. Discuss the motivation and importance of such decision making.

   Target completion date: October 10, 2023

2. **Framing the decision:** (10 points) Frame one decision by identifying objectives, attributes, and alternatives, as discussed in Module 3.
   - *Objectives:* Describe the fundamental objectives and means objectives.
   - *Attributes:* Identify the attributes essential towards achieving the means objectives. Identify appropriate scales for quantifying the attributes. Evaluate whether the attributes are comprehensive, measurable, and relevant (see Slide 27 of Module 3). Select four to six attributes from your list for developing an overall utility function. At least three attributes should have uncertainty.
   - *Alternatives:* List and describe the alternatives to be compared.

   Target completion date: October 17, 2023

3. **Assess single-attribute utility functions for the selected attributes.** (30 points) Use the procedure discussed in Module 06 for each selected attribute.
   - Identify the relevant qualitative characteristics (monotonicity, risk aversion).
   - Specify quantitative restrictions: (i) choose an appropriate functional form based on the qualitative characteristics, and (ii) determine points on the utility function by asking lottery questions).
   - Estimate the parameters of the utility function. List the estimated utility function.
   - Check for consistency to uncover any discrepancies in the utility function, and to ensure that the risk preferences have been correctly modeled.

   In the report, clearly document the process, including the lottery questions utilized, qualitative characteristics, and plots of utility functions.

   Target completion date: October 24, 2023

4. **Assess multi-attribute utility function.** (20 points) Use the procedure discussed in Module 07 to assess a multi-attribute utility function for the decision framed in Task 2 above.
   - Check whether additive independence conditions are satisfied.
   - Assuming that the additive independence conditions are satisfied, assess the scaling constants and determine the multi-attribute utility function.

Target completion date: October 31, 2023

5. **Collect Information on Alternatives.** (15 points) For each alternative, quantify the uncertainty (probability distributions) associated with achieving each attribute. Provide the relevant data of the alternatives with respect to the attributes you identified. Cite your sources. Discuss assumptions and situations where there is a lack of information.

   Target completion date: November 07, 2023

6. **Evaluate the expected utility of each alternative.** (5 points) Using the expected utilities, determine the most preferred alternative. Tabulate the results.

   Target completion date: November 14, 2023

7. **Discussion and Learning.** (10 points) Discuss the outcome of your decision-making problem. Discuss the validity of the outcome. Discuss the major assumptions of the entire process. Discuss your insights from this project. What did you learn?

   Target completion date: November 20, 2023

Table of Contents

Decision Situation

Background

I recently purchased a car for commuting purposes now that I have joined Graduate school at Purdue University for Mechanical Engineering. I live relatively far out in Lafayette, which makes commuting to and from campus inconvenient if I use the Citybus service, considering I often work late into the night in my part-time job. I also wished for a convenient form of transportation in case I wish to purchase groceries or run errands, without the expenses incurred by using Uber/Lyft everywhere I needed to go. Lastly, I wished to make the occasional trip to Chicago or Indianapolis during breaks or for weekend trips, and a ~100$ round-trip bus ticket using any of the bus services available nearby was unsustainable for me. Thus, in August, prior to the start of the semester, I purchased a 2016 Toyota Avalon and have been using it since.

Decision Context

As a first time car owner, I was very inexperienced with the buying process and had to read a large amount before investing in my first car, in turn making several key decisions through online research and my father's guidance. This process was very interesting, and I used several key concepts I learned about in this class in my search (although in a less mathematical and more of a logical sense). I wish to now come back to this decision that I was attempting to make in July and August and see if I can help past Abhishek from late July in his car buying decision, and to see if what I learn in this project supports my car purchasing choice or indicates to me that there could have been a better option.

Framing the Decision

Fundamental Objectives

This decision had 4 overarching fundamental objectives that I had identified prior to purchasing my vehicle. Although there are several fundamental objectives that can be decided upon, these were the ones my father and I identified as the most important based on my need as a new graduate student and a working professional once I complete my degree.

<u>Fundamental Objective 1: Costs involved in the vehicle.</u>

Since I am a new graduate student with little in the way of personal savings and my primary means of income being limited to a part-time job and an allowance sent over by my parents, it is imperative that the vehicle not be very expensive to own. By this, I do not only mean the sticker price on the car, but also the costs involved in ongoing expenses, such as fuel, maintenance and insurance costs. A cheaper car would allow for a lower fuel cost (since it would use regular gas instead of premium) and lower insurance costs, but in turn limit my purchase options and force me to purchase an older car or one with higher miles on it, impacting the vehicle maintenance costs. A relatively more expensive car would give me more purchase options and would not require immediate maintenance of expensive parts, but would also increase my ongoing costs. I would also need to account for depreciation and resale value of my car in buying it (since some brands and makes depreciate more than other, a Maserati Ghibli compared to a Porsche 911 despite being in the same price range for example)

<u>Fundamental Objective 2: Technology and Safety features</u>

Since I am a relatively new driver and have very little experience driving in snow and ice (having learned to drive in Texas), it is essential for my car to have modern technology and safety assists. These would include blind spot monitoring, cruise control, anti-lock brakes and traction control. I would also like certain characteristics of the car to be more beginner friendly, for example I would prefer an AWD, 4WD or FWD car over an RWD car due to it being easier to drive these cars in the winter and not face unexpected oversteer or loss of traction. I would also prefer an automatic transmission over a manual due to the convenience of not having to frequently shift during in-city driving, as well as the increased mileage offered. Lastly, I would like creature comforts if available such as bluetooth connectivity and a reverse camera, along with charging options to recharge my phone while driving.

<u>Fundamental Objective 3: Performance Characteristics</u>

Since my father and I enjoy taking road trips whenever my parents come over to the US, it is key that my car be fun and comfortable to drive on long roadtrips. I would prefer it to be able to easily overtake large haulers and slower vehicles on highways at speeds higher than 65 mph, thus requiring a relatively larger displacement straight 4 or V6 engine. I would also prefer to not have an SUV due to its higher centre of gravity and unpleasant handling dynamics. I would prefer to not buy an EV since I am buying a used vehicle and battery replacements for EVs at battery end of life would be prohibitively expensive for me.

Fundamental Objective 4: Vehicle design

I would prefer a smaller car to ensure that it is easier to park and has better visibility, thus preferring a hatchback or a sedan over a large SUV or a pickup truck. I would also like to maximise cargo space while minimising vehicle size to be easily able to carry things. I also enjoy a comfortable ride and would like a larger leg room in a vehicle if possible, and would like the vehicle to seat a minimum of 4 people comfortably.

These fundamental objectives are summarised in the following fundamental objective hierarchy:

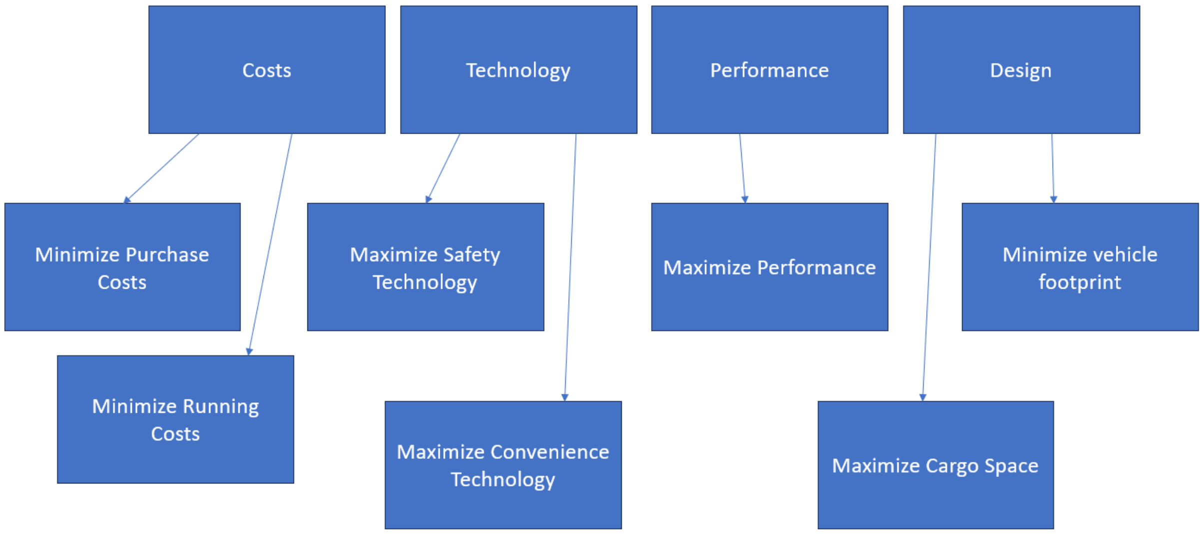

Figure 1: Fundamental Objective Hierarchy

Means Objectives

In order to meet these fundamental objectives in my decision problem, I identified a few means objectives to achieve the same.

Means Objective 1: Minimise Purchase Cost

To ensure I stay within my budget and reduce the initial impact the purchase of this car will have on my finances, I aim to minimise the purchase cost of the vehicle. This would also ensure I pay lesser tax and excise on the vehicle on a yearly basis.

Means Objective 2: Minimise Vehicle Upkeep Cost

To ensure the vehicle would not have a massive impact on my finances on a month by month basis, I would aim to minimise the upkeep costs on the car, such as repairs, insurance and fuel. I would also like to minimise the excise tax I would have to pay for the vehicle, which is an expense dependent on means objective 1.

Means Objective 3: Maximise Reliability

As I am a student who will need to commute to and from campus everyday for classes and part-time work, a car that frequently breaks down or needs to spend a significant amount of time in the shop is unsustainable for me. I also would like to minimise the repair cost of the vehicle and would like to learn how to do basic maintenance like oil changes and filter replacements on my own and not have to pay for the same. Thus a vehicle that is reliable through historical data, having few to no recalls and is easy to work with and learn to maintain would be ideal for me.

Means Objective 4: Maximise Safety and Technology features

I am a relatively new driver with little experience driving in snow and ice, I would like traction control, anti-lock braking and airbags as essential safety features for the vehicle. I would also like a blind-spot monitoring system and rear view camera for low visibility driving, although these are of medium level importance in my needs. Lastly, I would like creature comforts in the car, such as an infotainment system, bluetooth audio, charging ports and heated seats, but these are not essential and I may skip them if I need to.

Means Objective 5: Maximise Engine Displacement

I prefer the engine of my car to have a minimum displacement of two litres and have a straight 4 or V6 engine in order to have sufficient power while overtaking large vehicles on the freeway at speeds above 60 mph. I would prefer the highest engine displacement possible while maximising fuel efficiency. Since I am buying a used car which may need a battery replacement in case of it being a hybrid car I would prefer not to drive a vehicle with a hybrid powertrain, instead preferring a gasoline power vehicle. I would also not want exotic engine types, such as rotary engines due to them being harder to work with; and would prefer a naturally aspirated engine over a turbocharged engine due to them being more reliable in the long term.

Means Objective 6: Maximise Cargo Space

I will be making grocery runs, laundry runs and other such miscellaneous activities, for which I will need a sufficient amount of storage space in order to carry all the things that I might need. I would also prefer a large enough trunk space in order to store several bags in case I make trips to and from nearby cities to pick friends or family up from airports or if I end up making short road trips with friends. I would prefer to maximise cargo space while minimising the actual footprint of the car to make it easier to navigate in city traffic and find parking.

<u>Means Objective 7: Maximise Driver/Passenger Space</u>

Since I have relatively longer legs I would prefer my car to in turn have a larger legroom in order to comfortably seat me and three to four other passengers for shorter as well as longer trips. I would also prefer the car to have enough head room especially in the back, for taller passengers to be seated comfortably.

The means and fundamental objectives can be connected using a network map, which is as follows:

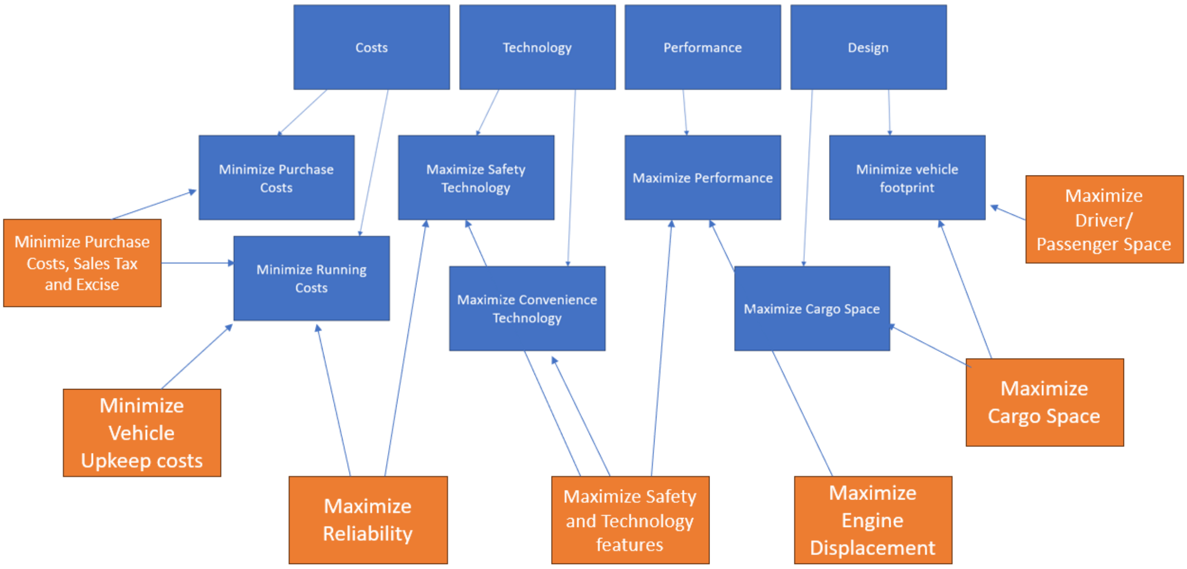

Figure 2: Means and Fundamental Objective Network Map

Essential Attributes and quantifying them

Now that I have established what are my fundamental and mean objectives for my particular decision situation, I will now identify quantitative measurements of how these objectives need to be met through specific attributes in a vehicle that I am looking to purchase. The attributes that I will be looking for in particular shall be explained below along with their scales of measurement (if quantitative) or categories (if qualitative) that I will use to rank options in.

<u>Attribute 1: Vehicle Initial Purchase Cost (including taxes and registration)</u>

The initial Purchase cost including taxes and registration is fairly easy to quantify due to dealerships and Online Car Sales sites showcasing what the sticker price of the vehicle is. I can then calculate taxes and registration based on the cost of the car(to calculate sales tax) and the model year of the vehicle and its MSRP to calculate the registration fee. The unit of measurement for this attribute will be US dollars, and the range I am considering for it is between $6,000 and $15,000. The ideal price I would consider for my car would be a middling value between these, in around $11,000. This attribute has uncertainty, due to it being possible to negotiate with the dealer to achieve a lower price.

Attribute 2: Vehicle maintenance costs

The vehicle's maintenance costs may be quantified using online data. For example, the insurance costs of the vehicle may be approximated by considering the vehicle's make and model and accounting for my age as a driver and obtaining insurance quotes from insurance providers available online. The cost of fuel in using the vehicle on a monthly basis may be approximated by obtaining the vehicle's mileage and the amount of miles that I will be driving in a month. Repair costs may be obtained through price lists of nearby auto shops or garages in Lafayette and West Lafayette. The unit of measurement for this attribute will be US dollars, and the range I'm considering for it is between $200 and $600. The ideal maintenance cost I would consider for my vehicle would be in the lower end of this range. This attribute contains uncertainty due to the possibility of there arising an unexpected and expensive repair that I may not have noticed while purchasing the car. It is also possible for me to drive around more or less each month depending on my needs, and for me to be switching insurance providers to achieve a more competitive rate.

Attribute 3: Vehicle fuel economy

The vehicle's fuel economy will be fairly straightforward to quantify using EPA fuel economy data for both in-city and highway driving. The unit of measurement for this attribute will be miles per gallon, and the range I am considering for this would be between 15 and 40 miles per gallon. The ideal vehicle fuel economy that I would consider would be at the highest end of this range at 40 miles per gallon, as I am a student and would prefer lower fueling cost if possible.For this attribute I shall mainly be considering the combined EPA fuel economy value since I shall be doing both City and highway driving with this vehicle. Since vehicle fuel economy reduces slightly as vehicles age, for the purposes of this project I shall be considering the vehicle's fuel economy as uncertain and slightly lower than the given EPA values.

Attribute 4: Engine Displacement

The vehicle's engine displacement will be fairly straightforward to quantify using available online data on car information websites. The unit of measurement for this attribute will be litres, and to primary categories of engines will be accounted for, those being I4 and V6 engines. due to me preferring conventional engines instead of exotic ones I shall be immediately rejecting other engine types. The range I am considering for the engine displacement would be between 2 to 4 litres, with the ideal engine displacement for my use being around 4 l since that would allow for a very high level of performance on highways (and be very fun to drive!). Since the size of the engine is constant for the particular vehicle's make and model, it is a deterministic attribute due to there being no uncertainty in it.

Attribute 5: Safety & Technology suite

For the purposes of this project I would consider traction control, anti-lock brakes, and airbags as essential safety features. These features are found in most modern cars, so eliminating any options that lack these features should not largely affect my choices available. For other non-essential features such as blind spot monitoring, a rear view camera, infotainment systems, Bluetooth audio, charging ports and heated seats, I would assign a ranking value of what is most preferred based on my needs, allowing me to quantify each car's safety and technological suite as a whole instead of accounting for each individual feature. This would also allow me to make the decisions on getting higher trim versus lower trim cars of the same make and model, which may differ in the model year of the same generation but may have drastically different features. Once the safety and technological suite of each vehicle choice has been quantified, I do not expect there to be any uncertainty within it due to the options available for each particular vehicle and its trim level being known. Thus, I shall consider this attribute deterministic.The ideal choice for this particular attribute would have all the described features.

Attribute 6: Cargo Space

Similar to the vehicle's engine displacement data, its cargo space should be fairly easy to quantify by simply pulling data from online sources. Particular vehicle types should however provide a variance in the amount of actual cargo space that is available. For example, a small crossover SUV may have lesser cargo space than a full size sedan while simply looking at numbers available online. Due to being able to fold the back seats however, it may be possible to carry large or unwieldy items better in the back of the small SUV than in the boot of the sedan. The unit of measurement for this attribute will be cubic feet, and the range I am considering will be between 10 and 30 cubic feet. The greater this attribute, the better since I will be able to store more things in the vehicle, especially while transporting luggage along with passengers. Since cargo size is constant for a particular vehicle's make, model and trim, it is a deterministic attribute due to there being no uncertainty in it and values can be obtained directly online from the manufacturer or reliable sources.

Attribute Selection

From these six identified attributes, I narrowed down four particular attributes as being the most important in my decision situation of purchasing a car, these being: Vehicle initial purchase cost ($\mathbf{X_1}$), Vehicle maintenance cost ($\mathbf{X_2}$), Vehicle fuel economy ($\mathbf{X_3}$), and Engine displacement ($\mathbf{X_4}$). I shall be using these attributes to develop single-attribute utility functions for each as well as a combined multi-attribute utility function throughout section 3 of this report. A summary table for the same is attached below:

Fig 3: Table summarising attributes to be developed

Attribute	Unit of Measurement	Range	Certainty Type	Objective
Vehicle initial purchase cost ($\mathbf{X_1}$)	US Dollars ($)	6,000 to 15,000	Uncertain	Minimise
Vehicle maintenance cost ($\mathbf{X_2}$)	US Dollars ($)	200 to 600	Uncertain	Minimise
Vehicle fuel economy ($\mathbf{X_3}$)	EPA MPG (Combined)	15 to 40	Uncertain	Maximise
Engine displacement ($\mathbf{X_4}$)	Litres	2 to 4	Certain	Maximise

Alternatives to compare

Using Carfax.com to cross reference vehicles in the greater Lafayette and greater Chicago area (since those would be the most convenient places for me to travel to buy a car), I identified four alternatives to compare in this particular decision situation at the time of July 2023. They are listed below:

1: 2018 Subaru Legacy 2.5i Premium(150,000 miles)
2: 2016 Toyota Avalon Touring 3.5 V6 (180,000 miles)
3: 2013 Honda Accord EX-L 2.4 I4 (120,000 miles)
4: 2014 Toyota Camry XLE 3.5 V6 (170,000 miles)

The selected attributes above for all these alternatives are discussed within section 4, Outline of alternatives.

Utility Assessment

Single-attribute Utility Functions

Attribute 1: Vehicle initial purchase cost (**X_1**)

<u>Qualitative Characteristics</u>

The utility function for the initial purchase price of the car should be continuous so that I can estimate the utility of any purchase cost between $6,000 and $15,000. I would expect the utility function to reflect a risk averse attitude, as I would be unwilling to pay a large amount in dealership markups for a used car and would rather expect the cost of the car to be similar to what I calculated using data available on carfax.com. I also expect risk aversion to increase as the price increases as an increase in cost over the expected purchase cost of the car on the higher end of the scale to affect my finances more and cause a larger hit to my budget than on the lower end of the budget.The utility function should also be monotonic since a lower cost should provide higher utility.

<u>Quantitative Restriction</u>

In order to establish quantitative restrictions for this utility function, I used the Five Points Bisection Method as described in the lecture notes. I gave the best purchase cost, which in this case would be $6,000 a utility value of one, and the worst purchase cost which would be $15,000 a utility of zero. I then conducted certainty equivalent gambles to bisect this utility function into a utility of 0.25, 0.5 and 0.75, identifying five points to which I can use Excel to plot a scatter plot and infer a polynomial curve and get a utility function.

This process is summarised in the following equations:

$$u_1(X_1^0) = u_1(\$15,000) = 0$$

$$u_1(X_1^1) = u_1(\$6,000) = 1$$

$$\langle X_1^1,\ 0.5,\ X_1^0 \rangle \sim X_1^{0.5}$$

$$\langle \$6,000,\ 0.5,\ \$15,000 \rangle \sim \$9,600$$

$$\langle X_1^{0.5}, 0.5, X_1^0 \rangle \sim X_1^{0.25}$$

$$\langle \$9,600, 0.5,\ \$15,000 \rangle \sim \$11,760$$

$$\langle X_1^1, 0.5, X_1^{0.5} \rangle \sim X_1^{0.75}$$

$$\langle \$6,000,\ 0.5,\ \$9,600 \rangle \sim \$7,440$$

<u>Utility Function</u>

After obtaining these 5 points, I used Excel to plot and extrapolate a utility function as a third order polynomial. This is attached below:

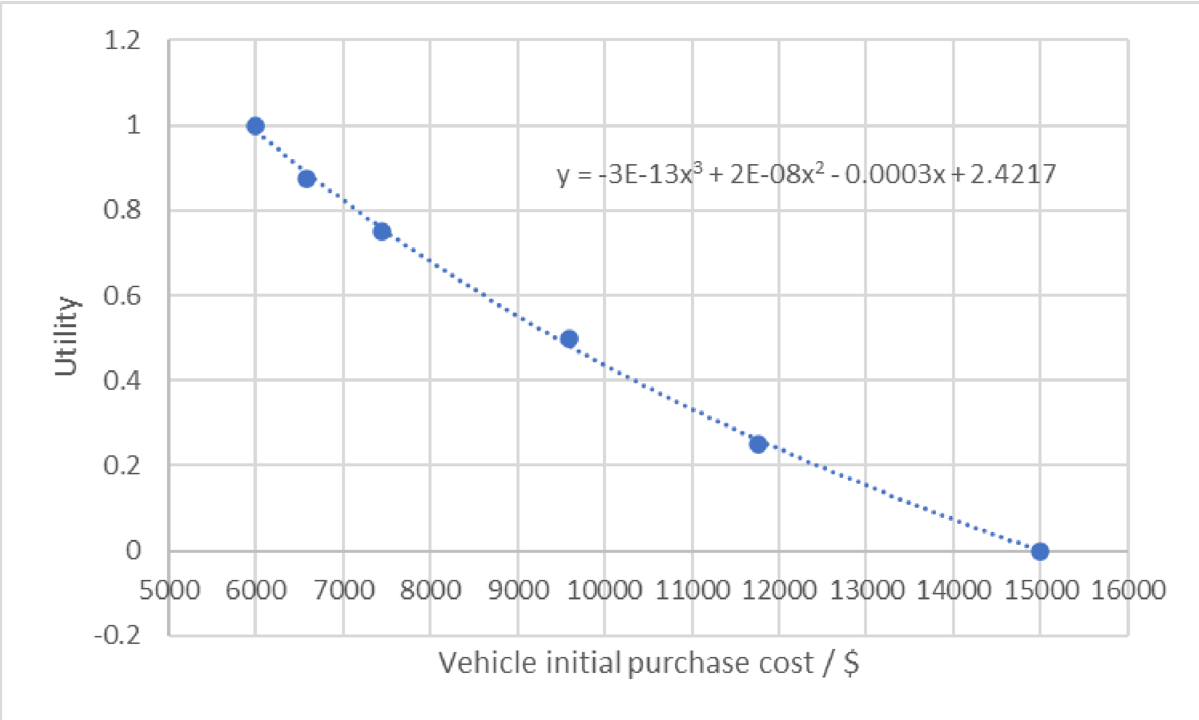

Figure 4: Vehicle initial purchase cost Utility function

Consistency Check

As a consistency check, I confirmed the utility function is a continuous function that is monotonically decreasing. It is also risk averse since it is concave with a greater initial decrease in utility which decreases as the purchase cost increases (gradient of the function is inversely proportional to purchase cost). I also conducted another certainty equivalent gamble that may be seen in the figure above to verify the new point is consistent with the obtained utility function. The lottery is as follows:

$$\langle X_1^{1}, 0.5, X_1^{0.75} \rangle \sim X_1^{0.875}$$

$$\langle \$6,000,\ 0.5,\ \$7,440 \rangle \sim \$6,576$$

The point obtained for the same lines up with the function as expected. This indicates that my design thought process has been captured in the utility function, allowing me to get an idea of how increasing the final purchase price of the vehicle changes my utility.

Attribute 2: Vehicle maintenance cost ($\mathbf{X_2}$)

Qualitative Characteristics

The utility function of the vehicle maintenance cost per month should be continuous so I can estimate the utility of any maintenance cost between \$200 and \$600 a month. I would expect the utility function to have a risk averse attitude, since a fixed if slightly higher monthly expense would be easier to budget for on my income as a student rather than the ups and downs of a lottery. Similar to the initial purchase cost of the vehicle, I would expect the risk aversion to increase as the monthly maintenance cost increases due to a smaller maintenance cost per month allowing me more of a buffer in my budget to account for a lottery while a higher maintenance cost per month will not allow me the same luxury. The utility function should be monotonic since a lower monthly cost should provide higher utility.

Quantitative Restriction

I used an identical Five Point Bisection method for this attribute to calculate the qualitative restrictions. Again, the maintenance cost of $200 was assigned a utility of 1 and a cost of $600 was assigned a utility of 0. The equations for the same are as follows:

$$u_2(X_2^0) = u_2(\$600) = 0$$

$$u_2(X_2^1) = u_2(\$200) = 1$$

$$\langle X_2^1,\ 0.5,\ X_2^0 \rangle \sim X_2^{0.5}$$

$$\langle \$200,\ 0.5,\ \$600 \rangle \sim \$360$$

$$\langle X_2^{0.5}, 0.5, X_2^0 \rangle \sim X_2^{0.25}$$

$$\langle \$360, 0.5,\ \$600 \rangle \sim \$456$$

$$\langle X_2^1, 0.5, X_2^{0.5} \rangle \sim X_2^{0.75}$$

$$\langle \$200,\ 0.5,\ \$360 \rangle \sim \$264$$

Utility Function

In an identical process to attribute 1, after obtaining these 5 points, I used Excel to plot and extrapolate a utility function as a third order polynomial. It is attached below:

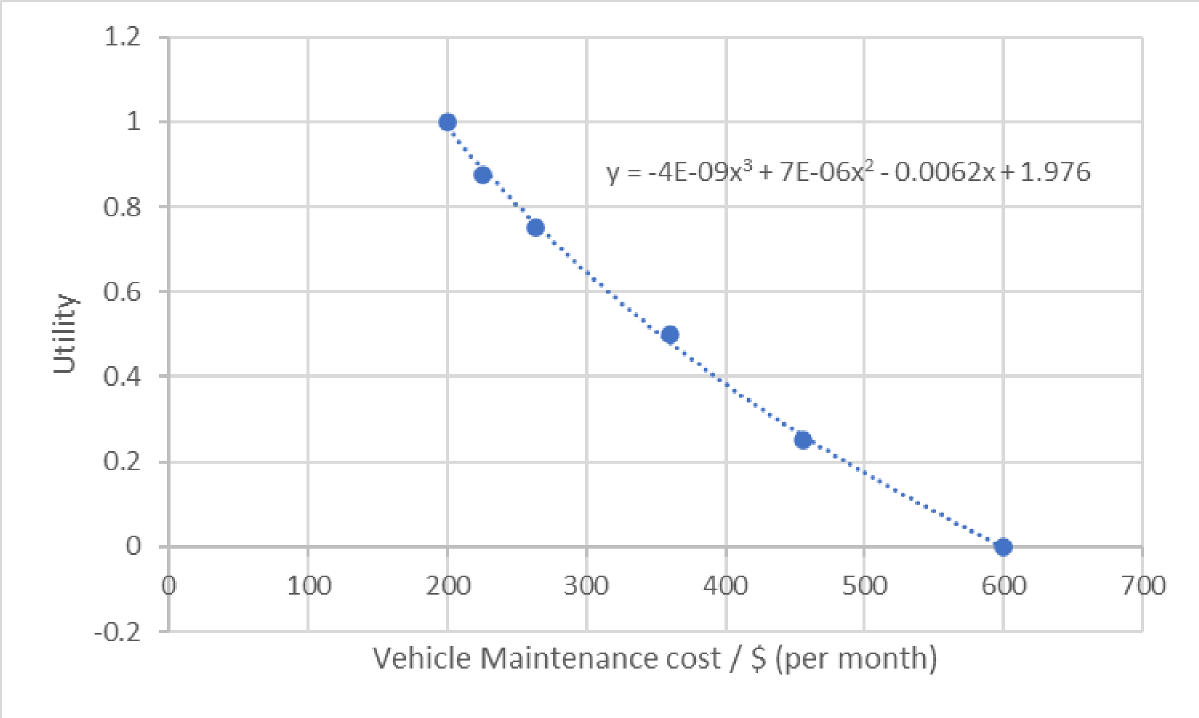

Figure 5: Vehicle maintenance cost Utility function

Consistency Check

As a consistency check, I confirmed the utility function is a continuous function that is monotonically decreasing. It is also risk averse since it is concave with a greater initial decrease in utility which decreases as the maintenance cost increases (gradient of the function is inversely proportional to maintenance cost). I also conducted another certainty equivalent gamble that may be seen in the figure above to verify the new point is consistent with the obtained utility function. The lottery is as follows:

$$\langle X_2^{1}, 0.5, X_2^{0.75} \rangle \sim X_2^{0.875}$$

$$\langle \$200,\ 0.5,\ \$264 \rangle \sim \$225.6$$

The point obtained for the same lines up with the function as expected. This indicates that my design thought process has been captured in the utility function, allowing me to get an idea of how increasing the maintenance cost of the vehicle changes my utility.

Attribute 3: Vehicle fuel economy (**X**$_3$)

Qualitative Characteristics

The ability function of the vehicle's fuel economy should be continuous so that I can estimate the utility of any possible value between 15 and 40 miles per gallon. I would not expect this utility function to reflect a risk averse attitude since the EPA values calculated are for ideal circumstances and actual driving, especially during winters or city driving where there will be a large amount of red lights, idling and starts from a full stop would mean my actual mileage will often be far lower than the EPA estimates. The actual money I will be paying for gas will also be highly affected by external factors, due to these external factors affecting gas prices. Thus, the amount of money I will be paying for filling up my gas tank will be highly variable due to these factors and I would prefer a less conservative utility function to reflect the same. The utility function should still remain monotonic however, similar to the previous attributes as a higher mileage should provide higher utility.

Quantitative Restriction

I used an identical Five Point Bisection method for this attribute to calculate the qualitative restrictions. However, the fuel economy of 40 mpg was assigned a utility of 1 while a fuel economy of 15 mpg was assigned a utility of 0, inverse of the previous attributes since fuel economy and utility are directly proportional. The equations for the same are as follows:

$$u_3(X_3^0) = u_3(15) = 0$$
$$u_3(X_3^1) = u_3(40) = 1$$
$$\langle X_3^1,\ 0.5,\ X_3^0 \rangle \sim X_3^{0.5}$$
$$\langle 15,\ 0.5,\ 40 \rangle \sim 25$$
$$\langle X_3^{0.5}, 0.5, X_3^0 \rangle \sim X_3^{0.25}$$
$$\langle 25, 0.5, 15 \rangle \sim 19$$
$$\langle X_3^1, 0.5, X_3^{0.5} \rangle \sim X_3^{0.75}$$
$$\langle 40,\ 0.5,\ 25 \rangle \sim 31$$

Utility Function

In an identical process to the past 2 attributes, after obtaining these 5 points, I used Excel to plot and extrapolate a utility function as a third order polynomial. It is attached below:

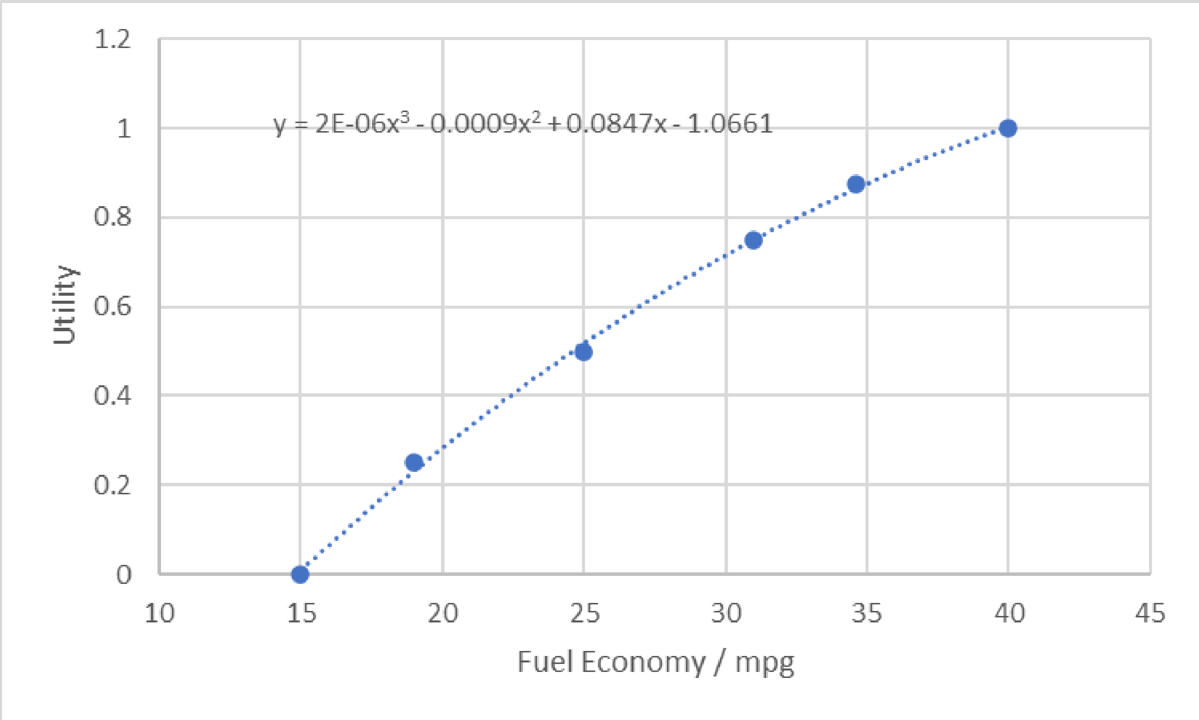

Figure 6: Vehicle fuel economy Utility function

Consistency Check

As a consistency check, I confirmed the utility function is a continuous function that is monotonically increasing. It is also not risk averse since it is concave outwards with a greater increase in utility (most increase in gradient) around the middle as expected. I also conducted another certainty equivalent gamble that may be seen in the figure above to verify the new point is consistent with the obtained utility function. The lottery is as follows:

$$\langle X_3^1, 0.5, X_3^{0.75} \rangle \sim X_3^{0.875}$$

$$\langle 40,\ 0.5,\ 31 \rangle \sim 34.6$$

The point obtained for the same lines up with the function as expected. This indicates that my design thought process has been captured in the utility function, allowing me to get an idea of how increasing the fuel economy of the vehicle changes my utility.

Attribute 4: Engine displacement (**X**$_4$)

Qualitative Characteristics

The engine displacement of the vehicle, since it is an attribute with no uncertainty, and has to be continuous so that I can estimate the utility of any possible value between two and four litres, needs to be modelled as a linear function. I would expect this function to be risk neutral, and monotonically increasing since a higher engine size should provide me with higher utility

Quantitative Restriction

I used an identical Five Point Bisection method for this attribute to calculate the qualitative restrictions. Again, the engine size of 4L was assigned a utility of 1 and an engine size of 2L was assigned a utility of 0. The equations for the same are as follows:

$$u_4(X_4^0) = u_4(2) = 0$$

$$u_4(X_4^1) = u_4(4) = 1$$

$$\langle X_4^1,\ 0.5,\ X_4^0 \rangle \sim X_4^{0.5}$$

$$\langle 4,\ 0.5,\ 2 \rangle \sim 3$$

$$\langle X_4^{0.5}, 0.5, X_4^0 \rangle \sim X_4^{0.25}$$

$$\langle 3, 0.5,\ 2 \rangle \sim 2.5$$

$$\langle X_4^1, 0.5, X_4^{0.5} \rangle \sim X_4^{0.75}$$

$$\langle 4,\ 0.5,\ 3 \rangle \sim 3.5$$

Utility Function

Now that the five points that I need to extrapolate this linear function are known, I repeated the process for the previous attributes using Excel to give me a linear equation. This is attached below:

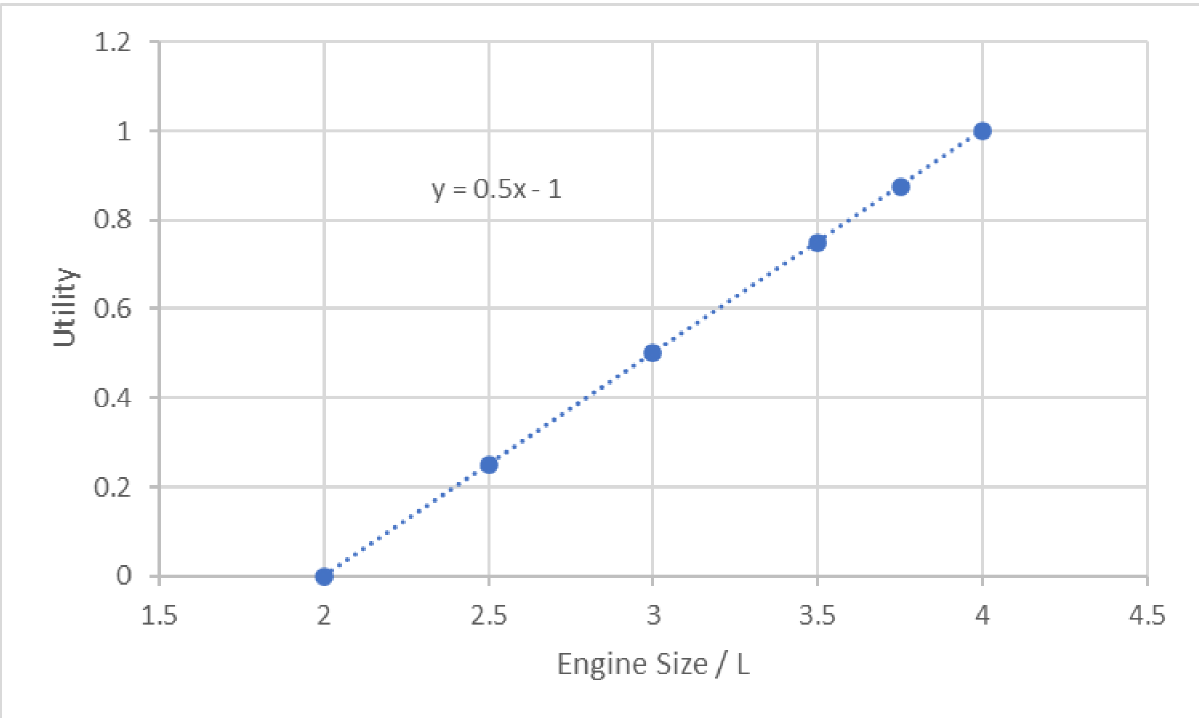

Figure 7: Vehicle engine size Utility function

Consistency Check

As a consistency check, I confirmed the utility function is a continuous linear function that is monotonically increasing. It is also risk neutral as expected. I also conducted another certainty equivalent gamble that may be seen in the figure above to verify the new point is consistent with the obtained utility function. The lottery is as follows:

$$\langle X_4^1, 0.5, X_4^{0.75} \rangle \sim X_4^{0.875}$$
$$\langle 4,\ 0.5,\ 3.5 \rangle \sim 3.75$$

The point obtained for the same lines up with the function as expected. This indicates that my design thought process has been captured in the utility function, allowing me to get an idea of how increasing the engine size of the vehicle changes my utility.

Multi-attribute Utility Functions

Additive Independence condition check

Now that the single-attribute utility functions have been established, I will now first need to check attribute pairs whether they are additively independent or not to decide the form of the multi-attribute utility function. If the attribute pairs fail the additive independence check, I will move on to the mutual utility independence check.

Initial purchase cost (X_1) vs maintenance cost (X_2):

In order to test whether the initial purchase cost and maintenance cost are additively independent I did the following Lottery comparison:

$$\langle (X_1^1,\ X_2^1),\ 0.5,\ (X_1^0,\ X_2^0)\ \rangle \sim \langle (X_1^1,\ X_2^0),\ 0.5,\ (X_1^0,\ X_2^1)\ \rangle$$
$$\langle (\$6,000,\ \$200),\ 0.5,\ (\$15,000,\ \$600) \rangle \sim \langle (\$6,000,\ \$600),\ 0.5,\ (\$15,000,\ \$200) \rangle$$

The first lottery has two possible outcomes, being the best one which is the cheapest possible car with the cheapest maintenance, and the worst outcome which is the most expensive car with the most expensive maintenance per month. The second lottery also has two possible outcomes, the first being a cheap car with high maintenance and the second being an expensive car with cheap maintenance. Since I am indifferent to both lotteries, it indicates to me that both these attributes are mutually independent to one another since the initial purchase cost does not affect my preference for maintenance cost and vice versa.

Initial purchase cost (X_1) vs fuel economy (X_3):

I followed the same testing methodology as the previous comparison to check whether the initial Purchase cost and fuel economy are additively independent. The lottery for the same is as follows:

$$\langle (X_1^1,\ X_3^1),\ 0.5,\ (X_1^0,\ X_3^0)\ \rangle \sim \langle (X_1^1,\ X_3^0),\ 0.5,\ (X_1^0,\ X_3^1)\ \rangle$$
$$\langle (\$6,000,\ 40),\ 0.5,\ (\$15,000,\ 15) \rangle \sim \langle (\$6,000,\ 15),\ 0.5,\ (\$15,000,\ 40) \rangle$$

The first lottery has two possible outcomes, being the best one which is the cheapest possible car with the best fuel economy, and the worst outcome which is the most expensive car with the worst fuel economy. The second lottery also has two possible outcomes, the first being a cheap car with bad fuel economy and the second being an expensive car with good fuel economy. Since I am indifferent to both lotteries, it indicates to me that both these attributes are mutually independent to one another since the initial purchase cost does not affect my preference for fuel economy and vice versa.

Initial purchase cost (X_1) vs engine displacement (X_4):

The same process is repeated here and the lottery is as follows:

$$\langle (X_1^1, X_4^1),\ 0.5,\ (X_1^0, X_4^0) \rangle \sim \langle (X_1^1, X_4^0),\ 0.5,\ (X_1^0, X_4^1) \rangle$$
$$\langle (\$6,000,\ 4),\ 0.5,\ (\$15,000,\ 2) \rangle \sim \langle (\$6,000,\ 2),\ 0.5,\ (\$15,000,\ 4) \rangle$$

The first lottery has two possible outcomes, being the best one which is the cheapest possible car with the highest engine displacement, and the worst outcome which is the most expensive car with the least engine displacement. The second lottery also has two possible outcomes, the first being a cheap car with a small engine and the second being an expensive car with a large engine. Since I am indifferent to both lotteries, it indicates to me that both these attributes are mutually independent to one another since the initial purchase cost does not affect my preference for engine size and vice versa.

Maintenance cost (X_2) vs fuel economy (X_3):

The same process is repeated here and the lottery is as follows:

$$\langle (X_2^1, X_3^1),\ 0.5,\ (X_2^0, X_3^0) \rangle \sim \langle (X_2^1, X_3^0),\ 0.5,\ (X_2^0, X_3^1) \rangle$$
$$\langle (\$200,\ 40),\ 0.5,\ (\$600,\ 15) \rangle \sim \langle (\$200,\ 15),\ 0.5,\ (\$600,\ 40) \rangle$$

The first lottery has two possible outcomes, being the best one which is the cheapest maintenance cost with the best fuel economy, and the worst outcome which is the most expensive maintenance cost with the worst fuel economy. The second lottery also has two possible outcomes, the first being a cheap to run car with bad fuel economy and the second being a car with expensive maintenance but good fuel economy. Since I am indifferent to both lotteries, it indicates to me that both these attributes are mutually independent to one another since the maintenance cost does not affect my preference for fuel economy and vice versa.

Maintenance cost (X_2) vs engine displacement (X_4):

The same process is repeated here and the lottery is as follows:

$$\langle (X_2^1, X_4^1), 0.5, (X_2^0, X_4^0) \rangle \sim \langle (X_2^1, X_4^0), 0.5, (X_2^0, X_4^1) \rangle$$
$$\langle (\$200, 4), 0.5, (\$600, 2) \rangle \sim \langle (\$200, 2), 0.5, (\$600, 4) \rangle$$

The first lottery has two possible outcomes, being the best one which is the cheapest maintenance cost with the largest engine size, and the worst outcome which is the most expensive maintenance cost with the smallest engine size. The second lottery also has two possible outcomes, the first being a cheap to run car with a small engine and the second being a car with expensive maintenance but a large engine. Since I am indifferent to both lotteries, it indicates to me that both these attributes are mutually independent to one another since the maintenance cost does not affect my preference for engine displacement and vice versa.

Fuel economy (X_3) vs engine displacement (X_4):

The same process is repeated here and the lottery is as follows:

$$\langle (X_3^1, X_4^1), 0.5, (X_3^0, X_4^0) \rangle \sim \langle (X_3^1, X_4^0), 0.5, (X_3^0, X_4^1) \rangle$$
$$\langle (40, 4), 0.5, (15, 2) \rangle \sim \langle (40, 2), 0.5, (15, 4) \rangle$$

The first lottery has two possible outcomes, the best one being the greatest fuel economy with the largest engine size, and the worst outcome which is the least fuel economy with the smallest engine size. The second lottery also has two possible outcomes, the first being a high fuel economy vehicle with a small engine and the second being a car with low fuel economy but a large engine. Since I am indifferent to both lotteries, it indicates to me that both these attributes are mutually independent to one another since the fuel economy does not affect my preference for engine displacement and vice versa.

Scaling constants assessment

Now that all the attributes have been compared and it has been established that all of them are additively independent, I am able to obtain the form of the multi-attribute utility function to be as follows:

$$U(X_1, X_2, X_3, X_4) = k_1 u_1(X_1) + k_2 u_2(X_2) + k_3 u_3(X_3) + k_4 u_4(X_4)$$

In this equation, u_1, u_2, u_3 and u_4 are the single-attribute utility functions that I obtained in the earlier sub-section and k_1, k_2, k_3 and k_4 are scaling constants I need to obtain in order to scale and weigh each utility function relative to the complete decision. The scaling constants thus sum up to 1, with their equation being as follows:

$$1 = k_1 + k_2 + k_3 + k_4$$

I shall now need to develop three lottery questions in order to develop a set of 4 total equations in order to solve for these 4 variables. I will do so by considering combinations of attributes that are equally preferred in order to create equations in terms of u and k, of which u may be substituted in for each term from the equations derived in the previous sub-section. The lottery questions I will ask are as follows:

<u>Lottery Question 1:</u>

The first lottery question is as follows:

$$\langle \$15,000,\ \$360,\ 25,\ 3\ \rangle \sim \langle X_1\ ,\ \$400,\ 20,\ 2.5\ \rangle$$

$$X_1 = ?$$

In this situation the first lottery offers average utility for maintenance cost, fuel economy and engine size while having the worst case purchase price scenario. The second lottery has an unknown purchase price but offers relatively low utility for maintenance cost, fuel economy and engine size. Considering a value for the unknown purchase price which allows these two lotteries to be indifferent to one another will allow me to substitute the single attribute utility function values into this lottery and have an equation for the K values. In this case a very high purchase price is something that will affect my decision greatly due to having a limited budget as a student, meaning a slightly lower than average purchase price of $8,000 will counteract the slightly lower utility of all the other attributes. Substituting the single attribute utility values for each attribute with a purchase price of $8,000 yields the following equation:

$$U\langle \$15,000,\ \$360,\ 25,\ 3\ \rangle\ =\ U\langle \$8,000,\ \$400,\ 20,\ 2.5\ \rangle$$

$$k_1u_1(\$15,000)\ +\ k_2u_2(\$360)\ +\ k_3u_3(25)\ +\ k_4u_4(3) =$$

$$k_1u_1(\$8,000)\ +\ k_2u_2(\$400)\ +\ k_3u_3(20)\ +\ k_4u_4(2.5)$$

Substituting the values for each utility function and simplifying the equation yields the following equation:

$$0.481 * k_2 +\ 0.518 * k_3 +\ 0.5 *\ k_4 =$$

$$0.68 * k_1 +\ 0.383 * k_2 +\ 0.282 * k_3 +\ 0.25 *\ k_4$$

Thus,

$$0.68 * k_1 -\ 0.098 * k_2 -\ 0.236 * k_3 - 0.25 *\ k_4 = 0$$

Lottery Question 2:

The second lottery question is as follows:

$$\langle \$9,600,\ \$600,\ 25,\ 3\ \rangle \sim \langle\ \$11,000,\ X_2,\ 20,\ 2.5\ \rangle$$

$$X_2 = ?$$

In this situation the first lottery offers average utility for purchase cost, fuel economy and engine size while having the worst case maintenance cost scenario. The second lottery has an unknown maintenance cost but offers relatively low utility for purchase price, fuel economy and engine size. Considering a value for the unknown maintenance cost which allows these two lotteries to be indifferent to one another will allow me to substitute the single attribute utility function values into this lottery and have an equation for the K values. In this case a very high maintenance is something that will affect my decision greatly due to having a limited monthly budget as a student, meaning a slightly lower than average monthly maintenance of $300 will counteract the slightly lower utility of all the other attributes. Substituting the single attribute utility values for each attribute yields the following equation:

$$U\langle \$9,600,\ \$600,\ 25,\ 3\ \rangle\ =\ U\langle \$11,000,\ \$300,\ 20,\ 2.5\ \rangle$$

$$k_1u_1(\$9,600)\ +\ k_2u_2(\$600)\ +\ k_3u_3(25)\ +\ k_4u_4(3) =$$

$$k_1u_1(\$11,000)\ +\ k_2u_2(\$300)\ +\ k_3u_3(20)\ +\ k_4u_4(2.5)$$

Substituting the values for each utility function and simplifying the equation yields the following equation:

$$0.481 * k_1 +\ 0.518 * k_3 +\ 0.5 *\ k_4 =$$

$$0.333 * k_1 +\ 0.647 * k_2 +\ 0.282 * k_3 +\ 0.25 *\ k_4$$

Thus,

$$-\ 0.148 * k_1 +\ 0.129 * k_2 +\ 0.282 * k_3 -\ 0.25 *\ k_4\ =\ 0$$

Lottery Question 3:

The third lottery question is as follows:

$$\langle \$9,600,\ \$360,\ 15,\ 3\ \rangle \sim \langle\ \$11,000\ ,\ \$400,\ X_3,\ 2.5\ \rangle$$

$$X_3 = ?$$

In this situation the first lottery offers average utility for purchase cost, maintenance cost and engine size while having the worst case fuel economy scenario. The second lottery has an unknown fuel economy but offers relatively low utility for purchase price, maintenance cost and engine size. Considering a value for the unknown fuel economy which allows these two lotteries to be indifferent to one another will allow me to substitute the single attribute utility function values into this lottery and have an equation for the K values. In this case a very high fuel economy is something that will not affect my decision greatly as I can simply choose to be more efficient with my driving, condensing multiple trips into one larger trip or by simply driving less, meaning a only a very slightly higher than average fuel economy of 29 mpg will counteract the slightly lower utility of all the other attributes. Substituting the single attribute utility values for each attribute yields the following equation:

$$U\langle \$9,600,\ \$360,\ 15,\ 3\ \rangle\ =\ U\langle \$11,000,\ \$400,\ 29,\ 2.5\ \rangle$$

$$k_1u_1(\$9,600)\ +\ k_2u_2(\$360)\ +\ k_3u_3(15)\ +\ k_4u_4(3) =$$

$$k_1u_1(\$11,000)\ +\ k_2u_2(\$400)\ +\ k_3u_3(29)\ +\ k_4u_4(2.5)$$

Substituting the values for each utility function and simplifying the equation yields the following equation:

$$0.481 * k_1 +\ 0.481 * k_2 + 0.5 *\ k_4 =$$

$$0.333\ * k_1 +\ 0.383 * k_2 +\ 0.679 * k_3 + 0.25 *\ k_4$$

Thus,

$$-\ 0.148 * k_1 -\ 0.098 * k_2 +\ 0.679 * k_3 - 0.25 *\ k_4 = 0$$

Thus a system of 4 equations is found, which is as follows:

$$1 =\ k_1 +\ k_2 +\ k_3 +\ k_4$$

$$0.68 * k_1 -\ 0.098 * k_2 -\ 0.236 * k_3 - 0.25 *\ k_4 = 0$$

$$-\ 0.148 * k_1 +\ 0.129 * k_2 +\ 0.282 * k_3 - 0.25 *\ k_4 = 0$$

$$-\ 0.148 * k_1 -\ 0.098 * k_2 +\ 0.679 * k_3 - 0.25 *\ k_4 = 0$$

Solving for the k values gives the following values:

$$k_1 = 0.211,\ k_2 = 0.334,\ k_3 = 0.191,\ k_4 = 0.263$$

Substituting these values into the template multi-attribute utility function (which is in the form of all variables being additive), yields the actual multi-attribute utility function I can then use to evaluate the alternatives I have selected, and is as follows:

$$U(X_1, X_2, X_3, X_4)\ =\ 0.211u_1(X_1)\ +\ 0.334u_2(X_2)\ +\ 0.191u_3(X_3)\ +\ 0.263u_4(X_4)$$

Consistency check

In considering the consistency of the k values I have obtained, I consider it fairly accurate in what I desire for my ideal first vehicle where a low monthly maintenance cost and relatively bigger engine are the attributes that I feel most strongly about, followed by the initial purchase cost and fuel economy. Thus the multi attribute utility function has been weighted according to my needs and capturing what I consider important. All the scaling constants are also positive, following my preferences and in turn the single-attribute utility functions.

Outline of Alternatives

Identification of Alternatives

Vehicle initial purchase cost

Using carfax.com and messaging individual dealerships selling the cars I had shortlisted, I was able to obtain a base price for each vehicle not accounting for any other fees or taxes. On top of that, since the cars I had shortlisted are all to be registered in Indiana, I calculated Indiana sales tax on top of the purchase price of the vehicle. I do expect the final cost to vary slightly from this calculated final value however, since I have not accounted for dealership discounts or negotiations that may reduce the final list price, nor any markups or additional fees I may have to pay to the dealership as part of the purchasing process. Note that I do not have sources for this data anymore, since most of these vehicles have since been sold. The data that is attached below is from my own calculations back in the month of July that I have reused, and may or may not reflect current market prices for these makes and models. This data is summarised in the table below:

Fig 8: Table summarising alternative purchase costs

Alternative	2018 Subaru Legacy	2016 Toyota Avalon	2013 Honda Accord	2014 Toyota Camry
Sticker price/$	13,000	12,500	14,000	12,000
Sales tax/$	910	875	980	840
Total cost/$	13910	13375	14980	12840

I assumed this uncertainty of cost to be normally distributed, and would expect it to be no more than $500 more or less than the calculated post-tax value of the vehicle. This was simulated for all alternatives in an excel plot using the NORM.DIST() function, which is attached below:

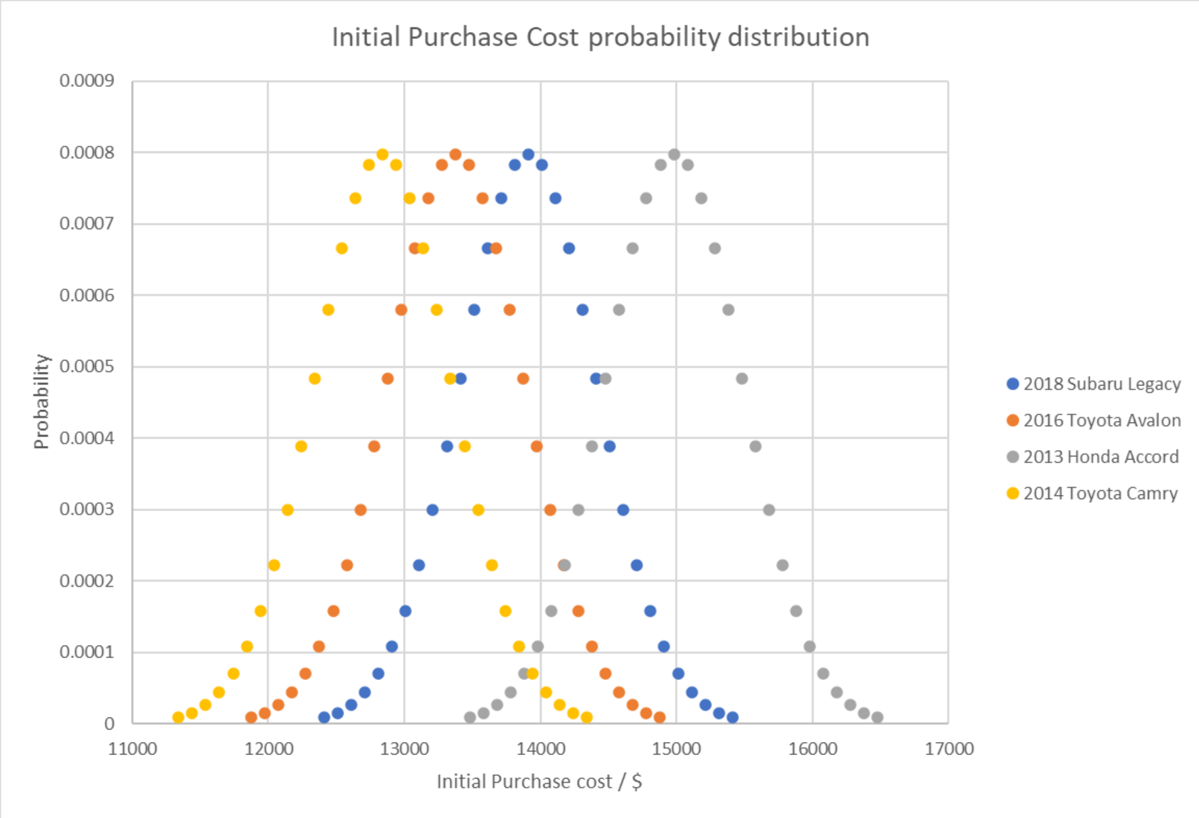

Figure 9: Vehicle initial purchase cost probability distribution

Vehicle maintenance cost

To assess how much I would be paying as part of running my vehicle, I divided my costs into four major categories. The first category would be fuel costs. This is variable since I may drive more or less every month depending on what my needs are and if I make any special trips out of the norm of my daily schedule that add to my monthly mileage, or if someone else pays for fuel. This can be simulated by assuming a mean of 1000 miles driven every month, with an assumed standard deviation of 150 miles.

The second category would be insurance, which would be fairly constant on a month to month basis for the first 6 months, after which it could go up or down. For the sake of simplicity, I am going to assume this estimate for the first 6 months is my insurance cost (since I am expecting this insurance cost to be a conservative estimate and will fall after my first 6 months of vehicle ownership and once I reach the age of 25). I obtained multiple estimates from several insurance providers and decided to go with the estimate that would give me the lowest per month cost. Through this, the lowest insurance cost I obtained was through Progressive for my particular age group and zip code for all four alternatives. Note that these values are estimates, thus I cannot directly attach a source for these values.

The third category would be repair costs. I do not expect any immediate maintenance charges that would be unexpected and expensive, since I will get a full inspection of the vehicle I am about to purchase from a third-party mechanic before

purchasing it. However, I will have to account for repair costs over the course of a year. This can be obtained through estimates available online and cross referenced with the vehicles reliability rating, any recalls that may have occurred, as well as user experience on forums such as Reddit to ensure I am able to calculate a reasonable repair budget for each car. The repair budget may also differ if I end up doing my own repairs and maintenance, such as me learning how to do my own oil changes instead of going to a shop to get it done. This would reduce my labour costs and in turn reduce my projected repair budget. Again, these values are also estimates, but I was able to attach a few sources in the appendix.
The fourth and last category is vehicle excise tax that I will have to pay as part of renewing the vehicle's registration every year. This is an expense that will reduce every year as the vehicle gets older, as defined in the Indiana vehicle excise tax fee table. I have estimated the same assuming I will be registering the car in 2023, using the vehicles MSRP as defined by the manufacturer's website found online. I am not including one-time registration fees or any additional plate fees that I may encounter. The source of this is the Indiana BMV tax website, which I have attached in the appendix.
I have summarised all these costs in the table below:

Fig 10: Table summarising alternative monthly costs

Alternative	**2018 Subaru Legacy**	**2016 Toyota Avalon**	**2013 Honda Accord**	**2014 Toyota Camry**
Fuel costs for 1000 miles driven ($3.5 per gallon)/$	120.69	145.83	116.67	145.83
Insurance Costs/$	145	130	120	125
Repair costs/$	33.33	25.83	25	24.17
Excise tax/$	121	129	36	42
Total costs not including variable costs/$	420.02	430.66	297.67	337

Simulating a probability distribution for my monthly mileage, I obtained the following graph using the Excel NORM.DIST() function:

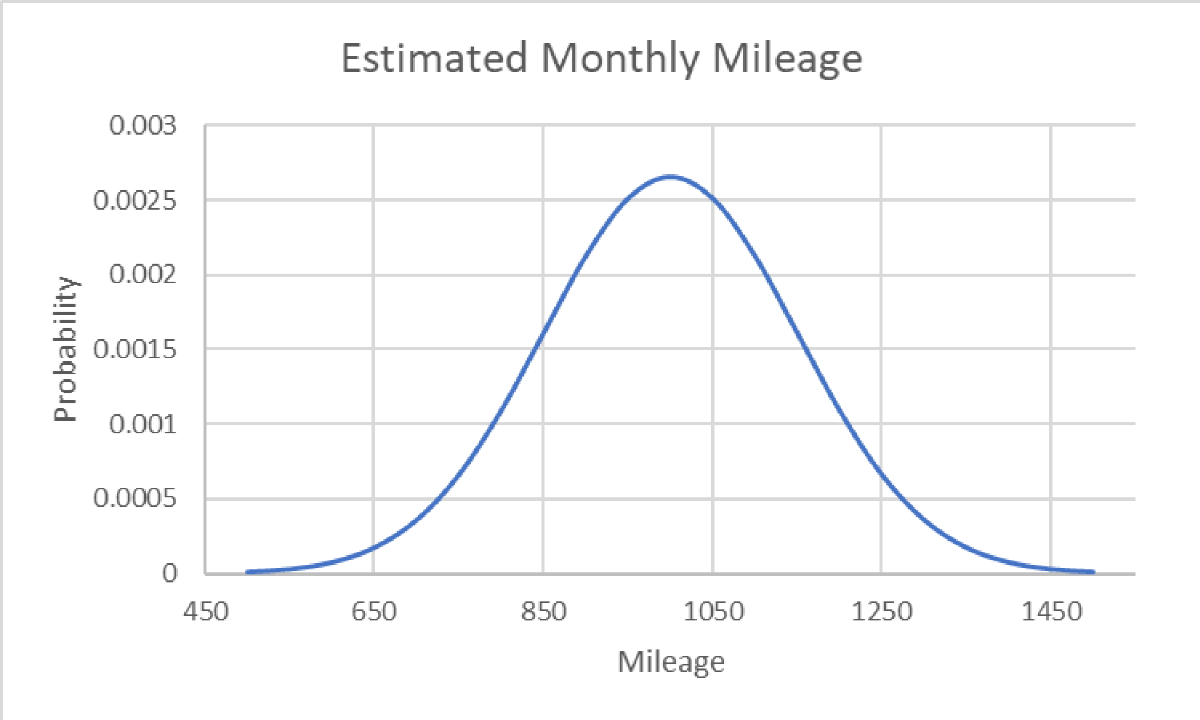

Figure 11: Vehicle mileage probability distribution

Using this I was able to formulate a graph for the monthly costs as mileage is varied for each alternative, which is as follows:

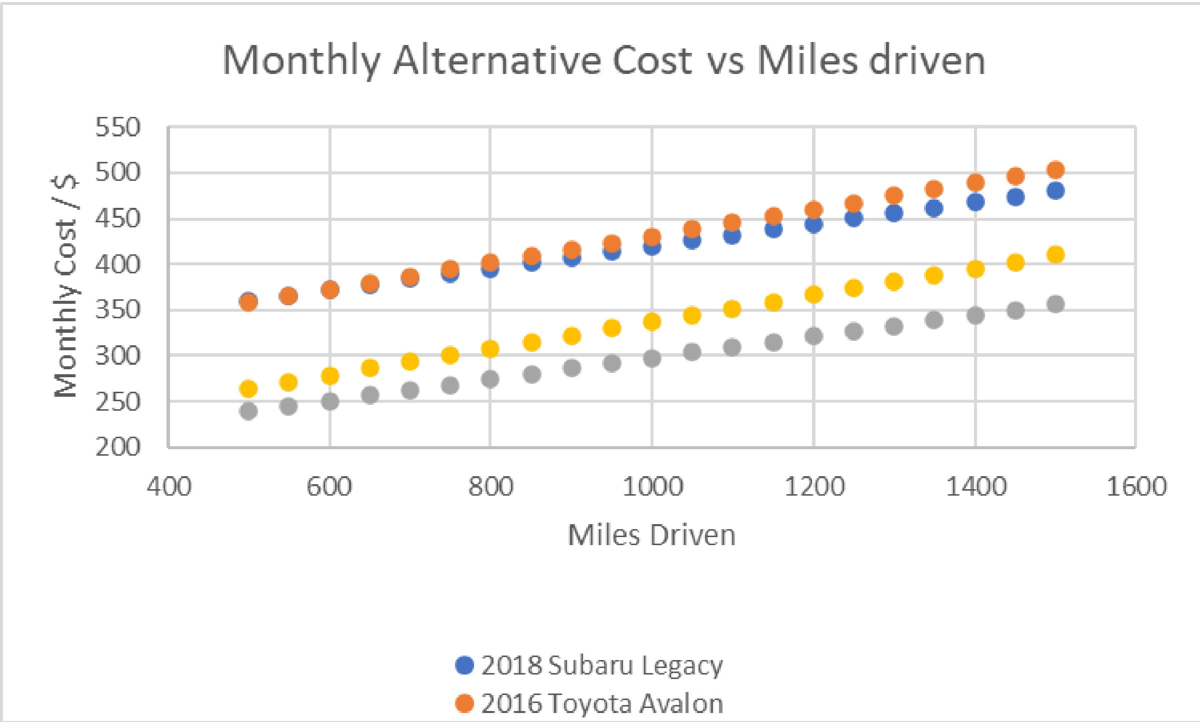

Figure 12: Monthly alternative cost vs mileage (yellow is Camry, grey is Accord)

Vehicle fuel economy

For calculating the vehicle's fuel economy I first took fuel economy data from the EPA. This would be the baseline value of the vehicle's fuel economy when it was new. Since vehicles lose fuel economy over time as their engines become less efficient, I decided to model the reduction in the vehicles fuel efficiency as a normal distribution, the mean of which is modelled as a linearly decreasing function of y = -0.035x, where y is the fuel efficiency as the vehicle ages and x being the age of the vehicle (this function was obtained by assuming a 35% efficiency reduction over 10 years). Since the vehicle ages are known, I calculated the adjusted mileage accounting for vehicle age. EPA data sources are attached in appendix 1. The findings for the same are summarised in the table below:

Fig 13: Table summarising fuel economy data

Alternative	**2018 Subaru Legacy**	**2016 Toyota Avalon**	**2013 Honda Accord**	**2014 Toyota Camry**
City MPG	25	21	26	21
Highway MPG	34	30	35	30
Combined MPG	29	24	30	24
Age-adjusted combined MPG	28.825	23.755	29.65	23.685

Engine displacement

Updating engine displacement data for all the available alternatives was fairly straightforward since the vehicle's engine displacement is an inherent property of the vehicle itself, depending on its make, model and year. Since I already know all this information, I was able to collect and tabulate the engine displacement for all the alternatives in the table below:

Fig 14: Table summarising engine displacement data

Alternative	**2018 Subaru Legacy**	**2016 Toyota Avalon**	**2013 Honda Accord**	**2014 Toyota Camry**
Engine Displacement	2.5 litres	3.5 litres	2.4 litres	3.5 litres
Engine Type	Straight 4	V6	Straight 4	V6

Sources have been attached in Appendix 1. One thing to note for this attribute is that the Camry and the Avalon both have an identical engine with the 3.5 L 2GR-FE V6, thus their expected utility value for this attribute is equal since both vehicles weigh a similar amount and are both mated to a traditional 6-speed automatic transmission meaning both vehicles will be approximately equal in ease of highway driving. The Subaru Legacy and the Honda Accord however have different engines, having a 2.5 and 2.4 L straight four respectively. Both of these are made to a CVT transmission, so these two vehicles will have a different expected utility than the two alternatives from Toyota.

Alternative Expected Utility

The expected utility of each alternative may now be calculated using the general multi-attribute utility function obtained previously, along with the uncertainties for the first two attributes that were calculated in the previous section. This allows us to obtain a final expected utility equation which is as follows:

$$U(X_1, X_2, X_3, X_4) = 0.211 * \int_{-\infty}^{\infty} u_1(X_1)P_1(X_1)dX_1 + 0.334 * \int_{-\infty}^{\infty} u_2(X_2)P_2(X_2)dX_2 + 0.191u_3(X_3) + 0.263u_4(X_4)$$

Attributes 3 and 4 are either deterministic or have their uncertainty modelled as a linear function with known variables, allowing for their utility to be calculated as direct delta functions as previously. This is why the above equation may have a few missing terms, which have been accounted for through the reason above.
I used Excel to calculate the utility of each section, since the functions of the same had already been input in the workbook I was using, as well as the probability functions. Using this, the expected utility calculations have been summarised in the following table:

Fig 15: Expected Utility for each alternative

Alternative	**Initial Purchase Cost Expected Utility**	**Vehicle Maintenance Cost Expected Utility**	**Vehicle fuel economy Expected Utility**	**Engine displacement Expected Utility**	**Total Expected Utility**
2018 Subaru Legacy	0.017035	0.112778	0.128362	0.06575	**0.323925**
2016 Toyota Avalon	0.025937	0.104975	0.088348	0.19725	**0.41651**
2013 Honda Accord	0.0000346	0.218285	0.134187	0.0526	**0.405037**
2014 Toyota Camry	0.035164	0.18084	0.087744	0.19725	**0.500998**

After calculating the total expected utility for each alternative, the 2014 Toyota Camry was chosen as the most preferred alternative, followed by the 2016 Toyota Avalon, the 2013 Honda Accord and the 2018 Subaru Legacy as the least preferred alternative.

Discussion and Learnings

Outcome, Validity

By calculating the total expected utility for each alternative, I was able to define the 2014 Toyota Camry as the most preferred alternative, followed by the 2016 Toyota Avalon, the 2013 Honda Accord and the 2018 Subaru Legacy, based on the attributes of initial purchase cost, vehicle maintenance cost, vehicle fuel economy and engine displacement. The Camry performs reasonably well in all attributes, as well as having the highest utility for the initial Purchase cost and engine displacement for which it is tied with the Avalon. The Avalon follows, being very close in utility to the Accord, though it beats out the Accord with its initial purchase price by a large margin. The Subaru comes in last due to not performing well in its initial cost as well as engine displacement while having a relatively high maintenance cost.

I do believe the results of this project should be valid, considering the order of preference of the alternatives lined up with what I intended on purchasing. However, this scenario differs from what actually happened due to an external factor that was not accounted for in this project, which is the likelihood of someone else buying the vehicle first. This relates to another shortcoming of this project, that not all variables were accounted for due to time limitations compared to the increasing complexity of adding more variables. Along with all the attributes listed above but not tested, there were other variables such as mileage, which while being dependent on make and model, were still not accounted for. For example, I would be comfortable purchasing a high mileage vehicle from Toyota or Honda, but would be hesitant to do so for a Jeep due to the former's reputation of being reliable, and the latter's reputation of being very unreliable.

Lastly, certain costs do not take into account my own efforts to reduce them, for example with vehicle maintenance. I have the choice of either going to an auto shop for oil changes or to learn how to do so on my own. Initially this would require a time and equipment investment, but would be cheaper in the long term. On the other hand, oil changes at Walmart or similar places would be cheaper short-term but more expensive in the long term. Thus there is a chance of the probability distribution of the maintenance cost to vary beyond my calculations.

Assumptions and Learnings

A major assumption in my project has been that the 4 selected attributes are independent of one another, but I have not accounted for other attributes that may affect them. For example, reliability is dependent on a vehicle's make, model, how old it is and how many miles it has run. Mileage is also dependent on the weather and my driving style. Not accounting for these additional variables does make my analysis simpler but indicates the utility function is not a complete picture yet.

Another major assumption is modelling uncertainty as either normal distributions or linear functions. Although in real life a large variety of events fall under normal distributions, this is an uncertainty that can be modified by small changes in my daily schedule such as me leaving earlier or later halfway through semesters due to a change in class schedules. This cannot be easily accounted for in the uncertainty modelling, and could significantly affect my actual costs and fuel efficiency.

Through this project I was able to understand that even relatively simple decisions can be very complex when considered in a mathematical sense, due to there being several variables to account for. Despite the expected utility method being ideal for a decision situation like this (due to meeting all of Hazelrigg's criteria), I found it difficult to scale it up to a decision that may require several uncertain attributes to consider. In comparing my own decision making process to this method, even though a near identical outcome was achieved, I feel like accounting for all variables can be done easier through a human element rather than a mathematical element in a time frame that allows for quick decisions to be made.

Appendix 1: Sources

1: Gas mileage reduction over a vehicle's ageing process:
https://zappysautowashes.com/blogs/news/surprise-surprise-your-gas-mileage-is-getting-worse

2: Gas mileage data for alternatives:
https://www.fueleconomy.gov/feg/bymodel/2018_Subaru_Legacy.shtml
https://www.fueleconomy.gov/feg/bymodel/2016_Toyota_Avalon.shtml
https://www.fueleconomy.gov/feg/bymodel/2014_Toyota_Camry.shtml
https://www.fueleconomy.gov/feg/bymodel/2013_Honda_Accord.shtml

3:Decision making process
George A. Hazelrigg (2003) Validation of engineering design alternative selection methods, Engineering Optimization, 35:2, 103-120, DOI: 10.1080/0305215031000097059.

4: Cost to own estimates:
https://www.edmunds.com/subaru/legacy/2018/cost-to-own/
https://repairpal.com/cars/toyota/avalon/2016
https://repairpal.com/cars/toyota/camry/2014
https://repairpal.com/cars/honda/accord/2013

5: Vehicle specs
https://www.cars.com/research/subaru-legacy-2018/
https://www.cars.com/research/toyota-avalon-2016/
https://www.cars.com/research/toyota-camry-2014/
https://www.cars.com/research/honda-accord-2013/

6:Pricing data
https://www.carfax.com/

7: BMV Excise Tax data
https://www.in.gov/bmv/fees-taxes/vehicle-registration-fees-and-taxes/excise-tax-information/

Appendix 2: K value calculation (via Symbolab)

$a+b+c+d=1,\ 0.68a-0.098b-0.236c-0.25d=0,\ -0.148a+0.129b+0.282c-0.25d=0,\ -0.148a-0.098b+0.679c-0.25d=0$

Solution

$a=0.21128\ldots,\ c=0.19119\ldots,\ b=0.33438\ldots,\ d=0.26313\ldots$

Solve by: Solve by substitution

$$\begin{bmatrix} a+b+c+d=1 \\ 0.68a-0.098b-0.236c-0.25d=0 \\ -0.148a+0.129b+0.282c-0.25d=0 \\ -0.148a-0.098b+0.679c-0.25d=0 \end{bmatrix}$$

Isolate a for $a+b+c+d=1$: $a=1-b-c-d$

Substitute $a=1-b-c-d$

$$\begin{bmatrix} 0.68(1-b-c-d)-0.098b-0.236c-0.25d=0 \\ -0.148(1-b-c-d)+0.129b+0.282c-0.25d=0 \\ -0.148(1-b-c-d)-0.098b+0.679c-0.25d=0 \end{bmatrix}$$

Simplify

$$\begin{bmatrix} -0.778b-0.916c-0.93d+0.68=0 \\ 0.277b+0.43c-0.102d-0.148=0 \\ 0.05b+0.827c-0.102d-0.148=0 \end{bmatrix}$$

Isolate b for $-0.778b-0.916c-0.93d+0.68=0$: $b=-\frac{458c+465d-340}{389}$

Substitute $b=-\frac{458c+465d-340}{389}$

$$\begin{bmatrix} 0.277\left(-\frac{458c+465d-340}{389}\right)+0.43c-0.102d-0.148=0 \\ 0.05\left(-\frac{458c+465d-340}{389}\right)+0.827c-0.102d-0.148=0 \end{bmatrix}$$

Simplify

$$\begin{bmatrix} 0.10386\ldots c+0.09410\ldots-0.43311\ldots d=0 \\ 0.76813\ldots c-0.16176\ldots d-0.10429\ldots=0 \end{bmatrix}$$

Isolate c for $0.10386\ldots c+0.09410\ldots-0.43311\ldots d=0$: $c=\frac{0.43311\ldots d-0.09410\ldots}{0.10386\ldots}$

Substitute $c=\frac{0.43311\ldots d-0.09410\ldots}{0.10386\ldots}$

$$\left[0.76813\ldots\cdot\frac{0.43311\ldots d-0.09410\ldots}{0.10386\ldots}-0.16176\ldots d-0.10429\ldots=0\right]$$

Simplify

$$\left[3.04130\ldots d-0.80026\ldots=0\right]$$

Isolate d for $3.04130\ldots d-0.80026\ldots=0$: $d=0.26313\ldots$

For $c=\frac{0.43311\ldots d-0.09410\ldots}{0.10386\ldots}$

Substitute $d=0.26313\ldots$

$$c=\frac{0.43311\ldots\cdot 0.26313\ldots-0.09410\ldots}{0.10386\ldots}$$

$$\frac{0.43311\ldots\cdot 0.26313\ldots-0.09410\ldots}{0.10386\ldots}=0.19119\ldots$$

$c=0.19119\ldots$

For $b=-\frac{458c+465d-340}{389}$

Substitute $c=0.19119\ldots, d=0.26313\ldots$

$$b=-\frac{458\cdot 0.19119\ldots+465\cdot 0.26313\ldots-340}{389}$$

$$-\frac{458\cdot 0.19119\ldots+465\cdot 0.26313\ldots-340}{389}=0.33438\ldots$$

$b=0.33438\ldots$

For $a=1-b-c-d$

Substitute $c=0.19119\ldots, b=0.33438\ldots, d=0.26313\ldots$

$a=1-0.33438\ldots-0.19119\ldots-0.26313\ldots$

Simplify

$a=0.21128\ldots$

The solutions to the system of equations are:

$a=0.21128\ldots, c=0.19119\ldots, b=0.33438\ldots, d=0.26313\ldots$