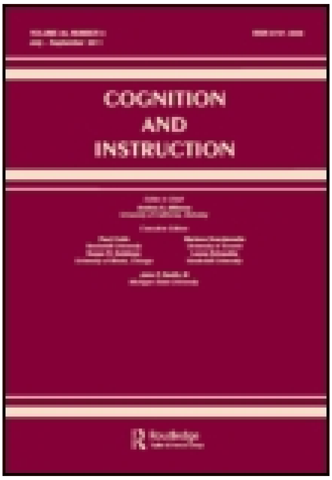

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Development of a Cognitive-Metacognitive Framework for Protocol Analysis of Mathematical Problem Solving in Small Groups

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A framework is presented that explicitly delineates the roles of metacognition and cognition within small-group heuristic problem solving in mathematics. This framework is used to describe the videotaped behaviors of 27 seventh-grade students of varying ability working in small groups to solve a mathematical problem. The results suggest the importance of metacognitive processes in mathematical problem solving in a small-group setting. A continuous interplay of cognitive and metacognitive behaviors appears to be necessary for successful problem solving and maximum student involvement. Within the groups, students returned several times to such problem-solving episodes as reading, understanding, exploring, analyzing, planning, implementing, and verifying. Stimulated-recall interviews held after completion of the task underscored an additional dimension of importance. Attitudes, particularly those of high-ability students, seemed to affect the interactions and the problem-solving behaviors of fellow group members. The framework shows promise of being a powerful tool for the future study of mathematical problem solving in a small-group setting.

Recently there has been a growing movement toward the use of small groups for mathematics instruction (Crabill, 1990; Johnson & Johnson, 1990; Lindquist, 1989; Noddings, 1989; Rosenbaum, Behounek, Brown, & Burcalow, 1989; Slavin, 1990). Research has shown that, under certain conditions, small-group approaches show positive effects on achievement in mathematics (Davidson, 1985; Davidson & Kroll, 1991). Support for small-group work now comes from the National Council of Teachers of Mathematics (1989) in its publication, *Curriculum*

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and Evaluation Standards for School Mathematics. The assumption is that through the use of small-group approaches the mathematical problem-solving abilities of students will be improved. Although there is optimism about the efficacy of small-group techniques, proponents acknowledge that little is known about the ways in which the activities used in these small-group approaches produce their positive effects (Bossert, 1988; Slavin, 1989–1990).

Over the last 2 decades, many researchers have studied problem solving in mathematics from a cognitive information-processing perspective. Recent summaries of studies investigating mathematical problem solving (Garofalo & Lester, 1985; Schoenfeld, 1987; Silver, 1987) suggest that a primary source of difficulty in problem solving may lie in students' inabilities to actively monitor and subsequently regulate the cognitive processes engaged in during problem solving. Small problem-solving groups provide natural settings for interpersonal monitoring and regulating of members' goal-directed behaviors. It may be that variables characteristic of these settings are responsible for the positive effects observed in small-group mathematics problem solving. In this exploratory study, we examine the cognitive processing that occurs as individuals engage in mathematical problem solving in small-group settings. Through this investigation, we hope to learn more about how levels of cognitive processes interact and contribute to the successful outcomes of problem solving within small groups.

To examine the interactions between two levels of cognitive processes (i.e., cognitive and metacognitive) observed in the problem-solving behaviors of students working in small groups on mathematics problems, we synthesized a framework for protocol analysis. The procedure has been used to analyze videotapes of students engaged in mathematical problem solving in small-group settings. The framework is derived from research on mathematical problem solving and on cognitive processes discussed in the following three sections.

MATHEMATICAL PROBLEM SOLVING

In mathematics, Polya's (1945) conception of mathematical problem solving as a four-phase heuristic process (understanding, planning, carrying out the plan, and looking back) has served as a standard for investigating problem-solving competence. More recently, Schoenfeld (1983) devised a model for analyzing problem-solving moves that was derived from Polya's. Schoenfeld's model incorporated, within Polya's structure, findings from research on problem solving by information-processing theorists. The model described mathematical problem solving in five episodes: reading, analysis, exploration, planning/implementation, and verification. Garofalo and Lester (1985) built on Polya's and Schoenfeld's structures by developing a framework for analyzing metacognitive aspects of performance on a wider range of mathematical tasks. The four broad component processes—orientation, organization, execution, and verification—are related to

Polya's four phases but are more broadly defined. The four components incorporate Schoenfeld's categories of reading and analysis taken together, planning, implementation, and verification, respectively. Exploration was not specified in the Garofalo and Lester framework. Although Garofalo and Lester indicated the distinctive metacognitive behaviors that may be associated with each category, more research is needed to analyze the specific cognitive processes inherent in mathematical problem solving.

Furthermore, in the application of his framework, Schoenfeld discovered that expert mathematicians returned several times to different heuristic episodes. For example, in one case, an expert engaged in the following sequence of heuristics: read, analyze, plan/implement, verify, analyze, explore, plan/implement, verify. In contrast, the sequence of heuristics for a novice problem solver was just read and explore. More research is needed to examine the sequence of heuristic episodes characteristic of novice problem solvers working in small groups.

COGNITIVE PROCESSES

Current studies of cognitive development focus on cognitive processes as well as on the mechanisms by which development in the processes occurs. Prominent in research on cognitive mechanisms have been strategy selection (e.g., in the solution of computational problems; see Siegler, 1988; Siegler & Shrager, 1984), processing efficiency (e.g., Kail, 1986; Sternberg, 1977), and social scaffolding (e.g., reciprocal teaching; see Palincsar, 1986; Palincsar & Brown, 1984).

Regarding cognitive processes, one lively area of research has focused on the knowledge, monitoring, evaluation, and overseeing that individuals use during any problem-solving endeavor. The term commonly used in the psychological literature for these cognitive processes is *metacognition* (e.g., Brown, 1978; Brown, Bransford, Ferrara, & Campione, 1983; Flavell, 1981; Jacobs & Paris, 1987). For example, Flavell (1981) defined *metacognition* as "knowledge or cognition that takes as its object or regulates any aspect of cognitive endeavor. Its name derives from this 'cognition about cognition' quality" (p. 37). The definition implies that metacognition includes reflection on cognitive activities as well as decisions to modify these activities at any time or place during a given cognitive enterprise. Flavell draws our attention to the dual nature of cognitive processes deployed during any given cognitive enterprise when he stated, "We develop cognitive actions or strategies for *making* cognitive progress and we also develop cognitive actions or strategies for *monitoring* cognitive progress. The two might be thought of as cognitive strategies and metacognitive strategies" (p. 53).

COGNITIVE PROCESSES IN MATHEMATICAL PROBLEM SOLVING

In his framework, Schoenfeld (1983) devised a scheme of parsing protocols into episodes and executive decision points. The executive decision points served as the mechanisms by which the problem-solving process was kept on track. Although his framework focused on the points at which metacognitive decisions may be considered and on their importance in the problem-solving process, in the analyses of protocols, Schoenfeld did not specify the cognitive levels of the episodes themselves.

To examine the problem-solving behaviors and cognitive processes of individuals as they work in small groups, we developed a framework that synthesizes the research on mathematical problem solving with that of cognitive theorists.

DESCRIPTION AND DEVELOPMENT OF FRAMEWORK

The framework for the protocol analysis of problem solving in mathematics developed for this study was designed to differentiate explicitly between cognitive and metacognitive problem-solving behaviors observed within the different episodes of problem solving. Our framework attempts to show a synthesis of the problem-solving steps identified in mathematical research by Garofalo and Lester, Polya, and Schoenfeld, and of cognitive and metacognitive levels of problem-solving behaviors studied within cognitive psychology, in particular, by Flavell (1981).

Schoenfeld's (1985b) framework was used as a foundation in our development. His framework partitioned a problem-solving protocol into "macroscopic chunks of consistent behavior called episodes. An episode is a period of time during which an individual or a problem-solving group is engaged in one large task" (p. 292). The episodes were categorized as *read, analyze, explore, plan/implement*, and *verify*. Through the determination that decisions at the control level would be those that affected the allocation or utilization of problem-solving resources, Schoenfeld allowed for junctures between episodes where these decisions would be most likely to occur. Furthermore, he made specific indications when overt signs of management activity occurred. His framework focused mainly on decision-making behaviors, specifically on statements made about the problem-solving process, at the executive level. A limitation of his framework, Schoenfeld admitted, was that he did not identify statements made about the problem (the more "local" indications of metacognitive behavior). He claimed that, as a result, he was unable to address the important role that consistent monitoring and evaluation of solutions play in the problem-solving process (1985b, p. 293).

Schoenfeld's framework was taken as a starting point; changes were then made to serve the purposes of the present investigation: to delineate explicitly the type and level of cognitive processes individuals use as they work with others in a small-group setting and to understand the mechanisms by which these processes facilitate problem solving. Following Schoenfeld, episodes were used to categorize the behaviors of the individual students within the group. Through the context of the verbal interactions that occurred within the small groups, however, it became clear that several modifications to Schoenfeld's episodes were needed. First, the episode of plan/implement was separated into two distinct episodes. This seemed advisable because the two episodes did not always occur sequentially in the small-group setting. In fact, quite often, a student proposed a plan that was immediately rejected by the other group members. In such cases, no implementation occurred. Second, it became apparent that we had to expand the episodic categories for the coding of student behaviors in groups to include *understanding the problem* and *watching and listening*. The frequent comments students made regarding the conditions of the problem, recognized by Polya as so important in the problem-solving process, served as our reason for including understanding the problem as a distinct episode. Furthermore, the verbal interaction that took place within the small group implied that at certain times students were watching and listening to one another.

Each of the eight problem-solving episodes (read, understand, analyze, explore, plan, implement, verify, watch and listen) was categorized as cognitive or metacognitive. Conceptually, one can distinguish the dual nature of cognitive processing, but operationally the distinction is often blurred. For example, cognition is implicit in any metacognitive activity, and metacognition may be present during a cognitive act, although perhaps not apparent. For this reason, none of the episodes was categorized as purely cognitive or purely metacognitive. The distinction was based on the predominant process observed.

Our working distinction of cognition and metacognition was similar to Garofalo and Lester's (1985, p. 164) description, "Cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done." Metacognitive behaviors can be exhibited by statements made about the problem or statements made about the problem-solving process. Cognitive behaviors can be exhibited by verbal or nonverbal actions that indicate actual processing of information. This distinction between cognitive and metacognitive actions corresponds to those of Flavell (1981) as well. See Table 1 for an outline of the categorization of episodes. A rationale for these categorizations follows.

Episodes of analyzing and planning are, by their very natures, predominantly metacognitive behaviors. Schoenfeld (1985b) stated that:

> In analysis an attempt is made to fully understand a problem, to select an appropriate perspective and reformulate the problem in those terms, and to introduce for

TABLE 1
Framework Episodes Classified by Predominant Cognitive Level

Episode	*Predominant Cognitive Level*
Read	Cognitive
Understand	Metacognitive
Analyze	Metacognitive
Explore	Cognitive and metacognitive
Plan	Metacognitive
Implement	Cognitive and metacognitive
Verify	Cognitive and metacognitive
Watch and listen[a]	

[a]Level not assigned.

> consideration whatever principles or mechanisms might be appropriate. The problem may be simplified or reformulated. (p. 298)

Any statements revealing such thought processes would necessarily be statements made about the problem or about the problem-solving process. Similarly, episodes of planning would be evidenced by statements made about how to proceed in the problem-solving process. Episodes of understanding the problem were categorized as predominantly metacognitive, because this category was assigned only when students made comments that reflected attempts to clarify the meaning of the problem. That is, if a student was making a comment about the meaning of a problem, he or she was also making a comment about the problem. Although it is true that some of the things one does to understand a problem are cognitive, in a coding scheme that relies on the verbal comments of students, it is impossible to decipher the understanding that is being derived during the actual doing of the problem. Behaviors coded as reading were categorized as predominantly cognitive, because they exemplify instances of doing. Behaviors coded as exploring, implementing, and verifying were sometimes categorized as cognitive and sometimes as metacognitive. As Schoenfeld (1987) documented, exploration at the cognitive level alone often results in unchecked "wild goose chases" (p. 210). When exploration is guided by the monitoring of either oneself or one's groupmate, that behavior can be categorized as exploration with monitoring or exploration with metacognition. As a consequence of such monitoring, either self or group regulation of the exploration process can occur, thereby keeping the exploration controlled and focused. The same analysis applies for implementation and verification, which can occur with or without monitoring and regulation. The lack of verbalization during episodes categorized as watching and listening made it impossible to infer a level of cognition. Therefore, these episodes were not categorized as either cognitive or metacognitive. Nonetheless, this last category may still be an important dimension in the process of problem solving in a small-group setting. See the Appendix for a detailed description of the framework.

Figure 1 illustrates the variety of sequences of behavior that could occur during the problem-solving session. Specific examples of the protocol analysis follow the explanation of the mathematical problem.

Unlike previous models, this framework delineates the type and level of processes used as individuals solve mathematical problems in small-group settings. It thereby enables the researcher to examine the role of cognition and metacognition within the heuristic framework of mathematical problem solving in a small-group setting.

METHOD

Subjects

The subjects for this study were 27 seventh-grade students (11 girls, 16 boys) who attended an urban public middle school in the borough of Queens, New York City. The students were selected from three average-ability mathematics classes

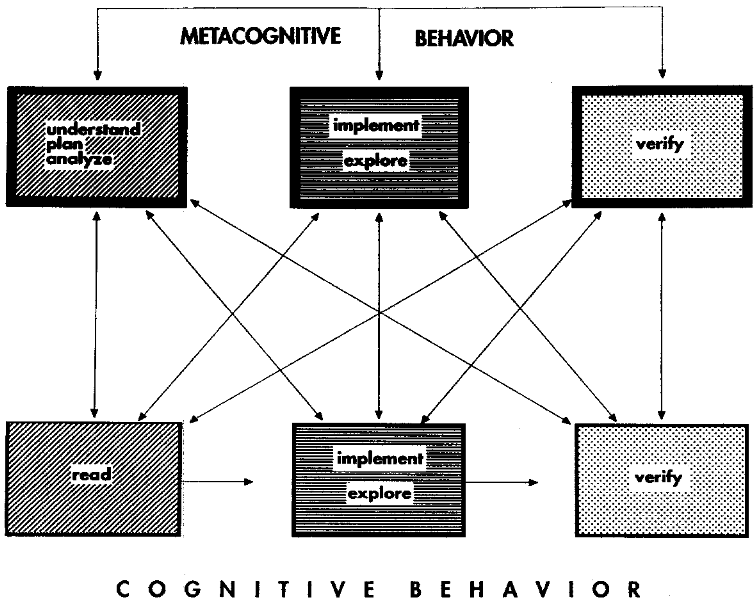

FIGURE 1 A cognitive-metacognitive model. Phases of the problem-solving enterprise.

TABLE 2
Mathematics Problem-Solving Ability:
Percentile Scores on the Metropolitan Achievement Test

	Group					
Students	*1A*	*1B*	*2A*	*2B*	*3A*	*3B*
Quartile (Q)						
Q_4	94	97	99	97	99	94
	82	86	82	86		
	82	77		86		
Q_3	69		73	59	69	63
	50			55	59	55
						50
Q_2			33		25	25
Q_1		18				
Range	44	79	66	42	74	69

taught by two teachers. Only 1 of these students had prior experience in a problem-solving workshop. One week before the study, teachers divided their classes into groups of 4 or 5 students of heterogeneous ability in mathematics. The purpose of this preliminary group experience was to make the students familiar with their group members and the process of group work. In each of the three classes, two groups of students were randomly selected for observation and videotaping, for a total of six groups: Groups 1A, 1B, 2A, 2B, 3A, and 3B. Each group was heterogeneous in mathematical problem-solving ability as measured by the problem-solving section of the Metropolitan Achievement Test. Table 2 lists the students' national percentile scores. The scores are listed according to the groups to which the students belonged and the appropriate quartile in which their scores fell. The groups having the same number (e.g., 1A and 1B) were members of the same class. Each group contained students differing in ability, sex, race, and ethnic background.

Of all six groups, Groups 1A and 2B were the only two whose members all scored above the 50th percentile. In general, these groups were most homogeneous and higher in ability.

Groups 3A and 3B were similar in that they each had only one student who scored in the upper quartile. In fact, each of these students was at the high end of the upper quartile. The students next in ability were in the middle of the third quartile. These groups each contained members at the lowest end of the second quartile. In general, these two groups contained a higher percentage of lower ability students than the other groups, and they had the widest range between the highest ability student and the second highest ability student.

Procedure

Instruments

Students' mathematical problem-solving ability was estimated from their most recent score on the Metropolitan Achievement Test (intermediate form). The test is administered annually in all middle schools. Scores on each student's school record card are recorded as percentiles. Informal interviews were used to obtain mathematics teachers' perceptions of each sample member's ability, attitude, and classroom behavior. Mathematics grades from teachers were a second index of achievement.

Problem Solving in Small Groups

As the students entered the class on the day of the study, they were instructed by the teacher to sit with their groups. They were asked to solve a mathematical problem by working with their group members.

The students were not given a time limit for working on the problem. The teacher called the class together when it was clear that most of the groups had solved the problem and that the few that had not solved the problem seemed unable to proceed further. The problem-solving session lasted between 15 and 20 min.

Videotape of Group Work

In each of the three classes, the investigator (first author) and a research assistant videotaped the two randomly selected groups for the full time they worked on the problem. The videotape was used to provide a permanent record for the coding of problem-solving behaviors. Each of the videotapes was transcribed.

Stimulated-Recall Interview

Within 1 week of the videotaping session, each student participated in a private, stimulated-recall interview as he or she watched the videotape of himself or herself working in the group at six specific times: (a) before his or her group was given the problem, (b) when his or her group was given the problem, (c) when the child began to work on the problem in his or her group, (d) when he or she was deeply involved in working on the problem in his or her group, (e) when the child thought that his or her group had arrived at the solution, and (f) after his or her group had finished working on the problem. These episodes were located by the investigator who viewed each tape and indicated the number on the VCR counter that corresponded to each episode. Students were asked to recall their thoughts during these times in the problem-solving session. The investigator and the research assistant conducted the interviews, which were audiotaped and transcribed.

These interviews provided opportunities to learn about (a) attitudes regarding the task, (b) contributions and participations as group members, and (c) awareness of the problem-solving episodes in which they or their group was engaged (an aspect of metacognition not easily identified through observance of behavior alone).

Coding of Problem-Solving Behaviors

To reach consensus on the categories that best described the observable behaviors, the authors and a research assistant randomly selected one videotape to be used as a pilot tape. The coders worked on the application of the framework until there was strong agreement on how the categories were to be defined and what behaviors were representative of these categories. The interrater reliabilities were high: 93% agreement between the two authors and 91% agreement between the research assistant and one of the authors. In addition, during the actual coding process, if any one of the observers was doubtful about how to categorize a certain behavior, all three observers watched the episode in question and agreed on the appropriate category.

The six videotaped groups consisted of either four or five students. The authors and a research assistant viewed the videotape of each group as it worked on solving the problem. Each viewer watched the behavior of one or two students. The tape was viewed in 1-min intervals, after which each viewer coded the heuristic episode and the cognitive level that best represented the behavior of the student(s) she was observing. Students often exhibited several behaviors during the 1-min time interval. Each behavior was indicated in sequence.

Group Problem-Solving Task

The students were asked to solve the following problem: "A banker must make change of one dollar using 50 coins. She must use at least one quarter, one dime, one nickel, and one penny. How many of each coin must she use to do this?"

The banking problem was selected for several reasons. First, because it cannot be solved using a strict algorithmic procedure, it lends itself to a variety of less structured problem-solving approaches. Second, because students are familiar with money, it was likely that they would understand and be interested in solving the problem. Third, the teachers judged it as an appropriate problem for the ability level of their students.

Before describing the problem-solving behaviors of the students and giving actual protocol examples, we examine the banking problem in light of the proposed framework. We present an outline of several approaches that could be used (many of which the students did use) to solve this problem. These approaches are categorized by episodes—although the episodes are numbered, they are not assumed to be sequential in their occurrence—and cognitive levels as follows:

Episode 1: Reading the problem (cognitive). The student must read or listen to someone else read the problem.
Episode 2: Understanding the problem (metacognitive). The student must understand that there are three conditions that must be met when solving this problem.

1. There must be a total of 50 coins.
2. The value of the coins must be one dollar.
3. There must be at least one quarter, one dime, one nickel, and one penny.

Episode 3: Analyzing the problem (metacognitive). If the students attempt to analyze the problem, there are many ways it can be done. For example:

1. The problem can be reformulated by using the condition that one of each type of coin must be used. That is, four coins (one quarter, one dime, one nickel, and one penny) have the value 25 + 10 + 5 + 1 or 41 cents. This reduces the old problem to a new one having one less condition. That is, now one must find any 46 coins that total 59 cents. No longer is there a restriction about the type of coins that must be used.
2. Because quarters, dimes, and nickels are all multiples of five, their sums, in any combination, will be a multiple of five. Because the sum must be 100, and 100 is a multiple of five, the number of pennies used must also be a multiple of five.
3. For fifty coins to be worth only $1.00, most of the coins selected will have to be pennies.
4. Not many quarters can be used, because four quarters are equivalent to one dollar and 50 coins must be used.

Episode 4: Planning (metacognitive). If the students attempt to plan an approach for solving the problem, there are many ways it can be done. For example:

1. Divide $1.00 into four quarters. Leave one quarter, and keep breaking down the remaining quarters until there are 50 coins.
2. Make a chart using headings of quarter, dime, nickel, and penny. Start with one coin of each type, and then continue adding coins until there are 50 coins totaling $1.00.
3. Start with 50 pennies, and then exchange the pennies for the other coins.
4. Manipulate actual coins to get an idea of how to solve the problem.

Episode 5: Exploring (cognitive and metacognitive). This problem lends itself to a guess-and-test problem-solving approach. This is a form of exploration. If a student is making guesses, testing the guesses, and then making new guesses based on the results of the old ones, he or she is monitoring and regulating the exploration (metacognitive). This, in fact, is an effective technique for solving this particular problem. If, however, the student is merely making a series of random guesses, the student is embarking on an unmonitored exploration (cognitive alone) that is unlikely to result in a solution.

Episode 6: Implementing (cognitive and metacognitive). If a student has devised a plan for solving the problem, he or she is likely to try to implement the plan. If the student does this systematically by monitoring and regulating the implementation (metacognitive), the student is likely to find that the plan either was good and has led him or her toward a solution or was not good and has led him or her to relinquish the implementation and try to devise another plan. If the implementation is not monitored (cognitive alone), however, the student may get buried in the implementation of a poor plan that is unlikely to lead to a solution.

Episode 7: Verifying (cognitive and metacognitive). For an effective verification to take place, the student must be able to take his or her final solution and check that the number of coins is 50, that the total value of the coins is $1.00, and that one of each type of coin is used. This process entails the ability to add numbers (cognitive) and the ability to monitor the results to check that they meet the conditions of the problem (metacognitive).

Episode 8: Watching and listening (uncategorized). For students to exchange ideas that may facilitate the problem-solving process, they must be willing and able to listen and watch each other.

Protocol Examples

We give several examples of group discussion protocols during different phases of the problem-solving process. The coding of each member's behaviors is given as well. Because the coding was based on the behaviors viewed as well as heard, an overview scenario of the behaviors of the students within the groups is also given. The coding decisions were made on the basis of the overriding behaviors of the students rather than on the basis of each individual statement made.

This first protocol presents the behaviors and the statements of four students (R,O,S,K) in Group 1B during the beginning segment of their problem-solving session (all four students silently read the problem from the blackboard). Student R took the lead in analyzing and devising plans for solving the problem. He was the only one in the group, however, who did practically no writing. All he did was talk about the problem. The other three students did some implementing of his plan while they seemed to be struggling to understand the requirements of the problem. It seemed that they were attempting to solve the problem using Student R's lead before they really understood what the problem asked them to do (evidenced by the statements they made). The protocol follows:

1. S: How many dollars?
2. R: One dollar. We have to use at least one of each to make one dollar. Let's try to get rid of the biggest coins first.

3. S: (Shakes head in agreement and begins to write.)
4. R: Let's get rid of the dimes too so we can get rid of the biggest coins first.
5. K: The dimes?
6. R: Yeah, so you have 35 cents.
7. K: Work with the nickels first.
8. R: No, 'cause we have to use the quarters, dimes, and the nickels.
9. O: Okay, one quarter—(as she writes).
10. R: We have to use a quarter and a dime, which adds up to 35 cents. We have 35 cents already.
11. K: One quarter, one dime (as he writes).
12. O: You know it has to equal one dollar.
13. R: We used two coins already. Let's use five pennies.
14. K: A nickel—
15. R: That's 40 cents.
16. S: Use two quarters.
17. R: No, let's use two of the biggest ones—a quarter and a dime. That's 35 cents.
18. S: We should use pennies then.
19. K: There has to be pennies?
20. S: There has to be 50 coins.
21. K: Fifty coins? One dollar? Okay. (They all start writing.)

Student R. In Statement 2, this student clarifies to himself and others the conditions of the problem (understanding). By deciding to "try to get rid of the biggest coins first," he has immediately launched into a plan. Statements 4, 6, 8, and 10 show that he is sticking to his plan and, in his head, is implementing (cognitive) his plan. By stating that he has accumulated 35 cents (Statements 6 and 10) and by declaring that they have used two coins already (Statement 13), he demonstrates that he is keeping track of or monitoring his implementation (implementing: metacognitive). His suggestion to use five pennies (Statement 13) shows that he is about to go off track from his original plan to use the "biggest coins" first. However, he is kept in line by Student K, who interprets his "five pennies" as a nickel (Statement 14). When Student S suggests the use of two quarters, he rejects it by emphasizing his original plan. Within this segment, Student R's behaviors were coded as follows: reading, understanding, planning, implementing (cognitive and metacognitive).

Student S. In her very first remark in Statement 1, this student shows that she is trying to understand the requirements of the problem (understanding). She shakes her head in agreement to Student R's plan, and, from her subsequent writing, one would assume that she is implementing Student R's plan (implementing: cognitive). Having used the three largest coins, she suggests the use of two quarters (Statement 16). There is no obvious plan at work at this point, and this suggestion seems to mark the beginning of exploring (metacognitive followed by cognitive) of the problem. Student R reminds her of his plan, and it appears from Statement 18 that she, having used all of the largest coins, is ready to use the pennies. In Statement 20, she is again trying to clarify the conditions of the problem for herself and her group. Within this segment, Student S's behaviors were coded as follows: reading, understanding, implementing (cognitive), exploring (metacognitive and cognitive).

Student K. In Statement 5, this student seems to be trying to understand Student R's suggested plan. He then contributes his own plan, which is immediately rejected by Student R (Statement 7). Although his idea of working with the nickels first takes the form of a plan, the fact that it is at such a local level (within Student R's plan), with no apparent rationale behind it, suggests that the appropriate coding is exploring (metacognitive). In Statement 14, he interprets Student R's suggestion to use five pennies as meaning to use one nickel. This monitoring of Student R's implementation helped the group stick to the original plan (implementation: metacognitive). When he asks if pennies must be used (Statement 19), the student shows that he did not yet fully understand the conditions of the problem. During this segment, the behaviors of Student K were coded as follows: reading, exploring (cognitive), implementing (metacognitive), understanding.

Student O. This student was mostly engaged in listening and writing. She only made two comments–once in Statement 9, when she was following Student R's directions, and then again in Statement 12, when she was trying to clarify the conditions of the problem to herself. Within this segment, the behaviors of Student O were coded as follows: reading, watching and listening, implementing (cognitive), understanding.

The second protocol presents the behaviors and the statements of four students (C, D, S, W), referred to as Group 3A, during the middle segment of their problem-solving session. The students have just reached the conclusion that they can reformulate the initial problem by first meeting the conditions of using one of each coin. The students have been working cooperatively, and each student has been engaged in the process of trying to solve the problem. We join them as they try to make sense of where they stand by exploring the revised problem.

1. W: Wait! Wait! We already have 41 cents with 4 coins.
2. C: How much more do we need?
3. W: Now we need 46 coins.
4. S: 46 coins and we need, um–
5. C: 59?
6. W: 59 cents.

7. S: Yeah.
8. D: So use all the pennies.
9. W: Forty-what coins? Forty-six.
10. C: Forty-six coins.
11. W: 59 and 46, what is it?
12. S: No, but you're getting confused 'cause this is the number of cents and this is the number of amount of coins.
13. W: No, but I mean what if we use 46 pennies?
14. D: It's a dollar five.
15. C: Yeah, but we have to use nickels and pennies.
16. D: Yeah.
17. W: All right.
18. S: Maybe we could use 5 nickels and then 41 pennies.
19. W: Try it.
20. S: So 5 nickels is 25 plus 41.
21. C: (Working on his own) Sixty-six again! We already did that. Forty-one plus twenty-five.

Student W. In Statements 1, 3, 6, 9, and 11, this student engages in an assessment of the status of the problem solution. He is figuring out how many coins and how many cents he must have after he already has used 41 cents with 4 different coins. Because we are joining him in the middle of an exploration, this behavior was coded as exploring (metacognitive). In Statement 11, he reveals his confusion of coins and monetary value and, although he does not openly admit it, he is straightened out by student S's comment. In Statements 13, 17, and 19, he makes suggestions and gives encouragement for further exploration of the problem. His suggestion to use 46 pennies was exploratory. Within this segment, the behaviors of Student W were coded as exploring (metacognitive).

Student S. This student's clarifying comment in Statement 12 exemplifies the type of higher level statement that can keep a group's effort on track. In effect, she has monitored the exploration of Student W (exploring: metacognitive). From Statements 18 and 20, it is clear that she has joined in the exploratory efforts of the group by both giving suggestions and making her own calculations (exploring: cognitive and metacognitive). During this segment, the behaviors of Student S were coded as exploring (cognitive and metacognitive).

Student C. During the beginning of this segment, this student was listening to Student W. Afterward, in Statements 2, 5, and 10, he interacted with Student W in exploring the problem. Student C appears to finish the thoughts and sentences that Student W begins (exploring: metacognitive). Although the ideas are not initially his, he adds to the clarification of the issues by helping Student W along. In Statement 15, he reminds the group of the conditions of the problem (although he was incorrect by recalling the need to use both nickels and pennies, because by then the students had already used one of each coin). Whether or not his statement was valid, he was engaging in efforts to clarify the conditions of the problem; thus, his statement was coded as understanding. Finally, in Statement 21, he joins the group in exploring the problem both cognitively (by working on Student S's suggestion) and metacognitively (by recognizing that the result was incorrect and, in fact, one that they had already reached). Within this segment, the behaviors of Student C were coded as follows: watching and listening, understanding, exploring (metacognitive and cognitive).

Student D. During the beginning of this segment, this student was listening to the remarks made by the other group members. In Statement 8, he suggested using "all the pennies." Although his suggestion sounded somewhat arbitrary and was, therefore, coded as exploring (metacognitive) rather than as planning, it had the potential to set the group on the right track. However, instead of adding the 46 pennies to the 41 cents that were already set aside, he added the 46 pennies to the 59 cents that was the sum to be sought (Statement 14). This gave him a total of "a dollar five," with which he could not work. Unfortunately, nobody in the group noticed his error. Within this segment, the behaviors of Student D were coded as follows: watching and listening, exploring (metacognitive and cognitive).

The third protocol presents the behaviors and the statements of four students (D, J, P, T), referred to as Group 2A, and the teacher (G) during the last segment of their problem-solving session. In this group, Student J appeared to be the prime problem solver. Because she did not have a pen or pencil, she dictated her ideas to Student P, who struggled to keep up with her suggestions. Student D busily worked on his own explorations, whereas Student T, the least involved, intermittently reminded fellow group members of the conditions of the problem. We join these students after their initial analysis that they need to find 59 cents using 46 coins. They have been lost in exploration for approximately 7 min.

1. J: Listen, if we have to use one of each, already we have 41 cents. We have 4 coins right? That means we need how many more coins? We need 46 more coins. So 46 coins and 41 cents. We have to break it down into nickels and pennies and everything else.
2. T: Pennies. Use all pennies.
3. J: Yeah, but that's too many. If we use all pennies, we wouldn't have enough.
4. D: It all depends on the pennies [Student T]. I bet you it all depends on the pennies.
5. T: I know. (Student P is holding the pencil, not knowing what to write. Student S does not have a pencil or paper.)
6. J: I'm confused now. (Thinks awhile and then instructs Student P) Put 40 pennies down.

7. P: 40 pennies?
8. J: Yeah—put 40 pennies. Now put one quarter, one dime. So that's 25, 35, (45, 55, 65 to herself) 75 cents. So we need 7 more coins.

(P is writing and trying to calculate to catch up with J's ideas.)

9. J: Use 45.
10. D: (Interrupting J) I got it! I got it! Look, 20 pennies, 1 nickel, 5 dimes, and 25 cents equals a dollar.
11. P: That's only . . . (counting up the number of coins)—
12. T: You've got to use 50 coins.
13. D: Oh, I was so close.
14. K: (Getting back to work with P) All right, so how much do we have here now? We have 44 coins. You got 80 cents. (Calculating to herself) That's 46. We need 4 more coins: 42, 43, 44, 45, 46. No, two. (P looks confused) Add another dime (calculating to herself).
15. D: It would have been easier if they told us how much money do we give the banker.
16. J: Put another dime in.

(T reads the problem to D.)

17. J: (Calculating what P has written) That's 50 coins. Look so that's . . . (calculating to herself) I got it! I got it! Look, look, 45 pennies, 2 nickels, 2 dimes, and a quarter. That's it.
18. D: Yea [Student J]!
19. T: Yes [Student J]!
20. D: Champion! We have it. Yea!
21. P: (Still looking bewildered, trying to figure out what he has written on the paper.)
22. J: (To P) What are you doing? Two nickels. . . .
23. P: Ten cents is two nickels.

(The teacher walks by.)

24. D: Miss G, we think we got it.
25. J: (To Miss G) Forty-five pennies, 2 nickels, 2 dimes, and 1 quarter.
26. G: But how come [Student P] is saying no?
27. P: Four, five, six . . . (Talking out loud as he still tries to figure it out.)
28. D: (To P) Carry the one.
29. P: I did.
30. J: (Pointing to the numbers on P's paper) 45, 55, 65, 75–75 and one quarter is a dollar. We got it.

Student J. In Statement 1, this student is clearly analyzing the problem. Student T comes up with a suggestion that she rebuffs in Statement 3. The student's suggestion was done at the local level and could, therefore, be categorized as a suggestion for exploration. Student J's statement would thus be categorized as monitoring the exploration (exploring: metacognitive). In Statement 6, Student J launches her own exploration by telling Student P to "put 40 pennies down." She calculates and monitors her own suggestion in Statement 8 (exploring: metacognitive and cognitive). By Statement 9, she hits on the correct number of pennies but is interrupted by Student D. When she returns to her ideas in Statement 14, she forgets about her suggestion of using 45 pennies and evaluates the status of the problem with her original idea of using 40 pennies. From this point on, she does most of the work in her head (Student P cannot keep up with her pace). Finally, in Statement 17, she checks her own exploration and discovers that she has solved the problem. She states her answer to the teacher and impatiently tries to help Student P verify the solution. Her final verification is in Statement 30. Her declaration, "We got it," shows that she has verified and checked the solution against the conditions of the problem. She has not merely added numbers but has also monitored the meaning of those numbers. This is an example of verifying at the metacognitive level. Within this segment, the behaviors of Student J were coded as follows: analyzing, exploring (metacognitive and cognitive), verifying (metacognitive and cognitive).

Student P. At this point in the problem-solving session, this student was acting as a secretary. Primarily, he was taking instructions from Student J. At one point, he took a moment to monitor Student D's incorrect proclamation that he had solved the problem (Statement 11). Because Student D's solution came out of extensive exploration, Student P's comment would be categorized as exploring (metacognitive). Because most of his behaviors entailed writing numbers that he did not seem to understand fully, his behavior was coded as exploring (cognitive). At the end of the session, Student P was attempting to verify Student J's solution. This was only at the cognitive level, however, because he still did not seem to have a grip on the problem. Within this segment, the behaviors of Student P were coded as exploring (metacognitive and cognitive).

Student D. This student picked up on Student T's suggestion that the number of pennies used would be very important (Statement 4). This was coded as analyzing. He strayed from the group and went off into his own calculations until he declared his solution in Statement 10. His calculations, compounded by his monitoring of the calculations that led him to believe that he had solved the problem, served as a rationale for coding these behaviors as exploring (metacognitive and cognitive). Students P and T alerted him to the fact that he had satisfied only some of the conditions of the problem. He understood their point and, in Statement 15, stated his wishes for a change in the wording of the problem.

At the end, he cheered for Student J. He watched the calculations that Student P was making as he tried to verify Student J's solution. During this segment, the behaviors of Student D were coded as analyzing, exploring (metacognitive and cognitive), watching and listening.

Student T. This student did not do any calculations during the entire problem-solving session. He spent most of his time watching and listening to the other students. He gave an important suggestion in Statement 2 to "use all pennies." Because this suggestion came at the local level, it was categorized as exploring (metacognitive). After that, his only other statement was to remind Student D of the conditions of the problem. Within this segment, the behaviors of Student T were coded as watching and listening, exploring (metacognitive), understanding.

RESULTS

Results of Problem Solving in Small Groups

The coding for each of the six groups observed was done on charts such as those shown in Figures 2, 3, and 4. The behavior of each student was categorized in two ways: by episode and by cognitive level. The episode or type of problem-solving behavior in which the student was engaged (read, understand, analyze, explore, plan, implement, verify, watch and listen) was recorded in the appropriate row. An asterisk indicates metacognitive behaviors. All other behaviors, except those categorized as watch and listen, were considered to be cognitive. Those categorized as watch and listen were not assigned a cognitive level.

Students' behaviors were coded in 1-min intervals. The time intervals are listed along the bottoms of Figures 2 through 4. Behaviors (episodes) are listed on the vertical axis and are displayed throughout the charts. Students are distinguished from one another by their first initial. For example, in Figure 3, during the first minute, Student C first read the problem, then tried to understand the problem, and then began to do some exploratory work. During Minute 2, Student C watched and listened to what the other students were doing and saying and then resumed exploratory work. Charting each student's behavior in this way yields a picture of each individual's behavior. As an added outcome, a picture of the group's behavior as a whole seems to emerge. (The protocol example of Group 3A can be located in Figure 3 during approximately the fifth to seventh minutes.)

Each figure contains a small table summarizing the behaviors of each student in the group. These behaviors are categorized as metacognitive, cognitive, and watch and listen. By counting the number of metacognitive behaviors coded for one student and dividing it by the total number of behaviors coded in the group, a profile can be obtained of each group member's metacognitive contributions

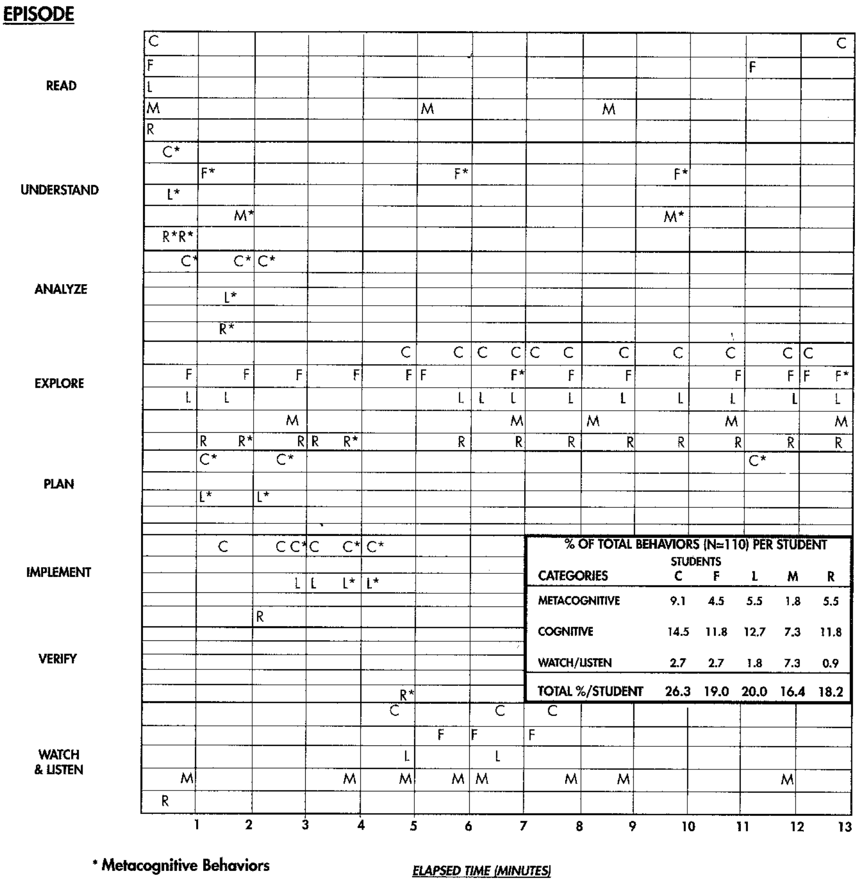

FIGURE 2 Diagram of behaviors in Group 1A.

within the group. The same is done for each member's cognitive behaviors and for watch and listen behaviors.

Metacognitive and Cognitive Behaviors

Table 3 shows the number and percentage of behaviors coded as metacognitive, cognitive, and watch and listen. Of 442 behaviors coded, 38.7% were metacognitive, 36.0% were cognitive, and 25.3% were in the watch and listen category, an undetermined cognitive level.

The metacognitive behaviors as a percentage of the total behaviors coded ranged from a low of 26.3% in Group 1A (the only group that did not solve the problem) to a high of 51.6% in Group 2A. The cognitive behaviors as a percentage of the total behaviors coded ranged from a low of 23.2% in Group 3B to a high

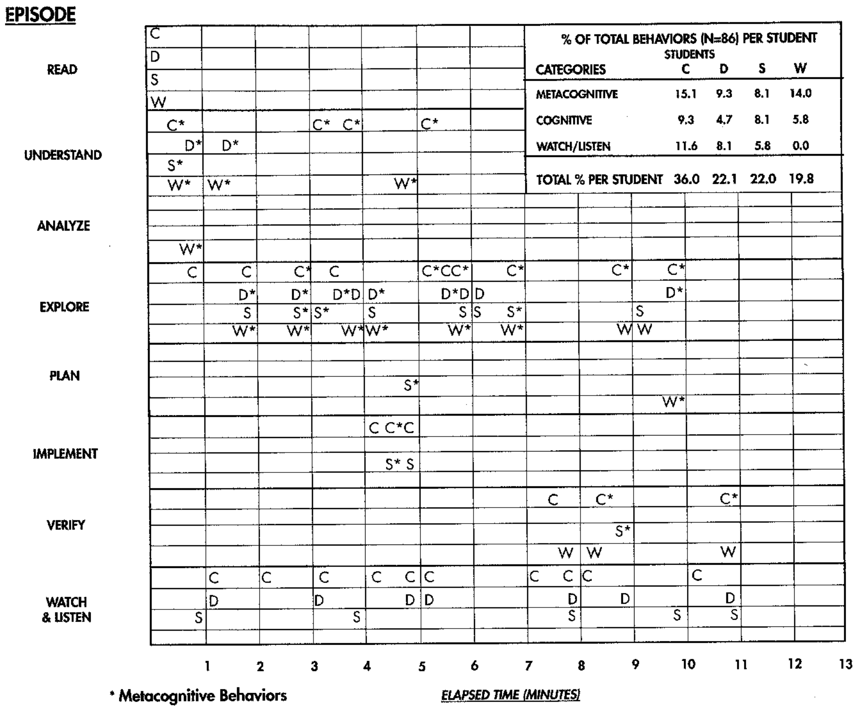

FIGURE 3 Diagram of behaviors in Group 3A.

of 58.2% in Group 1A. The ratio of metacognitive to cognitive behaviors ranged from a low of .45 in Group 1A to a high of 2.00 in Group 1B.

Problem-Solving Episodes and Cognitive Levels

Table 4 lists the percentage of cognitive and metacognitive behaviors coded by category for each group. Across all groups, there were 171 behaviors coded as metacognitive. Of these, 62 were in the category exploring (metacognitive), and 55 were in the category understanding. In other words, the greatest percentage of metacognitive behaviors was in exploring (36.3% of all metacognitive behaviors) and in understanding (32.2% of all metacognitive behaviors). In each of these categories, Group 1A had the lowest percentage of these behaviors—exploring (metacognitive), 3.6%, and understanding, 8.2%.

Across all groups, there were 159 behaviors coded as cognitive. Of these, 96 were in the category exploring (60.4% of all cognitive behaviors), and 38 were in the category reading (23.9% of all cognitive behaviors). In other words, the greatest percentage of cognitive behaviors was in exploring, followed by reading.

Of all the episodes coded, the exploring episode (metacognitive and cognitive together) was coded the greatest percentage of times in each group. The percent-

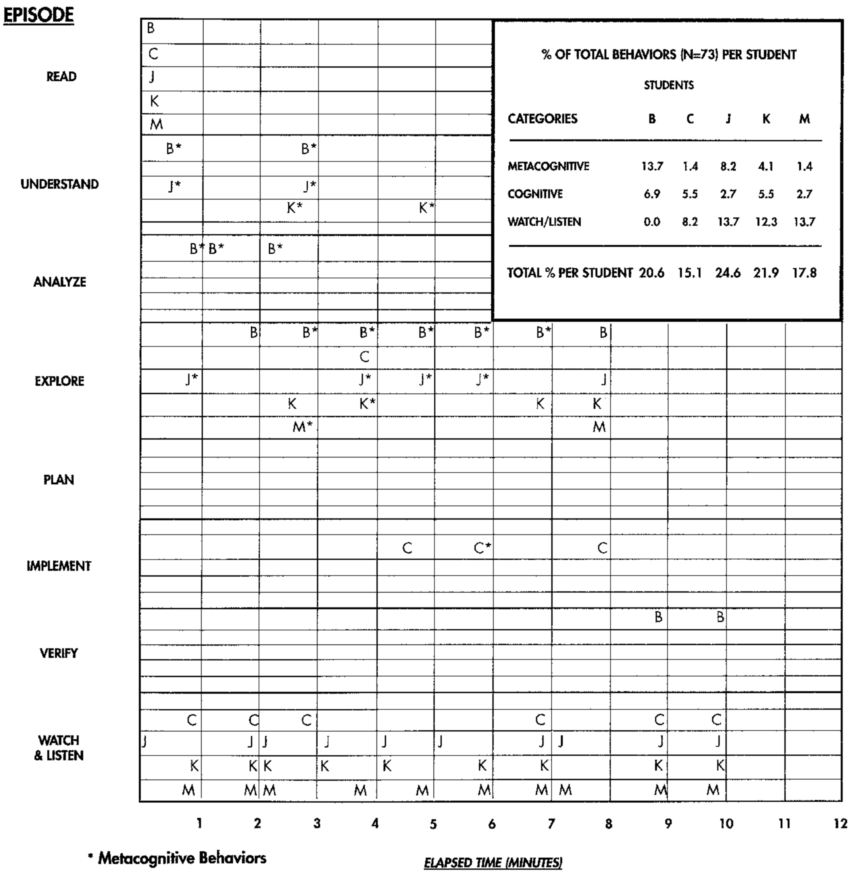

FIGURE 4 Diagram of behaviors in Group 3B.

age of total exploring episodes ranged from 25.3% in Group 1B to 48.0% in Group 1A. That is, in each group, at least one quarter of the episodes coded was in the exploring category.

In total, 112 coded behaviors were in the watch and listen category. Group 3B had the largest percentage of such behaviors (47.9%), and Group 1A had the smallest percentage (15.5%).

We elaborate on these findings in the Discussion section.

Stimulated-Recall Interviews of High-Ability Students

Although all students in the study were interviewed, this article focuses on the interviews of the highest ability student in each group, because they had the greatest potential to solve the mathematical problem. A summary of the narratives that

TABLE 3
Number (and Percentage) of Metacognitive, Cognitive, and Watch-and-Listen Behaviors Per Group

	Group						
Behavior Category	*1A*	*1B*	*2A*	*2B*	*3A*	*3B*	*Total*
Metacognitive	29 (26.3)	32 (47.7)	32 (51.6)	17 (38.7)	40 (46.5)	21 (28.8)	171 (38.7)
Cognitive	64 (58.2)	16 (23.9)	20 (32.2)	18 (40.9)	24 (28.0)	17 (23.2)	159 (36.0)
Watch and listen	17 (15.5)	19 (28.4)	10 (16.1)	9 (20.5)	22 (25.6)	35 (47.9)	112 (25.3)
Total	110 (100.0)	67 (100.0)	62 (100.0)	44 (100.0)	86 (100.0)	73 (100.0)	442 (100.0)

Note. Groups 1A, 2B, and 3B had five members; Groups 1B, 2A, and 3A had four members.

TABLE 4
Percent Distribution of Cognitive, Metacognitive, and Watch-and-Listen Behaviors by Problem-Solving Group

	Group					
Behavior Category	*1A*	*1B*	*2A*	*2B*	*3A*	*3B*
Metacognitive						
Understand problem	8.2	17.9	21.0	11.4	11.6	8.2
Analyze	4.5	8.9	8.1	2.3	1.2	4.1
Explore	3.6	11.9	16.1	15.9	25.6	15.1
Plan	4.5	4.5	4.8	0.0	2.3	0.0
Implement	4.5	3.0	0.0	0.0	2.3	1.4
Verify	0.9	1.5	1.6	9.1	3.5	0.0
Cognitive						
Read	8.2	6.0	12.9	18.2	4.7	6.8
Explore	44.5	13.4	14.5	18.2	15.1	11.0
Implement	5.5	0.0	0.0	0.0	3.5	2.7
Verify	0.0	4.5	4.8	4.5	4.7	2.7
Watch and listen	15.5	28.4	16.1	20.5	25.6	47.9

focused on the students' attitudes about solving the problem in the small-group setting is presented next.

The highest ability members of Groups 2A, 2B, and 3A all revealed insecurities about their own abilities to solve the problem. They expressed their anxieties about the possibility of not being able to solve the problem on their own. They all expressed the desire to receive helpful input from their group members. In contrast, the highest ability members of Groups 1A, 1B, and 3B expressed their desires to work independently. One student revealed her belief that she would proceed more quickly by working alone. She also stated that she was "stubborn" and liked to do things her own way. Another claimed that he preferred to work alone, because that was the way he was "trained" and that is how one is expected to work on an exam. He also intimated that he lacked respect for the abilities of his group members.

The effect of these attitudes on the interactions that occurred within the groups is addressed in the Discussion section.

DISCUSSION

Framework and Cognitive Levels

The framework provides useful information with respect to when, where, and in what frequency group members use processes at the cognitive and metacognitive levels and how these levels of thought may affect the problem solution. It is possible that a certain balance of both cognitive and metacognitive processes within a group is necessary for the problem-solving efforts to result in solution. Indeed, it is interesting that, in this study, the only group that did not solve the problem was the group with the lowest percentage of episodes at the metacognitive level and the highest percentage of episodes at the cognitive level. In fact, in this group, the ratio of metacognitive to cognitive behaviors was lower than any of the ratios of metacognitive to cognitive behaviors of the other five groups. It is also of interest to note that, during the exploratory phase of solving this problem, the unsuccessful group had, by far, the lowest percentage of metacognitive behaviors of all the groups.

Role of Metacognitive Behaviors

The current literature supports the importance of metacognitive behaviors such as active monitoring and subsequent regulation of cognitive processes during the act of problem solving (Baker & Brown, 1982; Flavell & Wellman, 1977; Garofalo & Lester, 1985; Schoenfeld, 1985b; Silver, 1987). When examining the specific instances of these types of metacognitive statements, one gets a better understanding of the ways in which they serve to enhance and propel the problem-solving process.

For example, the statement made by Student R in Group 1B (not reported in the earlier protocol), "We used every coin so far so we don't have to worry about it any more. So we have 41 cents, and we have 46 coins to use. We have to use more pennies," serves to help the group understand the status of the problem solution and the direction in which to go to continue the solution process. Other such statements made by different students were: "No, but you're getting confused 'cause this is the number of cents and this is the number of amount of coins"; and "Listen, if we have to use one of each, already we have 41 cents. We have 4 coins right? That means we need how many more coins? We need 46 more coins. So 46 coins and 41 cents. We have to break it down into nickels and pennies and everything else." Such statements often change the flow of conversation and appropriately redirect the efforts of the group members.

In a different way, the more "local" monitoring statements such as "No, that wouldn't work," "It's gotta be 50 coins," and "Use a lot of pennies" serve to control the group and to keep it from going off on wrong tangents by reminding the group members of the conditions of the problem that must be met and by suggesting the next small steps to take. As revealed by the transcriptions of the videotapes, these statements were made by all students who were caught up in the flow of the problem-solving process.

An example of what happens when there is an absence of consistent monitoring and regulating of the problem-solving process can be seen in Group 1A, which did not solve the problem. Student C declared the incorrect plan of using only nickels and pennies after they had 1 quarter and 1 dime. (The correct solution required 2 dimes.) If her plan had been monitored by another metacognitive statement such as "That won't work" or "Maybe we should try using more dimes also," the group might have had a chance of getting back on track.

These results support the importance of metacognitive processes in mathematical problem solving in small-group settings. The framework developed in this study proved to be an effective tool for capturing the metacognitive behaviors characteristic of effective group work and of effective problem solving.

Role of Cognitive Behaviors

In this study, cognitive activity was evident in all groups. We have seen the important role of metacognitive statements; however, without the presence of students who were able to follow through or implement the metacognitive statements, the problem solving could not have been advanced or completed. For example, in the protocol of Group 3A, the students enacted a plan proposed by one of the group members. After completing the computation, they noticed that their solution was not satisfactory. Through the combined cognitive efforts of performing the calculations and metacognitive efforts of evaluating their solution, the students determined that they had to take a new approach, and thus the problem-solving process was advanced. The interrelationship between metacognitive and cognitive processes is complex, and an appropriate interplay between the two is necessary for successful problem solving to occur.

Role of Watching and Listening

The role of watching and listening is an important variable to consider when studying individuals solving problems in a small-group setting. Watching and listening are as much a part of communication as speaking is, and as Patton, Giffin, and Patton (1989) claimed, "Communication is the essence of the small-group experience" (p. 11).

Although in this study it was not possible to assign a cognitive level to such behavior, watching and listening play a major role in the group process. In fact, the degree of watching and listening behaviors of students may be the defining variable of whether students are engaged in a group interaction at all. For example, in Group 1A the students hardly listened to one another. Perhaps if they had, someone could have helped Student C change her inappropriate plan. Of all six groups, Group 1A had the lowest percentage of watch-and-listen behaviors. The inability of the students to share meanings prevented their group from functioning as a productive unit.

In contrast, most of the students in Group 3B were watching and listening while one person was doing the majority of the work. This is a different version of poor group functioning. Student B assumed a leadership role, and, because of his reported lack of respect for his fellow group members, he dominated the discussion with his own ideas. Research shows that such leadership styles discourage and inhibit the other group members from offering their input (Yukl, 1981). Of all six groups, Group 3B had the highest percentage of watch-and-listen behaviors, none of which was contributed by Student B.

In contrast to the just-mentioned situations, the balance of watching and listening behaviors that occurred in Group 3A contributed to the fruitful interactions that took place. For example, after watching and listening for several minutes, Student D was able to contribute the helpful idea of using mostly pennies. One main advantage of working in a group is that students are able to benefit from group members' ideas. By listening to other people's ideas, one's own ideas are inspired. The extensive degree of interaction that occurred in Group 3A showed that each student was engaged in watching the activities and listening to the ideas of one another.

These examples support the group process theories that show the importance of communication skills for the effective functioning of the group. The balance of watching and listening behaviors during the group problem-solving process is an important issue to be given consideration.

Small-Group Setting

Observing Individuals in a Group Versus Observing a Group

The framework developed for this study was used to observe individual students as they worked in a small-group setting. The presence of group members affects the behaviors of the individual students in various ways and to different degrees. Moreover, in a classroom where the students are arranged in small groups, each group behaves in unique ways. The interactions of individuals working in small groups can be represented on a continuum that ranges from students who (although seated in a group) work independently and do not communicate with others to students who do interact with others in the solution of the problem. Between these two extremes is a multitude of possible scenarios. Figures 5a, 5b, 5c, and 5d depict several situations that can occur. In this study concerning only six groups, we saw a wide range of behavior patterns. For example, Group 1A was most like that depicted in Figures 5a and 5d where the students tended to work independently. Group 3A was a highly interactive group as depicted in Figure 5b, and Group 3B was the "one-man show" as depicted in Figure 5c.

In the literature, the term *group* has proved difficult to define. In fact, according to social psychologist Theodore Newcomb (1951), the term has never achieved a standard meaning. Definitions range from such loose requirements as a group being merely a collection of people (Homans, 1950) to more restrictive definitions that specify size and specific types of within-group communication (Patton et al., 1989). Generally, studies such as this one, which involve decision-making or problem-solving groups, adapt the more restrictive definitions. Patton et al. (1989) listed five conditions necessary for effective group work: (a) two or more people, (b) interdependence, (c) a common goal, (d) communication, and (e)

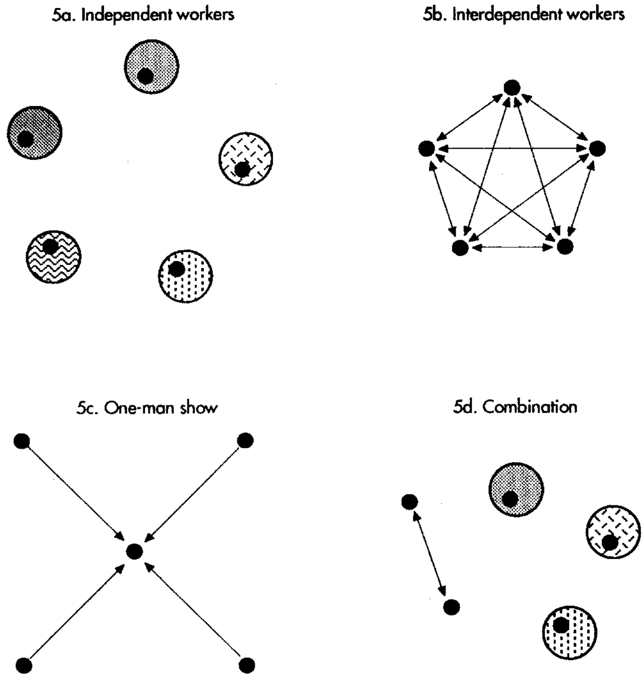

FIGURE 5 Patterns of group interactions.

norms. In the scenario depicted by Figure 5a in which each student works independently, there is little communication and thus ineffective group function. One might then question whether, in fact, this should really be considered a group (in the restrictive sense) or just five individuals seated in a group. At the other extreme, when the involvement of each individual becomes so integrated with the interactions of the other group members, the presence of individuals seems to disappear into the overall tapestry of the group. We have tried to examine the role of cognition and metacognition (as we have defined them) in problem solving in small-group settings by coding the behaviors of individual students. As the group behaviors tend to approach the scenario depicted in Figure 5b on the continuum, however, the behaviors of each of the group members become so interdependent that the group appears to take on its own "collaborative cognition," and the presence of individuals is almost lost.

Cognitive Processes Within a Small Group

The small-group format seems to encourage a spontaneous verbalization that allows the students to externalize their ideas for critical examination. The questioning, elaboration, explanation, and feedback to which these ideas are subjected may be the mechanisms that account for problem solution. Research suggests that the small-grouping schemes that produce the most positive outcomes are structured to maximize these types of behaviors (Slavin, 1989–1990). That is, the most successful grouping strategies are carefully structured to ensure positive interdependence and individual accountability of group members. These structures serve as motivation for students to engage in the active participation within the group that creates an optimum setting for monitoring and regulating behaviors to occur. These are the metacognitive behaviors we have coded in the proposed framework. On this basis, it is reasonable to suspect that the most successful groups, in terms of both solving the problem and getting active involvement of all the group members, should be those with the highest percentages of metacognitive behaviors. In this study, the groups were not structured, although the behaviors of students in Group 3A resembled what would be expected in a group structured for positive interdependence and individual accountability. The results of this exploratory study seem to lend support for the idea that groups having all members actively involved have high incidences of metacognitive behaviors that may be the mechanisms by which problem solving is facilitated.

The diagrams in Figures 2, 3, and 4 represent the behaviors of the students in Groups 1A, 3A, and 3B that are highlighted here because of their contrasting characteristics. A comparison of these charts reveals an observable difference between Group 1A (Figure 2) and the other two groups (Figures 3 and 4). In Groups 3A and 3B, the exploration that occurred was monitored and regulated by group members (as indicated by the asterisks) throughout the duration of the problem-solving session. In Group 1A, there are only a few instances of monitored or regulated exploration. Interestingly, only Group 1A did not solve the problem, and, of all six groups, Group 1A had the lowest percentage of episodes of exploration at the metacognitive level (3.6%; see Table 4). In fact, Group 1A had the lowest total percentage of metacognitive behaviors (26.2%). A related, but not so obvious, difference between Group 1A and the other groups is the smaller percentage of students who were watching or listening to other students (15.5%). The inserted table in Figure 2 clearly shows that none of the students had a high percentage of watching and listening behaviors. In general, one sees this group functioning as individuals who, after the first 3 min, were all engaging in their own private unguided explorations. This is supported by the fact that the highest percentage of each student's individual behaviors was at the cognitive level.

The individualistic, unmonitored explorations of the members of Group 1A contrast with the apparent united effort of the students in Group 3A. (This is indicated in Figure 3 by the presence of all four group members' initials with asterisks in the exploration episodes.) The exploratory activities of this group were characterized by the idea sharing of all group members, accompanied by the subsequent monitoring and regulating of one another's ideas and approaches. The inserted table in Figure 3 shows that the greatest percentage of each student's behavior was metacognitive. This contrasts with the behavior of the individual students in the other groups. From the observers' perspective, it appeared that this metacognitive behavior helped the students discover the solution to which each group member contributed.

Although the members of Group 3B also arrived at the solution, their main problem-solving activities appear to have been dominated by a few individuals (see Figure 4). Specifically, Student B was the driving force in this group. Note that this was the one student who had prior problem-solving experience. The asterisks indicate that Student B was sharing his ideas about the problem with the other students. However, aside from Student B, who did no watching and listening, the highest percentage of behaviors for each of the other students in the group was that of watching and listening. These data reflect the fact that one person dominated this group while the other group members mostly watched and listened.

Ability Levels

Why do groups take on such different behaviors? Why do the proportions of cognitive, metacognitive, and watch-and-listen behaviors differ so greatly among groups? What are the conditions that might bring a group to the highest level of group interaction and group productivity? Research efforts have provided abundant clues to the variables that are likely to impact positively or negatively on a group's performance. High on the list of influential variables are the communication skills, personalities, and intentions of the group members. In this small study, we can begin to investigate further a few ideas. For example, one may wonder how influential are the ability levels and the personalities and attitudes of the group members.

According to the ability data (Table 2), several groups had similar academic profiles (Groups 1A and 2B, Groups 3A and 3B, and Groups 1B and 2A). Despite these similarities, the groups functioned rather differently. Some variables that may have contributed to these differences were the personalities and attitudes of the highest ability member in each group.

For example, the students in Group 1A hardly worked together and were unsuccessful in solving the problem, whereas in Group 2B, aside from one student, the group members were very interactive and succeeded in solving the problem. In Group 1A, the highest ability member was admittedly unwilling to work with the other members of her group. She got fixed on one faulty plan and was not receptive to feedback from her peers. In contrast, in Group 2B, the highest ability member was admittedly insecure about her ability to solve the problem and expressed the desire to receive input from other group members.

A similar contrast existed between the behaviors of Groups 3A and 3B, despite their academic similarities. All the members in Group 3A were highly interactive and cooperative in solving the problem. In contrast, Group 3B was totally dominated by the highest ability member who solved the problem single-handedly and discouraged input from his fellow group members. This student admittedly did not like, or see the purpose of, working with others. He had a competitive attitude and wanted to be the one in his group to solve the problem. Although he was able to solve the problem, he never helped any of his group members and was uninterested in their understanding of the problem. As might be expected, the highest ability member of Group 3A had a very different attitude about working in a group. She expressed a nervousness about the possibility of failing to be able to solve the problem. She said she was anxious to work with her group members in hopes that they would be able to help. Although, in the group, she was the one who made the most high-level metacognitive statements, she was totally influenced by and submerged in the problem solving in which the group was engaged.

In four of the six groups in this study, the highest ability student in each of the groups was primarily responsible for solving the problem. The two groups that did not fit this pattern were Group 1A, which did not solve the problem, and Group 3A, which was so interactive that it was impossible to assign credit to any one person in the group for having solved the problem.

It seems reasonable to conclude that the personalities and attitudes of the highest ability group members have a very powerful effect on the subsequent behaviors of each of the members of the group.

Framework and Heuristic Episodes

The framework was a useful tool for investigating the occurrence and frequency of heuristic episodes. It was evident that the episodes occurred intermittently in all six groups. In all groups, the students returned several times to different episodes. Most often, they returned to the words of the problem to gain a clearer understanding. They could often be heard reminding one another of the conditions that had to be met in the solution of the problem. In fact, all of the groups returned several times to the understanding episode. Most of the groups returned to reading several times. Figure 1 illustrates the ways in which the episodes can occur during a problem-solving session. One would imagine that each time the students returned to an episode they brought new insights with them. So, although they were at the same episode, they were there with a higher level of comprehension.

Exploring (cognitive and metacognitive together) was the behavior that was coded the greatest percentage of times. The exploratory phase, however, was most often accompanied by several other episodes as well. Exploration often led students to have an idea for a plan that, when deemed acceptable by the group members, led to an implementation. Often the implementation was fruitless, and the students returned to their exploration. In several cases, the exploration and analysis of the problem occurred intermittently. The exploration might have allowed the students to gain the familiarity with the problem that is a prerequisite for analysis. In several groups, it appeared that the exploration sparked the analysis, which then sparked further exploration, and then more analysis. This sequence of behaviors looks very similar to the pattern of problem-solving behaviors of the expert mathematician that Schoenfeld (1985b) described in his study.

Framework and Type of Mathematical Problem

There is no question that the type of problem selected for the students to solve was an important variable in this study. Our banking problem was responsible for the high incidence of exploratory behaviors, because it lends itself to a trial-and-error approach. If the problem had been solvable through a more algorithmic approach, there probably would have been more evidence of systematic planning. In general, the type of problem used in this study affected the kinds of problem-solving approaches that might have been observed had a different problem been used. We acknowledge the important influence of context and content on problem-solving behavior, and we admit that the analysis of a single episode is a serious limitation of this study. The main purpose of this research, however, was to find out if the framework used for analyzing the behaviors of individuals as they worked on solving a mathematical problem in a small group showed evidence of being an effective tool. Once establishing that, we intend to apply it to multiple problems.

Study of Thought Processes

Researchers have begun to explore the small-group protocol (as opposed to a single-student or a "think-aloud" protocol) as a vehicle for studying mathematical problem solving, because observers are able to hear the thoughts of the students without interfering in the process (Hart, 1985; Noddings, 1982, 1985; Schoenfeld, 1985a; Silver, 1985). Unfortunately, there still exists a major difficulty in any research that aims to study thought processes. Even in a small group, thoughts are not always verbalized and, therefore, are not easily accessible to the observer. In this study, the only behaviors that could be categorized as metacognitive were those that were audible to the observers. If students were writing silently during episodes of exploration, implementation, and/or verification, their behaviors were categorized as cognitive. It is very possible that students were monitoring their work at these moments and that their metacognitive behaviors were overlooked. It is also possible that silent plans were overlooked as well.

Furthermore, when students work in small groups, they often watch and listen to one another. Because there is no verbalization at these times, it is impossible to assign a cognitive level to this activity. This problem adds an unknown variable to the results and has an effect on the percentages of reported cognitive and metacognitive behaviors. The stimulated-recall interviews were used to gain partial insight into these silent moments.

Compounding the methodological problem of coding was the theoretical complexity of the interplay of cognitive and metacognitive actions in problem solution. One source of the difficulty was identified by Brown (1978) as the difficulty of differentiating metacognitive from cognitive behaviors at both the theoretical and the empirical levels. And, as Sternberg (1985) pointed out, the difficulty is further compounded by the challenge of isolating the relative contributions of both cognitive and metacognitive actions to overall task performance.

CONCLUSION AND IMPLICATIONS

The purpose of this study was to examine the role of cognition and metacognition within the heuristic framework of mathematical problem solving in a small-group setting. A modification of Schoenfeld's framework was developed to delineate explicitly the type and level of processes used as individuals solved mathematical problems in small-group settings. Data analysis suggests the feasibility and usefulness of this framework for research of mathematical problem solving in small groups. With further study, the framework may be of pedagogical use to teachers.

The analysis of problem-solving behavior in the small-group protocols did provide some justification for the differentiation of metacognitive processes from cognitive processes. This is an important distinction with implications at both theoretical and practical levels. Different processes serve different important functions, and future research is needed to gain a better understanding of how interrelationships among processes affect the efficiency and effectiveness of problem solving. The framework also allowed an examination of the occurrence and frequency of heuristic episodes within small groups. The data suggest that, throughout the problem-solving session, students go back and forth using different heuristics intermittently. This behavior seems to play an important role in successful problem solving.

The stimulated-recall interviews provided insight about the attitudes students brought to their groups. Specifically, the attitudes of the high-ability group members manifested themselves in the subsequent behaviors of the group members. These results can provide important information to teachers and researchers who are interested in finding ways to maximize the effectiveness and efficiency of mathematical problem solving in small-group settings.

Most important, the framework shows promise as a powerful tool for the future study of individuals solving mathematical problems in a small-group setting. It can be used to study some of the key questions raised by this study:

1. What is the balance of cognitive, metacognitive, and watch-and-listen behaviors that is most favorable for productive group problem solving?
2. What is the balance of individual group members' contributions that is most favorable for productive group problem solving?

3. When is group problem solving really group problem solving? How should group problem solving be defined?
4. What role does the sequence of heuristic episodes play in the solution of a mathematical problem?
5. What effect do different types of problems have on the heuristic processes used in group problem solving?
6. What is the role of dialogue in small-group settings? More specifically, is there evidence of social scaffolding that occurs naturally in well-structured groups?

Research on questions such as these can begin to be addressed through the application of the framework developed here.

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APPENDIX

Cognitive-Metacognitive Framework for Protocol Analysis of Problem Solving in Mathematics

The following framework outlines the interactive relationship between metacognitive and cognitive processes in mathematical problem solving. The episodic categories are described theoretically and empirically. The level or levels of cognition associated with each category are indicated as well. Note that during the course of problem solving these episodes need not occur in the order listed, may occur several times, and may indeed be bypassed completely.

Episode 1: Reading the problem (cognitive)

Description: The student reads the problem.

Indicators: The student is observed as reading the problem or listening to someone else read the problem. The student may be reading the problem silently or aloud to the group.

Episode 2: Understanding the problem (metacognitive)

Description: The student considers domain-specific knowledge that is relevant to the problem. Domain-specific knowledge includes recognition of the linguistic, semantic, and schematic attributes of the problem in his or her own words and represents the problem in a different form.

Indicators: The student may be exhibiting any of the following behaviors: (a) restating the problem in his or her own words; (b) asking for clarification of the meaning of the problem; (c) representing the problem by writing the key facts or by making a diagram or list; (d) reminding himself or herself or others of the requirements of the problem, for example, "Remember, we must use the exact number that is asked for in the problem"; (e) stating or asking himself or herself whether he or she has done a similar problem in the past; and (f) discussing the presence or absence of important pieces of information.

Episode 3: Analyzing the problem (metacognitive)

Description: The student decomposes the problem into its basic elements and examines the implicit or explicit relations between the givens and goals of the problem.

Indicators: The student is engaging in an attempt to simplify or reformulate the problem. An attempt is made to select an appropriate perspective of the problem and to reformulate it in those terms. Examples of statements reflecting that such analysis is occurring are: "After you use all the given information, it becomes an easy problem of addition," and "Because the total is a multiple of five, I think the answer must be divisible by five."

Episode 4: Planning (metacognitive)

Description: The student selects steps for solving the problem and a strategy for combining them that might potentially lead to problem solution if implemented. The student may also select a representation for the information in the problem. In addition, the student may assess the status of the problem solution and make decisions for change if necessary.

Indicators: The student describes an approach that he or she intends to use to solve the problem. This may be in the form of steps to be taken or strategies to be used. Examples of statements that reflect planning include the following: "Let's use the given information first and see what the problem looks like after that"; "Let's work backwards by estimating an answer and see how it must be adjusted to fit the problem"; "Let's draw a chart and fill in the numbers"; "Let's think of a different way to go about this"; and "Let's check back to see where we went wrong."

Episode 5a: Exploring (cognitive)

Description: The student executes a trial-and-error strategy in an attempt to reduce the discrepancy between the givens and the goals.

Indicators: The student engages in a variety of calculations without any apparent structure to the work. There is no visible sequence to the operations performed by the student.

Episode 5b: Exploring (metacognitive)

Description: The student monitors the progress of his or her or others' attempted actions thus far and decides whether to terminate or continue working through the operations. This differs from analysis in that it is less well

structured, and it is further removed from the original problem. If one comes across new information during exploration, he or she may return to analysis in the hope of using that information to better understand the problem.
Indicators: (a) The student draws away from the problem to ask himself or herself or someone else what has been done during the exploration. Examples of such statements are: "What are you doing?" and "What am I doing?" (b) The student gives suggestions to other students about what to try next in the exploration. An example of such a comment is: "It's getting too big; try it with one less." (c) The student evaluates the status of the exploration. Examples of such statements are: "This isn't getting us anywhere," and "I think that's the answer!"

Episode 6a: Implementing (cognitive)

Description: The student executes a strategy that grows out of his or her understanding, analysis, and/or planning decisions and judgments. Unlike exploration, the student's actions are characterized by a quality of systematicity and deliberateness in transforming the givens into the goals of the problem.

Indicators: The student appears to be engaging in a coherent and well-structured series of calculations. There is evidence of an orderly procedure.

Episode 6b: Implementing (metacognitive)

Description: The student engages in the same kind of metacognitive process as in the exploring (metacognitive) phase of problem solving, monitoring the progress of his or her attempted actions. Unlike the exploratory phase, however, the metacognitive decisions build on, check, or revise those previously considered decisions. Furthermore, the student may consider a reallocation of his or her problem-solving resources, given the time constraint within which the problem must be solved.

Indicators: During the implementation phase, the student draws away from the work to see what has been done or where it is leading. The following examples of statements reflect this: "Okay, I used all the given conditions, and now I will start adding what is left"; "Wait. You forgot to use the second point"; and "This is taking too long. Try skipping the odd numbers."

Episode 7a: Verifying (cognitive)

Description: The student evaluates the outcome of the work by checking computational operations.

Indicators: The student redoes the computational operations he or she did before to check that it was done correctly.

Episode 7b: Verifying (metacognitive)

Description: The student evaluates the solution of the problem by judging whether the outcome reflected adequate problem understanding, analysis, planning, and/or implementation. Should the student discover a discrepancy in this comparison search, he or she engages in new decision making for correcting the faulty metacognitive and/or cognitive processing that led to the incorrect solution. The ability to adjust one's thinking on the basis of evaluative information is another indication of self-regulatory competence. Should the evaluation of problem solution indicate an adequacy of or congruence with metacognitive and cognitive processing, the mental reiteration ends.

Indicators: After the student has decided that the solution or part of the solution has been obtained, he or she may review the work in several ways: (a) The student checks the solution process to see whether it makes sense. For example, "When we simplified the problem, did we use all of the given information?" (b) The student checks to see if the solution satisfies the conditions of the problem. For example, "Does our answer satisfy both of the properties that were asked for?" (c) The student explains to a groupmate how the solution was obtained. For example, "I knew it had to be a big number, so I started with the largest numbers first."

Episode 8: Watching and listening (uncategorized)

Description: This category only pertains to students who are working with other people. The student is attending to the ideas and work of others.

Indicators: The student appears to be listening to a group member who is talking or watching a group member who is writing.