1. Find the root of $\log x = 1.2$ using newton raphson method.?

$F(x) = \log x - 1.2 = 0 \quad F'(x) = \frac{1}{x}$.

$F(0) = 0$, $F(1) = \bar{1}.2$, $F(2) = -0.899$

$F(3) = \bar{}0.723 \qquad F(4) = \bar{}0.598$

⋮

$F(15) = \bar{}0.024 \quad F(16) = 4.12 \times 10^{-3}$.

Take $x_0 = 16$.

$\underline{n=0}$ $x_1 = 16 - \frac{4.12 \times 10^{-3}}{0.0625} = 15.93408$

$\underline{n=1}$ $x_2 = 15.93408 - \frac{2.327 \times 10^{-3}}{0.0628}$

$= \underline{15.897}$

$\underline{n=2}$ $x_3 = 15.897 - \frac{1.3152 \times 10^{-3}}{0.0629}$.

$= 15.876$.

$\underline{n=3}$ $x_4 = 15.876 - 0.0118 = 15.8642$.

$\therefore$ approximate root is $\underline{15.876}$.

2. Find the root of $\cos x = xe^x$ using newton iteration method?

$F(x) = \cos x - xe^x = 0$

$F'(x) = -\sin x - xe^x - e^x = 0$

$F(0) = 1$, $F(1) = -2.178$, $F(2) = -15.19$

Take $x_0 = 2$.

$\underline{n=0}$, $x_1 = 0 - \frac{1}{-1} = 1$

$\underline{n=1}$, $x_2 = 1 - \frac{-2.1779}{-6.2780} = \underline{\underline{0.6531}}$

$\underline{n=2}$, $x_3 = 0.6531 - \frac{-0.4607}{-3.7841} = \underline{\underline{0.5314}}$

$\underline{n=3}$, $x_4 = 0.5314 - \frac{-0.0419}{-3.1121} = \underline{\underline{0.5179}}$

$n=4$, $x_5 = 0.5179 - \frac{-0.0486}{-3.2798} = 0.503$

approximate root $= \underline{\underline{0.52}}$

(3) Find the root of $x^3 - 2x - 5 = 0$ using NRM ?.

$F(x) = x^3 - 2x - 5 = 0.$

$F'(x) = 3x^2 - 2$

$F(0) = -5, \ F(1) = -6, \ F(2) = -1,$

$F(3) = 16.$

Take $x_0 = 3.$

$\underline{n=0}, \ x_1 = 3 - \frac{16}{25} = \underline{\underline{2.7975}}.$

$\underline{n=1}, \ x_2 = 2.7975 - \frac{11.2983}{21.478}$

$= \underline{\underline{2.2715}}.$

$\underline{n=2}, \ x_3 = 2.2715 - \frac{2.1773}{13.479}$

$= \underline{\underline{2.1099}}$

$n=3, \ x_4 = 2.1099 - \frac{0.1728}{11.355}$

$= 2.095$

$\therefore$ the approximate root $= 2.1$

Ⓐ Solve using Gauss Elimination Method.

$$2x + 3y - z = 5$$
$$4x + 4y - 3z = 3$$
$$2x - 3y + 2z = 2 \,?$$

$$[AB] = \begin{bmatrix} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ 2 & -3 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & -6 & 3 & -3 \end{bmatrix} \qquad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$= \begin{bmatrix} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 0 & 6 & 18 \end{bmatrix} \qquad R_3 \to R_3 - 3R_2$$

$$\begin{bmatrix} 2 & 3 & -1 & \cancel{5} \\ 0 & -2 & -1 & \cancel{-7} \\ 0 & 0 & 5 & \cancel{18} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 18 \end{bmatrix}$$

$$2x + 3y - z = 5$$
$$-2y - z = -7$$
$$6z = 18$$

$\Rightarrow x = 1,\ y = 2,\ z = 3$

⑤ Solve using gauss elimination method.

$3x + 4y + 5z = 18,\ 2x - y + 8z = 13,$
$5x - 2y + 7z = 20$?

$$[AB] = \begin{bmatrix} 3 & 4 & 5 & 18 \\ 2 & -1 & 8 & 13 \\ 5 & -2 & 7 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & 5 & 18 \\ 0 & -11 & 14 & 3 \\ 0 & -26 & -4 & -30 \end{bmatrix} \quad \begin{matrix} R_2 \to 3R_2 - 2R_1 \\ R_3 \to 3R_3 - 5R_1 \end{matrix}$$

$$= \begin{bmatrix} 3 & 4 & 5 & 18 \\ 0 & -11 & 14 & 3 \\ 0 & 0 & 408 & 408 \end{bmatrix} \quad R_3 \to -11R_3 - 26R_2$$

$$3x + 4y + 5z = 18.$$

$$-11y + 14z = 3.$$

$$408z = 408.$$

$\Rightarrow$

$$x = 3,\ y = 1,\ z = 1$$

⑥ Solve using Gauss Elimination method.

$$2x - y + 3z + \omega = 9.$$

$$3x + y - 4z + 3\omega = 3.$$

$$5x - 4y + 3z - 6\omega = 2.$$

$$x - 2y - z + 2\omega = -2$$

$$[AB] = \begin{bmatrix} 2 & -1 & 3 & 1 & 9 \\ 3 & 1 & -4 & 3 & 3 \\ 5 & -4 & 3 & -6 & 2 \\ 1 & -2 & -1 & 2 & -2 \end{bmatrix}$$

$$[AB] = \begin{bmatrix} 2 & -1 & 3 & 1 & 9 \\ 0 & 5 & -17 & 3 & -21 \\ 0 & -3 & -9 & -17 & -41 \\ 0 & -3 & -5 & 3 & -13 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 3 & 1 & 9 \\ 0 & 5 & -17 & 3 & -21 \\ 0 & 0 & -96 & -76 & -268 \\ 0 & 0 & -76 & 24 & -128 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 3 & 1 & 9 \\ 0 & 5 & -17 & 3 & -21 \\ 0 & 0 & -96 & -76 & -268 \\ 0 & 0 & 0 & -8080 & -8080 \end{bmatrix}$$

<u>Solve</u>

$$2x - y + 3z + \omega = 9$$

$$5y - 17z + 3\omega = -21$$

$$-96z - 76\omega = -268$$

$$-8080\omega = -8080$$

$$\underline{\omega = 1}$$

$$2x - y + 3z = 8.$$
$$5y - 17z = -24.$$
$$-96z = -192.$$
$$x = 2, \; y = 2, \; z = 2.$$

$\therefore x = 2, y = 2, z = 2, \omega = 1$

(7) Solve using Gauss Siedel Iteration method ?

$$2x + y + z = 4. \quad \text{--- (1)}$$
$$x + 2y + z = 4. \quad \text{--- (2)}$$
$$x + y + 2z = 4. \quad \text{--- (3)}$$

$$x = (4 - y - z)/2 \quad \text{--- (4)}$$

$$y = (4 - x - z)/2 \quad \text{--- (5)}$$

$$z = (4 - x - y)/2 \quad \text{--- (6)}$$

Approximation I

$x_1 = 2, \; y_1 = (4-2-0)/2 = \underline{1}$

$z_1 = \underline{\underline{0.5}}$

Approximation 2.

$x_2 = \dfrac{(4-1-0.5)}{2} = \underline{\underline{1.25}}$

$y_2 = (4-1.25-0.5)/2 = \underline{1.125}$

$z_2 = (4-1.25-1.125)/2 = \underline{\underline{0.8125}}$

Approximation 3

$x_3 = (4-1.125-0.8125)/2 = 1.0313$

$y_3 = (4-1.0313-1.125)/2 = 1.8437$

$z_3 = (4-1.0313-1.8437)/2 = 0.5685$

Approximation 4

$x_4 = (4-1.8437-0.5685)/2 = \underline{0.7969}$

$y_4 = (4-0.7969-0.5685)/2 = \underline{\underline{1.3203}}$

$z_4 = (4-0.7969-1.3203)/2 = \underline{\underline{1.8828}}$

Approximation 5

$x_5 = (4 - 1.3203 - 1.8828)/2 = 0.398$

$y_5 = (4 - 0.398 - 1.8828)/2 = 0.8596$

$z_5 = (4 - 0.398 - 0.8596)/2 = 1.3712$

Approximation 6

$x_6 = (4 - 0.8596 - 1.3712)/2 = 0.8846$

$y_6 = (4 - 0.8846 - 1.3712)/2 = 0.8721$

$z_6 = (4 - 0.8846 - 0.8721)/2 = 1.121$

Approximation 7

$x_7 = (4 - \overset{0.8721}{\cancel{0.8846}} - 1.121)/2 = 1.00$

$y_7 = (4 - 1 - 1.121)/2 = 0.9395$

$z_6 = (4 - 1 - 0.9395)/2 = 1.0$

Approximation 8

$x_8 = (4 - 0.9395 - 1)/2 = 1.0$

$y_8 = (4 - 1.0 - 1.0)/2 = 1.$

$$z_8 = (4-1-1)/_2 = 1$$

$$\Rightarrow x = 1,\ y = 1,\ z = 1$$

(8) Solve using Gauss Siedel Iteration method.

$$5x - y + z = 10$$
$$2x + 4y = 12$$
$$x + y + 5z = -1 \ ?$$

$$5x - y + z = 10 \quad \text{—(1)}$$
$$2x + 4y + 0z = 12 \quad \text{—(2)}$$
$$x + y + 5z = -1 \quad \text{—(3)}$$

$$x = (10 + y - z)/_5 \quad \text{—(4)}$$
$$y = \frac{12 - 2x}{4} \quad \text{—(5)}$$
$$z = (-1 - x - y)/_5 \quad \text{—(6)}$$

Approximation 1

$$x_1 = (10)/_5 = 2,\quad y_1 = \frac{12 - 2\times 2}{4} = 2$$

$z_1 = (-1-2-2)/5 = \underline{\underline{-1}}$

$x_1 = 2,\ y_1 = 2,\ z_1 = -1$

Approximation 2

$x_2 = (10 + y_1 - z_1)/5$
$= (10 + 2 + 1)/5 = \underline{\underline{2.6}}$

$y_2 = \dfrac{(12 - 2x_2)}{4} = \dfrac{(12 - 2\times 2.6)}{4}$
$= 1.7$

$z_2 = (-1 - x_2 - y_2)/5 = \dfrac{-1-2.6-1.7}{5}$
$= -1.06$

Approximation 3.

$x_3 = (10 + 1.7 + 1.06)/5 = 2.552.$

$y_3 = (12 - 2\times 2.552)/4 = 1.724.$

$z_3 = (-1 - 2.552 - 1.724)/5$
$= -1.0552$

$x_3 = 2.552,\ y_3 = 1.724,\ z_3 = -1.0052$

Approximation 4

$x_4 = (10 + 1.724 + 1.0052)/5$
$= \underline{2.546}$

$y_4 = (12 - 2 \times 2.546)/4 = \underline{\underline{1.727}}$

$z_4 = (-1 - 2.546 - 1.727)/5 = -1.\underline{\underline{055}}$

Approximation 5

$x_5 = (10 + 1.727 + 1.055)/5 = \underline{\underline{2.556}}$

$y_5 = (12 - 2(2.556))/4 = \underline{\underline{1.722}}$

$z_5 = (-1 - 2.556 - 1.722)/5 = -1.\underline{\underline{0556}}$

Approximation 6

$x_6 = (10 + 1.722 + 1.0556)/5 = \underline{\underline{2.55}}$

$y_6 = (12 - 2 \times 2.55)/4 = \underline{\underline{1.72}}$

$z_6 = (-1 - 2.55 - 1.72)/5 = \underline{\underline{-1.05}}$

$\Rightarrow x = 2.55,\ y = 1.72,\ z = -1.05$

(9) A third degree polynomial passes through the points $(0,-1), (1,1), (2,1)$ & $(3,-2)$. Find the polynomial using NFIM and find the value @ 1.5 ?.

x	0	1	2	3
y	-1	1	1	-2.

Value of x	value of y	Ist diff.	2nd diff.	3rd diff.
$0 = x_0$	$y_0 = -1$			
		$\Delta y_0 = 2$.		
$1 = x_1$	$y_1 = 1$		$\Delta^2 y_0 = -2$.	
		$\Delta y_1 = 0$		$\Delta^3 y_0 = -1$.
$2 = x_2$	$y_2 = 1$		$\Delta^2 y_1 = -3$	
		$\Delta y_2 = -3$		
$3 = x_3$	$y_3 = -2$.			

$x = x_0 + ph.$ $\quad h = 1,\ x_0 = 0.$

$$p = \frac{x - x_0}{h} = \frac{x}{1} \Rightarrow \underline{p = x}.$$

$$y(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$f(x) = -1 + (x)(2) + \frac{(x)(x-1)(-2)}{2!} + \frac{x(x-1)(x-2)}{3!}(-1)$$

$$\Rightarrow f(x) = -1 + 2x - (x^2 - x) - \frac{(x^2 - x)(x-2)}{3!}$$

$$= -1 + 2x - \frac{2x^2 + 2x}{2} - \frac{(x^3 - x^2 - 2x^2 + 2x)}{6}$$

$$= (-x^3 - 3x^2 + 16x - 6)\frac{1}{6}$$

$$= \frac{-1}{6}(x^3 + 3x^2 - 16x + 6)$$

$f(x)$ when $x = 1.5 = \underline{1.3125}$

(10) Using NFIM, find the no. of students whose weight is b/w 60 & 70, from the following table of data?

wt. in lbs	0-40	40-60	60-80	80-100	100-120
no. of students	250	120	100	70	50

The corresponding cumulative frequency table is.

wt. (x)	40	60	80	100	120
no. of students (y)	250	370	470	540	590

x	y	1st diff	2nd diff	3rd diff	4th diff
40	250				
		$\Delta y_0 = 120$			
60	370		$\Delta^2 y_0 = -20$		
		$\Delta y_1 = 100$		$\Delta^3 y_0 = -10$	
80	470		$\Delta^2 y_1 = -30$		$\Delta^4 y_0 = 20$
		$\Delta y_2 = 70$		$\Delta^3 y_1 = 10$	
100	540		$\Delta^2 y_3 = -20$		
		$\Delta y_3 = 50$			
120	590				

$x = x_0 + ph \quad h = 20 \quad x_0 = 40$

$x = 70$

$$p = \frac{x - x_0}{h} = \frac{70 - 40}{20}$$

$$= \frac{30}{20} = 1.5$$

$$y(x) = 250 + (1.5)(120) + \frac{(1.5)(0.5)(-20)}{2} + \frac{(1.5)(0.5)(-0.5)(-10)}{3!} + \frac{(1.5)(0.5)(-0.5)(-1.5)(2}{4!}$$

$= 250 + 180 + -7.5 + 0.625 + 0.46875$

$= 423.594.$

No. of people with wt. less than 70 lbs is 424. Since the no. of people with wt. less than 60 lbs is 370, no. of people with wt. in b/w 60 & 70 is $424 - 370 = \underline{54}$

(11) Find the 3rd degree polynomial passing through (3,0), (4,24), (5,60) & (6,120) using NBIF?

x	y	1st diff.	2nd diff.	3rd diff.
3	0			
		$\nabla y_1 = 24$		
4	24		$\nabla^2 y_1 = 12$	
		$\nabla y_2 = 36$		$\nabla^3 y_1 = 12$
5	60		$\nabla^2 y_2 = 24$	
		$\nabla y_3 = 60$		
6	120			

$x = x_n + ph$ $\quad p = \underline{\underline{x-6}}$

$$y(x) = 120 + (x-6)60 + (x-6)(x-5)\frac{24}{2} +$$

$$\frac{(x-6)(x-5)(x-4)}{6}(12)$$

$$= 120 + 60x - 360 + (x^2 - 11x + 30)12 +$$

$$(x^2 - 11x + 30)(x-4)(2)$$

$$= -240 + 60x + 12x^2 - 132x + 360 +$$

$$(x^3 - 11x^2 + 30x - 4x^2 + 44x - 120)2$$

$$= 120 - 72x + 12x^2 + (x^3 + 15x^2 - 74x - 120)2$$

$$= 120 - 72x + 12x^2 + 2x^3 + 30x^2 - 148x - 240$$

$=$

$=$

$$2x^3 - 18x^2 + 78x - 120$$

(12) Find the cubic polynomial which takes the following values & hence evaluate F(4)?

x	0	1	2	3
y	1	2	1	10

x	y	1st diff.	2nd diff.	3rd diff.
0	1			
1	2	$\nabla y_1 = 1$	$\nabla^2 y_1 = -2$	
2	1	$\nabla y_2 = -1$	$\nabla y_2 = 10$	$\nabla^3 y_1 = 12$
3	10	$\nabla y_3 = 9$		

$x = x_n + ph \Rightarrow p = x - 3$

$$y(x) = 10 + (x- \ )9 + \frac{(x- \ )(x- \ )}{\cancel{2}}\cancel{10}^{5} + \frac{(x- \ )(x- \ )(x- \ )}{\cancel{6}}\cancel{(12)}^{2}$$

$= 10 + 9x - 90 + 5(x^2 - 19x + 90) + (x^2 - 19x + 90)(x-8)\,2$

$= 9x - 80 + 5x^2 - 95x + 450 + 2(x^3 - 19x^2 + 90x - 8x^2 + 152x - 720)$

$= 5x^2 - 86x + 370 + 2(x^3 - 27x^2 + 242x - 720)$

$$= 5x^2 - 86x + 370 + 2x^3 - 54x^2 - 484x - 1440.$$

$$= 2x^3 - 49x^2 - 570x - 1070.$$

$$F(x) = 2x^3 - 7x^2 + 6x + 1.$$

~~$F(4) = -4006$~~ $F(4) = \underline{\underline{41}}$

(13). Using Legrange's formula, find the 3rd degree polynomial satisfying the following data.

x	1 x_0	3 x_1	5 x_2	7 x_3
y	24 y_0	120 y_1	336 y_2	720 y_3

?.

$$F(x) = \frac{(x-3)(x-5)(x-7)}{(1-3)(1-5)(1-7)}(24) +$$

$$\frac{(x-1)(x-5)(x-7)}{(3-1)(3-5)(3-7)}(120) + \frac{(x-1)(x-3)(x-7)}{(5-1)(5-3)(5-7)}(336)$$

$$+ \frac{(x-1)(x-3)(x-5)}{(7-1)(7-3)(7-5)}(720)$$

$$= (x-3)(x-5)(x-7)(-1/2) +$$
$$(x-1)(x-5)(x-7)(15/2) +$$
$$(x-1)(x-3)(x-5)(15) + (x-1)(x-3)(x-7)(-21)$$

$$= (x^2 - 8x + 15)(x-7)(-0.5) +$$
$$(x^2 - 6x + 5)(x-7)(7.5) +.$$

$(x^2 - 4x + 3)(x-5)(15)$.

$= (x^3 - 8x^2 + 15x - 7x^2 + 56x - 105)(-0.5) +$
$(x^3 - 6x^2 + 5x - 7x^2 + 42x - 35)(7.5) +$
$(x^3 - 4x^2 + 3x - 5x^2 + 20x - 15)(15)$

$= (x^3 - 15x^2 + 71x - 105)(-0.5) +$
$(x^3 - 13x^2 + 47x - 35)(7.5) +$
$(x^3 - 9x^2 + 23x - 15)(15)$

$= x^3(-0.5 + 7.5 + 15) +$
$x^2(-15 \times -0.5 + -13 \times 7.5 + -9 \times 15) +$
$x(71 \times -0.5 + 47 \times 7.5 + 23 \times 15) +$
$(-105 \times -0.5 + -35 \times 7.5 + -15 \times 15) +$

$= -21x^3 + 189x^2 - 483x + 315 +$
$22x^3 - 225x^2 + 662x - 435$

$= x^3 - 36x^2 + 179x - 120$.

(14) Find the polynomial $F(x)$ using Legrange's formula and hence find $f(3)$ for

x	0 (x_0)	1 (x_1)	2 (x_2)	5 (x_3)
y	2 (y_0)	3 (y_1)	12 (y_2)	147 (y_3)

?

$F(x) = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) +$

$$\frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3)+\frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12)+$$

$$\frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}147$$

$$= (x-1)(x-2)(x-5)(-0.2)+$$
$$(x-0)(x-2)(x-5)(0.75)+$$
$$(x-0)(x-1)(x-5)(-2)+$$
$$(x-0)(x-1)(x-2)\quad(2.45)$$

$$= (x^2-3x+2)(x-5)(-0.2)+$$
$$(x^2-2x)(x-5)(0.75)+(x^2-x)(x-5)(-2)+$$
$$(x^2-x)(x-2)\quad(2.45)$$

$$= (x^3-\underline{3x^2}+2x-\underline{5x^2}+15x-10)(0.2)+$$
$$(x^3-2x^2-5x^2+10x)(0.75)+$$
$$(x^3-x^2-5x^2+5x)(-2)+$$
$$(x^3-x^2-2x^2+2x)\quad(2.45)$$

$$= (x^3-8x^2+17x-10)(0.2)+$$
$$(x^3-7x^2+10x)(0.75)+$$
$$(x^3-6x^2+5x)(-2)+$$
$$(x^3-3x^2+2x)\quad(2.45)$$

$= x^3(0.2 + 0.75 + -2 + 2.45) +$

$x^2(-8 \times 0.2 + -7 \times 0.75 + -6 \times -2 + -3 \times 2.45) + x(17 \times 0.2 + 10 \times 0.75 + 5 \times -2 + 2 \times 2.45) + (-10 \times 0.2)$

$= 1.4x^3 - 2.2x^2 + 5.8x - 2$

$\therefore F(3) = \underline{\underline{33.4}}$

(15) From the following table, find the value of x corresponding to $y = 12$ using Legrange's formula of inverse interpolation?

x	1.2	2.1	2.8	4.1	4.9	6.2
y	4.2	6.8	9.8	13.4	15.5	19.6

$F(y) = F(12) =$

$$\frac{(12-6.8)(12-9.8)(12-13.4)(12-15.5)(12-19.6)}{(4.2-6.8)(4.2-9.8)(4.2-13.4)(4.2-15.5)(4.2-19.6)}(1.2)$$

$$+ \frac{(12-4.2)(12-9.8)(12-13.4)(12-15.5)(12-19.6)}{(6.8-4.2)(6.8-9.8)(6.8-13.4)(6.8-15.5)(6.8-19.6)}(2.1)$$

$$+ \frac{(12-4.2)(12-6.8)(12-13.4)(12-15.5)(12-19.6)}{(9.8-4.2)(9.8-6.8)(9.8-13.4)(9.8-15.5)(9.8-19.6)}(2.8)$$

$+$

$$\frac{(12-4.2)(12-6.8)(12-9.8)(12-15.5)(12-19.6)}{(13.4-4.2)(13.4-6.8)(13.4-9.8)(13.4-15.5)(13.4-19.6)}(4.1)$$

$$+\frac{(12-4.2)(12-6.8)(12-9.8)(12-13.4)(12-19.6)}{(15.5-4.2)(15.5-6.8)(15.5-9.8)(15.5-13.4)(15.5-19.6)}(4.9)$$

$$+\frac{(12-4.2)(12-6.8)(12-9.8)(12-13.4)(12-15.5)}{(19.6-4.2)(19.6-6.8)(19.6-9.8)(19.6-13.4)(19.6-15.5)}(6.2)$$

0.4471

$= 0.0227 + -0.2341 + \quad + \; 3.4193 + -0.9642 + 0.0552.$

$= 2.7453$

(16) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ with $h = \frac{1}{6}$ using trapezoidal rule? $f(x) = \frac{1}{1+x^2}$

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
y	1	0.973	0.9	0.8	0.6923	0.5902	0.5

$$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{6}\times\frac{1}{2}\left\{1+0.5+2(0.973+0.9+0.8+0.6923+0.5902)\right\}.$$

$$= \frac{1}{12}\left\{1.5+7.911\right\} = \underline{\underline{0.7843}}$$

(17) Evaluate $\int_0^1 xe^x dx$ using Simpson's rule, with $n=4$. Compare the result with actual value.

$f(x) = xe^x$

$h = \frac{1-0}{4} = \underline{\underline{0.25}}$

x	0	0.25	0.5	0.75	1
y	0 y_0	0.321 y_1	0.8244 y_2	1.5878 y_3	2.7183 y_4

$$\int_0^1 xe^x dx = \frac{0.25}{3}\left\{2.7183 + 2\times(0.8244) + 4\times(0.321+1.5878)\right\}.$$

$$= \frac{0.25}{3}\left\{2.7183 + 1.6488 + 7.6352\right\}.$$

$= \cancel{1.4532}$ 1.0002

$$\int_0^1 xe^x\,dx = xe^x - \int e^x\,dx$$

$$= [xe^x - e^x]_0^1 =$$

$$= e^1 - e^1 + e^0 = 1$$

(18) Evaluate $\int_0^{\pi/2} \sqrt{\sin x}\,dx$ using trapezoidal rule?

take $n = 4$. $h = \frac{\pi/2}{4} = \frac{\pi}{2} \times \frac{1}{4} = \frac{\pi}{8}$

$F(x) = \sqrt{\sin x}$

x	0	$\pi/8$	$2\pi/8$	$3\pi/8$	$4\pi/8$
y	0	0.6186	0.8409	0.9612	1

$$\int_0^{\pi/2} \sqrt{\sin x}\,dx = \frac{\pi}{8} \cdot \frac{1}{2} \{0 + 1 + 2 \times (0.6186 + 0.8409 + 0.9612)\}$$

$$= \frac{\pi}{16}(1 + 4.8418) = 1.147$$

(19) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by (1) trapezoidal rule (2) Simpson's rule
(3) verify the results by actual integration.

Take $n=5$.

$h = \frac{6}{5} = 1.2$

x	0	1.2	2.4	3.6	4.8	6.0
y	1 (y_0)	0.4098 (y_1)	0.1479 (y_2)	0.07163 (y_3)	0.0416 (y_4)	0.027 (y_5)

(1) Trapezoidal Rule :-

$$\int_0^6 \frac{dx}{1+x^2} = \frac{1.2}{2}\left\{1+0.027+2\left(0.4098+0.1479+0.07163+0.0416\right)\right\}$$

$= 1.421316$

(2) Simpson's Rule :-

$$\int_0^6 \frac{dx}{1+x^2} = \frac{1.2}{3}\left\{1+0.027+2\times(0.1479+0.0416)+4\times(0.4098+0.07163)\right\}$$

$$= \frac{1.2}{3}\left\{1.9257 + 0.379 + \cancel{2.03372}\ 1.027\right\}.$$

$$= \cancel{1.3779}\ \underline{1.59}$$

(3) Actual Integration :-

$$\int_0^6 \frac{dx}{1+x^2} = \left(\tan^{-1} x\right)_0^6$$

$$= \tan^{-1}(6) - \tan^{-1}(0)$$

$$= \underline{1.4056}$$

(20) Fit a parabola of the form $y = a + bx^2$ to the following data

x	1	2	3	4	5
y	0.43	0.83	1.4	2.33	3.42

$y = a + bx^2$ — (1) Let $X = x^2$

The normal equations corresponding to (1) are $\Sigma y = na + b\Sigma X$.

$$\Sigma Xy = a\Sigma X + b\Sigma X^2$$

From the following table, we get,

$\Sigma X = 55$, $\Sigma Xy = 139.13$,

$\Sigma y = 8.41$, $\Sigma X^2 = 979$.

x	y	xy	$x^2 = X$	$x^4 = X^2$
1	0.43	0.43	1	1
2	0.83	3.32	4	16
3	1.4	12.6	9	81
4	2.33	37.28	16	256
5	3.42	85.5	25	625

Hence the normal equations becomes

$$8.41 = 5a + 55b$$

$$139.13 = 55a + 979b$$

$\Rightarrow a = 0.3108$ and
$b = 0.1247$

Hence the required parabola is $y = \underline{0.3108 + 0.1247\, x^2}$

Q1) Fit a parabola by the method of least squares to the following data.

x	1	2	3	4	5
y	5	12	26	60	97

Let the equation of the parabola be of the form $y=a+bx+cx^2$. Then, the corresponding normal equations are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = \Sigma x(a) + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2 y = \Sigma x^2 a + b\Sigma x^3 + c\Sigma x^4.$$

x	y	x^2	x^3	x^4	xy	x^2y
1	5	1	1	1	5	5
2	12	4	8	16	24	48
3	26	9	27	81	78	234
4	60	16	64	256	240	960
5	97	25	125	625	485	2425

$\Sigma x = 15$, $\Sigma x^2 = 55$, $\Sigma x^3 = 225$,

$\Sigma x^4 = 979$, $\Sigma y = 200$, $\Sigma xy = 832$,

$\Sigma x^2 y = 3672$, $n = 5$.

Hence the normal equations changes to

$$200 = 5a + 15b + 55c$$
$$832 = 15a + 55b + 225c$$
$$3672 = 55a + 225b + 979c.$$

On solving the above eqtns, we get, $a = 10.4$, $b = -11.086$ and $c = 5.7143$

Hence the required parabola is

$$y = 10.4 - 11.086x + 5.7143x^2$$

(22) Use the principle of least squares to fit a straight line to the following data

x	0	5	10	15	20
y	7	11	16	20	26

?

Let the eqtn. of the required line be of the form $y = ax + b$.

Then, the corresponding normal equations are;

$$\Sigma y = a\Sigma x + nb.$$

$$\Sigma xy = a\Sigma x^2 + b\Sigma x$$

From the following table, we get, $\Sigma x = 50$. $\Sigma y = 80$.

$\Sigma x^2 = 750$.

$\Sigma xy = 1035$ $n = 5$.

x	y	x^2	xy
0	7	0	0
5	11	25	55
10	16	100	160
15	20	225	300
20	26	400	520

Hence the normal eqtns change to;

$80 = 50a + 5b$

$1035 = 750a + 50b$ on solving these eqtn, we get, $a = 0.94$, $b = 6.6$

Hence the required line is

$$\underline{\underline{y = 0.94x + 6.6}}$$

Q(23) Find the coefficient of relation for the following table.

x	10	14	18	22	26	30
y	18	12	24	6	30	36

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\cdot\sqrt{n\sum y^2 - (\sum y)^2}}$$

x	y	xy	x^2	y^2
10	18	180	100	324
14	12	168	196	144
18	24	432	324	576
22	6	132	484	36
26	30	780	676	900
30	36	1080	900	1296
120	126	2772	2680	3276

$n = 6$.

$$r = \frac{6 \times 2772 - 120 \times 126}{\sqrt{6 \times 2680 - (120)^2}\cdot\sqrt{6 \times 3276 - (126)^2}}$$

$$= \frac{1512}{40.988 \times 61.482} = \underline{0.5999}$$

Q (24). From the following data, obtain the 2 regression equations

x	6	2	10	4	8
y	9	11	5	8	7

x	y	$u = x - \bar{x}$	$v = y - \bar{y}$	u^2	v^2	uv
6	9	0	1	0	1	0
2	11	-4	3	16	9	-12
10	5	4	-3	16	9	-12
4	8	-2	0	4	0	0
8	7	2	-1	4	1	-2
30	40	0	0	40	20	-26

$$\bar{y} = \frac{9+11+5+8+7}{5} = 8$$

$$\bar{x} = \frac{6+2+10+4+8}{5} = \frac{30}{5} = 6$$

$$b_{xy} = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$

$$b_{uv} = \frac{n\sum uv - (\sum u)(\sum v)}{n\sum v^2 - (\sum v)^2}$$

$$= \frac{5 \times -26 - 0}{5 \times 20} = \underline{\underline{-1.3}} = b_{xy}$$

$$b_{vu} = \frac{n\sum uv - (\sum u)(\sum v)}{n\sum u^2 - (\sum u)^2}$$

$$= \frac{5 \times -26}{5 \times 40} = \underline{\underline{-0.65}} = b_{yx}$$

Regression line of x on y is

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$= r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$$x - 6 = -1.3(y - 8)$$

$$x - 6 = -1.3y + 10.4$$

$$\underline{\underline{x = -1.3y + 16.4}}$$

Regression line of y on x is

$$y-\bar{y} = b_{yx}(x-\bar{x}) = r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$$

$$y-8 = -0.65(x-6)$$

$$y = 8+0.65\times 6 - 0.65x.$$

$$\Rightarrow y = 11.9 - 0.65x$$

(25) From the following pair of regression lines, find $\bar{x}$, $\bar{y}$ and r,

$$5x-6y+90=0.$$
$$15x-8y-130=0.$$

consider the 2 lines of regression $5x-6y+90=0$ & $15x-8y-130=0$ that passes through $(\bar{x},\bar{y})$.

then

$$5\bar{x}-6\bar{y}+90=0$$
$$15\bar{x}-8\bar{y}-130=0.$$

on solving the above equations we get, $\bar{x}=30$, $\bar{y}=40$.

The equations of the lines of regression are :-

$$5x - 6y + 90 = 0.$$

$$6y = 5x + 90.$$

$$y = \frac{5}{6}x + \frac{90}{6}. \quad —(i)$$

$$\Rightarrow b_{yx} = \frac{5}{6}$$

Illy $15x - 8y - 130 = 0.$

$$15x = 8y + 130$$

$$x = \frac{8}{15}y + \frac{130}{15}$$

$$\Rightarrow b_{xy} = \frac{8}{15}.$$

$$\cancel{r^2 = b_{xy} b_{yx}}$$

$$b_{xy} \cdot b_{yx} = r\frac{\sigma_x}{\sigma_y} \cdot r\frac{\sigma_y}{\sigma_x} = r^2 \cdot \frac{\sigma_x}{\sigma_y} \cdot \frac{\sigma_y}{\sigma_x}$$

$$\Rightarrow r^2 = b_{xy} b_{yx} = \frac{8}{15} \cdot \frac{5}{6} = 0.44$$

$$\Rightarrow r = 0.667$$

(26) From the following data, find
I. the 2 regression eqtns
II. coefficient of correlation b/w. the marks in economics & statistics

marks in econo. (x)	25	28	35	32	31	36	29	38	34	32
mark in stats (y)	43	46	49	41	36	32	31	30	33	39

$\bar{x} = 32$

$\bar{y} = 38$

x	y	$u = x - \bar{x}$	$v = y - \bar{y}$	u^2	v^2	uv
25	43	-7	5	49	25	-35
28	46	-4	8	16	64	-32
35	49	3	11	9	121	33
32	41	0	3	0	9	0
31	36	-1	-2	1	4	2
36	32	4	-6	16	36	-24
29	31	-3	-7	9	49	21
38	30	6	-8	36	64	-48
34	33	2	-5	4	25	-10
32	39	0	1	0	1	0

$\Sigma x = 320$, $\Sigma y = 380$.

$\Sigma u = 0$, $\Sigma v = 0$, $\Sigma v^2 = 398$

$\Sigma uv = -93$, $\Sigma u^2 = 140$

$$r\frac{\sigma_x}{\sigma_y} = b_{xy} = b_{uv} = \frac{n\Sigma uv - \Sigma u \cdot \Sigma v}{n\Sigma v^2 - (\Sigma v)^2}$$

$$= \frac{10(-93)}{10(398)} = \frac{-93}{398} = -0.2337$$

$$r\frac{\sigma_y}{\sigma_x} = b_{yx} = b_{vu} = \frac{n\Sigma vu - \Sigma v \cdot \Sigma u}{n\Sigma u^2 - (\Sigma u)^2}$$

$$= \frac{10\cdot(-93)}{10(140)} = \frac{-93}{140} = -0.6643$$

$$r\frac{\sigma_x}{\sigma_y} \cdot r\frac{\sigma_y}{\sigma_x} = b_{xy} \cdot b_{yx}$$

$$\Rightarrow r^2 = \frac{-93}{398} \times \frac{-93}{140}$$

$$\Rightarrow r = 0.394$$

Here the coefficient of correlation b/w x and y is 0.394.

line of regression of x on y is

$$x-\bar{x} = b_{xy}(y-\bar{y})$$

$$x-32 = -0.2337(y-38)$$

$$\Rightarrow x = 32 - 0.2337 \times -38 - 0.2337y$$

$$\Rightarrow x = 40.8806 - 0.2337y \quad \text{--- (1)}$$

line of regression of y on x is

$$y-\bar{y} = b_{yx}(x-\bar{x})$$

$$y-38 = -0.6643(x-32)$$

$$\Rightarrow y = 38 + 0.6643 \times 32 - 0.6643x$$

$$\Rightarrow y = 59.2576 - 0.6643x \quad \text{--- (2)}$$

(27) From the following data, find the most likely value of x when $x = 24$ for $r = 0.58$.

	y	x
mean	965.8	18.1
SD	36.4	2.0

Given: $r = 0.58$, $\bar{y} = 965.8$.

$\bar{x} = 18.1$, $\sigma_x = 2$, $\sigma_y = 36.4$.

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = (0.58)\left(\frac{2}{36.4}\right)$$

$$= \underline{\underline{0.319}}$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = (0.58)\left(\frac{36.4}{2}\right)$$

$$= \underline{\underline{10.556}}$$

line of regression of x on y is

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

ie $x - 18.1 = 0.319(y - 965.8)$

$\Rightarrow x = 0.319y - 0.319 \times 965.8 + 18.1$

$\Rightarrow \underline{\underline{x = 0.319y - 12.678}}$ — (1)

line of regression of y on x is

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

ie $y - 965.8 = 10.556(x - 18.1)$

$\Rightarrow y = 10.556x - 10.556 \times 18.1 + 965.8$

$\Rightarrow y = 10.556x + 774.7364$

the value of y corresponding to $x = 24$ is;

$y = 10.556(24) + 774.7364$

$= 1028.0804$

(28) From the following data, find y when $x = 45$, $n = 10$ and $\Sigma(x-\bar{x})(y-\bar{y}) = 1220$.

	x	y
mean	53	142
SD	130	165

?

Given: $\bar{x} = 53$, $\bar{y} = 142$, $\sigma_y = 165$

$\sigma_x = 130$, $n = 10$.

$$r = \frac{\sigma_{xy}}{\sigma_x \cdot \sigma_y} = \frac{\frac{1}{n}\Sigma(x-\bar{x})(y-\bar{y})}{130 \times 165}$$

$$r = \frac{\frac{1220}{10}}{130 \times 165} = \frac{122}{130 \times 165} = 5.688 \times 10^{-3}$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = 5.688\times10^{-3}\times\frac{130}{165}$$

$$= 4.4815\times10^{-3}$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = 5.688\times10^{-3}\times\frac{165}{130}$$

$$= 7.2194\times10^{-3}$$

Regression line of x on y is

$$x-\bar{x} = b_{xy}(y-\bar{y})$$

$$x-53 = 4.4815\times10^{-3}(y-142)$$

$$\Rightarrow x = 4.4815\times10^{-3}y + 52.364.$$

Regression line of y on x is.

$$y-\bar{y} = b_{yx}(x-\bar{x})$$

$$y-142 = 7.2194\times10^{-3}(x-53).$$

$$\Rightarrow y = 7.2194\times10^{-3}x + 141.6174$$

Hence the value of y when $x = 45$ is

$$y = 7.2194\times10^{-3}\times45 + 141.6174$$

$$= 141.9423$$

29. Calculate the rank correlation coefficient for the following data.

Ranks in x:	1	2	3	4	5	6	7
Ranks in y:	4	3	1	2	6	5	7

R_x	R_y.	$d = R_x - R_y$	d^2.
1	4	-3	9
2	3	-1	1
3	1	2	4
4	2	2	4
5	6	-1	1
6	5	1	1
7	7	0	0

$\Sigma d^2 = 9+1+4+4+1+1+0 = 20$.

rank correlation coefficient : $\gamma = 1 - \dfrac{6 \Sigma d_i}{n(n^2-1)}$

$n = 7 \Rightarrow \gamma = 1 - \dfrac{6 \times 20}{7(49-1)}$

$r = 1 - 0.3571 = \underline{0.6429}$

(30) Find the rank correlation coeff. for the following data.

x: 52 53 42 60 45 41 37 38 25 2

y: 65 68 43 38 77 48 35 30 25 5

(31) Find the rank correlation coeff. for the following data.

x: 45 56 39 54 45 40 56 60 30 36

y: 40 36 30 44 36 32 45 42 20 36

(30)

x	y	R_x	R_y	d.	d^2.
52	65	3	3	0	0
53	68	2	2	0	0
42	43	5	6	-1	1
60	38	1	7	-6	36
45	77	4	1	3	9
41	48	6	5	1	1
37	35	8	8	0	0
38	30	7	9	-2	4
25	25	10	10	0	0
27	50	9	4	5	25

$\Sigma d^2 = 76 \quad n = 10$

$$r = 1 - \frac{6 \times \Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 76}{10(100-1)}$$

$$= 1 - 0.4606 = \underline{\underline{0.5394}}$$

(31) In x-series, 56 appears 2 times, its common rank is $\frac{2+3}{2} = 2.5$. ||ly 45 repeats twice, hence its common rank is $\frac{5+6}{2} = 5.5$.

In y-series, 36 repeats thrice, hence, its common rank is $\frac{5+6+7}{3} = 6$.

$$CF(56) = \frac{m(m^2-1)}{12} = \frac{2(4-1)}{12} = \frac{1}{2}.$$

$$CF(45) = \frac{m(m^2-1)}{12} = \frac{2(4-1)}{12} = \frac{1}{2}$$

x	y	R_x	R_y	d	d^2
45	40	5.5	4	1.5	2.25
56	36	2.5	6	-3.5	12.25
39	30	8	8	0	0
54	44	4	2	2	4
45	36	5.5	6	-0.5	0.25
40	32	7	7	0	0
56	45	2.5	1	1.5	2.25
60	42	1	3	-2	4
30	20	10	9	1	1
36	36	9	6	3	9

$$CF(36) = \frac{3(9-1)}{12} = \underline{\underline{2}}$$

$$\Sigma d^2 = 35$$

$$r = 1 - \frac{6 \times (\Sigma d + CF(s))}{n(n^2-1)}$$

$$= 1 - \frac{6 \times (35 + 2 + 0.5 + 0.5)}{10(99)}$$

$$= 1 - 0.2303 = \underline{\underline{0.7697}}$$

(32) The following table represents the joint probability distribution of X and Y. Find (i) $P(X \le 1)$ (ii) $P(Y \le 3)$ (iii) $P(X \le 1, Y \le 3)$ (iv) $P(X \le 1 / Y \le 3)$ (v) $P(Y \le 3 / X \le 1)$ (vi) $P(X + Y \le 4)$.

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

(33)

X \ Y	1	2	3
1	1/12	0	1/18
2	1/6	1/9	1/4
3	0	1/5	2/15

The above table represents the joint distribution of X and Y. Find the marginal & conditional distribution.

(32) $P(X \leq 1) = P(X=0) + P(X=1)$

$$= \sum_{j=1}^{6} P_{0j} + \sum_{j=1}^{6} P_{1j}$$

$$= P_{01} + P_{02} + P_{03} + P_{04} + P_{04} + P_{05} + P_{06} +$$
$$P_{11} + P_{12} + P_{13} + P_{14} + P_{15} + P_{16}.$$

$$= 0 + 0 + 1/32 + 2/32 + 2/32 + 3/32 +$$
$$1/16 + 1/16 + 1/8 + 1/8 + 1/8 + 1/8$$

$$= \frac{7}{8} = \underline{\underline{0.875}}$$

$P(Y \leq 3) = P(Y=1) + P(Y=2) + P(Y=3)$

$$= \sum_{i=0}^{2} P_{i1} + \sum_{i=0}^{2} P_{i2} + \sum_{i=0}^{2} P_{i3}$$

$$= 0 + 1/16 + 1/32 + 0 + 1/16 + 1/32 +$$
$$1/32 + 1/8 + 1/64.$$

$$= \frac{23}{64} = \underline{\underline{0.3594}}$$

$$P(X \le 1, Y \le 3) = \sum_{i=0}^{1} \sum_{j=1}^{3} P_{ij}$$

$$= P_{01} + P_{02} + P_{03} + P_{11} + P_{12} + P_{13}$$

$$= 0 + 0 + 1/32 + 1/16 + 1/16 + 1/8$$

$$= 9/32 = \underline{0.28125}$$

$$P(X \le 1 / Y \le 3) = \frac{P(X \le 1, Y \le 3)}{P(Y \le 3)}$$

$$= \frac{9/32}{23/64} = \frac{18}{23} = \underline{\underline{0.7826}}$$

$$P(Y \le 3 / X \le 1) = \frac{P(X \le 1, Y \le 3)}{P(X \le 1)}$$

$$= \frac{9/32}{7/8} = \frac{9}{28} = \underline{0.32143}$$

$$P(X + Y \le 4) = P_{01} + P_{02} + P_{03} + P_{04} +$$

$$P_{11} + P_{12} + P_{13} + P_{21} + P_{22}$$

$$= 13/32 = \underline{0.40625}$$

33: marginal distribution of $X = f_X(x)$

X	1	2	3
$f_X(x)$	$\frac{5}{36}$	$\frac{19}{36}$	$\frac{1}{3}$

marginal distribution of $Y = f_Y(y)$

Y	1	2	3
$f_Y(y)$	$\frac{1}{4}$	$\frac{14}{45}$	$\frac{79}{180}$

$$\sum_{i=1}^{3} \sum_{j=1}^{3} P_{ij} = f(x,y)$$

$=$

$f(X/Y) =$

$X \backslash Y$	1	2	3
1	$1/3$	0	$10/79$
2	$2/3$	$5/14$	$45/79$
3	0	$9/14$	$24/79$

$f(y/x)$

$x \backslash y$	1	2	3
1	3/5	0	2/5
2	6/19	4/19	9/19
3	0	3/5	2/5

(34) The joint pdf of X & Y is given by

$$f(x,y) = \begin{cases} x+y, & \forall\ 0<(x,y)<1 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) $P(X \leq 1/2)$

(ii) $P(X+Y \leq 1)$ (iii) $f_X(x), f_Y(y)$

(marginal distribut.)

(35) The joint density of X and Y is given by

$$f(x,y) = \begin{cases} ke^{-(2x+3y)}, & \forall\ x>0,\ y>0 \\ 0, & \text{otherwise} \end{cases}$$

Then find k? Are X and Y Independent?

(36) The joint density of X and Y is given by $f(x,y) = \begin{cases} C(2x+3y), \; \forall \; 0 \le (x,y) \le 1 \\ 0, \text{ otherwise.} \end{cases}$

Find: (i) C (ii) $P(0 < X < \frac{1}{2}, \frac{1}{4} < Y < \frac{1}{2})$

(iii) $f_X(x), f_Y(y)$ (iv) $f(Y/X)$.

(v) $P(Y > \frac{1}{2} / X = \frac{1}{4})$.

---

(34) $f(x) = \begin{cases} x+y, \; \forall \; 0 < x, y < 1 \\ 0, \text{ otherwise.} \end{cases}$

(i.) $P(X \le \frac{1}{2}) = \int_0^1 \int_0^{1/2} (x+y)\, dx\, dy$

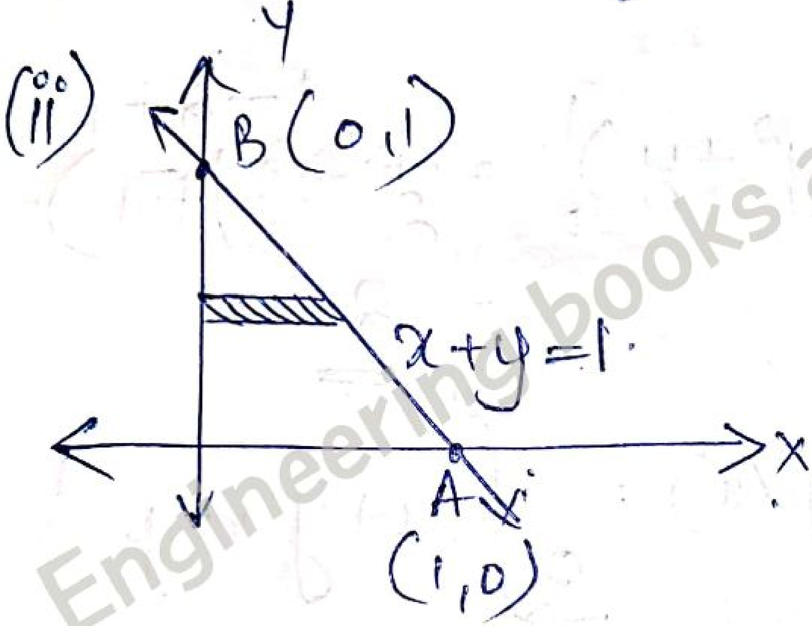
$$= \int_0^1 \left(\frac{x^2}{2} + yx\right)_0^{1/2} dy.$$

$$= \int_{-\infty}^{1} \left(\frac{1}{8} + \frac{y}{2}\right) dy.$$

$$= \int_0^1 \left(\tfrac{1}{8} + \tfrac{y}{2}\right) dy$$

$$= \left(\frac{y}{8} + \frac{y^2}{4}\right)_0^1 = \frac{1}{8} + \frac{1}{4} = \underline{\underline{\frac{3}{8}}}$$

(ii)



$x+y \leq 1$

$x+y=1$

$x=0 \Rightarrow y=1$

$y=0 \Rightarrow x=1$

$$P(X+Y \leq 1) = \int_0^1 \int_0 (x+y)\, dx\, dy$$

$$= \int_0^1 \left(\frac{x^2}{2} + xy\right)_0 \cdot dy$$

$$= \int_0^1 \left[\frac{(y-1)^2}{2} + y(\quad)\right] dy$$

$$= \int_0^1 (y^2 + 1 - 2y \quad 2y^2 - 2y)\, dy$$

$$= \frac{1}{2}\int_0^1 (-y^2 + 1)\,dy$$

$$= \frac{1}{2}\left(-\frac{y^3}{3} + y\right)_0^1$$

$$= \frac{1}{2}\left(-\frac{y^3}{3} + y\right)_0^1 = \frac{1}{2}\left(-\frac{1}{3} + 1\right)$$

$$= 1/3$$

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(iii) $f_X(x) = f(x) = \int_0^1 (x+y)\,dy$

$$= \left(xy + \frac{y^2}{2}\right)_0^1 = x + \frac{1}{2}$$

$$f_Y(y) = f(y) = \int_0^1 (x+y)\,dx$$

$$= \left(\frac{x^2}{2} + xy\right)_0^1 = y + \frac{1}{2}$$

(35) $f(x,y) = \begin{cases} ke^{-(2x+3y)} & \forall\, x>0,\, y>0 \\ 0, & \text{otherwise} \end{cases}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1.$$

$$\int_{0}^{\infty}\int_{0}^{\infty} ke^{-(2x+3y)}\,dx\,dy = 1.$$

$$\int_{0}^{\infty}\left[\frac{ke^{-(2x+3y)}}{(-2)}\right]_0^{\infty} dy = 1$$

$$\int_{0}^{\infty}\left(\frac{ke^{(-\infty-3y)}}{-2} - \frac{ke^{-3y}}{-2}\right)dy = 1.$$

$$\int_{0}^{\infty} \frac{ke^{-\infty}\cdot e^{-3y}}{-2}\,dy + \int_{0}^{\infty}\frac{ke^{-3y}}{2}\,dy = 1.$$

$$\int_{0}^{\infty}\frac{ke^{-3y}}{2}\,dy = 1.$$

$$\left(\frac{ke^{-3y}}{-3\times 2}\right)_0^{\infty}=1.$$

$$\frac{ke^{-3\infty}}{-3\times 2}-\frac{ke^{0}}{-3\times 2}=1.$$

$$\Rightarrow \frac{k}{3\times 2}=1 \Rightarrow \underline{\underline{k=6}}$$

$$f(x)=\int_0^{\infty} 6e^{-(2x+3y)}\,dy.$$

$$=\left(\frac{6e^{-(2x+3y)}}{-3}\right)_0^{\infty}=\underline{\underline{2e^{-2x}}}$$

$$f(y)=\int_0^{\infty} 6e^{-(2x+3y)}\,dx.$$

$$=\left(\frac{6e^{-(2x+3y)}}{-2}\right)_0^{\infty}=3e^{-3y}.$$

$$f(x)\cdot f(y)=(2e^{-2x})\cdot(3e^{-3y})$$

$$=(2\times 3)\cdot(e^{-2x}\cdot e^{-3y})$$

$$=6e^{-(2x+3y)}=f(x,y).$$

Hence X & Y are independent.

(36) (i) $$\int_0^1\int_0^1 c(2x+3y)\,dx\,dy = 1$$

$$\Rightarrow \int_0^1 c\left(\frac{2x^2}{2}+3xy\right)_0^1 dy = 1$$

$$\Rightarrow \int_0^1 c\left(x^2+3xy\right)_0^1 dy = 1$$

$$\Rightarrow \int_0^1 c\,(1+3y)\,dy = 1$$

$$\Rightarrow c\left(y+\frac{3y^2}{2}\right)_0^1 = 1$$

$$\Rightarrow c\left(1+\frac{3}{2}\right) = 1 \Rightarrow c = \frac{2}{5} = 0.4$$

(ii) $P\left(0<X<\frac{1}{2}, \frac{1}{4}<Y<\frac{1}{2}\right) =$

$$\int_0^{1/2}\int_{1/4}^{1/2} \frac{2}{5}(2x+3y)\,dy\,dx$$

$$= \int_0^{1/2} \frac{2}{5}\left(2xy+\frac{3y^2}{2}\right)_{1/4}^{1/2} dx$$

$$= \int_0^{1/2} \frac{2}{5}\left\{ x + \frac{3}{8} - \frac{x}{2} - \frac{3}{32} \right\} dx$$

$$= \int_0^{1/2} \left(\frac{x}{2} \cdot \frac{2}{5} + \frac{9}{32} \cdot \frac{2}{5} \right) dx$$

$$= \left(\frac{x^2}{4}\left(\frac{2}{5}\right) + \frac{9x}{80} \right)_0^{1/2}$$

$$= \left(\frac{x^2}{10} + \frac{9x}{80} \right)_0^{1/2}$$

$$= \frac{1}{40} + \frac{9}{160} = \frac{13}{160} = 0.08125$$

(iii) $$f_X(x) = \int_0^1 \frac{2}{5}(2x + 3y)\,dy$$

$$= \frac{2}{5}\left(2xy + \frac{3y^2}{2}\right)_0^1 = \frac{2}{5}\left(2x + \frac{3}{2}\right)$$

$$= \frac{4x}{5} + \frac{3}{5} = \underline{\frac{4x+3}{5}}$$

$$f(y) = \int_0^1 \frac{2}{5}(2x + 3y)\,dx = \frac{2}{5}\left(x^2 + 3xy\right)_0^1$$

$$= \frac{2}{5}(1 + 3y) = \frac{2}{5} + \frac{6y}{5} = \underline{\frac{2+6y}{5}}$$

(iv) $f(Y/X) = \dfrac{f(x,y)}{f_x(x)} = \dfrac{\frac{2}{5}(2x+3y)}{\frac{(4x+3)}{5}}$

$$= 2\left[\frac{2x+3y}{4x+3}\right]$$

(v) $P\left(Y>\frac{1}{2}\ X=\frac{1}{4}\right) = \int_{1/2}^{1} \frac{2}{5}(2x+3y)\,dy$ @ $x=\frac{1}{4}$

$$= \frac{2}{5}\left(2xy+\frac{3y^2}{2}\right)_{\frac{1}{2}}^{1}$$

$$= \frac{2}{5}\left[-2\cdot\frac{1}{4}\cdot\frac{1}{2}-\frac{3}{2}\cdot\left(\frac{1}{2}\right)^2+2\times\frac{1}{4}+\frac{3}{2}\right]$$

$$= \frac{2}{5}\left(-\frac{1}{4}-\frac{3}{8}+\frac{2}{4}+\frac{3}{2}\right) = 11/20$$

$$P\left(X=\frac{1}{4}\right) = f_x\left(x=1/4\right) = \frac{4}{5}\times\frac{1}{4}+\frac{3}{5} = \frac{4}{5}$$

$$\Rightarrow P\left(Y>\frac{1}{2}/X=\frac{1}{4}\right) = \frac{P\left(Y>\frac{1}{2},\ X=\frac{1}{4}\right)}{P\left(X=\frac{1}{4}\right)}$$

$$= \left(11/20\right) \div \left(\frac{4}{5}\right) = \frac{11}{16}$$

(37) Find the mean and SD of the following normal distribution $f(x) = ke^{-2x^2+10x}$.

(38) If X is normally distributed with mean 8 & SD = 4, find

(i) $P(5 \le X \le 10)$

(ii) $P(10 \le X \le 15)$ (iii) $P(X \ge 15)$

(iv) $P(X \le 5)$.

(39) The weekly wages of 1000 work men are normally distributed around a mean of ₹70 with a SD. of ₹5. Estimate the no. of workers whose weekly wages will be

(i) b/w ₹69 & ₹72.

(ii) less than ₹69.

(iii) more than ₹72.

(40) At an exam. 10% of the students got less than 30 marks & 97% got less than 62 marks, Assuming normal distribution find μ & σ.

(41) In an exam, 44% of the candidates obtained marks below 55 & 6% got above 80 marks. Assuming normal distribution, find μ & σ.

(37) $f(x) = ke^{-2x^2+10x}$

$= ke^{-2(x^2-5x)}$

$= ke^{-2\left(x^2-5x+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2\right)}$

$= ke^{-2\left(x^2-5x+(5/2)^2\right)-(5/2)^2}$

$= ke^{-2(x-2.5)^2+2(2.5)^2}$

$= ke^{12.5}e^{-2(x-2.5)^2}$

$= Ce^{-\frac{(x-2.5)^2}{\left(\frac{1}{2}\right)}}$

$\Rightarrow \mu = 2.5$ & $2\sigma^2 = \frac{1}{2}$

$2\sigma^2 = \frac{1}{2} \Rightarrow \sigma^2 = \frac{1}{4}$

$\Rightarrow \sigma = \frac{1}{2} = 0.5$

(38) $\mu = 8, \sigma = 4$

when $x = 5$, $z = \frac{5-8}{4} = -0.75$

when $x = 10$, $z = \frac{10-8}{4} = 0.5$

when $x = 15$, $z = \frac{15-8}{4} = 1.75$

$P(5 \le X \le 10)$

$= P(-0.75 \le Z \le 0.5)$

$= P(-0.75 \le Z \le 0) + P(0 \le Z \le 0.5)$

$= 0.2734 + 0.1915$

$= 0.4649$

$P(10 \le X \le 15)$

$= P(0.5 \le Z \le 1.75)$

$= -P(0 \le Z \le 0.5) + P(0 \le Z \le 1.75)$

$= -0.1915 + 0.4599$

$= 0.2684$

$P(X \le 5) = P(Z \le -0.75)$

$= 0.5 - P(0 \le Z \le 0.75)$

$= 0.5 - 0.2734$

$= 0.2266$

$P(X \ge 15)$

$= P(Z \ge 1.75)$

$= 0.5 - P(0 \le Z \le 1.75)$

$= 0.5 - 0.4599$

$= 0.0401$

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(29) mean, $\mu = ₹70$
SD, $\sigma = ₹5$.
To find $P(69 < X < 72)$,
$P(X < 69)$ and $P(X > 72)$.

When $x = 69$, $z = \frac{69-70}{5} = -0.2$.

When $x = 72$, $z = \frac{72-70}{5} = 0.4$.

$P(69 < X < 72)$
$= P(-0.2 < z < 0.4)$
$= P(-0.2 < z < 0) + P(0 < z < 0.4)$
$= 0.0793 + 0.1554$
$= 0.2347$

$P(X < 69) = P(z < -0.2)$
$= 0.5 - P(-0.2 < z < 0)$
$= 0.5 - 0.0793 = 0.4207$

$P(X > 72) = P(z > 0.4)$
$= 0.5 - P(0 < z < 0.4)$
$= 0.5 - 0.1554$
$= 0.3446$.

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∴ No. of workers with weekly wages:

(1) b/w ₹69 & ₹72 are 234.7 = $\underline{235}$ approx.

(2) less than ₹ are 420.7 ie $\underline{\underline{421}}$ approx.

& (3) greater than ₹72 are 344.6 ie $\underline{\underline{345}}$. approx.

(40) $P(X<30) = \frac{10}{100}$

$P(X<62) = \frac{97}{100}$.

$$z_1 = \frac{30-\mu}{\sigma}, \quad z_2 = \frac{62-\mu}{\sigma}.$$

$$P(X<30) = P\left(Z<\frac{30-\mu}{\sigma}\right) = 0.1$$

$$= 0.5 - P\left(0<Z<\frac{30-\mu}{\sigma}\right) = 0.1$$

$$\Rightarrow P\left(0<Z<\frac{30-\mu}{\sigma}\right) = 0.4.$$

Similarly, $P(X<62)$

$$= P\left(Z < \frac{62-\mu}{\sigma}\right) = 0.97.$$

$$P\left(Z<\frac{62-\mu}{\sigma}\right) = 0.5 - P\left(0<Z<\frac{62-\mu}{\sigma}\right)$$

$$= 0.97$$

$$\Rightarrow P\left(0<Z<\frac{62-\mu}{\sigma}\right) = -0.47.$$

From the table,

$$Z_1 = \frac{30-\mu}{\sigma} = -1.29$$

$$Z_2 = \frac{62-\mu}{\sigma} = 1.89.$$

ie $30-\mu = -1.29\sigma$ &

$62-\mu = 1.89\sigma$.

On rearranging & solving the above 2 eqtns, we get,

$$\sigma = 10.063 \text{ and } \mu = 42.981$$

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(41). $P(X<55) = \frac{44}{100}$

$P(X>80) = \frac{6}{100}$

$z_1 = \frac{55-\mu}{\sigma}, \quad z_2 = \frac{80-\mu}{\sigma}$

$P(X<55) = P(Z<z_1)$

$= P\left(Z < \frac{55-\mu}{\sigma}\right)$

$= 0.5 - P\left(0 < Z < \frac{55-\mu}{\sigma}\right) = 0.44$

$\Rightarrow P\left(0<Z<\frac{55-\mu}{\sigma}\right) = 0.06.$

$P(X>80) = P(Z>z_2)$
$= P\left(Z > \frac{80-\mu}{\sigma}\right)$

$= 0.5 - P\left(0<Z<\frac{80-\mu}{\sigma}\right) = 0.06.$

$\Rightarrow P\left(0<Z<\frac{80-\mu}{\sigma}\right) = 0.44.$

From the table,

$z_1 = -0.16 \;\&\; z_2 = 1.56.$

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$\therefore \frac{55-\mu}{\sigma} = -0.16$ & $\frac{80-\mu}{\sigma} = 1.56.$

$\Rightarrow 55 = \mu - 0.16\sigma \quad -(1)$

$80 = \mu + 1.56\sigma \quad -(2)$

On solving (1) & (2), we get,

$\mu = 57.326$ and $\sigma = 14.535.$

Questions

1.) A random variable X has the following probability function:

x:	0	1	2	3	4	5	6	7
$p(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find the value of k, Evaluate $P(X<6)$, $P(X\geq 6)$, $P(0<x<5)$?

2.) If a RV 'x' has the pdf

$$f(x) = \begin{cases} (x+1)/2, & -1<x<1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the mean & variance of X.?

3) A RV 'x' has the pdf

$$f(x) = \begin{cases} 2x, & 0<x<1 \\ 0, & \text{otherwise.} \end{cases}$$ Find

(i) $P(X<\frac{1}{2})$. (ii) $P(\frac{1}{4}<x<\frac{1}{2})$?

5) ~~4)~~ In a large consignment of electric bulbs, 10% are defective. A random sample of 20 is taken for inspection. Find the

probablity that ?
(1) All are good bulbs. ?
(2) Atmost 3 defective bulbs. ?
(3) Exactly 3 defective bulbs. ?

5) Find the binomial distribution for which mean is 4 and variance is $4/3$?

6) Fit a binomial distribution to the following frequency distributi

x:	0	1	3	4
f:	28	62	10	4

7) If 'x' is a poisson variate s.t. $P(x=1) = 3/10$ and $P(x=2) = \frac{1}{5}$, find $P(x=0)$ and $P(x=3)$?

8) An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population, what is the probablity that, not more than

2 of its clients are involved in such an accident the next year ?

9) A RV 'x' has a uniform distribution over $(-3, 3)$. Compute (i) $P(x < 2)$ (ii) $P(|x| < 2)$ (iii) $P(|x-2| < 2)$ (iv) Find k for which $P(x > k) = \frac{1}{3}$.

10) Find the root of the equation $x^3 - x - 11 = 0$ using bisection method correct to 3 decimal places.

11) Find the root of the equation $x^3 - 4x - 9 = 0$ using regular falsi method correct to 3 decimal places.

Answers

1. $\sum_x f(x) = 1$

$\Rightarrow k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 9k + 10k^2 = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$ — ①

on solving ①, $k = \frac{1}{10}, -1$

Neglect Negative value, ie $k = \frac{1}{10}$.

$P(X<6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= 0 + 1/10 + 2/10 + 2/10 + 3/10 + 1/100$

$= 81/100$

$P(X \geq 6) = 1 - P(X<6) = 1 - \frac{81}{100} = \frac{19}{100}$

$P(0 < x < 5) = \sum_{x=1}^{4} f(x) =$

$1/10 + 2/10 + 2/10 + 3/10 = 0.8$

2) $f(x) = \begin{cases} \frac{(x+1)}{2}, & -1 < x < 1. \\ 0, & \text{otherwise} \end{cases}$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{-1}^{1} x\,\frac{(x+1)}{2}\,dx$$

$$= \int_{-1}^{1} \left(\frac{x^2+x}{2}\right) dx = \left[\frac{x^3}{6} + \frac{x^2}{4}\right]_{-1}^{1}$$

$$= \frac{1}{6} + \frac{1}{4} + \frac{1}{6} - \frac{1}{4} = \frac{2}{6} = \underline{\frac{1}{3}}$$

$$E\,\cancel{Var}(X^2) = \int_{-\infty}^{\infty} x^2 f(x) = \int_{-1}^{1} x^2 \cdot \left(\frac{x+1}{2}\right) dx$$

$$= \frac{1}{2}\int_{-1}^{1} (x^3 + x^2)\,dx = \frac{1}{2}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{4} + \frac{1}{3} - \frac{1}{4} + \frac{1}{3}\right] = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$$

$$= \underline{\underline{\frac{1}{3}}} \qquad Var(X) = E(X^2) - (E(X))^2 = \underline{\underline{0.22}}$$

3) $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

$$P(X < 1/2) = \int_{0}^{1/2} 2x\,dx = \left(x^2\right)_{0}^{1/2}$$

$$= \frac{1}{4}.$$

$$P\left(\frac{1}{4}<x<\frac{1}{2}\right)=\int_{\frac{1}{4}}^{\frac{1}{2}} 2x\,dx=\left[x^2\right]_{\frac{1}{4}}^{\frac{1}{2}}$$

$$=\frac{1}{4}-\frac{1}{16}=\underline{\underline{\frac{3}{16}}}$$

5) Let X = no. of defective bulbs

p = probability (chosen bulb is defective) $=\frac{10}{100}$

$n = 20$.

$$\lambda = np = 20 \times \frac{10}{100} = \underline{\underline{2}}$$

$$P(X=0) = e^{-2}\cdot(2)^0 = e^{-2} = \underline{0.135}$$

$$P(X\leq 3) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$$

$$= e^{-2}+e^{-2}(2)+\frac{e^{-2}(2)^2}{2!}+\frac{e^{-2}(2)^3}{3!}$$

$$= 0.135+0.271+0.271+0.180$$

$$= 0.857.$$

$$P(X=3) = \frac{e^{-2}(2)^3}{3!} = \underline{\underline{0.1804}}$$

6) mean $= np = 4$
Variance $= npq = \frac{4}{3}$

$4q = \frac{4}{3} \Rightarrow q = \frac{1}{3}$

$n\frac{2}{3} = \cancel{4}\,2$

$p = 1-q = \frac{2}{3}$

$\Rightarrow n = 6$.

$P(x) = 6C_x\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{6-x}$.

7) $P(X=1) = e^{-\lambda}\cdot(\lambda) = \frac{3}{10}$

$P(X=2) = \frac{e^{-\lambda}(\lambda)^2}{2!} = \frac{1}{5}$.

$\frac{e^{-\lambda}\cdot\lambda}{e^{-\lambda}\lambda^2}\cdot(2) = \frac{3/10}{1/5} = \frac{3}{2}$

$\frac{1}{\lambda}\cdot 2 = \frac{3}{2} \Rightarrow \lambda = \frac{4}{3}$

$P(X=0) = e^{-4/3} = 0.2636$

$P(X=3) = \frac{e^{-4/3}\cdot(4/3)^3}{3!} = 0.10412$

(8) Let $p = 0.01\% = \frac{0.01}{100}$ &

$n = 1000$

$\Rightarrow \lambda = \frac{0.01}{100} \times 1000 = 0.1$.

$P(X \leq 2) = P(X=0) + P(X=2) + P(X=1)$

$= e^{(-0.1)} + e^{(-0.1)}(0.1) + \frac{e^{(-0.1)}(0.1)^2}{2!}$.

$= 0.9048 + 0.0905 + 4.52 \times 10^{-3}$

$= \underline{\underline{0.9998}}$

(9) $(a, b) = (-3, +3)$

$f(x) = \frac{1}{b-a} = \frac{1}{3+3} = \underline{\underline{1/6}}$

$P(X<2) = \int_{-3}^{2} \frac{1}{6} \cdot dx = \left(\frac{x}{6}\right)_{-3}^{2}$

$= 2/6 + 3/6 = \underline{\underline{\frac{5}{6}}}$

$P(|x|<2) = P(-2 < x < 2)$

$= \int_{-2}^{2} \frac{dx}{6} = (x/6)_{-2}^{2}$.

$$= \frac{x}{6} + \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$$

$$P(|x-2|<2) = P(-2<x-2<2)$$

$$= P(0<x<4)$$

$$= P(0<x<3) = \int_0^3 \frac{dx}{6}$$

$$= \left(\frac{x}{6}\right)_0^3 = \frac{3}{6} = \frac{1}{2}$$

(10) $f(x) = x^3 - x - 11 = 0$

put $x = 0, 1, 2, \ldots$

$f(0) = -11$, $f(1) = -11$, $f(2) = -5$.

$f(3) = 13 \ldots$

$a = 2;\ b = 3.$ $\quad x_1 = \frac{a+b}{2}.$

a	b.	x_1	$f(x_1)$.
2	3.	2.5	2.125.
2	2.5.	2.25	-1.859 -
2.25	2.5.	2.375.	0.0215.
2.25	2.375.	2.3125.	-0.2460
2.3125	2.375.	2.3438	-0.4684
2.3438	2.375	2.3594	-0.2252

2.3594	2.375	2.3672	-0.1023
2.3672	2.375	2.3711	-0.0405
2.3711	2.375	2.3731	-8.741×10^{-3}
2.3731	2.375	2.3741	7.16×10^{-3}
2.3731	2.3741	2.3736	-7.923×10^{-4}
2.3736	2.3741	2.3739	3.979×10^{-3}
2.3736	2.3739	2.3738	2.38×10^{-3}

approx. root of $f(x)$ is
$$x = \underline{\underline{2.374}}$$

(11) $f(x) = x^3 - 4x - 9 = 0$

put $x = 0, 1, 2, \ldots$

$f(0) = -9$, $f(1) = -12$

$f(2) = -9$, $f(3) = 6$

$a = 2$, $b = 3$

a	b	x_i	$f(x_i)$
2	3	$2 - \frac{1(-9)}{15} = 2.6$	-1.824
2.6	3	$2.6 - \frac{0.4(-1.824)}{7.824}$	

		$= 2.693$	-0.242
2.693	3	$2.693 - \frac{0.307(-0.242)}{6.242}$	
		$= 2.705$	-0.2745
2.705	3	$2.705 - \frac{0.295(-0.2745)}{6.2745}$	
		$= 2.718$	0.2073
2.705	2.718	$2.705 - \frac{0.013(0.2073)}{5.7927}$	
		$= \underline{\underline{2.705}}$	$\underline{\underline{-0.0274}}$

$$\text{root} = \underline{\underline{2.705}}$$

(6) $\bar{x} = \frac{62+30+16}{28+62+10+4} = \frac{108}{104} = \underline{\underline{1.0385}}$

$n = 4 \qquad \bar{x} = np$

$1.0385 = 4p \Rightarrow p = 0.2596$

$q = 1 - p = 0.7404$

x	$P(x)$	$N \cdot P(x)$	
0	0.3005	31.25	31
1	0.4215	43.836	44
2	~~0.0518~~ 0.2217	23.05	23
3	0.0518	5.38	5
4	4.542×10^{-3}	0.472	1
			104
			$\underline{\underline{N}}$

(9) $p(x > k) = \frac{1}{3}$

$$\int_k^3 \frac{1}{6} dx = \frac{1}{3} \Rightarrow \left(\frac{x}{6}\right)_k^3 = \frac{1}{3}$$

$$\Rightarrow \frac{3}{6} - \frac{k}{6} = \frac{1}{3}$$

$$\Rightarrow \frac{k}{6} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\Rightarrow \underline{\underline{k = 1}}$$