UNIVERSITY COLLEGE OF ENGINEERING
KARIAVATTOM, THIRUVANANTHAPURAM
FIRST SERIES TEST, JUNE 2018
FOURTH SEMESTER BTECH DEGREE EXAMINATION
(S_4 - CS)

ENGINEERING MATHEMATICS – III

Time : 2 hrs **Marks : 50**

Answer all questions

I. A discrete random variance X has the following probability distribution.

X:	0	1	2	3	4	5	6	7	8
$f(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Find a, $P(X<3)$ and variance.

II. A continuous random variance X has a pdf $f(x)=3x^2$, $0\le x\le 1$. Find a and b such that (i) $P(X\le a)=P(X>a)$ (ii) $P(X>b)=0.05$.

III. Define binomial distribution. Find the mean and variance of binomial distribution.

IV. Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) atleast one boy (iii) No girl (iv) atmost two girls.

V. A Car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as P.D with mean 1.5. Calculate the proportion of days on which (i) there is no demand and (ii) some demand is refused.

VI. Fit a binomial distribution to the following data.

x:	0	1	2	3	4
f:	4	10	15	9	2

VII. By Gauss elimination method, solve

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

VIII. By Gauss – Seidel iteration method solve

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$

IX. Explain Lagrange interpolation.

X. Using Lagrange's Formula, fit a polynomial to the data.

x:	0	1	3	4
y:	−12	0	6	12

Find the value of y when $x = 2$.

(10 x 5 = 50 marks)

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(Pages : 4) **D – 4331**

Fifth Semester B.Tech. Degree Examination, January 2018
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV (ERHBF)

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions from Part – A and **one full** question from **each** Module in Part – B.*

PART – A (10×4=40 Marks)

1. If $f(x) = \frac{x+1}{2}$, for $-1 < x < 1$

   0, otherwise, is the pdf of x. Find the mean and variance.

2. Find the binomial distribution for which mean 4 and variance $\frac{8}{3}$.

3. The amount of time that a watch will run without having to be reset is a random variable having an exponential distribution with mean 90 days. Find the probability that the watch have to be set in less than 24 days.

4. Find the mean and standard deviation of the normal distribution

   $f(x) = ce^{-\frac{1}{24}\left(x^2-6x+4\right)}, -\infty < X < \infty$.

5. Convert the equation $y = ax^3 + bx^2$ to a linear form and write the corresponding normal equations.

6. Find the arithmetic mean and correlation coefficient from the lines of regression $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$.

7. A sample of 50 items taken from a population with standard deviation 16 gave a mean 52.5. Find 90% confidence interval of the population mean.

P.T.O.

8. Describe briefly the steps in testing a statistical hypothesis.

9. The joint density of X and Y is given by

$f(x, y) = c(6 - x - y),\ 0 < x < 2,\ 2 < y < 4$

$\quad 0,\qquad$ elsewhere. Find c and $P(X < 1, Y < 3)$.

10. Find the mean and variance of the stationary process with no periodic components and $R(\tau) = 16 + \dfrac{9}{1+6\tau^2}$.

PART – B

Answer **one full** question from **each** Module.

Module – I

11. a) If $f(x) = \dfrac{c}{x^2+1}, -\infty < X < \infty$ is a pdf, find c and the distribution function F(x). 6

b) Six dice are thrown 729 times. How many times do you expect atleast 3 dice to show 5 or 6 ? 7

c) In a normal distribution 31% of the items are under 45 and 8% are above 64. Find its mean and standard deviation. 7

OR

12. a) If X has a uniform distribution in (–k, k), k > 0 find k such that $P[|x| < 1] = P[|x| > 1]$. 6

b) If X is a normal variable with mean 30 and standard deviation 5. Find $P(26 \le X \le 40)$ and $P(X \ge 45)$. 7

c) The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 2000 taxi drivers find the number of drivers with 7

i) no. of accidents in a year and

ii) more than 3 accidents in a year.

Module – II

13\. a) Fit a parabola of the form $y = a + bx + cx^2$ to the following data : 10

X :	1941	1951	1961	1971	1981
Y :	8	10	12	10	16

Also find y when x = 1976.

b) Compute the correlation coefficient from the following data : 10

x :	77	54	27	52	14	35	20	25
y :	35	58	60	40	50	40	35	56

Also find y when x = 24.

OR

14\. a) Calculate the regression lines from the following data : 10

x :	33	56	50	65	44	38	44	50	15	26
y :	50	35	70	25	35	58	75	60	55	26

b) In sample of 300 units of a manufactured product, 65 units were found to be defective and in another sample of 200 units there were 35 defectives. Is there significant difference in the proportion of defectives in the sample at 5% level of significance ? 10

Module – III

15\. a) The joint pdf of X and Y are given by $f(x, y) = c(x + 2y)$ where $x = 0, 1, 2$ and $y = 0, 1, 2, 3$. 7

i) Find c.

ii) Find $P(x \le 1)$ and $P(x + y \le 3)$

b) If $X(t) = A + Bt$ where A and B are independent random variables with $E(A) = p$, $E(B) = q$, $Var(x) = \sigma_A^2$ and $Var(B) = \sigma_B^2$, find $R(t_1, t_2)$ and $C(t_1, t_2)$. 7

c) The tpm of a markov chain $\{X_n\}$ have 3 states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$

and the initial distribution is $P(0) = (0.7, 0.2, 0.1)$

Find : 6

i) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$ and

ii) $P(X_2 = 3)$

OR

16\. a) If $f(x, y) = \begin{cases} ke^{-(2x+y)}, & x \geq 0, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$ is a joint pdf find :

i) k

ii) marginal pdf's of x and y. 7

b) If $X(t) = r\cos(\omega t + \phi)$ where the random variables r and ϕ are independent and ϕ is uniform in $(-\pi, \pi)$. Find $R(t_1, t_2)$. 7

c) Find the power spectrum if $R(\tau) = e^{-\alpha\tau} \cos\beta\tau$. 6

(Pages : 4) **E – 4168**

Fourth Semester B.Tech. Degree Examination, August 2018
(2013 Scheme)
13.401 : PROBABILITY, RANDOM PROCESSES AND NUMERICAL TECHNIQUES (FR)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries 4 marks.

1. The probability distribution of a random variable is given by

   $P(X = x) = k/2^x$, $x = 0, 1, 2, 3, 4$

   Find k and $P(X \neq 3)$.

2. The joint probability density function of a two-dimensional random variable (X, Y) is

$$f(x, y) = \begin{cases} c, & x^2 + y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   Find :

   i) c and

   ii) $P(X^2 + Y^2 \leq \frac{1}{2})$

3. Is a Poisson process stationary in any sense ? Justify your answer.

4. Find a real root of the equation $x^3 - 2x - 5 = 0$ lying between 2 and 3 by Regula-Falsi method. Carry on upto four approximations.

5. Using Simpson's rule, find $\int_0^2 \frac{1}{1+x} dx$ by dividing [0, 2] into 8 equal sub intervals. **(5×4=20 Marks)**

P.T.O.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

MODULE – I

6\. a) The number of hits to a popular website during a 1– minute interval is a Poisson random variable with mean 2.4. The server hosting the page is configured to handle a maximum of 4 requests per minute. What is the probability that the server will receive more than 4 request in a 1 minute period ? 7

b) In certain type of radio communications the phase difference X between the transmitter and receiver is modelled as a uniform random variable in $[-\pi, \pi]$. Find. 7

i) $P(X > \pi/2)$ and ii) $P(X > \pi/2 | X > 0)$.

c) The time that a machine will run without repair is exponentially distributed with mean 150 days. Find the probability that such a machine will

i) have to be repaired in less than 100 days.

ii) not have to be repaired in atleast 175 days. 6

7\. a) If X follows normal distribution with mean – 3 and variance 4, find

i) $P(1 \leq X \leq 2)$ ii) $P(X \geq -1.5)$

iii) $P(|X + 3| \leq 2)$. 10

b) 5-bit secret code words are sent across a noisy channel. Each bit has probability 0.05 of being received in error independently of others. If no more than 2 bits are received in error, the code word can be correctly decoded. What is the probability that a code word received can be correctly decoded ? 10

MODULE – II

8\. a) The joint distribution of a two-dimensional random variable (X, Y) is given by

$p(x, y) = c(2x + 3y),\ x = 0, 1, 2,\ y = 1, 2, 3.$ 12

Find :

i) The value of c

ii) The marginal distributions

iii) Are X and Y independent.

b) Discuss the classification of random process with examples. 8

9. a) Prove that the random process X (t) is defined by X (t) = a sin (ωt + θ), where a and ω are constants and θ is a random variable uniformly distributed in (0, 2π) is a WSS process. 10

b) Random variables X and Y have a joint probability density function given by

$$f(x,y) = \begin{cases} x+y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the covariance cov(X, Y) and the correlation coefficient ρXY. 10

MODULE – III

10. a) The customers of a post office arrive according to a Poisson process with rate 15 per hour. The post office works from 8:30 A.M. to 4.30 P.M. 12

i) What is the probability that at least 6 customers arrive before noon (i.e. between 8.30 A.M. and 12.00) ?

ii) What is the probability that all customers arrive in the afternoon (i.e., between 12.00 to 4.30 P.M.) ?

iii) What is the probability that no customer arrives during both the tea break (10.30 A.M.–11.00 A.M.) and the lunch break (1:00 P.M.–2.00 P.M.) ?

b) In each of the following examine whether f (ω) could be the Power Spectral Density (PSD) of a wide sense stationary process. Explain your reasoning. 8

i) $$f(\omega) = \begin{cases} \dfrac{\sin\omega}{\omega} & \omega \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

ii) $$f(\omega) = \begin{cases} \pi & \text{if } |\omega| < 1 \\ 0, & \text{otherwise} \end{cases}$$

11. a) The power spectral density of a wide sense stationary process X (t) is 12

$$S_X(\omega) = \begin{cases} 1+\omega^2, & |\omega| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the autocorrelation function.

b) A random process is defined as X (t) = U where U is a random variable which is uniformly distributed in [0, 1]. Is X (t) mean ergodic ? Justify. 8

MODULE – IV

12. a) Using Newton's forward difference interpolation formula, find the value f (4). Given that f (3) = 15, f (5) = 24, f (7) = 45, f (9) = 60 and f (11) = 86. 10

b) Apply Gauss-Seidel method to solve the equations : 10

$$10x - 2y + z = 12$$

$$x + 9y - z = 10$$

$$2x - y + 11z = 20.$$

Carry on upto four approximations.

13. a) Using Newton-Raphson method, compute a real root of $e^{2x} - x - 6 = 0$ lying between 0 and 1 correct to five decimal place. 7

b) Using Newton's backward difference interpolation formula, find the value f (9). Given that f (2) = 10, f (4) = 42, f (6) = 55, f (8) = 125 and f (10) = 180. 7

c) Apply Lagrange's interpolation formula to find y (2), given that y (0) = 1, y (1) = 0 and y (3) = 10. 6

(Pages : 4)

C – 2857

Fourth Semester B.Tech. Degree Examination, June 2017
(2013 Scheme)
13.401 : ENGINEERING MATHEMATICS – III (AT)
Probability and Random Processes

Time : 3 Hours Max. Marks : 100

PART – A

(Answer **all** questions. **Each** question carries 4 marks.)

1. State and prove Memoryless property of exponential distribution.

2. A continuous random variable has a p.d.f. $f(x) = 3x^2, 0 \leq x \leq 1$. Find a and b such that
   i) $P(X \leq a) = P(X > a)$ and
   ii) $P(X > b) = 0.05$.

3. If X and Y are random variables such that $Y = aX + b$, where a and b are real constants, show that the correlation coefficient $r(X, Y)$ between them has magnitude one.

4. If the transition probability matrix of a Markov chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, find the steady state distribution of the chain.

5. Prove that if a Gaussian process is WSS, it is also SSS.

PART – B

(Answer **one full** question from **each** Module. **Each** question carries **20** marks.)

Module – I

6. a) A random variable X has the following probability distribution.

x	:	0	1	2	3	4
p(x)	:	c	2c	2c	c^2	$5c^2$

Find the value of c. Evaluate $P(X < 3)$, $P(0 < X < 4)$. Determine the distribution function of X. Find the mean and variance of X.

b) A continuous random variable X is said to have a uniform distribution over the interval [a, b].

i) Show that the variance of X is $\frac{(b-a)^2}{12}$.

ii) If mean and variance of X are 1 and $\frac{4}{3}$ respectively, and three independent observations of X are made, what is the probability that all of them are negative ?

7. a) Find mean and variance of the binomial distribution.

b) In an examination, the candidates are awarded the following grades depending on the marks scored by them :

distinction : $\geq$ 80%, first class : 60% $\leq$ marks < 80%, second class : 45% $\leq$ marks < 60%, third class : 30% $\leq$ marks < 45%, fail : < 30%. It was found that 8% of the students failed and 8% have scored distinction. Find the average marks obtained by the candidates. Deduce the percentage of students placed in the second class. Assume normal distribution of marks.

Module – II

8. a) The joint p.d.f. of random variables X and Y is given by

$$f(x, y) = \begin{cases} 3(x+y), & \text{for } 0 < x \leq 1, 0 < y \leq 1, x + y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the marginal p.d.f. of X and Cov(X, Y).

b) The resistors r_1, r_2, r_3 and r_4 are independent random variables and is uniform in the interval (450, 550). Using central limit theorem, find $P(1900 \leq r_1 + r_2 + r_3 + r_4 \leq 2100)$.

9. a) Define a random process. Explain how you would classify a random process. Give an example to each type of random process.

   b) If $X(t) = \sin(\omega t + Y)$, where Y is uniformly distributed in $(0, 2\pi)$. Show that $X(t)$ is wide-sense stationary.

Module – III

10. a) A random process $\{X(t)\}$ is given by $X(t) = A \cos pt + B \sin pt$, where A and B are independent random variables such that $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = \sigma^2$. Find the power spectral density of the process.

    b) Prove the following properties of power spectral density function :

       i) The spectral density function of a real random process is an even function.

       ii) The spectral density of a process $\{X(t)\}$, real or complex, is a real function of ω and non-negative.

       iii) The spectral density and the autocorrelation function of a real WSS process form a Fourier cosine transform pair.

11. a) A gambler has Rs. 2. He bets Re. 1 at a time and wins Re. 1 with probability 1/2. He stops playing if he loses Rs. 2 or wins Rs. 4.

       i) What is the transition probability matrix of the related Markov chain ?

       ii) What is the probability that he has lost his money at the end of 5 plays ?

       iii) What is the probability that the game lasts for more than 7 plays ?

    b) i) Express the autocorrelation function of the process $\{X'(t)\}$ in terms of the autocorrelation function of the process $\{X(t)\}$.

       ii) The autocorrelation function for a stationary process X(t) is given by $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean of the random variable $Y = \int_0^2 X(t)dt$ and variance of $X(t)$.

Module – IV

12. a) Define Poisson process and derive the probability law for Poisson process $\{X(t)\}$.

    b) Write Little's formulae and obtain the steady state probabilities for (M/M/1) with finite capacity model.

13. a) There are 3 Data Entry Operators (DEOs) in an office. Each DEO can type an average of 6 documents per hour. If documents arrive for being typed at the rate of 15 documents per hour.

    i) What fraction of the time all the DEOs will be busy ?

    ii) What is the average number of documents waiting to be typed ?

    iii) What is the average time a document has to spend for waiting and for being typed ?

    iv) What is the probability that a document will take longer than 20 minutes waiting to be typed ?

b) It is given that $R_X(T) = e^{-|T|}$ for a certain stationary Gaussian random process $\{X(t)\}$. Find the joint pdf of the random variables $X(t)$, $X(t + 1)$ and $X(t + 2)$.

Fifth Semester B.Tech. Degree Examination, November 2013
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV (ERFBH)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries 4 marks.

1. The following is the p.d.f. of a random variable X :

x :	0	1	3	7	13
P(X = x) :	k	2k	2k	3k	k^2

Find k and mean of X.

2. The number of hits to a popular website during a 1-minute interval is a Poisson random variable with mean 2.4. The server hosting the page is configured to handle a maximum of 4 requests per minute. What is the probability that the server will receive more than 4 request would arrive in a 1 minute period.

3. 1000 light bulbs with mean length of life 120 days are installed in a factory. Their length of life is assumed to follow normal distribution with S.D. 20 days. If it is decided to replace all the bulbs together, what interval should be allowed between replacements if not more than 10% should expire before replacement ?

4. If a random variable X is uniformly distributed over $(-\alpha, \alpha)$, find α so that $P(|X| < 1) = P(|X| > 1)$.

5. The mean of a sample of size 20 from a normal population with S.D. 8 was found to be 81.2. Find a 90% confidence interval for the population mean.

6. Can $y = 2.8x + 5$ and $x = 3 - 0.5y$ be the estimated regression equation of Y on X and X on Y respectively ? Explain your answer.

7. In a city 325 men out of 600 were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers ?

8. Define wide sense stationary and strict sense stationary processes. Show that every strict sense stationary process is also wide sense stationary.

9. Explain how the auto correlation function, power and power spectral density of a wide sense stationary process are related to one another.

10. Is a Poisson process stationary in any sense ? Justify your answer.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

MODULE – I

11. a) In testing a certain kind of truck tire over a rugged terrain, it is found that 25% of the trucks fail to complete the test run without a blowout. Of the next 15 trucks tested, find the probability that i) from 3 to 5 blowouts ; ii) fewer than 4 have blowouts ; iii) more than 5 have blowouts.

b) The spot speeds at a particular location are normally distributed with a mean of 51.7 km/hour and a standard deviation of 8.3 km/hour. What is the probability that i) the speed exceeds 65 km/hour ? ii) and the speed lie between 40 km/hour and 70 km/hour ? iii) What is the 85^{th} percentile speed ?

c) The time that a machine will run without repair is exponentially distributed with mean 200 days. Find the probability that such a machine will i) have to repair in less than 100 days, ii) not have to repair in atleast 250 days.

12. a) If $f(x) = \begin{cases} \dfrac{x+1}{2} & \text{for } |x| \leq 1 \\ 0 & \text{elsewhere} \end{cases}$ is the p.d.f. of X, find the mean and variance.

b) The marks obtained by a batch of students in Mathematics are approximately normally distributed with mean 60 and standard deviation 5. If 5 students are selected at random from this group, what is the probability that atleast two of them will score above 80 ?

c) Buses arrived at a specified stop at 15 minute intervals starting at 7 am. A passenger arrives at the stop at random time between 7 and 7.30 am. Find the probability that he waits i) less than 5 minutes, ii) atleast 12 minutes.

MODULE – II

13. a) Calculate the correlation coefficient for the following data :

x :	65	66	67	67	68	69	70	72
y :	67	68	65	68	72	72	69	71

b) A certain injection administered to each of 12 patients resulted in the following increases of blood pressure 5, 2, 8, – 1, 3, 0, 6, – 2, 1, 5, 0, 4. Can it be concluded that the injuction, in general, accompanied by an increase in blood pressure. (Use 5% level of significance).

c) In two colleges affiliated to a university 46 out of 200 and 48 out of 250 candidates failed in an examination. If the percentage of failure in the university is 18%, examine whether the colleges differ significantly.

14. a) Using the principle of least squares, fit a parabola of the form $y = ax^2 + bx + c$ for the following data :

X :	0	1	2	3	4	5
Y :	14	18	22	27	38	40

b) Two types of batteries are tested for their length of life and the following results were obtained :

	No. of Sample	Mean (Hrs.)	Variance
Battery A	10	500	100
Battery B	15	560	121

Is there a significance difference in the two means ?

c) The mean breaking strength of the cables supplied by a manufacturer is 1800 with SD of 100. By a new technique in the manufacturing process, it is claimed that the braking strength of the cable has increased, to test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 1% level of significance assuming that the S.D. has not changed ?

MODULE – III

15. a) The joint distribution of a two-dimensional random variable (X, Y) is given by $p(x, y) = c(2x + 3y)$, $x = 0, 1, 2$, $y = 1, 2, 3$. Find i) the value of c ii) the marginal distributions. Also find the probability distribution of $X + Y$.

b) A random experiment consists of tossing a fair die repeatedly and let $X(n)$ denote the number that turns up at the n^{th} toss. Find the mean and autocorrelation of the random process $X(n)$. Is the process stationary ?

c) $X(t)$ and $Y(t)$ are independent, zero mean, wide sense stationary processes and $Z(t) = X(t) + Y(t)$. Show that i) $R_{ZZ}(\tau) = R_{XX}(\tau) + R_{YY}(\tau)$ and ii) $S_{ZZ}(\omega) = S_{XX}(\omega) + S_{YY}(\omega)$.

16. a) A particular hospital keeps records of the arrival of patients in the emergency room. Those records show that, the number of arriving patients constitutes a Poisson process with an average rate of 6 patients per hour. For a given day, starting at 12:00 midnight,

i) Find the probability that exactly 15 patients arrive between 12:00 midnight and 2:30 A.M.

ii) What is the expected number of patients to arrive in one day ?

iii) Find the probability that the first patient does not arrive until 12:30 A.M.

iv) What is the expected time until the first patient arrives ?

b) It is observed that customers of three different brands (A, B and C) of toothpastes have a tendency to switch brands in their next purchase. The following transition probability matrix shows the probability that a customer may or may not change brand in a subsequent purchase.

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$$

i) Find the proportion of the customers who prefer each of the brands, A, B and C.

ii) A survey conducted among selected customers showed that 30% of them will go for brand A, while 20% will go for brand B and 50% will go for brand C in the current purchase. What will be the probability distribution of the preferences of these selected customers for the various brands at the second purchase after the current ?

(Pages : 4) **A – 3928**

Fourth Semester B.Tech. Degree Examination, June 2016

(2008 Scheme)

Branch : Electronics and Communication

08.401 : ENGINEERING MATHEMATICS III – PROBABILITY AND RANDOM PROCESSES (TA)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries 4 marks. **(10×4=40 Marks)**

1. Find c for which $f(x) = \begin{cases} cxe^{-x}, & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$ is a probability density function.

2. If X is a Poisson variate such that P[X = 2] = 9P[X = 4] + 90 P[X = 6], find the standard deviation.

3. If X is a random variable such that E(X) = 3 and E(X^2) = 13. Use Chebyshev's inequality to find a lower bound for P(–2 < x < 8).

4. Show that the coefficient of correlation lies between –1 and 1.

5. Let X(t) = A sint + B cost be a process, where A and B are independent random variables with zero mean and equal variance. Show that the process is wide-sense stationary.

6. Find the mean and variance of the stationary process with

$$R_X(\tau) = 16 + \frac{9}{1 + 6\tau^2}.$$

7. If X(t) = Y sin wt where Y is uniformly distributed in the interval (–1, 1) is the sine wave random process, show that X(t) is not wide-sense stationary.

P.T.O.

8. If X(t) is the random telegraph signal process with $E[X(t)] = 0$ and $R(\tau) = e^{-2\lambda|\tau|}$. Find the mean and variance of the time average of X(t) over $(-T, T)$.

9. Define Mean-Ergodic process. State the Mean-Ergodic theorem.

10. If the autocovariance function of a process X(t) is given by $C(\tau) = \sigma^2 \cos w t$, prove that X(t) is mean ergodic.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) In a lot of 500 solenoids, 25 are defective, find the probabilities of a sample of 20 solenoids chosen at random may have

i) no defectives

ii) two defectives

iii) not more than two defectives and

iv) 2 or 3 defectives.

b) Find the mean and variance of the uniform distribution.

c) The joint probability mass function of X and Y are given by

$f(x, y) = \frac{1}{48}(x + 2y)$ where $\begin{matrix} x = 0, 1, 2 \\ y = 0, 1, 2, 3 \end{matrix}$ and find

i) $P[X \leq 1]$

ii) $P(X \leq 1, Y \leq 2)$

iii) $P[X \leq 1 | Y \leq 2]$

iv) $P[X^2 + Y^2 \leq 4]$

12. a) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as P.D. with mean 1.5. Calculate the proportion of days on which

i) there is no demand and

ii) some demand is refused.

b) In a certain examination, the percentage of candidates passing and getting distinctions were 45 and 9 respectively. Evaluate the average marks obtained by the candidate, the minimum pass and distinction marks being 40 and 75 respectively. (Assume the distribution to be normal).

c) State central limit theorem. Let $X_1, X_2, ..., X_{20}$ be independent Poisson random variables with mean 1. Use central limit theorem to approximate $P\left[\sum_{i=1}^{20} X_i > 15\right]$.

Module – II

13. a) If $X(t) = r \cos(wt + \phi)$ where the random variable r and ϕ are independent and ϕ is uniform in $(-\pi, \pi)$. Find the autocorrelation $R(t_1, t_2)$.

b) If $X(t) = A \cos wt + B \sin wt$ where A and B are independent normal random variables with zero mean and variance σ^2. Prove that X(t) is a strict-sense stationary process of order 2.

14. a) Show that the random telegraph signal process is wide-sense stationary.

b) Suppose that customers arrive at a shop according to Poisson process with a mean arrival rate of 5 per minute, find the probability that during a time interval of 3 minutes

i) exactly 10 customers arrive

ii) more than 4 customers arrive.

Module – III

15. a) If X(t) is a wide-sense stationary process with $E[X(t)] = 2$ and $R(\tau) = 4 + e^{-|\tau|/10}$, find the mean and variance of $S = \int_0^1 X(t)\,dt$.

b) If $R(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T} & \text{if } |\tau| < T \\ 0 & \text{otherwise} \end{cases}$, find the spectral density S(w).

c) Let X_n, $n \geq 0$ be Markov chain with three states 0, 1, 2 and a transition matrix

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$ and $P\{X_0 = i\} = \frac{1}{3}$ for i = 0, 1, 2. Determine

i) $P[X_2 = 2, X_1 = 1 | X_0 = 2]$

ii) $P\{X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\}$

16. a) If X(t) = a sin (bt + Y), find the autocorrelation function $R(\tau)$ and spectral density S(w).

b) Suppose that X(t) is a normal process with $\mu(t) = 3$ and $C(t_1, t_2) = 4e^{-0.2|t_1 - t_2|}$, find

i) $P[X(5) \leq 2]$

ii) $P[|X(8) - X(5)| \leq 1]$

c) Suppose that the probability of a dry day following a rainy day is $\frac{1}{3}$ and the probability of a rainy day following a dry day is $1/2$. Given that May 1 is a dry day. What is the probability that May 3 is a dry day ?

(Pages : 4) **A – 4165**

Fourth Semester B.Tech. Degree Examination, June 2016
(2013 Scheme)
13.401 : PROBABILITY, RANDOM PROCESSES AND NUMERICAL TECHNIQUES (FR)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Let X be a random variable with E(X) = 1 and E[X(X – 1)] = 4. Find V(X) and V(2 – 3X).

2. The joint pdf of the random variables X and Y is given by
$$f(x,y)=\begin{cases} kxye^{-(x^2+y^2)}, x>0, y>0 \\ 0, \text{ otherwise}\end{cases}$$
Find the value of k.

3. For any two random variables X and Y, show that Cov(aX, bY) = abCov(X,Y) where a and b are constants.

4. Define the power spectral density (psd) function of a stationary process. Write the Wiener-Khinchin relations.

5. Find the interpolating polynomial to find y from the following data.

x :	0	1	2	5
y :	2	3	12	147

P.T.O

PART – B

Answer **one full** question from **each** module. **Each** question carries **20** marks.

Module – I

6. a) If the number of telephone calls coming to a telephone exchange between 9 AM and 10 AM and between 10 AM and 11 AM are independently distributed Poisson variables with parameters 2 and 6 respectively, find the probability that more than 5 calls arriving between 9 AM and 11 AM to that exchange.

   b) The duration of a telephone call in minutes follows exponential distribution with mean 4. Find the probability that a call will last (i) more than 8 minutes (ii) between 3 and 6 minutes and (iii) less than 2 minutes.

   c) In a normal population with mean 15 and standard deviation 3.5, it is found that 647 observations exceed 16.25. What is the total number of observations in the population ?

7. a) In a family of 5 children, the probability that there are 3 boys is twice as the probability that there are two boys in the family. Out of 2500 such families with 5 children each, in how many families do you expect to have (i) exactly two boys (ii) no boys and (iii) no girls.

   b) Buses arrive at a specified stop at 15 minutes interval starting from 7AM. Thus they arrive at 7, 7.15, 7.30, 7.45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7.30 AM, find the probability that he waits (i) less than 5 minutes for a bus and (ii) at least 12 minutes for a bus.

   c) X and Y are two independent normally distributed random variables with means 45 and 44 and standard deviations 2 and 1.5 respectively. Find the probability that randomly chosen values of X and Y differ by 1.5 or more.

Module – II

8. a) The join pdf of the random variables X and Y is given by

$$f(x, y) = \begin{cases} 8xy, & 0 \le x \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) $P(X < \frac{1}{2} \text{ and } Y < \frac{1}{4})$ (ii) the marginal and conditional densities of X and Y. Are the variables independent ? Give reasons for your answer.

b) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. Let X be the number of white balls drawn and Y be the number of red balls drawn. Find the joint pdf of X and Y.

c) Let X be a uniformly distributed random variable over $(-1, 1)$ and $Y = X^2$. Show that X and Y are uncorrelated. Are they independent ?

9. a) Verify whether the random process $\{X(t)\}$ defined by $X(t) = Y \cos \omega t$ where Y is a uniformly distributed random variable in $(0, 1)$ and ω is a constant is an SSS process.

b) A stationary process $X = \{X(t)\}$ with mean 3 has autocorrelation function $R(\tau) = 16 + 9e^{-|\tau|}$. Find the standard deviation of the process.

c) If $X(t) = a\cos(\omega t + \theta)$ and $Y(t) = b\sin(\omega t + \theta)$ where a, b and ω are constants and θ is a random variable uniformly distributed in $(0, 2\pi)$, show that X(t) and Y(t) are jointly wide sense stationary.

Module – III

10. a) Define a Poisson Process. Is it an SSS process ? Justify your answer.

b) Define a mean ergodic process. Let $Z(t) = YX(t)$ where X(t) is a mean ergodic process with constant mean $\mu_X \neq 0$ and Y is a random variable independent of X(t) for every t with $E(Y) = 0$. Is Z(t) mean ergodic ?

11. a) The autocorrelation function of a wide sense stationary process is given by $R(\tau) = e^{-\alpha|\tau|}(1+\alpha|\tau|)$. Determine the power spectral density of the process.

b) The power spectral density of a wide sense stationary process X(t) with mean zero is given by

$$S(\omega) = \begin{cases} k & \text{if } |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. Show that X(t) and $X\left(t + \frac{\pi}{\omega_0}\right)$ are uncorrelated.

Module – IV

12. a) Find the smallest positive root of $x^2 - \log x - 12 = 0$ using Regula-falsi method correct to two decimal places.

b) Solve the following system of equations by Gauss elimination method correct to two decimal places.

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

c) Find a negative root of $x^3 - 2x + 5 = 0$ by Newton-Raphson method correct to two decimal places.

13. a) Using Newtons interpolation formula find y when (i) $x = 48$ and (ii) $x = 84$.

x :	40	50	60	70	80	90
y :	184	204	226	250	276	304

b) Find $\int_0^1 \frac{x^3}{1+x^3}\,dx$ dividing the interval of integration into 4 equal parts by

i) Trapezoidal rule

ii) Simpson's one third rule.

(Pages : 4) **A – 6366**

Fifth Semester B.Tech. Degree Examination, September 2016
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV (ERFBH)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. The pdf of a continuous random variable X is given by $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$.

   Find the probability that :

   i) $X \leq 1$

   ii) $X > 1$.

   Also find the distribution function of X.

2. The probability of error in transmission of a bit over a communication channel is 10^{-4}. What is the probability of more than three errors in transmitting a block of 1000 bits ?

3. Analog signal received at a detector is modeled as a normal random variable with mean 200 microvolts and variance 256 microvolts at a fixed point of time. What is the probability that the signal will exceed 240 microvolts ?

4. If X is uniformly distributed with mean 1 and variance 4/3, find $P[X < 0]$.

5. Two lines of regression are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ and variance of x is 12. Find $\bar{x}$, $\bar{y}$, r and σy.

6. In a random sample of 450 industrial accidents it was found that 230 were due to unsafe working conditions. Construct 95% confidence interval for population proportion.

7. Define the following terms :

   i) Null hypothesis

   ii) Level of significance

   iii) Critical region.

8. The joint pdf of X and Y is given by $f(x, y) = \begin{cases} K(2x+3y) & 0 \le x, y \le 1 \\ 0 & \text{Otherwise} \end{cases}$.

   Find K and the marginal distribution of X.

9. Define Auto correlation, Auto covariance and spectral density.

10. Customers arrive at a ticket counter according to a Poisson process with mean rate of 2 per minute. In an interval of five minutes, find the probability that the number of customers arriving is more than 3 ?

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) Find the mean and variance of the binomial distribution.

    b) The mileage which car owners get with a certain kind of radial type is a random variable having an exponential distribution with mean 40,000 km. Find the prob. that one of the tyres will last :

       i) atleast 20,000 km

       ii) atmost 30,000 kms.

    c) If X is uniformly distributed in (–3, 3), find $P[1X - 2| < 2]$.

12. a) Out of 800 families with 4 children each, how many families would be expected to have 2 boys and 2 girls, assuming equal probability for boys and girls.

    b) The marks obtained by in a certain subject follows normal distribution with mean 65 and SD 5. If 3 students are selected at random from this group, find the probability that atleast one of them would have scored above 75 ?

c) The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month :

i) Without a breakdown

ii) With only one breakdown.

Module – II

13. a) Find the lines of regression using the following data :

X :	60	61	62	62	63	64	65	67
Y :	61	62	57	61	65	65	61	64

b) The heights of 10 randomly selected students in a school in inches are 50, 52, 52, 53, 55, 56, 57, 58, 58 and 59. Test the hypothesis that mean height of students of the school is 54 inches.

14. a) Convert the equation $y = ax + bx^2$ to linear form and fit the same to the following data :

X :	1	2	3	4	5
Y :	5	7	9	10	11

b) The sales data of an item in six shops before and after a special promotional campaign are as follows :

Before campaign :	53	28	31	48	40	42
After campaign :	58	29	30	55	56	45

Test at 5% level of significance whether the campaign was a success.

Module – III

15. a) Distinguish between SSS and WSS. Give an example for each.

b) If $X(t) = P + Qt$, where P and Q are independent random variables with $E(P) = p$ and $E(Q) = q$, $Var(P) = \sigma_1^2$, $Var(Q) = \sigma_2^2$. Find $E(X(t))$, $R(t_1, t_2)$. Is the process $\{X(t)\}$ stationary in the wide sense.

c) Suppose the probability that a dry day following a rainy day is $1/3$ and the probability that a rainy day following a dry day is $1/2$ given that May 1 is a dry day, what is the probability that May 3rd is a dry day.

16. a) Find the spectral density function of the process with Auto correlation function $R(\tau) = 1 + e^{-\alpha|\tau|}$.

b) If $X(t) = A \sin t + B \cos t$ is a process where A and B are independent random variables with zero mean and equal variance σ^2. Find $E(X(t))$ and $R(t_1, t_2)$.

c) The tpm of a Markov Chain $\{X_n\}$ with states 1, 2, 3 is $P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$

Calculate :

i) $P[X_2 = 1/X_0 = 1]$

ii) $P[X_3 = 2/X_0 = 3]$.

(Pages : 3) **A – 4166**

Fourth Semester B.Tech. Degree Examination, June 2016
(2013 Scheme)
13.401 : ENGINEERING MATHEMATICS – III (AT)
(Probability and Random Process)

Time: 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. If X and Y are two continuous random variables with pdf $f_X(x)$ and $f_Y(y)$, then prove that $E[X + Y] = E[X] + E[Y]$.

2. If joint density function of X and Y is given by $f(x, y) = (1 - e^{-x})(1 - e^{-y})$, $x > 0$, $y > 0$, then prove that X and Y are independent.

3. Define power spectral density function.

4. Define Markov chain and one step transition probability matrix.

5. Find the mean and variance of uniform distribution.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

Module – I

6. a) If the random variable X takes the value 1, 2, 3 and 4 such that $2P[X = 1] = 3P[X = 2] = P[X = 3] = 5P[X = 4]$. Find the probability distribution.

   b) A pair of dice is thrown 4 times. If a getting doublet is considered a success, find the probability of 2 success.

7. a) Find the mean and variance of a continuous r.v.X, if it has density function.

$$f(x) = \begin{cases} 2(x-1), & 1<x<2 \\ 0, & \text{otherwise} \end{cases}$$

b) In a certain factory turning razar blades, there is a small chance of $\frac{1}{500}$ for any blade to be defective. The blades are in pockets of 10. Use Poisson distribution to calculate the approximate number of pockets containing :

i) no defective ii) one defective

iii) 2 defective blades respectively in a consignment of 10,000 packets.

Module – II

8. a) From the following joint distribution of X and Y, find the marginal distribution.

Y \ X	0	1	2
0	$3/28$	$9/28$	$3/28$
1	$3/14$	$3/14$	0
2	$1/28$	0	0

b) Explain different types of Random process.

9. a) The joint distribution of X and Y is given by $f(x, y) = \begin{cases} Kxy, & 0<x<1, \ 0<y<1 \\ 0 & \text{otherwise} \end{cases}$

i) Find K

ii) Find P $(X + Y < 1)$

iii) Are X and Y independent R.V's

b) If $X(t) = \sin(\omega t + y)$ where y is uniformly distributed in $(0, 2\pi)$, show that $X(t)$ is wide sense stationary.

Module – III

10. a) The auto correlation of stationary random process is given by

$R_{xx}(\tau) = a\, e^{-b|\tau|}$, $b > 0$. Find the spectral density function.

b) Let $\{X(t)\}$ be a wide sense stationary process with zero mean and auto correlation function $R_{xx}(\tau) = 1 - \frac{|\tau|}{T}$, where T is a constant. Find the mean and variance of the time average of $\{X(t)\}$ over (0, T).

11. a) Prove that the spectral density function of a real random process is an even function.

b) The initial process of the Markovian transition probability matrix is given by

$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$ with initial probabilities $p_1^{(0)} = 0.4$, $p_2^{(0)} = 0.3$, $p_3^{(0)} = 0.3$

find, $p_1^{(1)}$, $p_2^{(1)}$ and $p_3^{(1)}$.

Module – IV

12. In a given M|M|1 queuing system, the average arrivals is 4 customer's per minute and $\rho = 0.7$. What is (i) mean number of customer in the system (ii) mean number of customer in the queue (iii) probability that the server is idle (iv) mean waiting time W_s in the system (v) mean waiting time W_q in the queue ?

13. Find the mean and auto correlation of Random telegraph process.

(Pages : 4) **2304**

Fourth Semester B.Tech. Degree Examination, May 2013
(2008 Scheme)
Branch : Electronics and Communication
08.401 ENGINEERING MATHEMATICS III – PROBABILITY AND RANDOM PROCESSES (TA)

Time: 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions of Part – **A** and one **full** question **each** from Module – **I**, Module – **II** and Module – **III** of Part – **B**.*

PART – A

1. The probability function of an infinite discrete distribution is given by $P[X = x] = \frac{1}{2x}$ $(x = 1, 2, \ldots \infty)$ verify that the total probability is 1 and also find the mean and variance of the distribution.

2. The probability density function of a random variable X is

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 \le x < 2 \\ 0 & x \ge 2 \end{cases}$$ Find the cumulative distribution function of X.

3. A normal distribution has mean $\mu = 20$ and standard deviation $\sigma = 10$.

   Find $P(15 \le X \le 40)$.

4. The joint probability density function of a two-dimensional random variable (X, Y) is

$$f(x, y) = \begin{cases} \frac{8}{9}xy, & 1 < x < y < z \\ 0, & \text{otherwise} \end{cases}$$

   i) Find the marginal density function of X.

   ii) Find the conditional density function of Y given X = x.

P.T.O.

5. Show that the linear correlation coefficient lies between -1 and 1.

6. Define SSS process and WSS process. What is the difference between them?

7. Find the mean and variance of a Poisson process.

8. Given that the autocorrelation function for a stationary ergodic process with no periodic components is $R(\tau) = 25 + \frac{4}{1+6\tau^2}$

   Find the mean value and variance of the process $\{X(t)\}$.

9. State Wiener – Khinchin Theorem. If the autocorrelation function $R(\tau) = 1$ find the spectral density $S(w)$.

10. Let $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ be a stochastic matrix. Examine whether it is regular.

(10×4= 40 Mark

PART – B

Module – I

11. a) Out of 800 families with 4 children each, how many families would be expected to have

   i) 2 boys and 2 girls

   ii) atmost 2 girls

   iii) children of both sexes.

   b) If X has a distribution with probability density function $f(x) = e^{-x}$, $0 \leq x < \infty$. Use Chebychev's inequality to obtain the lower bound to the probability $P(-1 \leq x \leq 3)$ and compare it with the actual value.

   c) The joint probability density function of two random variables X and Y is given by $f(x, y) = kxy\ e^{-(x^2+y^2)}$, $x > 0, y > 0$.

   Find the value of k and also prove that X and Y are independent. **(7+7+6=20)**

12. a) An electrical firm manufactures light bulbs that have a life, before burn out, that is normally distributed with mean equal to 800 hrs and a standard deviation of 40 hrs. Find the probability a bulb burns

   i) more than 834 hrs

   ii) less than 900 hrs

   iii) between 778 and 834 hrs.

b) Find the coefficient of correlation between X and Y from the following data

X :	10	14	18	22	26	30
Y :	18	12	24	16	30	36

c) The joint probability distribution two random variables X and Y is given by

X \ Y	0	1	2
0	0.1	0.04	0.06
1	0.2	0.08	0.12
2	0.2	0.08	0.12

Examine whether X and Y are independent ? **(7+7+6=20)**

Module – II

13. a) Show that the random process

$X(t) = A \cos \lambda t + B \sin \lambda t$, where A and B are random variables is a wide-sense stationary if

i) $E(A) = E(B) = 0$　　ii) $E(A^2) = E(B^2)$ and　　iii) $E(AB) = 0$.

b) If $\{X(t)\}$ is a WSS process with autocorrelation function $R(\tau) = Ae^{-\alpha|\tau|}$ determine the second order moment of the random variable $X(8) - X(5)$.

c) On the average, a submarine on patrol sights 6 enemy ships per hour. Assuming that the number of ships sighted in a given length of time is a Poisson variate; find the probability of sighting

i) 6 ships in the next half an hour

ii) at least one ship in the next 15 minutes. **(7+7+6=20)**

14. a) Show that the random process $X(t) = A \cos(\omega_0 t + \theta)$ is a wide-sense stationary where A and ω_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$.

b) Two random process $\{X(t)\}$ and $\{Y(t)\}$ are defined by $X(t) = A\cos\omega t + B\sin\omega t$ and $Y(t) = B\cos\omega t - A\sin\omega t$. Show that X(t) and Y(t) are jointly wide-sense stationary, if $E(A) = E(B) = 0$, $E(A^2) = E(B^2)$; $E(AB) = 0$ and ω is a constant.

c) A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 5 minutes period.

(7+7+6=

Module – III

15. a) If $\{X(t)\}$ is a random signal process with $E[X(t)] = 0$ and $R(\tau) = e^{-2\lambda|\tau|}$. Find the mean and variance of the time average of $\{X(t)\}$ over $(-T, T)$. Is it mean-ergodic ?

b) Consider a Markov chain with state space $\{0, 1, 2\}$ and the transition probability matrix is $P = \begin{bmatrix} 0 & 1 & 0 \\ 1-p & 0 & p \\ 0 & 1 & 0 \end{bmatrix}$

i) Find P^2 and show that $P^2 = P^4$ ii) Find P^n, $n \geq 1$

c) Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}, \alpha > 0$.

(7+7+6=

16. a) Define irreducible Markov chain. Show that the matrix

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

is the transition probability matrix of an irreducible Markov chain ?

b) If $\{X(t)\}$ is a WSS process with $E[X(t)] = 2$ and $R(\tau) = 4 + e^{-|\tau|/10}$

Find the mean and variance of $S = \int_0^1 X(t)dt$.

c) Find the power spectral density of the random binary transmission process whose autocorrelation function is

$$R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T} & \text{for } |\tau| \leq T \\ 0 & \text{elsewhere} \end{cases}$$

(7+7+6=2

5709

Fifth Semester B.Tech. Degree Examination, November 2012
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV (ERFBH)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions, **all** questions carry **equal** marks.

1. Find the constant 'C', so that $f(x) = C\left(\frac{2}{3}\right)^x$; $x = 1, 2, 3, \ldots$ satisfies the p.d.f of a discrete random variable X.

2. A random variable X is defined as the sum of the number of faces when two dice are thrown. Find the mean of X.

3. If X follows a Poisson distribution and $P(X = 0) = P(X = 1) = k$. Find 'k'.

4. Four coins were tossed simultaneously. What is the probability of getting (i) 2 heads (ii) at most 2 heads ?

5. Convert the equation $y = ax + bx^2$ to a linear form and write the corresponding normal equations to fit it.

6. The mean of a sample of size 20 form a normal population $N(\mu, 8)$ was found to be 81.2. Find a 90% confidence interval for μ.

7. A normal population has a mean of 6.8 and S.D of 1.5. A sample of 400 members gave a mean of 6.75. Is the difference significant.

8. What is the difference between a random variable and a random process ?

9. The joint distribution of X and Y is given by $f(x, y) = \frac{x + y}{21}$; $x = 1, 2, 3$; $y = 1, 2$. Find the marginal distribution.

10. Find the mean and variance of the stationary process with autocorrelation $R_x(\tau) = 16 + \frac{9}{1 + 6\tau^2}$.

P.T.O.

PART – B

Answer **one full** question from **each** module.

Module – I

11. a) Let X have the p.d.f. $f(x) = \begin{cases} \dfrac{x+2}{18}, & \text{if } -2 < x < 4 \\ 0, & \text{otherwise} \end{cases}$. Find its mean and variance.

b) Five unbiased coins are tossed and no. of heads are noted. The experiment is repeated 64 times and the following data is obtained.

No. of heads	0	1	2	3	4	5	Total
Frequencies	3	6	24	26	4	1	64

Fit a binomial distribution to the following data.

c) Six dice are thrown 729 times. How many times do you expect at least 3 dice to show a five or six.

OR

12. a) If X is a normal variate with mean 30 and S.D. 5. Find the probabilities that (i) $26 \leq X \leq 40$ (ii) $X \geq 45$ (iii) $|X - 30| > 5$.

b) The distribution function of a random variable X is given by $F(x) = 1 - (1 + x)e^{-x}$, $x \geq 0$. Find its (i) p.d.f. (ii) mean and (iii) variance.

c) The daily consumption of milk in excess of 20,000 gallons is approximately exponentially distributed with $\lambda = 3000$. The city has a daily stock of 35,000 gallons. What is the probability that of 2 days selected at random, the stock is insufficient for both days ?

Module – II

13. a) Fit a straight line to the following data using method of least squares.

x :	0	1	2	3	4
y :	1	1.8	3.3	4.5	6.3

b) Find the least sample size required if the length of the 95% confidence interval for the mean of a normal population with S.D 5 should be less than 6.

c) The heights of students studying in college classes is believed to be distributed with S.D 1.5. A sample of 400 students have their mean height 4.75 ft. Does this contradict the hypothesis that the mean height of students is 4.48 ft ?

OR

14. a) The average income of persons was Rs. 120 with a S.D. of Rs. 10 in a sample of 100 people in a city. For another sample of 150 persons, the average income was Rs. 220 with S.D. of Rs. 12. The S.D. of incomes of the people of the city was Rs. 11. Test whether there is any significant difference between the average incomes of the localities.

b) Intelligence tests of two groups of boys and girls gives the following results :

Girls	Mean = 84	S.D. = 10	N = 121
Boys	Mean = 81	S.D. = 12	N = 81

i) Is the difference in mean scores significant.

Module – III

15. a) The two dimensional random variable (X, Y) has the joint density function

$f(x, y) = \frac{x + 2y}{27}$, x = 0, 1, 2 and y = 0, 1, 2. Find the conditional distribution of Y for X = x. Also find the conditional distribution of X given Y = 1.

b) The joint p.d.f of X and Y is given by $f(x, y) = kxy\, e^{-(x^2 + y^2)}$, x > 0, y > 0. Find the value of k and prove also that X and Y are independent.

c) Define power spectrum density function. Find power spectrum density of a random process if its autocorrelation function is $R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & \text{if } |\tau| < T \\ 0, & \text{otherwise} \end{cases}$.

OR

16. a) Define a random process and give an example of a random process.

b) S.T the Poisson process $\{X(t)\}$ given by the probability la

$$P\{X(t)=n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, \; n = 0, 1, 2, \ldots \text{ is not stationary.}$$

c) The T.P.M. of a Markov chain $\{X_n\}$ having 4 states 0 1, 2, 3 is

$$P = \begin{bmatrix} 0.2 & 0.8 & 0.0 & 0 \\ 0 & 0.2 & 0.8 & 0 \\ 0 & 0 & 0.2 & 0.8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the initial distribution is $P(0) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. Find $P[X_3 = 2]$.

(Pages : 4) **1343**

Fifth Semester B.Tech. Degree Examination, May 2012
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV (ERFBH)

Time : 3 Hours Max. Marks : 100

*Instruction : Answer **all** questions from Part **A** and **one full** question from each Module of Part **B**.*

PART – A (4×10=40 Marks)

1. If $f(x) = \begin{cases} K(2x+3), & 0 < x < 2 \\ 0 & , \text{otherwise} \end{cases}$ is a probability density function, find K and F(x).

2. Six coins are tossed 3200 times, using Poisson distribution to obtain approximate probability of getting 6 heads x times.

3. Find the mean and variance of uniform distribution.

4. The mileage which a Car owner gets with a certain kind of tyre is a random variable having an exponential distribution with mean 40,000 kms. Find the probability that one of these tyres will last atleast 30,000 kms.

5. Two lines of regression are $5x - 6y + 90 = 0$ and $15x - 8y = 130$, find $\bar{x}$, $\bar{y}$ and γ.

6. A random sample of 900 items with mean 3.5 is drawn from a population with SD 2.61. Find the 95% confidence interval for the mean.

7. Define (i) Type I error (ii) Type II error (iii) Power of a test and (iv) Critical region.

8. Suppose X(t) is a process with $\mu(t)=3$ and $R(t_1, t_2) = 9 + 4e^{-|t_1-t_2|/5}$. Find variance and covariance of X(5) and X(8).

9. Find the average power of the random process if its spectral density in

$$S(w)=\frac{1}{1+w^2}.$$

10. Define (i) Markov process (ii) Markov chain (iii) Poisson process and (iv) Transition probability matrix.

PART – B

Answer **one full** question from **each** Module. **(3×20=60 Marks)**

Module – I

11. a) A machine manufacturing screws is known to produce 5% defectives. In a random sample of 15 screws, what is the probability that there are (i) exactly three defectives (ii) not more than 3 defectives.

b) In a normal distribution, 5% of the items are under 60 and 40% are between 60 and 65. Find the mean and SD of the distribution.

c) If X has a uniform distribution in (– k, k), k > 0, find k such that P[| X | < 1] = P[| X | > 1].

OR

12. a) Fit a Poisson distribution to the following data :

x :	0	1	2	3	4
f :	63	28	6	2	1

b) In a test on 3000 electric bulbs, it was found that the life of a particular make was normally distributed with an average of 3040 and SD of 60 hours. Estimate the number of bulbs likely to burn for

i) more than 3250 hours

ii) less than 2850 hours and

iii) more than 2920 hours but less than 3160 hours.

c) The time in hours required to repair a machine is exponentially distributed with mean 30 hours. What is the probability that the required time

i) exceeds 35 hours

ii) in between 25 hours and 34 hours

iii) atmost 20 hours.

Module – II

13. a) Use the principle of least squares to fit a straight line to the following data :

x :	0	5	10	15	20
y :	7	11	16	20	26

b) In a sample of 20 persons from a town it was seen that 4 are suffering from T.B. Find a 90% confidence limits for the proportion of T.B. patients in the town.

c) Random samples of sizes 500 and 600 are found to have means 11.5 and 10.5 respectively. Can the samples be regarded as samples drawn from the same population whose SD is 6.

OR

14. a) Find the coefficient of correlation from the following data :

x :	20	22	23	25	25	28	29	30	30	34
y :	18	20	22	24	21	26	26	25	27	29

b) A sample of 10 items gave a mean 4.2 and a SD 2.78. Find a 99% confidence limits for the population mean.

c) Random samples of sizes 500 and 600 are found to have means 11.5 and 10.5 respectively. Can the samples be regarded as samples drawn from the same population whose SD is 6.

15. a) The joint pdf of X and Y is given by $f(x,y) = \begin{cases} x+y, & 0 < x,y < 1 \\ 0 &, \text{elsewhere} \end{cases}$

Find : (i) $P[X \le 1/2]$ (ii) $P[X+Y \le 1]$ (iii) the marginal distributions of X and Y.

b) Show that the random process $X(t) = A \sin(\omega t + \theta)$ where A and ω are constants, θ is uniformly distributed in $(0, 2\pi)$, is WSS.

c) The power spectral density of a random process $\{X(t)\}$ is given by $S(\omega) = \begin{cases} 1+\omega^2, & \text{for } |\omega| \le 1 \\ 0 &, \text{elsewhere} \end{cases}$. Find its auto correlation function.

OR

16. a) The joint density of X and Y is given by

$$f(x, y) = \begin{cases} xy^2 + \frac{1}{8}x^2, & 0 \le x \le 2, 0 < y < 1 \\ 0 &, \text{elsewhere} \end{cases}$$

. Compute (i) $P(X > 1)$ (ii) $P[X+Y \le 1]$ (iii) $P[X < Y]$.

b) Find the spectral density of the random process whose autocorrelation function is $R(\tau) = \frac{1}{2}e^{-|\tau|}$.

c) The tpm of a Markov chain $\{X_n\}$ having 4 states 0, 1, 2, 3 is

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.2 & 0 & 0.8 & 0 \\ 0 & 0.2 & 0 & 0.8 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and the initial distribution is $P[X_0 = i] = 0.25$ for $i = 0, 1, 2, 3$.

Find :

i) $P[X_2 = 2, X_1 = 1 \mid X_0 = 2]$

ii) $P[X_2 = 2, X_1 = 1, X_0 = 2]$

iii) $P[X_2 = 3]$

Name

Fourth Semester B.Tech. Degree Examination, May 2011
(2008 Scheme)
Branch : Electronics and Communication
P 10.201/08.401 : ENGINEERING MATHEMATICS – III
Probability and Random Processes (TA)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Suppose a fair die is rolled twice and let X be a random variable taking the value of first roll minus the value of the second roll. Obtain the probability distribution of X.

2. If X is uniformly distributed in (–3, 3), find (i) E(X) (ii) $P(|X-2|<2)$.

3. If the number of items produced in a factory during a week is a random variable with mean 500 and variance 100, what can be said about the probability that the production in a particular week will be between 400 and 600 ?

4. If the joint probability density function of (X, Y) is $f(x, y) = 2, 0 < x < y < 1$, find the marginal density functions of X and Y.

5. Define a stationary process. Prove that a first order process has a constant mean.

6. Prove that the auto correlation function $R(\tau)$ of a random process $\{X(t)\}$ is maximum at $\tau = 0$.

7. A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in a 4 minute period.

8. Consider the Markov chain with TPM given by

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1/2 & 1/8 & 1/8 & 1/4 \end{array}$$

Show that it is ergodic.

9. Prove that the spectral density function of a real random process is an even function.

10. Define a linear time invariant system. Give an example.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) The amount of time a computer functions before breaking down is a continuous random variable with probability density function $f(x) = \lambda e^{-x/100}$ for $x > 0$ and 0 for $x < 0$. What is the probability that (i) The Computer will function between 50 and 150 hours before breaking down ? (ii) It will function less than 100 hours ?

b) Find 'k' for which the following is a probability distribution. Also find the mean and variance.

X	:	0	1	2	3
P(X)	:	k/2	k/3	(k + 1)/3	(2k – 1)/6

c) If the probability that a person hits the target on any trial is 0.8, what is the probability that he will finally hit the target, (i) On the 4^{th} trial and (ii) In fewer than 4 trials ?

12. a) A manufacturer produces airmail envelops. The weight of these envelops follows a normal distribution with mean 1.95 gm and standard deviation 0.025 gm. The envelops are sold in lots of 1000. How many envelops in a lot may be heavier than 2 gms ?

b) If the joint distribution function of X and Y is given by

$F(x, y) = (1 - e^{-x})(1 - e^{-y})$ for $x, y > 0$ and 0 otherwise

i) Find the marginal densities of X and Y.

ii) Are X and Y independent ?

iii) Find $P(1 < X < 3, 1 < Y < 2)$.

Module – II

13. a) Describe the various classifications of Random processes with examples.

b) Prove that the autocorrelation $R(t_1, t_2)$ of an SSS process is a function of $t_1 - t_2$.

c) Show that the random process $X(t) = \sin(\omega t + Y)$ where Y is uniformly distributed in $(0, 2\pi)$ is wide sense stationary.

14. a) Find the mean and autocorrelation of a Poisson process.

b) Let two random processes $\{X(t)\}$ and $\{Y(t)\}$ be defined as $X(t) = A\cos\omega t + B\sin\omega t$ and $Y(t) = B\cos\omega t - A\sin\omega t$ where ω is a constant and A and B are random variables such that $E(A) = E(b) = 0$, $E(AB) = 0$, $E(A^2) = E(B^2)$. Prove that $\{X(t)\}$ and $\{Y(t)\}$ are jointly wide sense stationary.

Module – III

15. a) Define a Markov chain and give an example.

b) A gambler has Rs. 2. He bets Re. 1 at a time and wins Re. 1 with probability $^1/_2$. He stops playing if he losses Rs. 2 or wins Rs. 4.

i) What is the TPM of the related Markov chain ?

ii) What is the probability that he has lost his money at the end of 5 plays ?

iii) What is the probability that the game lasts more than 7 plays ?

16. a) If $\{X(t)\}$ is a process for which $C(\tau) = q e^{-a|\tau|}$, show that X(t) is mean ergodic.

b) The autocorrelation function of a process is given by $R(\tau) = a^2 e^{-2t|\tau|}$. Determine the power density spectrum of the process.

c) If X(t) is a Gaussian process with $\mu(t) = 10$ and $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$, prove that $P[X(10) \leq 10] = 0.3085$ and $P[|X(10) - X(6)| \leq 4] = 0.5222$.

Fifth Semester B.Tech. Degree Examination, October/November 2011
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV (ERFBH)

Time : 3 Hours Max. Marks : 100

*Instruction : Answer **all** questions from Part **A** and **one** full question from **each** Module of Part **B** it carries **20** marks.*

PART – A (10×4=40 Marks)

1. Find k for which the following is a probability distribution function. Also find the mean.

x :	0	1	2	3
f(x) :	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k+1}{3}$	$\frac{2k-1}{6}$

2. Find the mean and standard deviation of the normal distribution

   $f(x) = ke^{-2x^2+10x}$.

3. If X is uniformly distributed random variable with mean 1 and variance $4/3$. find $P[X<0]$.

4. Define exponential distribution and hence find its mean.

5. Show that the correlation coefficient lies between –1 and 1.

6. What are the criteria of a good estimator ? Explain.

7. A random sample of 16 newcomers gave a mean height of 1.67 m and a SD of 0.16 m. Is the mean height of the newcomers is significantly different from the mean height of previous years which is 1.6 m.

8. The joint density of X and Y is given by

$$f(x,y) = \begin{cases} e^{-(x+y)} & \text{for } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find $P\left[\frac{1}{2} < X < 2, 0 < Y < 4\right]$.

9. Find the mean and variance of the stationary process $R_x(\tau) = 25 + \frac{4}{1+6\tau^2}$.

10. Show that the random process $X(t) = A\cos(\omega t + \theta)$ is evolutionary if A and ω are constants and θ is uniformly distributed in $(0, \lambda)$.

PART – B

Answer **one** full question from **each** Module.

MODULE – I

11. a) Fit a Binomial distribution to the following data.

x :	0	1	2	3	4	5
f :	1	10	24	35	18	8

b) If 20% of the memory chips made in a certain factory are defective, what is the probability that in a lot of 100 randomly chosen for inspection, atmost 15 will be detective ? Use normal distribution.

c) If 1500 values approximate a Poisson distribution with mean 2.25, how many of them would have values 3, 5 and 7 ?

OR

12. a) Find k for which $f(x) = \begin{cases} kx(1-x), & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

is a pdf. Find the mean and standard deviation.

b) Show that Poisson distribution is a limiting case of Binomial distribution.

c) If X is normally distributed with mean 4 and SD 3, find $P[2.5 \leq X \leq 5.5]$.

a) Fit a parabola $y = a + bxy$ to the following data

x :	−4	1	2	3
y :	4	6	10	8

b) A random sample of size 16 has 53 as mean. The sum of the squares of the deviations from mean is 150. Obtain 95% confidence limits of the mean of the population.

c) The marks secured by a sample of 36 students of a college in a class give a mean of 55 and a SD of 10. Test the hypothesis that the mean of the college is 50 at 5% level of significance.

OR

. a) Find the two regression lines from the following data.

x :	1	2	3	4	5	6	7	8	9	10
y :	10	12	16	28	25	36	41	49	40	50

b) Find the least sample size required if the length of the 90% confidence interval for the mean of a normal population with SD 8 should be less than 10.

c) In a sample of 600 men from city A, 400 are consumers of wheat. In 900 from city B, 450 are consumers of wheat. Do these data reveal a significant difference between town A and town B, so far as the proportion of wheat consumers is concerned.

MODULE – III

15. a) The joint density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{x}{4}(1 + 3y^2), & 0 < x < 2, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find i) $f_X(x)$ ii) $f_Y(y)$ iii) $f(x|y)$

b) Find the mean and autocorrelation of the Poisson process $X(t) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}$ where k = 0, 1, 2.....

c) Find spectral density of the process $X(t)$ with autocorrelation function $R(\tau) = e^{-\alpha|\tau|}$.

OR

16. a) Show that the process $X(t) = \sin(\omega t + \phi)$ where ϕ is uniformly distributed in $(0, 2\pi)$, is WSS.

b) Find the average power of the random process if $S(w) = \frac{10w^2 + 35}{(w^2+4)(w^2+9)}$.

c) The tpm of a Markov chain $\{X_n\}$ having 3 states 1, 2 and 3 is

$$P = \begin{bmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{bmatrix}$$ and the initial distribution is P(0) = (0.7, 0.2, 0.1).

Find $P(X_3 = 2, X_2 = 1, X_1 = 1, X_0 = 2]$.

(Pages : 4) **1629 A**

Fourth Semester B.Tech. Degree Examination, May 2010
(2008 Scheme)
08-401 : ENGINEERING MATHEMATICS – III
(Probability and Random Processes) (TA)
Branch : Electronics and Communication Engineering

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Let X be a random variable taking values 1, 2 and 3 each with equal probability. If $Y = 2x + 1$, find (i) the probability distribution of Y (ii) E(Y).

2. Find the mean and variance of a random variable which is uniformly distributed on the interval [a, b].

3. The joint probability density function of a bivariate random variable (X, Y) is

$$f(x, y) = \begin{cases} 1/\pi & \text{if } x^2 + y^2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X^2 + Y^2 \leq 1/2)$

4. A computer generates 100 random numbers which are uniformly distributed between 0 and 1. Find approximately the probability that their sum is at least 50.

5. How is a random process different from a random variable ? Explain with examples.

6. The autocorrelation function of an ergodic process is $R_x(\tau) = \dfrac{25\tau^2 + 36}{4\tau^2 + 4}$. Find the mean, variance and total power of the process.

7. Find the autocorrelation of a Poisson process.

8. If X (t) and Y(t) are independent zero mean wide sense stationary processes, and $Z(t) = X(t) + Y(t)$, show that Z(t) is wide sense stationary.

9. The transition probability matrix of a Makov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$. Find the steady state distribution.

10. A random process X(t) consists of 3 member functions $x_1(t) = 1$, $x_2(t) = -3$ and $x_3(t) = \sin(2\pi t)$, each occurring with equal probability. Is this process mean ergodic ? Justify.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) A continuous random variable has cumulative distribution function.

$$F(x) = \begin{cases} 1 - e^{-x}, & x \geq 0 \\ 0, & \text{Otherwise.} \end{cases}$$

Find (i) $P(3 < X < 5)$ and (ii) $P(X > 5/X < 7)$

b) A communication system consists of 5 components each of which may fail independently with probability 0.15. The system will work if at least 3 of the components operate correctly. i) What is the probability that the system will work ? ii) What is the expected number of components working correctly ?

c) Prove that the probability mass function of a binomial random variable with parameters n and p approaches that of a Poisson random variable as $n \to \infty$ such that np stays constant.

12. a) The breaking strength of a certain kind of rope (in kg.) is normally distributed with mean 45 kg and standard deviation 1.8 kg. The rope is labelled grade A if its breaking strength is more than 43 kg and grade B otherwise. The manufacturer of the rope will get a profit of Rs. 1000 for each coil of grade A rope and of Rs. 400 per coil of grade B rope. Find the expected profit per coil.

b) The joint probability density function of a two-dimensional random variable (X, Y) is

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x, y \geq 0 \\ 0 & \text{Otherwise} \end{cases}$$

Find i) $P(X \leq 1, Y \leq 1)$ ii) $P(X + Y \leq 1)$ iii) Are X and Y independent ?

Module – II

13. a) X_n is a random process defined by $X_n = Z_1 + Z_2 + \ldots Z_n$, $n = 1, 2, \ldots$, where Z_i are independent random variables, each taking value -1 or 1 with equal probability.

i) Find the mean, autocorrelation and autocovariance of X_n.

ii) Is the process stationary ?

b) State and prove any two characteristic properties of the autocorrelation function of a wide sense stationary process.

14. a) X(t) is a zero mean, wide sense stationary (WSS) process and $Y(t)=X(t)\cos(w_0 t+\Theta)$ where w_0 is a constant and Θ is a ramdom variable which is uniformly distributed in $[0, 2\pi]$ and is independent of X(t). Show that

i) $R_{xy}(\tau) = 0$ ii) $Y(\tau)$ is WSS

b) The number of failures which occur in a computer network follows a Poisson process. On the average one failure occur in every 4 hours. What is the probability of

i) at most one failure in the first 8 hours ?

ii) at most one failure in the first 8 hours and at least two failures in the next 8 hours ?

Module – III

15. a) The autocovariance function of a wide sense stationary process X(t) is $C(\tau) = e^{-|\tau|}$. Prove that X(t) is mean -ergodic.

b) Three boys A, B and C are throwing a ball to one another. A always throws the ball to B and B always thows the ball to C, but C is just as likely to throw the ball to B as to A.

i) Regardings the process as Markov, set up the transition matrix.

ii) If initially the ball is with C what is the probability that the ball is with B after two passes ?

iii) Find the probability of finding the ball with each of them in the long run.

16. a) In each of the following examine whether f(w) could be the power spectral density (PSD) of a wide sense stationary process. Explain your reasoning.

i) $f(w) = \begin{cases} \sin w / w & w \neq 0 \\ 0, & \text{otherwise} \end{cases}$

ii) $f(w) = \begin{cases} \pi, & \text{if } |w| < 1 \\ 0, & \text{otherwise} \end{cases}$

If f(w) is a valid PSD find the corresponding autocorrelation function.

b) Show that if the input to a linear time invariant system is a wide sense stationary (WSS) process, then the output process is also WSS.

(Pages : 3)

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Fifth Semester B.Tech. Degree Examination, December 2008

(2003 Scheme)

03.506 : PROBABILITY AND RANDOM PROCESSES (TA) (Elective – I)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions.

1. A binary symmetric channel has transmitting a "0" with probability $P[X=0]=\frac{1}{2}$. The symbol will be received with probability $P[Y=0]=0.9$. Obtain the following
   a) $P[Y=1 \mid X=0]$
   b) $P[Y=1 \mid X=1]$
   c) $P[X=0, Y=0]$
   d) $P[X=1, Y=0]$

2. A fair coin is flipped three times. What is the probability of obtaining two heads and one tail ?

3. State the relationship between probability density function and probability distribution function. Obtain the mean and variance of a uniformly distributed random variable, between the limits [a, b].

4. State the properties of joint pdf of two random variables X and Y.

5. State and prove Baye's Theorem.

6. The PDF of a random variable X is given by $F_X(x) = (1-e^{-x})\,u(x)$. Find the probability of the event $\{X < 1 \text{ or } X > 2\}$.

7. Compute E [X] when X is a Bernoulli r.v. that is,

$$X = \begin{cases} 1, & P_x(1) = p > 0 \\ 0 & P_x(0) = 1-p > 0 \end{cases}$$

P.T.O.

8. Define recurrent states and transient states of a finite state Markov chain.

9. A process S (t) has the auto correlation function $R(\tau) = 2\alpha e^{-\alpha|\tau|}$. Determine its predictor.

10. Differentiate between filtering and prediction. **(10×4=40 Marks)**

(10×6=60 Marks)

PART – B

Answer **any two** questions from **each** Module.

MODULE – I

11. A biased coin is tossed till a head appears for the first time. What is the probability that the number of required tosses is old ? 10

12. A coin is flipped three times and the r.v. X denotes the total number of heads that turn up. The probability of a head in one flip of the coin is p.

a) What values can the r.v. X take ?

b) What is the pmf of x ?

c) Derive the cumulative distribution function of X.

d) What is the probability that $X > 1$? 10

13. A r.v. Φ is uniformly distributed on the interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ find the pdf of $X = \tan\Phi$. Find the mean and variance of X. 10

MODULE – II

14. a) State six properties of joint pdf.

b) Test the function $g(x, y) = \begin{cases} b\, e^{-x} \cos y \\ 0, \quad \text{otherwise} \end{cases}$ $, 0 \le x \le 2$ and $0 \le y \le \pi/2$ for a valid pdf and obtain the condition. **(4+6=10)**

15. Discrete r.v.s. X and Y have a joint distribution function

$$
\begin{aligned}
F_{X,Y}(x, y) = \; & 0.1\, u(x+4)\, u(y-1) + 0.15\, u(x+3)\, u(y+5) \\
& + 0.17\, u(x+1)\, u(y-3) + 0.05\, u(x)\, u(y-1) \\
& + 0.18\, u(x-2)\, u(y+2) + 0.23\, u(x-3)\, u(y-4) \\
& + 0.12\, u(x-4)\, u(y+3)
\end{aligned}
$$

Find a) the marginal distributions $F_X(x)$ and $F_Y(y)$

b) sketch the two functions. **(5+5=10)**

16. The joint pdf of 4 r.v.s. X_i, $i = 1, 2, 3$ and 4 is

$f_{x_1,x_2,x_3,x_4}(x_1, x_2, x_3, x_4) \prod_{i=1}^{4} \exp(-2|x_i|)$ find densities.

a) $f_{x_1,x_2}(x_1, x_2, x_3 | x_4)$

b) $f_{x_1,x_2}(x_1, x_2 | x_3, x_4)$ **(5+5=10)**

MODULE – III

17. For a random process $X(t) = A\cos(\pi t)$ A is a Gaussian r.v. with zero mean and variance σ_A^2.

a) Find the density functions of X (0) and X (1)

b) Is X (t) stationary in any sense ? **(5+5=10)**

18. a) Prove the Parseval's theorem on r.v. X (t).

b) Determine which of the following functions can and can not be valid PSDS. For those that are not, explain why.

a) $\dfrac{w^2}{w^6 + 3w^2 + 3}$ b) $\dfrac{w^2}{w^4 + 1} - \delta(w)$ **(5+5=10)**

19. If X (t) is a stationery process find the power spectrum of $Y(t) = A_0 + B_0 X(t)$ in terms of PSD of X (t) if A_0 and B_0 are real constants. **10**