

# CS6660 Assignment

Every question carries 10 marks, Total 50 Marks. Submission Deadline : 4 Dec 2023

Answer all parts of a question together. Write/type the solutions neatly and turn them in.

Strictly no plagiarism. Plagiarized submissions will be penalized heavily. Check this link to understand departments stand regarding plagiarism:  
<https://cse.iith.ac.in/academics/plagiarism-policy.html>

## 1 Linear Algebra

1. Let  $V, W$  be two vector spaces over  $\mathbb{R}$  with  $\dim(V) = \dim(W) = n$ . Show that  $V$  is isomorphic to  $W$ .
2. Let  $v = (v_1, v_2)$  and  $w = (w_1, w_2)$  be any two vectors in  $\mathbb{R}^2$  over  $\mathbb{R}$ . We define

$$\langle v, w \rangle := v_1 w_1 - (v_1 w_2 + v_2 w_1) + 2v_2 w_2.$$

For example, if  $v = (1, 2)$  and  $w = (3, 4)$ , then  $\langle v, w \rangle = 1 \cdot 3 - (1 \cdot 4 + 2 \cdot 3) + 2 \cdot 2 \cdot 4 = 3 - (4 + 6) + 16 = 9$ . Show that this is indeed a valid inner product by verifying that it satisfies all the properties of a general inner product.

## 2 Probability

3. Suppose there is a covid test for which the probability of false positive (incorrectly showing positive when the patient is not infected) is 0.1, and the probability of false negative is 0.2. Further assume that 10% of the population is infected by covid. Suppose a random person tests positive, then what is the probability that he/she is infected by covid?
4. Let  $X$  be a continuous random variable. The density function for  $X$  is given by:

$$f_X(x) = \begin{cases} 1/x & \text{if } C \leq x \leq e \\ 0 & \text{otherwise} \end{cases}$$

- (a) For what value of  $C$  is  $f_X(x)$  a valid density function?
  - (b) What is the expectation of  $X$ ?
  - (c) What is the probability that  $X > 2$ ?
5. (a) Suppose the world cup streaming website gets a million ( $10^6$ ) hits in expectation every day. Provide an upper bound on the probability that it gets more than 2 million hits on a given day using Markov's inequality.
- (b) Suppose the expected number of daily hits on the streaming website is a million with variance  $10^{10}$ . Provide a tighter upper bound on the probability that it gets more than 2 million hits on a given day, using Chebyshev's inequality.