Robustness evaluation of a minimal RBF neural network for nonlinear-data-storage-channel equalisation

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Abstract: The authors present a performance-robustness evaluation of the recently developed minimal resource allocation network (MRAN) for equalisation in highly nonlinear magnetic recording channels in disc storage systems. Unlike communication systems, equalisation of signals in these channels is a difficult problem, as they are corrupted by data-dependent noise and highly nonlinear distortions. Nair and Moon (1997) have proposed a maximum signal to distortion ratio (MSDR) equaliser for data storage channels, which uses a specially designed neural network, where all the parameters of the neural network are determined theoretically, based on the exact knowledge of the channel model parameters. In the present paper, the performance of the MSDR equaliser is compared with that of the MRAN equaliser using a magnetic recording channel model, under conditions that include variations in partial erasure, jitter, width and noise power, as well as model mismatch. Results from the study indicate that the less complex MRAN equaliser gives consistently better performance robustness than the MSDR equaliser in terms of signal to distortion ratios (SDRs).

1 Introduction

Digital magnetic recording technology has made considerable progress in the last decade, especially in the areas of storage capacity and data transfer [1]. However, high recording densities and nonlinearity in the read-back process cause partial erasure, peak jitter and width variations in the read-back signal [2–4]. The channel characteristics also vary with head-medium separation, temperature and the relative velocity of the medium with respect to the head. There is also noise arising from the read-electronics, which is usually modelled as additive, white and Gaussian. Because of these reasons, it is necessary to use a nonlinear equaliser in the detection process to overcome the inter-symbol interference (ISI) problem in the recording channels.

Because of their ability to approximate nonlinear functions, multi-layer feedforward neural networks have been proposed as nonlinear equalisers for recording channels [5, 6]. However their use is severely restricted by the long training times and the lack of a methodology for the network-architecture selection. To overcome this problem, Nair and Moon [7, 8] have recently proposed a neural-network equaliser, in which the network architecture and the weight parameters are derived theoretically to obtain a maximum signal to distortion ratio (MSDR).

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In [7], the MSDR's decision boundaries are shown to be geometrically close to those obtained from a neural network trained by the backpropagation (BP) algorithm. It should be noted that the MSDR method does not require any training. The number of hidden layers and hidden neurons are worked out as part of the theoretical calculation, unlike a BP network where these have to be selected using a trial and error process. Hence, whenever the channel characteristics are known, Nair and Moon have shown that their theoretically designed neural network is to be preferred to other BP neural net-based equalisers.

Recently, a new RBF network-learning algorithm, called the minimal resource allocation network (MRAN), was developed by Yingwei *et al.* [9, 10]. This sequential learning algorithm employs a scheme for adding and pruning the RBF's hidden neurons, so as to achieve a parsimonious network. The MRAN was first introduced in [11] for equalisation problems in communication channels and in [12] for equalisation of magnetic recording channels. In [12], the MRAN was trained and tested using the same channel model and parameters such as partial erasure, jitter etc. Its performance was found to be superior to that of the MSDR equaliser for these conditions. The MSDR equaliser design requires an exact knowledge of the channel model (impulse response) parameters, the partial erasure parameter and the noise power. However, the MRAN equaliser only requires input–output training data to build the network. In reality, there will be always some mismatch between the actual channel parameters and the ones used in the equaliser design, and also the parameters may be time varying. In this paper, the robustness of the MRAN equaliser for nonlinear data recording channel problems is presented. The MRAN is first trained using nominal values of these parameters and then tested when these parameters are varied over a wide range. The performance of the MRAN is compared with that of the MSDR equaliser under the same conditions, and a comparison between the MRAN and MSDR equaliser complexities is also highlighted.

2 Magnetic recording-channel modelling

In this Section, a very brief introduction to the magnetic storage channel is presented. For details, refer to [2]. The input signal $\{b_k\}$ of the magnetic channel is a bipolar, $\{1, -1\}$ sequence. The differentiating nature of the recording channel is incorporated in the sequence of the magnetic flux transition $\{a'_k\}$, which is represented by the equation $a'_k = b_k - b_{k-1}$. At high densities, a magnetic transition undergoes partial erasure in all cases, except when it is the last one in a cluster of an odd number of consecutive transitions. The output signal can be expressed as

$$a_k = (1 - \chi)a'_k \tag{1}$$

where χ denotes the partial erasure parameter. The read channel is assumed to have a Lorentzian-type transition response $h(t, w)$, where t is the time and w is the width parameter. Assuming a front-end filter matched to $h(t, w)$ and a symbol-rate sampler, the digital magnetic channel model along with the equaliser is shown in Fig. 1.

The read-back response sequence can be written as

$$Z_k = \sum_{i=-L}^{L} (h_i a_{k-i} + (h_i^t \Delta t_{k-i} + h_i^w \Delta w_{k-i}) a_{k-i}) + n_k \tag{2}$$

where h_i is the sampled auto-correlation function of $h(t, w)$; h_i^t and h_i^w are the sampled derivatives of $h(t, w)$ with respect to t and w, respectively [13]. The former represents magnetic transition 'jitter' and the latter, 'width variation', both of which arise from the transition noise. In our study, the quantities of Δt_k and Δw_k are modelled as independent and identically distributed Gaussian random variables. n_k is the sampled additive white Gaussian noise (AWGN) sequence representing the electronics noise. The aim of the equaliser is to classify the corrupted read-back data sequence into the three classes of $\{-2, 0, +2\}$ based on the magnetic transition sequence of a'_k.

3 Minimal resource-allocation network (MRAN)

The MRAN is a minimal radial basis function neural network developed by Yingwei *et al.* [9, 10].

The output of an MRAN has the following form:

$$f(\boldsymbol{X}_k) = \alpha_0 + \sum_{n=1}^{h} \alpha_n \phi_n(\boldsymbol{X}_k) \tag{3}$$

where $\phi_n(\boldsymbol{X}_k)$ is the response of the nth-hidden neuron to the input vector $\boldsymbol{X}_k$ and α_n is the weight connecting the nth-hidden unit to the output unit. α_0 is the bias term and h represents the number of hidden neurons in the network. $\phi_n(\boldsymbol{X}_k)$ is a Gaussian function given by

$$\phi_n(\boldsymbol{X}_k) = \exp\left(-\frac{1}{(\sigma_n)^2} \|\boldsymbol{X}_k - \boldsymbol{\mu}_n\|^2 \right) \tag{4}$$

where $\boldsymbol{\mu}_n$ is the centre and σ_n is the width of the Gaussian function. $\|\|$ denotes the Euclidean norm. In the MRAN algorithm, the network begins with no hidden neurons. As each lot of input or output training data $(\boldsymbol{X}_k, d_k)$ is received, the network is built up based on certain growth criteria. The algorithm adds/prunes hidden neurons, as well as adjusting the existing network parameters. A brief outline of the various steps in the MRAN is given below. For details see [9, 10].

Step 1. Obtain an input $\boldsymbol{X}_k$ and calculate the network output $f(\boldsymbol{X}_k)$.

Step 2. Create a new hidden neuron if the following conditions are met:

(i) The error $\| f(\boldsymbol{X}_k) - d_k \|$ exceeds a threshold value (e_{min}),

(ii) the root mean square of the error (e_{rms}) computed over a window (N_w) of past data exceeds a threshold (e_{min1}), and

(iii) the new input is sufficiently far $(>\varepsilon_{max})$ from the centres of the existing neurons.

Step 3. If the conditions in step 2 are not met, adjust the weights and widths of the existing RBF network using an extended Kalman filter (EKF). In addition, a pruning strategy is also adopted:

(i) If a hidden neuron's normalised contribution to the network output for a certain number of consecutive inputs (S_w) is below a threshold (δ), then that neuron is pruned.

(ii) The dimensions of EKF are adjusted and the next input is processed.

This paper presents the use of the MRAN equaliser when the channel parameters $(h_i, h_i^t$ and $h_i^w)$, partial erasure (χ), jitter, width and AWGN vary. The MRAN is first trained using nominal values of these parameters and then tested when these parameters are varied over a wide range. The MRAN's performance is also compared with that of the MSDR equaliser under these conditions.

4 Performance comparison of MRAN and MSDR equalisers

In this Section, the realistic recording channel model, discussed in Section 2, is used to compare the SDR performance of the MRAN equaliser with that of the MSDR equaliser [7].

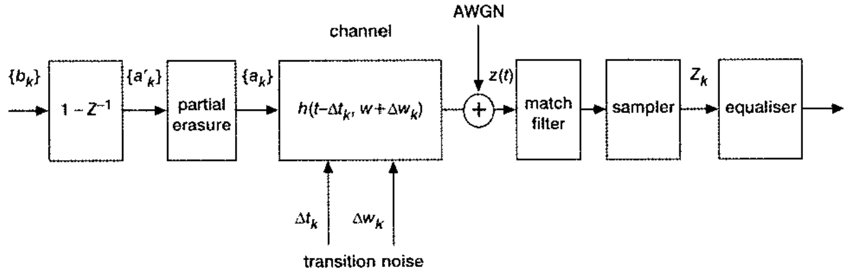

Fig. 1 *Nonlinear magnetic channel model with equaliser*

4.1 Problem description

The impulse response matrices for the channel considered are given by [7]

$$\boldsymbol{h} = \begin{bmatrix} 0.1480 & 0.7132 & 0.2574 \end{bmatrix}' = \begin{bmatrix} h_{-1} & h_0 & h_1 \end{bmatrix}' \quad (5)$$

$$\boldsymbol{h}^t = \begin{bmatrix} 0.2476/T & 0.1356/T & -0.3512/T \end{bmatrix}' = \begin{bmatrix} h^t_{-1} & h^t_0 & h^t_1 \end{bmatrix}' \quad (6)$$

$$\boldsymbol{h}^w = \begin{bmatrix} 0.2593/T & -1.6230/T & -0.0996/T \end{bmatrix}' = \begin{bmatrix} h^w_{-1} & h^w_0 & h^w_1 \end{bmatrix}' \quad (7)$$

where $'$ denotes the transpose.

The read-back signals are given by the following equation:

$$Z_k = \begin{bmatrix} a_{k+1} & a_k & a_{k-1} \end{bmatrix} \begin{bmatrix} h_{-1} \\ h_0 \\ h_1 \end{bmatrix} + \begin{bmatrix} a_{k+1}\Delta t_{k+1} & a_k \Delta t_k & a_{k-1}\Delta t_{k-1} \end{bmatrix} \begin{bmatrix} h^t_{-1} \\ h^t_0 \\ h^t_1 \end{bmatrix} + \begin{bmatrix} a_{k+1}\Delta w_{k+1} & a_k \Delta w_k & a_{k-1}\Delta w_{k-1} \end{bmatrix} \begin{bmatrix} h^w_{-1} \\ h^w_0 \\ h^w_1 \end{bmatrix} + n_k \quad (8)$$

The discrete magnetic channel model is depicted in Fig. 2.

To be consistent with the MSDR method [7], the observation vector $\boldsymbol{z}$ for the equaliser has a length of four, $\boldsymbol{z} = [z_{k+1}\ z_k\ z_{k-1}\ z_{k-2}]'$ and the magnetic transition vector $\boldsymbol{a}$ is defined as $[a_{k-2}\ a_{k-1}\ a_k\ a_{k+1}\ a_{k+2}\ a_{k+3}]'$. The equaliser and the detector are designed to classify the received vectors into three classes, labelled by the current magnetic transition a_k.

4.2 The MSDR equaliser

In the MSDR equaliser, the structure of the neural network is derived as follows. The network has four layers consisting of an input, output and two hidden layers. The observation vector length is four and therefore there are four nodes in the input layer. Since there are three classes, there are always three output nodes in the fourth layer. Assuming that the observation vectors ($\boldsymbol{z}$) formed 15 clusters in the Euclidean space and the cores of the clusters are defined by the vectors $[a'_{k-1}\ a'_k\ a'_{k+1}]$ there are 15 nodes in the third layer [7]. There exist 72 linear classifiers to separate the clusters in the different classes. Thus, the number of nodes in the second layer is 72. Hence, the structure of the neural network is 4–72–15–3. The number of connections between the first layer and second layer is 288, whereas the connections between 2&3 and 3&4 are 144 and 15, respectively.

The signal to distortion ratio is computed as follows [7]. First, the mean signal at the equaliser output for transitions of 0, − and + polarities is found. Let the mean of the equalised signal corresponding to $a'_k = \pm 2$ be r_a and $-r_b$, respectively. Then, the signal to distortion ratio (SDR) is expressed as

$$SDR1 = r_a^2/4e^2 \quad (9)$$

$$SDR2 = r_b^2/4e^2 \quad (10)$$

$$SDR = \min\,(SDR1, SDR2) \quad (11)$$

$$e^2 = E\{e_k^2\} = E\{(y_k - d_k)^2\} \quad (12)$$

where the error sequence e_k is defined as the difference between d_k, the desired undistorted channel output, and y_k, the actual equaliser output. When the SDR increases, the probability of error for practical distortion distributions decreases.

4.3 Performance comparison under parameter variations and mismatched conditions

In [12], the performance of the MRAN equaliser was evaluated by comparing its SDR to that of the MSDR equaliser for nominal channel and noise parameters. The results showed that the MRAN's signal to distortion ratio (SDR) is consistently higher than that of the MSDR equaliser. Here, the robustness of the MRAN equaliser with respect to channel model and noise parameters is analysed in detail. In all simulations, MRAN uses four input units and a single continuous output unit to represent the three levels $\{-2, 0, +2\}$ of the output.

4.3.1 Partial erasure parameter variation: The effect of partial erasure variation on the detection performance was first investigated. The MRAN was trained with 2000 data bits with noise values at $\sigma_n^2 = 0.004$, $\sigma_t^2 = 0.01T^2$, $\sigma_w^2 = 0.000625T^2$ and $\chi = 0.4$. The values of the parameters used in the MRAN were: $e_{min} = 0.5$, $e_{min1} = 0.8$, $\varepsilon_{max} = 0.4$, the size of the two sliding windows N_w and S_w is 80 and the pruning threshold $\delta = 0.001$. The resulting MRAN network had 13 hidden neurons. Under the same noise condition, the parameters of the MSDR network with a structure of 4–72–15–3 was also obtained. 10^5 data bits with the partial erasure parameter χ varying from 0–0.7 were then used to test the performance of the resulting MRAN and the MSDR

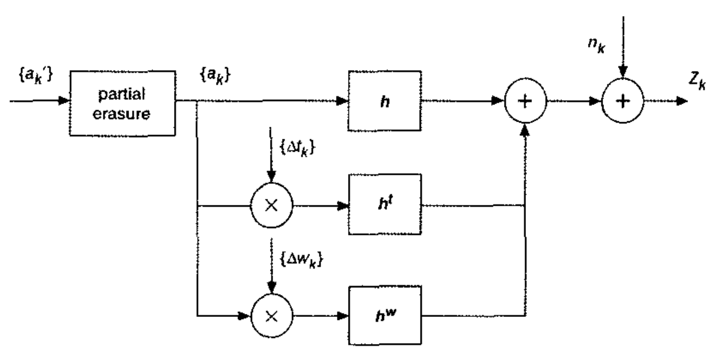

Fig. 2 *Discrete magnetic channel model*

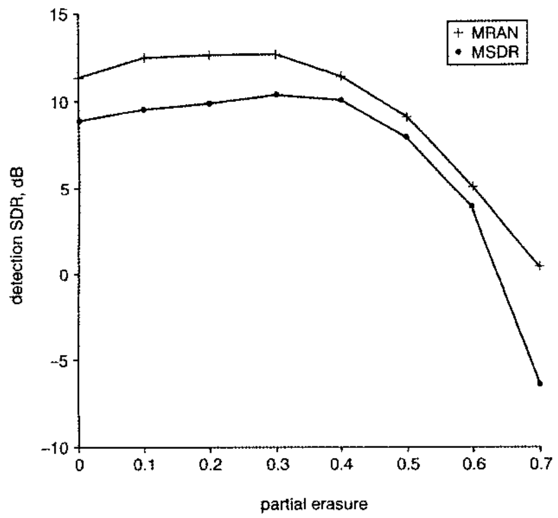

Fig. 3 *Detection SDR for channel with partial-erasure-parameter-variation*

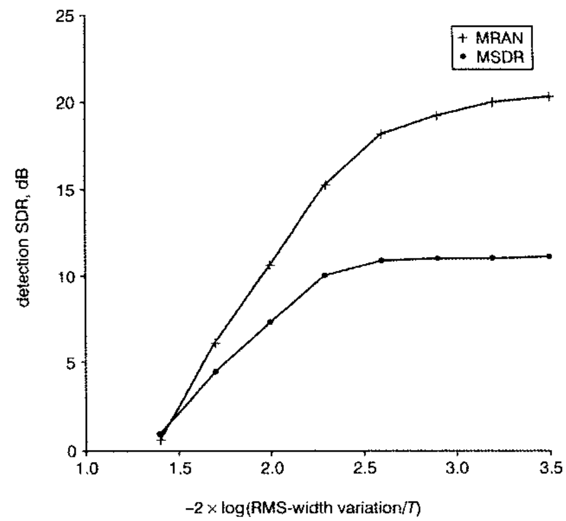

Fig. 5 *Detection SDR for channel with width variation*

network. Fig. 3 shows the detection SDR results. It is seen that when the partial erasure parameter varies, the performance of the MSDR equaliser degrades more compared to the MRAN equaliser. In the presence of severe partial erasure ($\chi = 0.7$), the MRAN equaliser has an advantage of about 6 dB over the MSDR equaliser.

4.3.2 Jitter variation: For this case, the training is done for the jitter noise intensity parameter defined as $-2 \times \log(\text{RMS jitter}/T) = 1.4$. The MSDR equaliser is designed under exactly the same conditions. Data with a varying jitter noise intensity are then used for testing the two equalisers and the results are given in Fig. 4. Under severe jitter, both equalisers have similar performances. In other cases, the MRAN has a clear performance advantage. The SDR curve of the MSDR method is limited on the upper side by the residual ISI, whereas the MRAN has a performance improvement of more than 4 dB.

4.3.3 Width parameter variation: For training, 2000 samples were used where the width variation noise was set to $\sigma_w^2 = 0.005$ (which corresponds to $-\log \sigma_w^2/T^2 = 2.3$). In order to compare the performance, 10^5 test samples were used and the resulting performance is shown in Fig. 5. It is clear that the performance of the MRAN equaliser is superior to the MSDR equaliser over the entire range.

In summary, when the equalisation for the data storage channel is performed under a large parameter variation, a performance loss is observed for both the MRAN and MSDR equalisers. However, the MRAN equaliser is consistently less sensitive to the parameter variation than the MSDR equaliser.

4.3.4 Noise power variation: To study the noise power variation, the partial-erasure parameter was set to zero for a channel corrupted by AWGN. Then MRAN algorithm was trained with 2000 data samples at 15 dB SNR. The resulting network was tested with 10^5 test data

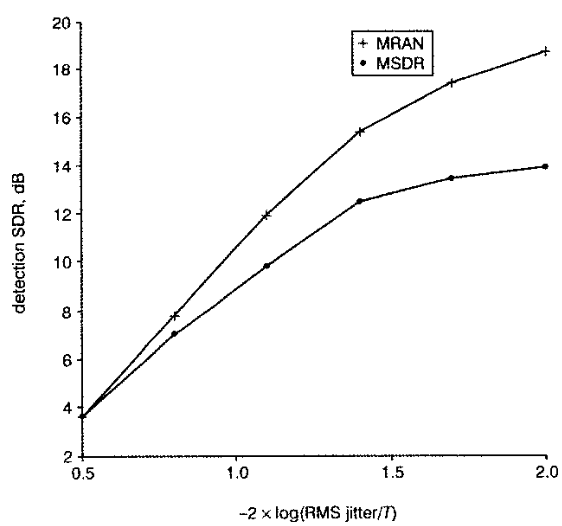

Fig. 4 *Detection SDR for channel with jitter variation*

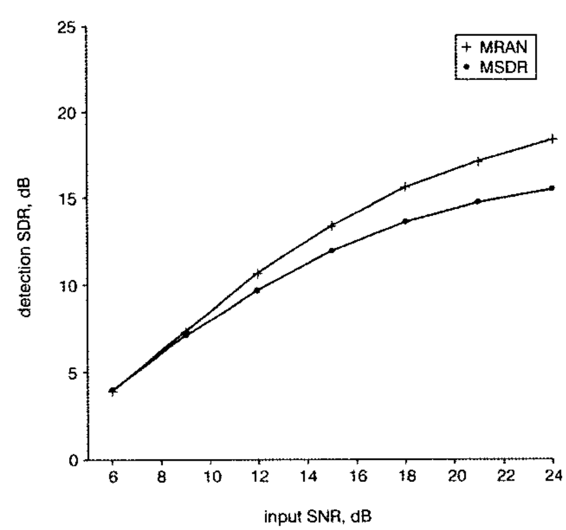

Fig. 6 *Detection SDR for channel with variation in AWGN*

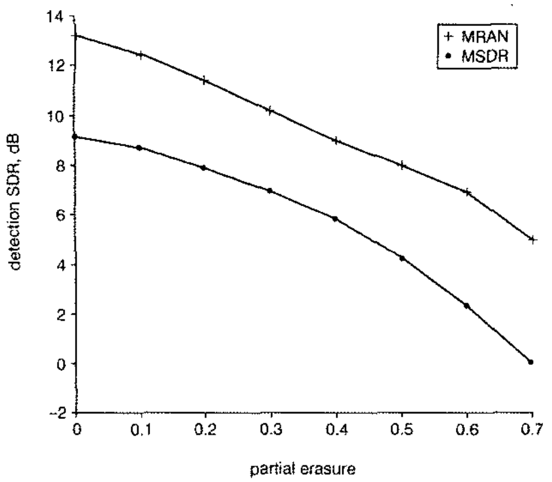

Fig. 7 *Detection SDR for channel with partial erasure parameter variation, under model mismatch*

for SNR varying from 6 to 24 dB, and the obtained SDR curve is shown in Fig. 6. It is observed that at low SNR, the MRAN's performance is very close to that of the MSDR equaliser, whereas at high SNR the MRAN gives a better performance.

4.4 Channel transition response mismatch:

When the transition response of the channel varies significantly, good detection schemes are needed that adapt to this variation and still give a reasonable performance. In order to investigate this, both MRAN and MSDR equalisers were designed using the nominal values for the transition response parameters ($\boldsymbol{h}$, $\boldsymbol{h}'$, $\boldsymbol{h}^w$). Their performance was then tested varying all the transition response parameters individually and collectively. The results for the case of a 30% variation in all the parameters together, i.e. $(\boldsymbol{h}+30\%)+(\boldsymbol{h}'+30\%)+(\boldsymbol{h}^w+30\%)$ are presented.

Fig. 7 shows the detection SDR when the partial erasure parameter is varied from 0 to 0.7. It is clear from the Figure that the MRAN equaliser provides a higher SDR than the MSDR equaliser.

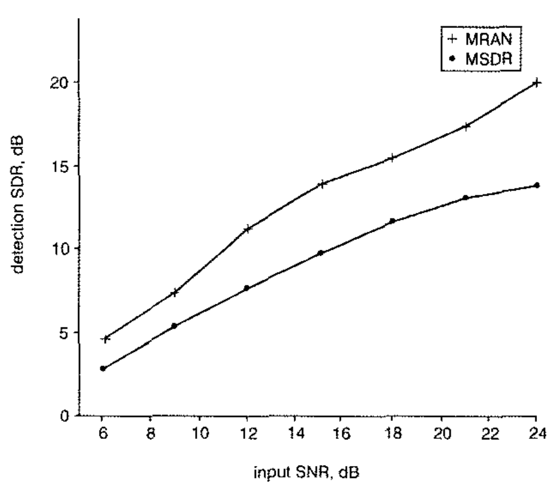

Fig. 8 *Detection SDR for channel with AWGN variations under model mismatch*

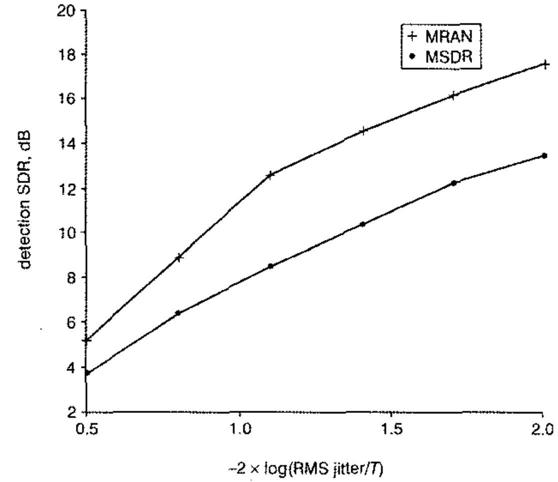

Fig. 9 *Detection SDR for channel with jitter variations under model mismatch*

The parameters of the MRAN are set as: $\varepsilon_{max}=1$, $\varepsilon_{min}=0.4$, the size of the sliding windows $S_w=50$ and the pruning threshold $\delta=0.001$. Figs. 8–10 show the SDR curves of the MRAN and MSDR equalisers for the cases of AWGN, jitter and width variations. In all these cases, the MRAN equaliser produces a higher SDR than the MSDR equaliser [7].

In all these simulations, the different threshold parameters in the MRAN are selected by trial and error best for to give results the training data. The selected values are then used in the testing operation.

4.5 Equaliser complexity

In this Section, the complexity of the MRAN and MSDR equalisers are analysed by comparing the network size and the CPU time taken by both the methods to perform equalisation on a Pentium III/800 PC. Table 1 shows these for the channel problem described in Section 4.1. It can be seen that the MRAN equaliser has a more compact

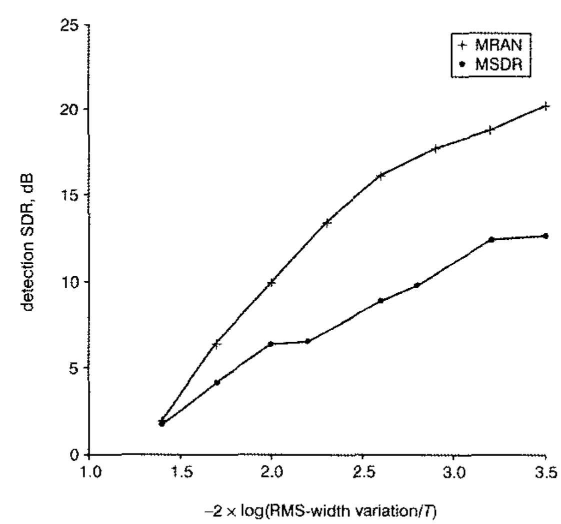

Fig. 10 *Detection SDR for channel-width variation under model mismatch*

Table 1: Complexity comparison

	MRAN	MSDR
Network size	4–13–1 RBF	4–72–15–3 feedforward
Equalisation time, ms	1.02	9.03

network and takes less time to perform equalisation, meaning that its complexity is less than that of the MSDR equaliser. However, it should be noted that the MSDR equaliser does not need any training whereas the MRAN requires training (one iteration).

5 Conclusions

This paper has presented the performance robustness of the MRAN equaliser for a digital magnetic recording system under parameter variations including the channel-transition response. The MRAN's performance in terms of the SDR is compared with that of the MSDR equaliser under these conditions. In the presence of data-dependent noise, such as jitter, and width variations, and also in the case of partial erasure channels, the MRAN's signal to distortion ratio (SDR) has been shown to be consistently higher than that of the MSDR. Also under channel characteristic mismatch, the MRAN performs better than the MSDR with less complexity.

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