**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Z = (X - *μ* )/ *σ* = (X - 45)/8.0

|  |
| --- |
| Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4% |
| The probability that the service manager cannot meet his commitment = 100-73.4 = 26.6%  or 0.2676 |

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans : probability of employees older than 44= Pr(X>44)

|  |  |
| --- | --- |
|  | Pr(X > 44) = 1 - Pr(X ≤ 44). |
|  | Z = (X - *μ* )/ *σ* = (X - 38)/6 |
|  | Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1)=84.1345% |
|  | So the desired probability is 100-84.1345=15.86% |

Therefore the statement that “More employees at the processing center are older than

44 than between 38 and 44” is TRUE.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Probabilty of employees less than age of 30 = Pr(X<30)

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%

So the no of employess having probability 0.912 or employess under age 30

= 0.0912\*400=36.48( or 36 employees).

Therfore the statement is true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random

variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 )

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X

and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

* 2X1~ N(2 u,4 σ^2) and X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 µ, 2σ^2 )
* 2X1-(X1+X2)
* N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: Since we need to find out the values of a and b, which are symmetric about the mean, such

that the probability of random variable taking a value between them is 0.99

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is

0.01 (ie. 1-0.99)

The Probability towards left from a = -0.005 (ie. 0.01/2)

The Probability towards right from b = +0.005 (ie. 0.01/2)

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

Thus option D is correct

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: 5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: *Probability of Division 1 making a loss P(X<0) is* 0.0477903522728147

*Probability of Division 2 making a loss P(X<0) is* 0.040059156863817086