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Lite

① Underflow is the condition where the result of calculations is a number of more precise absolute value than the computer can actually represent in memory on its CPU-means magnitude is too small to be represented with numbers.

Overflow error occurs when a result has a magnitude too big to be represented with the number of bits available.

② $f(x) = x\sin x + \cos x = 0$ | $f'(x) = \dfrac{x\cos x + \sin x \cdot 1}{-\sin x}$

$x_0 = \pi$ | $\boxed{f'(x) = x\cos x}$

Using
①

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \pi - \left\{\frac{\pi\sin\pi + \cos\pi}{\pi\cos\pi}\right\}$$

$$x_1 = \pi - \frac{1}{\pi} = 2.823$$

(ii)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.823 - \left\{\frac{2.823 \cdot \sin 2.823 + \cos 2.823}{2.823 \cdot \cos 2.823}\right.$$

$$x_2 = 2.823 + 0.022 = 2.801$$

(iii)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 2.801 - \left\{\frac{2.801 \cdot \sin 2.801 + \cos 2.801}{2.801 \cdot \cos 2.801}\right\}$$

$$x_3 = 2.798$$

(iv)

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \Rightarrow 2.798 - \left\{\frac{2.798 \cdot \sin 2.798 + \cos 2.798}{2.798 \cdot \cos 2.798}\right\}$$

$$x_4 = 2.798$$

$\because x_3 = x_4 = 2.798$

$\therefore$ Required root is <u>2.798</u>

(3) False position method = Regula Falsi method

$$f(x) = x^3 + x^2 + x + 7 = 0$$

$$f(a) = f(-3) = -1\cancel{4}1 \cancel{-1+6} \Rightarrow -27 + 9 - 3 + 7 \Rightarrow -14$$

$$f(b) = f(-2) = -8 + 4 - 2 + 7 \Rightarrow -4 + 5 \Rightarrow +1$$

$\because f(a) \cdot f(b) < 0$ where $a = -3$ & $b = -2$

$$c = \frac{a f(b) - b \cdot f(a)}{f(b) - f(a)} \quad \{f(a) = -14, f(b) = +1\}$$

$$c = -2.066$$

$$f(c) = \cancel{2.066}\,(2.066)^3 + (-2.066)^2 + (-2.066) + 7$$

$$f(c) \Rightarrow 0.383$$

$\because f(a) \cdot f(c) < 0 \therefore$ roots lie between <u>-14</u> to <u>0.383</u>

$a = -14, b = -2.066$

(4) $\cot x = e^x$

$f(x) = \cot x - e^x = 0$

$x = \cot^{-1}(e^x)$

$e^x = \cot x$

$x = \log(\cot x)$

$f(-2) = 0.322$

$f(-1) = -1.009$

$\varphi(x) = \cot^{-1}(e^x)$

$\varphi'(x) = \dfrac{-e^x}{e^{2x}+1}$

$|\varphi'(-2)| = -6.38 < 1$

$x_0 = \dfrac{-1-2}{2} = -1.5$

$x_1 = 3.519$

$x_2 = 0.023$

$x_3 = +0.767$

$x_4 = \cdot 364$

$x_5 = 0.545$

$x_6 = 0.455$

$x_7 = 0.498$

$x_8 = 0.471$

$x_9 = 0.482$

$x_{10} = 0.482$

(5) (i) for square root

let N be the number

if $x \rightarrow$ assumed number

for root,

$$\boxed{root = 0.5*(x+(N/x))}$$

we will keep on finding root until

$|root - x|$ not less than tolerance level

(ii) Reciprocal method

from newton raphson,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

let $x = \frac{1}{N}$

$+ f(x) = \frac{1}{x} - N$

$f'(x) = -\frac{1}{x^2}$

$$x_{i+1} = x_i - \left(\frac{\frac{1}{x_i} - N}{-\frac{1}{x_i^2}}\right) \Rightarrow x_i + \left(\frac{1 - Nx_i}{x_i}\right)(x_i)^2$$

$$\boxed{x_{i+1} = x_i(2 - Nx_i)}$$

(6) $f(x) = x\tan x + 1 = 0$

using newton raphson method.

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

$f(-6) = -0.746$

$f(-6.2) = 0.483$

$x_0 = \frac{-6-6.2}{2} \Rightarrow -6.1$

$f(-6.1) = -0.130$

$f'(x) = x\sec^2 x + \tan x$

(1) $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= -6.1 - \left\{\frac{-0.130}{-9.161}\right\}$

$x_1 = -6.112$

$f(x_1) = -0.056$

$f'(x_1) = -8.118$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = -6.118$

$f(x_2) = -0.019$

$f'(x_2) = -7.639$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = -6.120$

$f(x_3) = -0.007$

$f'(x_3) = -7.483$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$x_4 = -6.120$

$\therefore$ Required root is -6.120

(7) $r = 3h(h^6 - 2)$

$dr = \frac{dr}{dh} \times dh$

$= \frac{d}{dh}[3h(h^6-2)]\, dh$

$= (21h^6 - 6)\, dh$

~~multiply~~ divide by r and multiple b 100

$\frac{dr}{r} = \frac{(21h^6-6)\, dh}{r}$

$\frac{dr}{r} \times 100 = \frac{(21h^6-6)\, dh \times 100}{(3h^7 - 6h)}$

$\because h = 1$

→ error in h in %

$= \left(\frac{21-6}{3-6}\right)\left(\frac{dh}{h} \times 100\right)$

$= \frac{\cancel{15}^{5}}{-\cancel{3}} \times 5\% = -25\%$

$\therefore$ % error in $r = \left|\frac{dr}{r} \times 100\right| = |-25\%| = 25\%$

(11) P

(9) $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\frac{dA}{A} = \frac{2\pi r}{A} \times dr \Rightarrow \frac{2\pi r}{\pi r^2} \times dr$

$\frac{dA}{A} = \frac{2}{r} dr$

$\Rightarrow$ Percentage error in r is 2 in A.
$\Rightarrow$ % error in diameter would be 4 times A

$\frac{dA}{A} = \frac{2}{\frac{D}{2}} dr \Rightarrow \frac{dA}{A} = \frac{4}{D} dr$

$\therefore \frac{dr}{D}$ or % of error in D $< 0.025\%$

(10) $x = .4845, y = .4800$

$\frac{x^2 - y^2}{x+y} = \frac{(.4845)^2 - (.4800)^2}{.4845 + .4800} = 0.004845$

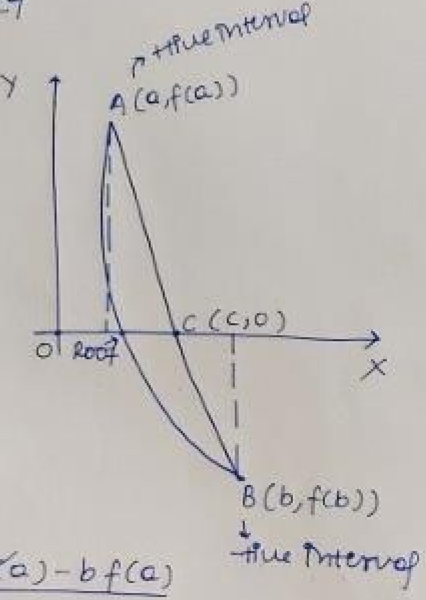
$x - y = 0.0045$

error $= \frac{0.0045 - 0.0044}{0.0045} = 0.022 \times 100$

$= \underline{2.2\%}$

(11) Regula Falsi Method

Slope of AB = Slope of AC



$$\frac{f(b)-f(a)}{b-a} = \frac{0-f(a)}{c-a}$$

$$c-a = \frac{-f(a)(b-a)}{f(b)-f(a)}$$

$$c-a = \frac{-f(a)b + af(a)}{f(b)-f(a)}$$

$$c = a + \frac{af(a) - bf(a)}{f(b)-f(a)}$$

$$c = \frac{af(b) - af(a) + af(a) - bf(a)}{f(b)-f(a)}$$

$$\boxed{c = \frac{af(b) - bf(a)}{f(b) - f(a)}}$$

Rate of convergence of Regula Falsi method is 1.6

(12) (i) $(5-x)e^x$ near $x=5$

$f(x) = (5-x)e^x$

$f(-2) = 0.947$
$f(-3) = 0.398$
$f(-4) = 0.164$
$f(4.9) = 13.428$
$f(5.01) = -1.499$

$a = 5.01$, $b = 4.9$

$f(a) = -1.499$, $f(b) = 13.428$

I $c = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = \underline{4.998}$

$f(c) = (5-4.998)\times e^{4.998} \Rightarrow 0.296$

$\therefore$ new $b = 4.998$ and $f(b) = 0.296$

II $c = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = 4.999$

$f(c) = (5-4.999)e^{4.999} \Rightarrow 0.148$

new $f(b) = 0.148$ and $b = 4.999$

III $c = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = 4.999$

$\therefore$ Required root is 4.999

(12)

(ii) $x^2 - \log_e x - 12 = 0$ (3 dec places)

$f(3) = -4.098$

$f(4) = 2.613$

$a = 3,\ f(a) = -4.098,\ b = 4,\ f(b) = 2.613$

I $\quad c = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = 3.610$

$f(c) = -0.251$

$\therefore$ new $a = 3.610$ & new $f(a) = -0.251$

II $\quad c = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = 3.644$

$f(c) = -0.014$

new $a = 3.644$ & $f(a) = -0.014$

III $\quad c = \dfrac{3.644 \times 2.613 + 4 \times 0.014}{2.613 + 0.014} = 3.645$

$f(c) = 0.007$

new $b = 3.645$ and $f(b) = 0.007$

IV $\quad c = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = 3.645$

$\therefore$ Required root is 3.645

(13) (1) $x^2 - 1 = \sin^2 x$

$f(x) = x^2 - \sin^2 x - 1 = 0$

$f(1) = -0.84$

$f(1.5) = 0.471$

$x_0 = \frac{1+1.5}{2} = 1.25$

$\varphi(x) = (\sin^2 x + 1)^{1/2}$

$\varphi'(x) = \frac{1}{2}(\sin^2 x + 1)^{-1/2} \cdot 2\sin x \cdot \cos x$

$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}\sin x \cdot \cos x}{(\sin^2 x + 1)^{1/2}} = \cancel{\frac{\sin x \cdot \cos x}{(1 - \cos^2 x + 1)^{1/2}}}$

$\varphi'(x) = \frac{\sin x \cdot \cos x}{(\sin^2 x + 1)^{1/2}}$

$x_1 = \frac{\sin x_0 \cdot \cos x_0}{(\sin^2 x_0 + 1)^{1/2}} = 0.149$

$x_2 = 0.127$

$x_3 = 0.111$

$x_4 = 0.099$

$x_5 = 0.089$

$x_6 = 0.074$

$x_7 = 0.074$

$x_8 = 0.068$

$x_9 = 0.063$

$x_{10} = 0.059$

$x_{11} = 0.058$

$x_{12} = 0.052$

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$x_{13} = 0.049$

$x_{14} = 0.046$

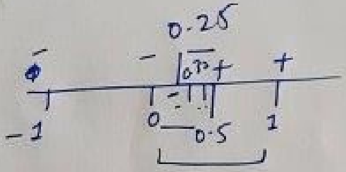
(14) (ii) $3x-(1+\sin x)^{1/2}=0$ (5 iterations)

Bisection method,

$f(x)=3x-(1+\sin x)^{1/2}=0$

$f(1)=1.158$

$f(-1)=-3.158$



$x_0=\frac{-1+1}{2}=0$

$f(x_0)=-1$, $x_1=\frac{1+0}{2}=0.5$

I $f(x_1)=0.283$

$x_2=\frac{0+0.5}{2}=0.25$

II $f(x_2)=-0.366$

$x_3=\frac{0.25+0.5}{2}=0.375$

III $f(x_3)=-0.043$

$x_4=\frac{0.375+0.5}{2}=0.437$

IV $f(x_4)=0.118$

$x_5=\frac{0.375+0.437}{2}=0.406$

V $\boxed{f(x_5)=0.036}$

(15)

$$\begin{bmatrix} 3 & 1 & -1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$

$R_2 = R_2 - \frac{1}{3} R_1$ + $R_3 = R_3 - \frac{2}{3} R_1$

$$\begin{bmatrix} 3 & 1 & -1 \\ 0 & -4/3 & 4/3 \\ 0 & 1/3 & 5/3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -10/3 \\ -2/3 \end{bmatrix}$$

$0 \cdot x + 0 \cdot y + 2 \cdot z = -3/2$ — (1)

$\boxed{z = -\frac{3}{4}}$

$0x + -\frac{4}{3} y + \frac{4}{3} z = -\frac{10}{3}$

$-\frac{4}{3} y + \frac{\cancel{4}}{\cancel{3}} \times \frac{-\cancel{3}}{\cancel{4}} = -\frac{10}{3}$

$\boxed{y = \frac{7}{4}}$

$3x + y - z = 1$

$\Rightarrow 3x = 1 - \frac{7}{4} + \left(\frac{-3}{4}\right)$

$\boxed{x = -\frac{1}{2}}$

(18)

$$\begin{bmatrix} 2 & 5 & 0 \\ 1 & 2 & -1 \\ -3 & -4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 1 \end{bmatrix}$$

$R_2 \to R_2 - \frac{1}{2}R_1$ $\quad$ $R_3 \to R_3 + \frac{3}{2}R_1$

$$\begin{bmatrix} 2 & 5 & 0 \\ 0 & -1/2 & -1 \\ 0 & 7/2 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -3/2 \\ 29/2 \end{bmatrix}$$

$R_3 \to R_3 + 7R_2$

$$\begin{bmatrix} 2 & 5 & 0 \\ 0 & -1/2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -3/2 \\ 4 \end{bmatrix}$$

$0x + 0y + 0z \neq 4$

LHS $\neq$ R.H.S $\Rightarrow$ No solution Exist.

(18)

$$12x_1 + 3x_2 - 5x_3 = 0$$
$$x_1 + 5x_2 + 3x_3 = 28$$
$$3x_1 + 7x_2 + 13x_3 = 76$$

$$x_1 = \frac{1 - 3x_2 + 5x_3}{12}$$

$$x_2 = \frac{28 - x_1 - 3x_3}{5}$$

$$x_3 = \frac{76 - 3x_1 - 7x_2}{13}$$

(I) $[x_1 \; x_2 \; x_3] = [1 \; 0 \; 1]$

$$x_1 = \frac{1 - 3(0) + 5(1)}{12} = 0.5$$

$$x_2 = \frac{28 - 0.5 - 3(1)}{5} = 4.9$$

$$x_3 = \frac{76 - 3(0.5) - 7(4.9)}{13} = 3.0923$$

$[x_1 \cdot x_2 \; x_3] = [0.5, 4.9 \; 3.0923]$

(II) $[x_1 \; x_2 \; x_3] = [1 \; 0 \; 1]$

$$|\epsilon_a^1| = \left|\frac{0.5 - 1}{0.5}\right| \times 100 = 100\%$$

$|\epsilon_a^2| = 100\%$

$|\epsilon_a^3| = 67.662\%$

max $= \epsilon_2$ 100%

II

$[x_1 \; x_2 \; x_3] = [0.5 \; 4.9 \; 3.0923]$

$$x_1 = \frac{1-3(4.9)+5(3.0923)}{12} = 0.14679$$

$$x_2 = \frac{28-8\,0.14679-3(3.0923)}{5} = 3.7153$$

$$x_3 = \frac{76-3(0.14679)-7(3.7153)}{13} = 3.8718$$

$[x_1 \; x_2 \; x_3] = [0.14679 \; 3.7153 \; 3.8718]$

$|\epsilon_a^1| = 240.61\%$

$|\epsilon_a^2| = 31.889\%$

$|\epsilon_a^3| = 18.874\%$

$\max(|\epsilon_a^1|; |\epsilon_a^2|, |\epsilon_a^3|) = 240.61\%$

~~Converging~~
Diverging