CBNST

Q $x \log x = 1.2$ Bisection

$x \log x - 1.2 = 0$

$f(0) = -1.2$, $f(1) =$

Regula - Falsi Method :- (False - position method)

Slope $= \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

Eqn:- $\dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{y - f(x_0)}{x - x_0}$

At intersection

$y = 0$, $x = x_2$

$\Rightarrow \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{0 - f(x_0)}{x_2 - x_0} \Rightarrow (x_2 - x_0)[f(x_1) - f(x_0)]$

$\Rightarrow -f(x_0)(x_1 - x_0)$

$\Rightarrow x_2[f(x_1) - f(x_0)] - x_0[f(x_1) - f(x_0)] \Rightarrow x_0 f(x_0) - x_1 f(x_0)$

$\Rightarrow x_2[f(x_1) - f(x_0)] - x_0 f(x_1) + \cancel{x_0 f(x_0)} = \cancel{x_0 f(x_0)} - x_1 f(x_0)$

$x_2[f(x_1) - f(x_0)] = x_0 f(x_1) - x_1 f(x_0)$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Q $x^3 - 4x - 9 = 0$

① $f(x) = 0$

② Finding the interval

$f(0) = -9$, $f(1) = -12$, $f(2) = -9$, $f(3) = +6$

$I \equiv (2, 3)$

③ IVT :- $f(2) \cdot f(3) < 0$

$I \equiv (2, 3)$

$x_0 = 2$, $f(x_0) = -9 = f(2)$

$x_1 = 3$, $f(x_1) = +6 = f(3)$

$= \dfrac{2(+6) - 3(-9)}{6 - (-9)} \Rightarrow \dfrac{12 + 27}{3} \Rightarrow \dfrac{39}{15} = 2.6$

$f(x_2) = f(2.6) = (2.6)^3 - 4(2.6) - 9 = -1.824$

$(\underset{-}{2}, \underset{-}{2.6}, \underset{+}{3}) \rightarrow f(2.6) \cdot f(3) < 0$ $\;\; I \equiv (2.6, 3)$

ii) $x_2 = 2.6,\ f(x_2) = 1.824 = f(2.6)$ $\qquad x_3 = \dfrac{x_2 \cdot f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$

$x_1 = 3,\ f(x_1) = f(3) = 6$

$$x_3 = \frac{(2.6)6 - 3(-1824)}{6-(-1824)} \Rightarrow x_3 = 2.69325$$

$f(2.69325) = -0.2372$

$(\overset{x_2}{2.6}, \overset{x_3}{2.69325}, \overset{x_1}{3}) \rightarrow f(2.69325) \cdot f(3) < 0$

$I = (2.69325, 3)$

iii) $$x_4 = \frac{x_3 \cdot f(x_1) - x_1 \cdot f(x_3)}{f(x_1) - f(x_3)}$$

$x_3 = 2.69325$ | $x_1 = 3$

$f(x_3) = -0.2372$ | $f(x_1) = 6$

2.705

$$\frac{2.69325 \times 6 - 3 \times (-0.2372)}{6-(-0.2372)}$$

$$\Rightarrow \frac{16.1595 + 0.7116}{6.2372} = \frac{16.8711}{6.2372} = 2.70492$$

~~f(2.7049) =~~

Q = $\boxed{xe^x = \cos x}$; $x = \sin x$; $x^4 - x - 10 = 0$

Q) $x \log_{10} x = 1.2$ by secant method

$f(x) = 0$

$x \log x - 12 = 0$

$f(1) = 0 - 1.2 = -1.2$, $\quad f(2) = 2 \times 0.30 - 1.2 = -0.6$

~~f(2) = 2×0.30 − 1.2 = −0.7~~ $\quad f(3) = 3 \times 0.477 - 1.2 = 1.431 - 1.2 = 0.2313637$

~~f(3) = 3×0.477 − 1.2~~

$f(2, 3)$

$x_0 = 2,\ f(x_0) = f(2) = -0.6$

$x_1 = 3,\ f(x_1) = f(3) = 0.2316$

$$x_2 = x_1 - \left[\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right] \times f(x_1)$$

$$= 3 - \left[\frac{3-2}{0.2316+0.6}\right] \times 0.231 \Rightarrow 3 - \left[\frac{0.2313637}{0.8313637}\right]$$

$\Rightarrow 3 - 0.2782942051 \Rightarrow 2.721705$

$$x_3 = 2.721705 - \left[\frac{2.721705 - 3}{-0.01649 - 0.23136}\right] \qquad * f(2.721705) = 1.183509 - 1.2 = -0.01649$$

$$\Rightarrow 2.721705 - \left[\frac{-0.278295}{-0.21487}\right] \Rightarrow 2.721705 - 1.295178$$

$$\Rightarrow 1.426527$$

$$x_4 = 1.426527 - \left[\frac{1.426527 - 2.721705}{-0.979915 - (-0.01649)}\right] \qquad * f(1.426527) = 0.220084 - 1.2 = -0.979915$$

$$= 1.426527 - \left[\frac{-1.295178}{-0.963425}\right] \Rightarrow 1.426527 - 1.344347 = 0.08218$$

$$x_5 = 0.08218 - \left[\frac{0.08218 - 1.426527}{-1.28918 - (-0.979915)}\right] \qquad f(0.08218) = -0.08918 - 1.2 = -1.28918$$

$$= 0.08218 - \left[\frac{-1.344347}{-0.309265}\right] \Rightarrow 0.08218 - 4.34685 = -4.26467$$

* Newton Raphson Method :-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\because \frac{df(x_n)}{d(x_n)} = f'(x_n)$$

x_0 = Initial guess / approx

x_{0th} = root

$f(x_{0th}) = 0$

Cases of failure :-

1. If $f'(x_n)$ is 0.
2. Wrong change of approx
3. Point of inflexion

Q $x^4 - x = 10$ to 3 decimal

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$f(x) = x^4 - x = 10$

$f'(x) = 4x^3 - 1$,

$f(1) = 1 - 1 - 10 = -10$, $f(2) = 16 - 1 - 10 = 5$

$f(1, 2)$ $f(1.5) = 5.0625 - 11.5 = -6.4375$

$$x_1 = 1.5 - \frac{(-6.4375)}{2.375}$$

$f'(x) = 3.375 - 1$

$= 2.375$

$x_1 = 1.5 + 2.7105 = 4.2105$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(4) = 314.300 - 14.2105$

$= 300.0895$

$$= 4.2105 - \frac{300.0895}{297.5802}$$

$f'(4.2105) = 298.5802 - 1$

$x_2 = 4.2105 - 1.00843 = 3.20207$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$f(3.20207) = 105.12918 - 13.20207$

$= 91.92711$

$f'(3.20207) = 32.83163 - 1 = 31.83163$

$$= 3.20207 - \frac{91.92711}{31.83163}$$

$\Rightarrow 3.20207 - 2.88791 \Rightarrow 0.31416$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$f(0.31416) = 0.009741 - 10.31416$

$= -10.304419$

$f'(0.31416) = 0.03100 - 1 = -0.969$

$$\Rightarrow 0.31416 - \frac{(-10.304419)}{(-0.969)}$$

$0.31416 - 10.634075$

Q) $3x = \cos x + 1$

$f(x) = 3x - \cos x - 1 = 0$ $\qquad$ $f'(x) = 3 + \sin x$

$f(0) = 0 - 1 - 1 = -2$

$f(1) = 3 - 0.540302305 - 1 = 1.459697694$

$f(2) = 6 - (-0.4161468) - 1 = 5.4161468$

$f(0,1)$

x_0 = Initial guess

$x_0 = \frac{a+b}{2} = \frac{1}{2} = 0.5$

$x_1 = 0.5 - \frac{(-0.37758)}{3.47943}$

$f(0.5) = 1.5 - 0.87758 - 1$
$= -0.37758$

$f'(0.5) = 3 + 0.47942 = 3.47943$

$x_1 = 0.5 + 0.108518$
$= 0.608518$

$x_2 = 0.60851 - \frac{0.00503}{3.57165}$

$f(0.60851) = 1.82553 - 0.82050 - 1$
$= 0.00503$

$f'(0.60851) = 3 + 0.57164$
$= 3.57165$

$= 0.60851 - 0.0014083$
$= 0.60710$

$x_3 = 0.60710 - \frac{0}{3.57048}$

$f(0.60710) = 1.8213 - 0.82130 - 1$
$= 0$

$f'(0.60710) = 3 + 0.57048$
$= 3.57048$

$x_3 = 0.60710$

$x_4 = 0.60710$

Q) $x = \sqrt{12}$

$x - \sqrt{12} = 0 \Rightarrow x = (12)^{1/2} \Rightarrow x^2 = 12$ $\qquad$ $f'(x) = 2x$

$x^2 - 12 = 0$ $\qquad$ $f(0) = 0 - 12 = 12$

$f(1) = 1 - 12 = -11$, $f(2) = 4 - 12 = -8$; $f(3) = 9 - 12 = -3$

$f(4) = 16 - 12 = 4$ $\qquad$ $f(3,4)$

$x_0 = 3.5$

$x_1 = 3.5 - \frac{0.25}{7}$

$f(3.5) = 12.25 - 12 = 0.25$

$f'(3.5) = 2 \times 3.5 = 7$

$= 3.5 - 0.03571$
$= 3.46429$

$x_2 = 3.46429 - \frac{0.00131}{6.92858}$

$f(3.46429) = 12.001301 - 12$
$= 0.001301$

$f'(3.46429) = 2 \times 3.46429$
$= 6.92858$

$= 3.46429 - 0.00018907$
$= 3.46410$

$x_3 = 3.46410 - \frac{0}{6.9282}$

$f(3.46410) = 11.99998 - 12$
$= 0$
$f'(3.46410) = 2 \times 3.46410$
$= 6.9282$

$x_3 = 3.46410$

$x_4 = 3.46410$

Q) $e^{-x} = \sin x$

$f(x) = e^{-x} - \sin x = 0$

$f'(x) = -e^{-x} - \cos x$

$f(0) = e^{-0} - \sin 0 = 1$
$f(1) = e^{-1} - \sin 1 = 0.350427$
$f(2) = e^{-2} - \sin 2 = 0.0100435$
$f(3) = e^{-3} - \sin 3 = -0.002548$

$f(2, 3)$ $\quad x_0 = 2.5$

$x_1 = x_0 \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 2.5 \frac{f(2.5)}{f'(2.5)}$

$f(2.5) = e^{-2.5} - \sin(2.5) = 0.038$
$f'(2.5) = -1.081133$

$\Rightarrow 2.5 \times \frac{0.038465}{-1.081133} \Rightarrow - 2.5 \times 0.0355784$

$\Rightarrow -0.088946$

$x_2 = -0.088946 \times \frac{1.091469}{-2.093020}$

$f(0.088946) = 1.091469$
$f'(0.088946) = -2.093020$

$= +0.088946 \times 0.5214804 \Rightarrow 0.046384$

$x_3 = 0.046384 \times \frac{0.953866}{-1.954675}$

$f(0.046384) = 0.953866$
$f'(0.046384) = -1.954675$

$= -0.046384 \times 0.487992 \Rightarrow -0.022635$

$x_4 = -0.022635 \times \frac{1.023288}{-2.022893}$

$f(-0.022635) = 1.023288$
$f'(-0.022635) = -2.022893$

$= 0.022635 \times 0.5058537 = 0.011449$

$x_5 = 0.011449 \times \frac{0.988416}{-1.988616}$

$f(0.011449) = 0.988416$
$f'(0.011449) = -1.988616$

$= 0.011449 \times (-0.497037 = 0.0056906$

$x_6 = -0.0056906 \times \frac{1.005806}{-2.0057068}$

$f(-0.0056906) = 1.005806$
$f'(-0.0056906) = -2.0057068$

$= 0.0056906 \times 0.5014720 = 0.0028537$

$x_7 = 0.0028537 \times$

$f(0.0028537) = 0.997100$
$f'(0.0028537)$

Iteration Method :-

$f(x)=0 \mid f(a)\cdot f(b)<0 \Rightarrow I=(a,b)$

$$x=\psi(x)$$

If $\psi(x)$ & $\psi'(x)$ continuous on (a,b) & $|\psi'(x)|<1$

$$\forall\ x\in(a,b)$$

Then $\boxed{x_{i+1}=\psi(x_i)}$ finding the soln.

$$|x_{i+1}-x_i| \le 0.0001$$

Q) $2x-\log_{10}x=7$

$$\boxed{x=\frac{7+\log_{10}x}{2}}$$

$f(x)=2x-\log_{10}x-7=0$

$f(0)=-7$

$f(1)=2-0-7$

$=-5$

$f(2)=4-0.30103-7$

$=-3.30103$

$f(3)=6-0.47712-7$

$=-1.47712$

$f(4)=8-0.60206-7$

$=0.39794$

$f(3,4)$

$$\psi'(x)=\frac{-1}{2x}$$

~~for~~ 3 $=\frac{-1}{2\times3}=\frac{-1}{6}=-0.16667$

~~for 4~~ $=\frac{-1}{2\times4}=\frac{-1}{8}=-0.125$

for 3.5 $=\frac{-1}{7}=$

Q Find the cube root of 15 correct to 4 significant digit?

$x = \sqrt[3]{15} \Rightarrow x = (15)^{1/3} \Rightarrow x^3 = 15$

$x = \frac{15}{x^2} \Rightarrow x = 15x^{-2}$

$\varphi'(x) = -30x^{-3} = \frac{-30}{x^3}$

$x^3 = 15 + 20x - 20x \Rightarrow 20x = 15 + 20x - x^3 \Rightarrow x = \frac{15+20x-x^3}{20}$

$\varphi(x) = \frac{20-3x^2}{20}$

$f(1) = \frac{15+20-1}{20} = \frac{34}{20} = 1.7$

$f(0) = \frac{15}{20} = 0.75$

$f(-1) = \frac{15-20-1}{20} = \frac{-6}{20} = -0.3$

$f(-1,0)$

$x_1 = \frac{20-3(-0.5)^2}{20} \Rightarrow \frac{20-0.75}{20} \Rightarrow \frac{19.25}{20} = 0.9625$

$x_2 = \frac{20-3(0.9625)^2}{20} \Rightarrow \frac{20-2.77922}{20} \Rightarrow \frac{17.22078}{20} \Rightarrow 0.861039$

$x_3 = \frac{20-3(0.861039)^2}{20} \Rightarrow \frac{20-2.22416}{20} \Rightarrow \frac{17.77584}{20} = 0.888792$

$x_4 = \frac{20-3(0.888792)^2}{20} \Rightarrow \frac{20-2.36985}{20} \Rightarrow \frac{17.630015}{20} \Rightarrow 0.8815075$

$x_5 = \frac{20-3(0.8815075)^2}{20} \Rightarrow \frac{20-2.33117}{20} \Rightarrow \frac{17.66883}{20} \Rightarrow 0.8834415$

$x_6 = \frac{20-3(0.8834415)^2}{20} \Rightarrow \frac{20-2.341406}{20} = \frac{17.658594}{20} \Rightarrow 0.8829297$

$x_7 = \frac{20-3(0.8829297)^2}{20} \Rightarrow \frac{20-2.3386946}{20} = \frac{17.6613054}{20} \Rightarrow 0.88306527$

$x_8 = \frac{20-3(0.88306527)^2}{20} \Rightarrow \frac{20-2.339412}{20} \Rightarrow \frac{17.660587}{20} \Rightarrow 0.88302935$

$x_9 = \frac{20-3(0.88302935)^2}{20} = \frac{20-2.339222}{20} \Rightarrow \frac{17.660778}{20} \Rightarrow 0.8830389$

$\cos(x) - \sec x$

57.3X

(1)

(2)

-0.0003288

-0.3288×10^{3}

$\boxed{10E3}$

CBNST Lab

Program 1

$X = 1.23459125$ | WAP to calc E_r, E_a, E_p using transaction & round off concept.

$X' = 1.2345$, $X' = 1.2346$

$E_a = |X - X'|$

$E_r = \left|\frac{X - X'}{X}\right|$

$E_p = E_r \times 100$

Program 2:- Write a program to calculate the root of any transcendental ~~error~~ eqn. using bisection method correct upto 3 decimal place.

Program 3:- WAP to find the soln. of any non-polynomial eqn. using regular falcy method correct upto 3 decimal place

Program No. (16, B) | Page No.

Name:
Course/Branch
Sem/Sec
Roll No:
Date:
1. Objective
2. Method/Algorithm:-
3. Program:
4. Output:

Sentence Case

12, Plain Text

Q4) Newton-Raphan

Simultaneous Linear Eqns

$a_{11}x_1 + a_{12}x_2 + \dots + a_{2n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$

a

Gauss Elimination :-

The unknown values are eliminated sequentially & the system is reduced to an upper triangular method from which the unknowns are obtained by back substitution.

$a_1x + b_1y + c_1z = d_1$ — (1)

$a_2x + b_2y + c_2z = d_2$ — (2)

$a_3x + b_3y + c_3z = d_3$ — (3)

1.55740

Step 1:- Eliminate x from (2) & (3)

$$R_2 \to R_2 - \frac{a_2}{a_1} R_1$$

$$R_3 \to R_3 - \left(\frac{a_3}{a_1}\right) R_1$$

$$a_1x + b_1y + c_1z = d_1$$
$$b'_2y + c'_2z = d'_2$$
$$b'_3y + c'_3z = d'_3 \quad \text{--- (4)}$$

Step 2 Eliminate y from (4)

$$R_3 \to R_3 - \left(\frac{b'_3}{b'_2}\right) R_2$$

$$a_1x + b_1y + c_1z = d_1$$
$$b'_2y + c'_2z = d'_2$$
$$c''_3z = d''_3$$

Step 3

$$c''_3z = d''_3$$
$$z = \frac{d''_3}{c''_3}$$

Put z in R_2 to get y:-

Put y & z in R_1 to get x:-

$10x - 7y + 3z + 5u = 6$
$-6x + 8y - z - 4u = 5$
$3x + y + 4z + 11u = 2$
$5x - 9y - 2z + 4u = 7$

This method will fail
- if any of the pivot a_i', become zero
In such cases we rewrite the eq^n. in a diff. order so that the pivot is non-zero.

The unknown are eliminated frequently & the diagonal is

$x + 4y - z = -5$ — (1)
$x + y - 6z = -12$ — (2)
$3x - y - z = 4$ — (3)

$$\begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & -6 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -12 \\ 4 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc:c} 1 & 4 & -1 & -5 \\ 0 & -3 & -5 & -7 \\ 0 & -13 & 2 & 19 \end{array}\right]$$

$x + 4y - z = -5$
$-3y - 5z = 7$
$-13y + 2z = 19$ — (4)

$R_3 \to R_3 - \frac{13}{3} R_2$

$x + y - 6z = -12$
$- \quad x + 4y - z = -5$
$-3y - 5z = -7$

$x + 4y - z = -5$
$-3y - 5z = -7$
$\frac{712}{3} = \frac{148}{3}$

$z = \frac{148}{7} = 2.0845$

$y = -1.1408$

Gauss Jordan

$$a_1x + b_1y + c_1z = d_1 \qquad x \to R_1$$
$$a_2x + b_2y + c_2z = d_2 \qquad y \to R_2$$
$$a_3x + b_3y + c_3z = d_3 \qquad z \to R_3$$

Step 1:- Remove x from R_2 & R_3 :-

$$R_2 \to R_2 - \left(\frac{a_2}{a_1}\right) R_1$$
$$R_3 \to R_3 - \left(\frac{a_3}{a_1}\right) R_1$$

$$a_1x + b_1y + c_1z = d_1$$
$$b_2'y + c_2z = d_2$$
$$b_3'y \quad c_3z = d_3$$

Step 2:- Remove y from R_1 & R_3

$$R_3 \to R_3 - \left(\frac{b_1}{b_2'}\right) R_2$$
$$R_1 \to R_1 - \left(\frac{b_1}{b_2'}\right) R_2$$

$$a_1'x \qquad + c_1'z = d_1'$$
$$b_2'y + c_2'z = d_2'$$
$$c_3''z = d_3''$$

Step 3:- Remore z from R_1 & R_2

$$R_1 \to R_1 - \left(\frac{c_1'}{c_3''}\right) R_3$$
$$R_2 \to R_2 - \left(\frac{c_2'}{c_3''}\right) R_3$$

Gauss-Siedal Iteration

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$$
$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$$
$$z = \frac{1}{c_3}(d_3 - a_3x - b_3y)$$

$$x = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0)$$
$$y = \frac{1}{b_2}(d_2 - a_2x_0 - c_2z_0)$$
$$z = \frac{1}{c_3}(d_3 - a_3x_0 - b_3y_0)$$

Jacobi's Iteration method

$$x_2 = \frac{1}{a_1}(d_1 - b_1y_1 - c_1z_1)$$
$$y_2 = \frac{1}{b_2}(d_2 - a_2x_1 - c_2z_1)$$
$$z_2 = \frac{1}{z_2}(d_3 - a_3x_1 - b_3y_1$$

(x_0, y_i, z_i)

$$x_{i+1} = \frac{1}{a_1}(d_1 - b_1y_i - c_iz_i)$$
$$y_{i+1} = \frac{1}{b_2}(d_2 - a_2x_{i+1} - c_2z_i)$$
$$z_{i+1} = \frac{1}{c_3}(d_3 - a_3x_{i+1} - b_3y_{i+1})$$

ill condition system of linear eqn.

Q
$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25 \qquad x_0 = y_0 = z_0 = 1$$

$$x_1 = \frac{1}{20}(17 - y_0 + 2z_0)$$
$$y_1 = \frac{1}{20}(-18 - 3x_1 + z_0)$$
$$z_1 = \frac{1}{20}(25 - 2x_1 + 3y_1)$$

$x_1 = 0.8500$ (1)
$y_1 = -1.0275$ (-1)
$z_1 = 1.0109$ (1)

Ill Conditioned System of Linear Eqns

If small change in the coefficient of unt eq^n. results in the large change of the unknown then the system is termed as ill conditioned system.
This problem occurs when the determinant of coefficient matrix is nearer to zero.

Unit -2

Finite Differences & Interpolation

The calculus of finite differences deals with the study of change of y w.r.t change in independent var x provided the function is continuous.

Forward Difference:- (Descending diff', Δ)

$$\Delta y_i = y_{i+2} - y_i$$
$$\Delta f(x_i) = f(x_i + h) - f(x_i)$$

$$\Delta y_1 = y_2 - y_1$$
$$\Delta y_2 = y_3 - y_2$$
$$\vdots$$
$$\boxed{\Delta y_n = y_{n+1} - y_n}$$

Second order forward diff:-

$$\Delta^2 y_i = \Delta \cdot \Delta y_i = \Delta(y_{i+2} - y_i)$$
$$\boxed{\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i}$$
$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$$
$$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$$

$$\Delta^3 y_i = \Delta^2 y_{i+1} - \Delta^2 y_i$$
$$\Delta^4 y_i = \Delta^3 y_{i+1} - \Delta^3 y_i$$
$$\Delta^5 y_i = \Delta^4 y_{i+1} - \Delta^4 y_i$$
$$\vdots$$
$$\boxed{\Delta^n y_i = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i}$$

$$\Delta y_0 = y_1 - y_0$$
$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0 = y_2 - y_1 - (y_1 - y_0) = y_2 - 2y_1 + y_0$$
$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0 = (y_3 - 2y_2 + y_1) - (y_2 - 2y_1 + y_0) = y_3 - 3y_2 + 3y_1 - y_0$$
$$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0 = (y_4 - 3y_3 + 3y_2 - y_1) - (y_3 - 3y_2 + 3y_1 - y_0)$$
$$= y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$$

$$\boxed{\Delta^n y_0 = y_n - {}^nC_1 y_{n-1} + {}^nC_2 y_{n-2} - \dots\dots + (-1)^n y_0}$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0		$\Delta^2 y = \Delta y_1 - \Delta y_0$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$	
		$\Delta y_0 = y_1 - y_0$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$		$\Delta^5 y_0 = \Delta^4 y_1 - \Delta^4 y_0$
x_0+h	y_1		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_1 = \Delta^3 y_2 - \Delta^3 y_1$	
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_0$		$\Delta^4 y_1 = \Delta^3 y_2 - \Delta^3 y_1$
x_0+2h	y_2		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$			
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2$		
x_0+3h	y_3					
x_0+4h	y_4					
x_0+5h	y_5					

Properties:-

If a is constant then $\Delta K = 0$ & Δ is commutative w.r.t. constant K

$$\Delta K = K \Delta$$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta K = K - K = 0$$

$$\Delta[K(f(x))] = K \cdot \Delta(f(x))$$

L.H.S:-

$$K \cdot f(x+h) - K \cdot f(x)$$

$$K[f(x+h) - f(x)]$$

$$K\Delta[f(x)] \quad \text{R.H.S}$$

3 operand Δ is distributive:-

$$\Delta[f(x)+g(x)] = \Delta \cdot f(x) + \Delta \cdot g(x)$$

$$[f(x+h)+g(x+h)] - [f(x)+g(x)]$$

$$f(x+h) + g(x+h) - f(x) - g(x)$$

4. $\Delta[\alpha f(x) + \beta g(x)] = \alpha . \Delta f(x) + \beta . \Delta g(x)$

5. $\Delta^m(\Delta^n f(x)) = \Delta^n(\Delta^m f(x)) = \Delta^{m+n} f(x) = \Delta^{n+m} f(x)$

6. $\Delta[f(x) \cdot g(x)] = f(x+h) . \Delta g(x) + g(x) . \Delta f(x)$

7. $\Delta\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\ \Delta f(x) - f(x) . \Delta g(x)}{g(x+h) - g(x)}$

Properties:-

i) E is distributive:-

$$f(x) = u(x) + v(x)$$

$$E[f(x)] = E[u(x)] + E[v(x)]$$

$$E[u(x) + v(x)] = E[u(x)] + E[v(x)]$$

$$u(x+h) + v(x+h)$$

ii) E is commutative w.r.t const:-

$$EK = KE$$

iii) E is distributive:-

$$E^{m+n} f(x) = f(x + (m+n)h)$$

$$= f(x + mh + nh)$$

$$\mu[f(x)] = \frac{f(x+\frac{h}{2}) + f(x-\frac{h}{2})}{2}$$

$$= E^{1/2} f(x) + E^{-1/2} f(x)$$

$$\mu . f(x) = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right] f(x)$$

∇, δ & μ are expressed in form of

- $\underset{f(x)}{\Delta} = E - \underset{f(x)}{1}$

$$\Delta[f(x)] = (E-1)[f(x)]$$

L.H.S $\quad f(x+h) - f(x)$

$$E f(x) - f(x)$$

$$f(x)[E-1]$$

- $\nabla E = E\nabla = \Delta$

$\cancel{\Delta f(x) = f(x+h) - f(x)}$

$\cancel{\nabla f(x)}$

$$\nabla E f(x)$$

$$= \nabla[f(x+h)]$$

$$= f(x+h) - f(x)$$

$$= \Delta f(x)$$

1. $\nabla = \Delta E^{-1}$
2. $\mu^2 = 1 + \frac{\delta^2}{4}$
3. $\nabla = \frac{E-1}{E}$
4. $\Delta \cdot \nabla = \Delta - \nabla$
5. $\delta = E^{1/2} - E^{-1/2}$

Newton Backward Interpolation :-

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

$x = x_n + ph$

$$E_n(x) = \frac{h^{n+1} p(p+1)(p+2)\ldots(p-n)}{(n+1)!} f^{n+1}(\alpha)$$

where $\alpha \in I \equiv$ Boundary values

diff of $f(n) = \alpha$ $n+1$

Find sin 54° using Newton Backward I.P formula for given values:-

x	$y=f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
30°	0.5000					
		0.0736				
35°	0.5736		-0.0044			
		0.0692		-0.0005		
40°	0.6428		-0.0049		-0.0001	
		0.0643		-0.0004		0.0001
45°	0.7071		-0.0053		0.0000	
		0.0589		-0.0004		
50°	0.7660		-0.0057			
		0.0532				
55°	0.8192					

$x = x_n + ph$

$54 = 55 + p(5)$

$p = -\frac{1}{5}$ $\qquad -1 < p < 0$

$\boxed{p = -0.2}$ $\qquad \boxed{0.80903}$

6 Central Diff Inter-Polation Formula :-

Gauss Forward Inter Polation :-

$$y_P = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_{-1} + \frac{(P+1)P(P-1)}{3!}\Delta^3 y_{-1} + \frac{(P+1)P(P-1)(P-2)}{4!}\Delta^4 y_{-2} + \cdots$$

Observations:-

① y_0 - - - - - - - $\Delta^2 y_{-1}$ - - - - - - $\Delta^4 y_{-2}$ - - - - - - $\Delta^6 y_{-3}$ central line

Δy_0 $\Delta^3 y_{-1}$ $\Delta^5 y_{-2}$

② $0 < P < 1$

③ $x = x_0 + ph$

Use gauss forward formula to obtain $f(32)$ & given values $f(25) = 0.2707$, $f(30) = 0.3027$, $f(35) = 0.3386$, $f(40) = 0.3794$

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
25	0.2707			
		0.0320		
			0.0039	
30	0.3027			0.0010
		0.0359		
			0.0049	
35	0.3386			
		0.0408		
40	0.3794			

$$x = x_0 + ph$$

$$32 = 30 + P(5)$$

$$\boxed{P = 0.4}$$

3.0985

3.098 5|6 328

3.0986

Gauss backward IP :-

$$y_P = y_0 + P\,\Delta y_{-1} + \frac{(P+1)P}{2!}\Delta^2 y_{-1} + \frac{(P+1)P(P-1)}{3!}\Delta^3 y_{-2} + \frac{(P+2)(P+1)P(P-1)}{4!}\Delta^4 y_{-2} + \dots$$

Observations :-

① $y_0 \rightarrow \Delta y_{-1} \searrow \Delta^2 y_{-1} \nearrow \Delta^3 y_{-2} \searrow \Delta^4 y_{-2} \nearrow \Delta^5 y_{-3}$

Central line

② $-1 < P < 0$

③ $x = x_n + ph$

① y at given x :- $x_n > x$ & nearer

② $y = f(x)$
 x_n = below & nearer to mid point.

Error :-

$$E_{2n}(x) = \frac{h^{2n+1}(P^2-1^2)(P^2-2^2)\dots(P^2-n^2)}{(2n+1)!}$$

where $\alpha \in I \equiv$ boundary values

Q Find the value of $(1.06)^{19}$ using Gauss backward IP formula using given data.

$(1.06)^{10} = 1.79085$

$(1.06)^{15} = 2.39656$

$(1.06)^{20} = 3.20714$

$(1.06)^{25} = 4.29187$

$(1.06)^{30} = 5.74349$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	1.79085				
		0.60571			
15	2.39656		0.20487		
		0.81058		0.06928	
20	y_0 3.20714		0.27415		0.02346
		1.08473		0.09274	
25	4.29187		0.36689		
		1.45162			
30	5.74349				

$x = x_n + ph$

$19 = 20 + p(5)$

$p = \frac{-1}{5} = -0.2$

Stirling's I formula:-

$$y_p = y_0 + P\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{P^2}{2!}\Delta^2 y_{-1} + \frac{P(P^2-1)}{3!}\left(\Delta^3 y_{-1} + \Delta^3 y_{-2}\right)$$
$$+ \frac{P^2(P^2-1)}{4!}\Delta^4 y_{-2} + \ldots\ldots$$

Observations

① $y_0 - - \begin{bmatrix} \Delta y_{-1} \\ - - - \\ \Delta y_0 \end{bmatrix} - - - \Delta^2 y_{-1} - \begin{bmatrix} \Delta^3 y_{-2} \\ - - - \\ \Delta^3 y_{-1} \end{bmatrix} - \Delta^4 y_{-2} - \begin{bmatrix} \Delta^5 y_{-3} \\ - - - \\ \Delta^5 y_{-2} \end{bmatrix}$ central line

② $0 < P < 1$

③ $x = x_0 + ph$

y at given x

$x_0 < x$ & nearer

$y = f(x)$

$x_0 =$ above & nearer midpoint

* Bessel's I. Formula :-

$$y_p = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\left[\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right] + \frac{(P-\frac{1}{2})P(P-1)}{3!}\Delta^3 y_{-1}$$
$$+ \frac{(P+1)P(P-1)(P-2)}{4!}\left[\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2}\right] + \ldots\ldots$$

Observations

$\rightarrow y_0 \ldots\ldots \begin{bmatrix} \Delta^2 y_{-1} \\ \\ \Delta^2 y_0 \end{bmatrix} - - - \Delta^3 y_{-1} \begin{bmatrix} \Delta^4 y_{-2} \\ \\ \Delta^4 y_{-1} \end{bmatrix} - - - \Delta^5 y_{-2} \begin{bmatrix} \Delta^6 y_{-3} \\ \\ \Delta^6 y_{-2} \end{bmatrix}$ - - - central line

Δy_0

$x = x_0 + ph$ | $0 < P < 1$ | $\frac{y \text{ at given } x}{x_0 > x \text{ \& nearer}}$ | $y = f(x)$, $x_0 =$ above & nearer to mid