

# Secure MPC Lecture Notes

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## 1 Secure MPC (Multi-party company)

- Privacy-preserving Data Mining
- Membership
- Privacy-preserving ML
- MPC is easy if we could trust someone.
- A trusted party could get all the data and compute the value of the function at given inputs and then return it back.
- This is the ideal solution.

### 1.1 Problem:

- Creates a single point of failure.
- Trust is very rare and volatile.
- If there is trust in the world, then our problem is solved.

### 1.2 MPC Protocol

- Goal: to emulate the role of trusted party
- $P_i$  should not learn anything beyond what it can form  $(y, x_i)$

## 2 Secure Multi-Party Computation: Part II - A Toy NPC Protocol

### 2.1 Sum Function:

- Goal: to securely compute  $S = b_1 + \dots + b_n, b_i \in \{0, 1\}$
- $P_i$  should not learn anything beyond what it can form  $(S, b_i)$
- Parties connected by pair-wise private and authentic channel
- Each  $P_i$  has a private bit  $b_i \in \{0, 1\}$
- Goal: to securely compute  $S = b_1 + \dots + b_n$
- $S \in \{0, \dots, n\}$ 
  - Parties agree upon a public modulus  $M > n$
  - All operations are done over  $\mathbb{Z}_M$
- $k$  is randomly sampled from  $\mathbb{Z}_5$ , and  $c_1$  is defined to be the sum of this random value with its input  $b_1$ .

- $c_1 = b_1 + k$
- Now,  $c_2 = c_1 + b_2 = k + b_1 + b_2$
- $c_3 = c_2 + b_3 = k + b_1 + b_2 + b_3$
- $c_4 = c_3 + b_4 = k + b_1 + b_2 + b_3 + b_4$
- $S = c_1 + c_2 + c_3 + c_4 + k$
- Does  $P_i$  learn anything beyond what it can learn from  $(S, b_i)$ ?
- $View_i$  = 'information' learnt by  $P_i$  during the protocol execution
- Security goal:  $P_i$  should not learn anything beyond what it can from  $(S, b_i)$

## 2.2 Toy MPC Protocol: Possible Attacks

- What happens if two parties  $P_i$  and  $P_{i+2}$  collude together in the MPC protocol?
  - The input  $b_i$  will be learnt
  - Not possible in the ideal solution
- Hence, the MPC protocol is not as secure as the ideal solution, if two parties collude.

# 3 Secure Multi-Party Computation: Part III - Maurer's MPC Protocol

## 3.1 The Arithmetic Circuit Abstraction

- The protocol is a generic MPC protocol for securely computing any function over a finite Ring  $(\mathbb{R}, +, \cdot)$
- Simplifying assumptions (without loss of generality):
  - Each  $P_i$  has a single input value from  $\mathbb{R}$
  - Single function-output, to be learnt by everyone.
  - The function is a deterministic function
- There is the publicly known circuit for computing the function  $f$ .
- The arithmetic-circuit abstraction is without loss of generality.
  - Any (efficient) algorithm/computation can be represented by an efficient Boolean circuit  $B_{cir}$  consisting of AND and NOT gates.
  - $B_{cir}$  can be simulated by an equivalent (efficient) arithmetic circuit  $A_{cir}$  over  $(\mathbb{R}, +, \cdot)$
  - Bit 0  $\leftrightarrow$  additive identity of  $\mathbb{R}$
  - Bit 1  $\leftrightarrow$  multiplicative identity of  $\mathbb{R}$
  - $\neg b \leftrightarrow "1" - "b"$  over  $\mathbb{R}$
  - $a \text{ AND } b \leftrightarrow "a" \cdot "b"$  over  $\mathbb{R}$
- Example of circuit emulation over Boolean ring  $\mathbb{R} = \{(0, 1), \text{XOR}, \cdot\}$  — all operations are modulo 2.
- Why arithmetic circuits and not boolean circuits?
  - There are instances where we can represent in a compact manner using arithmetic circuits.
- Computing  $f(x_1, \dots, x_n)$  is equivalent to evaluating the corresponding arithmetic circuit over  $x_1, \dots, x_n$ .
- If  $x_1, \dots, x_n$  are made public, then the circuit can be evaluated locally (without any interaction) by the parties themselves.
- MPC protocol will also perform circuit-evaluation such that corrupt parties

### 3.2 Modelling Corruption in an MPC Protocol

- Bad people work together to cause maximum harm.
- To be To model this, we assume that there is a single monolithic/centralized entity called adversary, who 'controls' a 'certain' number of parties.
- Threshold model: at most  $t$  corrupt parties, where  $t < n$ . It is not publicly known who the corrupt parties are.
- $t = 2 \Rightarrow$  Possible corruption scenarios is 4 choose 2 = 6 cases.
- Question: What if there was only 1 corrupt party?
- Ans: In that case, the cases will be a subset of the case we are considering.
- Unbounded: Corrupt parties are computationally unbounded.
- Passive/Semi-honest: Adversary is a passive observer, eavesdrops on the corrupted parties.
- Static: Adversary corrupts parties at the onset of a protocol.

### 3.3 $(n, t)$ Secret Sharing Scheme [Shamir 1979, Blakley 1979]

- A set of parties  $P = \{P_1, \dots, P_n\}$  connected by pair-wise private and authentic channels.
- A designated dealer  $D \in P$ , with a secret input  $s \in S$ .
- Goal: to distribute a share  $s_i$  of  $s$  to every party  $P_i$ :
  - Should be impossible for any set of  $t$  or less number of share-holders to pool their shares and reconstruct back  $s$ 
    - \* Even if the  $t$  parties are given unlimited time and resources
  - Should be possible for any set of  $(t + 1)$  or more share-holders to pool their shares and reconstruct back  $s$ .
- Less than  $t + 1$  shares: no information about the secret
- $t + 1$  or more shares: complete secret

## 4 Secure MPC via Secret-Shared Circuit Evaluation

- Secure MPC equivalent to secret-shared circuit evaluation over a ring
- Inputs:  $(n, t)$ -secret shared
- Intermediate gates:
  - $(n, t)$  secret-shared gate input(s)  $\Rightarrow (n, t)$  secret-shared gate output
- Output gate: publicly reconstruct.
- Any generic MPC protocol follows the above template.
- An instantiation of  $(n, t)$  secret-shared.
- Protocols for addition and multiplication over  $(n, t)$  secret-shared inputs.

### 4.1 $(n, t)$ Replicated Secret Sharing (RSS): [ISN87]

- Idea: divide the secret randomly such that every candidate adversarial set of size  $t$  misses at least one share
- All operations over a finite Ring  $(\mathbb{R}, +, \cdot)$
- $s = s_1 + s_2 + s_3$
- $G_i = P - Z_i$
- A piece  $s_i$  can be replicated to multiple entities depending on the group.

## 4.2 (n, t)-RSS: Sharing Protocol

- $k = |n \text{ choose } t|$
- $Z = \{Z_1, \dots, Z_k\}$ : all subsets of  $t$  parties
- $G_i = P - Z_i$
- $Sh_{RSS}(s)$ 
  - $D$  on having input  $s \in \mathbb{R}$  does the following
  - Randomly pick  $s_1, \dots, s_k \in \mathbb{R}$ , such that  $s = s_1 + \dots + s_k$
  - For  $i = 1, \dots, k$  send  $s_i$  to every party in  $G_i$ .
  - For  $j = 1, \dots, n$  party  $P_j$  outputs its share  $S_j$  where  $S_j = \{s_i\}_{P_j \in G_i}$
  - Privacy: if  $D$  is honest, then any subset of  $t$  shareholders learn no information about  $s$ .
  - Adversary will be missing at least one piece  $s_i$  which will be random for it.

## 4.3 (n, t)-RSS: Correctness

- Claim: Any subset  $A$  of  $t + 1$  parties can reconstruct  $s$
- Union of the shares of  $A \Rightarrow \{s_1, \dots, s_k\}$
- If the union of the shares of  $A$  misses  $s_i$  then it implies that  $A = Z_i$ . Contradiction: size of  $A$  is  $t + 1$ , but the size of these  $Z_i$  (adversaries) is  $t$ .

## 4.4 Secure MPC via Secret-Shared Circuit Evaluation

- Secure MPC equivalent to secret-shared circuit evaluation over a ring.

## 4.5 (n, t)-RSS: Linearity Part I

- $a = a_1 + \dots + a_i + \dots + a_k$
- $b = b_1 + \dots + b_i + \dots + b_k$
- $a + b = (a_1 + b_1) + \dots + (a_i + b_i) + \dots + (a_k + b_k)$

## 4.6 (n, t)-RSS: Linearity Part II

- $a = a_1 + \dots + a_i + \dots + a_k$ ; public constant  $c$ .
- $a + c = (a_1 + c) + a_2 + \dots + a_k$  (Gives replicated share of  $a + c$ )
- Non-interactive process
- Similarly, if we tell all  $G_i$  to multiply their share with  $c$ , then we have the replicated share of  $a \cdot c$
- But, to get the replicated share of  $a \cdot b$ , we can't ask each of the groups to multiply  $a_i$  with  $b_i$ . That wouldn't result in  $a \cdot b$ .
- To get this done, we would require interaction. Non-linear operation  $\rightarrow$  multiplication.

## 4.7 (n, t)-RSS: Evaluating Multiplication Gate with $t < \frac{n}{2}$

- Idea: for each summand  $a_i \cdot b_j$ , identify the designated party who will  $(n, t)$ -RSS-Share  $a_i \cdot b_j$ .
- $G_i = P - Z_i \rightarrow$  Parties having the share  $a_i$
- $G_j = P - Z_j \rightarrow$  Parties having the share  $b_j$
- $G_i \cap G_j = P - (Z_i \cup Z_j)$
- $G_i \cap G_j = P - (Z_i \cup Z_j) \neq \emptyset$
- Least indexed party from  $G_i$  and  $G_j$  ...

## 4.8 Downside of RSS-Based MPC?

- What will be the share-size of each party?
  - $O(k)$ , where  $k$  is  $n$  choose  $t$ .
  - This is exponential in  $n$  if  $t < \frac{n}{2}$

## 5 Secure Multi-Party Computation: Part IV - Shamir's Secret Sharing Scheme

### 5.1 Polynomials Over a Field

- Let  $(\mathbb{F}, +, \cdot)$  be a field
- Every non-zero element has a multiplicative inverse, this property is not there in rings.
- Definition: a  $t$ -degree polynomial  $f(X)$  over  $\mathbb{F}$  is of the form  $f(X) = a_0 + a_1 \cdot X + \dots + a_t \cdot X^t$
- Example: consider the field  $(\mathbb{Z}_7, +_7, \cdot_7) \rightarrow$  addition/multiplication modulo 7.
- $f(X) = 6 + 2X + 3X^4$
- $f(1) = (6 + 2 + 3) \bmod 7 = 4$
- Properties:
  - Theorem (Abstract algebra): a  $t$ -degree polynomial over  $\mathbb{F}$  has at most  $t$  roots.
  - Theorem (Abstract algebra): two distinct  $t$ -degree polynomials over  $\mathbb{F}$  can have at most  $t$  common values
  - Let  $f(X) = X^2 + 4X + 2$
  - Let  $g(X) = X^2 + 3X + 3$  (over  $(\mathbb{Z}_7, +_7, \cdot_7)$ )
  - $f(1) = g(1) = 0$
  - Point  $(1, 0)$  is common

### 5.2 Lagrange's Polynomial Interpolation

- Theorem: Let  $(x_1, y_1), \dots, (x_{d+1}, y_{d+1})$  be pairs of elements from  $\mathbb{F}$ , where  $x_1, \dots, x_{d+1}$  are distinct. Then, there exists a unique  $d$ -degree polynomial  $f(X)$  over  $\mathbb{F}$ , such that  $f(x_i) = y_i$ , for  $1 \leq i \leq d+1$ .
- Idea: Express the unknown  $f(X)$  as a linear combination of  $d+1$   $d$ -degree polynomials  $\delta_1(X), \dots, \delta_{d+1}(X)$
- $$f(X) = y_1 \cdot \delta_1(X) + \dots + y_i \cdot \delta_i(X) + \dots + y_{d+1} \cdot \delta_{d+1}(X)$$
- $\delta_1(x_1) = 1, \delta_i(x_i) = 1, \delta_{d+1}(x_{d+1}) = 1$
- $\delta_1(x_2) = \dots = \delta_1(x_{d+1}) = 0, \delta_i(x_1) = \dots = \delta_i(x_{i-1}) = \delta_i(x_{i+1}) = \dots, \delta_{d+1}(x_1) = \dots = \delta_{d+1}(x_d) = 0$
- Cannot do Lagrange's interpolation over a ring.

### 5.3 Properties of $t$ -degree Polynomial

- Let  $P^{s,t}$  = set of all  $t$ -degree polynomials over  $\mathbb{F}$ , with  $s$  as the constant term.
- Each  $f(X) \in P^{s,t}$  is of the form  $f(X) = s + a_1 \cdot X + \dots + a_t \cdot X^t$ , where each  $a_1, \dots, a_t \in \mathbb{F}$
- $$|P^{s,t}| = |\mathbb{F}|^t$$
- Ex:  $t = 2, \mathbb{F} = (\mathbb{Z}_3, +_3, \cdot_3)$  and  $s = 1$

- $1, 1 + X, 1 + X^2, 1 + X + X^2, 1 + 2X + X^2$
- $1 + 2X, 1 + 2X^2, 1 + X + 2X^2, 1 + 2X + 2X^2$
- These are in  $P^{1,2}$
- $\{(x_{i_1}, y_{i_1}), \dots, (x_{i_t}, y_{i_t})\} \rightarrow$  arbitrary values from  $\mathbb{F}$ :
- $$x_{i_1} \neq \dots \neq x_{i_t} \neq 0$$
- For any given  $s \in \mathbb{F}$ , there is a unique polynomial from  $P^{s,t}$  passing through  $\{(0, s), (x_{i_1}, y_{i_1}), \dots, (x_{i_t}, y_{i_t})\}$
- Input:  $s \in \mathbb{F}$
- Pick  $f(X) \in_r P^{s,t}$
- Output:  $f(x_1), \dots, f(x_n) \rightarrow y_i = f(x_i)$
- $x_1 \neq \dots \neq x_n \neq 0$  and publicly known
- This is known as  $Exp_{Shamir}$
- How much information about the input  $s$  is learnt through any subset of  $t$  output values?
- $\Pr_{f(X) \in_r P^{s,t}}[(f(x_1) = y_1) \text{ AND } \dots \text{ AND } (f(x_t) = y_t)] = \frac{1}{|P^{s,t}|}$
- $\Pr_{g(X) \in_r P^{s',t}}[(g(x_1) = y_1) \text{ AND } \dots \text{ AND } (g(x_t) = y_t)] = \frac{1}{|P^{s',t}|}$
- Since both have the same probability, there is no way to distinguish between them.
- In summary, we are hiding in the constant term of the polynomial

#### 5.4 Shamir's (n, t) Secret-Sharing Scheme

- Public set-up: finite field  $(\mathbb{F}, +, \cdot)$ , with  $|\mathbb{F}| > n$  and publicly known, non-zero distinct elements  $x_1, \dots, x_n \in \mathbb{F}$
- $Sh_{Shamir}(s)$ 
  - Randomly pick  $a_1, \dots, a_t \in \mathbb{F}$
  - Define the polynomial  $f(X) = s + a_1 \cdot X + \dots + a_t \cdot X^t$
  - $$f(X) \in_r P^{s,t}$$
  - For  $i = 1, \dots, n$ , compute the share  $s_i = f(x_i)$
- Correctness: any set of  $(t+1)$  shares suffice to uniquely interpolate back  $t$ -degree polynomial  $f(X)$
- Privacy: Any set of  $t$  shares ...

## 6 Secure Multi-Party Computation: Part V - BGW MPC Protocol [Michael Ben-Or, Shafi Goldwasser, Avi Wigderson]

### 6.1 Why do we need to reduce the degree here?

- The degree-of-sharing may blow up so much that we don't have sufficient shares to reconstruct the output
- 100 immediate successive multiplications, would result in the  $(n, 102t)$ -SSS, and we don't have enough shares to reconstruct it back.
- To reconstruct  $y =$  "successive multiplication 100 times", we would need ...

## 6.2 The Degree Reduction Problem

- No "additional information" about  $a$  and  $b$  should be leaked in the process.
- Requires interaction among the parties

BGW : Completeness Theorems for Non-Cryptographic Fault-Tolerant Distributed Computation (Extended Abstract). STOC 1988: 1-10

GRR : Simplified VSS and Fast-Track Multiparty Computations with Applications to Threshold Cryptography. PODC 1998:

## 6.3 GRR Method of Degree-Reduction with $t < \frac{n}{2}$

- $A(Z) - a_1, a_2, \dots, a_i, \dots, a_n (n, t) \rightarrow a$
- $B(Z) - b_1, b_2, \dots, b_i, \dots, b_n (n, t) \rightarrow b$
- $K(Z) - k_1, k_2, \dots, k_i, \dots, k_n (n, 2t) \rightarrow a \cdot b$
- Idea:  $ab$  is a linear function of  $k_1, \dots, k_n$
- $K(Z) = \delta_1(Z) \cdot k_1 + \dots + \delta_n(Z) \cdot k_n$
- $K(0) = \delta_1(0) \cdot k_1 + \dots + \delta_n(0) \cdot k_n$
- $ab = d_1 \cdot k_1 + \dots + d_n \cdot k_n$
- $\Rightarrow a \cdot b = d_1 \cdot k_1 (n, t)\text{-SSS} + \dots + d_n \cdot k_n (n, t)\text{-SSS}$

## 6.4 Shared Circuit-Evaluation in the Pre-Processing Model

- Also known as shared circuit-evaluation with correlated randomness
- Inputs: Randomly  $(n, t)$  - secret-shared
- Shared gate-evaluation: If gate-input(s) is/are randomly  $(n, t)$  secret-shared, then gate output is randomly  $(n, t)$  secret-shared.
- Linear gates: non-interactive
- Multiplication gates: ...

## 6.5 Evaluation of Multiplication Gates via Beaver Multiplication Triples

- Multiplication is the main operation which results in a 'bottle-neck'.
- $d = u - a; e = v - b;$
- $w = (u - a + a)(v - b + b)$
- $= de + db + ea + c \rightarrow$  linear function of  $(a, b, c)$
- Steps of the online phase protocol:
  - 1. Parties compute  $d = u - a; e = v - b.$
  - 2. Parties reconstruct  $d$  and  $e$  (Just need 2 public reconstructions)
- Privacy of gate-inputs:
  - $(a, b, c)$ : independent of gate-inputs
  - $d, e$ : one-time pad encryption of gate-inputs
- Need to have  $M$  multiplication-triples if there are  $M$  multiplication gates in the circuit
- Assume that pre-processing is done without knowing how this is done.

## 6.6 Generating Secret-Shared Multiplication-Triples

- Goal: to generate  $M$  number of  $(n, t)$ -secret-shared multiplication-triples  $(a, b, c)$  such that:
  - $a \in_r \mathbb{F}$
  - $b \in_r \mathbb{F}$
  - $c = a \cdot b$  (product of the first two elements sampled from  $\mathbb{F}$ )
  - $(a, b, c)$  remains random for the adversary
- Done in two stages:
  - Stage I: Generating  $(n, t)$ -secret-sharing of a random  $a$  and a random  $b$
  - Stage II: Computing  $(n, t)$ -secret-sharing of  $a \cdot b$  from  $(n, t)$ -secret-sharing of  $a$  and  $b$
- All instances of multiplication-protocol can be executed in parallel (whereas earlier multiplication used to happen sequentially and each step was dependent on the previous multiplication result)

## 6.7 Generating Shared Multiplication-Triples: Stage I

- Goal: Generating  $(n, t)$ -secret-sharing of a random  $a$  and a random  $b$
- $a$  and  $b$  should remain random for the adversary
- Naive method for generating  $(n, t)$ -secret-sharing of a random  $a$
- Each  $P_i$  picks a random value and  $(n, t)$ -secret-shares it.
- 

$$a^{(1)} + a^{(2)} + \dots + a^{(i)} + \dots + a^{(n)} = a$$

- The final  $a$  is defined to be the sum of all the  $n$  values
- $a$  is random as long as  $t < n$ .

## 6.8 More Efficient Methods for Pre-Processing Phase

- Hyperinvertible matrices: Perfectly-Secure MPC with Linear Communication Complexity
- A more efficient (and direct) method for generating shared multiplication-triples: An Efficient Framework for Unconditionally Secure Multiparty Computation.

# 7 Secure Multi-Party Computation: Part VII - GMW MPC Protocol [Goldreich, Micali, Wigderson: How to Play any Mental Game or ...]

## 7.1 Perfectly-Secure MPC Against Passive Corruptions: Summary

- Setup required: pair-wise private and authentic channels.
- Optimal resilience:
  - $t < \frac{n}{2}$
- Circuits over a field (for efficient protocols)
- Q. Can these "restrictions" be removed assuming a PPT adversary?



## 7.2 (n, t) Additive Secret-Sharing Scheme

- Threshold  $t < n$ . Worst case:  $t = n - 1$ .
- Intuition: secret  $s$  divided into  $n$  random shares, such that:
  - Sum of the shares is  $s$
  - Any subset of  $n - 1$  shares is independent of  $s$
- $Sh_{Add}(s)$ :
  - Select  $s_1, \dots, s_{n-1} \in_r \mathbb{R}$
  - Set  $s_n = s - (s_1 + \dots + s_{n-1})$
  - Output  $s_1, \dots, s_n$
- $Rec_{Add}$ :
- ...

## 7.3 Linearity of Additive Sharing: Summary

- Any (publicly-known) linear function of  $(n, t)$ -additively-shared values can be computed by applying the linear function on the shares.

## 7.4 1-out-of-2 Oblivious Transfer (OT)

- Michael O. Rabin. "How to exchange secrets by oblivious transfer" Technical Report TR-81, Aiken Computation Laboratory, Harvard University, 1981
- Sender's security:
  - A corrupt receiver should learn no additional information about  $m_{1-b}$
- Receiver's security:
- ...

## 7.5 Constructing OT Protocols

- OT protocols can be designed based on varieties of assumptions
  - Factoring problem
  - DDH problem
  - Trapdoor one-way permutations (ex, RSA problem)
  - Noisy communication channel
- Can't we design OT protocols unconditionally (without any assumptions)?
  - Unconditionally-secure OT  $\rightarrow$  unconditionally-secure

## 7.6 Multiplication Over Additive-Secret-Shared Values Using OT

- For simplicity,  $n = 2$ ,  $t = 1$  and Boolean ring.  $\mathbb{R} = (\{0, 1\}, \text{XOR}, \text{AND})$
- $a_1 \text{ XOR } a_2 = a$
- $b_1 \text{ XOR } b_2 = b$
- $c_1 \text{ XOR } c_2 = c$
- $c = a \text{ AND } b$
- $a \text{ AND } b = (a_1 \text{ XOR } a_2) \text{ AND } (b_1 \text{ XOR } b_2)$

- $= (a_1 \text{ AND } b_1 \text{ XOR } a_2 \text{ AND } b_1) \text{ XOR } (a_1 \text{ AND } b_2 \text{ XOR } a_2 \text{ AND } b_2) \rightarrow$  first component is  $c_1$  and second is  $c_2$ .
- 1-out-of-2 OT  $0 \rightarrow [] \leftarrow b_2$
- $a_1 \rightarrow [] \rightarrow a_1 \text{ AND } b_2$
- 1-out-of-2 OT  $b_2 \rightarrow [] \leftarrow 0$
- $a_2 \text{ AND } b_1 \leftarrow [] \rightarrow \dots$
- Problem with this is that  $b_2$  knows the value of  $b_2$  and so it can figure out what  $a_1$  is from the result of  $a_1 \text{ AND } b_2$ .
- A way around this is to pick two random bits  $r_0, r_1 \in \{0, 1\}$
- $r_0 \rightarrow [] \leftarrow b_2$
- $r_0 \text{ XOR } a_1 \rightarrow [] \rightarrow r_0 \text{ XOR } (a_1 \text{ AND } b_2)$
- $b_1 \rightarrow [] \leftarrow r_1$
- $r_1 \text{ XOR } (a_2 \text{ AND } b_1) \leftarrow [] \rightarrow r_1 \text{ XOR } a_2$
- $ab = (a_1 \text{ XOR } a_2) \text{ AND } (b_1 \text{ XOR } b_2)$
- $= a_1 \text{ AND } b_1 \text{ XOR } a_2 \text{ AND } b_1) \text{ XOR } (a_1 \text{ AND } b_2 \text{ XOR } a_2 \text{ AND } b_2)$
- $= (a_1 b_1 \text{ XOR } r_0 \text{ XOR } r_1 \text{ XOR } a_2 b_1) \text{ XOR } (r_1 \text{ XOR } r_0 \text{ XOR } a_1 b_2 \dots)$

## 7.7 n-Party Multiplication Over Additive-Secret-Shared Values Using OT

- Each pair of distinct parties  $(P_i, P_j)$  execute two OTs to generate additive-sharing of  $a_i b_j$  and  $a_j b_i$ .
- It's possible to extend this idea to a general ring.

# 8 Secure Multi-Party Computation: Part VIII - Yao's 2PC Protocol [Protocols for secure computations]

## 8.1 Yao's Protocol: The Setting

- Semi-honest (passive) corruption
  - Corrupt party does not deviate from protocol instructions.
- Extension for the  $n$ -party case possible but non-trivial. (DMR's protocol)
- Function  $f$  represented as a Boolean Circuit (AND, OR, NOT, XOR)
- Requires constant number of interactions between the parties. This is what makes Yao's protocol different from the previous ones discussed.
- The price to pay is that these are very expensive operations.

## 8.2 Yao's Garbled Circuit Construction

- GC Constructor (input  $a, b$ ); GC Evaluator (input  $c$ ). Operation  $Y = (a \text{ AND } b) \text{ OR } c$
- Constructor assigns two keys for the wire  $a$  and labels these keys. Does the same for  $b, c$ , and output wire.
- Garbling up the gate: each gate is assigned 4 boxes.

### 8.3 Replacing keyed-box with Cryptographic Mechanisms

- Each wire is assigned a pair of AES keys.
- Each box for the gate is double-encrypted (?).
- There is a small problem: When we give the two keys for each wire to open one of the boxes of the gate, we cannot know whether the output we get is a valid output or whether it is simply a ciphertext because decryption is just a computation. This was avoided when we had boxes and locks because each of the lock either opens or remains locked.
- Question: How is this issue resolved?
- Ans: "Special Correctness" resolves this issue. It enables the evaluator to directly pinpoint the cipher to the correct box and decrypt.
- $(G, E, D)$  has 'special correctness'
- for two distinct keys  $(k_1, k_2)$

### 8.4 The Last Thing: Obviously Transferring The Keys to The Evaluator

- ...

### 8.5 Yao's 2-Party Protocol

- Evaluator is given the labelling of the output wire since that is safe to reveal.
- GC:  $(C_1, C_2, C_3, C_4)$  + decoding info:  $(k_3^0, k_3^1)$
- The keys for  $x$ :  $k_0^1$ .
- OT is used for
- Evaluator will try to use the keys to get the legitimate output by checking all 4 cases.
- It already knows the output since it has the labelling.
- The second interaction will be asking for the output key to be correctly mapped to the right output.
- The first interaction will be in giving the keys to the evaluator.

### 8.6 Yao's Garbling - Recent Developments

- Point-and-permute [NPS99]: no 'special correctness' needed
- only one ciphertext needs to be decrypted
- Garbled Row Reduction (only 3 boxes/ciphertexts needed for each gate)

NPS99 : 4-to-3 ciphertexts

GNLP15, ZRE15 : 4-to-2 ciphertexts (optimal)

## 9 Courses Offered by the Speaker on NPTEL

- Foundations of Cryptography
- Secure Computation: Part I
- Secure Computation: Part II (Byzantine corruption, malicious corruption)