

COMBINATORIAL GAME THEORY (CS201 Pre-Project Report)

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1 Introduction

Combinatorial game theory (CGT) is a branch of applied mathematics and theoretical computer science that typically studies sequential games with perfect information. A game, in its simplest terms, is a list of possible "moves" that two players, called left and right, can make. The game position resulting from any move can be considered to be another game. This idea of viewing games in terms of their possible moves to other games leads to a recursive mathematical definition of games.

2 Description of Combinatorial Game

In a combinatorial game there are two players who take turns moving alternatively. Both players will have complete knowledge of the game state at all times ("no hidden information"), and the effect of each move will be entirely known before the move is made("no chance elements"). Play continues until the player whose turn it is to move has no legal moves available. Under normal play the last player to move wins. In mis'ere play the last player loses. Combinatorial games include well-known games like chess, checkers, Go, Arimaa, Hex, and Connect6.

3 Objective of this Project

The aim of doing this project on Combinatorial Game Theory is to understand in a more detailed way the principles involved in optimum game play of combinatorial games. Not only I will get to learn about games in general and how to understand them but I will also understand how to play them well.

4 Mathematical Background Required

- Basic knowledge of number theory, especially, fields like Modular Arithmetic and Counting.
- Knowledge of Abstract Algebra, especially, Groups is required.

5 Topics to Study

5.1 Basic techniques(heuristics) of Combinatorial games

- **Greedy Strategy:** A player following a greedy strategy always chooses the move that maximizes or minimizes some quantity related to the game position after the move has been made.
- **Symmetry:** Symmetry is an intuitively obvious strategy. Whenever your opponent does something on one part of the board you should mimic this move in another part. A strategy that maintains a simple symmetry is called Tweedledum-Tweedledee.
- **Change the Game:** Sometimes a game is just another game in disguise. The tactics of a particular game can be successfully applied to another game, i.e., A game can be played in the same manner in which another game is played.
- **Parity:** A number's parity is whether the number is odd or even. In lot of games, only the parity of a certain quantity is relevant — the trick is to figure out just what quantity! With the normal play convention that the last player with a legal move wins, it is always the objective of the first player to play to ensure that the game lasts an odd number of moves, while the original second player is trying to ensure that it lasts an even number of moves.
- **Give Them Enough Rope!:** If you are in a losing position, it pays to follow the Enough Rope Principle: Make the position as complicated as you can with your next move. Hopefully, your opponent will tie himself up in knots while trying to analyze the situation.
- **Strategy Stealing:** Strategy stealing is a technique whereby one player steals another player's strategy. This strategy proves quite handy at times in many games.

5.2 Outcome Classes

- Game positions and options.
- Impartial games like Partizan Endnim.

5.3 The Algebra of Games

- **Fundamental Theorem of Combinatorial Game Theory:** Let G be a short combinatorial game, and assume normal play. Either Left can force a win playing first on G or else Right can force a win playing second, but not both.
- **Bouton's Theorem:** Let G be a Nim position.
 - If G is a zero position, then every move from G leads to a nonzero position.
 - If G is not a zero position, then there exists a move from G to a zero position.
- Games form an abelian group with a partial order.
- If G is an impartial game, then G is either N or P.

5.4 Games to Study

I'll study rules and strategies of many games, namely, Tic-Tac-Toe, Nim, Hex. Also, if time permits, I'll study checkers, conect6, chess, etc.

5.5 Case Studies

- Long Chains in Dots & Boxes
- Partizan Endnim
- Domineering Rectangles

6 Resources

- **Lessons in play:** An introduction to combinatorial game theory by M. Albert, R.Nowakowski, D. Wolfe
- **Wikipedia link:** https://en.wikipedia.org/wiki/Combinatorial_game_theory