MATH-GA 2751.001: RISK AND PORTFOLIO MANAGEMENT

Spring 2021

Final exam

Due May 12, 2021 at 7:10 PM New York time

In an effort to reduce money supply and increase the US savings rate post-pandemic, the US Treasury is considering issuing two new types of treasury securities. Currently the Treasury offers conventional bonds that pay fixed semi-annual coupons with fixed redemption amount at maturity, the Treasury Inflation Protected Securities (TIPS) which also have fixed semi-annual coupon but the par amount increases with inflation so coupon payments increase over time and the investor gets back more than the original cost of the bond at maturity.

One new proposal is Treasury Equity Linked Securities (TELS) in which the par amount increases with the Nasdaq stock market index. These are expected to carry negative coupon rates since they are effectively call options on equities, so holders will have to pay money every six months, or have their par amount debited for the cost. This is expected to be popular among investors who want stock market appreciation without the risk of stock market declines.

The other new proposal is Treasury Unemployment Backed Securities (TUBS). The par amount of these securities is fixed, but the semi-annual coupon rate is equal to the average monthly unemployment rate over the preceding six months, minus a spread set at time of issue. These are expected to be popular among people who want higher income in times of recession, either to insure against unemployment or to hedge other investments.

Both new securities will initially be offered in five year maturities.

Table I

Date	Treasury	TIPS			TELS			TUBS	
	Cash flow	Cash flow	CPI	Principal	Cash flow	Nasdaq	Principal	Cash flow	Unemployment
2016-04-29	(998.55)	(1,022.45)	238.99	1,000.00	(1,630.17)	4,775.36	1,000.00	(1,000.00)	
2016-10-29	6.25	0.63	241.74	1,011.50	10.43	5,190.10	1,086.85	4.42	4.88%
2017-04-29	6.25	0.64	244.27	1,022.10	11.77	6,047.61	1,266.42	3.03	4.61%
2017-10-29	6.25	0.64	246.66	1,032.07	13.35	6,701.26	1,403.30	1.45	4.29%
2018-04-29	6.25	0.65	250.28	1,047.21	14.47	7,119.80	1,490.95	0.34	4.07%
2018-10-29	6.25	0.66	252.90	1,058.19	14.91	7,050.29	1,490.95	-	3.82%
2019-04-29	6.25	0.66	255.33	1,068.35	16.00	8,161.85	1,709.16	-	3.83%
2019-10-29	6.25	0.67	257.39	1,076.97	17.21	8,276.85	1,733.24	-	3.62%
2020-04-29	6.25	0.67	256.19	1,071.97	18.00	8,914.71	1,866.81	7.43	5.49%
2016-10-29	6.25	0.68	260.46	1,089.84	21.05	11,185.59	2,342.36	28.13	9.63%
2021-04-29	1,006.25	1,108.64	264.79	1,107.96	2,975.46	14,082.55	2,949.00	1,011.62	6.32%

Table 1 shows pro forma cash flows for securities issued five years ago. The 5-year treasury coupon was 1.25% and it was issued at a yield of 1.28%, so the price was $998.55 per $1,000 face. It paid $6.25 every six months until maturity, then $1,000 plus the last $6.25 coupon at maturity.

The 5-year TIPS carried a 0.125% coupon and sold at a yield of negative 0.32% for a price of $1,022.45 per $1,000 face. The principal amount went up and down with inflation (however it would not dip below $1,000 at maturity even if there were net deflation over the five years). The coupon was paid on the average monthly principal amount, the redemption value at maturity was based on the final principal amount.

The TELS were assumed to have been issued with a 2% coupon and a negative 8% yield, for a $1,630.17 price per $100 face. The principal amount goes up with the Nasdaq index, and did not go down when the Nasdaq went down. The coupon is paid on the average value over the period. The redemption value is the final principal amount.

The TUBS were assumed to be issued at par, with a coupon equal to the unemployment rate minus 4%, but floored at zero.

The file *NYU 2021 returns.csv* contains estimated daily returns on all four securities going back to 2003 as well as the returns on a one-month treasury bill to use as the risk-free rate of interest. Use it to answer the following questions.

1. What were the average annualized daily excess returns, annualized standard deviation of excess returns and annualized Sharpe ratios for the four securities?
2. How do the average annualized daily excess returns from (1) compare to the compound average annual excess growth rates for the four securities? What explains the differences?
3. Compute the covariance matrix, correlation matrix and inverse covariance matrix for the four excess return series. Do the results support Philips Curve ideas?
4. Compute the optimal Sharpe ratio portfolio with weights adding up to one. What is its Sharpe ratio?
5. Consider three factor portfolios. Mkt-R_f puts 25% in each of the four securities, and shorts 100% of the 1 month treasury bill. Momentum weights each of the four securities by 10 times its previous day's return. So if the 5-year treasury returned 1% yesterday, it gets a 10% portfolio weight today. If it returned -1%, it gets -10%. Any net long or short exposure is offset using the 1-month treasury bill, so if total exposure over the three securities is negative 7%, you go long 7% of the 1-month treasury bill to make the portfolio zero net investment. The value portfolio is computed using the weights in the *Final 2021 value signal.csv* file. These weights are computed using a fundamental economic model of the value of the four securities relative to price. This portfolio also has to be adjusted to zero net investment using the 1-month treasury bill. What are the annualized average returns, annualized standard deviation of return and annualized Sharpe ratios of the three factor portfolios?

Table 2

		Alpha	Mkt-R_f	Momentum	Value	r^2
Mkt-R_f	Coefficient					
	t-stat					
Momentum	Coefficient					
	t-stat					
Value	Coefficient					
	t-stat					

6. Is it possible for a factor portfolio to have a higher Sharpe ratio than the optimal Sharpe ratio portfolio from (4)? Why or why not?
7. Construct a table similar to the ones in the *Our Model Goes to Six* assignment. That is, you will run three regressions and put the results in a table like Table 2. Discuss the results briefly.
8. Compute indices for all four securities beginning at 1 on January 2, 2003 and growing at the return (the actual return, not the excess return). Starting on February 3, 2003, use the Black-Scholes model to price at-the-money 21-trading-day call options on each of the four indices. Exercise price equals current index value. Use the 1-month t-bill rate as the risk-free rate, and assume zero payout rate. Estimate the volatility using the previous 21 trading day returns. What is the average call option price from 2003 to 2021? How does it compare to the average discounted present value of value at expiry? What explains any difference?
9. Propose a reasonable modification to the Black-Scholes model that improves the match between average option price and average option discounted expiry value. The two averages don't have to match exactly, just be close enough that the difference is reasonably attributable to random noise. Ideally the adjustment should be simple and economically defensible, not just an arbitrary multiplier.