

Foundations of Audio Signal Processing

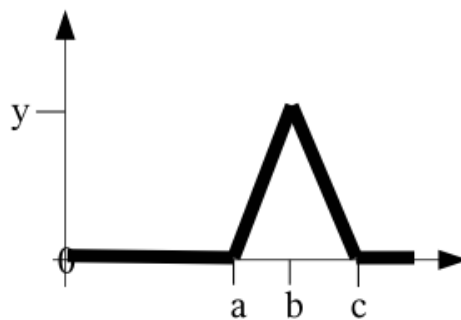
Exercise sheet 7

To be uploaded in eCampus till: 08-12-2018 22:00 (strict deadline)

Exercise 7.1

[3 points]

The following figure shows the Fourier transform \hat{f} of a function $f \in L^2(\mathbb{R})$:



Draw the Fourier transform of the scaled function $t \mapsto f(t/s)$ for $s \in \mathbb{R}_{>0}$, specifying explicitly the positions corresponding to a, b, c and y as functions of s .

Exercise 7.2

[2 + 2 + 2 + 2 + 2 = 10 points]

Let $x, y \in \ell^2(\mathbb{Z})$, $\lambda \in \mathbb{C}$, $k \in \mathbb{Z}$, $\omega_0 \in [0, 1]$. Prove the following properties of the Fourier transform.

- (a) Linearity: $\widehat{x + y} = \hat{x} + \hat{y}$ and $\widehat{\lambda x} = \lambda \hat{x}$.
- (b) Time shift: $\widehat{x_k}(\omega) = e^{-2\pi i \omega k} \hat{x}(\omega)$, where $x_k(n) := x(n - k)$.
- (c) Frequency shift: $\widehat{x^{\omega_0}}(\omega) = \hat{x}(\omega + \omega_0)$, where $x^{\omega_0}(n) := e^{-2\pi i \omega_0 n} x(n)$.
- (d) Frequency reversal: $y = \bar{x} \Rightarrow \hat{y}(\omega) = \overline{\hat{x}(-\omega)}$.
- (e) Time reversal: If, for all integers n , $y(n) = x(-n)$, then $\hat{y}(\omega) = \hat{x}(-\omega)$.

Exercise 7.3

[2 + 6 = 8 points]

When you dial a number on a push-button telephone, each number is associated to a superposition of two sine waves of different frequencies. The following table indicates which frequencies are associated to which number:

	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	6
852 Hz	7	8	9
941 Hz	*	0	#

This means that when dialing number 1 on the telephone, a sine wave of 1029 Hz and one of 697 Hz are generated and summed.

- (a) You are given a .wav file in which you can hear a person dialing a 6 digits number. Explain how you could find out which numbers have been dialed.
- (b) Implement a Matlab function that, given the .wav file "dialtones.wav" (which you can find together with the exercise sheet on eCampus), helps you finding out which 6 numbers have been dialed. Note that in the given .wav file, each tone lasts 1 second. Send the .m file and also state clearly in your solutions the numbers you found out.
(Hint: The function does not need to output the numbers, but you should be able to "read-off" the results from suitable plots.).