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Winter Term 2018/2019

Foundations of Audio Signal Processing
Exercise sheet 6

To be uploaded in eCampus till: 01-12-2018 22:00 (strict deadline)

Exercise 6.1

[$3+3+3+3=12$ points]

Consider a function $f \in L^2(\mathbb{R})$ and its Fourier transform $\hat{f}$. In addition, the real and imaginary part of the function are indicated respectively by $Re(f)$ and $Im(f)$. Prove the following properties:

(a) Let f be differentiable with $f' \in L^2(\mathbb{R})$, then

$$\widehat{f'}(\omega) = 2\pi i\omega \hat{f}(\omega)$$

holds for the Fourier Transform of the derivative of f.

(b) If $\hat{f}$ is differentiable and $g(t) := tf(t)$, then

$$\hat{f}'(\omega) = -2\pi i\hat{g}(\omega).$$

(c) If f is real, then the $Re(\hat{f})$ is even and the $Im(\hat{f})$ is odd.

(d) If f is real and even, then the $\hat{f}$ is real and even.

Exercise 6.2

[$4+6+1=11$ points]

Consider the following function $f(t) \in L^2([0,1])$:

$$f(t) = \begin{cases} -1 & \text{if } 0 \le t \le \frac{1}{2}, \\ 1 & \text{if } \frac{1}{2} < t \le 1 \end{cases}$$

(a) Calculate the Fourier coefficients $\langle f|A_k\rangle$ and $\langle f|B_k\rangle$, which are defined as follows:

$$\langle f|A_k\rangle = \sqrt{2}\int_0^1 f(t)\cos(2\pi kt)dt,$$

$$\langle f|B_k\rangle = \sqrt{2}\int_0^1 f(t)\sin(2\pi kt)dt.$$

(b) Write a function in Matlab that plots the signal $f(t)$ defined in (a) and its Fourier series expansion with respect to the Hilbert basis $\{1, A_k, B_k | k \in \mathbb{N}\}$. Analyze the changes for increasing k, by using an animated plot.

(c) Test the function you have created in (b) with:

   (i) $k = 5$,
   (ii) $k = 25$,
   (iii) $k = 50$.

   Which phenomenon can be observed with increasing k?