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**Winter Term 2018/2019**

# Foundations of Audio Signal Processing
# Exercise sheet 3

To be uploaded in eCampus till: 10-11-2018 22:00 (strict deadline)

## Exercise 3.1 [$2+2+2+2=8$ points]

Following the example put on the website of the course, please calculate (without using a calculator) the polar coordinate representation of the following complex numbers.

(a) $4 + i4\sqrt{3}$

(b) $\left(-1 + i\sqrt{3}\right)^4$

(c) $\dfrac{\left(-1 + i\sqrt{3}\right)^4}{4 + i4\sqrt{3}}$

(d) $2e^{\frac{\pi}{2}i}(1+i)$

## Exercise 3.2 [$2+4=6$ points]

A function $f : \mathbb{Z} \to \mathbb{C}$ is called *periodic*, if there exists a $p \in \mathbb{N}$, such that $f(n+p) = f(n)$ for all $n \in \mathbb{Z}$. For $\omega \in [0,1)$ we define the following discrete frequency signal: $f_\omega : \mathbb{Z} \to \mathbb{C}$ as $f_\omega(n) := e^{2\pi i \omega n}$.

(a) Draw figures of $f_{\frac{1}{2}}$, $f_{\frac{1}{3}}$, $f_{\frac{1}{4}}$, $f_{\frac{1}{8}}$ in the complex plane.

(b) Prove that: $f_\omega$ periodic $\Leftrightarrow \omega \in \mathbb{Q}$.

## Exercise 3.3 [4 points]

Given the Euler formula: $e^{ix} = \cos(x) + i\sin(x)$ and knowing that $\sin(x) = \dfrac{1}{2i}\left(e^{ix} - e^{-ix}\right)$ and that $\cos(x) = \dfrac{1}{2}\left(e^{ix} + e^{-ix}\right)$, please prove the following equation:

$$\cos^3(x) = \frac{1}{4}\cos(3x) + \frac{3}{4}\cos(x)$$