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Foundations of Audio Signal Processing
Exercise sheet 2

To be uploaded in eCampus till: 03-11-2018 22:00 (strict deadline)

Exercise 2.1

[2 + 2 + 2 = 6 points]

Given z, z_1, and $z_2 \in \mathbb{C}$ and $z = r \cdot e^{i\phi} = a + ib$.

(a) Calculate (without using a calculator) the polar coordinate representation of the following complex number:

$$2e^{\frac{\pi}{2}i}(1+i).$$

(b) Using polar coordinates, prove the following statement:

$$z\overline{z} = |z|^2$$

(c) Using the Euler formula $e^{i\alpha} = \cos(\alpha) + i\sin(\alpha)$ and the equation $e^{z_1+z_2} = e^{z_1}e^{z_2}$, prove the following statement:

$$\sin(\alpha) = \frac{1}{2i}\left(e^{i\alpha} - e^{-i\alpha}\right)$$

Exercise 2.2

[4 + 4 = 8 points]

(a) Calculate all n-th roots of unity and primitive roots of unity for $n \in \{4, 6\}$. In addition, illustrate your results in a figure.

(b) Prove that for all positive integers $n > 1$, the sum of all the n-th roots of unity is equal to 0.

Exercise 2.3

[2 + 3 + 2 = 7 points]

(a) Write a Matlab function that converts a complex number from polar to Cartesian coordinates and plots it.

(b) Write a Matlab function that draws all the primitive n-th roots of unity for a given $n \in \mathbb{N}$.

(c) Verify and visualize using Matlab the following statement: $\cos(\alpha) = \frac{1}{2}\left(e^{i\alpha} + e^{-i\alpha}\right)$.

Please remember that when handing in Matlab exercises you have to follow the rules indicated in exercise sheet 1.