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Winter Term 2018/2019

Foundations of Audio Signal Processing
Exercise sheet 4

To be uploaded in eCampus till: 17-11-2018 22:00 (strict deadline)

Exercise 4.1

[4 points]

Prove the theorem of Pythagoras:
If $x_1, ..., x_n$ are pairwise orthogonal elements of the Hilbert space V, then the energy of the sum of the x_i is the same as the sum of the single energies:

$$\| \sum_{j=1}^{n} x_j \|^2 = \sum_{j=1}^{n} \|x_j\|^2.$$

Explain the steps of your proof.

Exercise 4.2

[3 + 3 + 3 = 9 points]

Indicate whether the following mappings d: $\mathbb{C} \times \mathbb{C} \longrightarrow \mathbb{C}$ define a metric on $\mathbb{C}$. Prove your statement.

(a) $d(x, y) := |x - y|$

(b) $d(x, y) := |x| \cdot |y|$

(c) $d(x, y) := \begin{cases} 1, & \text{if } x \neq y, \\ 0 & \text{else.} \end{cases}$

Exercise 4.3

[4 points]

(a) Write a Matlab function which plots the unit circles in $\mathbb{R}^2$ for different p. p should be an input of the function and it can be a single number or a vector. In case p is a vector, all the unit circles for the chosen p should be plotted in one figure.

(b) Write a Matlab script which tests the function created in (a) with $p = (1, 2, 3, 4, 10)$.

Exercise 4.4

[4 + 2 = 6 points]

As you know from the lecture, a chirp signal is defined by:

$$t \mapsto \sin\left(2\pi\left(f_0 + \frac{k}{2}t\right)t\right),$$

with real constants f_0 and $k > 0$.

(a) Write a Matlab function that shows (with some plots) what happens either when increasing the constant f_0 keeping k fixed, or when increasing k keeping f_0 fixed. A user should be able to decide which variant has to be considered. The sampling frequency and the duration of the chirp signal have to be input arguments of the function.

(b) Submit a Matlab script which tests the function you have created in part (a). The chirp signal must have a sampling frequency of 1 kHz and must be 5 seconds long. Keep f_0 fixed and show what happens with $k \in \{2, 4, 6, 8, 10\}$.