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Winter Term 2018/2019

Foundations of Audio Signal Processing
Exercise sheet 8

To be uploaded in eCampus till: 14-12-2018 22:00 (strict deadline)

Exercise 8.1

[5 points]

The sinc-function serves as synthesis function for the sampling theorem. It is defined by

$$\operatorname{sinc}(t) := \begin{cases} 1, & \text{if } t = 0, \\ \frac{\sin(\pi t)}{\pi t}, & \text{if } t \neq 0. \end{cases}$$

Prove that the integer translates $t \mapsto \operatorname{sinc}(t-k), k \in \mathbb{Z}$, are pairwise orthogonal with respect to the inner product, i.e., $\langle \operatorname{sinc}(\cdot - k) | \operatorname{sinc}(\cdot - \ell) \rangle = \delta_{k\ell}$ for $k, \ell \in \mathbb{Z}$. Use the property that the Fourier transform defines an isometry on $L^2(\mathbb{R})$, i.e., for $f, g \in L^2(\mathbb{R})$: $\langle f | g \rangle = \langle \hat{f} | \hat{g} \rangle$ (Plancherel's theorem).

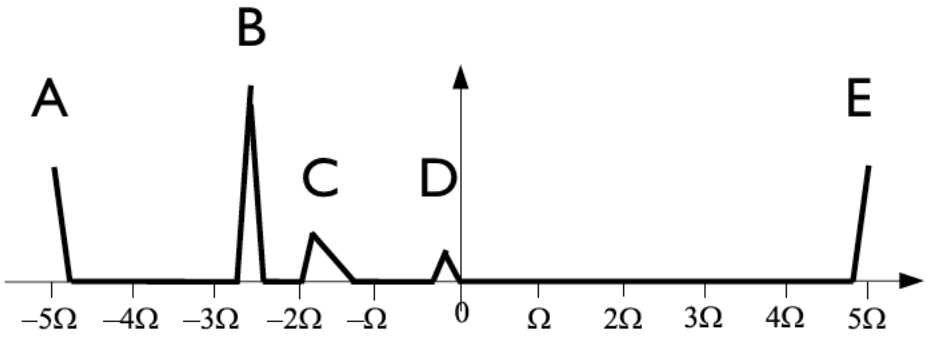
Exercise 8.2

[4 + 4 = 8 points]

The sampling theorem states that an Ω-bandlimited signal $f \in L^2(\mathbb{R})$ can be reconstructed from its sample values if a sample rate $\frac{1}{T} = 2\Omega$ is used. Let the continuous signal $f \in L^2(\mathbb{R}), f : \mathbb{R} \to \mathbb{C}$, be 5Ω-bandlimited. We consider the T-sampled version of f for $T := \frac{1}{2\Omega}$.

(a) Specify the Fourier transform $\hat{g}$ of the signal $g \in L^2(\mathbb{R})$ which is represented by the T-sampled version of f. Explain how you get to this result. Proceed as in the example from the lecture.

(b) Sketch $|\hat{g}|$ for the case that $|\hat{f}|$ has the following appearance:



Please add labels A-D to your sketch to indicate the resulting positions of the five non-zero spectral regions of $|\hat{f}|$.

Exercise 8.3 [8 points]

In the lecture you have seen that, according to the sampling theorem, you can reconstruct a signal from a sum of weighted sinc functions at equidistant points. Write a Matlab function that performs a signal approximation using sinc functions. Test your function with the test signal given in the file "random_signal.mat" which you find together with the exercise sheet. In this file you have a signal and the corresponding sampling frequency. Visualize your result and how you get to it as done on slide 11 of Chapter 5. You have to submit the Matlab function together with the plot and a script which shows how you called the function to get the submitted plot.