Date:

Q-13

A function $f(n) = O(g(n))$ if there exists N & c such that

$$f(n) < c\,g(n) \quad \text{when } n > N.$$

Using this definition transitivity is proved as below.

$f(n) = O(g(n))$ implies.

$$f(n) < c_1 g(n) \;,\; c_1 > 0 \;,\; n > N_1 \quad - (1)$$

$g(n) = O(h(n))$ implies:

$$g(n) < c_2 h(n) \;,\; c_2 > 0 \;,\; n > N_2 \quad - (2)$$

Now multiplying (2) by c_1 we get.

$$c_1 g(n) < c_1 c_2 h(n) \quad - (3) \quad (\because c_1 > 0)$$

Thus we get (from (1) & (3))

$$f(n) < c_1 c_2 h(n) \qquad c_1 > 0,\; c_2 > 0,\; n > \max(N_1, N_2)$$

$$\therefore\; f(n) < C\,h(n) \qquad c_1 c_2 = C > 0,\; n > \underbrace{\max(N_1, N_2)}_{N}$$

$$\therefore\; f(n) = O(h(n))$$

Thus proved.