

# Assignment No - B6

---

## 1 Aim

Implement OBST Tree search using HPC task sub-division. Merge the results to get final result.

## 2 Objective

- To understand concept of OBST.
- To effectively use multi-core or distributed, concurrent/Parallel environments.
- To develop problem solving abilities using Mathematical Modeling.
- To develop time and space efficient algorithms.

## 3 Software Requirements

- Linux or Windows Operating System
- Python

## 4 Mathematical Model

Let S be the system,  
S : (s, e, I, O, Fm, Ft, St)

Where,

s: Start State , i.e. taking input e: End State i.e. search result I: Set of inputs

I: size of array, array elements O:

Set of outputs

O: Sorted array, position of element

Fm: Main function or algorithm that gives specific Output i.e. main()

Ft: Failure State. i.e. Getting Exceptions as Output after inserting characters, string or float values as Input.

St : Success State. i.e. Getting the Expected Element and its position in array as output.

## 5 Theory

### 5.1 OSBT:

An optimal binary search tree is a binary search tree for which the nodes are arranged on levels such that the tree cost is minimum. For the purpose of a better presentation of optimal binary search trees, we will consider extended binary search trees, which have the keys stored at their internal nodes. Suppose n keys  $k_1, k_2, \dots, k_n$  are stored at the internal nodes of a binary search tree. It is assumed that the keys are given in sorted order, so that  $k_1 \leq k_2 \leq \dots \leq k_n$ . An extended binary search tree is obtained from the binary search tree by adding successor nodes to each of its terminal nodes as indicated in the following figure by squares.

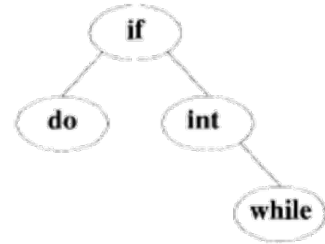
$$c(i, j) = \min_{i < k \leq j} \{c(i, k-1) + c(k, j) + p(k) + w(i, k-1) + w(k, j)\}$$

$$c(i, j) = \min_{i < k \leq j} \{c(i, k-1) + c(k, j)\} + w(i, j)$$

**Example 5.18** Let  $n = 4$  and  $(a_1, a_2, a_3, a_4) = (\mathbf{do}, \mathbf{if}, \mathbf{int}, \mathbf{while})$ . Let  $p(1 : 4) = (3, 3, 1, 1)$  and  $q(0 : 4) = (2, 3, 1, 1, 1)$ . The  $p$ 's and  $q$ 's have been multiplied by 16 for convenience. Initially, we have  $w(i, i) = q(i)$ ,  $c(i, i) = 0$  and  $r(i, i) = 0$ ,  $0 \leq i \leq 4$ . Using Equation 5.12 and the observation  $w(i, j) = p(j) + q(j) + w(i, j - 1)$ , we get

$$\begin{aligned}
w(0, 1) &= p(1) + q(1) + w(0, 0) = 8 \\
c(0, 1) &= w(0, 1) + \min\{c(0, 0) + c(1, 1)\} = 8 \\
r(0, 1) &= 1 \\
w(1, 2) &= p(2) + q(2) + w(1, 1) = 7 \\
c(1, 2) &= w(1, 2) + \min\{c(1, 1) + c(2, 2)\} = 7 \\
r(1, 2) &= 2 \\
w(2, 3) &= p(3) + q(3) + w(2, 2) = 3 \\
c(2, 3) &= w(2, 3) + \min\{c(2, 2) + c(3, 3)\} = 3 \\
r(2, 3) &= 3 \\
w(3, 4) &= p(4) + q(4) + w(3, 3) = 3 \\
c(3, 4) &= w(3, 4) + \min\{c(3, 3) + c(4, 4)\} = 3 \\
r(3, 4) &= 4
\end{aligned}$$

	0	1	2	3	4
0	$w_{00} = 2$ $c_{00} = 0$ $r_{00} = 0$	$w_{01} = 8$ $c_{01} = 8$ $r_{01} = 1$	$w_{02} = 12$ $c_{02} = 19$ $r_{02} = 1$	$w_{03} = 14$ $c_{03} = 25$ $r_{03} = 2$	$w_{04} = 16$ $c_{04} = 32$ $r_{04} = 2$
1		$w_{11} = 3$ $c_{11} = 0$ $r_{11} = 0$	$w_{12} = 7$ $c_{12} = 7$ $r_{12} = 2$	$w_{13} = 3$ $c_{13} = 3$ $r_{13} = 3$	
2			$w_{22} = 1$ $c_{22} = 0$ $r_{22} = 0$	$w_{23} = 3$ $c_{23} = 3$ $r_{23} = 3$	
3				$w_{33} = 1$ $c_{33} = 0$ $r_{33} = 0$	
4					$w_{44} = 1$ $c_{44} = 0$ $r_{44} = 0$



## 5.2 Advantage:

The major advantage of binary search trees over other data structures is that the related sorting algorithms and search algorithms such as in-order traversal can be very efficient, they are also easy to code.

## 5.3 Disadvantage:

- The shape of the binary search tree totally depends on the order of insertions and deletions, and can become degenerate.
- When inserting or searching for an element in a binary search tree, the key of each visited node has to be compared with the key of the element to be inserted or found.
- The keys in the binary search tree may be long and the run time may increase.
- After a long intermixed sequence of random insertion and deletion, the expected height of the tree approaches square root of the number of keys,  $N$ , which grows much faster than  $\log N$ .

## 5.4 Analysis:

The optimal binary search tree has a time complexity of  $O(n^3)$ . Its space efficiency is only  $O(n^2)$ . It can possibly be lowered to complexity  $O(n^2)$  with smarter recursive functions and smaller ranges of values.

# 6 Algorithm

Suppose  $n$  keys  $k_1, k_2, \dots, k_n$  are stored at the internal nodes of a binary search tree. It is assumed that the keys are given in sorted order, so that  $k_1 < k_2 < \dots < k_n$ . An extended binary search tree is obtained from the binary search tree by adding successor nodes to each of its terminal nodes as indicated in the following figure by squares.

$$c(i, j) = \min_{i < k \leq j} \{c(i, k-1) + c(k, j) + p(k) + w(i, k-1) + w(k, j)\}$$

$$c(i, j) = \min_{i < k \leq j} \{c(i, k-1) + c(k, j)\} + w(i, j)$$

## 7 Testing

### 7.1 Positive Testing

Sr. No.	Test Condition	Steps to be executed	Expected Result	Actual Result
1.	Enter number of <u>keys P.Q</u>	Press Enter	Display Binary Tree	Same as Expected

### 7.2 Negative Testing

Sr. No.	Test Condition	Steps to be executed	Expected Result	Actual Result
1.	Enter number of <u>keys P.Q</u> not in particular format	Press Enter	Tree is not display	Binary tree

## 8 Conclusion

From this assignment we have studied concept of OBST and developed time and space efficient algorithm using multicore , concurrent environment.

Roll No.	Name of Student	Date of Performance	Date of Submission
302	Abhinav Bakshi	8/3/16	15/3/16

## 9 Plagarism Report

Result:

61% Unique

<code>\documentclass[11pt]{article} \usepackage{graphicx} \begin{document}</code>	- Unique
<code>Div: C\\ \end{flushleft}• \begin{center} \begin{Large} \textsc{Assignment}</code>	- Unique
<code>\end{flushleft}• Using Divide and Conquer Strategies design</code>	- Unique
<code>\textbf{Description:} \end{flushleft}• Quick Sort, as the</code>	- Unique
not stable search, but it is very fast and requires very	- Unique
and Conquer(also called partition-exchange sort). This algorithm	- Unique
<code>\item Elements less than the Pivot element \item Pivot element</code>	- Unique
In the list of elements, mentioned in below example, we	- Unique
will be changed like this. [6 8 17 14 25 63 37 52]\\ Hence	- Unique
with all the elements smaller to it on its left and all	- Plagiarized
and [63 37 52] are considered as two separate lists, and	- Unique
the complete list is sorted.\\ Quicksort is a fast sorting	- Unique
but widely applied in practice. On the average, it has $O(n$	- Plagiarized
big data volumes.\\ \begin{flushleft} \textbf{Algorithm}	- Unique
strategy is used in quicksort. Below the recursion step	- Plagiarized
a pivot value. We take the value of the middle element as	- Plagiarized