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FLASH 7 BLOG TEAMS SUBMISSIONS GROUPS CONTESTS PROBLEMSETTING

flash 7's blog

Digit DP

By flash_7, history, 23 months ago, k, @

Wrote this article a long ago but during solving a problem recently thought of sharing this article publicly. Hope it will help some contestants to understand the idea clearly.

Digit dp is a very easy technique and also useful to solve many dynamic programming problems. Seeing the name "Digit DP" it's easy to guess that we are going to do something using the digits. Yes we are actually going to play with digits. Let's explain the concept using a classical problem.

Problem

How many numbers \mathbf{x} are there in the range \mathbf{a} to \mathbf{b} , where the digit \mathbf{d} occurs exactly \mathbf{k} times in x? There may have several solutions including number theory or combinatorics, but let's see how we can solve this problem using digit dp.

Solve for range (zero to a)

Using digit dp we always focus on building a number satisfying all the conditions. If we finally manage to build that number then we say, yes we have got one ;-) But how we'll build that number? For the time being let's say a is zero. So we need to find the total numbers which are not greater than **b** and also satisfy the given conditions.

Building a sequence of digits

Let's consider the number as a sequence of digits. Let's name the sequence sq. Initially sq is empty. We'll try to add new digits from left to right to build the sequence. In each recursive call we'll place a digit in our current position and will call recursively to add a digit in the next position. But can we place any of the digits from 0 to 9 in our current position? Of course not, because we need to make sure that the number is not getting larger than **b**.

Information we need to place a digit at the current position

Let's say during the building of the sequence, currently we are at position pos. We have already placed some digits in position from 1 to pos-1. So now we are trying to place a digit at current position pos. If we knew the whole sequence we have build so far till position pos-1 then we could easily find out which digits we can place now. But how?

You can see that, in the sequence sq the left most digit is actually the most significant digit. And the significance get decreased from left to right. So if there exist any position t (1<=t<pos) where sq[t] < b[t] then we can place any digit in our current position. Because the sequence has already become smaller than **b** no matter which digit we place in the later positions. Note, b[t] means the digit at position \boldsymbol{t} at number \boldsymbol{b} .

But if there was no t that satisfy that condition then at position pos, we can't place any digit greater than b[pos]. Because then the number will become larger than b.

Do we really need the whole sequence?

Now imagine, do we really need that whole sequence to find if a valid t exist? If we placed any digit in our previous position which was smaller than its corresponding digit in b then couldn't we just pass the information somehow so that we can use it later? Yes, using an extra parameter f1(true/false) in our function we can handle that. Whenever we place a digit at position t which is smaller than b[t] we can make f1 = 1 for the next recursive call. So

Pay attention

Before contest Codeforces Round #577 (Div. 2)

4 days

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whenever we are at any position later, we don't actually need the whole sequence. Using the value of $\mathbf{f1}$ we can know if the sequence have already become smaller than \mathbf{b} .

Extra condition

So far we focused on building the sequence **sq**, but we have forgotten that there is an extra condition which is, digit **d** will have to occur exactly **k** times in sequence **sq**. We need another parameter **cnt**. **cnt** is basically the number of times we have placed digit **d** so far in our sequence **sq**. Whenever we place digit **d** in our sequence **sq** we just increment **cnt** in our next recursive call.

In the base case when we have built the whole sequence we just need to check if cnt is equal to k. If it is then we return 1, otherwise we return 0.

Final DP States

If we have understood everything so far then it's easy to see that we need total three states for DP memoization. At which position we are, if the number has already become smaller than $\bf b$ and the frequency of digit $\bf d$ till now.

Solve for range (a to b)

Using the above approach we can find the total valid numbers in the range $\bf 0$ to $\bf b$. But in the original problem the range was actually $\bf a$ to $\bf b$. How to handle that? Well, first we can find the result for range $\bf 0$ to $\bf b$ and then just remove the result for range $\bf 0$ to $\bf a$ - $\bf 1$. Then what we are left off is actually the result from range $\bf a$ to $\bf b$.

How to solve for range a to b in a single recursion?

In the above approach we used an extra parameter f1 which helped us to make sure the sequence is not getting larger than b. Can't we do the similar thing so that the sequence does not become smaller than a? Yes of course. For that, we need to maintain an extra parameter f2 which will say if there exist a position t such that sq[t] > a[t]. Depending on the value of f2 we can select the digits in our current position so that the sequence does not become smaller than a. Note: We also have to maintain the condition for f1 parallely so that the sequence remains valid.

Please check this to find the sample code of our initial approach.

Problem List

- 1. Investigation
- 2. LIDS
- 3. Magic Numbers
- 4. Palindromic Numbers
- 5. Chef and Digits
- 6. Maximum Product
- 7. Cantor
- 8. Digit Count
- 9. Logan and DIGIT IMMUNE numbers
- 10. Sanvi and Magical Numbers
- 11. Sum of Digits
- 12. Digit Sum
- 13. Ra-One Numbers
- 14. LUCIFER Number
- 15. 369 Numbers
- 16. Chef and special numbers
- 17. Perfect Number
- 18. The Great Ninja War

Is there some other problems? Some suggestions are really welcomed:)





Um nik → How to read problem statements ©

xukuan → Did he plagiar anyone's code?

Sphener → Runtime error on cf server but code

runs fine locally. What could the issue be?



23 months ago, # |

← Rev. 2



I've had this problem for a while, and most probably it's solvable with Digit DP. Maybe it's not, but I haven't found a solution:



Given a integer X, find the number of integers i in [I, r] such that $i \le X$ and $rev(i) \le X$, where rev(i) is the number formed by reversing the digits of i. For example, rev(1560) = 651 and rev(156) = 651 (not 6510, 65100, etc)

Unfortunately I can remember neither the source nor the limits, but probably $X < 10^{18}$ or something.

→ Reply

23 months ago, # ^ I

<u></u> 0 🔻

I have found a solution but not fully sure about it.

First of all if X < I then the answer is zero. And i must be A = I where I = I minI must be I where I minI must be I where I must be I must be I where I is an animal problem for range I where I is an inverse I is an inverse I where I is an inverse I is an inverse I in I is an inverse I in I

In the first part, for each k (1 <= k < Len) we'll try to find the number of valid i which has k digit in it's representation. We can do this easily because if i <= X then rev(i) <= will also hold. As total number of digits is smaller. So we can just ignore that rev(i) <= X condition for this part.

Let's sum1 is the summation of result for each k (1 <= k < Len). Now each of the number x in sum1 will contribute to the final result twice. Because for each x we can add some trailing zeroes in x to make it a Len digit number. As we are adding some trailing zeroes the conditions i <= X and rev(i) <= X will still hold.



Now we'll solve the second part where we need to find the number of Len digit number which has no leading or trailing zeroes. I'll explain this with an example. Let's L = 372967524. Let's divide this L into 3 parts. P1 = 372, P2 = 9, P3 = 67524. Here P2 has only one digit.

Let's build a number i whose first 3 digit is fixed (same as P1). And at the 4th position we place a smaller digit than P2 (for example 8). So i now looks like this 3 7 2 8 _ _ _ _ . So i has already become less than X. It doesn't matter what we place in the remaining 5 positions. Condition i <= X will always satisfy. So now we focus on selecting 5 digits for the remaining position such that $rev(i) \le X$ satisfy.

The reverse of the trailing 5 digits of i is actually the 5 leading digits of rev(i). As we don't need to worry about i <= L condition any more, so now we'll do digit DP to find this first 5 leading digits of rev(i), comparing to the first 5 digits in X. We need to handle one more case here. As we have already placed the leading 4 digits in i whose reverse is actually the trailing 4 digits of rev(i), so after choosing the leading 5 digits in rev(i) we just need to check if the whole rev(i) satisfy the condition (<= X) or not

As i said above, we'll have to divide L in 3 parts. For each digit in L we'll select it as P2. And the remaining two parts as P1 and P2. Then we repeat the same approach each time. The summation of all these results(including the one in part1) is our final answer.

→ Reply



23 months ago, # |

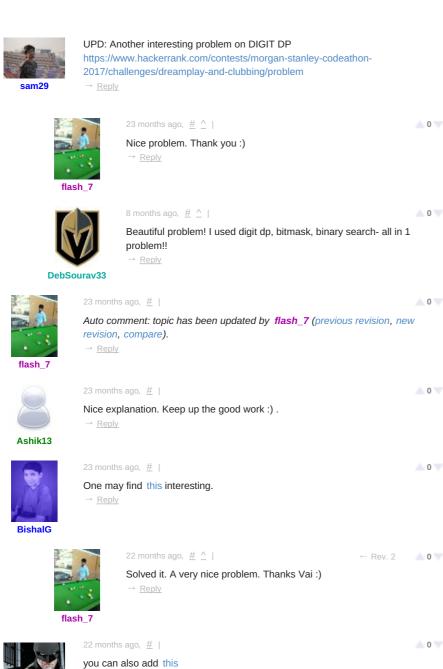
Check BOI(Baltic) 2013 Numbers, i really liked that one.

→ Reply

23 months ago, # |

← Rev. 2 **+11**

I think you can add this problem. I solved it using Digit DP. I really liked this problem.





22 months ago, # | How to solve LIDS?

xxxcxxx

flash 7

22 months ago, # $^{\wedge}$ | The maximum length of LIDS can be at max 10. So we can check for

<u>0</u>

each length k, in how many ways we can make a number whose LIDS is k. We can then print the result we found for the maximum k.

I'm describing the states we need to find the number of ways we can build an LIDS of length k.

- 1. At which position we are.
- 2. Has the sequence already become less than y? (0 or 1)
- 3. Has the sequence already become greater than x? (0 or 1)
- 4. Did we place at least one non zero digit so far? (To handle the first digit of LIDS)
- 5. Last digit we have placed in the sequence.
- 6. Number of total digit in the LIDS sequence.

Basically we build two sequence parallelly. One is definitely the whole sequence, another one is the LIDS sequence (Which is the sub sequence of the original sequence). When ever we select a digit we

want to place, it becomes the last digit of the original sequence till now. But we can also choose if we want to consider that digit in our LIDS sequence or not.



22 months ago, # ^ | Thank you.

<u>0</u>



11 months ago, # $^{\wedge}$ |

<u>0</u>

Why do need the fourth state ??@flash_7

sm247



11 months ago, # ^ |

A 0 V

To make sure we don't place any leading zeroes. Because if we build a number of length k with some leading zeroes then that number is not really a number of length k.

22 months ago, # |

← Rev. 3 **+6**

Some additional problems:-

Digit Sum

RAONE



demonsbane

LUCIFER

NUMTSN

GONE

Chef and special numbers



flash 7

22 months ago, # |

<u></u> 0 🔻

Auto comment: topic has been updated by flash_7 (previous revision, new revision, compare).

22 months ago, # $^{\wedge}$ |

A 0 V

I just wrote a topic about a doubt in one question here: http://codeforces.com/blog/entry/55105



joaquimnt_

I will read your topic now. Hope that helps me :)

Some questions that i believe that are solvable by Digit DP:

https://www.urionlinejudge.com.br/judge/pt/problems/view/1138 https://www.urionlinejudge.com.br/judge/pt/problems/view/2013 https://www.urionlinejudge.com.br/judge/pt/problems/view/1492



21 month(s) ago, # |

<u>0</u>

How to solve "palindromic numbers"?

21 month(s) ago, # ^ |

← Rev. 3 **0**

Initially we have the space of all numbers $S = \{0, 1, 2, 3, ..., 10^{17}\}$. This space is too big, so we want to reduce it.

From all that space we only need to consider the numbers with a special property (being palindrome) $P \subset S$.



egor.okhterov

Now let's consider another space where all of our original numbers are mapped into a different number

 $S_2 = \{f(0), f(1), f(2), f(3), ..., f(10^{17})\}.$

There is no point in cosidering this new space S_2 instead of original S if it has the same size.

Can you think of **any** mapping $f: S \longrightarrow S_2$ that will make the size of S_2 manageable?



4 months ago, # ^ |

A 0

Can't think of solution. Any more suggestions?





"So if there exist any position t (1 \leq t \leq pos) where sq[t] \leq b[t] then we can place any digit in our current position.



Because the sequence has already become smaller than b no matter which digit we place in the later positions" How is this true? If b = 5321 and current sq = 621, there exists a t such that sq[t] < b[t]. So in the current position pos, putting any digit will cause sq > b? Am I missing something?



20 months ago, # |

+5

Vaiya can you please sort the problems according to difficulty?









I didn't solve them all but here goes my order

- Investigation
- · Sum of Digits
- Digit Sum
- Bomb
- Ra-One Numbers
- LUCIFER Number
- G-One Numbers
- Digit Count
- Round Numbers
- Fast Bit Calculations
- Magic Numbers
- Maximum Product

If you need help then you can look at this .



How do I solve Maximum Product?





18 months ago, # $^{\wedge}$ |

Here's a gift :D

Target2018





__ -10 V



In the problem "Investigation" whats the logic behind taking the max remainder as 90 only? you have defined the dp states as dp[90][2][90][90]. why 90 ?Target2018

MotaSanyal





Can Anyone please provide solution for this tutorial also ? I am noob. Sorry for that.

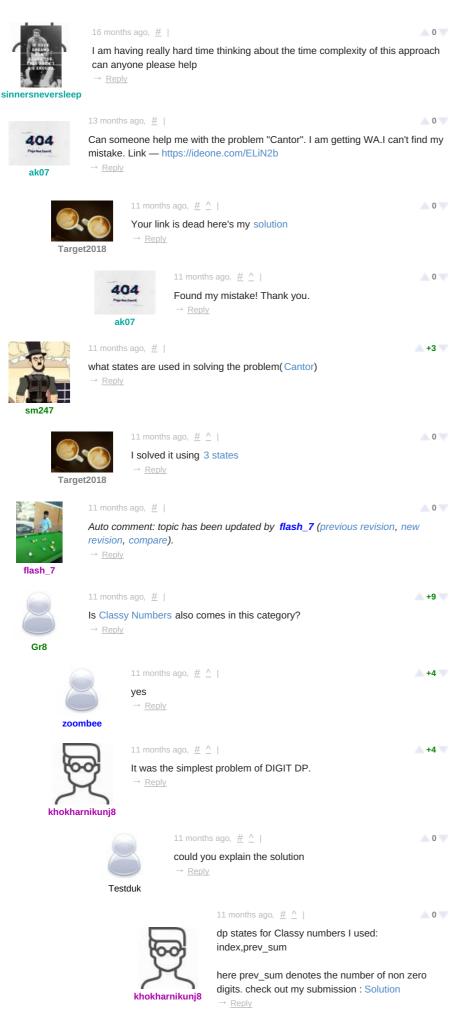


System_test_failed

FlyToTheSky

bounce_back20

Target2018



11 months ago, # ^ | ← Rev. 3

help you. There are quite a lot of cases to take care The main idea is that for each interval [L, R] the answer is F(R) - F(L - 1). Where F(x) equals the number of classy integers from 0 to x. How to you calculate F(x)? Just using Dynamic Programming. For a number x, the DP looks something like this: dp[i][j][k][0] = number of integers less than x that are i digits long, end in j and they have k digits bigger than 0. **TheRedLegend** dp[i][j][k][1] = number of integers equal to x that are i digits long, end in j and they have k digits bigger than 0. j is going to be between 0..9 and k in the range from 0..3, because a classy number contains no more than 3 non-zero digits. My dynamic has more states than the khokharnikunj8's but I think it's more intuitive. Hope it's going to help you. → Reply 11 months ago, # $^{\wedge}$ | A 0 https://www.hackerrank.com/topics/digit-dp this is a good tutorial 4 months ago, # ^ | A 0 Easy to understand solution using the same template as described in the post. Hope it helps:) solution



zoombee



I tried to solve the INVESTIGATION Link: https://vjudge.net/problem/LightOJ-1068 problem given above, i wrote the following solution https://code.hackerearth.com/aedc54l I am getting a wrong answer for test case : $1\,1000\,4$, the answer is given 64 but i am getting 74, could someone please help



Target2018

11 months ago, # ^ |

A 0 V

A 0

← Rev. 4

You can look at this now, I wrote the solution here. Term final is the day after tomorrow, so can't look at your code now.

11 months ago, # |



11 months ago, # | Superb explanation! Thanks. <u>0</u>



9 months ago, # |

▲ +1 ▼

Another really good question on digit dp: Link



9 months ago, # |

← Rev. 2 **△ 0** ▼

Another good problem in digit dp Balanced Number

CuteCoder07

Why the for loop start from 0? let a = 100, d = 5, k = 1

Please correct me if I'm wrong, because one of the base cases is pos==num.size(), so there will will be state with, 050 chosen right? this is representing number 50? and is that 005 means it's number 5?, because we return 1 when pos == num.size() so we have to fill each digit, this zero a bit difficult to understand.

→ Reply



8 months ago, # $^{\wedge}$ | \leftarrow Rev. 2 \wedge 0

I too have the same doubt. @flash_7 ,could you please explain this? If we want digit 0,two times then 005,050 are also counted but that shouldn't be the case.

→ Rep

8 months ago, # $^{\wedge}$ |

A +1 V

Yes, you are correct. 005 means 5 and 050 means 50. So the way i have written my code, it'll consider all the numbers $\mathrel{<=} N$ including the ones whose number of digits is less than the number of digits in N. So it'll work perfectly.



But when d = 0, my given code will produce an incorrect output. Because it'll count zero for the numbers who have leading zeroes. For example, for the number 005, it'll count zero two times. Which is wrong. Because we don't consider the leading zeroes in the decimal numbers. You have to handle this case separately.

How can we do that? We can use another boolean state, which will keep track if we have placed at least one non zero digits so far. If it's true in any state, then we'll count the zero in that state. Otherwise, we don't. Hope it helps you to understand.

→ Reply



8 months ago, # | Thanks bro :) .

+3

→ Reply

whoami_shubham

dreamplayDaddy

4 months ago, # |

6.Maximum Product by digitDp as i am no

by digitDp as i am not able to get what should be the states though i have already solved this question using greedy approach solution but i want to know how it can be solved using digitDp or more precisely is it solvable using digitDp approach or not?

→ Reply

4 months ago, # |

A 0 V

In question Investigation I have been using dp states as pos, flag, sum_of_digits, remainder_of_so_far_number.



So the dp size should be dp[32][2][10000][10000] but this gives RTE and dp[32][2][101][101] passes.

flash_7 can you please tell me how to solve problem

But when k>100 this should give the wrong answer but it is passing My submission https://vjudge.net/solution/18823741 I can't understand why such dp size works?

→ Reply

SAKIB_ALAMIN

just think about , what could be maximum sum of digits & does the value of k matters if it's greater than the sum of digits ?

→ Reply

4 months ago. # ^ I

4 months ago, # ^ |

+6



remainder_of_so_far_number can be greater than 100 upto value of k i.e.10000



so , if sod(sum of digits) is < 100 & k \geq 100 , these numbers will never be divisible by k according to this problem, right? so, if k is >= 100, we can simply print 0 , otherwise use dp , thus state gets reduced.

4 months ago, $\ensuremath{\#}$ ^ |



4 months ago, # ^ | <u></u> 0 🔻 Thanks:) Got it

harshit2202



4 months ago, # ^ | +6 great :-)

+6

<u>0</u>

<u>0</u>

← Rev. 2 **0**

SAKIB_ALAMIN



What should be the dp state in Palindromic Numbers?



2 months ago, # |

Problem Editorial

4 months ago, # |



A 0 V 2 months ago, # |



Can anyone please explain me as why this type of problem is called a DP, although we aren't using any memoisation, but rather doing pure recursive calls. Shouldn't it be rather termed as recursion? Please let me know if I am missing out something.

harshrai22



Thanks a lot, it really helped !! @flash_7

5 weeks ago, # |



2 weeks ago, # |



flash_7 How To Solve Palindromic Numbers ??

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