TSP distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
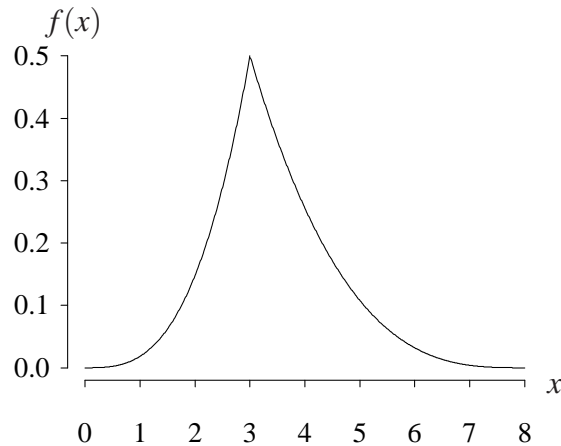
The shorthand $X \sim \text{TSP}(a, b, m, n)$ is used to indicate that the random variable X has the TSP distribution with parameters a, b, m, and n. A TSP random variable X has probability density function

$$f(x) = \begin{cases} \frac{n(x-a)^{n-1}}{(b-a)(m-a)^{n-1}} & a < x < m \\ \frac{n(b-x)^{n-1}}{(b-a)(b-m)^{n-1}} & m \le x < b, \end{cases}$$

for $n > 0$, and $a < m < b$. The probability density function with $a = 0, b = 8, m = 3$, and $n = 4$ is illustrated below.

The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \begin{cases} \frac{(x-a)^n (m-a)^{1-n}}{b-a} & a < x < m \\ -\frac{a+b(b-x)^n (b-m)^{-n} - b - m(b-x)^n (b-m)^{-n}}{b-a} & m \le x < b. \end{cases}$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \begin{cases} -\frac{-b+a+(x-a)^n (m-a)^{1-n}}{b-a} & a < x < m \\ \frac{(b-x)^n (b-m)^{1-n}}{b-a} & m \le x < b. \end{cases}$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \begin{cases} -\frac{n(x-a)^{n-1}(m-a)^{1-n}}{a-b+(m-a)(x-a)^n (m-a)^{-n}} & a < x < m \\ \frac{n}{b-x} & m \le x < b. \end{cases}$$

The moment generating and characteristic functions of X are mathematically intractable. The population mean and variance of X are

$$E[X] = \frac{b - m + mn + a}{n + 1}$$

1

$$V[X] = \frac{-2mnb + 2m^2n + a^2n + b^2n - 2mna + 2bm - 2ba + 2am - 2m^2}{(n+1)^2(n+2)}.$$

APPL verification: The APPL statements

```
X := [[x -> (n * (x - a) ^ (n - 1)) / ((b - a) * (m - a) ^ (n - 1)),
       x -> (n * (b - x) ^ (n - 1)) / ((b - a) * (b - m) ^ (n - 1))],
      [a, m, b],["Continuous", "PDF"]];
CDF(X)
SF(X);
HF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution function, survivor function, hazard function, population mean, variance, skewness, and kurtosis.