**Arctangent distribution** (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \arctan(\lambda, \phi)$ is used to indicate that the random variable X has the arctangent distribution with phase shift parameter ϕ and positive location parameter λ. An arctangent random variable X with parameters λ and ϕ has probability density function

$$f(x) = \frac{\lambda}{\left(\arctan\left(\lambda\phi\right) + 1/2\pi\right)\left(1 + \lambda^2\left(x - \phi\right)^2\right)} \qquad x \geq 0$$

for $\lambda > 0$ and $-\infty < \phi < \infty$.
The probability density function with three different choices of parameters is illustrated below.
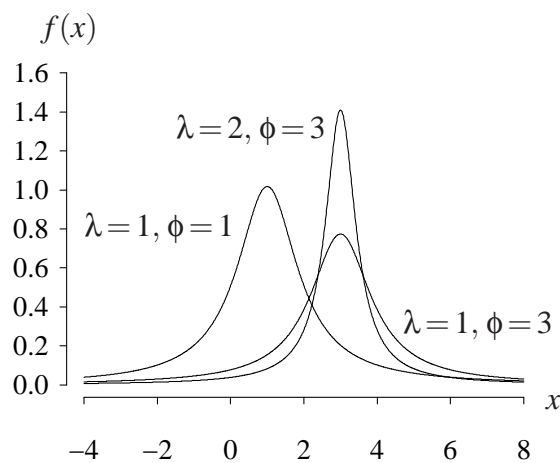
The cumulative distribution function of X is

$$F(x) = P(X \leq x) = 2\left(\frac{\arctan\left(\lambda\phi\right) - \arctan\left(-x\lambda + \lambda\phi\right)}{2\arctan\left(\lambda\phi\right) + \pi}\right) \qquad x \geq 0.$$

The survivor function of X is

$$S(x) = P(X \geq x) = \frac{\pi + 2\arctan\left(-x\lambda + \lambda\phi\right)}{2\arctan\left(\lambda\phi\right) + \pi} \qquad x \geq 0.$$

The hazard function of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{2\lambda}{\left(1 + \lambda^2 x^2 - 2\lambda^2\phi x + \lambda^2\phi^2\right)\left(\pi + 2\arctan\left(-x\lambda + \lambda\phi\right)\right)} \qquad x \geq 0.$$

The cumulative hazard function of X is

$$H(x) = \ln\left(2\arctan\left(\lambda\phi\right) + \pi\right) - \ln\left(\pi + 2\arctan\left(\lambda\left(-x + \phi\right)\right)\right) \qquad x \geq 0.$$

The inverse distribution function of X is

$$F^{-1}(u) = \frac{\lambda\phi + \tan\left(-\arctan\left(\lambda\phi\right) + u\arctan\left(\lambda\phi\right) + 1/2u\pi\right)}{\lambda} \qquad 0 < u < 1.$$

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The moments of *X* are undefined. It follows that the population mean, variance, skewness, and kurtosis of *X* are also undefined.

APPL verification: The APPL statements

```
X := ArcTanRV(lambda, phi);
CDF(X);
SF(X);
HF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, population mean, variance, skewness, kurtosis, and moment generating function.