

**A) Aggregate Method:**

In the aggregate method, first we have to calculate the total cost of performing a sequence of operations and then divide by the number of operations to get the amortized cost per operation.

**1. Cost Analysis:**

- i. We denote  $n$  as the number of elements inserted and  $m$  as the number of resizing operations (doubling the table size).
- ii. Let  $C_i$  be the cost of the  $i$ th insertion operation.
- iii. When inserting the  $i$ -th element, if a resize operation is not needed (i.e., the current size is sufficient), the cost is  $O(1)$ . If a resize operation is required, the cost is  $O(i)$  as it involves copying the existing elements to the new table of size  $2^k$  ( $k$  is the number of resizes performed.)

**2. Total Cost:**

- i.  $\sum_{i=1}^n C_i = O(n) + O(2) + O(4) + \dots + O(2^m) \Rightarrow$   
 $= O(n + 2 + 4 + \dots + 2^m)$   
 $= O(n + (2^{m+1}) - 1)$

Where we get

$$\sum_{i=1}^m 2^i = 2^{m+1} - 1$$

**3. Amortized Cost Per Operation:**

- i. Since the number of resizes is at most  $\log_2(n)$  (as each resize doubles the size), the amortized cost per insertion is  $O(1)$ .

**B) Accounting Method:****1. Cost Analysis:**

- i. Let  $C_i$  be the cost of the  $i$ th insertion operation.
- ii. assign a credit of  $k$  to each insertion operation, where  $k$  is chosen such that it covers both the immediate cost of insertion and the potential cost of future resizes. Since each resize operation (doubling the size) occurs when the table is full, and it doubles the size, we need to assign  $k = O(1)$  credits per element inserted.

**2. Amortized Cost Calculation:**

- i. The total amortized cost  $c_i$  of each insertion operation is  $c_i = C_i + k$
- ii. For each insertion operation, the immediate cost  $C_i$  is  $O(1)$  (for inserting into the dynamic table), and  $k$  is  $O(1)$  as well (to cover potential resizing).
- iii. Hence, the total amortized cost of  $n$  insertion operations is:  $\sum_{i=1}^n c_i = O(n) + O(n) = O(n)$
- iv. The amortized cost per insertion operation is  $O(1)$ .

both methods have an amortized cost per insertion operation of  $O(1)$