RISK NETWORKS: MANAGING RISK IN THE FINANCIAL SYSTEM

Sanjiv R. Das
Santa Clara University

http://srdas.github.io/

@ISB, Term 7, 2018

Network Models of Systemic Risk

Confluence of Graph Theory and Credit Modeling

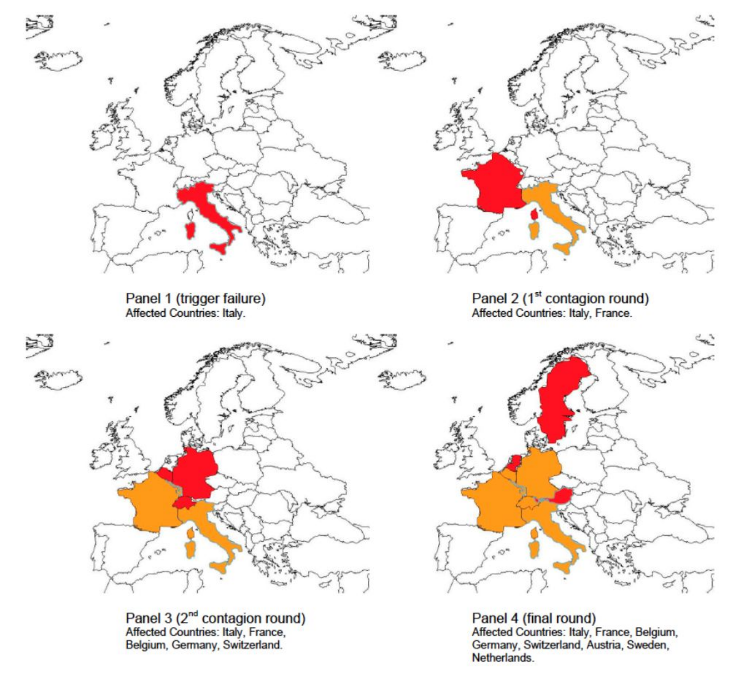
Systemic Risk

Magnitude (Large Impact)

Widespread

Ripple Effect

How to use open data to construct economy-wide risk measures?



Contagion Networks (Espinosa-Vega & Sole, IMF 2010)

Attributes of Systemic Risk Measures

Systemic risk is an attribute of the economic system and not that of a single entity. Its measurement should have two important features:

1. Quantifiability (Aggregation): must be measurable on an ongoing basis.
2. Decomposability (Attribution): Aggregate system-wide risk must be broken down into additive risk contributions from all entities in the system.

Financial institutions that make large risk contributions to system-wide risk are deemed “systemically important.”

Systemic Analysis

The Dodd-Frank Act (2010) and Basel III regulations characterize a systemically risky FI as one that is

1. Large;
2. Complex;
3. Interconnected;
4. Critical, i.e., provides hard to substitute services to the economy.

The DFA does not provide quantification guidance.

Systemic Analysis

Definition: the measurement and analysis of relationships across entities with a view to understanding the impact of these relationships on the system as a whole.

Challenge: requires most or all of the data in the system; therefore, high-quality information extraction and integration is critical.

Adrian, T. and M. K. Brunnermeier (2016). Covar. *American Economic Re-*

Allen, F. and E. Carletti (2013). What is systemic risk? *Journal of Money, Credit*

Billio, M., M. Getmansky, A. W. Lo, and L. Pelizzon (2012a). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of*

Das, S. R. (2016). Matrix metrics: Network-based systemic risk scoring. *Journal of*

Diebold, F. and K. Ylmaz (2014). On the network topology of variance decompositions: Measuring the connectedness of financial firms. *Journal of Econometrics 182*, 119–134.

Referen

Abbass, P., C
nectedness.
1–12.

Acemoglu, D., A. Ozda
in financial networks

Acharya, V., R. Engle,
to ranking and regu

Acharya, V., L. Pedersen, T. Ph
suring systemic risk. *Review*
New York University.

Acharya, V., P. Schnabl, and G. Suarez
Journal of Financial Economics 103,

Acharya, V. V., J. A. C. Santos, and T. Y
Bank of New York, August)). Syst
Economic Policy Review.

Adrian, T. and M. K. Brunnermeier (2016). Covar. *Am*
view 106(7), 1705–1741.

Adrian, T. and H. Shin (2010). Liquidity and leverage. *Jour*
mediation 19, 418–437.

Ahern, K. R. (2013)
ing Paper, USC-M

Allen, F. and E. Ca
and Banking 45,

Allen, L., T. Bali,
predict future eco

Anand, K., P. Gai,
system resilience.

Avdjiev
mark

Avrami
mode

Benoit,
surve

system with endogenous asset markets. *Journal of Financial Stat*

Bonacich, P. (1987). Power and centrality: A family of measures. *A*
of Sociology 92(5), 1170–1182.

Bonacich, P. and P. Lloyd (2001, July). Eigenvector-like measures

network connectedness. *Journal of Applied Econometrics*.

Diebold, F. and K. Ylmaz (2014). On the network topology of va
tions: Measuring the connectedness of financial firms. *Journal of*
119–134.

Kitwiwattanachai, C. (2015). Learning network structu
from cds data. Working Paper, University of Connectic

Laeven, L., L. Ratnovski, and H. Tong (2016). Bank size,
Some international evidence. *Journal of Banking and F*

Yang, T. Yorulmazer, and A. Alentorn (2007). Network models and
ability. *Journal of Economic Dynamics and Control 31*, 2033–2060.

a, F., B. Schwaab, S. J. Koopman, and A. Lucas (2016). The information in
temic risk rankings. *Journal of Empirical Finance 38*, 461–475.

Patton (2016). Time-varying systemic risk: Evidence from a
del of cds spreads. *Journal of Business and Economic Statis-*

lunov (2016). A comprehensive approach to measuring the
nic risk exposure and sovereign debt. *Journal of Financial*

(2009). Liquidity risk charges as a macroprudential tool.

a-Borboad, S. Martínez-Jaramillod, M. van der Leije, and
S. Thurner (2015). The multi-layer network nature of systemic risk and its implications for the costs of financial crises. *Journal of Financial Stability 20*, 70–81.

Saldías, M. (2013). Systemic risk analysis using forward-looking distance-to-default
series. *Journal of Financial Stability 9*, 498–517.

arcz, S. (2008). Systemic risk. *Georgetown Law Journal 97*, 193–249.

nov, J. (2016). What is the systemic risk exposure of financial institutions?
urnal of Financial Stability 24, 71–87.

ya, A. (2017). Firm size, ownership structure, and systematic liquidity risk:
e case of an emerging market. *Journal of Financial Stability forthcoming*.

iro (2017). An analysis of the literature on systemic
of Financial Stability 28, 91–114.

hweitzer (2014). Quantifying the impact of lever-
emic risk. *Journal of Financial Stability 15*, 43–52.

Chan-Lau, J., M. A. Espinosa-Vega, K. Giesecke, and J. Solé (2009). Assessing the systemic implications of financial linkages. IMF Global Financial Stability Report, Vol. 2.

Acharya, V., L. Pedersen, T. Philippon, and M. Richardson (2016, January). Measuring systemic risk. *Review of Financial Studies 30*(1), 2–47. Working Paper, New York University

Das, S. R., S. R. Kim, and D. N. Ostrov (2017). Dynamic systemic risk networks. Working Paper, Santa Clara University.

Billio, Getmansky, Lo, Pelizzon (2012)

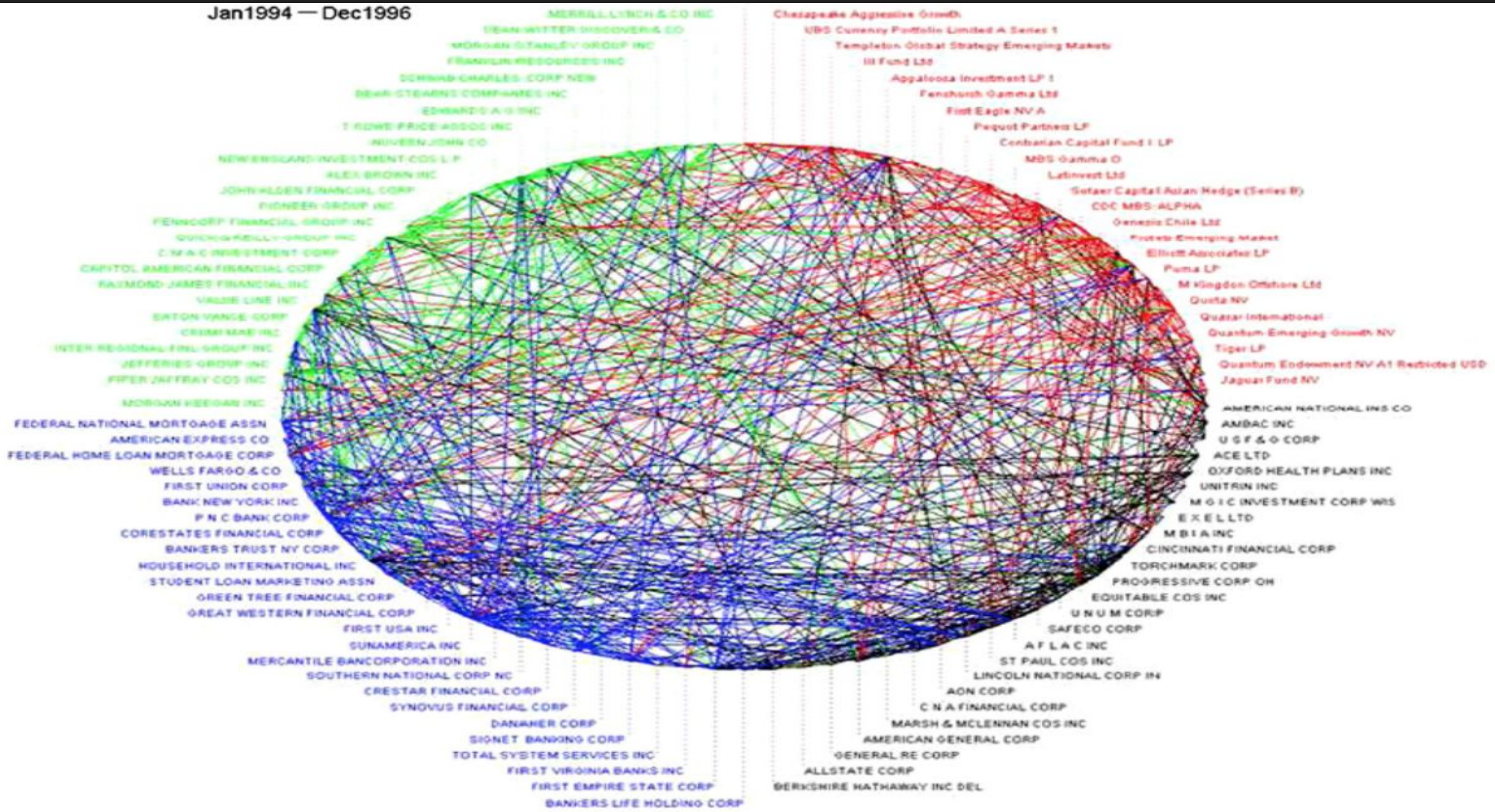
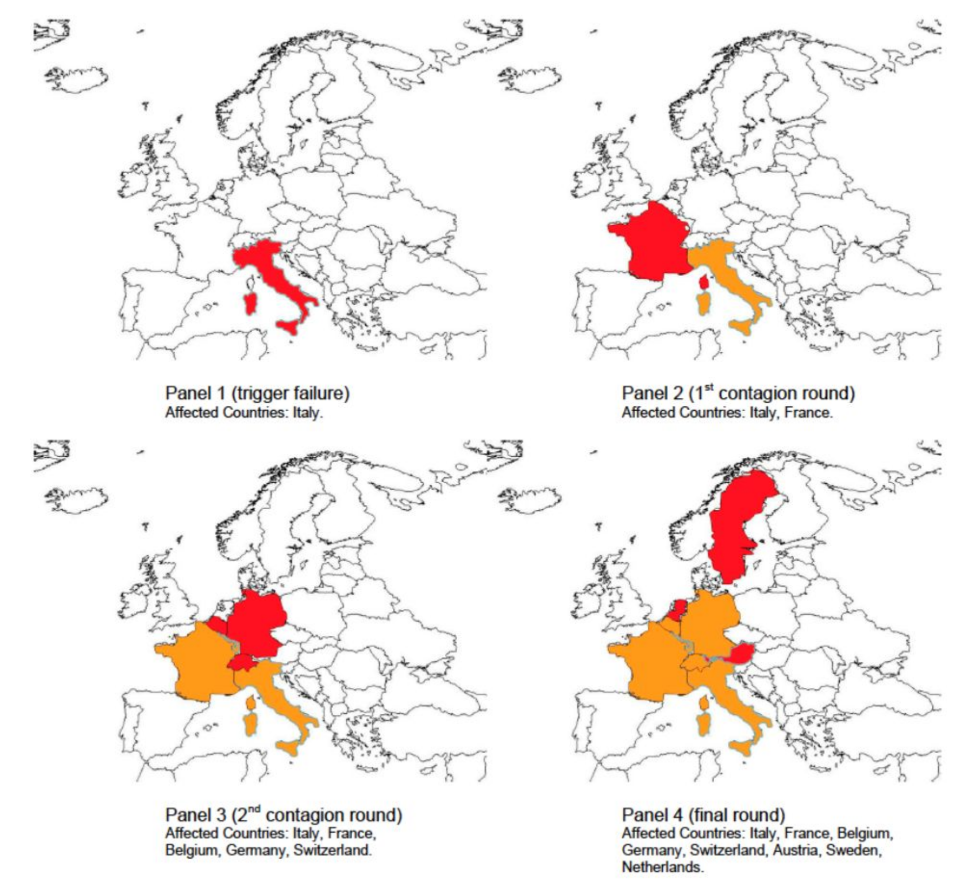


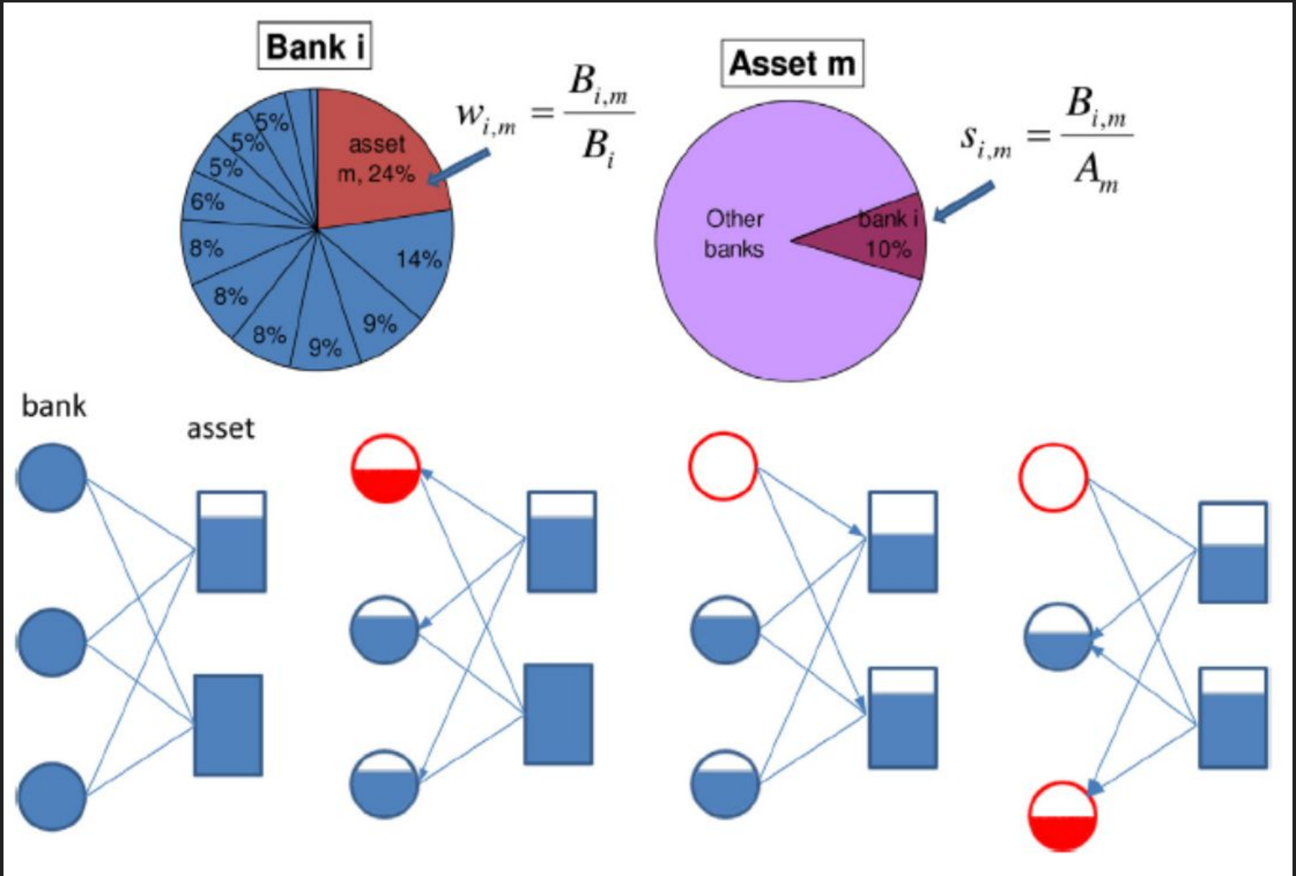
Fig. 2. Network diagram of linear Granger-causality relationships that are statistically significant at the 5% level among the monthly returns of the 25 largest (in terms of average market cap and AUM) banks, broker/dealers, insurers, and hedge funds over January 1994 to December 1996. The type of institution causing the relationship is indicated by color: green for broker/dealers, red for hedge funds, black for insurers, and blue for banks. Granger-causality relationships are estimated including autoregressive terms and filtering out heteroskedasticity with a GARCH(1,1) model.

Contagion Networks (Espinosa-Vega & Sole, IMF 2010)

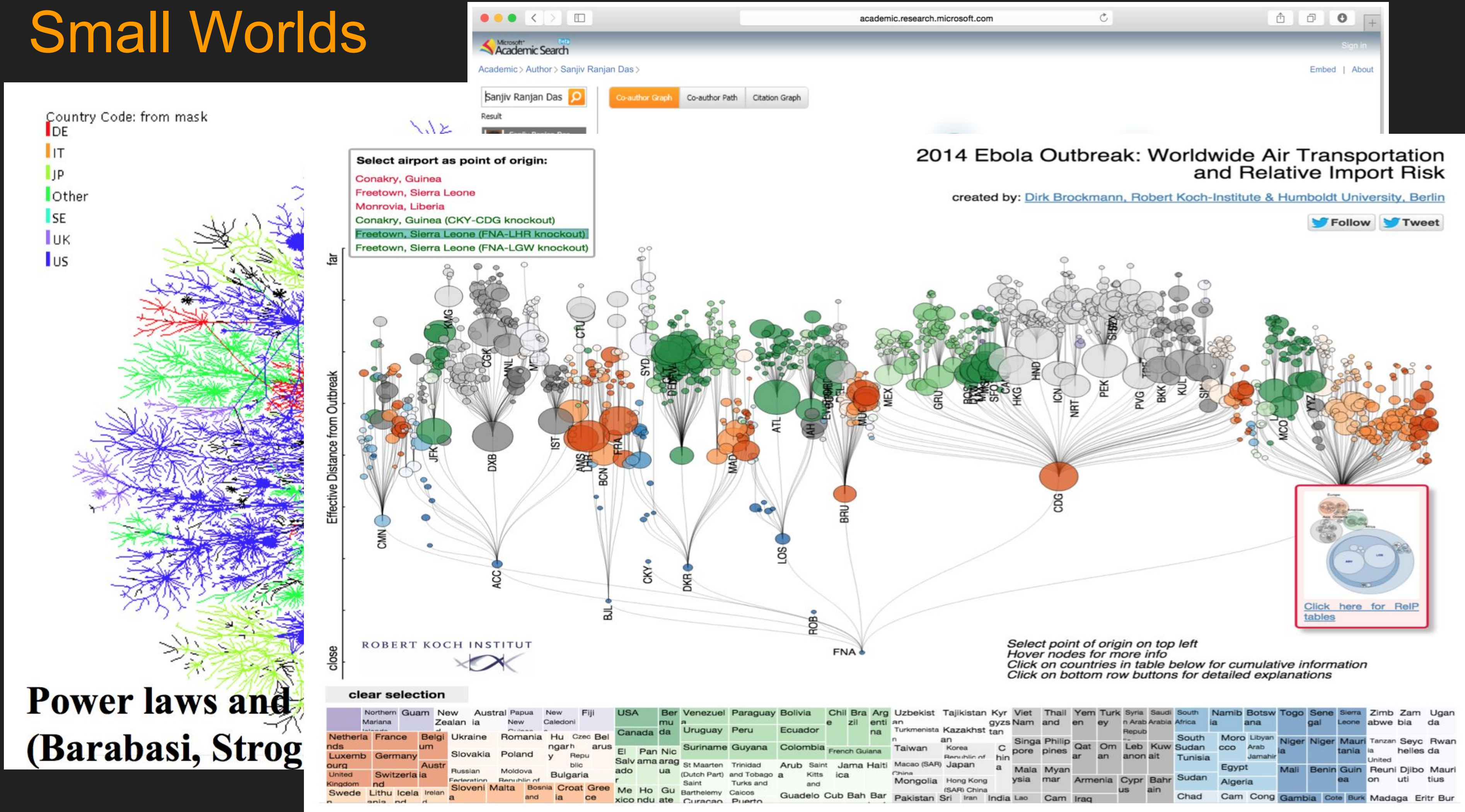


Bivalent Networks

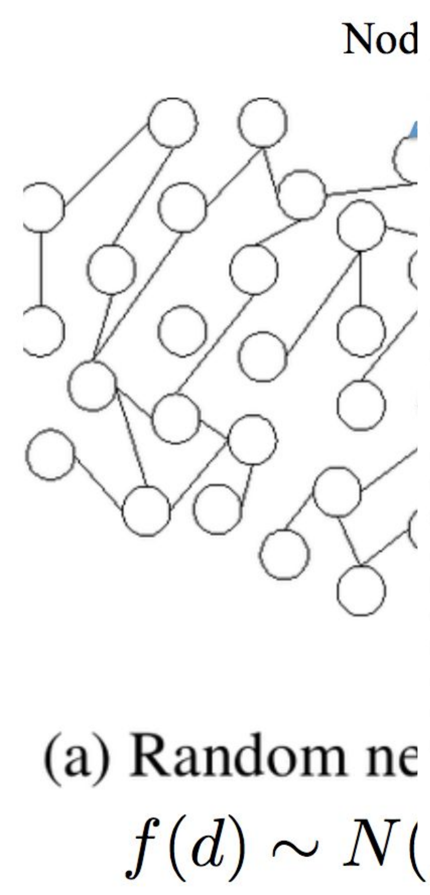
Levy-Carciente, Kennet, Avakian, Stanley, Havlin, JBF 2015

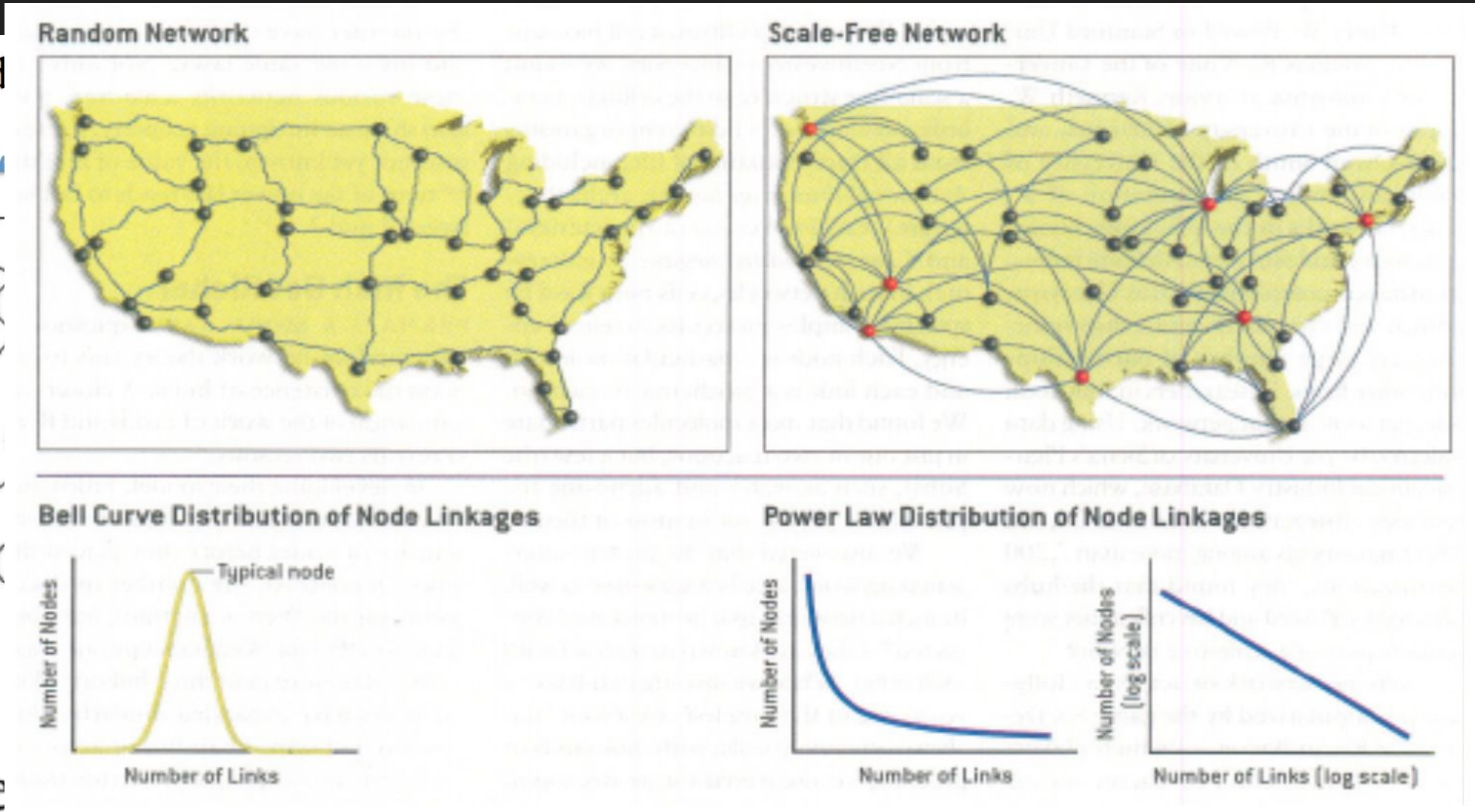


Small Worlds



Graph Theory

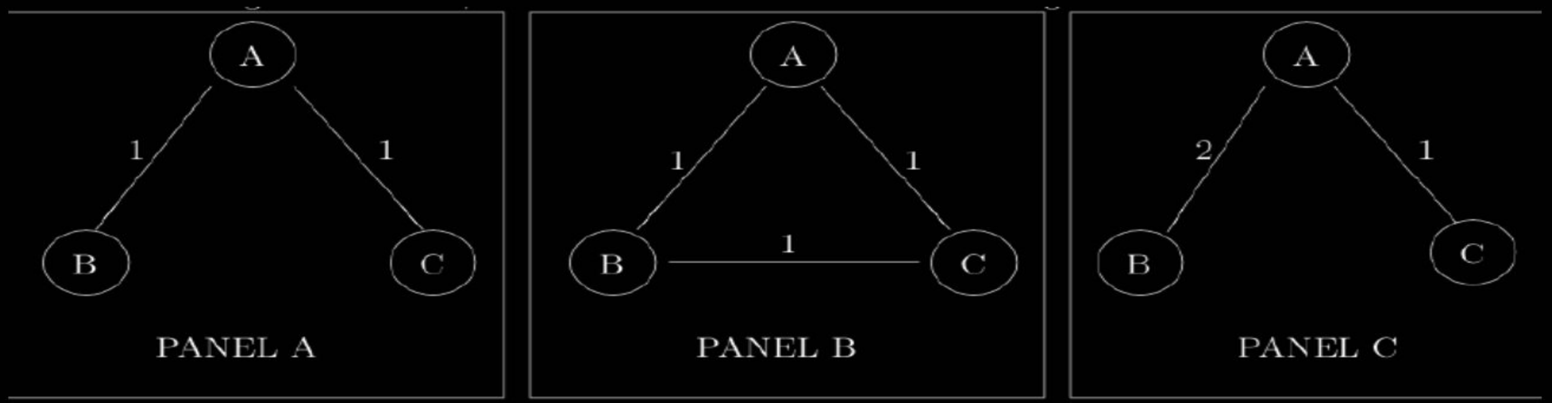




Barabasi, Sciam, May 2003

Centrality (Bonacich 1987)

- Similar to PageRank by Google.
- Adjacency matrix: $A_{ij} \in \mathcal{R}^{N \times N}$
- Influence: $x_i = \sum_{j=1}^{N} A_{ij} x_j$
- $\lambda \mathbf{x} = \mathbf{A} \cdot \mathbf{x}$
- Centrality is the eigenvector $\mathbf{x}$ corresponding to the largest eigenvalue.



$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Centrality scores = {0.71, 0.50, 0.50}

Centrality scores = {0.58, 0.58, 0.58}

Centrality scores = {0.71, 0.63, 0.32}

Diameter

Longest sh

The diame

Fragility

- Definition: how quickly will the failure of any one node trigger failures across the network? Is network malaise likely to spread or be locally contained?
- Metric:

$$R = \frac{E(d^2)}{E(d)},$$

where d is node degree.

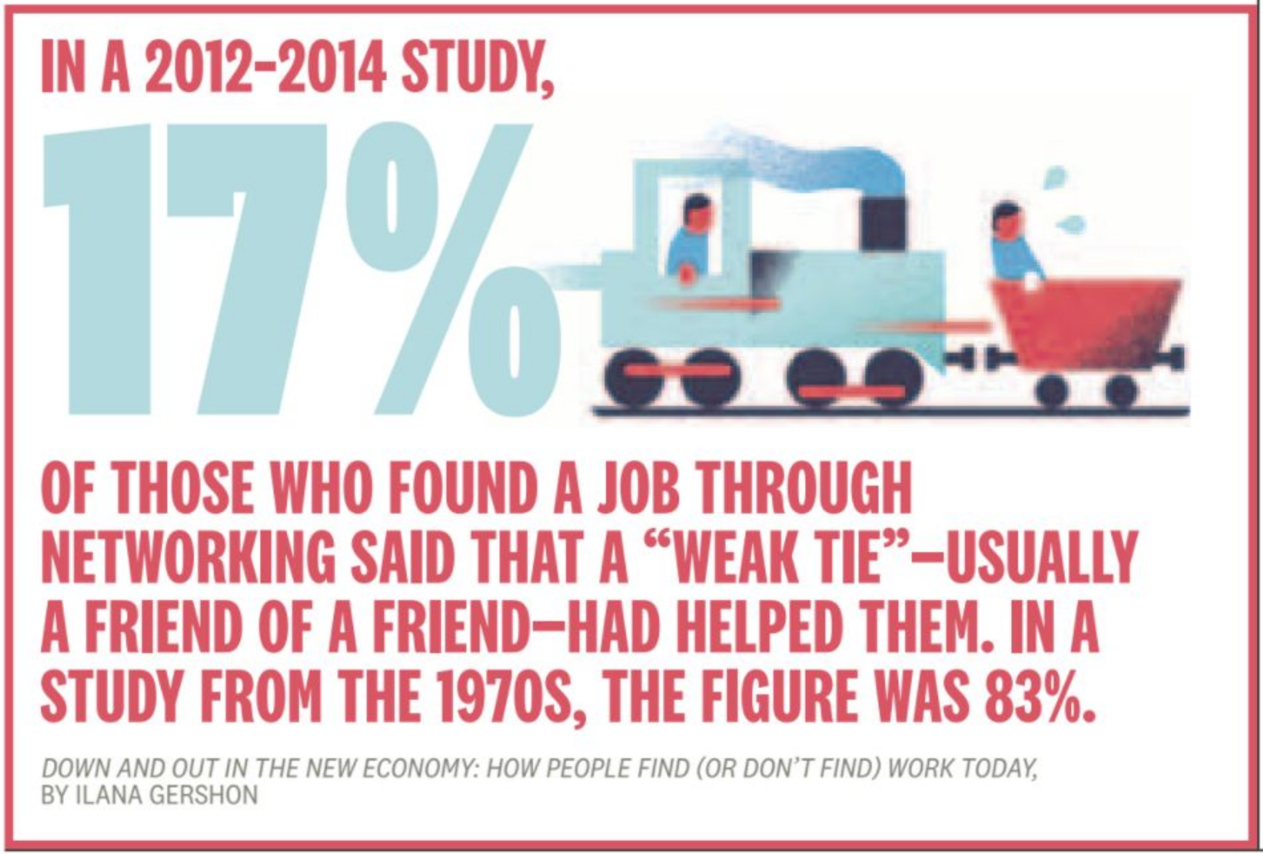
- Similar to a normalized Herfindahl Index.
- Fragility of the sample network $= 20$

Weak Ties

The Strength of Weak Ties[1]

Mark S. Granovetter
Johns Hopkins University

Analysis of social networks is sugges
and macro levels of sociological theo
by elaboration of the macro implicati
interaction: the strength of dyadic ti
of overlap of two individuals' frien
with the strength of their tie to on
principle on diffusion of influence ar
tunity, and community organization i
cohesive power of weak ties. Most n
with strong ties, thus confining the
defined groups. Emphasis on weak t
relations *between* groups and to analy
ture not easily defined in terms of pri



Communities

- Definitio
  commun
- Hard co
- Fast-gre
- Walk-tra

Consider a network of five nodes $\{A, B, C, D, E\}$, where the edge weights are as follows: $A : B = 6$, $A : C = 5$, $B : C = 2$, $C : D = 2$, and $D : E = 10$. Assume that a community detecti
$\{D, E\}$ to another, i.e., o
graph is

The Kronecker delta matr

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Implementation in R

```
> A = matrix(c(0,6,5,0,0,6,0,2,0,0,5,2,0,2,0,0,0,2,0,10,0,0,0,10,0),5,5)
> delta = matrix(c(1,1,1,0,0,1,1,1,0,0,1,1,1,0,0,0,0,0,1,1,0,0,0,1,1),5,5)
> print(Amodularity(A,delta))
[1] 0.4128
```

We now repeat the same analysis using the R package.

```
> g = graph.adjacency(A,mode="undirected",weighted=TRUE,diag=FALSE)
```

We then pass this graph to the walktrap algorithm:

```
> wtc=walktrap.community(g,modularity=TRUE,weights=E(g)$weight)
> res=community.to.membership(g,wtc$merges,steps=3)
> print(res)
$membership
[1] 0 0 0 1 1

$csize
[1] 3 2
```

Midas Project

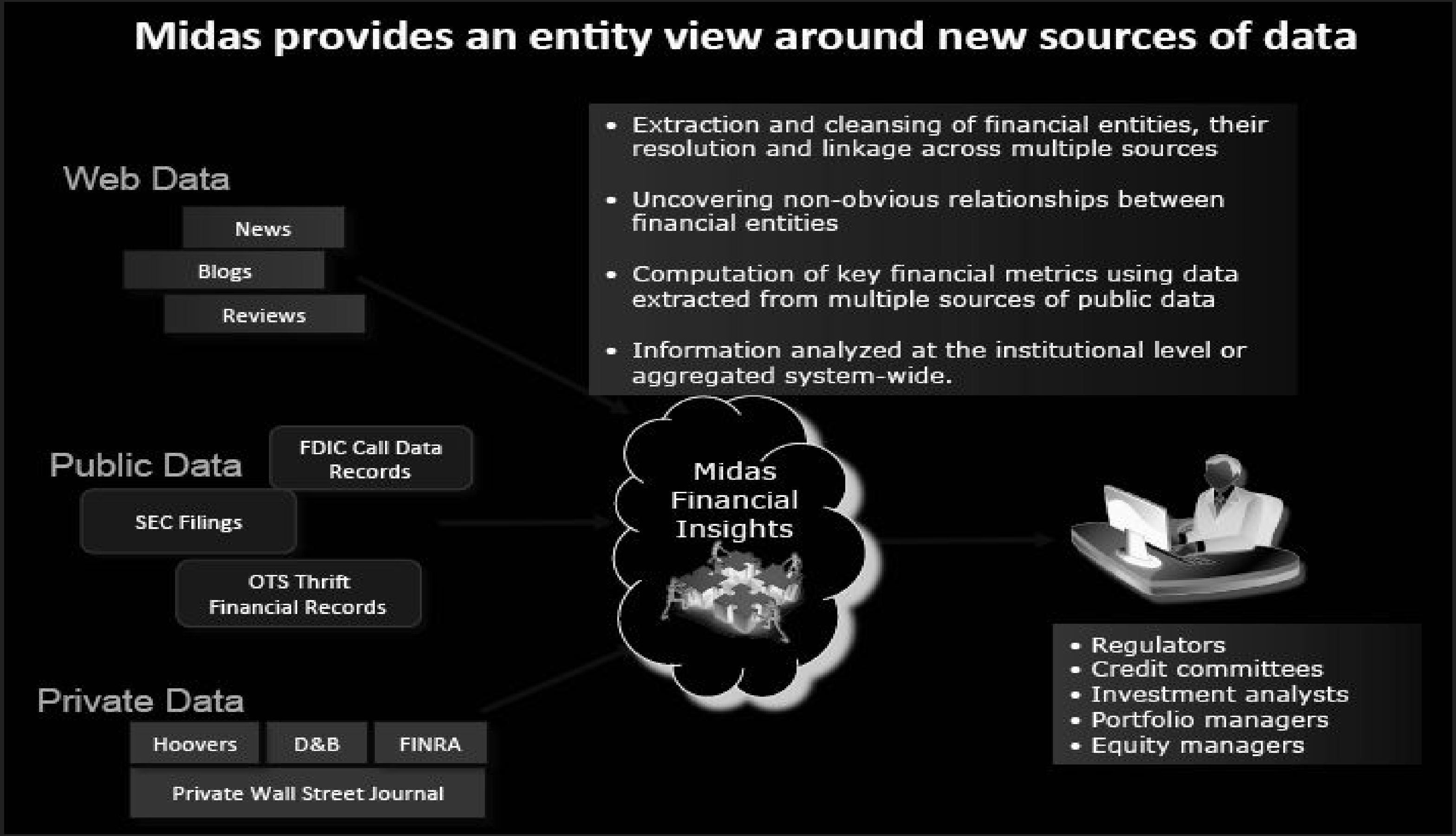
Joint work with IBM Almaden[2]

- Focus on financial companies that are the domain for systemic risk (SIFIs).
- Extract information from unstructured text (filings).
- Information can be analyzed at the institutional level or aggregated system-wide.
- Applications: Systemic risk metrics; governance.
- Technology: information extraction (IE), entity resolution, mapping and fusion, scalable Hadoop architecture.

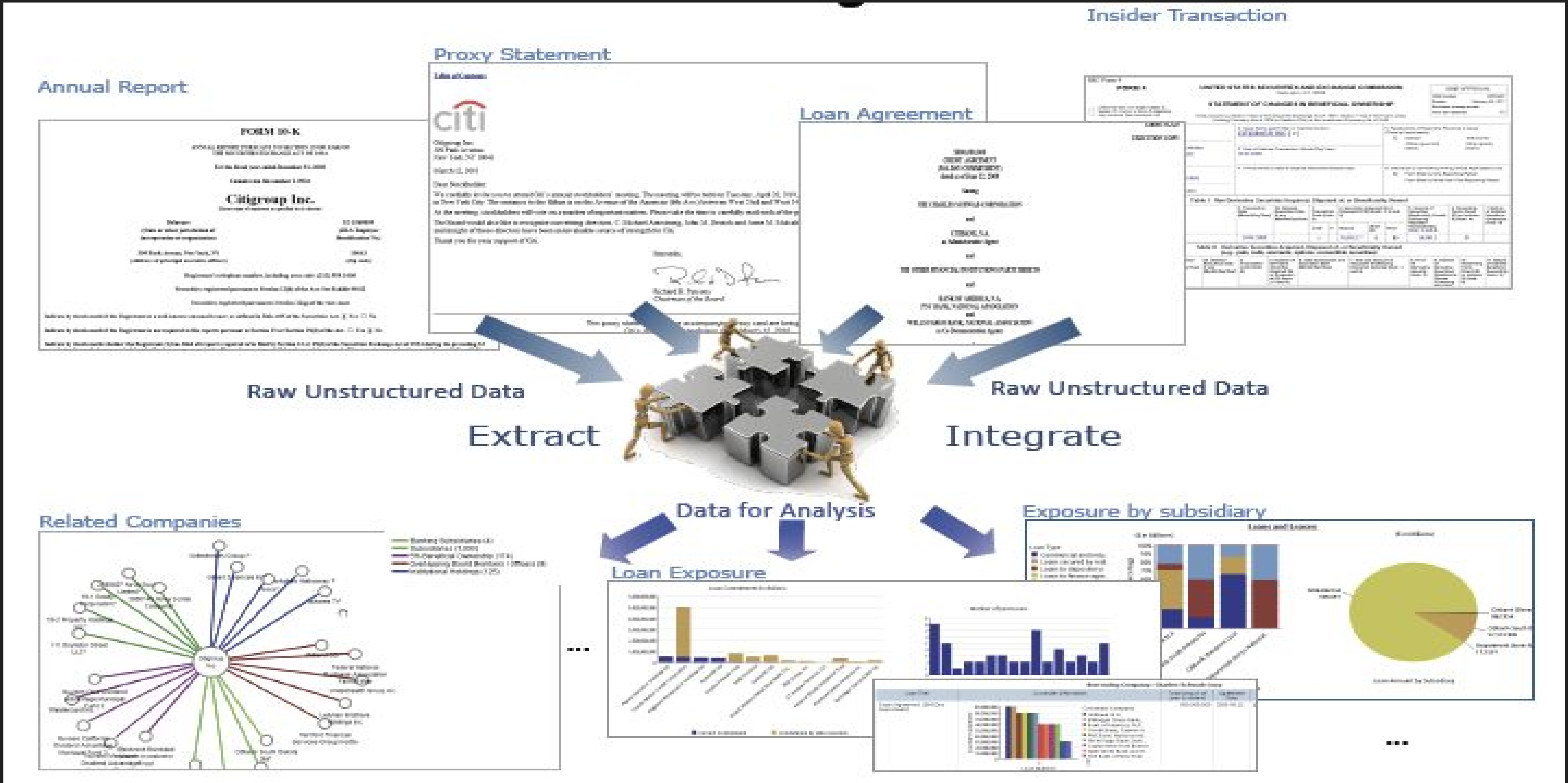
---

[2] "Extracting, Linking and Integrating Data from Public Sources: A Financial Case Study," (2011), (with Douglas Burdick, Mauricio A. Hernandez, Howard Ho, Georgia Koutrika, Rajasekar Krishnamurthy, Lucian Popa, Ioana Stanoi, Shivakumar Vaithyanathan), *IEEE Data Engineering Bulletin*, 34(3), 60-67. [Proceedings WWW2010, April 26-30, 2010, Raleigh, North Carolina.]

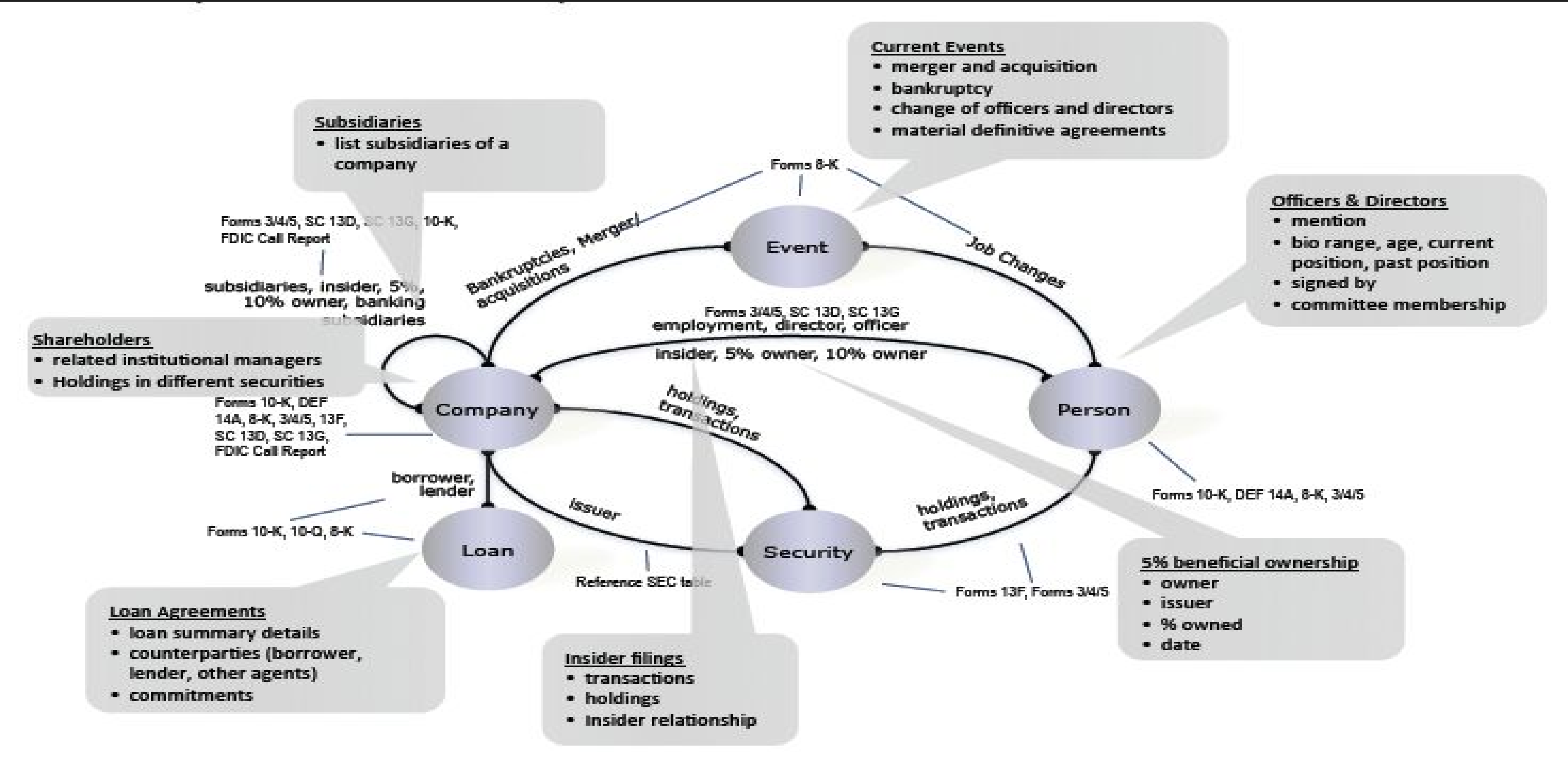
Entity View


Midas provides an entity view around new sources of data
Web Data
News
Blogs
Reviews
Public Data
FDIC Call Data Records
SEC Filings
OTS Thrift Financial Records
Private Data
Hoovers
D&B
FINRA
Private Wall Street Journal
• Extraction and cleansing of financial entities, their resolution and linkage across multiple sources
• Uncovering non-obvious relationships between financial entities
• Computation of key financial metrics using data extracted from multiple sources of public data
• Information analyzed at the institutional level or aggregated system-wide.
Midas Financial Insights
• Regulators
• Credit committees
• Investment analysts
• Portfolio managers
• Equity managers

Input and Output

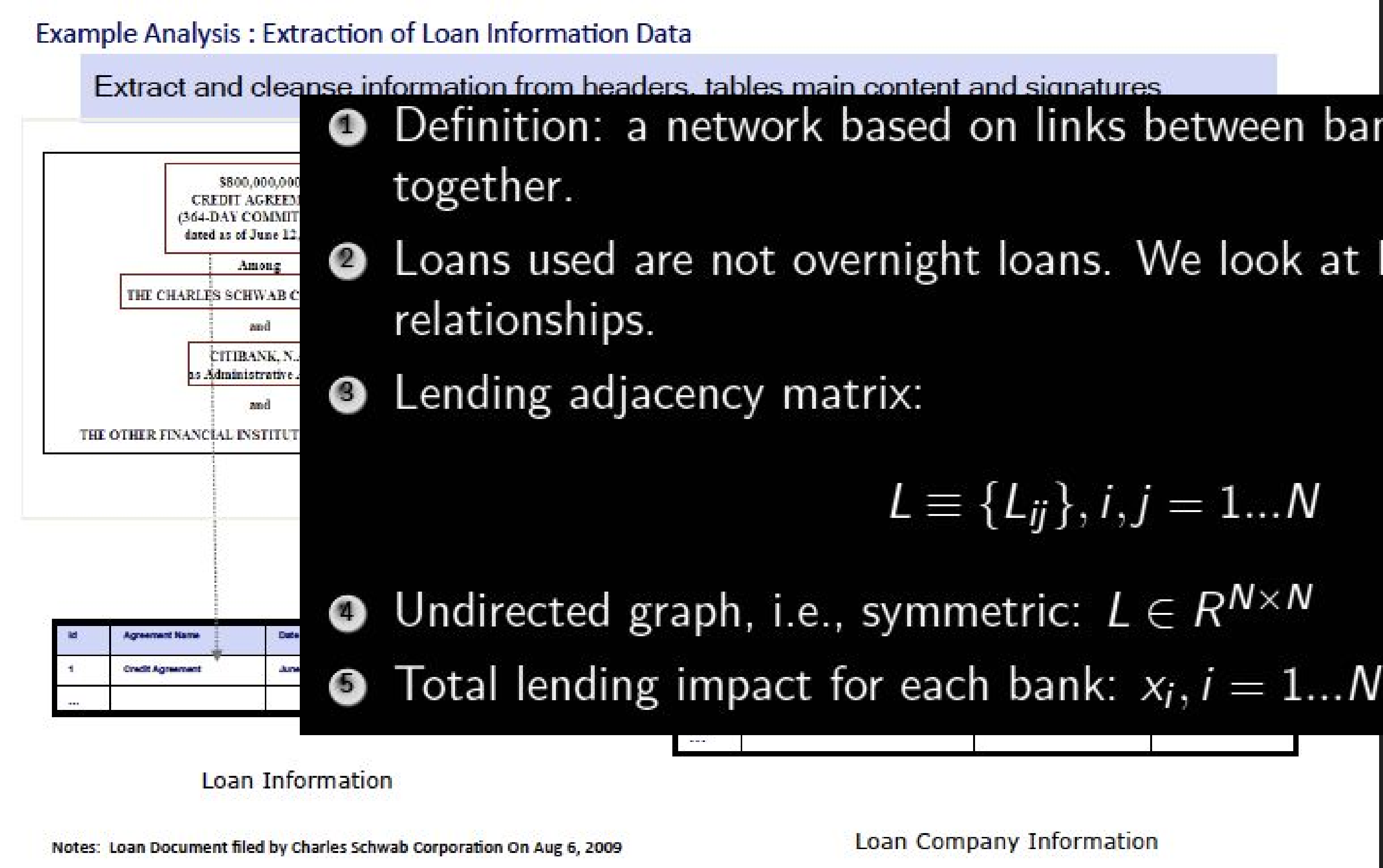


Data Model



Current Events
merger and acquisition
bankruptcy
change of officers and directors
material definitive agreements
Subsidiaries
list subsidiaries of a company
Officers & Directors
mention
bio range, age, current position, past position
signed by
committee membership
Forms 8-K
Event
Bankruptcies, Merger/ acquisitions
Job Changes
Forms 3/4/5, SC 13D, SC 13G, 10-K, FDIC Call Report
subsidiaries, insider, 5%, 10% owner, banking subsidiaries
Shareholders
related institutional managers
Holdings in different securities
Forms 3/4/5, SC 13D, SC 13G
employment, director, officer
insider, 5% owner, 10% owner
Company
Person
holdings, transactions
Forms 10-K, DEF 14A, 8-K, 3/4/5, 13F, SC 13D, SC 13G, FDIC Call Report
borrower, lender
issuer
Forms 10-K, DEF 14A, 8-K, 3/4/5
holdings, transactions
Forms 10-K, 10-Q, 8-K
Loan
Security
Reference SEC table
Forms 13F, Forms 3/4/5
5% beneficial ownership
owner
issuer
% owned
date
Loan Agreements
loan summary details
counterparties (borrower, lender, other agents)
commitments
Insider filings
transactions
holdings
Insider relationship

Loan Extraction



1. Definition: a network based on links between banks that lend together.
2. Loans used are not overnight loans. We look at longer-term lending relationships.
3. Lending adjacency matrix:

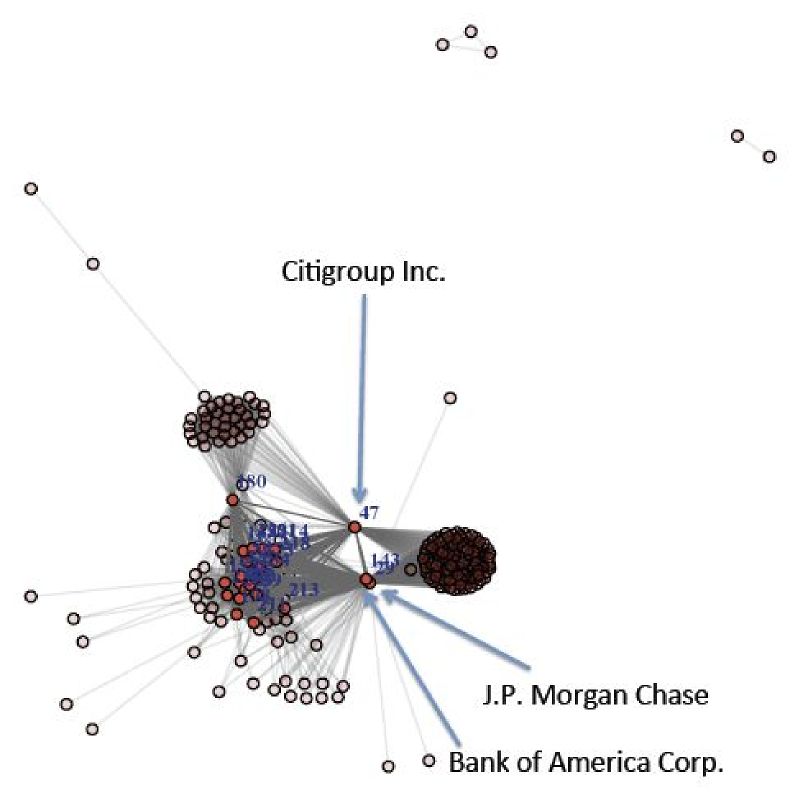
$$L \equiv \{L_{ij}\}, i, j = 1 \ldots N$$

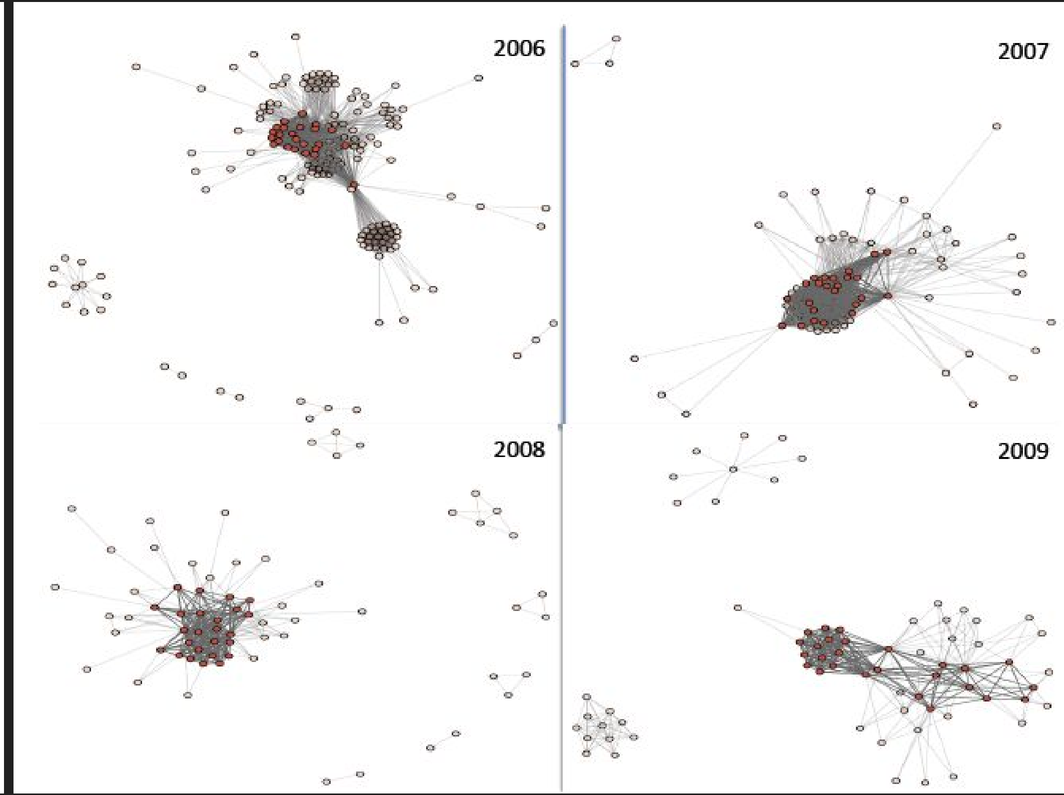
4. Undirected graph, i.e., symmetric: $L \in R^{N \times N}$
5. Total lending impact for each bank: $x_i, i = 1 \ldots N$

Data

- Five years: 2005 to 2009.
- Loans between FIs only.
- Filings made with the SEC.
- No overnight loans.
- Example: 364-day bridge loans, longer-term credit arrangement, Libor notes, etc.
- Remove all edge weights < 2 to remove banks that are minimally active. Remove all nodes with no edges. (This is a choice for the regulator.)

Loan Network 2005 - 2009





Systemically Important Financial Institutions (SIFIs)

Year	#Colending banks	#Coloans	Colending pairs	$R = E(d^2)/E(d)$	Diam.
2005	241	75	10997	137.91	5
2006	171	95	4420	172.45	5
2007	85	49	1793	73.62	4
2008	69	84	681	68.14	4
2009	69	42	598	35.35	4

(Year = 2005)

Node #	Financial Institution	Normalized Centrality
143	J P Morgan Chase & Co.	1.000
29	Bank of America Corp.	0.926
47	Citigroup Inc.	0.639
85	Deutsche Bank Ag New York Branch	0.636
225	Wachovia Bank NA	0.617
235	The Bank of New York	0.573
134	Hsbc Bank USA	0.530
39	Barclays Bank Plc	0.530
152	Keycorp	0.524
241	The Royal Bank of Scotland Plc	0.523
6	Abn Amro Bank N.V.	0.448
173	Merrill Lynch Bank USA	0.374
198	PNC Financial Services Group Inc	0.372
180	Morgan Stanley	0.362
42	Bnp Paribas	0.337
205	Royal Bank of Canada	0.289
236	The Bank of Nova Scotia	0.289
218	U.S. Bank NA	0.284
50	Calyon New York Branch	0.273
158	Lehman Brothers Bank Fsb	0.270
213	Sumitomo Mitsui Banking	0.236
214	Suntrust Banks Inc	0.232
221	UBS Loan Finance Llc	0.221
211	State Street Corp	0.210
228	Wells Fargo Bank NA	0.198

Stochastic Risk Networks

- We use the Merton (1974) model to extend the static Das (2016) model to a stochastic network setting (Das, Kim, Ostrov, 2016).
- We extend each node's properties to including size, in addition to the credit score.
- To do this we normalize the S measure.
- This model can be calibrated using the same methods used for the Merton model, or variants such as the Moody's KMV model.

Definitions

Model Data (standard Merton model inputs) for each firm:

- Equity price $= \mathbf{s} = \{s_1, s_2, ..., s_n\}$
- Equity volatility $= \boldsymbol{\sigma} = \{\sigma_1, \sigma_2, ..., \sigma_n\}$
- Number of shares $= \mathbf{m} = \{m_1, m_2, ..., m_n\}$
- Risk free rate $= r$

Model Variables (all derived from the Merton model):

- n = number of banks in the system
- $\mathbf{a}$ = n-vector with components a_i that represent the assets in bank i (derived from s, σ, m, r).
- $\boldsymbol{\lambda}$ = n-vector with components λ_i that represent the average yearly chance of bank i defaulting (from s, σ, r).
- $\mathbf{E}$ = $n \times n$ matrix with components E_{ij} that represent the probability that if bank j defaults, it will cause bank i to default (from s, σ, r).

Brief Recap of the Merton (1974) Model

Geometric Brownian motion as the stochastic process for each FI's underlying assets. For the n FIs in the system, we have:

$$\begin{aligned} da_i &= \mu_i a_i \, dt + v_i a_i \, dB_i, \qquad i = 1, 2, \ldots, n \\ da_i \, da_j &= \rho_{ij} \, dt, \qquad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, n. \end{aligned}$$

FI's equity, E_i, is but a call option on the assets, i.e.,

$$\begin{aligned} E_i &= a_i \Phi(d_{1,i}) - D_i e^{-r_f T} \Phi(d_{2,i}) \\ d_{1,i} &= \frac{\ln(a_i / D_i) + (r_f + v_i^2/2) T}{v_i \sqrt{T}} \\ d_{2,i} &= d_1 - v_i \sqrt{T} = \frac{\ln(a_i / D_i) + (r_f - v_i^2/2) T}{v_i \sqrt{T}}, \end{aligned}$$

Volatility of equity:

$$\sigma_i = v_i \frac{\partial E_i}{\partial a_i} \frac{a_i}{E_i}.$$

Systemic Risk Function

- Our systemic risk measures take the following functional form

$$\mathcal{S} = f(\boldsymbol{\lambda}, \mathbf{a}, \boldsymbol{\Sigma}),$$

  where a specific systemic risk model corresponds to a specific function f and specific definition for the connection matrix $\boldsymbol{\Sigma}$.

- the case where $\mathcal{S}$ is homogeneous in its default risks, $\boldsymbol{\lambda}$, which means, for any scalar $\alpha > 0$,

$$\alpha f(\boldsymbol{\lambda}, \mathbf{a}, \boldsymbol{\Sigma}) = f(\alpha\boldsymbol{\lambda}, \mathbf{a}, \boldsymbol{\Sigma}). \tag{3}$$

Risk Decomposition

- The impact of each institution on $\mathcal{S}$ is to decompose $\mathcal{S}$ into the sum of n components by differentiating equation (3) with respect to $\boldsymbol{\lambda}$, yielding the result of Euler's theorem:

$$\mathcal{S} = \frac{\partial \mathcal{S}}{\partial \boldsymbol{\lambda}} \boldsymbol{\lambda} = \sum_{i=1}^{n} \frac{\partial \mathcal{S}}{\partial \lambda_i} \lambda_i. \tag{4}$$

- Using each component, $\frac{\partial \mathcal{S}}{\partial \lambda_i} \lambda_i$, of the sum to define the corresponding *institution risk measure* of institution i.

Model

- Define $\mathbf{c}$ to be an n-vector with components c_i that represent bank i's credit risk. More specifically, we define

$$\mathbf{c} = \mathbf{a} \odot \boldsymbol{\lambda},$$

  where $\odot$ represents component multiplication; that is, $c_i = a_i \lambda_i$.
- The aggregate systemic risk created by the n banks in our system is

$$\boxed{R = \frac{\sqrt{\mathbf{c}^\top \mathbf{E} \mathbf{c}}}{\mathbf{1}^\top \mathbf{a}},} \tag{5}$$

  where $\mathbf{1}$ is an n-vector of ones, so the denominator $\mathbf{1}^\top \mathbf{a} = \sum_{i=1}^{n} a_i$ represents the total assets in the n banks.
- r is linear homogenous in $\boldsymbol{\lambda}$.

Useful Features

Property 1: All other things being equal, $\mathcal{S}$ should be minimized by dividing risk equally among the n financial institutions, and maximized by putting all the risk into one institution.

Property 2: $\mathcal{S}$ should increase as the financial institutions become more entwined.

Property 3: If all the assets, a_i, are multiplied by a common factor, $\alpha > 0$, it should have no effect on $\mathcal{S}$.

Property 4: Substanceless partitioning of a bank into two banks should have no effect on $\mathcal{S}$.

Three models: C, D, G

Define $\mathbf{\Sigma} = \mathbf{M}$, an $n \times n$ matrix where $M_{ij} \in [0,1]$ for all i and j and $M_{ii} = 1$ for all i. (Note that $\mathbf{M} \neq \mathbf{B}$, because values in $\mathbf{M}$ are from the continuum $[0,1]$, whereas $\mathbf{B}$ is binary and a function of $\mathbf{M}$ and threshold K.)

1. **Model C**: *Correlation based model.* $M_{ij} = \frac{1}{2}(\rho_{ij} + 1)$, where ρ_{ij} is the correlation between the daily asset returns of institutions i and j.
2. **Model D**: *Conditional default model.* M_{ij} is the annual conditional probability that institution j defaults if institution i fails.
3. **Model G**: *Granger causality model.*

$$r_i(t) = \delta_1 + \delta_2 \cdot r_i(t-1) + \delta_3 \cdot r_j(t-1) + \epsilon_i$$
$$r_j(t) = \delta_4 + \delta_5 \cdot r_j(t-1) + \delta_6 \cdot r_i(t-1) + \epsilon_j$$

Connectedness is $M_{ij} = 1 - p(\delta_6)$ and $M_{ji} = 1 - p(\delta_3)$, where $p(x)$ is the p-value for the hypothesis that the coefficient $x = \delta_6$ or δ_3 is equal to zero in the regressions. When $i = j$, we set $M_{ii} = 1$.

Homogeneity in default risks

- Define
$$\mathbf{c} = \mathbf{a} \odot \boldsymbol{\lambda},$$
where $\odot$ represents component multiplication; that is, $c_i = a_i \lambda_i$.
- Systemic risk, $\mathcal{S}$, is
$$\mathcal{S} = \frac{\sqrt{\mathbf{c}^T \mathbf{M} \mathbf{c}}}{\mathbf{1}^T \mathbf{a}}, \tag{6}$$
- The model is homogeneous in $\boldsymbol{\lambda}$, so, from equation (4), we have that
$$\mathcal{S} = \frac{\partial \mathcal{S}}{\partial \boldsymbol{\lambda}} \boldsymbol{\lambda} = \sum_{i=1}^{n} \frac{\partial \mathcal{S}}{\partial \lambda_i} \lambda_i,$$
where, from differentiating our system risk definition in equation (6), we obtain the n-dimensional vector
$$\frac{\partial \mathcal{S}}{\partial \boldsymbol{\lambda}} = \frac{1}{2} \frac{\mathbf{a} \odot [(\mathbf{M} + \mathbf{M}^T)\mathbf{c}]}{\mathbf{1}^T \mathbf{a} \sqrt{\mathbf{c}^T \mathbf{M} \mathbf{c}}}. \tag{7}$$

Property 1: $\mathcal{S}$ is minimized by dividing the credit risk equally among the n financial institutions, and maximized by putting all the credit risk into one institution

To make "all other things be equal," we set the total assets, $\sum_{i=1}^{n} a_i = \mathbf{1}^T \mathbf{a}$, constant, set the total credit risk, $\sum_{i=1}^{n} c_i = \mathbf{1}^T \mathbf{c}$, equal to a constant, c_{total}, and set M_{ij} equal to the same number, m, if $i \neq j$ while, of course, keeping $M_{ii} = 1$ for all i. For the singular case where $m = 1$, all the institutions act like a single institution, and so it makes no difference to $\mathcal{S}$ how the credit risk is spread among the institutions. But for the general case where $m < 1$, from the definition of $\mathcal{S}$ in equation (6), we see that maximizing or minimizing $\mathcal{S}$ now corresponds to maximizing or minimizing $\mathbf{c}^T \mathbf{M} \mathbf{c} = \sum_{i=1}^{n} c_i^2 + m \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j$, subject to the restriction that $\mathbf{1}^T \mathbf{c} = \sum_{i=1}^{n} c_i = c_{total}$.

Proof continued...

Since $m < 1$, it is clear that
$\sum_{i=1}^{n} c_i^2 + m \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j \leq \sum_{i=1}^{n} c_i^2 + \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j = \left(\sum_{i=1}^{n} c_i \right)^2 = c_{total}^2$. But if all the credit risk is put into one institution, we have
$\sum_{i=1}^{n} c_i^2 + m \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j = c_{total}^2$, the highest possible value, and so $\mathcal{S}$ is maximized when all the credit risk is concentrated into one financial institution.

Proof continued...

On the other hand, the Lagrange multiplier method tells us that we have minimized $\sum_{i=1}^{n} c_i^2 + m \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j$ subject to the restriction $\sum_{i=1}^{n} c_i = c_{total}$ when (denoting the Lagrange multiplier by λ),

$$\frac{\partial}{\partial c_k} \left(\sum_{i=1}^{n} c_i^2 + m \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j \right) = \lambda \frac{\partial}{\partial c_k} \sum_{i=1}^{n} c_i \text{ where } k = 1, 2, \ldots n$$

and

$$\sum_{i=1}^{n} c_i = c_{total}.$$

The first n equations give us that $c_1 = c_2 = \ldots = c_n = \frac{\lambda - 2mc_{total}}{2(1-m)}$. That is, when $\mathcal{S}$ is minimized, all c_i have the same value. The second equation then tells us that each $c_i = \frac{c_{total}}{n}$, and so we have that $\mathcal{S}$ is minimized by dividing the credit risk equally among the n institutions.

Property 2: $\mathcal{S}$ should increase as the institutions' defaults become more connected.

Consider the case where **a** and **c** are both held constant, so that $\mathcal{S}$ only depends on **M**, specifically through the expression

$$\mathbf{c}^T\mathbf{M}\mathbf{c} = \sum_{i=1}^{n}\sum_{j=1}^{n} c_i M_{ij} c_j$$

in the numerator of our model's definition of $\mathcal{S}$. Clearly, the bigger the values of M_{ij} are, the larger $\mathcal{S}$ becomes. Since M_{ii} must always equal 1, $\mathcal{S}$ is minimized when $\mathbf{M} = \mathbf{I}$, the identity matrix, and maximized when the components of the **M** matrix are all ones. We note that when $\mathbf{M} = \mathbf{I}$, $\sqrt{\mathbf{c}^T\mathbf{M}\mathbf{c}} = \sqrt{\sum_{i=1}^{n} c_i^2} = ||\mathbf{c}||_2$, the 2-norm of the vector **c**, whereas when **M** is all ones, $\sqrt{\mathbf{c}^T\mathbf{M}\mathbf{c}} = \sum_{i=1}^{n} c_i = ||\mathbf{c}||_1$, the 1-norm of the vector **c**.

Property 3: If all the assets, a_i, are multiplied by a common factor, $\alpha > 0$, it should have no effect on $\mathcal{S}$.

In our model, if we replace each a_i with αa_i, we then replace $\sqrt{\mathbf{c}^T \mathbf{M} \mathbf{c}}$ by $\alpha \sqrt{\mathbf{c}^T \mathbf{M} \mathbf{c}}$ and replace $\mathbf{1}^T \mathbf{a}$ with $\alpha \mathbf{1}^T \mathbf{a}$. Since the α then cancel in the expression for $\mathcal{S}$ from equation (6), we have the desired property that systemic risk is unchanged.

Property 4: Substanceless partitioning of an institution into two institutions should have no effect on $\mathcal{S}$.

If institution i's assets are artificially divided into two institutions of size γa_i and $(1-\gamma)a_i$ for some $\gamma \in [0,1]$, where both of these new institutions are completely connected to each other and both have the same connections with the other banks that the original institution did, then this division is without substantive meaning, so it should not affect the value of $\mathcal{S}$. Without loss of generality, we can let the index of the divided institution $i = n$, so, in our model, the new $(n+1)$-vector $\mathbf{c}$ is

$$\mathbf{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_{n-1} \\ \gamma c_n \\ (1-\gamma)c_n \end{bmatrix}$$

Proof continued...

The new $(n+1) \times (n+1)$ matrix $\mathbf{M}$ is

$$\mathbf{M} = \begin{bmatrix} 1 & M_{12} & \cdots & M_{1(n-1)} & M_{1n} & M_{1n} \\ M_{21} & \ddots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \ddots & \ddots & M_{(n-2)(n-1)} & \vdots & \vdots \\ M_{(n-1)1} & \cdots & M_{(n-1)(n-2)} & 1 & M_{(n-1)n} & M_{(n-1)n} \\ M_{n1} & \cdots & \cdots & M_{n(n-1)} & 1 & 1 \\ M_{n1} & \cdots & \cdots & M_{n(n-1)} & 1 & 1 \end{bmatrix},$$

where we note that $M_{(n+1)n} = M_{n(n+1)} = 1$ to reflect the fact that both of the new institutions are completely connected to each other. A quick computation shows that the new $\sqrt{\mathbf{c}^T\mathbf{M}\mathbf{c}}$ is equal to the old $\sqrt{\mathbf{c}^T\mathbf{M}\mathbf{c}}$, and since $a_1 + \ldots + a_n = a_1 + \ldots + a_{(n-1)} + \gamma a_n + (1-\gamma)a_n$, we also have that the new $\mathbf{1}^T\mathbf{a}$ is equal to the old $\mathbf{1}^T\mathbf{a}$. Therefore, the value of $\mathcal{S}$ in equation (6) is unchanged and our model has this desired property.

Model $\mathbf{R}$ not Homogenous in Default Risks

- $\mathbf{\Sigma} = \mathbf{M}$, where M_{ij} is the annual probability that financial institutions i and j both default.
- *Internal risk* for FI i: credit risk, $c_i = \lambda_i a_i$. We can also write this as $c_i = M_{ii} a_i$ since, by definition, $M_{ii} = \lambda_i$.
- *External risk* from FI i to FI j: probability that FI i will default multiplied by the probability that FI j will default given that FI i defaults multiplied by the assets in FI j. Equal to the probability that both FI i and FI j default multiplied by the assets in FI j, i.e., $M_{ij} a_j$.

Systemic Risk for Model R

- Define ρ_i, internal risk from FI i plus the sum of the external risks from FI i to each of the other FIs:

$$\rho_i = \sum_{j=1}^{n} M_{ij} a_j. \tag{8}$$

- Defining $\boldsymbol{\rho}$ to be the n-vector with components ρ_i, we can define the systemic risk to be

$$\mathcal{S} = \frac{\sqrt{\boldsymbol{\rho}^T \boldsymbol{\rho}}}{\mathbf{1}^T \mathbf{a}}. \tag{9}$$

- $\sum_{i=1}^{n} \rho_i \neq \mathcal{S}$, unlike the case where $\mathcal{S}$ is homogeneous in $\boldsymbol{\lambda}$ for which this equality holds due to equation (4).
- Connectedness risk measure from bank i to bank j is the external risk, $M_{ij} a_j$.

Property 1: $\mathcal{S}$ is minimized by dividing the risk equally among the n financial institutions, and maximized by putting all the risk into one institution.

Hold the total assets, $\sum_{i=1}^{n} a_i = \mathbf{1}^T \mathbf{a}$, constant and also the total risk, $\sum_{i=1}^{n} \rho_i = \mathbf{1}^T \boldsymbol{\rho}$, equal to a constant. If we replace $\mathbf{c}$ and $\mathbf{M}$ in the model from the previous section for $\mathcal{S}$ given in equation (6) with $\boldsymbol{\rho}$ and the identity matrix $\mathbf{I}$, we get our new model for $\mathcal{S}$ in equation (9). Therefore, the proof of Property 1 from the previous section with $m = 0$ also establishes Property 1 for the model of $\mathcal{S}$ in equation (9).

Note that if the numerator in the definition of $\mathcal{S}$ in equation (9) were $\sum_{i=1}^{n} \rho_i$, the 1-norm of $\boldsymbol{\rho}$, instead of $\sqrt{\boldsymbol{\rho}^T \boldsymbol{\rho}}$, the 2-norm of $\boldsymbol{\rho}$, we would lose property 1.

Property 2: $\mathcal{S}$ should increase as the institutions' defaults become more connected.

An increasing connection means M_{ij} is increasing which, from equation (8), means that ρ_i increases. As any ρ_i increases, we have from equation (9) that $\mathcal{S}$ increases, assuming, as we also did in the previous section, that $\mathbf{a}$ is held constant.

Property 3: If all the assets, a_i, are multiplied by a common factor, $\alpha > 0$, it should have no effect on $\mathcal{S}$.

In our model, if we replace each a_i with αa_i, we replace $\sqrt{\boldsymbol{\rho}^T \boldsymbol{\rho}}$ by $\alpha\sqrt{\boldsymbol{\rho}^T \boldsymbol{\rho}}$, and we replace $\mathbf{1}^T\mathbf{a}$ with $\alpha\mathbf{1}^T\mathbf{a}$. Since the α then cancel in the expression for $\mathcal{S}$ given in equation (9), we have the desired property that systemic risk is unchanged.

Property 4: Substanceless partitioning of an institution into two institutions should have no effect on $\mathcal{S}$.

This property does not hold. Let's say we artificially divide institution n's assets into two institutions, call them institution n_{new} and institution $(n+1)_{new}$, of size γa_n and $(1-\gamma)a_n$, where, since the division is artificial, $M_{n_{new}n_{new}} = M_{(n_{new}+1)n_{new}} = M_{n_{new}(n_{new}+1)} = M_{(n_{new}+1)(n_{new}+1)}$, which all equal M_{nn}, where n again represents the divided institution before it was divided, and, for any $i < n$, $M_{n_{new}i} = M_{(n_{new}+1)i} = M_{in_{new}} = M_{i(n_{new}+1)}$ equals $M_{ni} = M_{in}$.
From equation (8), we see that the ρ_i are unchanged for $i = 1, 2, ..., n$. However, there is now an extra $(n+1)^{th}$ component that has been added to the vector $\boldsymbol{\rho}$, where $\rho_{n+1} = \rho_n$, which must increase the norm of $\boldsymbol{\rho}$, which must increase the systemic risk $\mathcal{S}$ in equation (9). Therefore, artificial division of a financial institution increases $\mathcal{S}$ instead of having no effect on it.

Data

- Sample period: January 1992 to December 2015.
- Publicly traded financial institutions under major Standard Industrial Classification (SIC) groups 60 (depository institutions), 61 (non-depository credit institutions), and 62 (security and commodity brokers, dealers, exchanges, and services).
- Final sample: Panel dataset of 2,066,868 firm-days for 1,171 distinct financial institutions, we select the 20 largest institutions by total assets at various points across time.
- Working with more institutions does not pose computational difficulty; we choose only 20 institutions for clarity.
- Top 20 institutions consistently represent over 70% of the total worth of the assets in the 1,171 FIs.

Table 1

Table: 20 Largest Financial Institutions (FIs): June 30, 1995.

	Mean	P25	P50	P75
Book Value of Assets	120,061	66,302	107,548	125,522
Leverage	0.9407	0.9290	0.9390	0.9529
Market Capitalization, E	7,622	2,718	6,814	9,548
Equity Volatility, σ	0.2370	0.1941	0.2230	0.2711
Implied Market Value of Assets, a	114,788	65,456	108,927	124,465
Implied Volatility of Assets, v	0.0171	0.0101	0.0175	0.0224
% of all FIs' Total Assets:	77.34%			

Note: The dollar amounts are all in millions of dollars.

Table 2

Table: 20 Largest Financial Institutions (FIs): June 30, 2000

	Mean	P25	P50	P75
Book Value of Assets	354,319	235,274	276,039	417,851
Leverage	0.9475	0.9342	0.9518	0.9655
Market Capitalization, E	40,353	11,354	26,272	58,439
Equity Volatility, σ	0.4485	0.3522	0.4662	0.4999
Implied Market Value of Assets, a	357,125	243,392	265,194	418995
Implied Volatility of Assets, v	0.0531	0.0159	0.0394	0.0922
% of all FIs' Total Assets:	73.83%			

Note: The dollar amounts are all in millions of dollars.

Table 3

Table: 20 Largest Financial Institutions (FIs): June 29, 2007

	Mean	P25	P50	P75
Book Value of Assets	1,313,221	796,235	1,191,233	1,541,156
Leverage	0.9521	0.9443	0.9558	0.9673
Market Capitalization, E	61,228	3,472	39,833	88,437
Equity Volatility, σ	0.1956	0.1646	0.1999	0.2240
Implied Market Value of Assets, a	1,254,163	777,503	1,132,813	1,468,445
Implied Volatility of Assets, v	0.0092	0.0006	0.0073	0.0181
% of all FIs' Total Assets:	77.51%			

Note: The dollar amounts are all in millions of dollars.

Table 4

Table: 20 Largest Financial Institutions (FIs): June 30, 2015

	Mean	P25	P50	P75
Book Value of Assets	1,546,362	1,068,044	1,525,649	1,883,981
Leverage	0.9265	0.9063	0.9393	0.9432
Market Capitalization, E	70,998	1,823	52,161	88,712
Equity Volatility, σ	0.2287	0.1969	0.2156	0.2583
Implied Market Value of Assets, a	1,500,492	1,093,359	1,433,546	1,826,187
Implied Volatility of Assets, v	0.0096	0.0004	0.0104	0.0191
% of all FIs' Total Assets:	76.83%			

Note: The dollar amounts are all in millions of dollars.

Systemic Risk over time

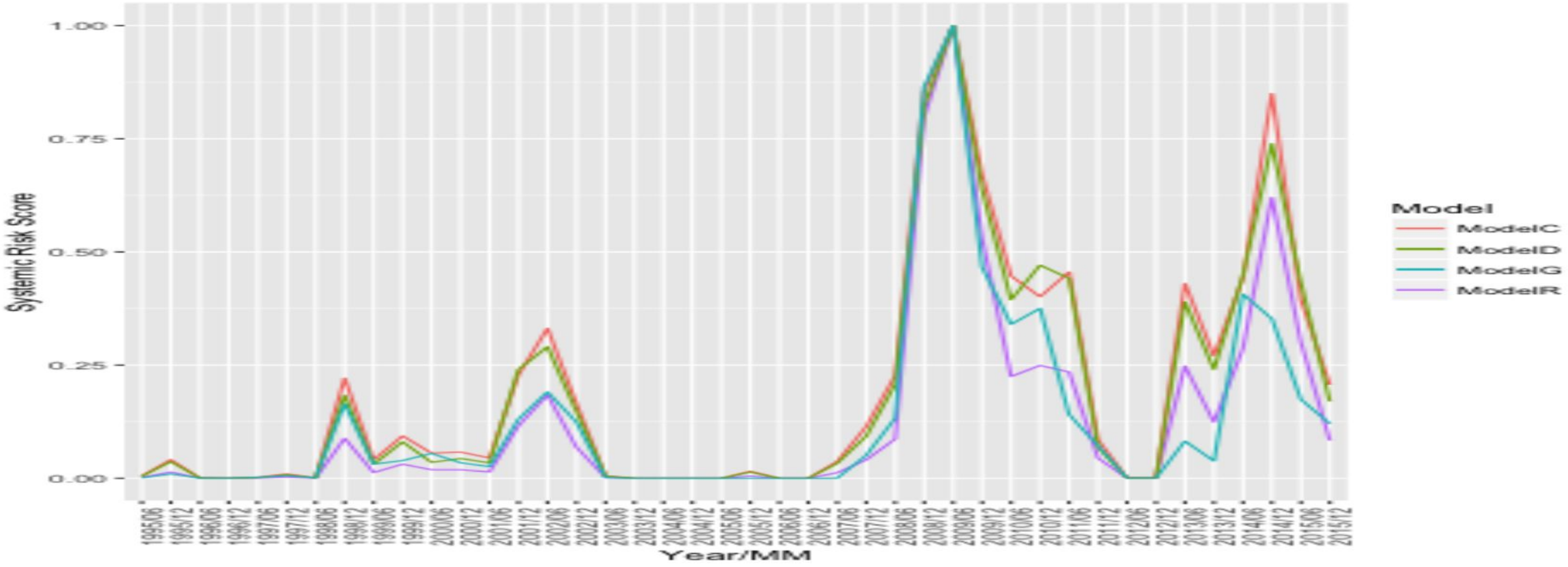


Figure: Systemic risk over time (1995–2015). The plot shows systemic risk computed from data for the top 20 financial institutions (by assets). All four models, **C**, **D**, **G**, and **R**, are represented. The average correlation between all four models' time series is 95%.

Risk by Institution

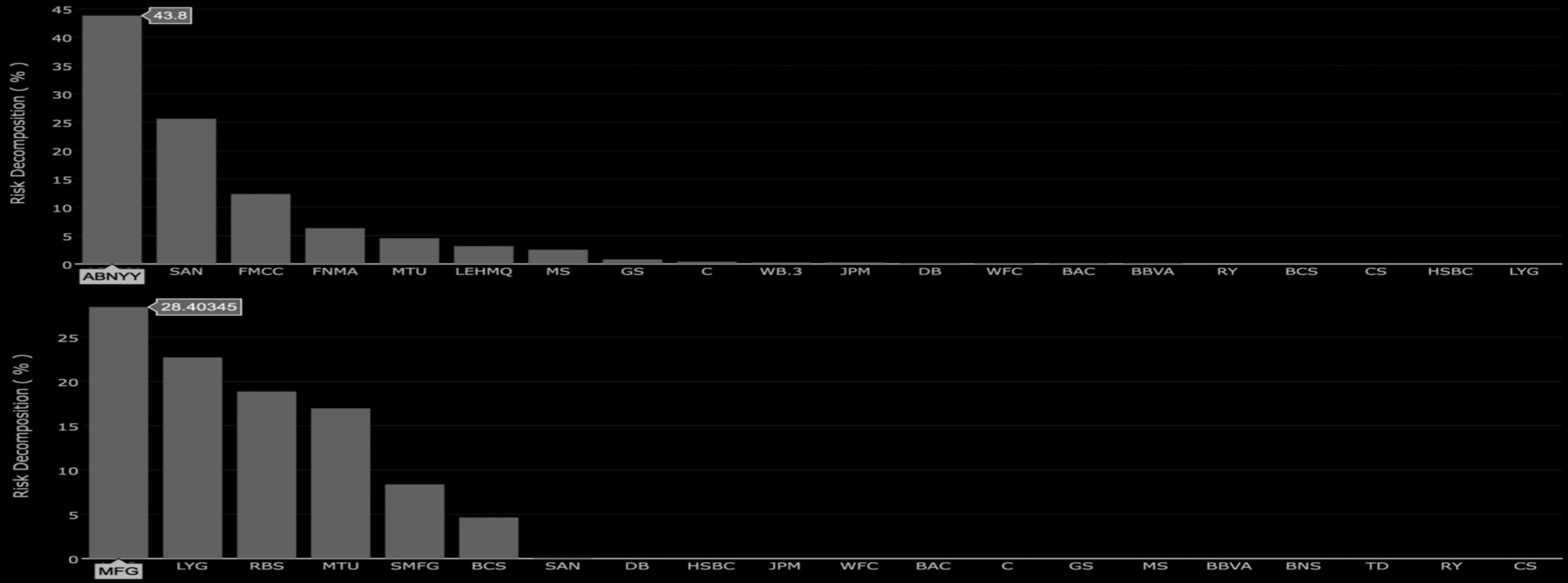


Figure: *Institution Risk Measures*. We display the institution risk measure using Model **G**. This decomposes the systemic risk by institution. The upper plot is for December 2007 and the lower one is for December 2014.

Top SIFIs over time

20051230	ABNYY	FNMA	FMCC	BCS	SAN	CS	LEHMQ	GS	C	JPM	
20060630	FNMA	LEHMQ	DB	GS	SAN	C	JPM				
20061229	LEHMQ	BSC.1	ABNYY	SAN	BBVA	MS	C	FNMA	DB	BAC	BCS
20070629	MTU	LEHMQ	MS	GS	ABNYY	FNMA					
20071231	ABNYY	SAN	FNMA	FMCC	MTU						
20080630	MTU	FNMA	FMCC	DB	C	MS					
20081231	MTU	FMCC	FNMA	C	BAC	DB					
20090630	MFG	FNMA	MTU	C	BAC	DB					
20091231	MFG	MTU	NMR	FNMA	FMCC	C					
20100630	MTU	MFG	NMR	CS	BBVA	DB	SAN				
20101231	MFG	MTU	BBVA	SAN	CS	DB					
20110630	MFG	MTU	MS	BBVA	SAN						
20111230	BAC	C	MS	DB	BBVA	WFC					
20120629	BAC	MS	C	JPM	BBVA						
20121231	BBVA	DB	BAC	C	MS	SAN					
20130628	LYG	MFG	SMFG	RBS	MS	BBVA					
20131231	LYG	SMFG	MFG	SAN	BCS	HSBC	JPM	DB	BAC	C	
20140630	LYG	BCS	SMFG	BBVA	BAC						
20141231	RBS	LYG	MFG	MTU	SMFG						
20150630	RBS	LYG	SAN	MTU	SMFG						
20151231	RBS	SAN	BCS	DB	MS	BAC					

Connectedness

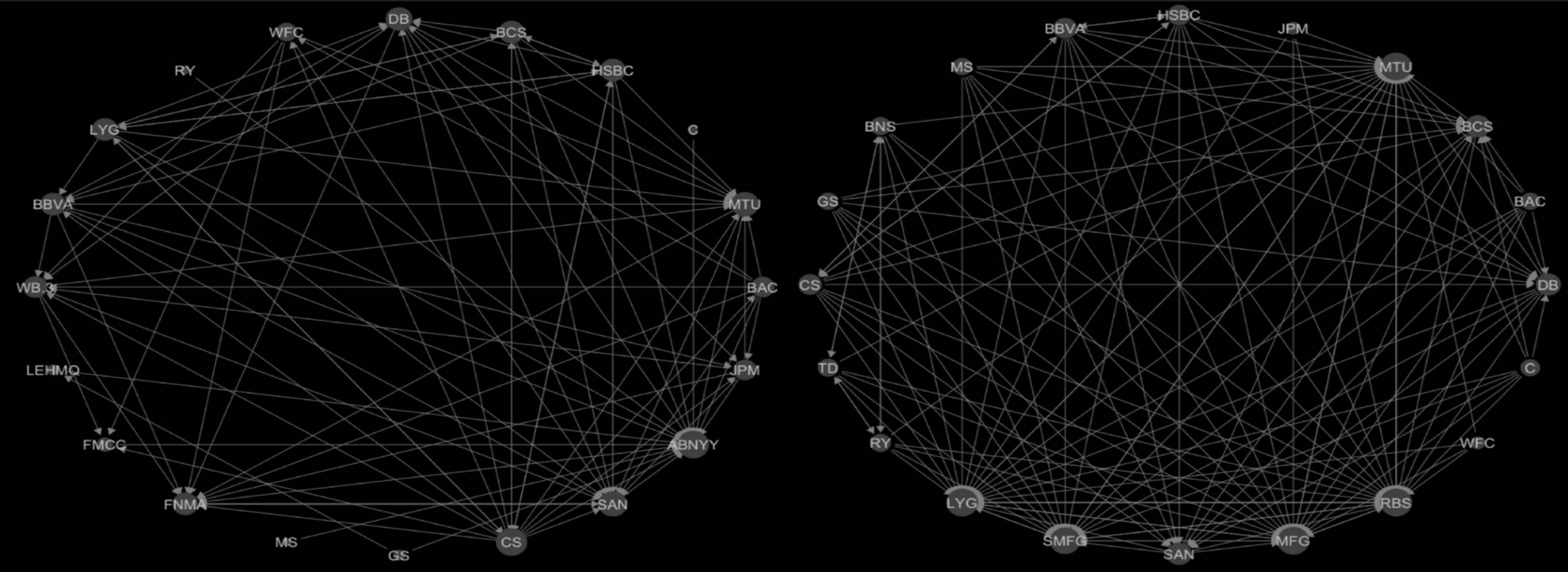


Figure: *Connectedness between institutions.* The links between institutions that are displayed have a connectedness risk measure above the threshold $K = 0.5$ according to Model **R**. The left plot is for December 2007, and the right one is for December 2014.

Top Risky Links

19950630	BT.2:C	C:BT.2	FFB:C	C:FFB	C:CMB.1
19951229	BPC.2:C	C:BPC.2	BPC.2:CMB.1	BPC.2:JPM	CMB.1:BPC.2
19960628	C:FNMA	FNMA:C	C:JPM	JPM:C	FMCC:JPM
19961231	LEHMQ:MS	MS:LEHMQ	C:FNMA	FMCC:FNMA	FNMA:C
19970630	GWF.:C	GWF.:FMCC	C:FNMA	GWF.:FNMA	BK:FNMA
19971231	LEHMQ:MS	MS:LEHMQ	SAN:NW.1	NW.1:SAN	BSC.1:SAN
19980630	SAN:C	BSC.1:C	C:SAN	BSC.1:FMCC	FMCC:C
19981231	LEHMQ:NW.1	SAN:NW.1	NW.1:LEHMQ	NW.1:SAN	NW.1:MS
19990630	BSC.1:SAN	LEHMQ:SAN	LEHMQ:MS	SAN:LEHMQ	SAN:C
19991231	SAN:NW.1	NW.1:SAN	LEHMQ:NW.1	NW.1:LEHMQ	BSC.1:NW.1
20000630	AFS:SAN	SAN:FNMA	SAN:FMCC	LEHMQ:MS	FMCC:FNMA
20001229	LEHMQ:MS	LEHMQ:SAN	SAN:LEHMQ	BSC.1:SAN	SAN:JPM
20010629	SAN:MS	MS:SAN	LEHMQ:MS	LEHMQ:SAN	SAN:LEHMQ
20011231	SAN:ABNYY	ABNYY:SAN	ABNYY:MS	MS:ABNYY	SAN:MS
20020628	IMI.2:ABNYY	SAN:ABNYY	ABNYY:SAN	IMI.2:SAN	ABNYY:IMI.2
20021231	SAN:ABNYY	IMI.2:SAN	ABNYY:SAN	SAN:JPM	SAN:IMI.2
20030630	ABNYY:JPM	JPM:ABNYY	ABNYY:FNMA	FNMA:ABNYY	LEHMQ:MS
20031231	FMCC:FNMA	FNMA:FMCC	JPM:FNMA	GS:MS	FNMA:JPM
20040630	DB:ABNYY	LEHMQ:MS	FNMA:DB	DB:FNMA	MS:LEHMQ
20041231	MS:GS	MS:LEHMQ	LEHMQ:MS	GS:MS	MS:FNMA
20050630	FMCC:FNMA	FNMA:FMCC	LEHMQ:MS	MS:GS	MS:LEHMQ

20051230	FMCC:ABNYY	ABNYY:FMCC	ABNYY:BCS	ABNYY:JPM	ABNYY:CS
20060630	FNMA:DB	DB:FNMA	LEHMQ:GS	GS:LEHMQ	FNMA:C
20061229	BSC.1:LEHMQ	LEHMQ:BSC.1	LEHMQ:MS	DB:C	DB:ABNYY
20070629	LEHMQ:MTU	MS:MTU	MTU:MS	LEHMQ:GS	MTU:LEHMQ
20071231	SAN:ABNYY	ABNYY:SAN	FMCC:ABNYY	FNMA:ABNYY	FNMA:SAN
20080630	FNMA:MTU	FMCC:MTU	MTU:FNMA	MTU:FMCC	FNMA:FMCC
20081231	FMCC:MTU	FNMA:MTU	C:MTU	MTU:DB	FMCC:DB
20090630	MFG:MTU	FNMA:MTU	FNMA:MFG	MTU:MFG	C:MTU
20091231	MFG:MTU	NMR:MTU	FNMA:MTU	FMCC:MTU	MTU:MFG
20100630	MFG:MTU	MTU:MFG	NMR:MTU	NMR:MFG	BBVA:MTU
20101231	MFG:MTU	MTU:MFG	BBVA:MTU	BBVA:MFG	SAN:MTU
20110630	MFG:MTU	MTU:MFG	MS:MTU	MS:MFG	BBVA:MTU
20111230	C:BAC	BAC:C	MS:BAC	BAC:DB	MS:DB
20120629	MS:BAC	C:BAC	BAC:C	BAC:JPM	JPM:BAC
20121231	BBVA:BAC	C:DB	MS:BAC	DB:C	BAC:BBVA
20130628	LYG:MFG	SMFG:MFG	MFG:SMFG	LYG:SMFG	MFG:LYG
20131231	LYG:SMFG	SMFG:LYG	SMFG:MFG	LYG:MFG	MFG:SMFG
20140630	LYG:BCS	SMFG:BCS	LYG:SMFG	BCS:SMFG	BCS:LYG
20141231	RBS:MTU	LYG:MTU	MFG:MTU	SMFG:MTU	RBS:MFG
20150630	LYG:RBS	SAN:RBS	RBS:SAN	LYG:SAN	RBS:LYG
20151231	RBS:SAN	SAN:RBS	SAN:BCS	RBS:BCS	BCS:SAN

Simplified Score for Systemic Risk

(For India, and Emerging Markets)

$$S = \frac{1}{n}\sqrt{C^\top \cdot A \cdot C} \geq 0$$

banks (normalization across time)

Adjacency matrix

$A(i,j)$ in (0,1)
$A(i,i) = 1$

Vector of credit risk scores {PD, rating, etc}. Higher = more risk

$C(i) > 0$

$S(C,A)$ is linear homogenous in C

Apply Euler's Formula

$$S = \frac{\partial S}{\partial C_1} C_1 + \frac{\partial S}{\partial C_2} C_2 + \ldots + \frac{\partial S}{\partial C_n} C_n = \sum_{i=1}^{n} \frac{\partial S}{\partial C_i} C_i$$

Risk Contribution

Risk Increment

$$\frac{\partial S}{\partial C} = \frac{1}{2n^2 S}\left[A \cdot C + A^{\top} \cdot C\right] \in \mathcal{R}^n$$

Closed vector form makes computation facile.

Risk Decomposition in closed form

$$\frac{\partial S}{\partial C_i} \cdot C_i = \frac{1}{2n^2 S} \cdot [A \cdot C + A^\top \cdot C] \odot C$$

$$S = \left[\frac{\partial S}{\partial C_i} \cdot C_i\right] \cdot 1$$

Total Systemic Risk Score

Data (India)

838 Indian firms from the Datastream Database -- explicitly identified as financial firms, are active firms, and have common equity that are major securities trading in a primary exchange in the local (Indian) market.

Reject (a) non-financial firms, (b) inactive (delisted) firms, (c) firms with only preferred stock, (d) foreign firms trading in Indian exchanges, and (e) Indian firms trading exclusively in either a minor exchange in India or a foreign exchange, (f) reject firms with less than 125 active trading days (six months).

Table 1: Bank Identification Data. This table contains a sampling of the bank name, and various other identification information.

MNEMONIC	ISIN	SEDOL	NAME	INDUSTRY	GVKEY	SIC
IN:ALN	INE428A01015	6708289	ALLAHABAD BANK	Bank	272772	6020
IN:CKB	INE476A01014	6580012	CANARA BANK	Bank	255701	6020
IN:ICG	INE090A01021	BSZ2BY7	ICICI BANK	Bank	223148	6020
IN:SBK	INE062A01020	BSQCB24	STATE BANK OF INDIA	Bank	203666	6020
IN:UBI	INE692A01016	6579634	UNION BANK OF INDIA	Bank	257156	6020
IN:TYA	INE865C01022	B0HXGC5	ADITYA BIRLA MONEY	Broker-Dealer	289796	6211
IN:ERE	INE143K01019	B56JDC8	ESSAR SECURITIES	Broker-Dealer	293675	6200
IN:KGC	INE929C01018	B03K039	K L G CAPITAL SERVICES	Broker-Dealer	289851	6211
IN:NKK	INE526C01012	B03J1D3	NIKKI GLOBAL FINANCE	Broker-Dealer	296350	6211
IN:UEI	INE519C01017	B5NH8B9	SUMMIT SECURITIES	Broker-Dealer	296724	6211
IN:BFS	INE918I01018	B2QKWK1	BAJAJ FINSERV	Insurer	288902	6300

Filtering the sample

- Based on International Securities Identification Number (ISIN) and/or Stock Exchange Daily Official List (SEDOL) identifiers, we match the Indian financial firms to the Compustat Global Database and obtain the corresponding GVKEYs and Standard Industrial Classification (SIC) codes.
- Based on SIC codes, we categorize firms as (a) Banks (SIC: 6000-6199), (b) Broker-Dealers (SIC: 6200-6299), (c) Insurers (SIC: 6300-6499), and (d) Others (all other SICs).
- Eliminate firms with no SIC code and firms classified as others (which include financial subsidiaries of non-financial corporations and specialized investment vehicles such as funds, REITs and securitized assets).
- Final screened sample consists of 387 Indian financial institutions -- 193 Banks, 191 Broker-Dealers and 3 Insurers.

Table 2: Industry groups, sample count.

	TOTAL	NUMBER WITH VALID			
INDUSTRY	NUMBER	RETURNS	RATINGS	DTD	PD
Bank	193	193	20	176	177
Broker-Dealer	191	191	0	177	177
Insurer	3	3	0	2	2
Total	387	387	20	355	356

Network Construction

Billio, Getmansky, Lo, Pelizzon (2012)

$$r_{j,t} = a + b \cdot r_{j,t-1} + c \cdot r_{i,t-1} + e_{j,t}$$

return

Significant,
p-value <0.01

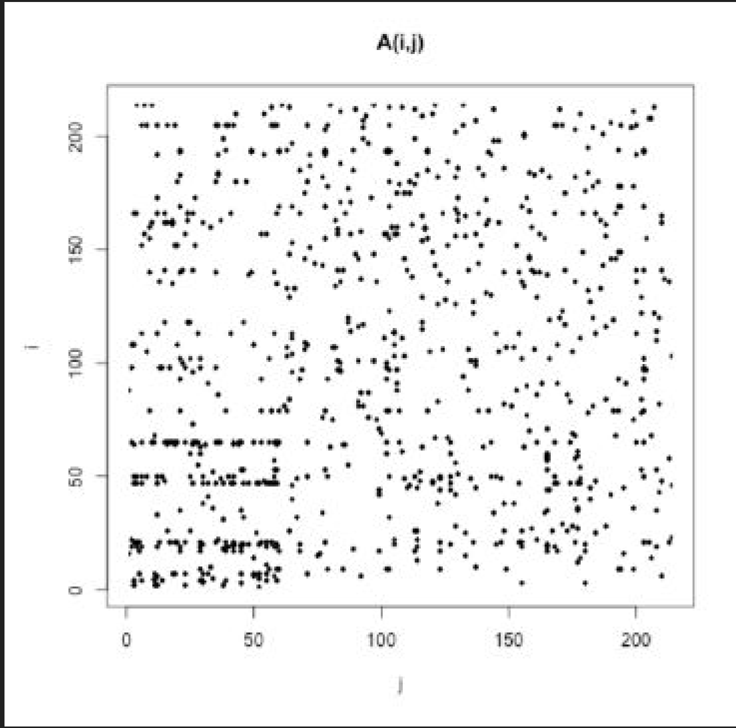
$$r_{j,t} = a + b \cdot r_{j,t-1} + c \cdot r_{i,t-1} + d \cdot r_{EW,t-1} + e_{j,t}$$

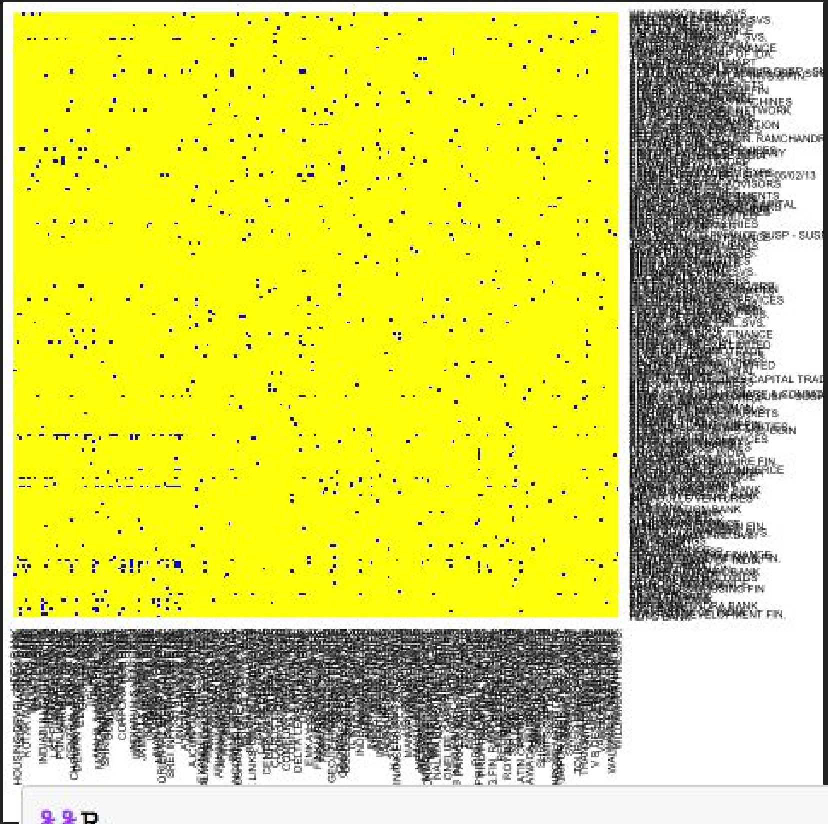
Lookback period = 130 days

Equally weighted return

Exclude banks that have more than ⅓ days with zero returns

Adjacency Matrix





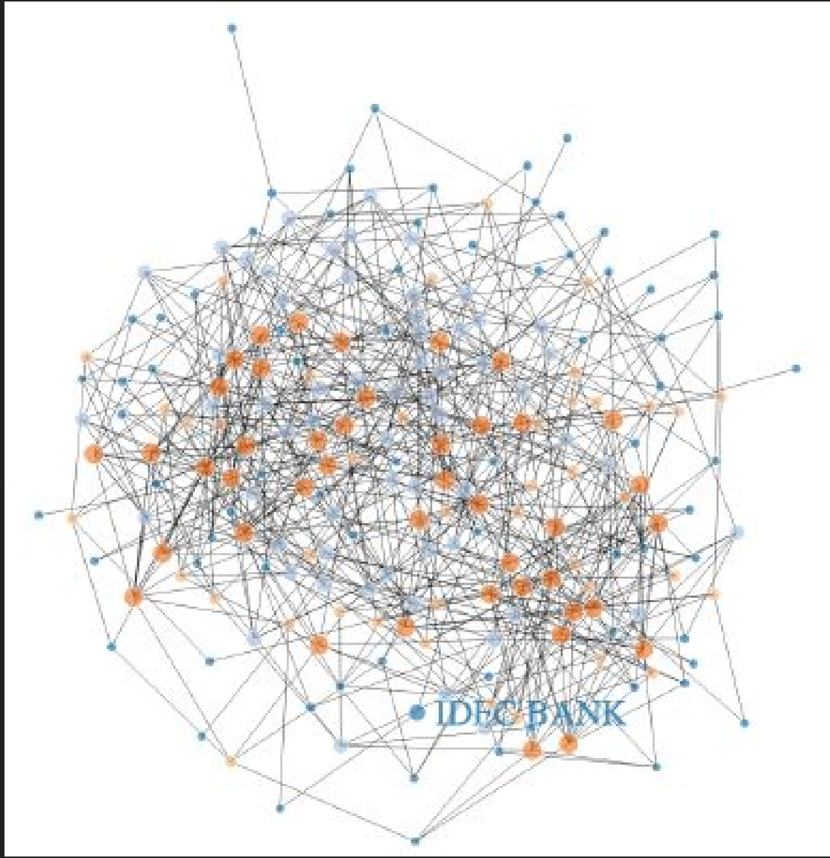
```
Pct of possible directed links:[1] 0.01827476
No of banks: [1] 214
No of regressions: [1] 45582
```

```
%%R
system.time(Amat <- genAdjMat(df_rets))
```

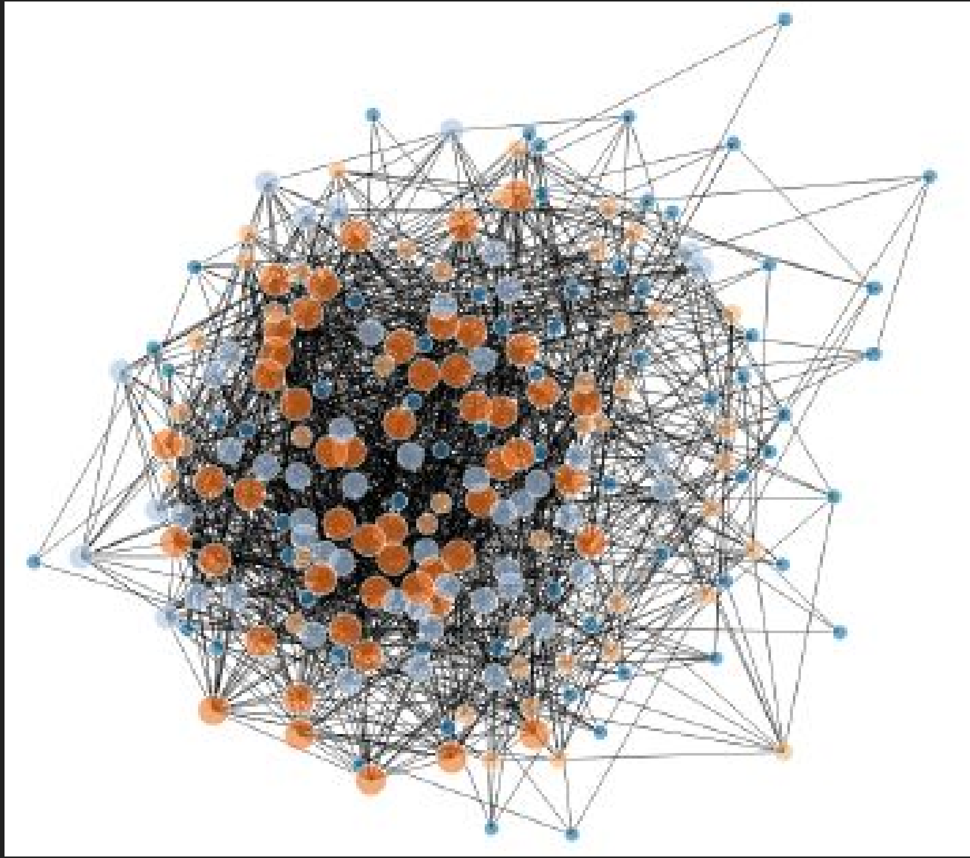
```
   user  system elapsed
 31.741   0.173  31.832
```

Network

P = 0.025



P = 0.050



Centrality

$$c_i = \sum_{j=1}^{n} A_{ij} c_j, \forall i$$

Eigenvalue Centrality

$$b_v = \sum_{\substack{i,j \\ i \neq j \\ i \neq v \\ j \neq v}} \left[\frac{g_{ivj}}{g_{ij}} \right]$$

Betweenness Centrality

Distribution of Centrality

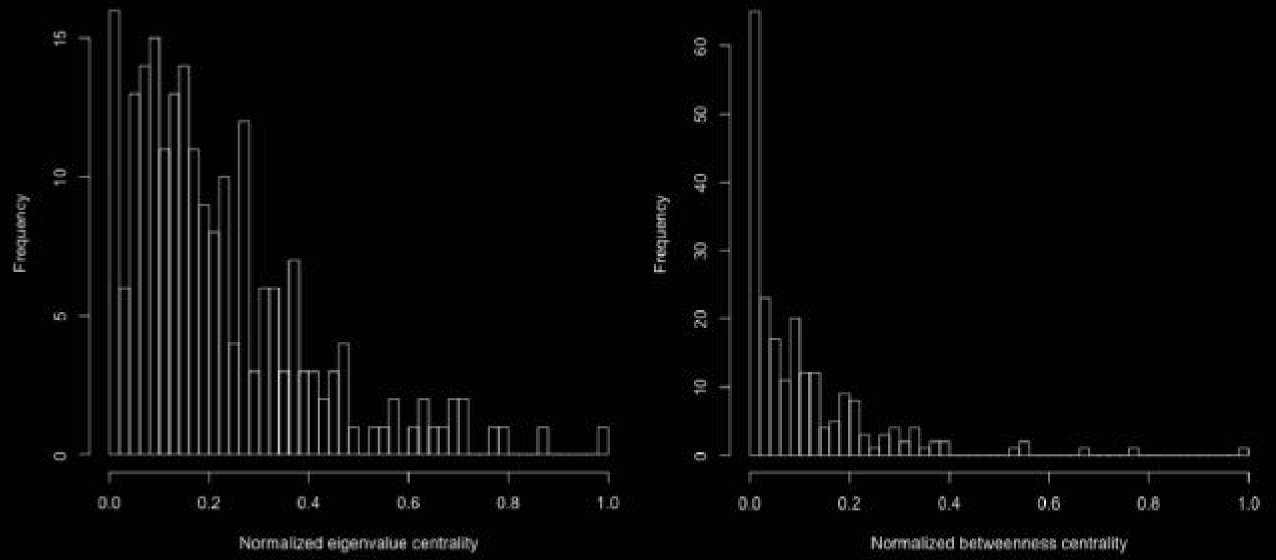


Figure 3: Distribution of Eigenvalue Centrality and Betweenness Centrality for all the nodes in the network, for Q4 2016. The centrality is normalized, so that it ranges from 0 to 1.

Top 20 Banks, Q4 2016

Bank	EVCENT	BCENT
PRITI MERCANTILE COMPANY	1.000000	0.217527
DHANLAXMI BANK	0.879521	0.289056
BANK OF MAHARASHTRA	0.797941	0.033656
INDIAN BANK	0.771766	0.033376
UCO BANK	0.710815	0.082385
UNITED BANK OF INDIA	0.708690	0.033280
RR FINL.CONSULTANTS	0.694695	0.135618
UNION BANK OF INDIA	0.687011	0.047011
CENTRAL BANK OF INDIA	0.675282	0.667370
IFCI	0.656577	0.053150
P N B GILTS	0.633888	0.248902
GLOBAL CAPITAL MARKETS	0.629967	0.375415
J M FINANCIAL	0.601884	0.132343
CORPORATION BANK	0.564888	0.000000
INTER GLOBE FINANCE	0.562848	0.533449
STATE BANK OF INDIA	0.548690	0.175723
BANK OF BARODA	0.539016	0.009665
S P CAPITAL FINANCING	0.497271	0.022460
SOUTH INDIAN BANK	0.476020	0.091634
TRANSWARRANTY FINANCE	0.472221	0.072575

Number of banks in the network

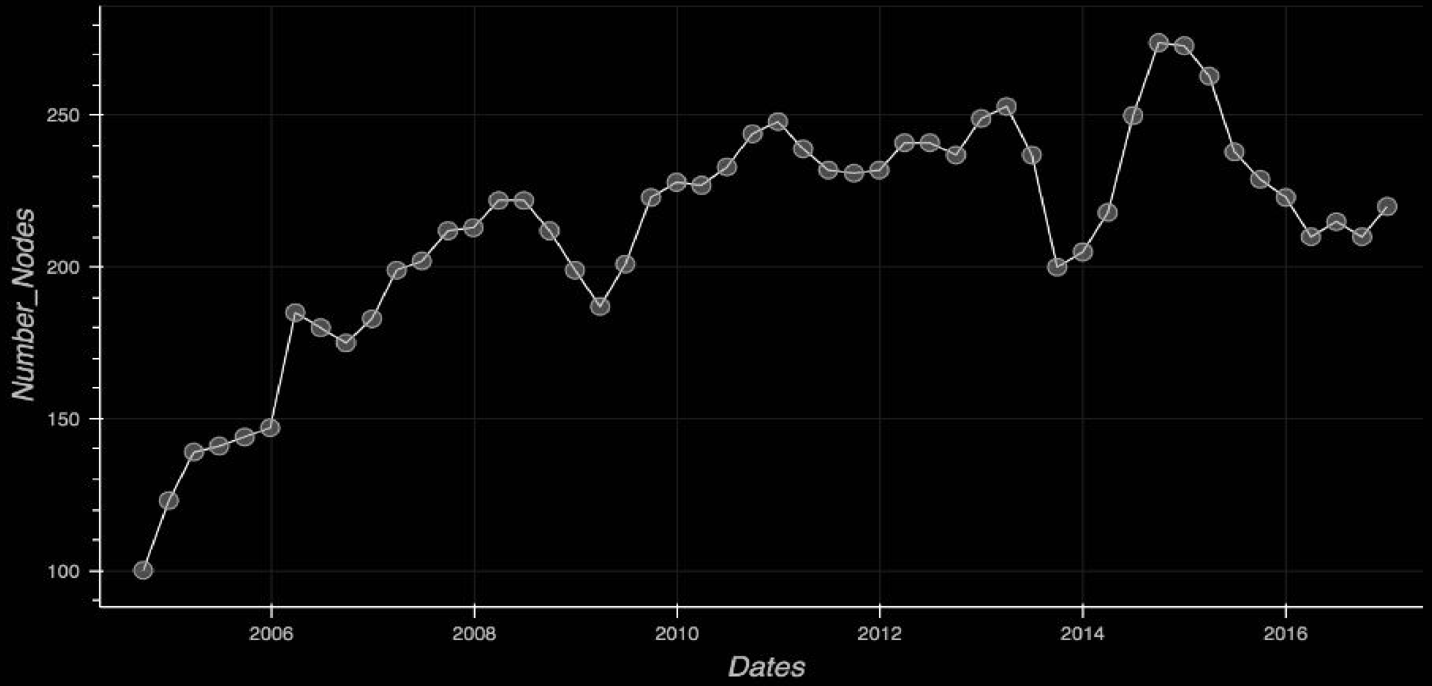
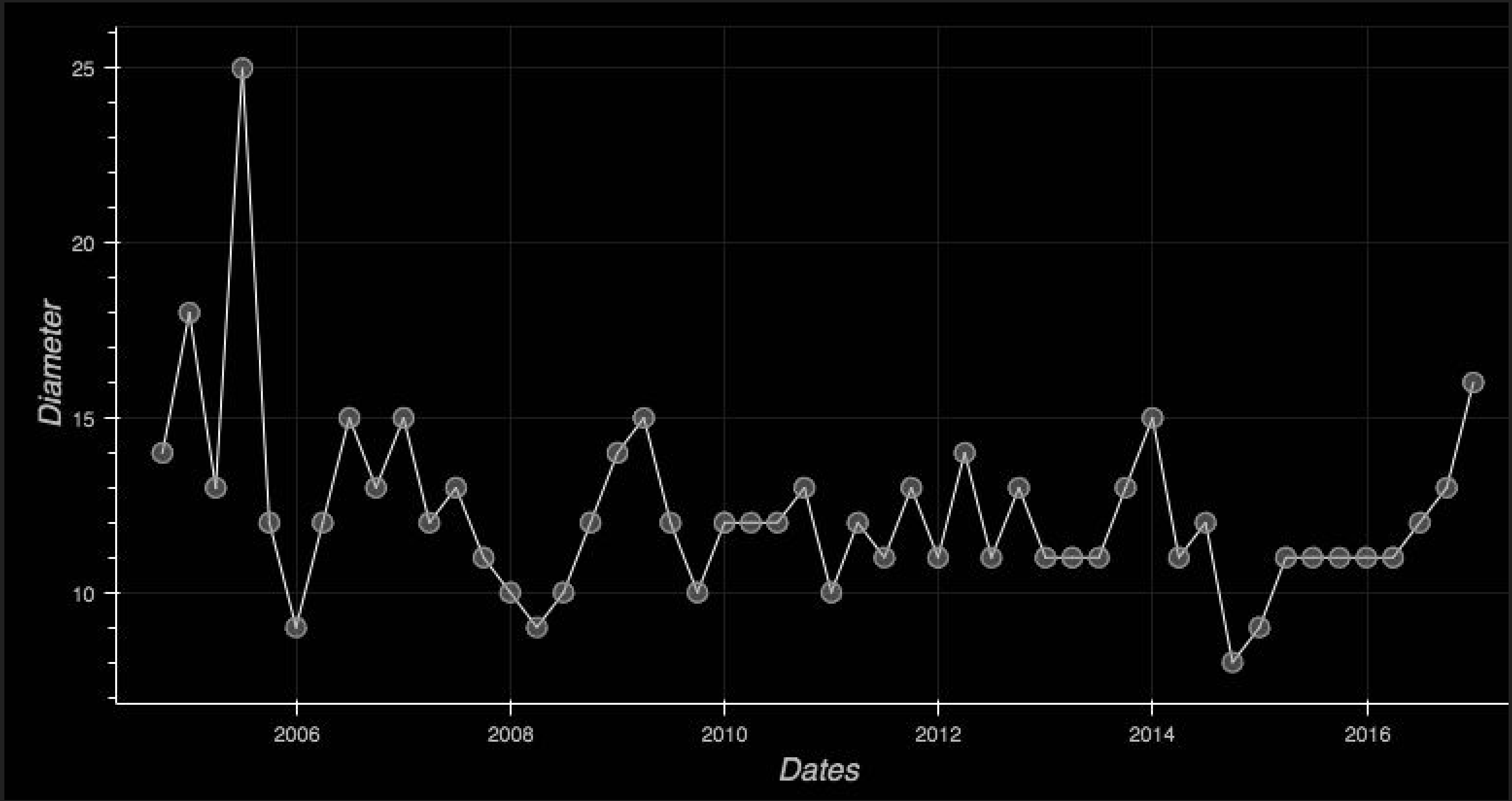
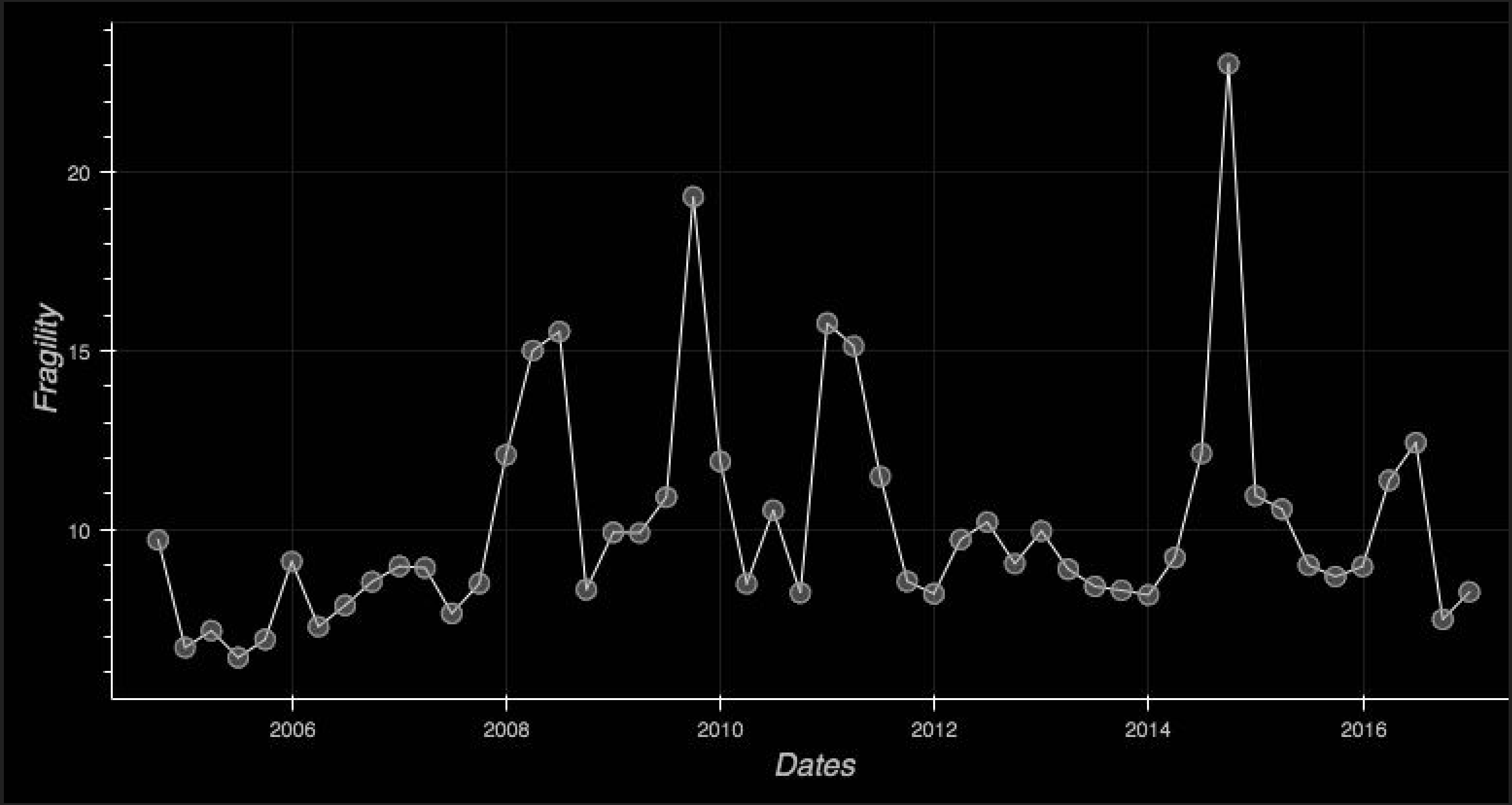


Figure 4: The number of banks in the network for all quarters between Q3 2004 and Q4 2016.

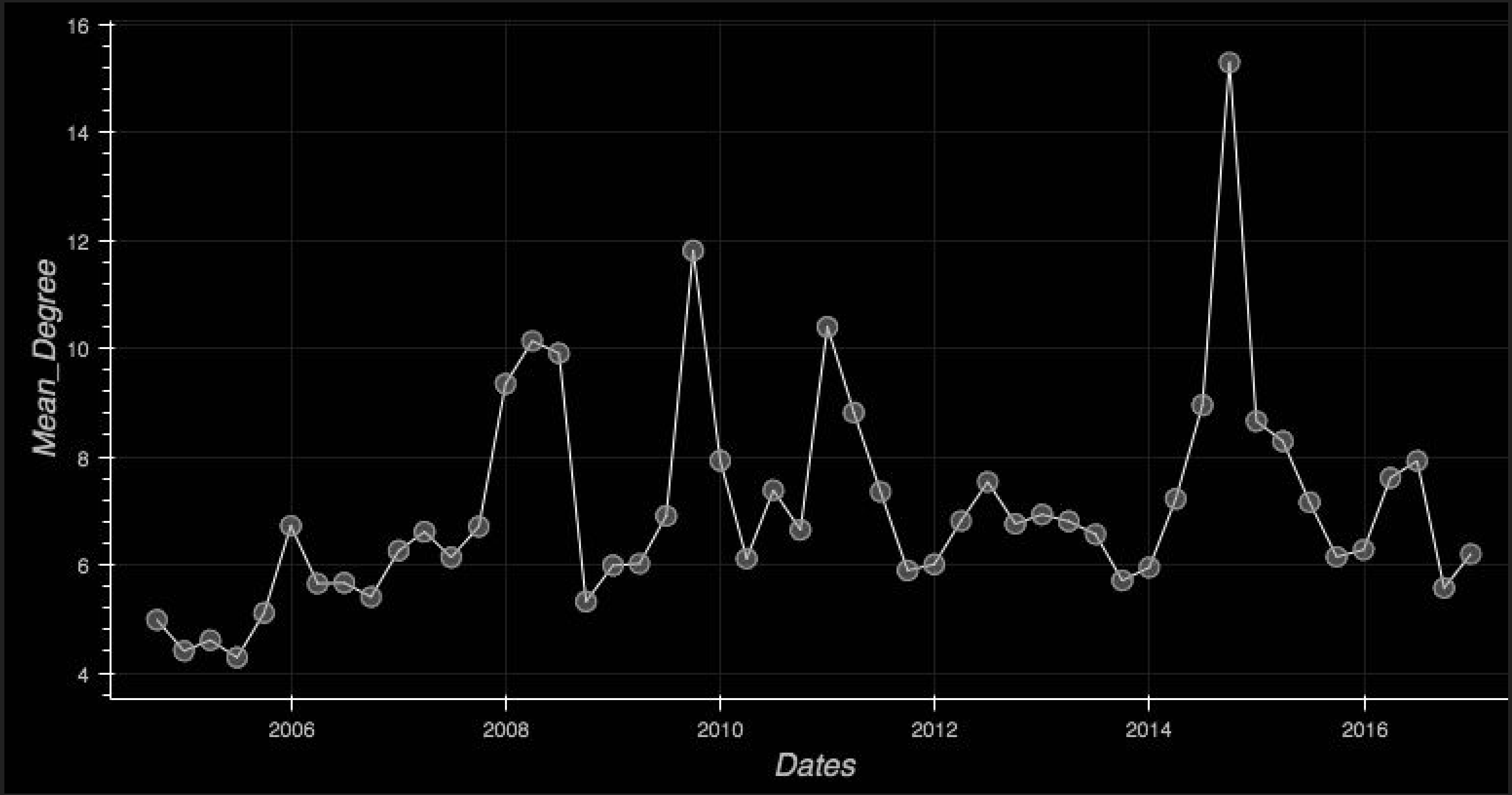
Diameter



Fragility

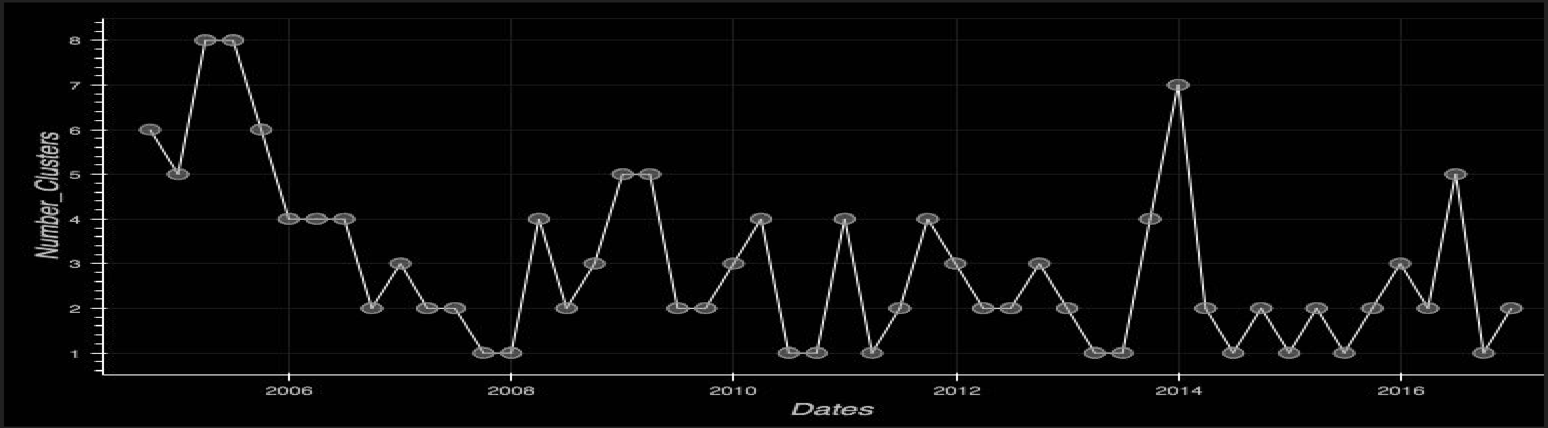


Degree

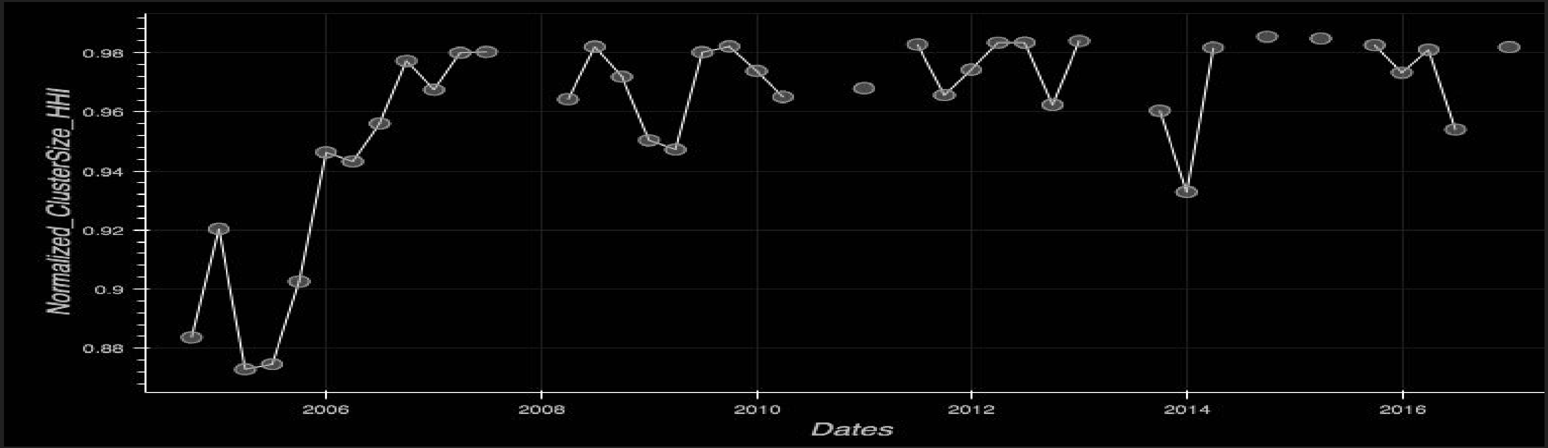


16
14
12
10
8
6
4
Mean_Degree
2006
2008
2010
2012
2014
2016
Dates

Clusters

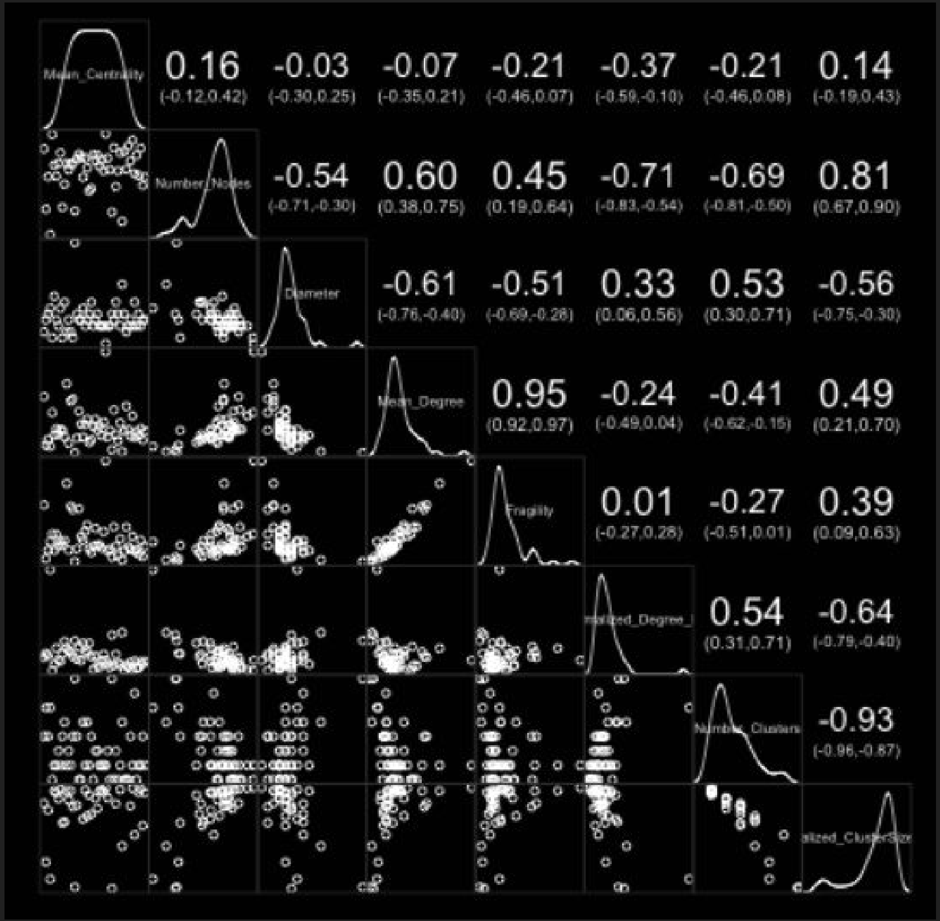


8
7
6
5
4
3
2
1
Number_Clusters
2006
2008
2010
2012
2014
2016
Dates



0.98
0.96
0.94
0.92
0.9
0.88
Normalized_ClusterSize_HHI
2006
2008
2010
2012
2014
2016
Dates

Correlations



Mean Centrality

Number of Nodes

Diameter

Mean Degree

Fragility

Normalized degree Herfindahl Index

Number of Clusters

Normalized cluster size Herfindahl

Probabilities of Default (PDs)

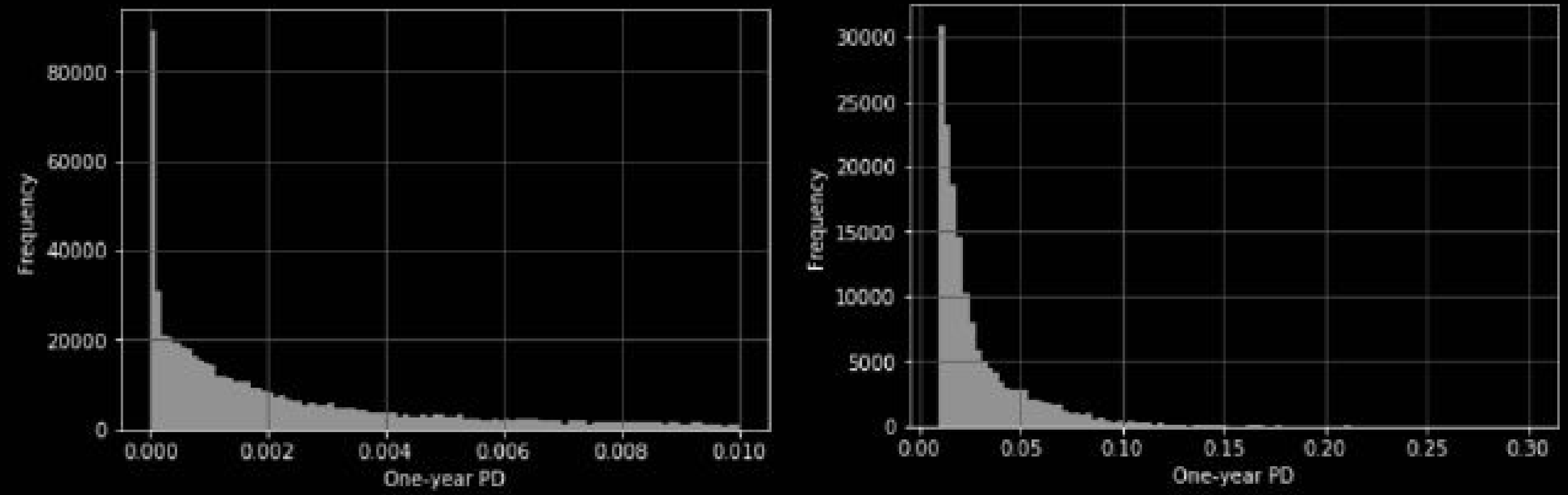


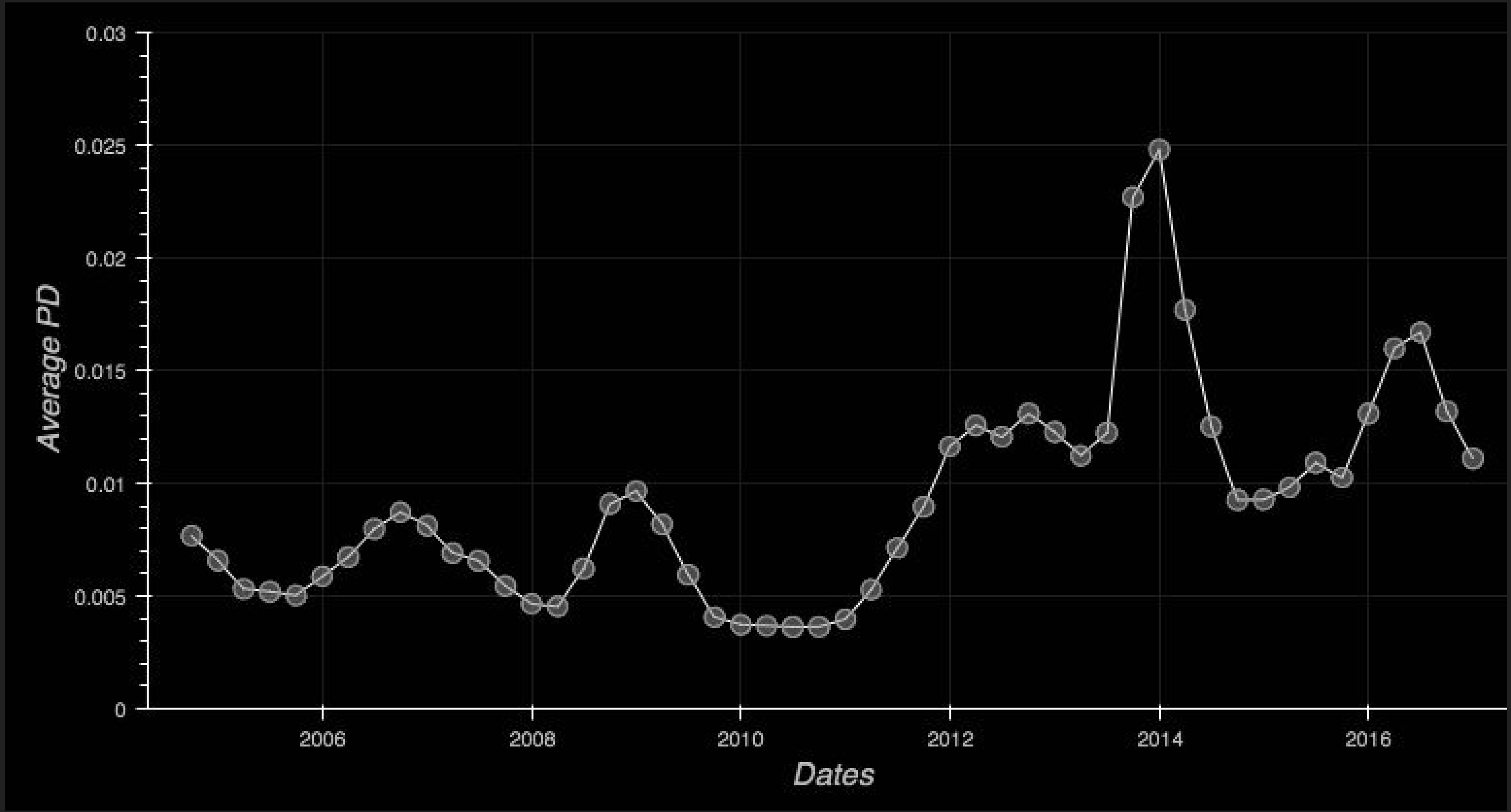
Figure 11: Distribution of PDs of all Indian FIs from 2004 to 2016. The first plot is the histogram of PDs that lie in the interval $(0, 0.01)$, and the second in the interval $(0.01, 0.30)$.

Highest PD = 26.36%

$C = 1 + 30\ PD$

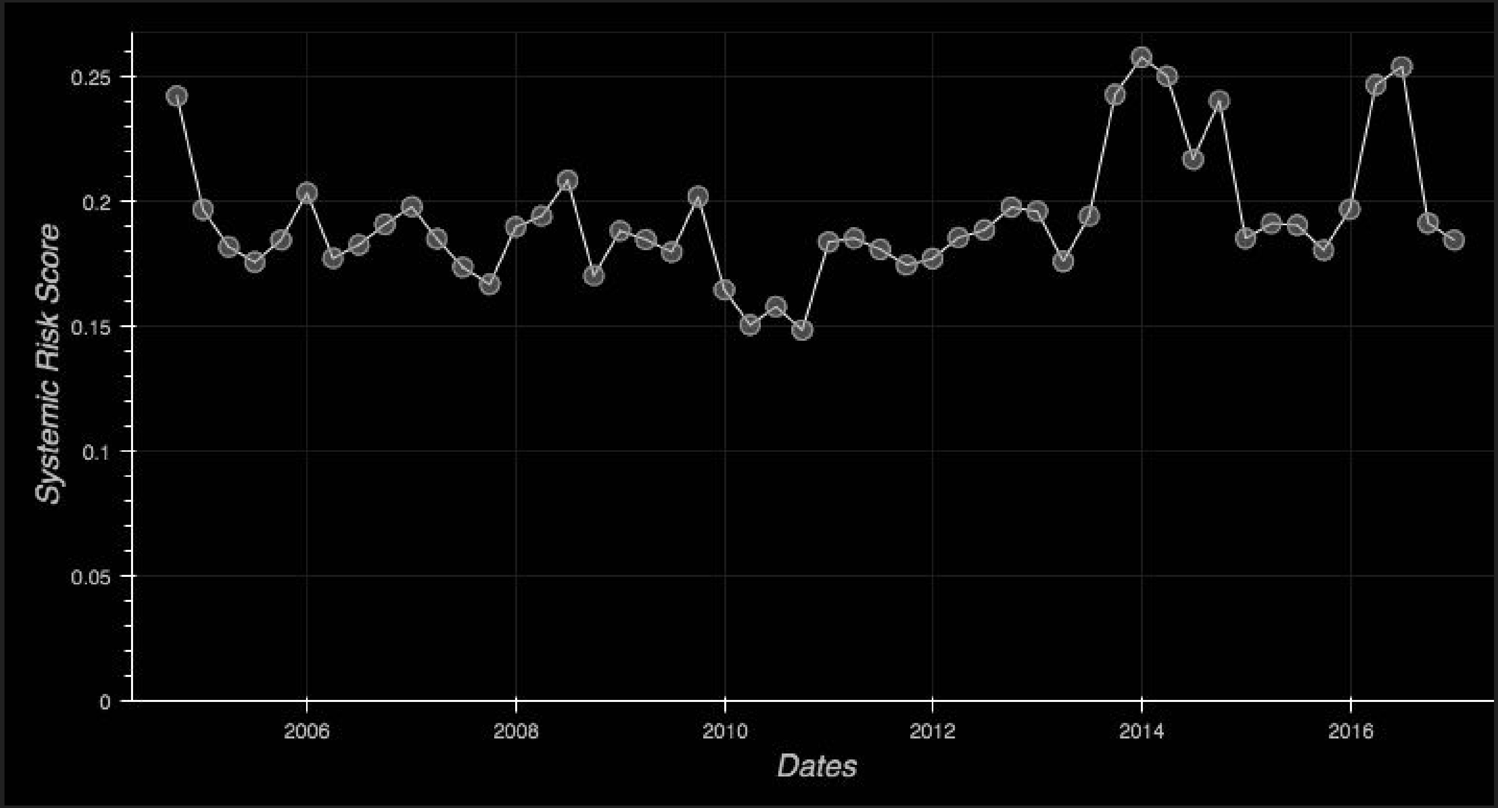
Since $PD < 0.30$, C lies in $(0,10)$

Mean PDs



0.03
0.025
0.02
0.015
0.01
0.005
0
Average PD
2006
2008
2010
2012
2014
2016
Dates

Systemic Risk Score (*S*)



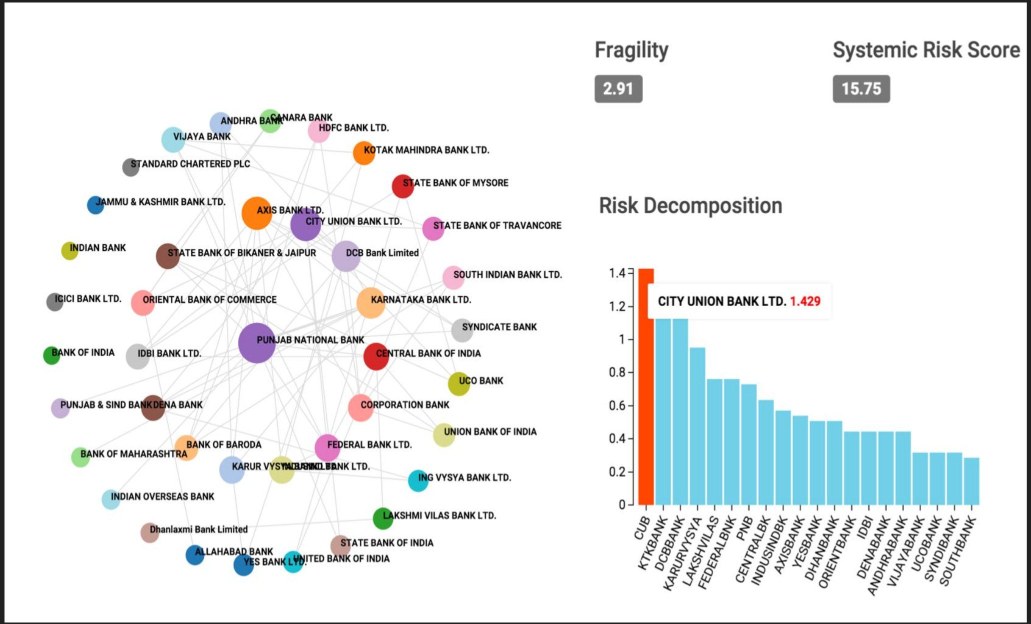
Correlation of PDs and S = 69.7%

Risk Contributions of top 20 banks

	2005-Q1		2016-Q1	
	Bank Name	Risk Decomp	Bank Name	Risk Decomp
1	PRIME SECURITIES	2.705139	BANK OF MAHARASHTRA	2.222866
2	STATE BANK OF INDIA	2.476634	UCO BANK	1.698109
3	UCO BANK	2.438924	POWER FINANCE	1.437113
4	CORPORATION BANK	1.882045	UNITED BANK OF INDIA	1.410672
5	GIC HOUSING FINANCE	1.771204	STATE BK.OF BIN.& JAIPUR SUSP - SUSP.15/03/17	1.388539
6	I N G VYSYA BANK SUSP - SUSP.15/04/15	1.696898	DENA BANK	1.343904
7	UNION BANK OF INDIA	1.607279	STATE BANK OF INDIA	1.335314
8	IFCI	1.597618	INDIAN OVERSEAS BANK	1.331388
9	SUNDARAM FINANCE	1.569000	BANK OF TRAVANCORE SUSP - SUSP.15/03/17	1.309907
10	P N B GILTS	1.492469	CIL SECURITIES	1.282169
11	DHANLAXMI BANK	1.328556	COMFORT COMMOTRADE	1.137495
12	JAMMU & KASHMIR BANK	1.322932	BANK OF BARODA	1.093183
13	INDIABULLS FINL.SVS. SUSP - SUSP.18/03/13	1.215547	ANDHRA BANK	1.066791
14	DEWAN HOUSING FINANCE	1.198211	DEWAN HOUSING FINANCE	0.994385
15	ALMOND GLOBAL SECURITIES	1.195593	ORIENTAL BK.OF COMMERCE	0.917884
16	DENA BANK	1.194755	JAGSONPAL FIN.& LSG.	0.917517
17	ANDHRA BANK	1.193921	ELIXIR CAPITAL	0.873306
18	INDUSIND BANK	1.163923	MAHA.& MAHA.FINL.SVS.	0.871946
19	MARGO FINANCE	1.163827	CUBICAL FINANCIAL SVS.	0.855089
20	UNITED CREDIT	1.148539	VAX HOUSING FINANCE	0.852056
	TOTAL	31.36301	TOTAL	24.33963

Explaining quarterly systemic risk

Network Link basis Ind. Variables	$p = 0.025$ (1)	$p = 0.025$ (2)	$p = 0.025$ (3)	$p = 0.01$ (4)
Intercept	0.1580 26.93	0.1498 9.57	0.0087 0.40	0.1571 3.69
Mean PD	3.8253 6.728		5.261 16.89	3.879 16.34
Mean Degree		0.0041 2.30	0.0193 3.56	0.0212 2.73
Degree HHI		6.8740 2.42	6.9690 2.66	4.3210 3.05
Mean Betw. Centrality			−0.0001 −4.87	−0.0001 −0.94
Diameter			0.0003 0.51	−0.0002 −0.90
Fragility			−0.0077 −2.29	−0.0049 −1.54
Num Clusters			−0.0013 −0.45	−0.0011 −2.63
Cluster HHI			0.0471 0.22	−0.0923 −2.50
R^2	0.485	0.160	0.931	0.903
Adj R^2	0.475	0.124	0.912	0.884
F-stat p-value	0.000	0.017	0.000	0.000



Previous system for systemic risk

Thank you !!