BITS Pilani
Pilani Campus

Mechatronics
(Merged - DEZG516/DMZG511/ESZG511)

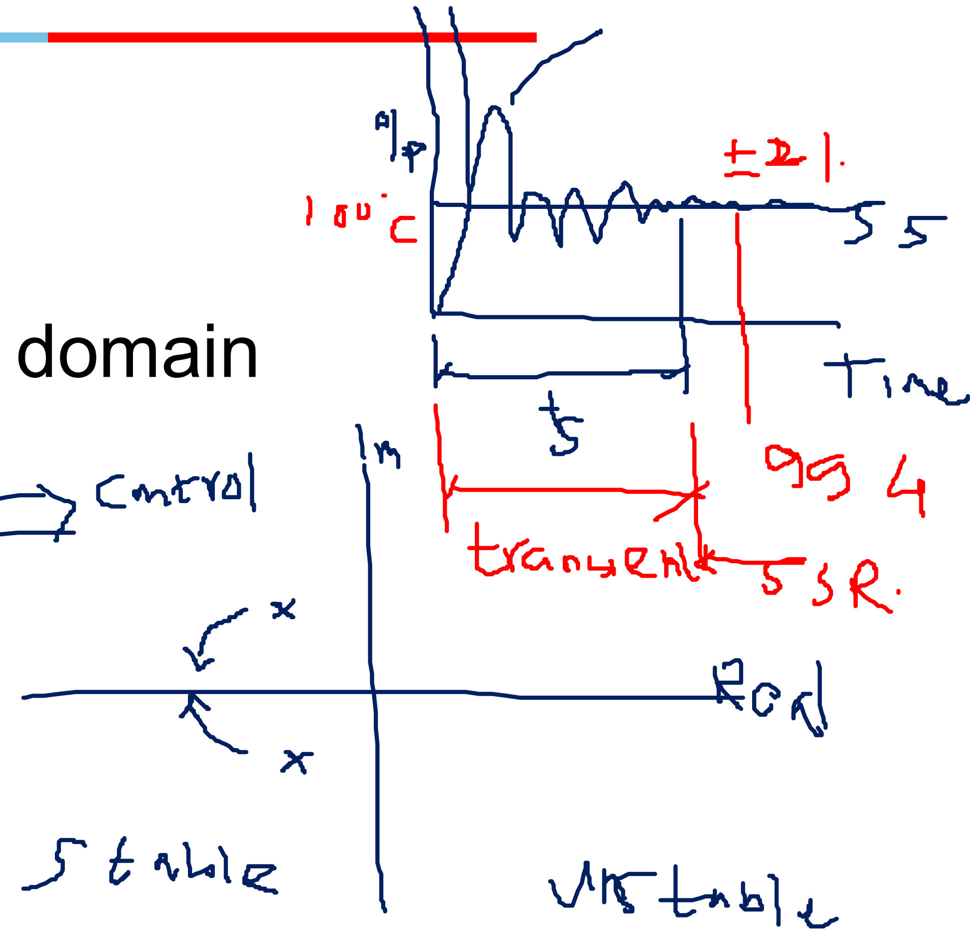
Lecture

Closed Loop Controllers

Till Now we have learnt

- System Modelling
- System analysis
  - Time and frequency domain
  - Stability
  - Poles and roots
  - Phasors

Control



Recap- system performance

- We studied the responses of typical first order and second order closed-loop systems to different inputs.

- First order system :

$$G_1(s) = \frac{1}{1+Ts}$$

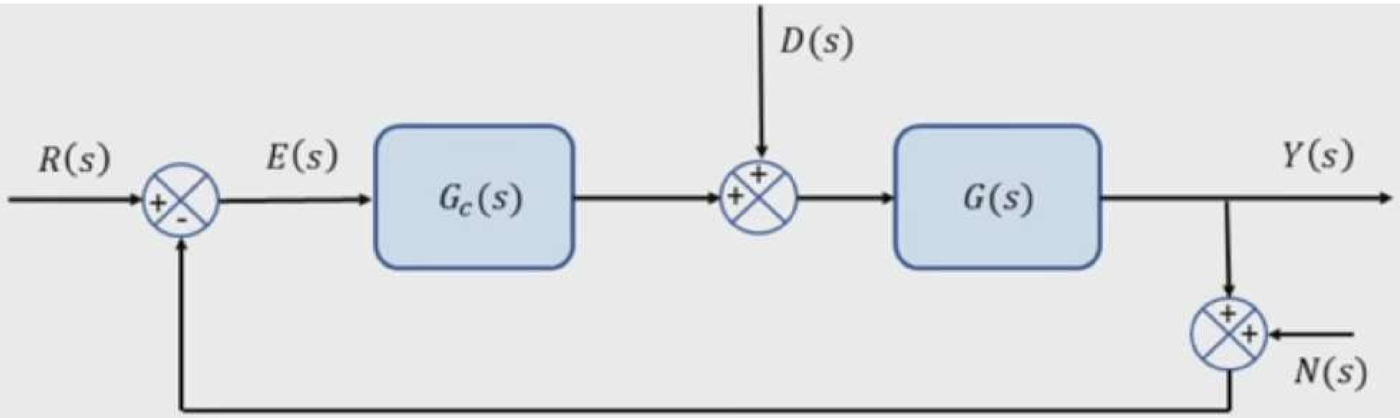
- Second order system :

$$G_2(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- We identified the features that captured the nature of these responses. For the second order system -

- Rise time $t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}\zeta}{\omega_n\sqrt{(1-\zeta^2)}}$
- Percentage Overshoot $M_p = 100e^{-\frac{\zeta\pi}{\sqrt{(1-\zeta^2)}}}\%$
- Delay time $t_d = \frac{1+0.7\zeta}{\omega_n}$
- Settling time $t_s = 4\tau = \frac{4}{\zeta\omega_n}$ (2% tolerance)

Recap



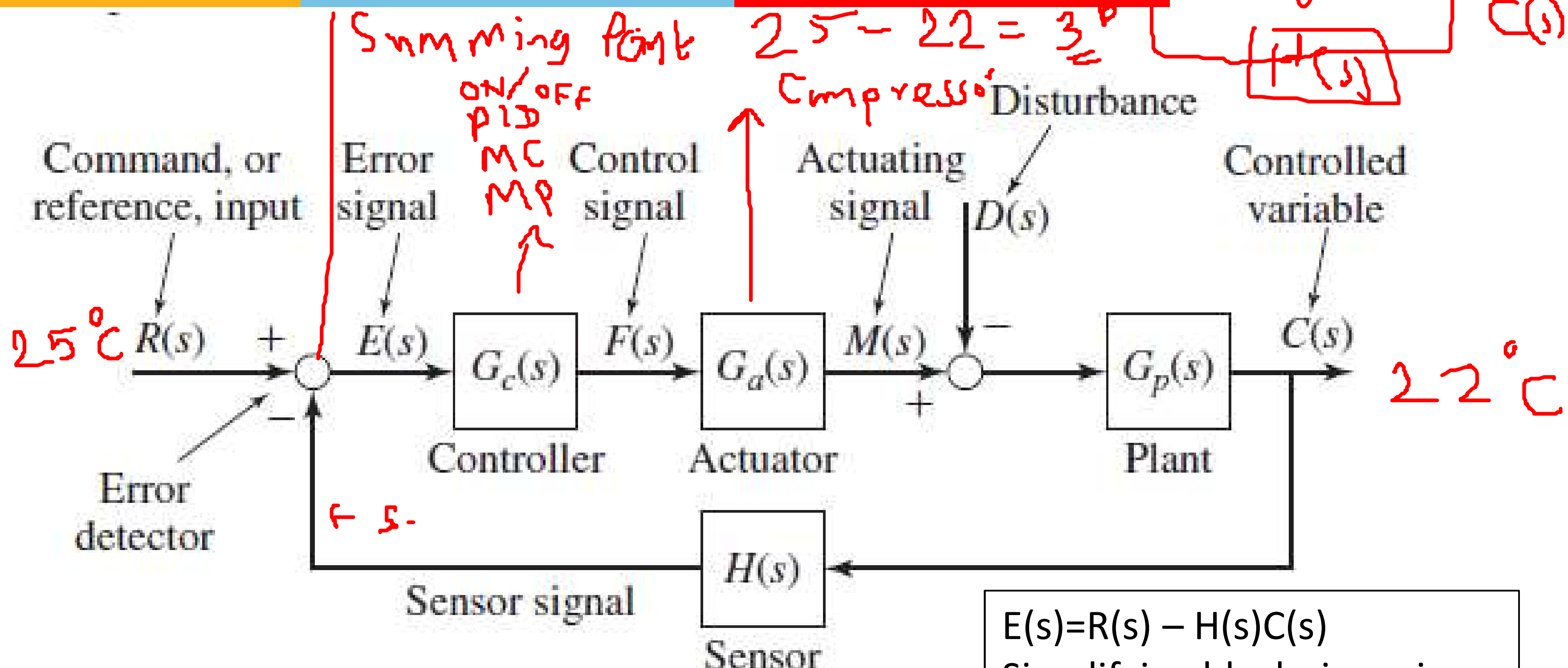
We have seen that feedback control is capable of

- Improving the stability
- Meeting the performance specifications
- Decreasing the sensitivity to parametric variations
- Improving disturbance rejection
- Attenuating the measurement noise

$R(s)$ is the reference input
$Y(s)$ is the controlled output
$D(s)$ is the disturbance input
$N(s)$ is the measurement noise

We especially care about $E(s)$, the difference between the reference input and the measured output

Control system terminology



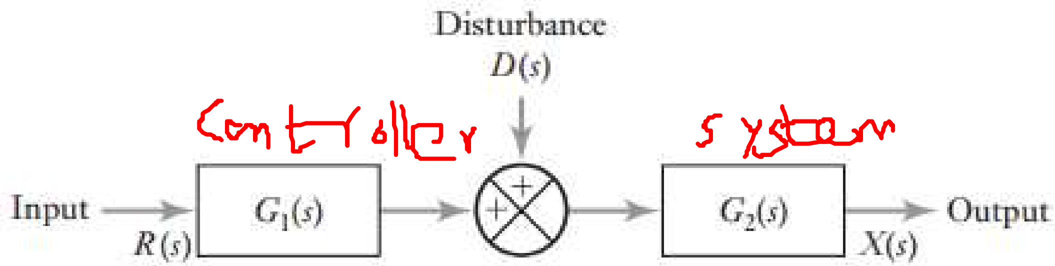
E(s)=R(s) – H(s)C(s)
Simplifying blocks in series, [Gc(S)Ga(S) - D(S)]Gp(S) is forward loop function

Function of controller – Thermo stat RTD / Thermo Couple

- **Maintain stability – gain / phase margin**
- **Follow set point – SSE, rise time, settling time**
- **Avoid/ Reject disturbances – Sensor, actuator, plant/process**

Open-Loop Control Systems

Open-Loop Control Systems utilize a controller or control actuator to obtain the desired response.
Open-loop control is often just a switch on–switch off form of control, e.g. an electric fire is either switched on or off in order to heat a room.

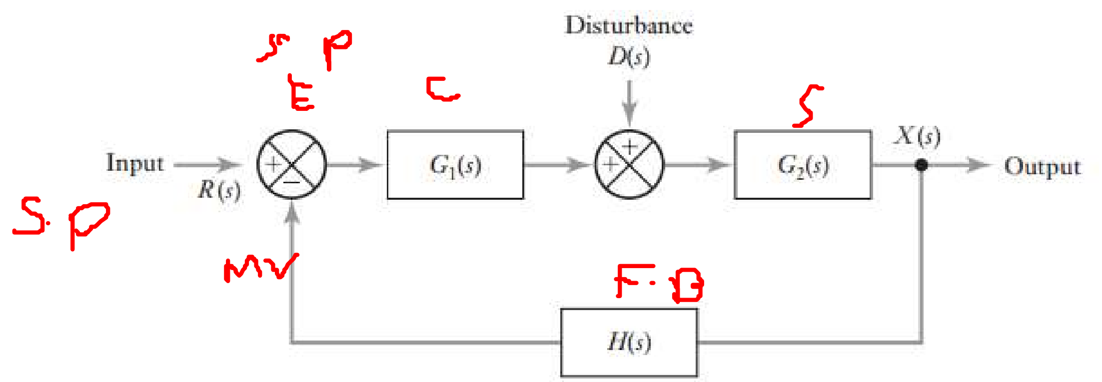


$$X(s) = G_2(s)[G_1(s)R(s) + D(s)] = G_1(s)G_2(s)R(s) + G_2(s)D(s)$$

Closed-Loop Control Systems

Closed-Loop Control Systems utilizes feedback to compare the actual output to the desired output response.

Closed-loop control systems , a controller is used to compare continuously the output of a system with the required condition and convert the error into a control action designed to reduce the error.



SP

MV = error

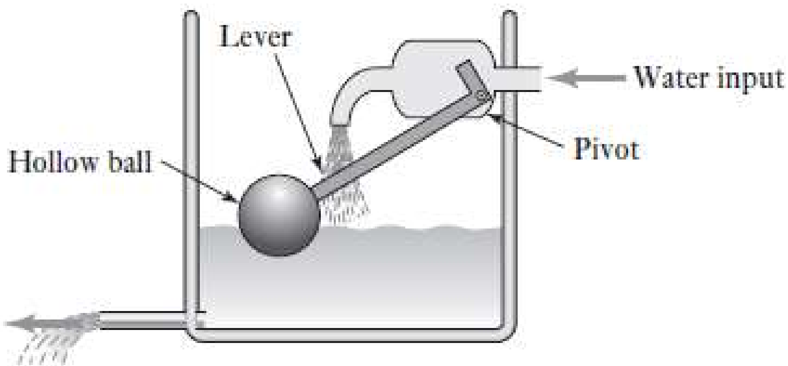
Controlled

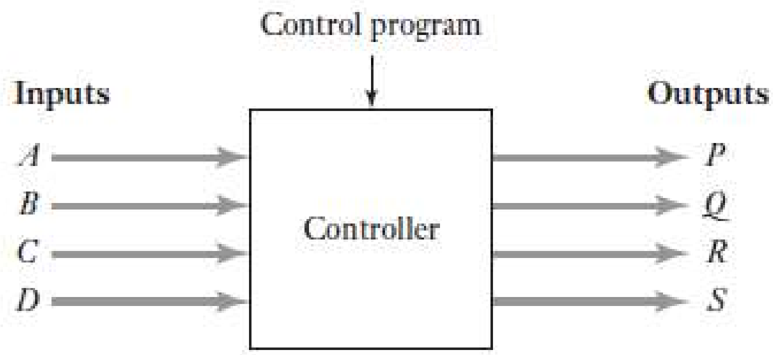
$$X(s) = G_2(s)\{G_1(s)[R(s) - H(s)X(s)] + D(s)\}$$

Thus

$$X(s) = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}R(s) + \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}D(s)$$

Continuous and discrete control process





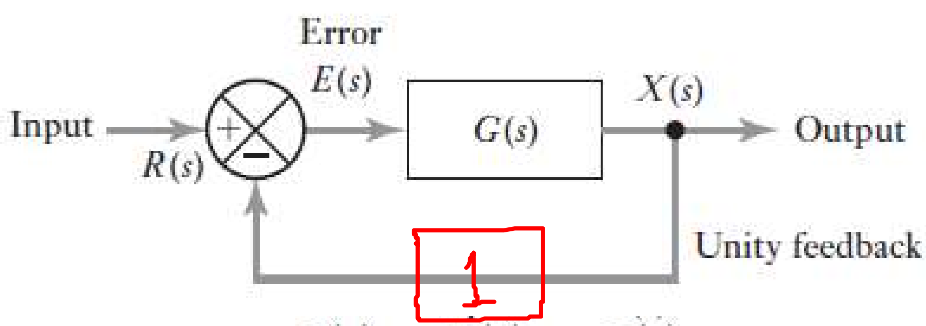
Control Modes

- Two Step (ON/OFF) controller
  - the controller is essentially just a switch which is activated by the error signal and supplies just an on/off correcting signal.
- Proportional Controller
  - produces a control action that is proportional to the error. The correcting signal thus becomes bigger, the bigger the error.
- Integral Controller
  - which produces a control action that is proportional to the integral of the error with time.
- Derivative Controller
  - produces a control action that is proportional to the rate at which the error is changing.

Terminology

- **Lag**
  - Time required for the system to make necessary response

- **Steady State error**



$$E(s) = R(s) - X(s)$$

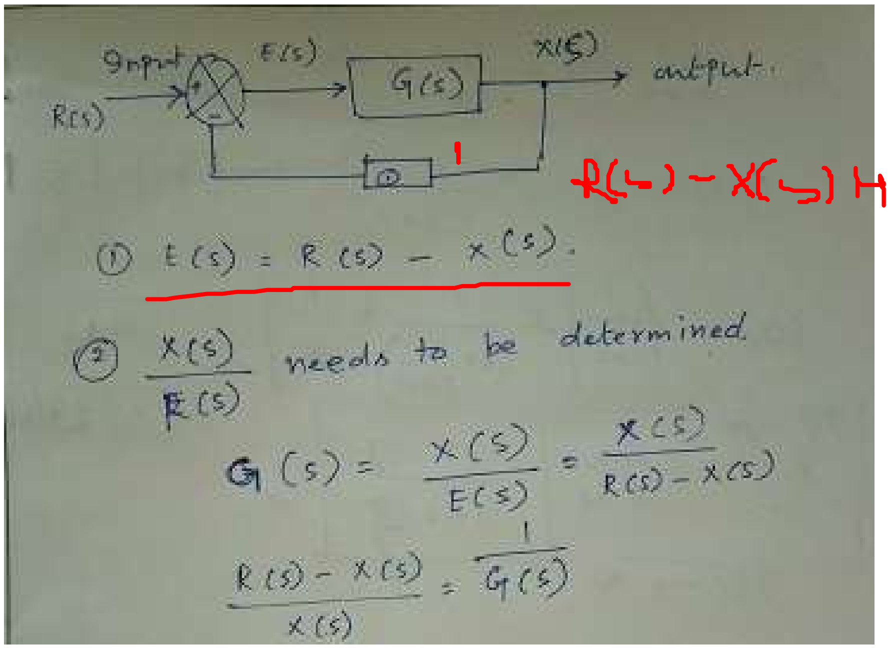
$$\frac{X(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{G(s)}{1 + G(s)}$$

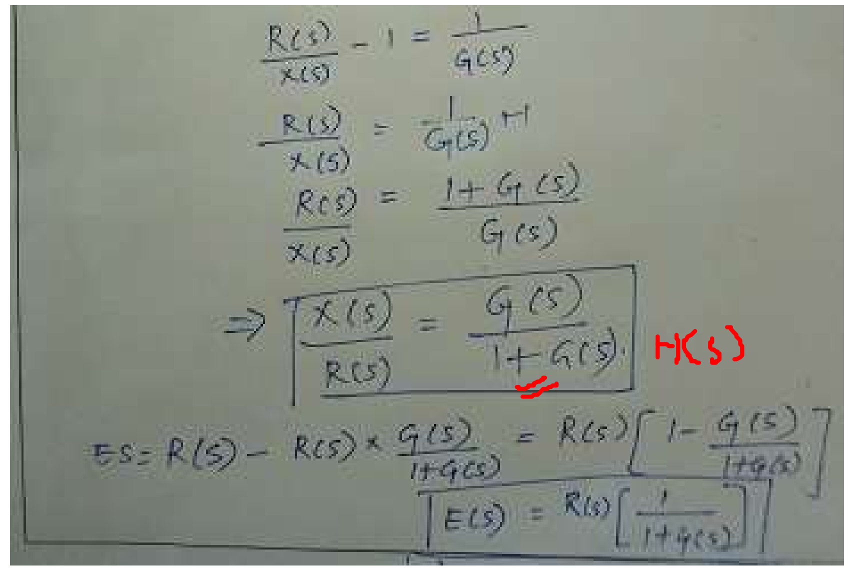
$$E(s) = R(s) - X(s) = R(s) - \frac{G(s)R(s)}{1 + G(s)} = \frac{1}{1 + G(s)}R(s)$$

The term **steady-state error** is used for the difference between the desired set value input and the output after all transients have died away.

Steady state error

$\frac{o/p}{i/p} = T.F$





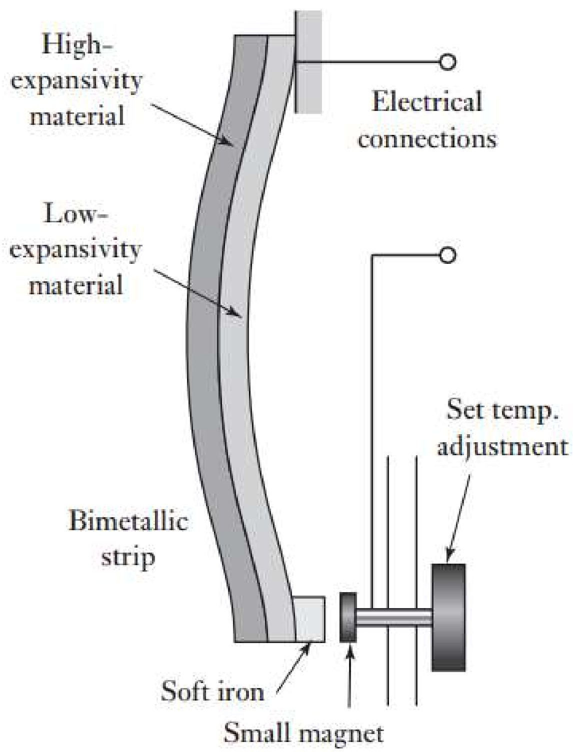
$$e_{SS} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

$$e_{SS} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \left[s \frac{1}{1 + k/(\tau s + 1)} \frac{1}{s} \right] = \frac{1}{1+k}$$

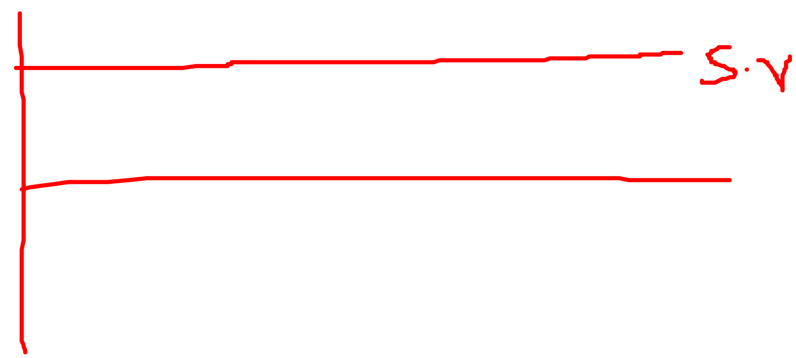
termed as Offset error

- **k is the gain of the controller for a system transfer function 1/τs+1**

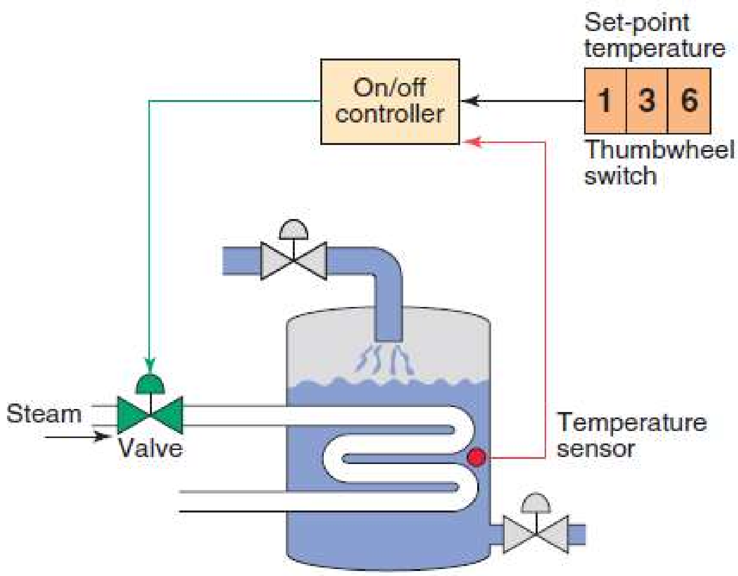
Control modes – On/Off



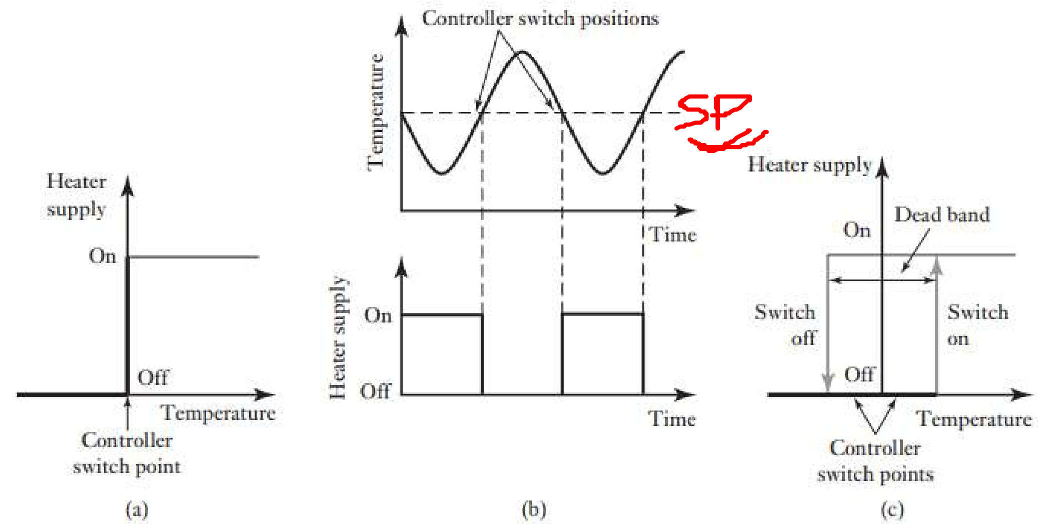
- Only 2 Positions ON/OFF
- Controller action is discontinuous.
- Oscillations of the controlled variable about set point value.
- There is lags in the time that the control system and the process take to respond.



Control modes – On/Off



On/off controlled liquid heating system.



The term **dead band** is used for the values between the on and off values.

A large dead band results in large fluctuations of the temperature about the set temperature;

A small dead band will result in an increased frequency of switching.

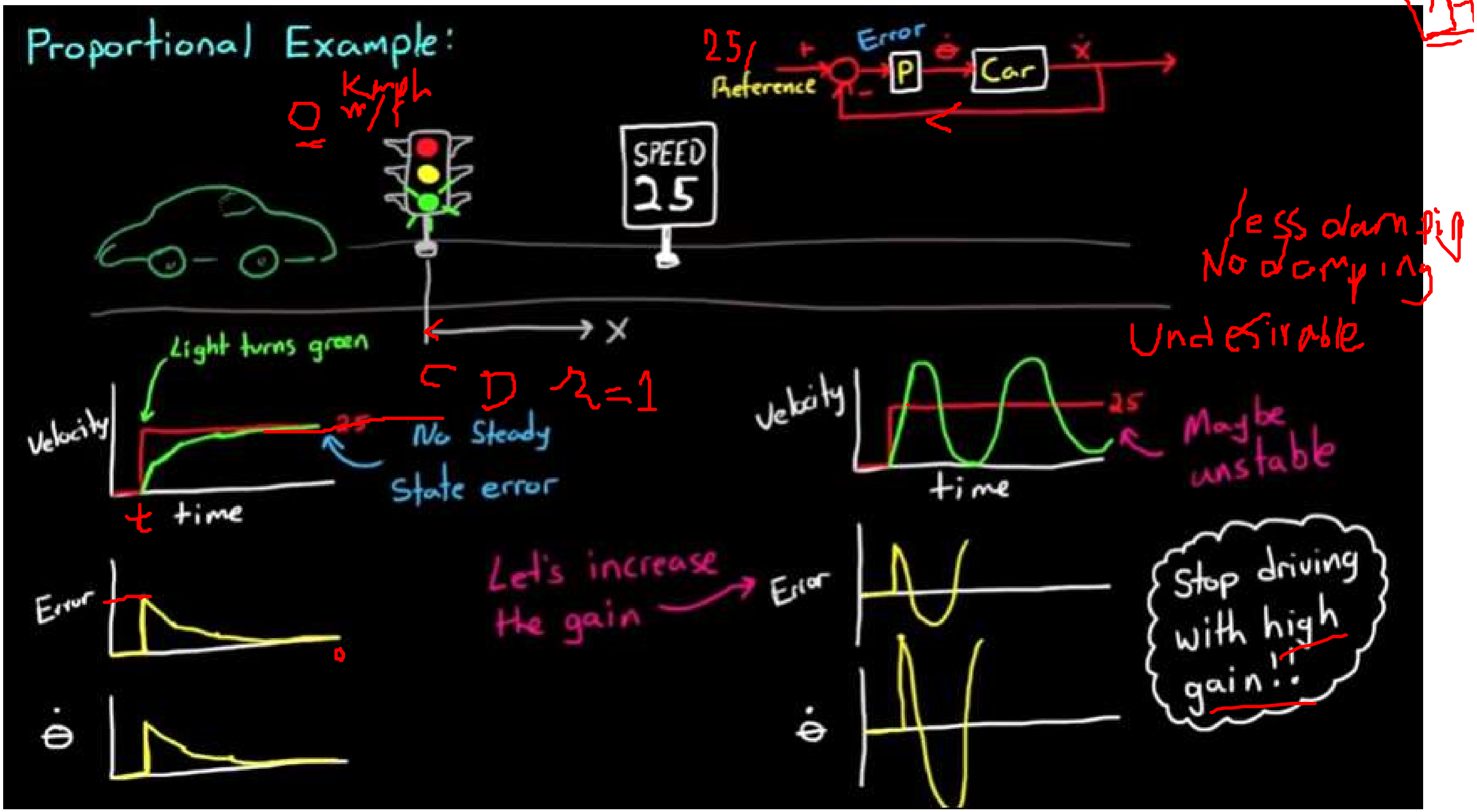
Control modes – Proportional (P)

- With the proportional mode, the size of the controller output is proportional to the size of the error: the bigger the error, the bigger the output from the controller
- the correction element of the control system, e.g. a valve/heater, will receive a signal which is proportional to the size of the correction required.

$\propto$

e ↑ o/p ↑

Control modes – Proportional (P)



Proportional Control

$$u(t) = u_P(t) = K_P e + \text{Offset}$$

o/p

gain × error + offset

+ domain

– In Proportional Control, the control signal, u, is directly proportional to the error, e.

– As the gain is increased the system responds faster to changes in set-point but becomes progressively under damped and eventually unstable.

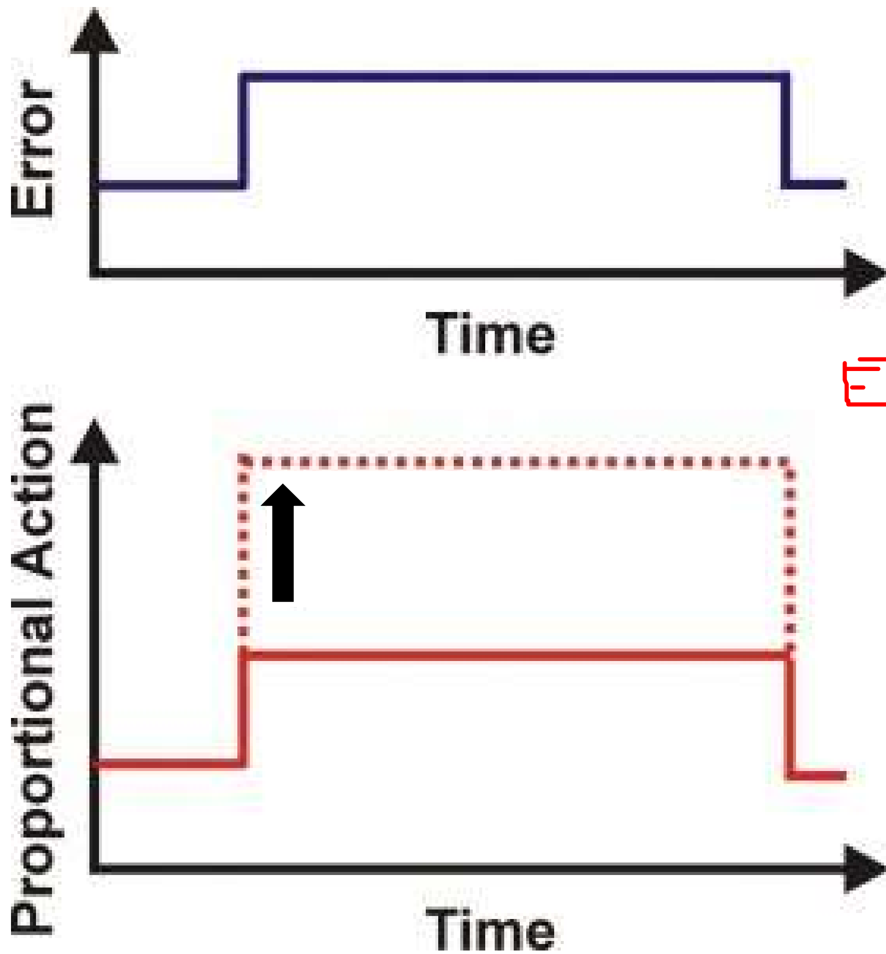
Proportional Control Action

K_p

C

$E(t)$ → $u(t)$

$u(t) = E(t) K_p$



P Control Signal

i/p

K_p

$E(s)$ → Controller → $u(s)$

o/p

Control modes – Proportional (P)

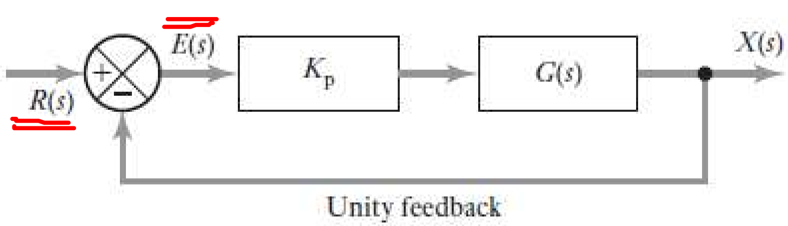
$$\text{controller output} = K_P e$$

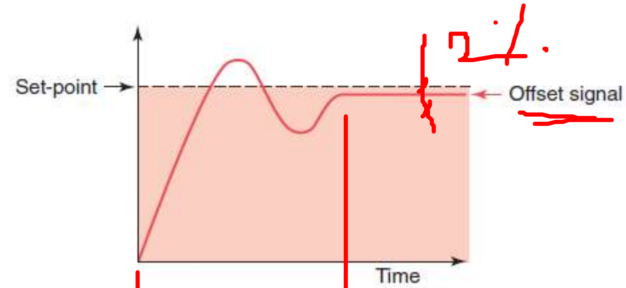
where e is the error and K_P a constant. Thus taking Laplace transforms,

$$\text{controller output}(s) = K_P E(s)$$

and so K_P is the transfer function of the controller.

$$E(s) = \frac{K_P G(s)}{1 + K_P G(s)} R(s)$$





and so, for a step input, the steady-state error is

$$e_{SS} = \lim_{s \to 0} sE(s) = \lim_{s \to 0}\left[s \frac{1}{1 + 1/K_P G(s)} \frac{1}{s}\right]$$

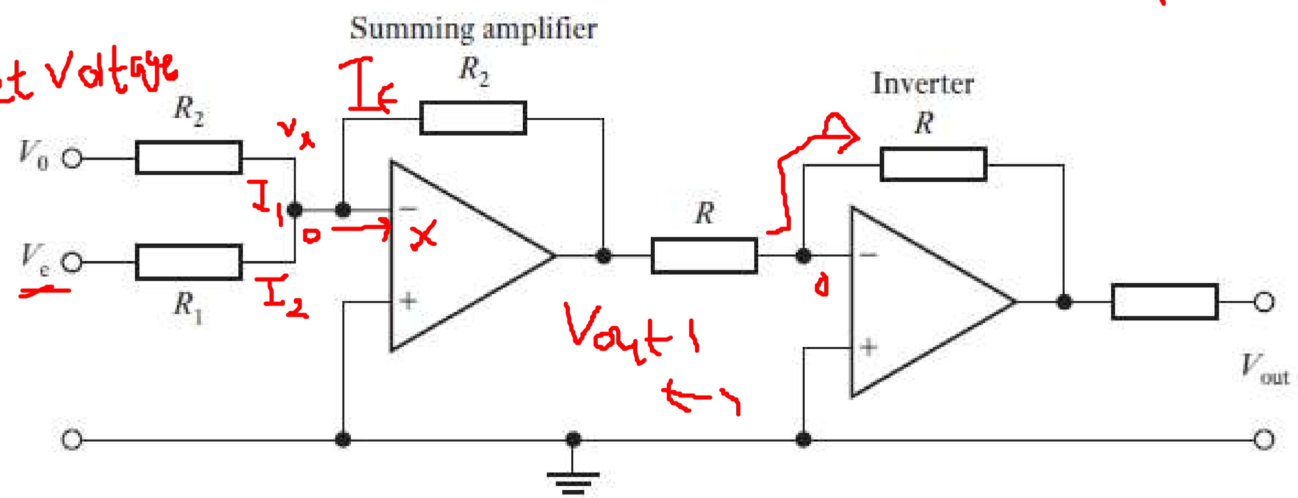
$t \to \infty$

- **Lower the value of Kp large SSE but stable responses**
- **Higher the value of Kp smaller SSE but Unstable responses**

Proportional (P) Hardware

$I_1 + I_2 = I_f$

Offset voltage



(+)

$$V_{out} = -\frac{R_2}{R_1}V_e - V_0$$

$$V_{out} = \frac{R_2}{R_1}V_e + V_0$$

$$\boxed{V_{out} = K_P V_e + V_0}$$

Summing Amplifier

Vo/R2 + Ve/R1 = -Vout1/R2

Vout1 = -Vo – R2/R1 Ve

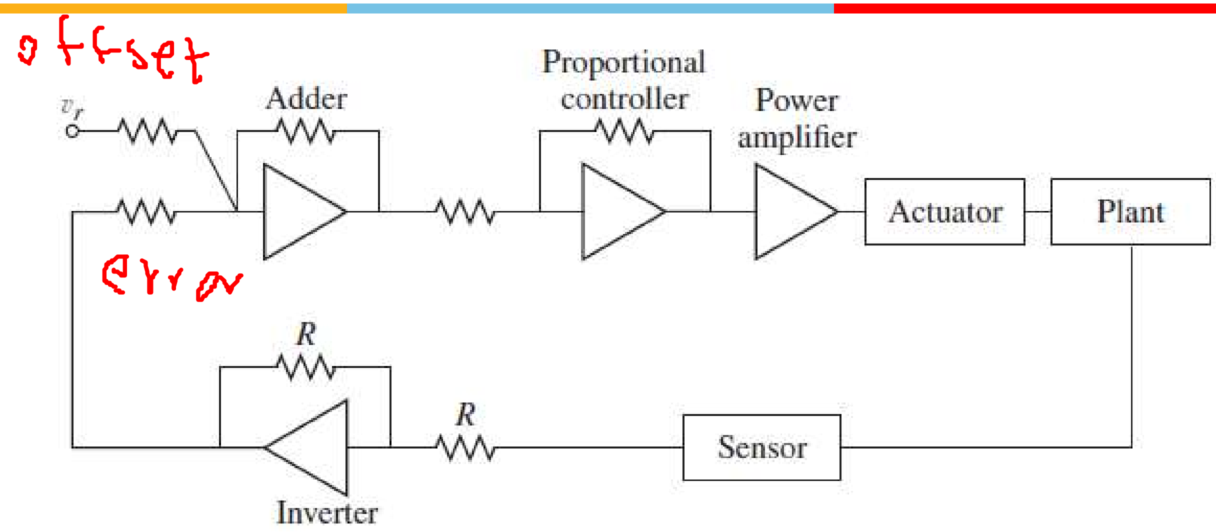
Inverter

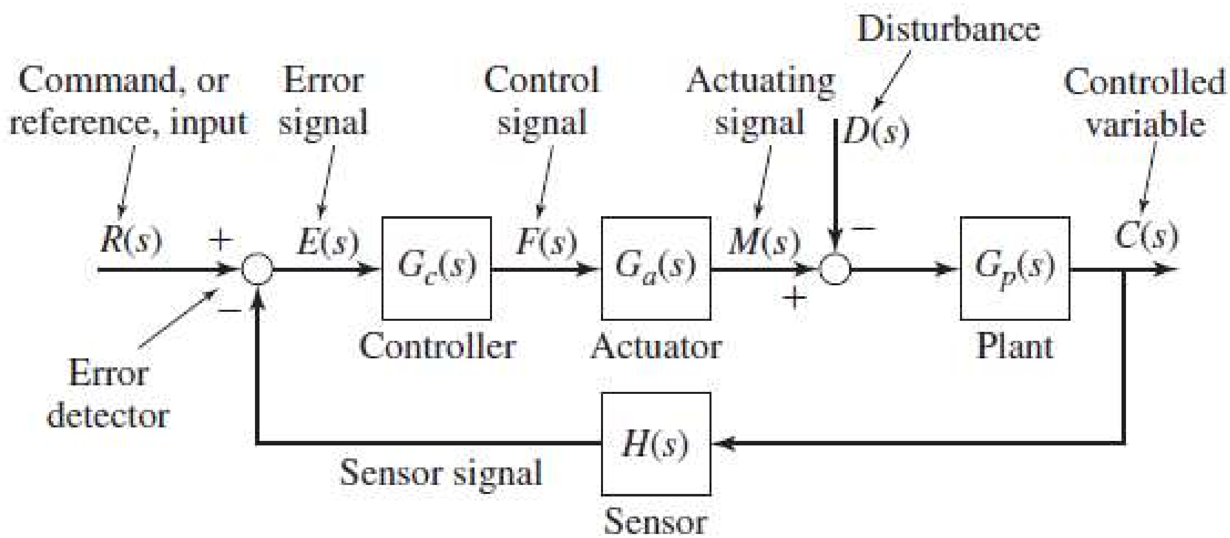
Vout1/R = -Vout/R

Vout1 = -Vout

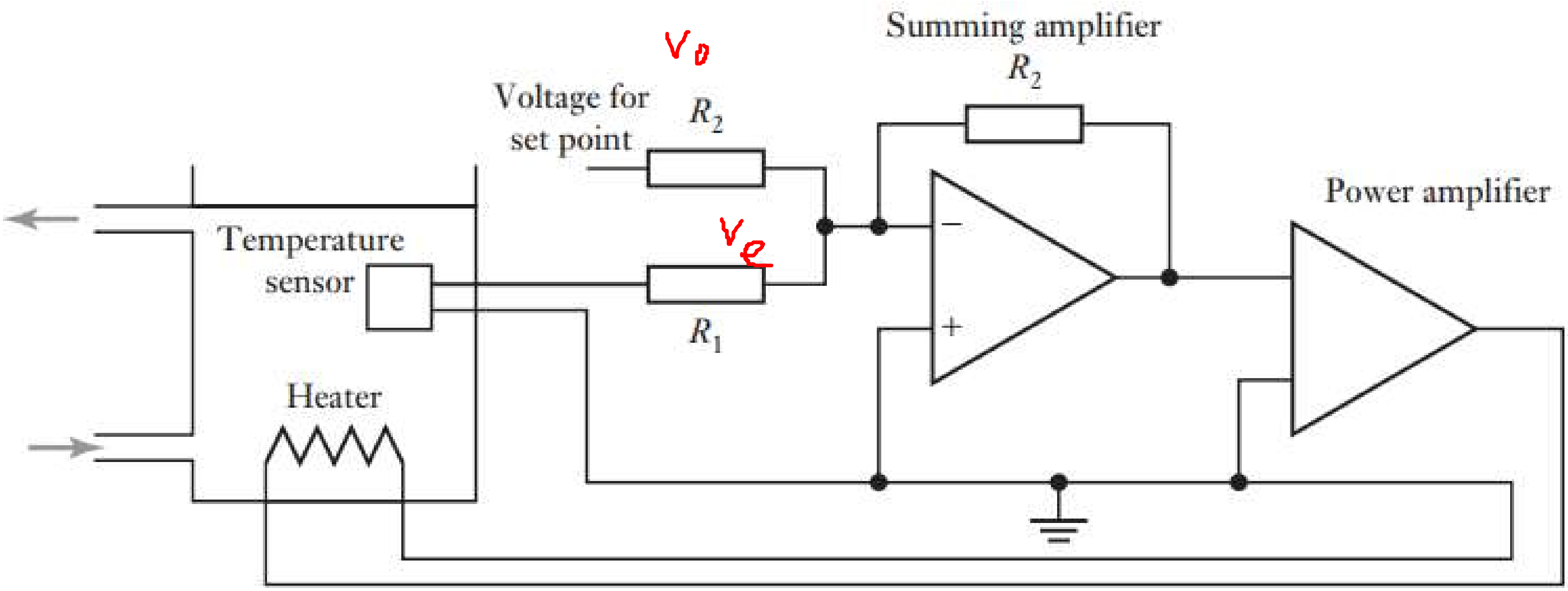
Vo + R2/R1 Ve = V out

Proportional (P) Hardware





Proportional Temperature Controller



Proportional Control

Advantages:

– Simple and easy to design and tune

– Rapid Response / Reduces Rise Time

– Reduces Steady State Error

Disadvantages:

– Not possible to eliminate Steady State Error / Offset

– Could lead to instability / rise in overshoot/ oscillations

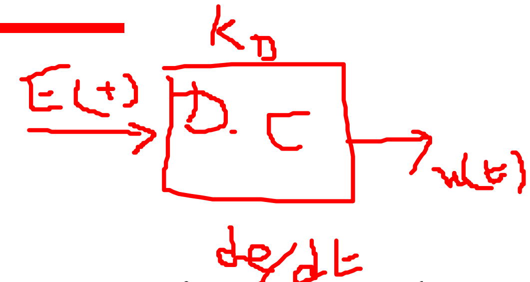
Applications:

– Float Valve, Thermostat etc

Derivative Control

$$u(t) = u_D(t) = K_D\left(\frac{de}{dt}\right)$$

$\propto$



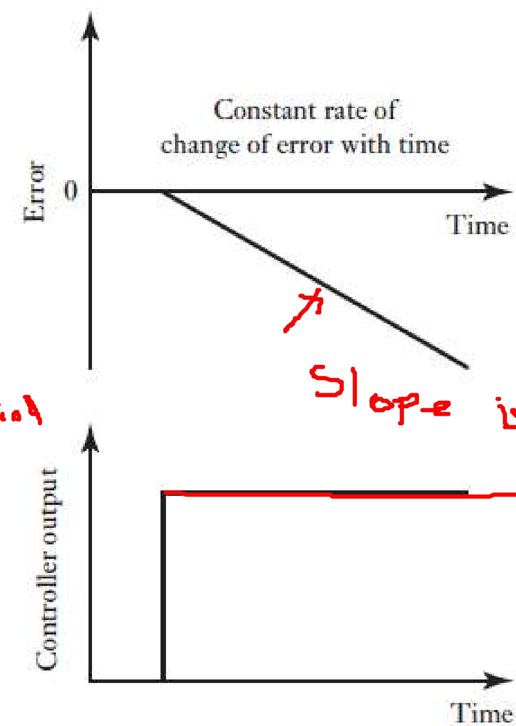
- Derivative control produces a control signal proportional to the rate at which the error is changing.
  - Also known as rate controller.
- While sudden/rapid change in error leads to a control signal of larger magnitude, gradual change leads to small magnitude.
- Even if the error is huge, the derivative control will generate no signal if the error is constant
  - Thus, not used alone; used with P control

Derivative (D)

$$\text{controller output} = K_D \frac{de}{dt}$$

K_D is the constant of proportionality. The transfer function is obtained by taking Laplace transforms, thus

$$\text{controller output}(s) = K_D s E(s)$$



1/p

Slope is constant

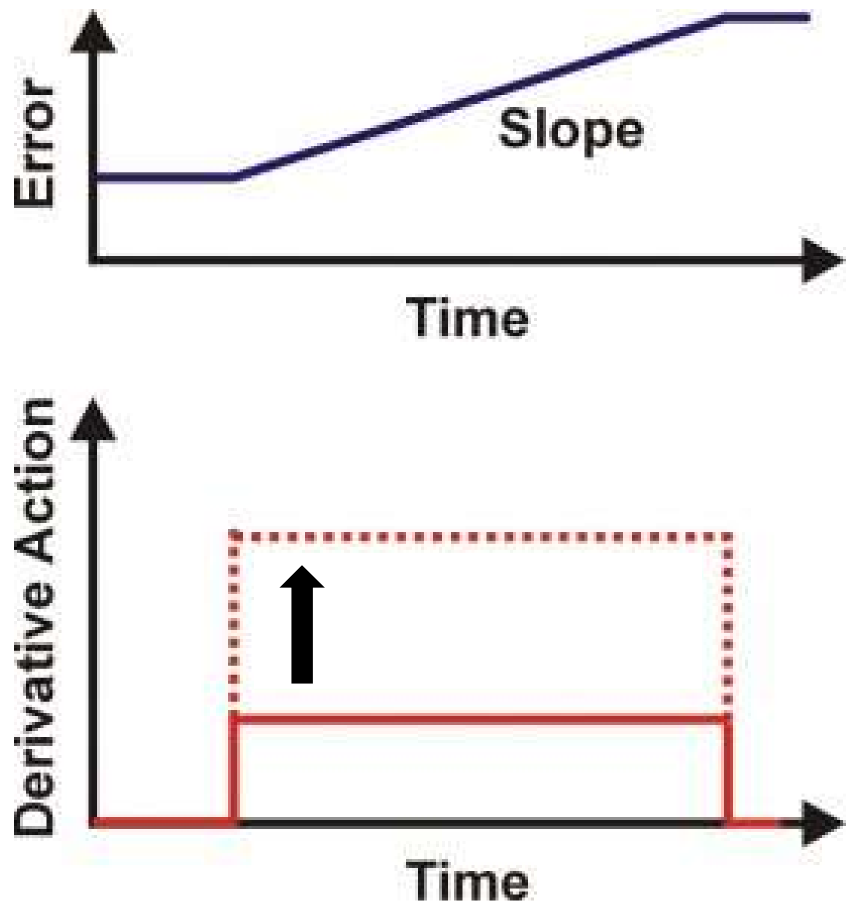
0/p

$\frac{de}{dt}$ = constant

$\frac{d(0)}{dt} = 0$

Derivative Control Action

P D

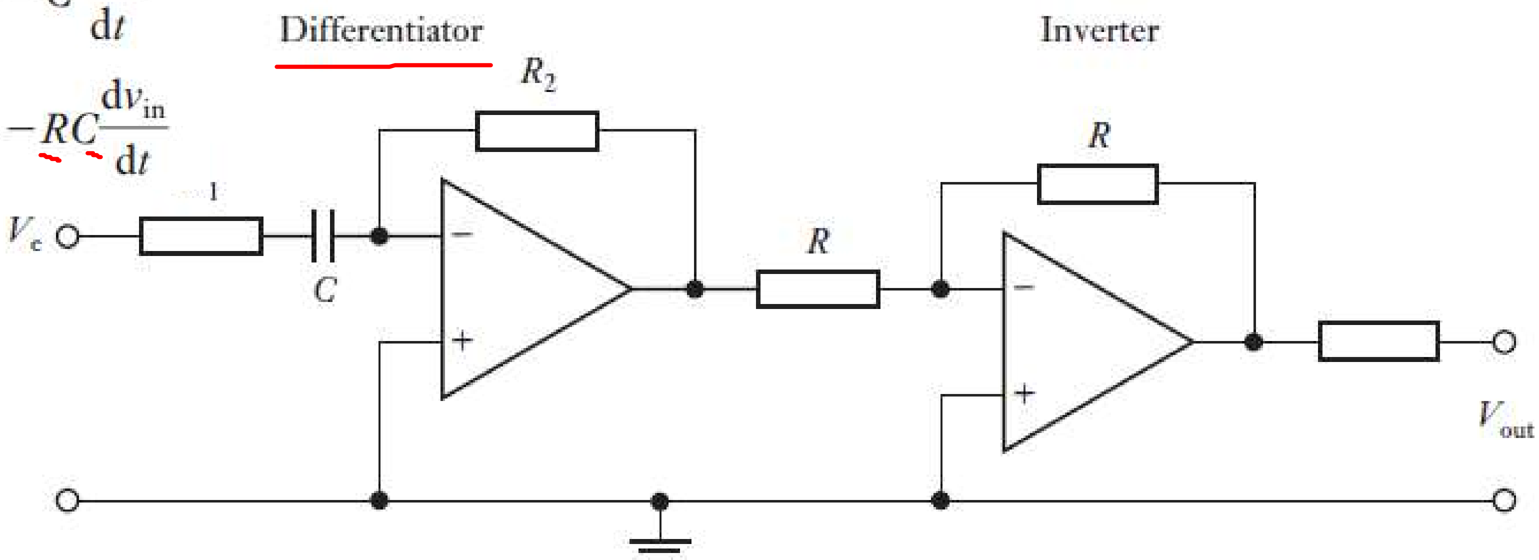


D Control Signal

Derivative (D) – Hardware

$$\frac{v_{out}}{R} = -C\frac{dv_{in}}{dt}$$

$$v_{out} = -RC\frac{dv_{in}}{dt}$$



The derivative time K_D is R_2C.

Derivative control is always combined with proportional control; the proportional part gives a response to all error signals, including steady signals, while the derivative part responds to the rate of change.

Does not respond to steady state errors!

Derivative Control

Advantages:

– Reduces Settling time; Adds lead

– Reduces Overshoot; Adds more stability

Disadvantages:

– Not possible to eliminate Steady State Error / Offset

– Not possible to use alone

– Excessive use may make the system slow

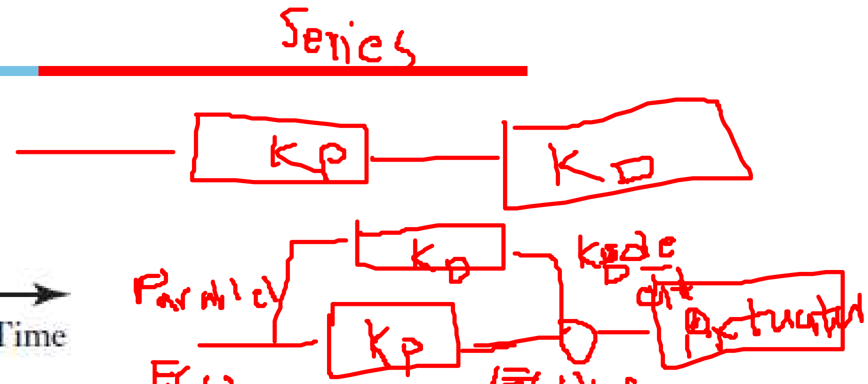
– Amplifies Noise- Derivative action can also be a problem if the measurement of the process variable gives a noisy signal, the rapid fluctuations of the noise resulting in outputs which will be seen by the controller as rapid changes in error and so give rise to significant outputs from the controller.

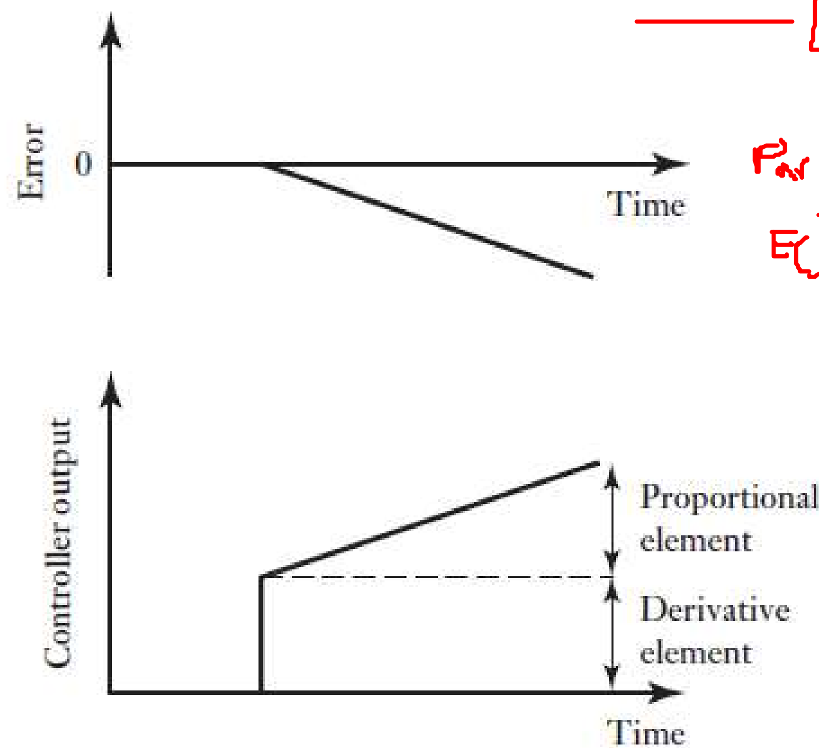
$\frac{de}{dt}$

Applications:

– In conjunction with P Control

PD control





Ideal for fast changing processes

+ = Control

Derivative control is always combined with proportional control; the proportional part gives a response to all error signals, including steady signals, while the derivative part responds to the rate of change.

PD control

P D

With proportional plus derivative control the controller output is given by

$$\text{controller output} = K_P e + K_D \frac{de}{dt}$$

⇒ Parallel Config

K_P is the proportionality constant and K_D the derivative constant, de/dt is the rate of change of error. The system has a transfer function given by

$$\text{controller output}(s) = K_P E(s) + K_D s E(s)$$

→ $E(s)[K_P + K_D s]$

Hence the transfer function is $K_P + K_D s$. This is often written as

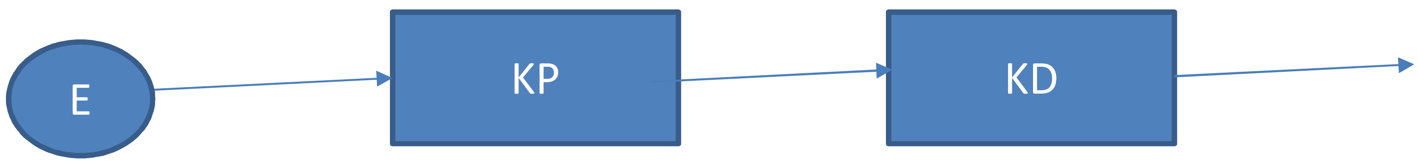
$$\text{transfer function} = K_D\left(s + \frac{1}{T_D}\right)$$

where $T_D = K_D / K_P$ and is called the derivative time constant.

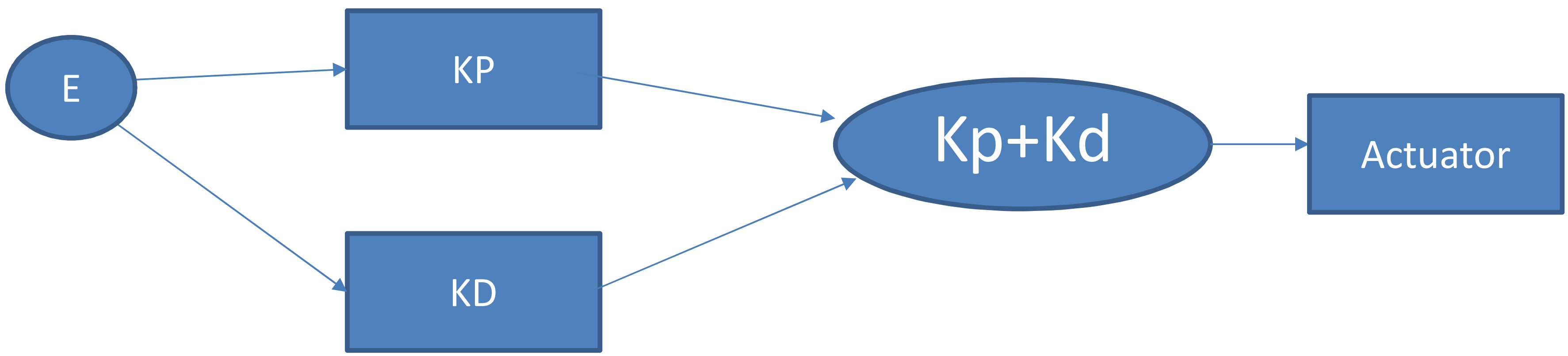
PD control

- There is an initial quick change in controller output because of the derivative action followed by the gradual change due to proportional action.
- This form of control can thus deal with fast process changes.

PD in series



PD in parallel – Preferred config.



Integral Control

The **integral mode** of control is one where the rate of change of the control output I is proportional to the input error signal e:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = K_{\mathrm{I}}e$$

K_{I} is the constant of proportionality and has units of $1/s$. Integrating the above equation gives

$$\int_{I_0}^{I_{\text{out}}} \mathrm{d}I = \int_0^t K_{\mathrm{I}}e\,\mathrm{d}t$$

$$I_{\text{out}} - I_0 = \int_0^t K_{\mathrm{I}}e\,\mathrm{d}t$$

I_0 is the controller output at zero time, I_{out} is the output at time t.

The transfer function is obtained by taking the Laplace transform. Thus

$$(I_{\text{out}} - I_0)(s) = \frac{1}{s}K_{\mathrm{I}}E(s)$$

Laplace transform of an integral (Refer Zill)!

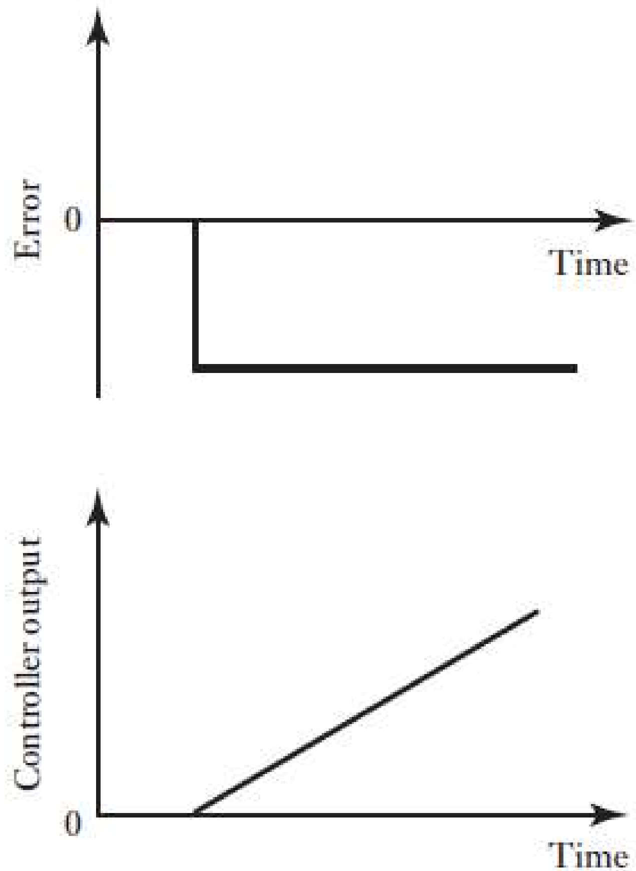
and so

$$\text{transfer function} = \frac{1}{s}K_{\mathrm{I}}$$

Transform of an Integral When $g(t) = 1$ and $\mathscr{L}\{g(t)\} = G(s) = 1/s$, the convolution theorem implies that the Laplace transform of the integral of f is

$$\mathscr{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}. \qquad (7)$$

Integral (I) control

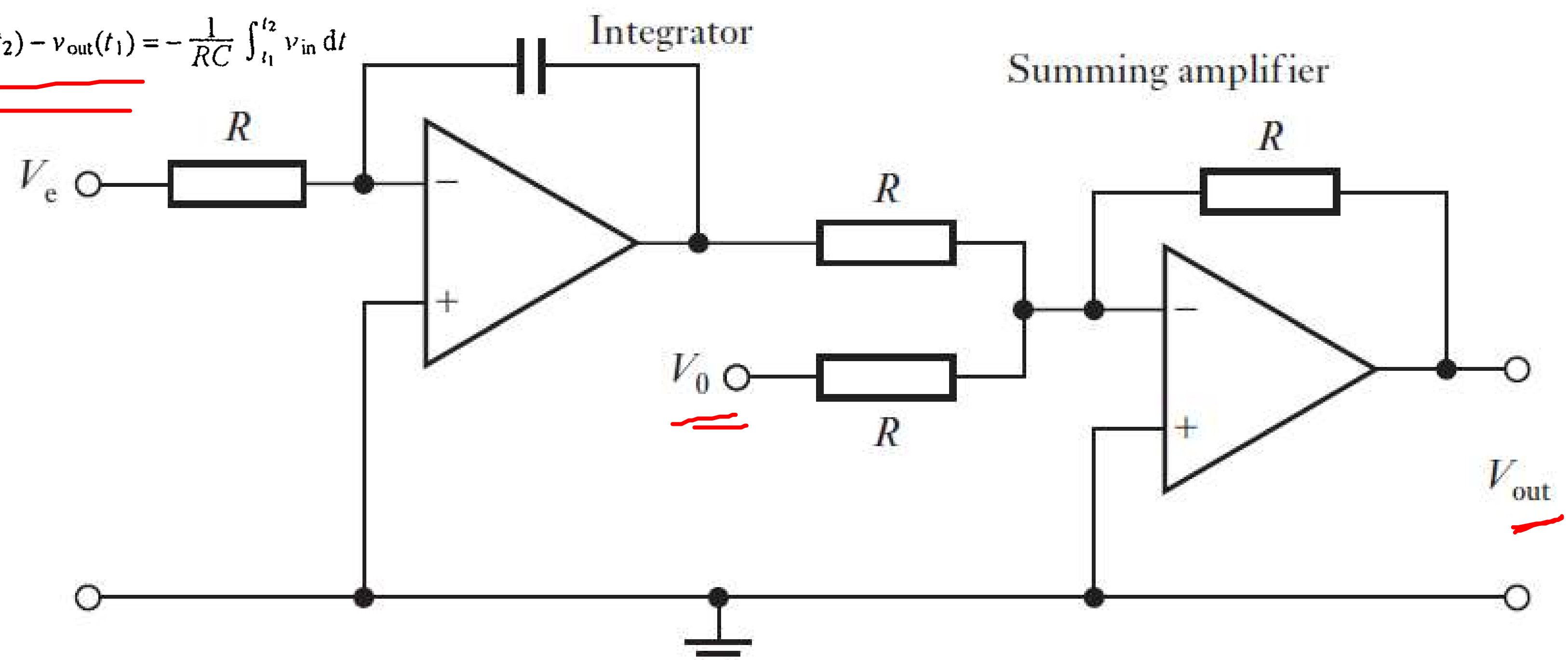


Integral (I) control - Hardware

$$dv_{out} = -\left(\frac{1}{RC}\right) v_{in}\, dt$$

Integrating both sides gives

$$v_{out}(t_2) - v_{out}(t_1) = -\frac{1}{RC}\int_{t_1}^{t_2} v_{in}\, dt$$



It consists of an operational amplifier connected as an integrator and followed by another operational amplifier connected as a summer to add the integrator output to that of the controller output at zero time. K_I is $1/R_IC$.

Integral Control

Advantages:

– Eliminates steady state error/offset

– Decreases Rise Time

Disadvantages:

– Causes Integral Wind Up

– Leads to minor increase in overshoot

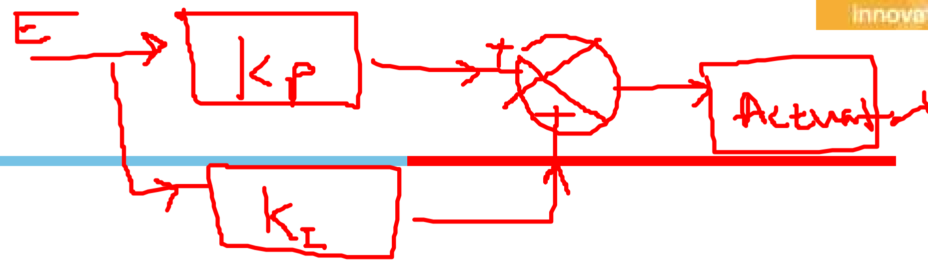
– Could make the system less stable

– Increases Settling time

Applications:

– In conjunction with P Control

P I control



$$\text{controller output} = K_P e + K_I \int e\,dt$$

⇒ ||el Config

where K_P is the proportional control constant, K_I the integral control constant and e the error e. The transfer function is thus

$$\text{transfer function} = K_P + \frac{K_I}{s} = \frac{K_P}{s}\left(s + \frac{1}{T_I}\right)$$

E(s)

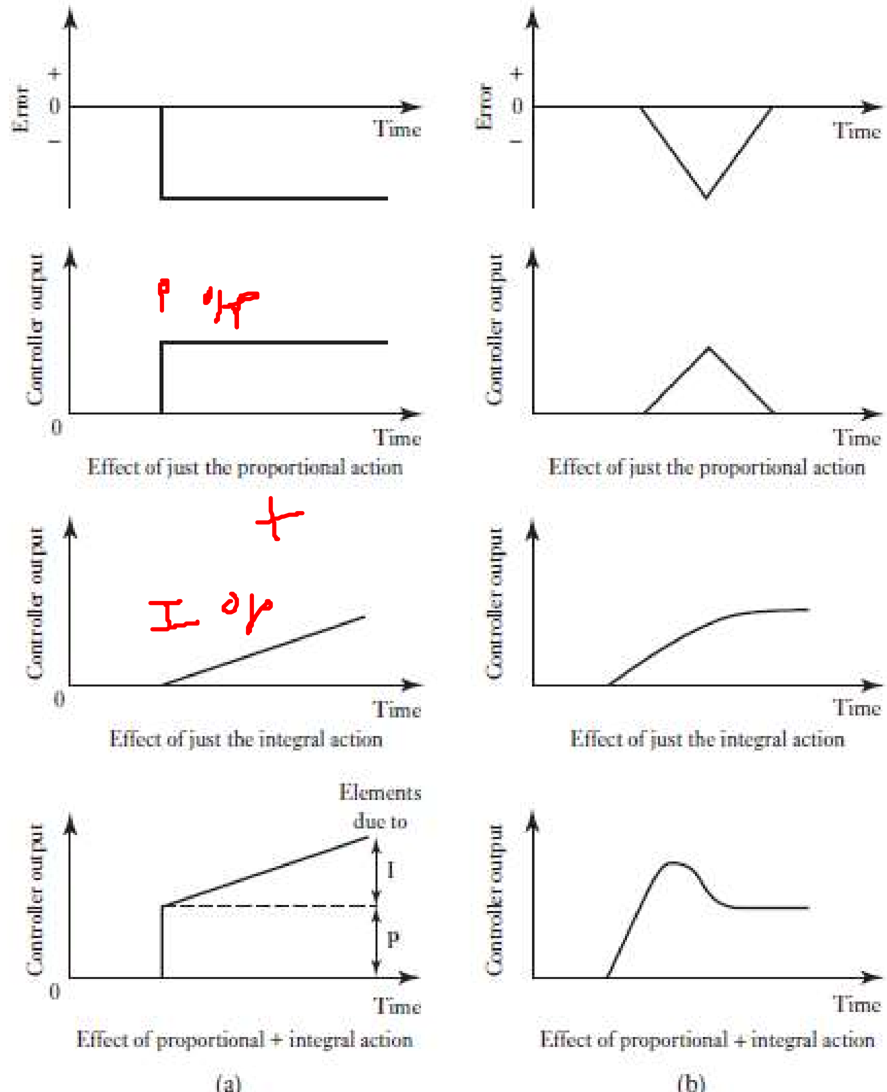
where $T_I = K_P/K_I$ and is the **integral time constant.**

P I control

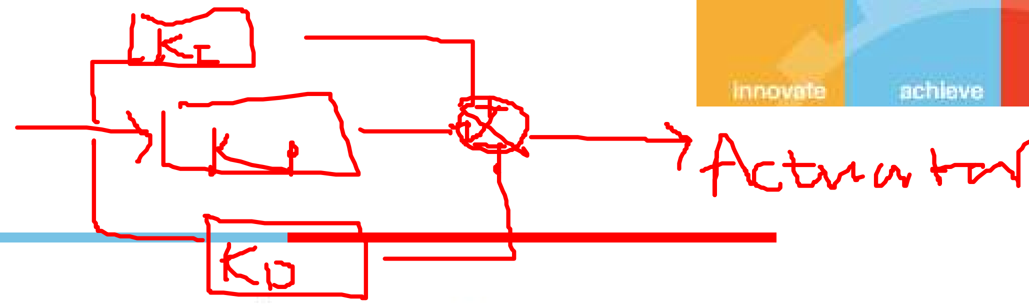
i/p

PI

PI =



P I D control

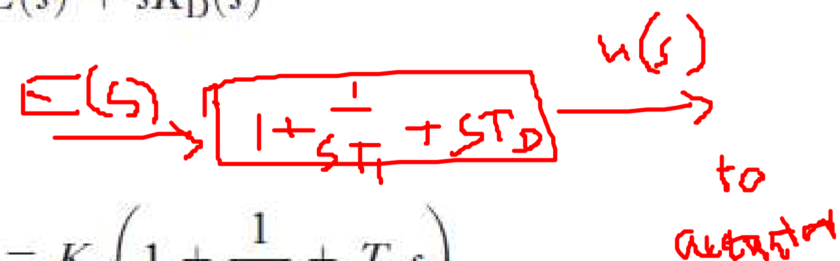


$$\text{controller output} = K_P e + K_I \int e\,dt + K_D \frac{de}{dt}$$

$\Rightarrow$ ||el config

where K_P is the proportionality constant, K_I the integral constant and K_D the derivative constant. Taking the Laplace transform gives

$$\text{controller output}(s) = K_P E(s) + \frac{1}{s} K_I E(s) + s K_D(s)$$



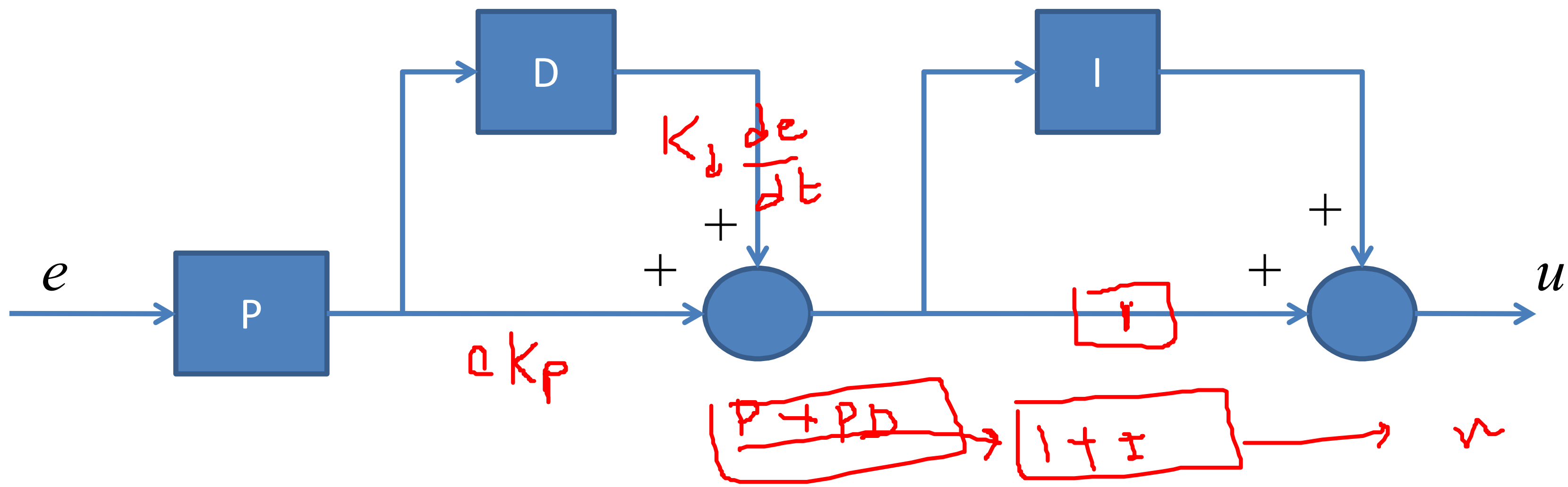
and so

$$\text{transfer function} = K_P + \frac{1}{s} K_I + s K_D = K_P \left(1 + \frac{1}{T_I s} + T_D s \right)$$

$$1 + \frac{1}{s}\frac{K_I}{K_P} + s\frac{K_D}{K_P}$$

$$1 + \frac{1}{sT_I} + sT_D$$

PID: Series / Interacting Form



- Derivate Action interacts with Integral Action (P+PD)(1+I)
  - Modification in derivative time constant affects integral action
- Commercially used controller

Transfer Function of Series Form

$$\text{Transer Function of PID in series} : (P + PD)(1 + I)$$

where,

$$P = \text{Proportional Controller}, \; I = \text{Integral Controller}$$

$$D = \text{Derivative Controller}$$

$$\boxed{TF = P + PD + PI + \cancel{PI}D}$$

$$\text{The term PID} \approx 0 \text{ since } T_i > 4T_d$$

where,

$$T_i = \text{Integral Time Constant}, \quad T_d = \text{Derivative Time Constant}$$

$$\therefore TF = P + PI + PD$$

Transfer Function of Series Form

Control Signal for PID in series :

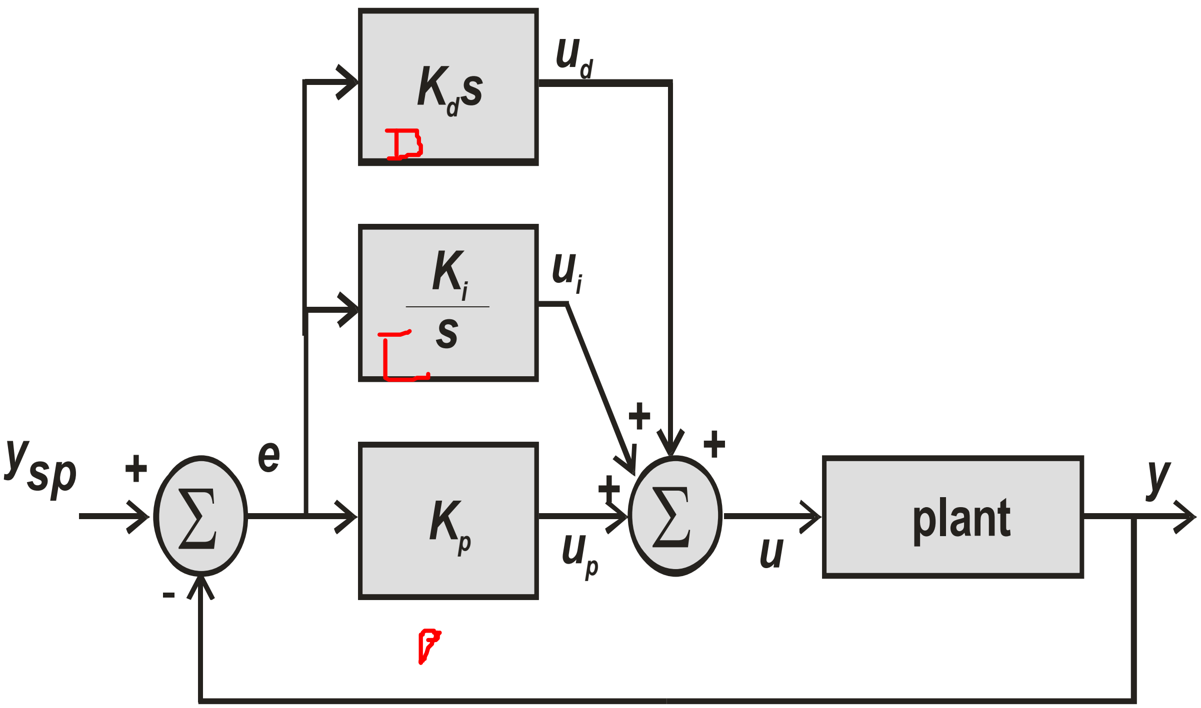
$$u(t) = \underbrace{u_P(t)}_{P} + \underbrace{u_P(t)u_I(t)}_{PI} + \underbrace{u_P(t)u_D(t)}_{PD}$$

$$= K_P e + K_P K_I \int e\,dt \; + K_P K_D \left({de}/{dt} \right)$$

Where,

$e = \text{Error} = \text{Difference between reference \& measured signal}$

PID: Parallel / Non-Interacting Form



- Ideal Form
- Derivative Action does not Interact with Integral Action

Transfer Function of Parallel Form

Transer Function :

$$H(s) = K_P + {K_I}/{s} + K_D s$$

P I D

Where,

K_P = Proportion al Gain, K_I = Integral Gain

K_D = Derivative Gain

Control Signal :

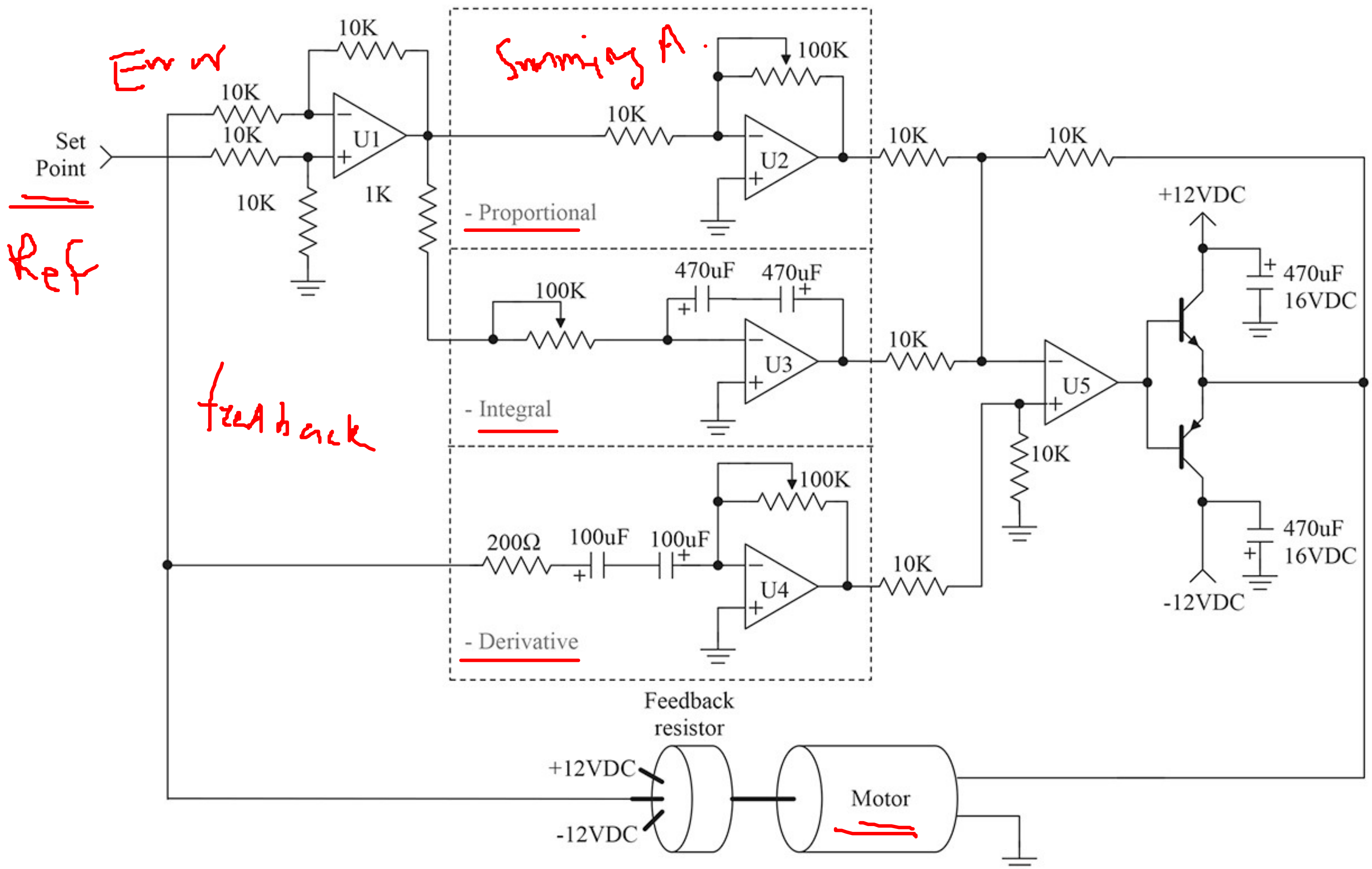
$$u(t) = u_P(t) + u_I(t) + u_D(t)$$

$$= K_P e + K_I \int e\,dt + K_D \left({de}/{dt}\right)$$

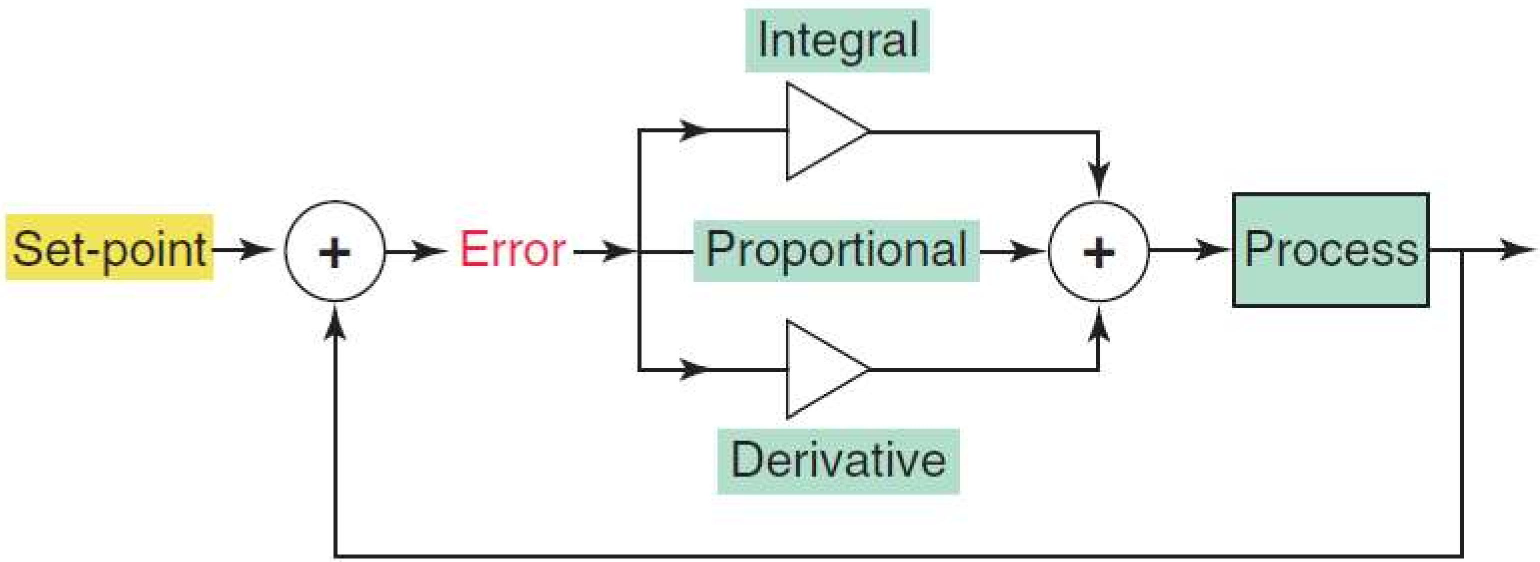
Where,

e = Error = Difference between reference & measured signal

P I D control - Hardware



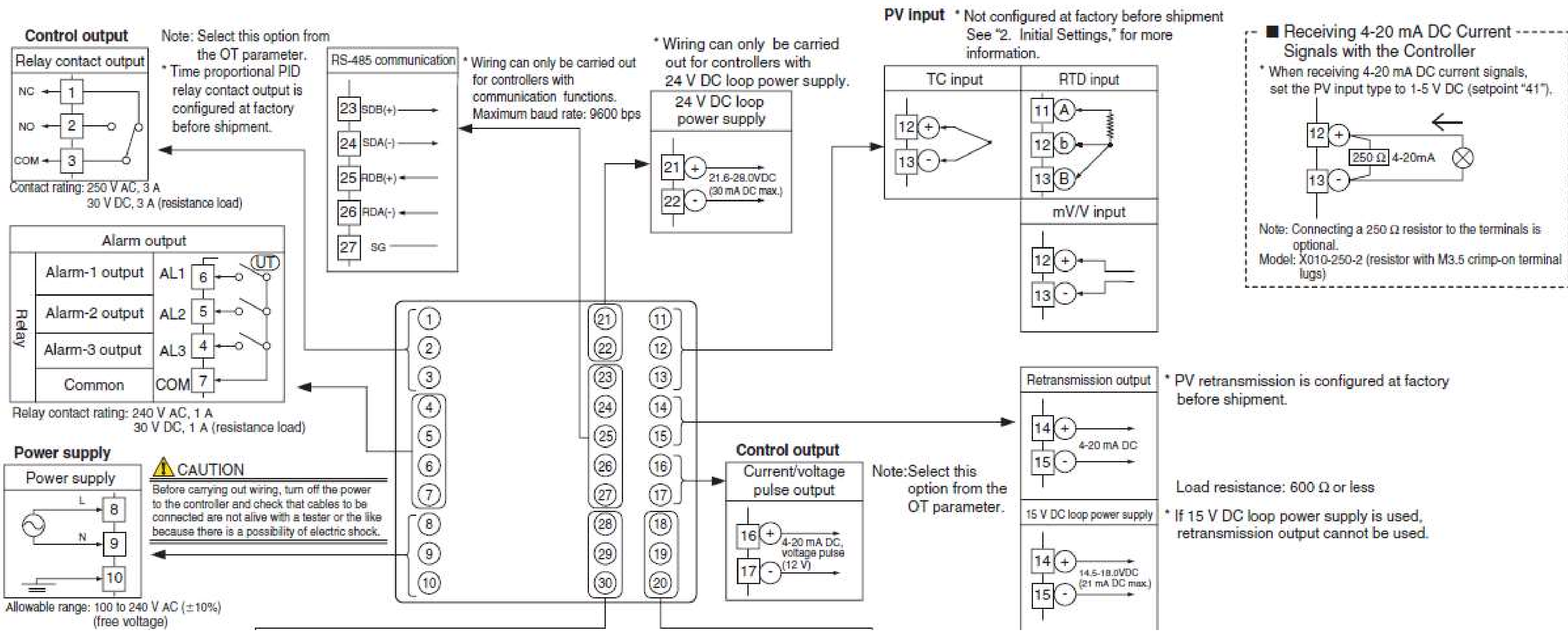
Typical PID combination



PID controller

Commercial PID

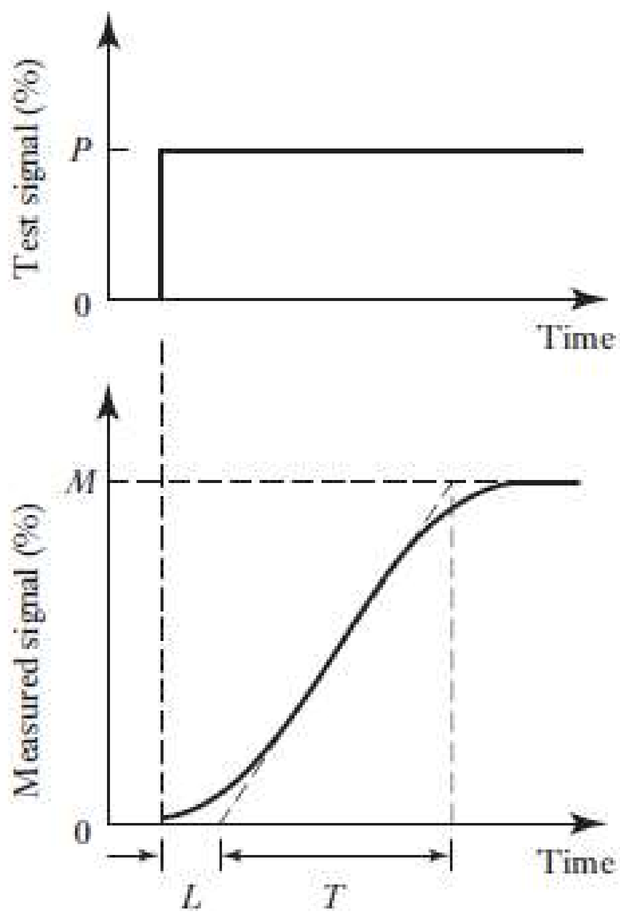
Commercial PID



PID Tuning guide

- The term tuning is used to describe the process of selecting the best controller settings.
- With a proportional controller this means selecting the value of K_P; with a PID controller the three constants K_P, K_I and K_D have to be selected.

Ziegler – Nichols – Process Reaction



Open the control loop, no control action is allowed

Control mode	K_P	T_I	T_D
P	P/RL		
PI	$0.9P/RL$	$3.33L$	
PID	$1.2P/RL$	$2L$	$0.5L$

R is maximum gradient(Slope) = M/T

The time between the start of the test signal and the point at which this tangent intersects the graph time axis is termed the lag L

Study the example problem given in Bolton

Ziegler – Nichols – Process Reaction

Table 22.1 Process reaction curve criteria.

Control mode	K_P	T_I	T_D
P	P/RL		
PI	$0.9P/RL$	$3.33L$	
PID	$1.2P/RL$	$2L$	$0.5L$

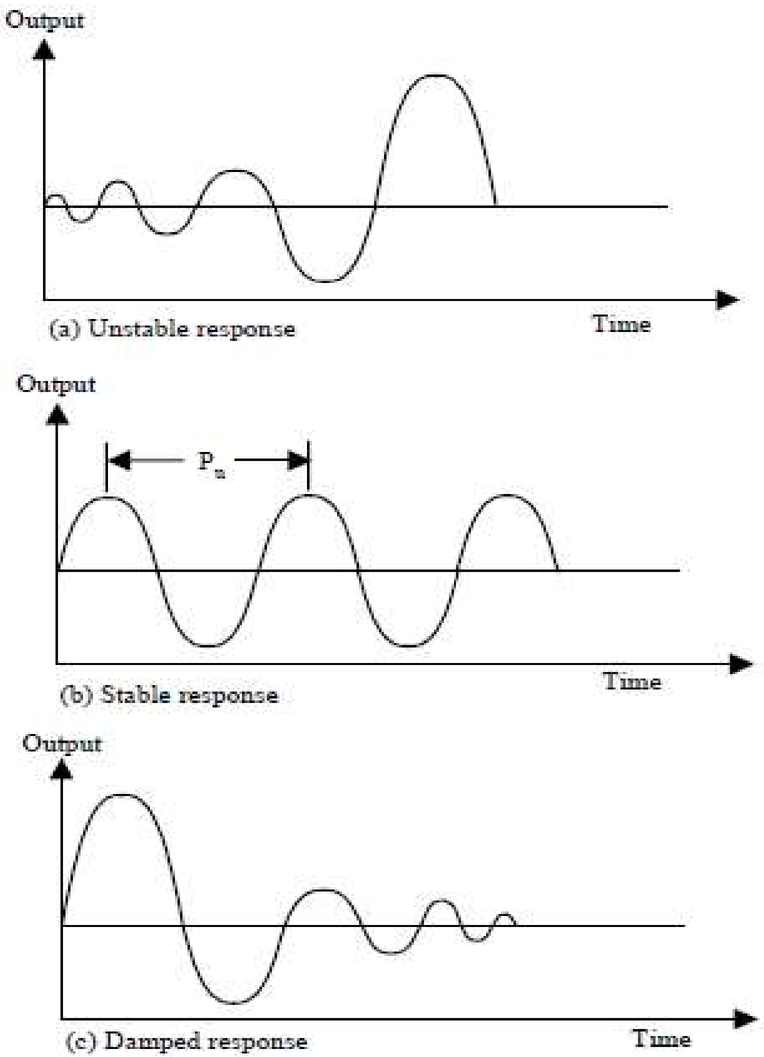
Consider the following example. Determine the settings required for a three-mode controller which gave the process reaction curve shown in Figure 22.19 when the test signal was a 6% change in the control valve position. Drawing a tangent to the maximum gradient part of the graph gives a lag L of 150 s and a gradient R of $5/300 = 0.017/\text{s}$. Hence

$$K_P = \frac{1.2P}{RL} = \frac{1.2 \times 6}{0.017 \times 150} = 2.82$$

$$T_I = 2L = 300\,\text{s}$$

$$T_D = 0.5L = 0.5 \times 150 = 75\,\text{s}$$

Ziegler-Nichols – Ultimate cycle



The method describes the procedure to find out constants like gain, Integral time, derivative time

K_p

T_i & T_D

Tuning Procedure -Ultimate cycle Method

Step 1:

Remove the integral and derivative action from the controller by setting

a) Derivative time to zero,
b) Integral time to zero
c) Proportional gain to one.

t_D & $T_i = 0$

$K_p = 1$

Step 2:

Run the system in automatic mode and control loop.

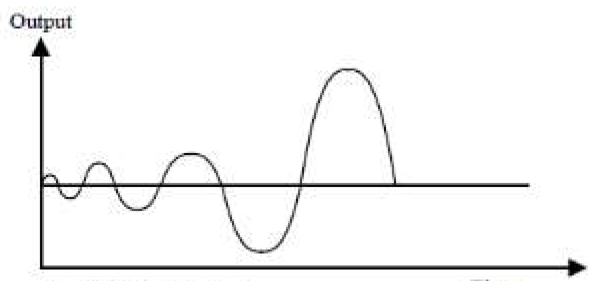
Tuning Procedure -Ultimate cycle Method

Step 3:

Upset the process (Say change the set point)

Step 4:

If the response curve does not damp but is unstable (Like below)



(a) Unstable response

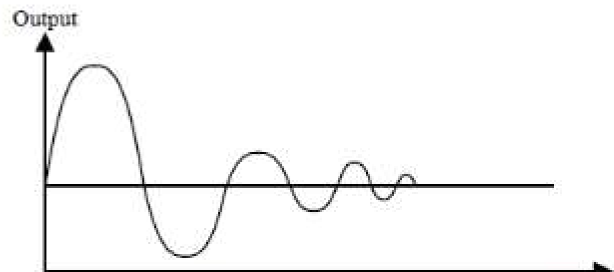
SP

then gain is too high. Reduce the gain. $K_p = 1$ ↓

Tuning Procedure

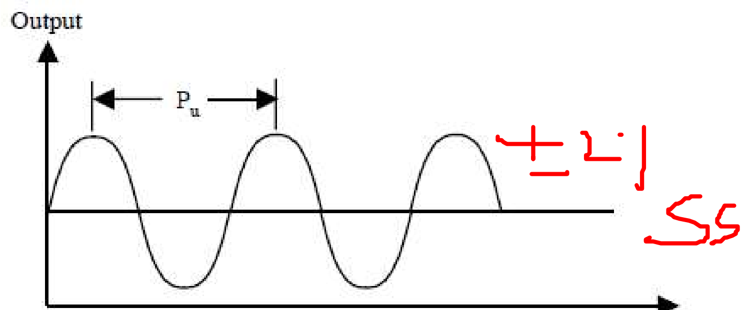
<u>Step 5:</u>

If the response curve damps out



(c) Damped response

then gain is too low. Increase the gain till you get the stable response.



(b) Stable response

Tuning Procedure -Ultimate cycle Method

Control mode	K_P	T_I	T_D
P	$0.5K_{Pc}$		
PI	$0.45K_{Pc}$	$T_c/1.2$	
PID	$0.6K_{Pc}$	$T_c/2.0$	$T_c/8$

Tuning Procedure

Record the value of Time (P_u) and ultimate gain (S_u) which generated stable response.

PI control:

$$K_c = 0.45 S_u$$

$$T_i = \frac{P_u}{1.2}$$

PD control:

$$K_c = 0.6 S_u$$

$$T_d = \frac{P_u}{8}$$

PID control:

$$K_c = 0.6 S_u$$

$$T_i = 0.5 P_u$$

$$T_d = \frac{P_u}{8}$$

Set the calculated parameters in the controller

Other tuning procedures

Manual:

- ✓ Operator estimates the tuning parameters required to give the desired controller response
- ✓ Proportional , integral, and derivative terms must be adjusted and tuned individually to a particular system using trial and error method.

Auto tune:

- The controller takes care of calculating and setting PID parameters
  - ✓ Measures sensor
  - ✓ Calculates error, sum of error, rate of change of error
  - ✓ Calculates desired parameter with PID equations
  - ✓ Updates control output

Thank you