Standard Errors

- when one is able to truly understand the concept of standard error, many of our most beloved inferential statistics (t tests, ANOVA, regression coefficients, correlations) become easy to understand.
- Inferential statistics is all about using information collected from samples to reach conclusions about the populations the samples came from. In the inferential statistics that we will see, the formulas are all basically the same: How large is the effect that you observe in your sample data compared to the amount of error you would expect to get just due to chance?*(In inferential statistics, **chance means random sampling error, or the error you would expect just due to random sampling.**)* As you will see when we discuss t test, F values, and correlation coefficients, the formulas for the inferential statistics all have this same general formula, with the observed effect in the sample as the numerator of the formulas and the error due to chance as the denominator.

What Is a Standard Error?

- **The standard error is the measure of how much *random* variation we would expect from samples of equal size drawn from the same population.**

The Conceptual Description of the Standard Error of the Mean

- Let's describe what is Sampling distribution first. We take out lots of samples from population and those samples have mean and standard deviation. We calculate all samples mean, this mean of all samples form a distribution. This distribution of means (taken from samples) are known as Sampling distribution of the mean.
- The mean of mean(sampling distribution) sample distribution is known as the **expected value of the mean**.
- It is called expected value because the mean of the sampling distribution of the mean is the same as the population mean.
- The Standard deviation of the Sampling distribution of the mean is called standard error.
- So the standard error is simply the standard deviation of the sampling distribution.

The final step in understanding the concept of standard error of the mean is to understand what this statistic tells us.

- The standard deviation tells us the average difference, or deviation, between an individual score in the distribution and the mean for the distribution.

- The standard error of the mean provides essentially the same information, except it refers to the average difference between the expected value (e.g., the population mean) and an individual sample mean.

  - So one way to think about the standard error of the mean is that it tells us how confident we should be that a sample mean represents the actual population mean.
  - Phrased another way, the standard error of the mean provides a measure of how much error we can expect when we say that a sample mean represents the mean of the larger population. That is why it is called a standard error.
  - Knowing how much error we can expect when selecting a sample of a given size from a population is critical in helping us determine whether our sample statistic, such as the sample mean, is meaningfully different from the population parameter, such as the population mean.
  - This is the foundation of all the inferential statistics.

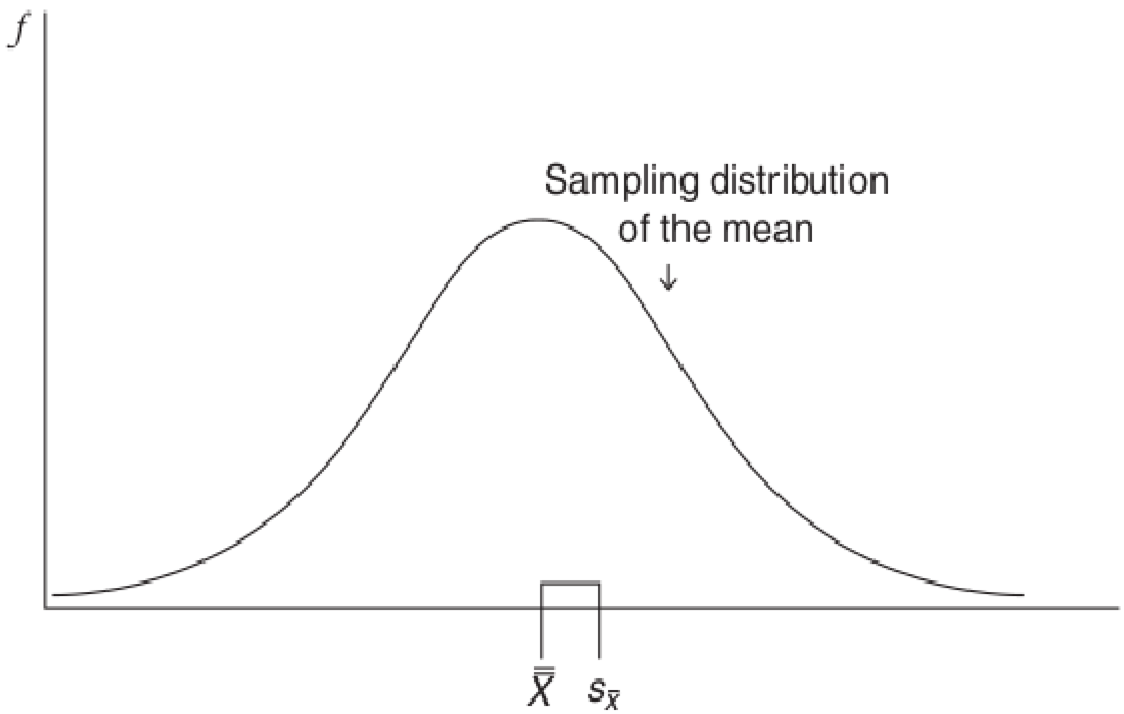

Figure 1: Sampling distribution of the mean with the expected value and the standard error shown

How to Calculate the Standard Error of the Mean

- Most of the time, researchers do not draw 1,000 samples of equal size from the population and then figure out the mean and standard deviation of this distribution of sample means.
- In fact, most of the time, researchers collect data from only a single sample, and then use this sample to make inferences about the population from which the sample was drawn. How can we make inferences about a larger population on the basis of a single sample?
- To do this, researchers must use what they know about their sample to make educated guesses, or estimates, about the population. I demonstrate this concept using the shoe-size example mentioned earlier. Suppose that I have a random sample of 100 women.
- Now, if this sample were truly selected at random (i.e., every adult woman in the country had an equal chance of being selected), my most logical assumption would be that this sample represents the larger population accurately.
- Therefore, I would have to assume that the mean shoe size of my sample (suppose it is 6) is also the mean shoe size of the larger population.
- Of course, I cannot know if this is true. In fact, as discussed earlier, there is good reason to believe that my sample may not represent my population well. But if the only information I have about adult women's shoe sizes comes from my sample of 100 women, my best guess about what the larger population of women looks like must be that they are similar to this sample of 100 women.
- **Now I am faced with a critical question**:

1. When I guess that the population of women in the country where I am conducting the study has an average shoe size of 6 (based on my sample average), how much error can I expect to have in this estimation?

2. In other words, what is the standard error?
3. To answer this question, I must examine two characteristics of my sample. First, how large is my sample? The larger my sample, the less error I should have in my estimate about the population. This makes sense because the larger my sample, the more my sample should look like my population, and the more accurate my estimates of my population will be. If there are 50 million women in the country where the study is being conducted and I use a sample of 25 million to predict their average shoe size, I would expect this prediction to be more accurate than a prediction based on a sample of 100 women. Therefore, the larger my sample, the smaller my standard error.
4. The second characteristic of my sample that I need to examine is the standard deviation. Remember that the standard deviation is a measure of how much variation there is in the scores in my sample. If the scores in my sample are very diverse (i.e., a lot of variation and therefore a large standard deviation), I can assume that the scores in my population are also quite diverse. So, if some women in my sample have a shoe size of 2 and others have a shoe size of 14, I can also assume that there is a pretty large variety of shoe sizes in my population. On the other hand, if all of the women in my sample have shoe sizes of either 5, 6, or 7, I can assume that most of the women in the larger population have an equally small variety of shoe sizes. Although these assumptions about the population may not be true (e.g., I may have selected a biased sample from the population), I must rely on them because this is all the information I have. So, the larger the sample standard deviation, the greater the assumed variation of scores in the population, and consequently the larger the standard error of the mean.

- The women in the larger population have an equally small variety of shoe sizes. Although these assumptions about the population may not be true (e.g., I may have selected a biased sample from the population), I must rely on them because this is all the information I have. So, the larger the sample standard deviation, the greater the assumed variation of scores in the population, and consequently the larger the standard error of the mean. (Note: In those instances where I know the population standard deviation, I can use that in my calculation of the standard error of the mean.)

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

or

$$s_{\overline{x}} = \frac{s}{\sqrt{n}}$$

- where σ is the standard deviation for the population.
- s is the sample estimate of the standard deviation.
- n is the size of the sample.