P452 - Computational Physics

Assignment 4

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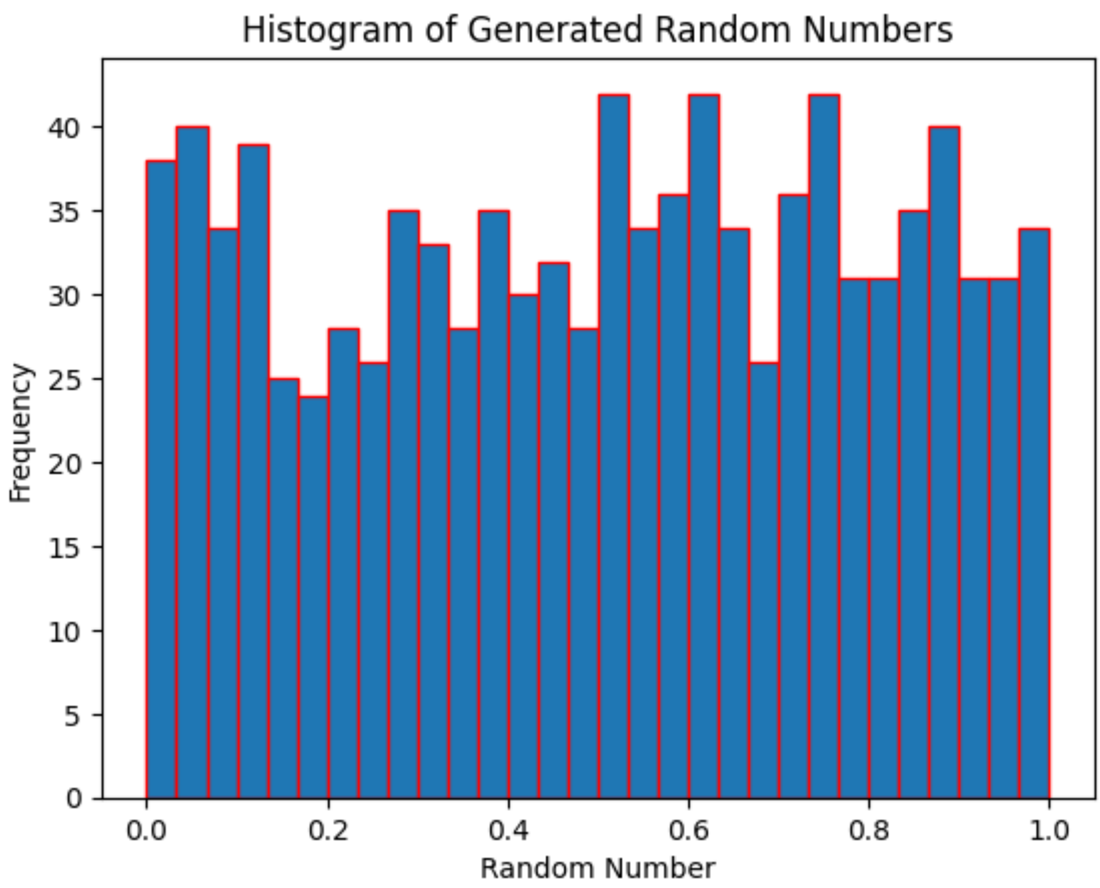
In [ ]:
```
import numpy as np
from matplotlib import pyplot as plt
from plotly import graph_objects as go

from utils import mlcgList, mlcg,mlc_generator
```

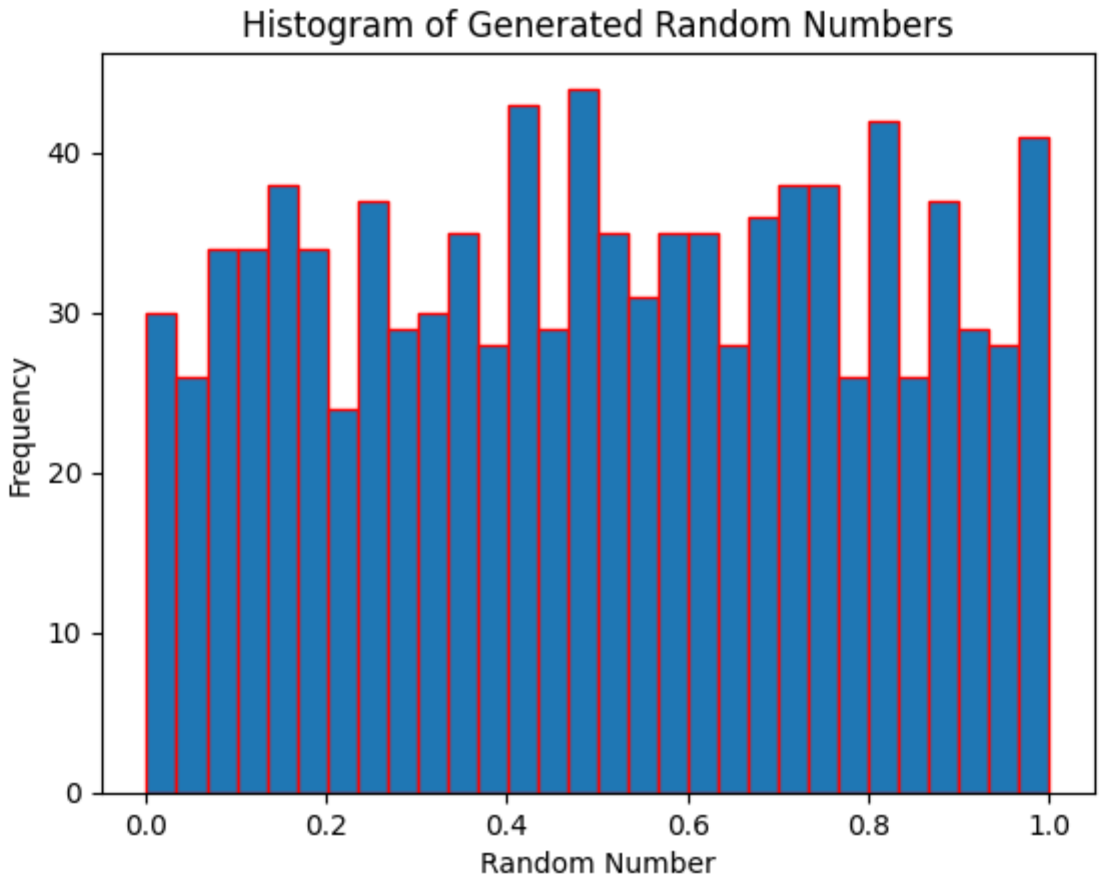
Question 1

In [ ]:
```
#(i) a= 65, m=1021

import matplotlib.pyplot as plt

# Assuming rnd_numbers is your list of random numbers
rnd_numbers = mlcgList(1000, (0,1), 65, 1021)

plt.hist(rnd_numbers, bins=30, edgecolor='red')
plt.xlabel('Random Number')
plt.ylabel('Frequency')
plt.title('Histogram of Generated Random Numbers')
plt.show()
```



In [ ]:

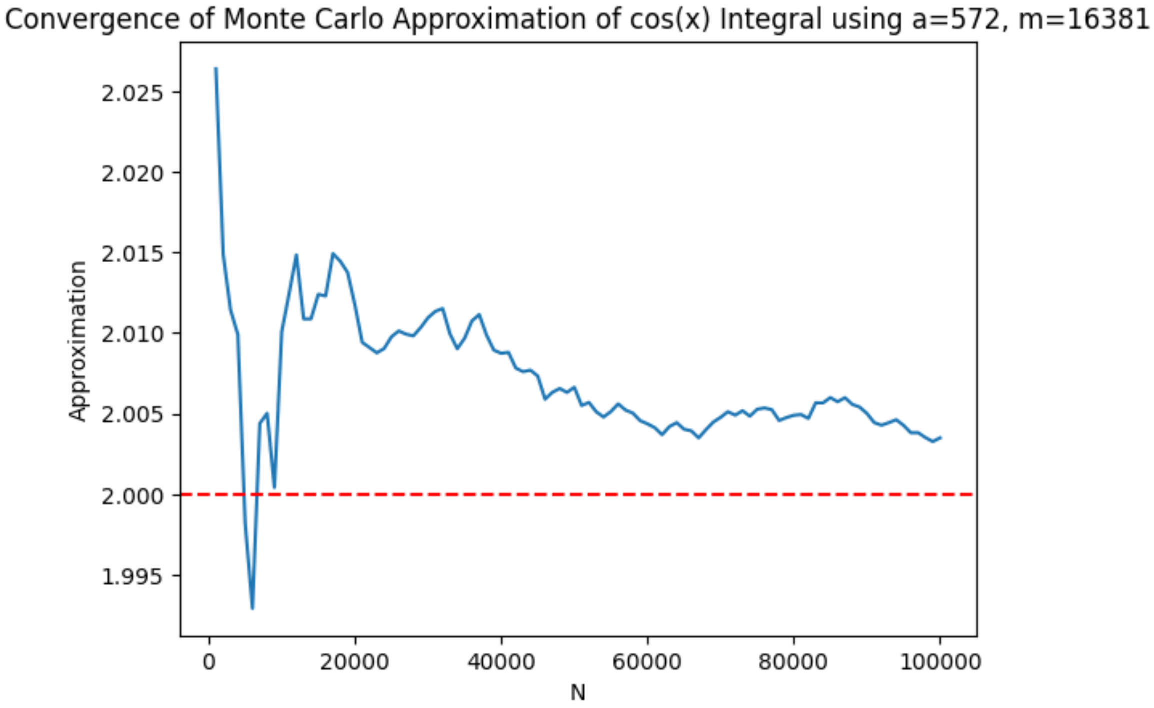
```
#(ii) a= 572, m=16381
rnd_numbers = mlcgList(1000, (0,1), 572, 16381)
plt.hist(rnd_numbers, bins=30, edgecolor='red')
plt.xlabel('Random Number')
plt.ylabel('Frequency')
plt.title('Histogram of Generated Random Numbers')
plt.show()
```



Question 2

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt

# Step 1: Generate N random numbers using the MLCG listed in previous pro


# Step 2: Use Monte Carlo to approximate the integral
def monte_carlo_cos_integral(mlcg_numbers):
    total = 0
    for num in mlcg_numbers:
        x = np.pi * (num - 0.5)  # transform to [-π/2, π/2]
        total += np.cos(x)
    return total / len(mlcg_numbers) * np.pi  # scale by the range width

# Step 3: Repeat for different N
N_values = range(1000, 100001, 1000)
approximations = []
for N in N_values:
    mlcg_numbers = mlcgList(N,(0,1), 572, 16381)
    approximation = monte_carlo_cos_integral(mlcg_numbers)
    approximations.append(approximation)

# Step 4: Plot the convergence
plt.plot(N_values, approximations)
plt.xlabel('N')
plt.ylabel('Approximation')
plt.title('Convergence of Monte Carlo Approximation of cos(x) Integral us
plt.axhline(y=2, color='r', linestyle='--')
```

```
plt.show()
```


Convergence of Monte Carlo Approximation of cos(x) Integral using a=572, m=16381
Approximation
2.025
2.020
2.015
2.010
2.005
2.000
1.995
0
20000
40000
60000
80000
100000
N

In [ ]:
```
approximations = []
for N in N_values:
    mlcg_numbers = mlcgList(N,(0,1), 65, 1021)
    approximation = monte_carlo_cos_integral(mlcg_numbers)
    approximations.append(approximation)

# Step 4: Plot the convergence
plt.plot(N_values, approximations)
plt.xlabel('N')
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plt.show()
```
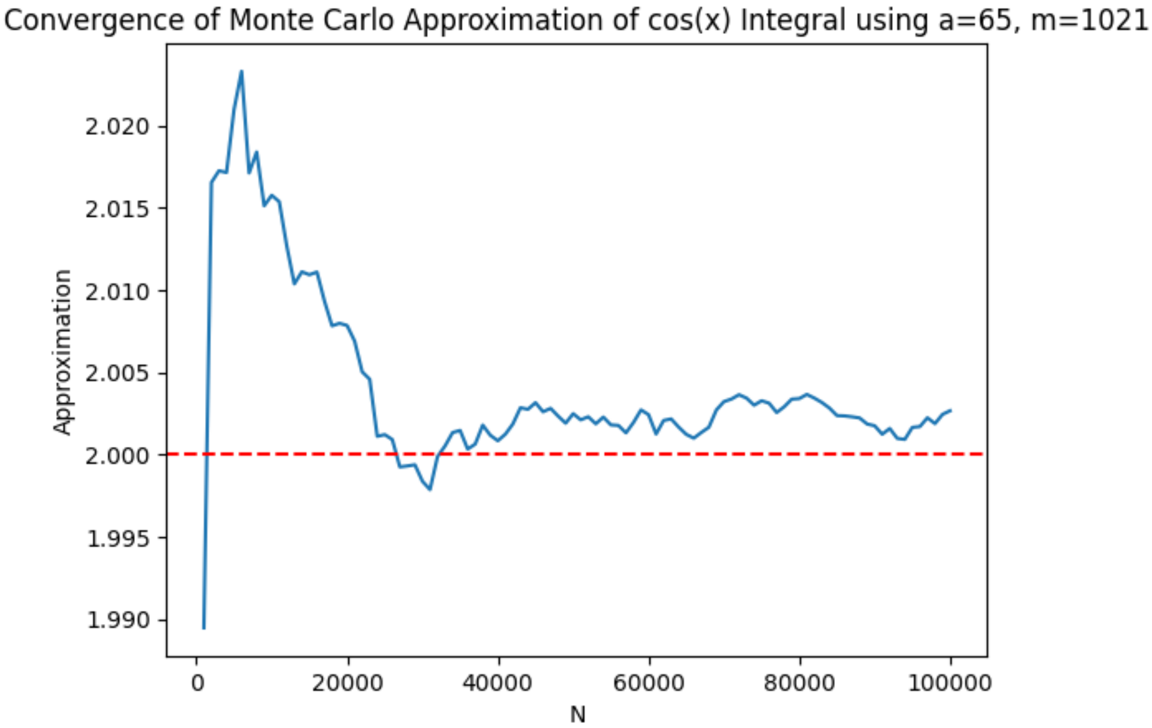
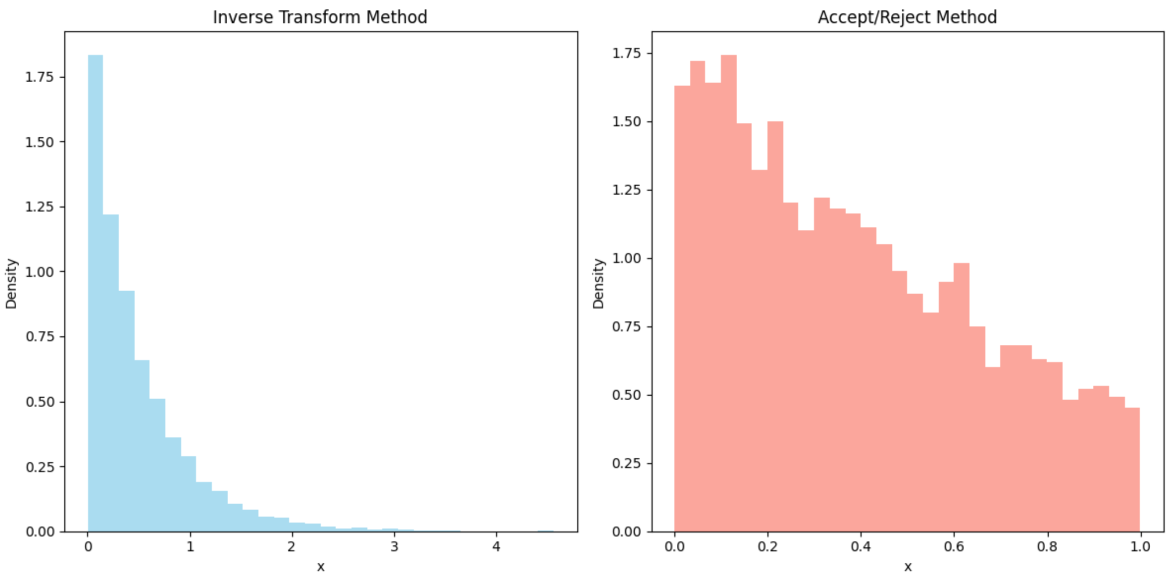



Question 3

In [ ]:
```
def target_distribution(x):
    return np.exp(-2 * x)

def inverse_cdf(y):
    return -np.log(1 - y) / 2

# Inverse Transform Method
def inverse_transform_sampling(n, generator):
    transformed_samples = []
    for _ in range(n):
        u = next(generator)
        x = inverse_cdf(u)
        transformed_samples.append(x)
    return transformed_samples

# Accept/Reject Method
def accept_reject_sampling(n, generator):
    samples = []
```

```
    M = 4  # Majorizing constant for the sampling distribution
    while len(samples) < n:
        x = next(generator)
        u = next(generator)
        if u <= target_distribution(x) / (M * (2 - x)):
            samples.append(x)
    return samples

num_samples = 3000

mlc_random_generator = mlc_generator(2**31, 1103515245)
samples_inverse_transform = inverse_transform_sampling(num_samples, mlc_r
samples_accept_reject = accept_reject_sampling(num_samples, mlc_random_ge
```

In [ ]:
```
plt.figure(figsize=(12, 6))

plt.subplot(1, 2, 1)
plt.hist(samples_inverse_transform, bins=30, density=True, color='skyblue
plt.title('Inverse Transform Method')
plt.xlabel('x')
plt.ylabel('Density')

plt.subplot(1, 2, 2)
plt.hist(samples_accept_reject, bins=30, density=True, color='salmon', al
plt.title('Accept/Reject Method')
plt.xlabel('x')
plt.ylabel('Density')

plt.tight_layout()
plt.show()
```



Question 4

In [ ]:
```
from scipy.integrate import simps # For comparison
from tabulate import tabulate # For tabulating the results

# Function to be integrated
def f(x):
    return np.exp(-2*x) / (1 + x**2)
```

```
# Simpson's method
def simpson_integration():
    x = np.linspace(0, 2, 1000)
    y = f(x)
    integral = simps(y, x)
    return integral

# importance sampling functions
def p1(x):
    return 0.5

def p2(x):
    return np.exp(-x)

def p3(x):
    return np.exp(-x/2) / (2 * (1 - np.exp(-1/2)))

# Monte Carlo integration function

def monte_carlo_integration(N, sampling_function, generator = mlc_random_
    integral_sum = 0
    for _ in range(N):
        x = next(generator) * 2  # Sample x from the range [0, 2] using t
        weight = f(x) / sampling_function(x)
        integral_sum += weight
    return integral_sum / N
```

In [ ]:

```
# Number of samples
N = 40000

integral_estimate_p1 = monte_carlo_integration(N, p1)
integral_estimate_p2 = monte_carlo_integration(N, p2)
integral_estimate_p3 = monte_carlo_integration(N, p3)

integral_simpson = simpson_integration()

# Tabulating the results
table_data = [
    ["p1(x)", integral_estimate_p1],
    ["p2(x)", integral_estimate_p2],
    ["p3(x)", integral_estimate_p3],
    ["Simpson's Method", integral_simpson]
]

print(tabulate(table_data, headers=["Sampling Function", "Monte Carlo Est
```

```
+--------------------+------------------------+
| Sampling Function  |   Monte Carlo Estimate |
+====================+========================+
| p1(x)              |               0.397526 |
+--------------------+------------------------+
| p2(x)              |               0.303063 |
+--------------------+------------------------+
| p3(x)              |               0.191775 |
+--------------------+------------------------+
| Simpson's Method   |               0.397675 |
+--------------------+------------------------+
```

```
/tmp/ipykernel_151139/3777958439.py:12: DeprecationWarning: 'scipy.integra
te.simps' is deprecated in favour of 'scipy.integrate.simpson' and will be
removed in SciPy 1.14.0
  integral = simps(y, x)
```

Efficiency of (p1(x)): The proximity of the Monte Carlo approximation using (p1(x)) to the actual value indicates the effectiveness of (p1(x)) in minimizing variance. This suggests a close resemblance between (p1(x)) and the integrand (f(x)), leading to more precise estimates.

Declining Approximations with (p2(x)) and (p3(x)): A decrease in estimates using (p2(x)) and (p3(x)) and their divergence from the actual value signifies the ineffectiveness of these sampling functions. This could be attributed to a misalignment between (p2(x)) and (p3(x)) with (f(x)), resulting in increased variance.

Variance Minimization: Efficient sampling functions, such as (p1(x)), should decrease variance compared to uniform sampling. Hence, (p1(x)) is anticipated to provide the most accurate estimates, while (p2(x)) and (p3(x)) may exhibit growing deviation due to increased variance.

In [ ]: