Newton's Laws: Their origin and implications

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Abstract

In this article I describe the origins of Newton's Laws, starting from ancient Greek notions of space time and how those ideas evolved. I also talk about the laws from a modern standpoint. How Newton's Laws had far-reaching implications in science and philosophy is also discussed.

1 Introduction

Newton's Laws really need no introduction. We've literally grown up with them. Sprayed across textbooks liberally, it has become so ubiquitous that all of us take for granted, and often overlook the revolutionary philosophical and scientific shift in human thinking that Newton brought about. To truly understand the splendour of Newton's work, we've to turn to the ones who first properly began to explore logic and philosophy. Though their ideas were later found to be incorrect, they must be understood for appreciating the scientific framework of Newton's day.

2 The Greeks

2.1 Zeno

The ancient Greeks were perennially confused and pondered about space, time and motion. One of the first among the ancients to deeply think about the problem of understanding motion was Zeno of Elea, who is famous for his *paradoxes*.

Zeno's paradoxes seem to show that motion is simply an illusion. Of his eight surviving paradoxes (which are presented in Aristotle's *Physics*), many are equivalent to one another. Three paradoxes of his are the most famous – that of Achilles and the tortoise, the dichotomy argument, and an arrow in flight.

I'll present the paradoxes in brief here:

- In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead. – Aristotle, Physics VI:9, 239b15

- That which is in locomotion must arrive at the half-way stage before it arrives at the goal. – Aristotle, Physics VI:9, 239b10

- If everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless. – Aristotle, Physics VI:9, 239b5

In the first paradox, Zeno tells us that Achilles (the runner) will never catch up with the tortoise; as the distance between Achilles and the tortoise will continue to decrease, but will never become zero.

In the second paradox, Zeno says that some one must complete half of the journey, then half of the remaining journey, and so on. This requires the mover to complete an infinite number of tasks, which Zeno deems impossible.

In the third paradox, Zeno says that when we observe an arrow in flight, at any instant of time, it seems to be at rest. So all motion is an illusion.

These above paradoxes show us how the Greeks had conceptual problems with the idea of infinitesimals, which was only properly defined in the 1600s by Newton and Leibnitz and put on a firm mathematical grounding by Auguste Louis Cauchy much later. Aristotle refutes the paradoxes by countering that time is not a succession of *nows*. Modern solutions involve the idea of infinitesimals and the concept that even after an infinite number of terms, the sum can still be finite.

2.2 Aristotle

Aristotle, considered by many as the father of modern scientific thought, was the son of a Greek doctor who served the king of Macedonia. He believed in observance of the material world rather than rely on commonplace opinion.

His theory of motion was limited by the era in which he lived. He believed that earth was the center of the universe and that gravity was just the tendency of everything to rush to its center. He believed that the natural state of all bodies was rest, that all bodies tended to return to rest and needed a mover to keep them in motion. For Aristotle this mover was Zeus. The concept of inertia did not exist then as we know of. He believed that everything moved in a straight line until something intervened to deflect or stop it. He famously remarked that a heavy body falls faster than a light one, which was refuted by Galileo centuries later.

3 Galileo

We approach the Newtonian ideas with Galileo Galilei's ideas and experiments of motion. Galileo could be regarded the father of *kinematics*, the study of moving objects. Galileo was one of the first to realise that the laws of reality could be described using mathematics. Though I'll only discuss his contribution to the study of motion, Galileo's contributions to astronomy were more immense. His support of the heliocentric theory (that the sun was the centre of the universe) caused anger in the Church.

Galileo's theoretical and experimental work on the motions of bodies, along with the largely independent work of Kepler and René Descartes, was a precursor of the classical mechanics developed by Sir Isaac Newton. Galileo developed the idea of *acceleration*. He rolled balls down inclined planes. He proved using these experiments that they are accelerated independently of their mass.

Galileo found the correct formula for relating distance covered for an uniformly accelerating body (it is proportional to the square of the time elapsed). He also concluded that objects retain their velocity unless a force – often friction – acts upon them, refuting the generally accepted Aristotelian hypothesis that objects "naturally" slow down and stop unless a force acts upon them. He established the concept of *inertia*, which was a groundbreaking idea. His principle of inertia states that "A body moving on a level surface will continue in the same direction at constant speed unless disturbed." This was later incorporated into Newton's First Law.

Galileo also put forward a principle of relativity, now known as **Galilean invariance**. It states that the fundamental laws of physics are the same in all inertial frames. Specifically, the term Galilean invariance today usually refers to this principle as applied to Newtonian mechanics, meaning that Newton's Laws hold true in all inertial frames. A frame is inertial with respect to another frame if it is moving with a constant velocity with respect to that frame.

4 Newton

> If I have seen further it is by standing on ye shoulders of giants.
> – Sir Isaac Newton, in a letter to Robert Hooke, February 1676.

Sir Isaac Newton had indeed seen further than all who had come before him. During his lifespan, he made revolutionary breakthroughs in mathematics, the science of optics, mechanics and gravitation. So powerful were his ideas, that they went virtually unchallenged for more than three hundred years till 1905, when Albert Einstein changed our worldview once again with his theory of relativity.

Though Newton's contributions to all these fields were immense, he is most famously remembered for his three laws of motion. The immense implications and philosophical shift which originated with Galileo and put on a firmer basis by Newton dominated scientific thought from the 1600s to the 1900s. These laws were first published in his magnum opus, the *Philosophiae Naturalis Principia Mathematica* in 1687.

4.1 The Three Laws of Motion

- **Law of Inertia** *Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*

  An object at rest will remain at rest unless acted upon by an external and unbalanced force. An object in motion will remain in motion unless acted upon by an external and unbalanced force.

  The Law of Inertia may seem obvious to most people in the present age, but it was not so in ancient times. As we saw, Aristotle had a markedly different notion of motion. This law was, in fact, first proposed by Galileo; it is a clear departure from Aristotelian ideas of motion, and forms the basis of the next two laws.

- **Law of acceleration** *Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.*

  The rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the same direction.

  Using modern notation of differentials, we can write Newton's second law in this form:

  $$\vec{F} = \frac{d(m\vec{v})}{dt}$$

  where $\vec{F}$ and $\vec{v}$ are the force and velocity vectors, while m and t are mass and time, respectively.

- **Law of reciprocal actions** *Actioni contrariam semper et qualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse quales et in partes contrarias dirigi.*

  All forces occur in pairs, and these two forces are equal in magnitude and opposite in direction.

  Newton used the third law to derive the law of conservation of momentum ($m\vec{v}$); however from a deeper perspective, the conservation of momentum is the more fundamental idea (derived via Noether's theorem from Galilean invariance), and holds in cases where Newtonian mechanics seems to fail, as in quantum mechanics.

Newton's ideas of inertial frames could be summarised as two axioms, which define his idea of space-time:

- There exists an absolute space, in which Newton's laws are true. An inertial frame is a reference frame in relative uniform motion to absolute space.
- All inertial frames share an universal time.

Newton's laws were verified by experiment and observation for over 200 years, and they are excellent approximations at the scales and speeds of everyday life. Newton's laws of motion, together with his law of universal gravitation and the mathematical techniques of calculus, provided for the first time a unified quantitative explanation for a wide range of physical phenomena.

However Newton's laws of motion while seemingly valid, did not give us a precise idea of space-time. It's true that the law of inertia paved the way for a more modern understanding of physical reality; by doing away with aged Aristotelian notions, and while a lot of experiments done on everyday objects seemed to show Newton correct, we still had no idea about the structure of space time itself.

To illustrate the kind of problems we can run into; let's consider a rope, whose two ends have small stones tied to each other. From common experience, we would say that the rope would stretch taut. Now let's consider that there's nothing else in the universe. Would the rope still stretch taut? For there was no reference frame that we could conceivably think of. However Newton countered with the idea of **absolute space** and **absolute time**. According to him, the rope would stretch taut because it was spinning with respect to absolute space. Also time flowed the same way in *all* frames. But in attempting to describe absolute space, Newton sidestepped it and himself acknowledged that he couldn't properly define it. He said "Absolute space, in its own nature, without reference to anything external, remains always similar and unmovable."

5 Conclusion

It is undoubtedly true that Newton had a great deal of impact on modern scientific thought. His laws of motion held true for more than three centuries, and still hold true today for everyday objects. More importantly, he along with Laplace and other scientists of his era established the deterministic worldview which said that if we know all the positions and momenta of objects in the universe at this instant, we could predict anything in the future. This certainty would not be challenged until Werner Heisenberg came up with his famous Uncertainty Principle which said that both the position and momenta cannot be determined simultaneously with accuracy. His ideas on absolute space and time were debated, but not seriously challenged till Ernst Mach came with his firmly relationist position. In his view, in an empty universe the Newton's rope would not stretch taut but lay slack, as there could be no differentiation between spinning and not spinning, as there were no benchmarks, no reference points at all.

Such a potent idea troubled many physicists and had a profound impact on Albert Einstein; however in his attempt to incorporate Machian notions into his equations, he failed, and introduced something that redefined the question itself. He introduced *absolute space-time*.

As we move away from Newtonian ideas to Einstein's notions of relativity, and even more bizarre theories of the 20th century, which challenge and move our intuition to a new level, Newton's famous quote aptly sums it up:

> "I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

6 References

- Ideas from Anaxagoras to Aristotle (`http://www.fsmitha.com/h1/ch10.htm`)
- Wikipedia (`http://en.wikipedia.org`) entries on Isaac Newton, Newton's Laws of Motion, Noether's Theorem, Galileo Galilei, Ernst Mach.
- The Fabric of the Cosmos, *Brian Greene* (2004)