CS 7646 - Project 1 Report:

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Question 1

An American wheel has 18 black, 18 red, 1’00’ and 1 ‘0’. Therefore, the probability of winning a bid is 18/ (18 + 18 + 1 + 1) = 18/38 = 9/19.

MARTINGALE strategy encompasses two independent strategies for winning and losing. In case of winning, the gambler increases the bet amount by $1 and doubles the bet amount in losing case. During a flip from losing to winning, the bet amount is reset to $1. The basic premise of this strategy is to let the gambler recover from all the previous losses and win an extra equal amount. However, the chances of losing during each bid is still the same since mathematically, each bid is independent and identically distributed. Therefore, the loss is exponential too. MARTINGALE loss branch exponential bid strategy increases the chance of winning/losing $80 much sooner than if we apply the linear strategy of betting the same $1 amount for losing cases.

The probability of making $80 within 1000 sequential bets using the MARTINGALE strategy is greater than the probability of making $80 using linear strategy of bidding the same $1 amount for both losing and winning cases (because MARTINGALE loss branch decision has exponential bid strategy).

Now, let’s first calculate probability of making at least $80 with linear strategy of bidding the same $1 amount for both losing and winning:
= probability of winning at least 80 times (each bet has $1 betting amount) out of 1000 times.
= 1 - probability of winning less than 80 times
= 1 - P(winning = 0 times) - P(winning = 1 times) - …… - P(winning = 79 times)
= 1 - C (1000, 0) * (9/19)^0 * (10/19)^1000 - C (1000, 1) * (9/19)^1 * (10/19)^999 - …… - C (1000, 79) * (9/19)^79 * (10/19)^921 >> 0.999999

Therefore, probability of making atleast $80 in 1000 tries with linear strategy >> 0.999999
=> probability of making at least $80 in 1000 tries with MARTINGALE strategy >> probability of making atleast $80 in 1000 tries with linear strategy >> 0.999999

Therefore, we can say that the chance of winning is almost 100%.

Question 2
The probability of winning is = 18/38 = 9/19. Therefore, for each bet, the expected value of win for each bet = bet amount * probability of winning = $1 * 9/19 = $ 9/19.

Now, let's assume the straight forward case where the gambler doesn't stop once he/she wins $80.

Similar to question 1, the expected value of win in 1000 tries with MARTINGALE strategy >> expected value of win in 1000 tries with linear strategy = 1000 * $ 9/19 = $ 473.68
(This relation will hold because in the case the gambler loses in MARTINGALE strategy, he/she starts betting using exponential strategy. Therefore, each bet at that stage is $1 * 2^n greater than just $1 where n is the number of consecutive losses. Therefore, the expected win of each bet will be $1 * 2^n * 9/19 which is >= $ 9/19.)

Now, since a gambler stops betting once he/she wins $80, and $80 < $ 473.68, we can say that the expected value of the gambler winning after 1000 sequential bets gets capped at $80.

We can also see this behavior clearly in figure 2 where the gambler is getting capped to $80 within 200 tries (and the graph then becomes flat).

Therefore, the estimated expected value of our winnings after 1000 sequential bets = $80.

Question 3
In question 1, we saw that the probability of winning in 1000 tries has a probability of almost 1. Therefore, each of the simulations' wins and the win value converges to $80. As soon as all the win value of the 1000 simulations converge to $80, the mean of 1000 simulations converges to $80 too, leading to standard deviation converging to 0.

Mathematically, this can be seen from the standard deviation formula:

$$SD = \sqrt{\frac{\sum \left(r_i - r_{avg}\right)^2}{n-1}}$$

In the above formula, since we have r_i and r_{avg} both converging to $80, the numerator becomes 0, leading to the std dev converging to $0.

Question 4
From my 1000 simulations, I found 609 simulations winning \$80. Therefore, the rough win rate I would say is 609/1000*100 = 60.9%
This can be explained based on the added condition in experiment 2 which says that the gambler doesn't have unlimited bank roll anymore as in experiment 1 and the bank roll now is fixed at \$256.
This leads to gamblers becoming bankrupt and will thus now not be able to recoup the lost bets in case he/she loses \$256. The fixed bank roll of \$256 now also puts a hard upper limit on the bet amount for each bid. This leads to delayed loss recouping in subsequent bids in case the gambler lost previously. This may not work in the favor of the gambler if in case he continues to accumulate subsequent losses and then eventually becoming bankrupt.

Question 5
As calculated in question 4, we know that the win rate is around 60.9%.
Therefore, the expected value after 1000 bids can be calculated as below:
P(win) * winning amount + P(loss) * losing amount
= 0.609 * \$80 + 0.391 * (-\$256)
= -51.376 \$

Question 6
The standard deviation reaches a maximum value and stabilizes thereafter. This is because the majority of the gamblers either lose -\$256 or win \$80 eventually. Therefore, based on the std deviation formula which is

$$SD = \sqrt{\frac{\sum \left(r_i - r_{avg}\right)^2}{n-1}}$$

After a significant number of bets (around 170), the r_{avg} converges to -51.376 \$ (calculated in Question 5) and r_i for gamblers who win, gets the winning value of \$80 and their r_i converges to \$80. The remaining gambler who loses, gets the r_i value of -\$256. This leads to maximization of the standard deviation function because two groups i.e. winners and losers diverge from the mean.

Question 7

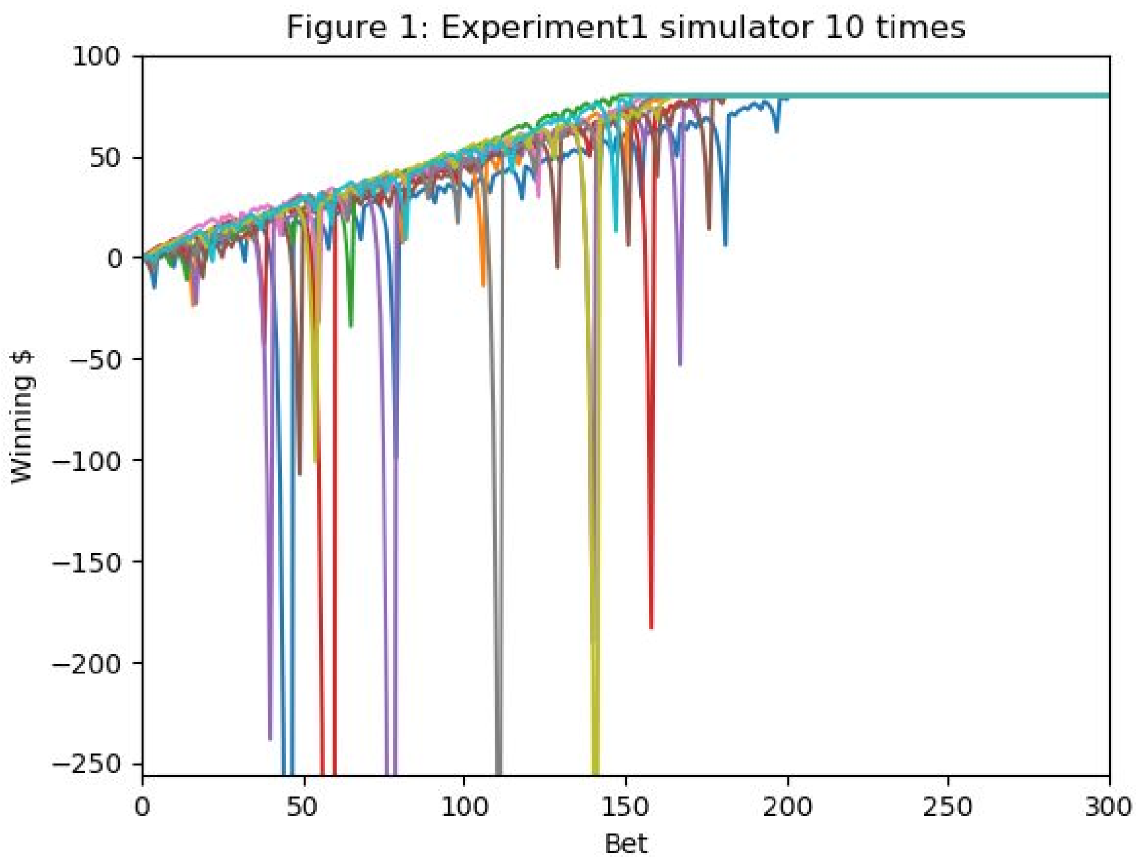
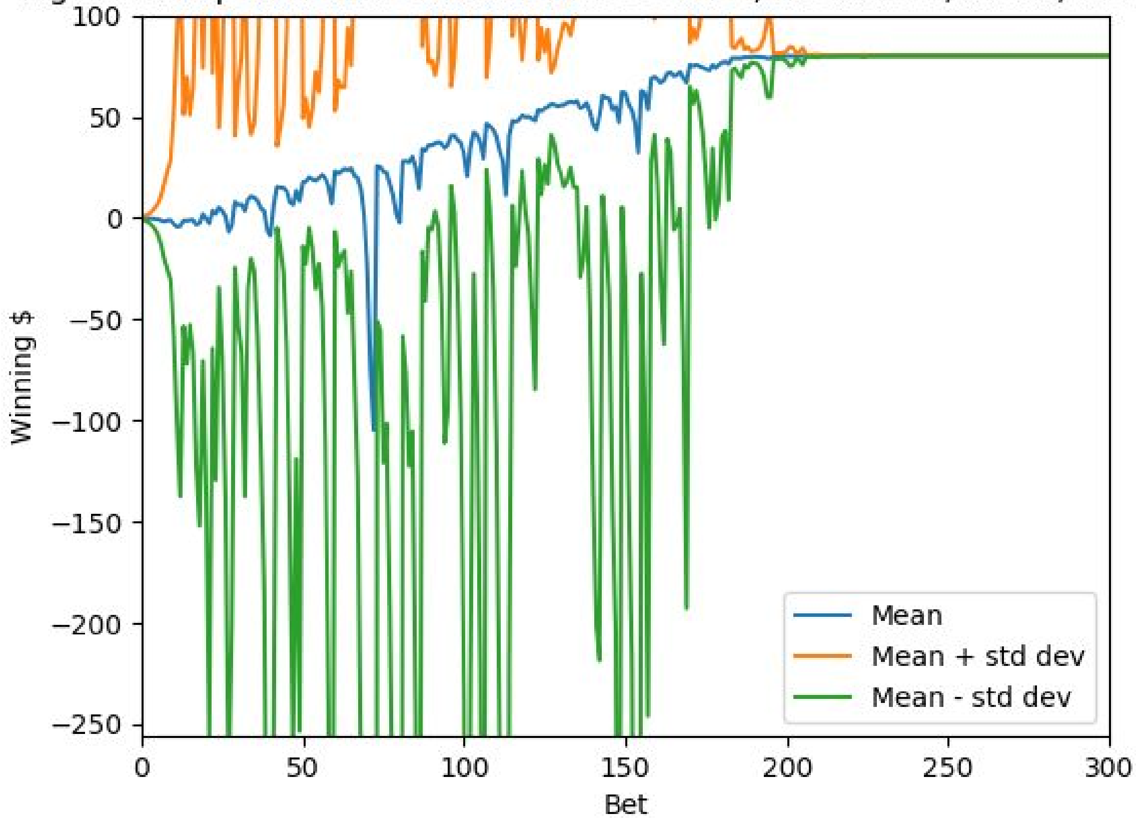


Figure 2: Experiment 1 simulator 1000 times, mean+std, mean, mean-std



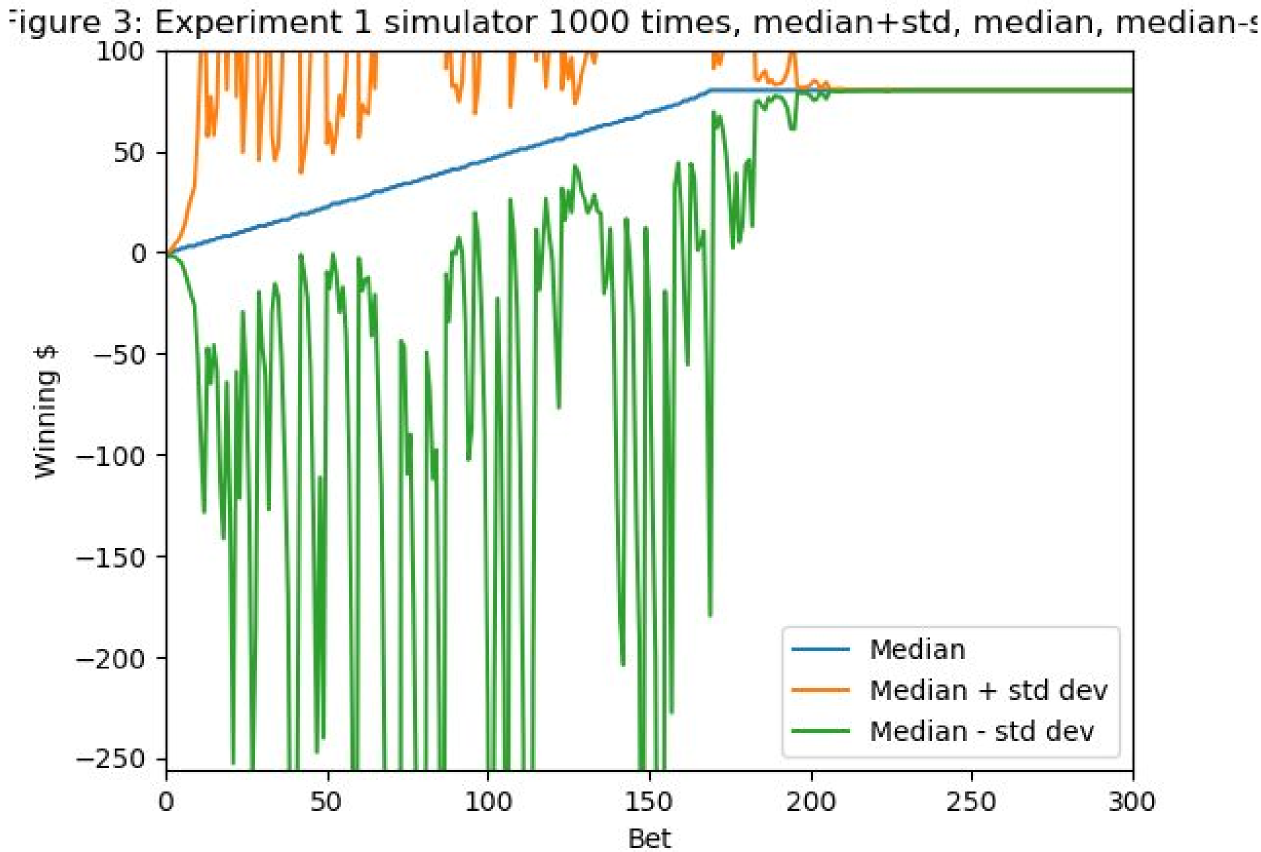

igure 3: Experiment 1 simulator 1000 times, median+std, median, median-s
Winning $
Bet
Median
Median + std dev
Median - std dev
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300

Figure 4: Experiment 2 simulator 1000 times, mean+std, mean, mean-std

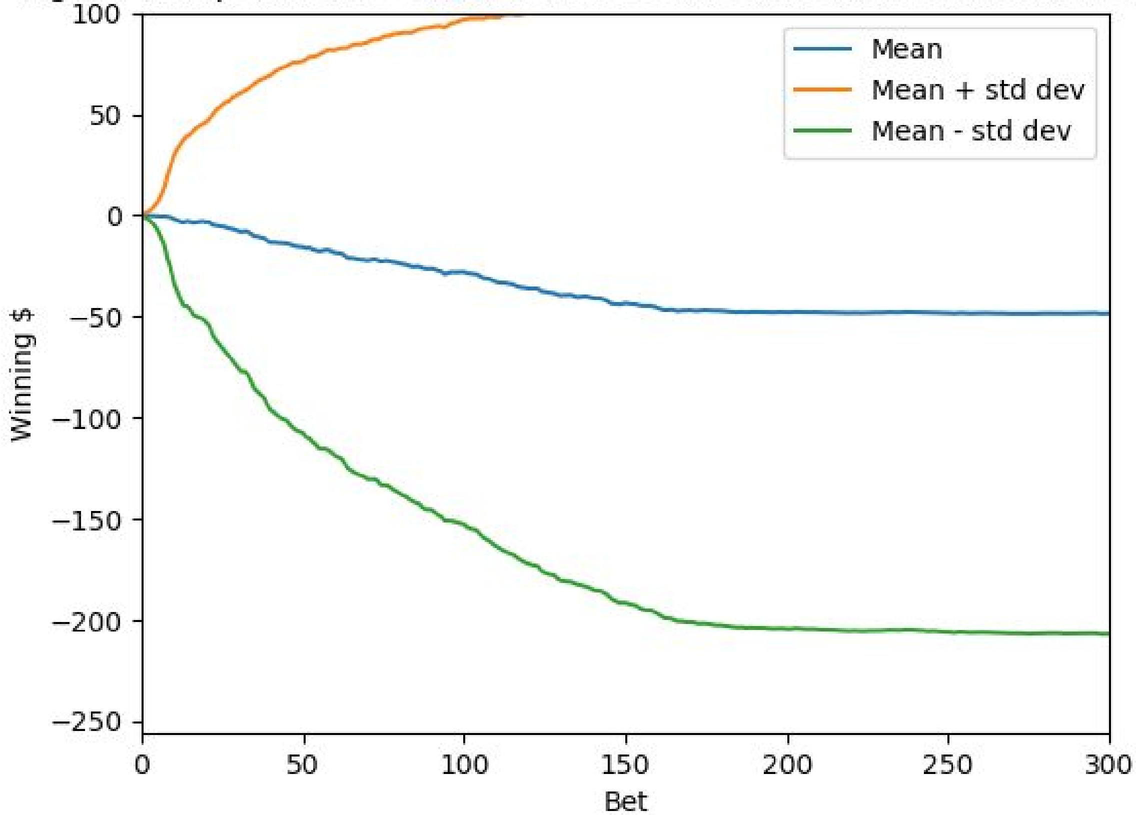


Figure 5: Experiment 2 simulator 1000 times, median+std, median, median-s