

Question 1:

Below is the OpenBUGS code that I used to fit the regression with tumor profile as covariate:

```

q1
model
{
  for(i in 1 : N) {
    times[i] ~ dweib(v,lambda[i])|(censor[i],)
    lambda[i] <- exp(beta0 + beta1*type[i])
    S[i] <- exp(-lambda[i]*pow(times[i],v));
    f[i] <- lambda[i]*v*pow(times[i],v-1)*S[i]
    h[i] <- f[i]/S[i]
    index[i] <- i    #for plots
  }
  beta0 ~ dnorm(0.0, 0.0001)
  beta1 ~ dnorm(0.0, 0.0001)
  v ~ dexp(0.001)
}

list( N=80, type=c(1,1,1,1,1,1,1,1,1, 1,1,1,1,1,1,1,1,1, 1,1,1,1,1,1,1,1,1,
1,1,1,1,1,1,1,1,1, 1,1,2,2,2,2,2,2,2, 2,2,2,2,2,2,2,2,2, 2,2,2,2,2,2,2,2,2,
2,2,2,2,2,2,2,2,2), censor=c(0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,
0,61,74,79,80,81,87,87,88,89, 93,97,101,104,108,109,120,131,150,231,
240,400,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0, 0,0,0,0,8,67,76,104,176,231),
times=c(1,3,3,4,10,13,13,16,16,24, 26,27,28,30,30,32,41,51,65,67,
70,72,73,77,91,93,96,100,104,157, 167,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA, NA,NA,1,3,4,5,5,8,12,13,
18,23,26,27,30,42,56,62,69,104, 104,112,129,181,NA,NA,NA,NA,NA,NA) )

<=>

INITS
list(v = 1, beta0 = 0, beta1=0)

```

With burning of 1000 samples, I got below node statistics on next 99000 samples:

Node statistics								
i	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
beta0	-3.195	0.5634	0.02517	-4.329	-3.177	-2.132	1001	100000
beta1	-0.5557	0.2852	0.01099	-1.105	-0.5523	-0.01844	1001	100000
v	0.8449	0.09834	0.003365	0.6608	0.8409	1.048	1001	100000

Clearly, from above 95% credible set for the slope β_1 is [-1.105, -0.01844].

Question 2:

Below is the OpenBUGS code that I used to fit Y by Poisson regression, with X as a covariate:

```
q2
model{
  for( i in 1:n ) {
    y[i] ~ dpois(mu[i])
    mu[i] <- exp( beta0 + beta1* x[i])
  }
  # Priors
  beta0 ~ dnorm(0, 0.001)
  beta1 ~ dnorm(0, 0.001)

  # for predicting missing X values.
  for(i in 1:n) {
    x[i] ~ dpois(2)
  }

  muystar <- exp( beta0 + beta1*4) #average number of broken packages at X=4
  ystar ~ dpois(muystar) #predicted number of broken packages at X=4
}

# data
list(n=15, y=c(NA, 16, 9, 17, 12, 22, 13, 8, NA, 19, 17, 11, 10, 20, 2), x = c(2, 1, 0, 2, NA, 3, 1, 0, 1, 2, 3, 0, 1, NA, NA))

# initial values
list(beta0=2, beta1=0)
```

With burning of 1000 samples, I got below node statistics on next 99000 samples:

Node statistics								
	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
beta0	2.17	0.1433	0.00285	1.881	2.174	2.44	1001	100000
beta1	0.2891	0.07437	0.001485	0.1421	0.2878	0.4367	1001	100000
deviance	109.6	3.288	0.03326	105.5	108.7	117.8	1001	100000
muystar	28.38	5.639	0.09077	18.72	27.84	40.97	1001	100000
x[5]	1.276	0.7843	0.003162	0.0	1.0	3.0	1001	100000
x[14]	2.549	0.827	0.004045	1.0	3.0	4.0	1001	100000
x[15]	0.1958	0.4302	0.002627	0.0	0.0	1.0	1001	100000
y[1]	15.68	4.184	0.01634	8.0	15.0	24.0	1001	100000
y[9]	11.73	3.591	0.02014	5.0	12.0	19.0	1001	100000
ystar	28.37	7.726	0.09013	15.0	28.0	45.0	1001	100000

(a) Using the OpenBUGS code mentioned above, I fit Y by Poisson regression, with X as a covariate.

From the node statistics, below are the values asked in the question:

Value of the bayesian estimate of slope: $\beta_1 = 0.2891$

Value of the bayesian estimate of intercept: $\beta_0 = 2.17$

The Bayesian estimate of the deviance of the fit = 109.6. The 95% CS of the deviance = [105.5, 117.8]

(b) The number of packages on average are expected will be broken if the number of shipment routes is $X = 4$ is given by the parameter muystar in the node statistics above. The Bayesian estimate of muystar is 28.38. Therefore, I expect 28.38 packages to on average broken if number of shipment routes is $X = 4$. The 95% CS of muystar is: [18.72, 40.97].

(c) The predicted number of packages on average that are expected to be broken if the number of shipment routes is $X = 4$ is given by the parameter ystar in the node statistics above. The Bayesian estimate of ystar is 28.37. The 95% CS of ystar is: [15, 45]. The Bayesian estimate of the predicted differ with expected by only 0.01. However, one thing to note here is that 95% CS of expected values is a subset of the 95% CS of predicted value.

Predicted value is estimated from using a poisson distribution which has the mean of the expected value. This leads to 95% CS of predicted value to be wider because the 95% CS of predicted value incorporates a greater standard error.

(d) From the node statistics above, below are the predicted bayesian estimated for each of the mentioned values:

Estimate of X_5 is given by $x[5]$ in the node statistic which is: 1.276.

Estimate of X_{14} is given by $x[14]$ in the node statistic which is: 2.549.

Estimate of X_{15} is given by $x[15]$ in the node statistic which is: 0.1958.

Estimate of Y_1 is given by $y[1]$ in the node statistic which is: 15.68.

Estimate of Y_9 is given by $y[9]$ in the node statistic which is: 11.73.